PHYS11 CH:5.1-5.3

Vector Analysis and Applications

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Introduction to Vector Analysis

- Vectors are essential in physics for describing quantities with magnitude and direction
- Key topics:
  - Vector addition and subtraction
  - Resolving vectors into components
  - Vector applications in real-world problems

Table of Contents

Trig Review

phys11-math-trigonometry-sohcahtoa.png

Figure: SOHCAHTOA mnemonic

phys11-math-sine-cosine-laws.p

Figure: Sine and Cosine Laws

Vector Addition and Subtraction

- Vector addition: - Tip-to-tail method - Parallelogram method
- Vector subtraction: - Add the negative of the vector: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$
- Resultant vector: $\vec{R} = \vec{A} + \vec{B}$
  - For n vectors: $\vec{R} = \vec{A} + \vec{B} + \vec{C} + ... + \vec{N}$

Vector Addition and Subtraction

- Magnitude of resultant:
  - General case: $|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$
  - For perpendicular vectors: $|\vec{R}| = \sqrt{A^2 + B^2}$
- Direction of resultant:
  - Using magnitudes and angle: $\tan\phi = \frac{B\sin\theta}{A+B\cos\theta}$
  - Using components: $\tan\phi = \frac{A_y+B_y}{A_x+B_x}$,

Resolving Vectors into Components

- Any vector can be resolved into x and y components
- $A_x = A\cos\theta$, $A_y = A\sin\theta$
- Magnitude: $A = \sqrt{A_x^2 + A_y^2}$
- Direction: $\tan\theta = \frac{A_y}{A_x}$

Table of Contents

Example 1: River Crossing Problem

Problem: A river flows SW to NE at 7.1 m/s. A boat moving at 13 m/s wants to reach a point due east.

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Solution:

- River velocity components: $v_{rx} = v_{ry} = 7.1 \cos 45 = 5.02$ m/s
- Boat must counteract river's y-component: $v_{by} = -5.02$ m/s
- Angle of boat's heading: $\theta = \arcsin(-5.02/13) \approx -22.6$

Answer: The boat should head 22.6° south of east.

Example 2: Displacement Problem

Problem: A person walks 10.0 m north, then 2.0 m east. Find the resultant displacement.

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Solution:

- Use Pythagorean theorem: $R = \sqrt{10.0^2 + 2.0^2} \approx 10.2$ m
- Angle: $\theta = \arctan(10.0/2.0) \approx 78.7$ north of east

Answer: $\vec{R} = 10.2$ m, 78.7° north of east

Table of Contents

Projectile Motion Basics

- Combination of horizontal and vertical motion
- Horizontal motion: constant velocity (neglecting air resistance)
- Vertical motion: constant acceleration due to gravity
- Key equations:
  - Range: $R = \frac{v_0^2 \sin(2\theta)}{g}$
  - Maximum height: $h_{\text{max}} = \frac{v_0^2 \sin^2 \theta}{2g}$
  - Time of flight: $t = \frac{2v_0 \sin \theta}{g}$

Example 3: Projectile Motion Problem

Problem: A water-balloon cannon fires at 30 m/s, 50° above horizontal. Find the range.

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Solution:

- Use range equation: $R = \frac{v_0^2 \sin(2\theta)}{g}$
- $R = \frac{(30 \text{ m/s})^2 \sin(2*50)}{9.8 \text{ m/s}^2} \approx 90.44$ m

Answer: The water balloon will fall approximately 90.44 m away.

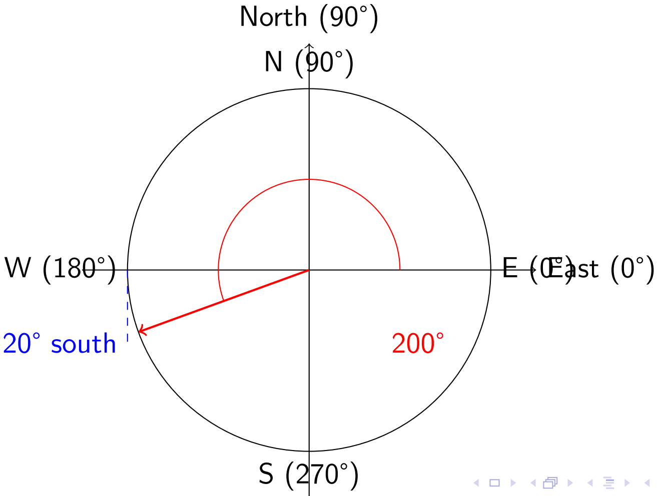
Conclusion

- Vector analysis is crucial for understanding motion in physics
- Key concepts covered:
  - Vector addition and subtraction
  - Resolving vectors into components
  - Applications in real-world problems
  - Basics of projectile motion
- These concepts help analyze complex motions and solve practical problems
- Practice with various examples to master vector analysis

Global Angles

Problem

What is the global angle of 20° south of west?

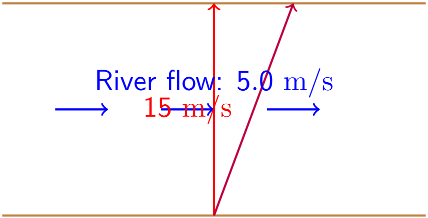


River Crossing Problem

Problem

A person attempts to cross a river in a straight line by navigating a boat at 15 $\mathrm{m/s}$. If the river flows at 5.0 $\mathrm{m/s}$ from left to right, what would be:

- The magnitude of the boat's resultant velocity?
- The direction relative to the straight line across the river?



River Crossing Solution

The correct solution:

- Resultant velocity: 15.8 $\mathrm{m/s}$
- Direction: 18.4° to the right

Using the Pythagorean theorem:

$$v_{\text{resultant}} = \sqrt{(5.0)^2 + (15.0)^2} = 15.8\ \mathrm{m/s}$$

The angle is given by:

$$\theta = \tan^{-1}\left(\frac{5.0}{15.0}\right) = 18.4^\circ$$