Section Summary

3.1 Kinematics in Two Dimensions: An Introduction

- The shortest path between any two points is a straight line. In two dimensions, this path can be represented by a vector with horizontal and vertical components.
- The horizontal and vertical components of a vector are independent of one another. Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

3.2 Vector Addition and Subtraction: Graphical Methods

- The **graphical method of adding vectors** $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A} + \mathbf{B} = \mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor, respectively.
- The **graphical method of subtracting vector** $\mathbf{B}$ from $\mathbf{A}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathrm{A}-\mathbf{B} = \mathbf{A} + (-\mathrm{B}) = \mathrm{R}$. Then, the head-to-tail method of addition is followed in the usual way to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is commutative such that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.
- The head-to-tail method of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- If a vector $\mathbf{A}$ is multiplied by a scalar quantity c, the magnitude of the product is given by $c\mathrm{A}$. If c is positive, the direction of the product points in the same direction as $\mathbf{A}$; if c is negative, the direction of the product points in the opposite direction as $\mathbf{A}$.

3.3 Vector Addition and Subtraction: Analytical Methods

- The analytical method of vector addition and subtraction involves using the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.
- The steps to add vectors $\mathbf{A}$ and $\mathbf{B}$ using the analytical method are as follows:

  Step 1: Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations

  $$\begin{aligned} A_x &= A \cos\theta \\ B_x &= B \cos\theta \end{aligned}$$

  and

$$A_y = A \sin\theta$$
$$B_y = B \sin\theta.$$

Step 2: Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, **R**:

$$R_x = A_x + B_x$$

and

$$R_y = A_y + B_y.$$

Step 3: Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector **R**:

$$R = \sqrt{R_x^2 + R_y^2}\ .$$

Step 4: Use a trigonometric identity to determine the direction, θ, of **R**:

$$\theta = \tan^{-1}(R_y/R_x).$$

3.4 Projectile Motion

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- To solve projectile motion problems, perform the following steps:
    1. Determine a coordinate system. Then, resolve the position and/or velocity of the object in the horizontal and vertical components. The components of position $\mathbf{s}$ are given by the quantities x and y, and the components of the velocity $\mathbf{v}$ are given by $v_x = v \cos\theta$ and $v_y = v \sin\theta$, where v is the magnitude of the velocity and θ is its direction.
    2. Analyze the motion of the projectile in the horizontal direction using the following equations:

       Horizontal motion($a_x = 0$)

       $$x = x_0 + v_x t$$

       $v_x = v_{0x} = \mathbf{v}_x$ = velocity is a constant.
    3. Analyze the motion of the projectile in the vertical direction using the following equations:

Vertical Motion

(assuming positive is up

$a_y = -g = -9.80 \mathrm{m/s}^2$)

$y = y_0 + \frac{1}{2}(v_{0y} + v_y)t$

$v_y = v_{0y} - \mathrm{gt}$

$y = y_0 + v_{0y}t - \frac{1}{2}\mathrm{gt}^2$

$v_y^2 = v_{0y}^2 - 2g(y - y_0)$.

4. Recombine the horizontal and vertical components of location and/or velocity using the following equations:

   $s = \sqrt{x^2 + y^2}$

   $\theta = \tan^{-1}(y/x)$

   $v = \sqrt{v_x^2 + v_y^2}$

   $\theta_\mathrm{v} = \tan^{-1}(v_y/v_x)$.

- The maximum height h of a projectile launched with initial vertical velocity v_{0y} is given by

  $h = \frac{v_{0y}^2}{2g}$.

- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by

  $R = \frac{v_0^2 \sin 2\theta_0}{g}$.

3.5 Addition of Velocities

- Velocities in two dimensions are added using the same analytical vector techniques, which are rewritten as

  $v_x = v \cos \theta$

  $v_y = v \sin \theta$

  $v = \sqrt{v_x^2 + v_y^2}$

$\theta = \tan^{-1}(v_y/v_x)$.

- Relative velocity is the velocity of an object as observed from a particular reference frame, and it varies dramatically with reference frame.
- **Relativity** is the study of how different observers measure the same phenomenon, particularly when the observers move relative to one another. **Classical relativity** is limited to situations where speed is less than about 1% of the speed of light (3000 km/s).