

## 7.1 Work: The Scientific Definition

### Learning Objectives

By the end of this section, you will be able to:

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

### What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be displacement in the direction of the force.

Formally, the work done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = |\mathbf{F}| (\cos \theta) |\mathbf{d}|,$$

7.1

where  $W$  is work,  $\mathbf{d}$  is the displacement of the system, and  $\theta$  is the angle between the force vector  $\mathbf{F}$  and the displacement vector  $\mathbf{d}$ , as in Figure 7.2. We can also write this as

$$W = Fd \cos \theta.$$

7.2

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we divide the motion into one-way one-dimensional segments and add up the work done over each segment.

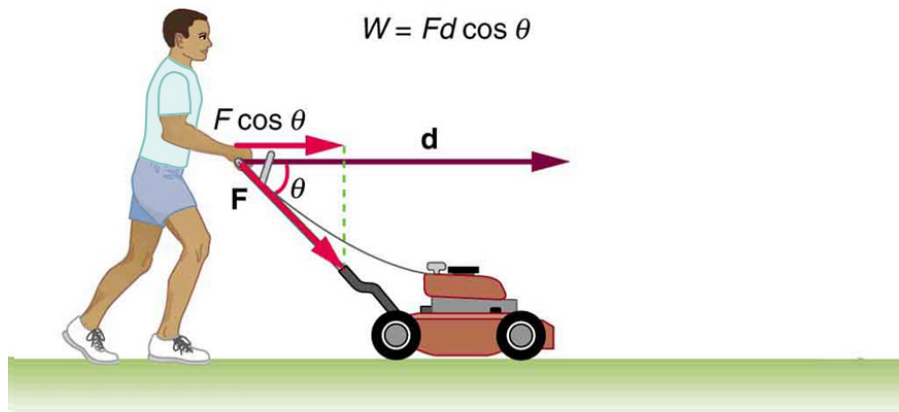
### What is Work?

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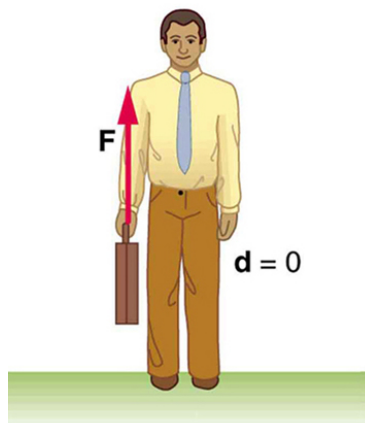
$$W = Fd \cos \theta,$$

7.3

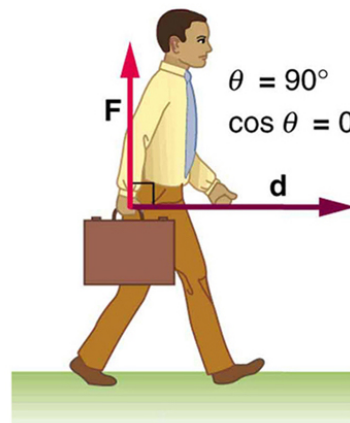
where  $W$  is work,  $F$  is the magnitude of the force on the system,  $d$  is the magnitude of the displacement of the system, and  $\theta$  is the angle between the force vector  $\mathbf{F}$  and the displacement vector  $\mathbf{d}$ .



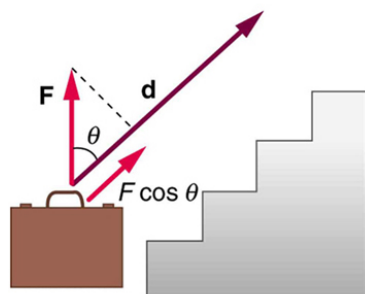
(a)



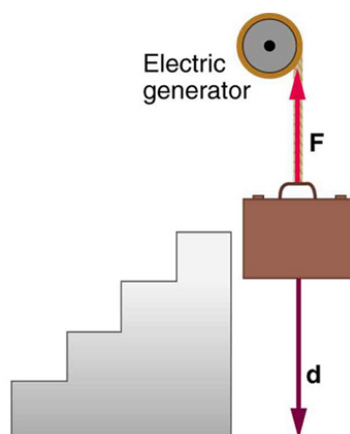
(b)



(c)



(d)



(e)

Figure 7.2 Examples of work. (a) The work done by the force  $\mathbf{F}$  on this lawn mower is  $Fd \cos \theta$ . Note that  $F \cos \theta$  is the component of the force in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no displacement. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. (d) Work *is* done on the briefcase by carrying it up stairs at constant speed, because there is necessarily a component of force  $\mathbf{F}$  in the direction of the motion. Energy is transferred to the briefcase and could in turn be used to do work. (e) When the briefcase is lowered, energy is transferred out of the briefcase and into an electric generator. Here the work done on the briefcase by the generator is negative, removing energy from the briefcase, because  $\mathbf{F}$  and  $\mathbf{d}$  are in opposite directions.

To examine what the definition of work means, let us consider the other situations shown in Figure 7.2. The person holding the briefcase in Figure 7.2(b) does no work, for example. Here  $d = 0$ , so  $W = 0$ . Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the “briefcase-Earth system”—see Gravitational Potential Energy for more details). There must be displacement for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in Figure 7.2(c) does no work on it, because the force is perpendicular to the motion. That is,  $\cos 90^\circ = 0$ , and so  $W = 0$ .

In contrast, when a force exerted on the system has a component in the direction of motion, such as in Figure 7.2(d), work *is* done—energy is transferred to the briefcase. Finally, in Figure 7.2(e), energy is transferred from the briefcase to a generator. There are two good ways to interpret this energy transfer. One interpretation is that the briefcase’s weight does work on the generator, giving it energy. The other interpretation is that the generator does negative work on the briefcase, thus removing energy from it. The drawing shows the latter, with the force from the generator upward on the briefcase, and the displacement downward. This makes  $\theta = 180^\circ$ , and  $\cos 180^\circ = -1$ ; therefore,  $W$  is negative.

## Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in newton-meters. A newton-meter is given the special name joule (J), and  $1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$ . One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

### Example 7.1

**Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn** How much work is done on the lawn mower by the person in Figure 7.2(a) if he exerts a constant force of 75.0 N at an angle  $35^\circ$  below the horizontal

and pushes the mower 25.0 m on level ground? Convert the amount of work from joules to kilocalories and compare it with this person's average daily intake of 10,000 kJ (about 2400 kcal) of food energy. One *calorie* (1 cal) of heat is the amount required to warm 1 g of water by 1°C, and is equivalent to 4.186 J, while one *food calorie* (1 kcal) is equivalent to 4186 J.

**Strategy** We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation  $W = Fd \cos \theta$ . The force, angle, and displacement are given, so that only the work  $W$  is unknown.

**Solution** The equation for the work is

$$W = Fd \cos \theta.$$

7.4

Substituting the known values gives

$$\begin{aligned} W &= (75.0 \text{ N})(25.0 \text{ m}) \cos (35.0^\circ) \\ &= 1536 \text{ J} = 1.54 \times 10^3 \text{ J}. \end{aligned}$$

7.5

Converting the work in joules to kilocalories yields  $W = (1536 \text{ J})(1 \text{ kcal}/4186 \text{ J}) = 0.367 \text{ kcal}$ . The ratio of the work done to the daily consumption is

$$\frac{W}{2400 \text{ kcal}} = 1.53 \times 10^{-4}.$$

7.6

**Discussion** This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we “work” all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.