18.2 Coulomb's law

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Coulomb's law verbally and mathematically
- Solve problems involving Coulomb's law

Teacher Support

Teacher Support The learning objectives in this section will help your students master the following standards:

- (5) The student knows the nature of forces in the physical world. The student is expected to:
  - (C) describe and calculate how the magnitude of the electrical force between two objects depends on their charges and the distance between them.

Section Key Terms

Teacher Support

Teacher Support This section presents Coulomb's law and points out its similarities and differences with respect to Newton's law of universal gravitation. The similarities include the inverse-square nature of the two laws and the analogous roles of mass and charge. The differences include the restriction of positive mass versus positive or negative charge.

[BL][OL]Discuss how Coulomb described this law long after Newton described the law of universal gravitation.

[AL]Ask why the law of force between electrostatic charge was discovered after that of gravity if gravity is weak compared to electrostatic forces.

More than 100 years before Thomson and Rutherford discovered the fundamental particles that carry positive and negative electric charges, the French scientist Charles-Augustin de Coulomb mathematically described the force between charged objects. Doing so required careful measurements of forces between charged spheres, for which he built an ingenious device called a *torsion balance*.

This device, shown in Figure 18.15, contains an insulating rod that is hanging by a thread inside a glass-walled enclosure. At one end of the rod is the metallic sphere A. When no charge is on this sphere, it touches sphere B. Coulomb would touch the spheres with a third metallic ball (shown at the bottom of the diagram) that was charged. An unknown amount of charge would distribute evenly between spheres A and B, which would then repel each other, because like charges repel. This force would cause sphere A to rotate away from sphere B, thus twisting the wire until the torsion in the wire balanced the electrical force. Coulomb then turned the knob at the top, which allowed him to rotate the thread, thus bringing sphere A closer to sphere B. He found that bringing sphere A twice as close to sphere B required increasing the torsion by a factor of four. Bringing the sphere three times closer required a ninefold increase in the torsion. From this type of measurement, he deduced that the electrical force between the spheres was inversely proportional to the distance squared between the spheres. In other words,

$F \propto \frac{1}{r^{2,}}$

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where r is the distance between the spheres.

An electrical charge distributes itself equally between two conducting spheres of the same size. Knowing this allowed Coulomb to divide an unknown charge in half. Repeating this process would produce a sphere with one quarter of the initial charge, and so on. Using this technique, he measured the force between spheres A and B when they were charged with different amounts of charge. These measurements led him to deduce that the force was proportional to the charge on each sphere, or

$F \propto q_{\mathrm{A}} q_{\mathrm{B},}$

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where q_{A} is the charge on sphere A, and q_{B} is the charge on sphere B.

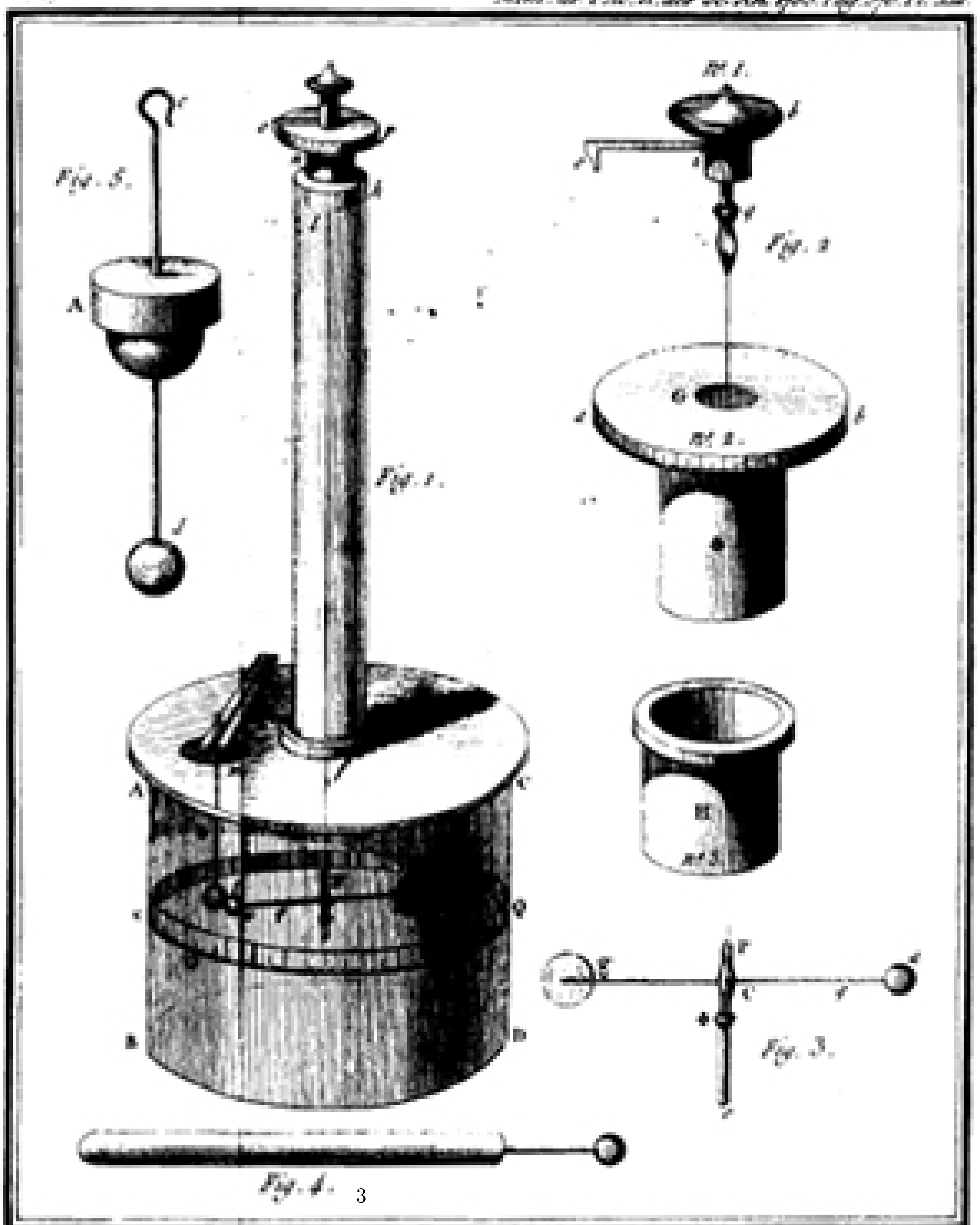

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Figure 18.15 A drawing of Coulomb's torsion balance, which he used to measure the electrical force between charged spheres. (credit: Charles-Augustin de Coulomb)

Combining these two proportionalities, he proposed the following expression to describe the force between the charged spheres.

$F = \frac{kq_1q_2}{r^2}$

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This equation is known as Coulomb's law, and it describes the electrostatic force between charged objects. The constant of proportionality k is called *Coulomb's constant*. In SI units, the constant k has the value $k = 8.99 \ \times \ 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^{2.}$

The direction of the force is along the line joining the centers of the two objects. If the two charges are of opposite signs, Coulomb's law gives a negative result. This means that the force between the particles is attractive. If the two charges have the same signs, Coulomb's law gives a positive result. This means that the force between the particles is repulsive. For example, if both q_1 and q_2 are negative or if both are positive, the force between them is repulsive. This is shown in Figure 18.16(a). If q_1 is a negative charge and q_2 is a positive charge (or vice versa), then the charges are different, so the force between them is attractive. This is shown in Figure 18.16(b).

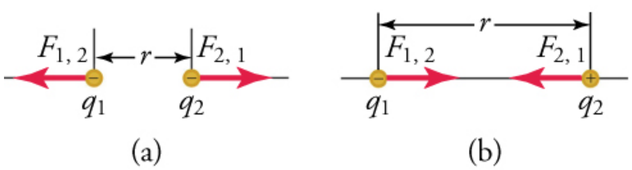

Figure 18.16 The magnitude of the electrostatic force F between point charges q_1 and q_2 separated by a distance r is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force $(F_{1,2})$ on q_1 is equal in magnitude and opposite in direction to the force $(F_{2,1})$ it exerts on q_2. (a) Like charges. (b) Unlike charges.

Teacher Support

Teacher Support Point out how the subscripts $_{1,\ 2}$ means the force on object 1 due to object 2 (and vice versa).

Note that Coulomb's law applies only to charged objects that are not moving with respect to each other. The law says that the force is proportional to the amount of charge on each object and inversely proportional to the square of the distance between the objects. If we double the charge q_1, for instance, then the force is doubled. If we double the distance between the objects, then the force between them *decreases* by a factor of $2^2 = 4$. Although Coulomb's law is true in general, it is easiest to apply to spherical objects or to objects that are much smaller than the distance between the objects (in which case, the objects can be approximated as spheres).

Coulomb's law is an example of an inverse-square law, which means the force depends on the square of the denominator. Another inverse-square law is Newton's law of universal gravitation, which is $F = Gm_1 m_2 / r^2$. Although these laws are similar, they differ in two important respects: (i) The gravitational constant G is much, much smaller than k
($G = 6.67 \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$); and (ii) only one type of mass exists, whereas two types of electric charge exist. These two differences explain why gravity is so much weaker than the electrostatic force and why gravity is only attractive, whereas the electrostatic force can be attractive or repulsive.

Finally, note that Coulomb measured the distance between the spheres from the centers of each sphere. He did not explain this assumption in his original papers, but it turns out to be valid. From *outside* a uniform spherical distribution of charge, it can be treated as if all the charge were located at the center of the sphere.

Watch Physics

Electrostatics (part 1): Introduction to charge and Coulomb's law This video explains the basics of Coulomb's law. Note that the lecturer uses d for the distance between the center of the particles instead of r.

Click to view content

Grasp Check

True or false—If one particle carries a positive charge and another carries a negative charge, then the force between them is attractive.

a. true
b. false

Snap Lab

Hovering plastic In this lab, you will use electrostatics to hover a thin piece of plastic in the air.

- Balloon
- Light plastic bag (e.g., produce bag from grocery store)

Instructions

Procedure

1. Cut the plastic bag to make a plastic loop about 2 inches wide.
2. Inflate the balloon.

3. Charge the balloon by rubbing it on your clothes.
4. Charge the plastic loop by placing it on a nonmetallic surface and rubbing it with a cloth.
5. Hold the balloon in one hand, and in the other hand hold the plastic loop above the balloon. If the loop clings too much to your hand, recruit a friend to hold the strip above the balloon with both hands. Now let go of the plastic loop, and maneuver the balloon under the plastic loop to keep it hovering in the air above the balloon.

Grasp Check

How does the balloon keep the plastic loop hovering?

a. The balloon and the loop are both negatively charged. This will help the balloon keep the plastic loop hovering.
b. The balloon is charged, while the plastic loop is neutral.This will help the balloon keep the plastic loop hovering.
c. The balloon and the loop are both positively charged. This will help the balloon keep the plastic loop hovering.
d. The balloon is positively charged, while the plastic loop is negatively charged. This will help the balloon keep the plastic loop hovering.

Worked Example

Using Coulomb's law to find the force between charged objects Suppose Coulomb measures a force of 20×10^{-6}N between the two charged spheres when they are separated by 5.0 cm. By turning the dial at the top of the torsion balance, he approaches the spheres so that they are separated by 3.0 cm. Which force does he measure now?

Strategy

Apply Coulomb's law to the situation before and after the spheres are brought closer together. Although we do not know the charges on the spheres, we do know that they remain the same. We call these unknown but constant charges q_1 and q_2 . Because these charges appear as a product in Coulomb's law, they form a single unknown. We thus have two equations and two unknowns, which we can solve. The first unknown is the force (which we call F_{f}) when the spheres are 3.0 cm apart, and the second is $q_1 q_2$.

Use the following notation: When the charges are 5.0 cm apart, the force is $F_{\mathrm{i}} = 20 \times 10^{-6}$N and $r_{\mathrm{i}} = 5.0$ cm $= 0.050$ m, where the subscript i means *initial*. Once the charges are brought closer together, we know $r_{\mathrm{f}} = 3.0$ cm $= 0.030$ m, where the subscript f means *final*.

Solution

Coulomb's law applied to the spheres in their initial positions gives

$F_{\mathrm{i}} = \frac{kq_1 q_2}{r_{\mathrm{i}}^2}.$

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Coulomb's law applied to the spheres in their final positions gives

$F_{\mathrm{f}} = \frac{kq_1 q_2}{r_{\mathrm{f}}^2}.$

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Dividing the second equation by the first and solving for the final force F_{f} leads to

$$\begin{aligned} \frac{F_{\mathrm{f}}}{F_{\mathrm{i}}} &= \frac{kq_1q_2/r_{\mathrm{f}}^2}{kq_1q_2/r_{\mathrm{i}}^2}. \\ &= \frac{r_{\mathrm{i}}^2}{r_{\mathrm{f}}^2} \\ F_{\mathrm{f}} &= F_{\mathrm{i}}\frac{r_{\mathrm{i}}^2}{r_{\mathrm{f}}^2}. \end{aligned}$$

18.10

Inserting the known quantities yields

$$\begin{aligned} F_{\mathrm{f}} &= F_{\mathrm{i}}\frac{r_{\mathrm{i}}^2}{r_{\mathrm{f}}^2} \\ &= \left(20 \times 10^{-6}\mathrm{N}\right)\frac{(0.050\ \mathrm{m})^2}{(0.030\ \mathrm{m})^2} \\ &= 56 \times 10^{-6}\mathrm{N} \\ &= 5.6 \times 10^{-5}\mathrm{N}. \end{aligned}$$

18.11

The force acts along the line joining the centers of the spheres. Because the same type of charge is on each sphere, the force is repulsive.

Discussion

As expected, the force between the charges is greater when they are 3.0 cm apart than when they are 5.0 cm apart. Note that although it is a good habit to convert cm to m (because the constant k is in SI units), it is not necessary in this problem, because the distances cancel out.

We can also solve for the second unknown $|q_1 q_2|$. By using the first equation, we find

$$
\begin{aligned}
F_{\mathrm{f}} &= \frac{kq_1q_2}{r_{\mathrm{i}}^2} \\
q_1q_2 &= \frac{F_{\mathrm{i}}r_{\mathrm{i}}^2}{k} \\
&= \frac{\left(20 \times 10^{-6}\mathrm{N}\right)\left(0.050\ \mathrm{m}\right)^2}{8.99 \times 10^9 \mathrm{N{\cdot}m^2/C^2}} \\
&= 5.6\ \times\ 10^{-18}\mathrm{C}^2
\end{aligned}
$$

18.12

Note how the units cancel in the second-to-last line. Had we not converted cm to m, this would not occur, and the result would be incorrect. Finally, because the charge on each sphere is the same, we can further deduce that

$q_1 = q_2 = \pm\sqrt{5.6\ \times\ 10^{-18}\mathrm{C}^2} = \pm 2.4\ \mathrm{nC}.$

18.13

Worked Example

Using Coulomb's law to find the distance between charged objects
An engineer measures the force between two ink drops by measuring their acceleration and their diameter. She finds that each member of a pair of ink drops exerts a repulsive force of $F = 5.5$ mN on its partner. If each ink drop carries a charge $q_{\mathrm{inkdrop}} = -1\ \times\ 10^{-10}\mathrm{C}$, how far apart are the ink drops?

Strategy

We know the force and the charge on each ink drop, so we can solve Coulomb's law for the distance r between the ink drops. Do not forget to convert the force into SI units: $F = 5.5\ \mathrm{mN} = 5.5\ \times\ 10^{-3}\mathrm{N}$.

Solution

The charges in Coulomb's law are $q_1 = q_2 = q_{\mathrm{inkdrop}}$, so the numerator in Coulomb's law takes the form $q_1q_2 = q_{\mathrm{inkdrop}}^2$. Inserting this into Coulomb's law and solving for the distance r gives

$$
\begin{aligned}
F &= \frac{kq_{\mathrm{inkdrop}}^2}{r^2} \\
r &= \pm\sqrt{\frac{kq_{\mathrm{inkdrop}}^2}{F}} \\
&= \pm\sqrt{\frac{\left(8.99 \times 10^9 \mathrm{N{\cdot}m^2/C^2}\right)\left(-1 \times 10^{-10}\mathrm{C}\right)^2}{5.5 \times 10^{-3}\mathrm{N}}} \\
&= \pm 1.3 \times 10^{-4}\mathrm{m}
\end{aligned}
$$

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or 130 microns (about one-tenth of a millimeter).

Discussion

The plus-minus sign means that we do not know which ink drop is to the right and which is to the left, but that is not important, because both ink drops are the same.

Practice Problems

11.

A charge of -4×10^{-9} C is a distance of 3 cm from a charge of 3×10^{-9} C . What is the magnitude and direction of the force between them?

a. 1.2×10^{-4} N, and the force is attractive
b. 1.2×10^{14} N, and the force is attractive
c. 6.74×10^{23} N, and the force is attractive
d. $-\hat{y}$, and the force is attractive

12.

Two charges are repelled by a force of 2.0 N. If the distance between them triples, what is the force between the charges?

a. 0.22 N
b. 0.67 N
c. 2.0 N
d. 18.0 N

Check Your Understanding

13.

How are electrostatic force and charge related?

a. The force is proportional to the product of two charges.
b. The force is inversely proportional to the product of two charges.
c. The force is proportional to any one of the charges between which the force is acting.
d. The force is inversely proportional to any one of the charges between which the force is acting.

14.

Why is Coulomb's law called an inverse-square law?

a. because the force is proportional to the inverse of the distance squared between charges
b. because the force is proportional to the product of two charges

c. because the force is proportional to the inverse of the product of two charges
d. because the force is proportional to the distance squared between charges