

PHYS12 CH:5 Motion in Two Dimensions

Vectors and Projectiles

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Outline

- 1 Introduction
- 2 Vector Addition: Graphical Methods
- 3 Vector Addition: Analytical Methods
- 4 Projectile Motion
- 5 Summary

The Mystery

How do you describe motion
when objects move in two directions at once?

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A football spiraling downfield...

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when objects move in two directions at once?

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Motion happens in multiple dimensions simultaneously.

Learning Objectives

By the end of this section, you will be able to:

- **5.1:** Describe the graphical method of vector addition and subtraction

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By the end of this section, you will be able to:

- **5.1:** Describe the graphical method of vector addition and subtraction
- **5.1:** Use the graphical method to solve physics problems

5.1 What Is a Vector?

The Dual Nature

A vector is a quantity with both magnitude AND direction.

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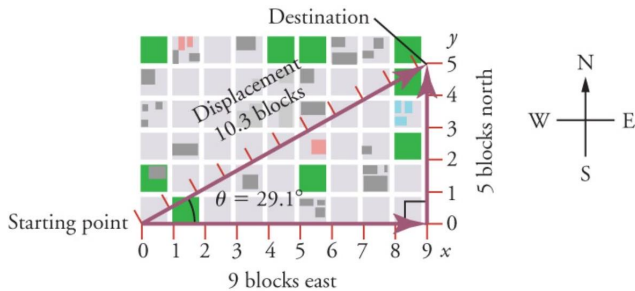
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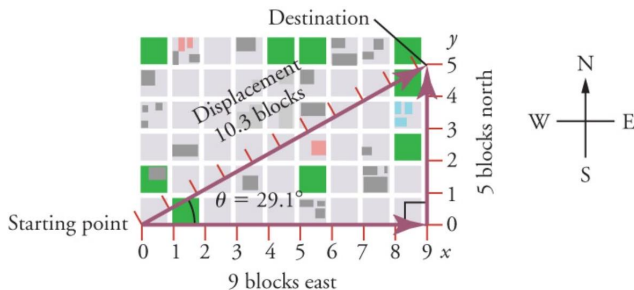
The Mental Model

Velocity is like a speedometer with a compass attached. Speed tells how fast. Velocity adds where.

5.1 The Journey to the Destination



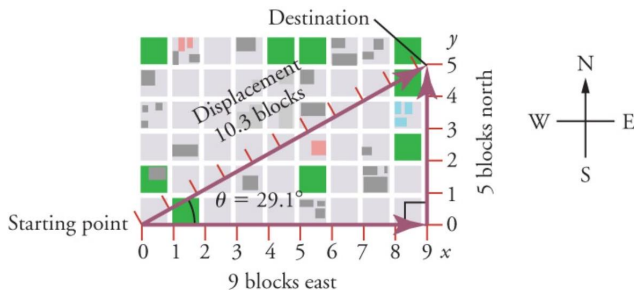
5.1 The Journey to the Destination



Path: 9 blocks east + 5 blocks north = 14 blocks walked

Displacement: 10.3 blocks at 29.1° north of east

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Civilian View vs. Reality

Civilian: "I walked 14 blocks."

Physicist: "Displacement was 10.3 blocks."

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- 1 Draw first vector to scale with protractor

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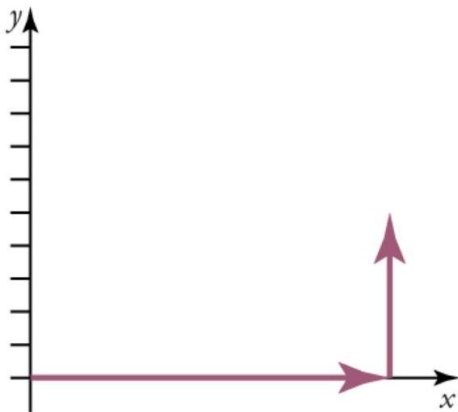
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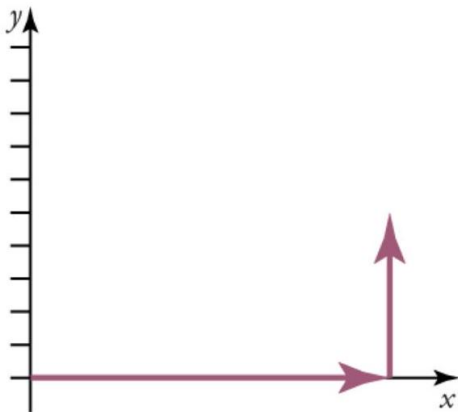
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Key insight: Order doesn't matter! $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

5.1 Building the Resultant



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The resultant vector connects start to finish.

5.1 Vector Subtraction

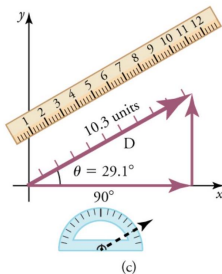
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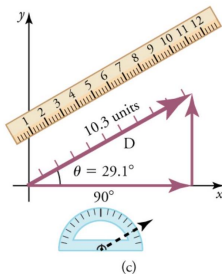
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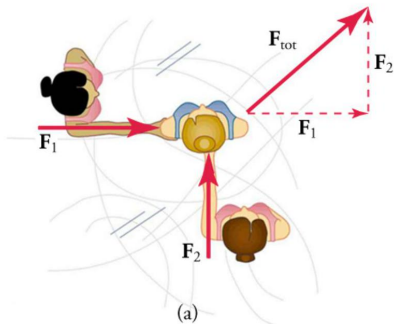
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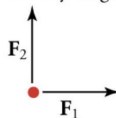


The negative vector: Same magnitude, opposite direction

5.1 Forces on Ice

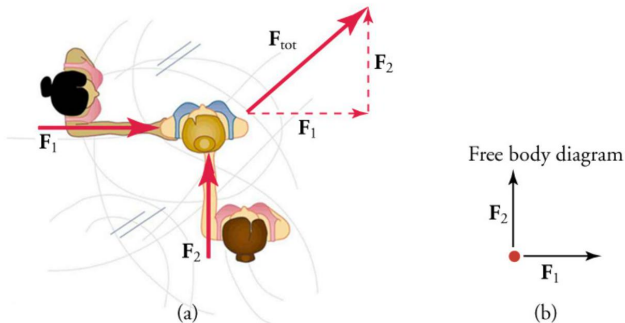


Free body diagram



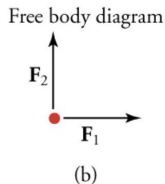
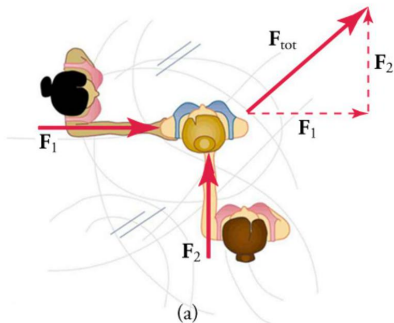
(b)

5.1 Forces on Ice



Two skaters push with 400 N each at right angles.

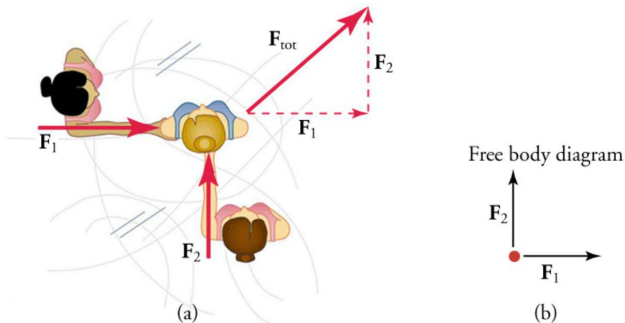
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Resultant force: $F_{tot} = \sqrt{(400)^2 + (400)^2} = 566 \text{ N}$

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Pythagorean theorem works when vectors are perpendicular!

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- **5.2:** Define components of vectors

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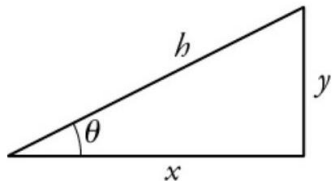
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- **5.2:** Use the analytical method to solve problems

5.2 The Power of Trigonometry

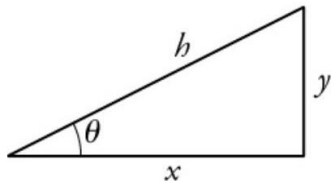


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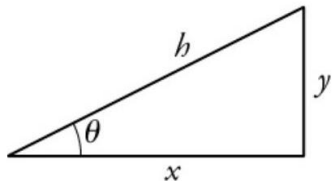
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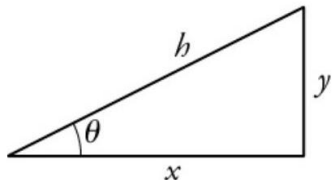
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5.2 Breaking Vectors Into Components

The Source Code

$$A_x = A \cos \angle \theta$$

$$A_y = A \sin \angle \theta$$

Every 2D vector can be split into x and y components.

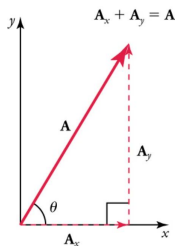
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Components match the original path: 9 east + 5 north!

5.2 Reverse: From Components to Vector

Reconstruction Formulas

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\angle\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

Pythagorean theorem gives magnitude, inverse tangent gives direction.

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The Paradox

Students confuse: $A_x + A_y = A$ (vector addition)
with $A = \sqrt{A_x^2 + A_y^2}$ (magnitude calculation)

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More accurate than graphical method - not limited by drawing precision!

Attempt: Two-Leg Journey

The Challenge (3 min, silent)

A person walks 53.0 m at 20.0° north of east, then 34.0 m at 63.0° north of east.

Given:

- $A = 53.0 \text{ m}$, $\angle\theta_A = 20.0^\circ$
- $B = 34.0 \text{ m}$, $\angle\theta_B = 63.0^\circ$

Find: Total displacement magnitude and direction

Can you decode this journey? Work silently.

Compare: Vector Addition Strategy

Turn and talk (2 min):

- 1 What's the first step - find components or add vectors?
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Reveal: The Navigation Solution

Self-correct in a different color:

Step 1: Find components of **A**

$$A_x = (53.0)(\cos 20.0^\circ) = 49.8 \text{ m}$$

$$A_y = (53.0)(\sin 20.0^\circ) = 18.1 \text{ m}$$

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Step 2: Find components of **B**

$$B_x = (34.0)(\cos 63.0^\circ) = 15.4 \text{ m}$$

$$B_y = (34.0)(\sin 63.0^\circ) = 30.3 \text{ m}$$

Reveal: Combining Components

Step 3: Add components

$$R_x = A_x + B_x = 49.8 + 15.4 = 65.2 \text{ m}$$

$$R_y = A_y + B_y = 18.1 + 30.3 = 48.4 \text{ m}$$

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$$R = \sqrt{(65.2)^2 + (48.4)^2} = \boxed{81.2 \text{ m}}$$

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Step 5: Find direction

$$\angle\theta = \tan^{-1}(48.4/65.2) = \boxed{36.6^\circ \text{ north of east}}$$

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- **5.3:** Apply kinematic equations and vectors to solve projectile problems

5.3 The Great Separation

Nature's Rule for Projectiles

Horizontal and vertical motions
are independent

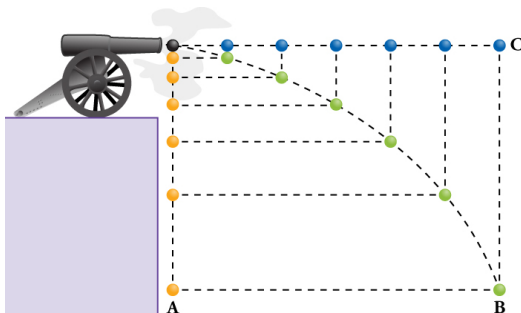
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We ignore air resistance in introductory physics.

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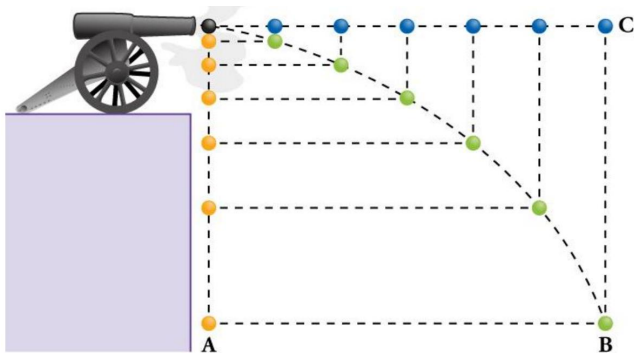
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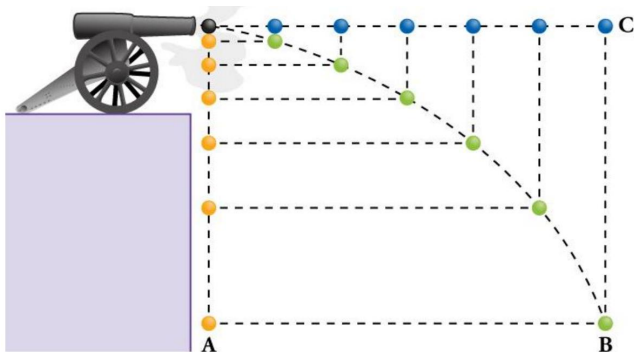
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- Velocity: $v_y = v_{0y} - g t$
- Position: $y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

Time t is the only variable connecting the two motions!

5.3 The Trajectory



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Ball kicked at angle θ follows parabolic path.

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Recombination formulas:

$$d = \sqrt{x^2 + y^2}, \quad \angle\theta = \tan^{-1}(y/x)$$

$$v = \sqrt{v_x^2 + v_y^2}, \quad \angle\theta_v = \tan^{-1}(v_y/v_x)$$

5.3 Maximum Height Formula

The Peak

$$h = \frac{v_{0y}^2}{2g}$$

Maximum height depends only on initial vertical **velocity**.

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Key insight: At maximum height, $v_y = 0$

What Your Brain Gets Wrong

Intuition: "Faster launch means higher peak."

Reality: "Only the *vertical* component matters."

Attempt: Fireworks Launch

The Challenge (3 min, silent)

A fireworks shell is launched at 70.0 m/s at 75° above horizontal.

Given:

- $v_0 = 70.0 \text{ m/s}$
- $\angle\theta = 75^\circ$

Find: Maximum height h

Can you predict where it explodes? Work silently.

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Reveal: Reaching the Sky

Self-correct in a different color:

Step 1: Find vertical **velocity** component

$$\begin{aligned}v_{0y} &= v_0 \sin \angle\theta \\ &= (70.0)(\sin 75^\circ) = 67.6 \text{ m/s}\end{aligned}$$

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$$\begin{aligned}v_{0y} &= v_0 \sin \angle\theta \\&= (70.0)(\sin 75^\circ) = 67.6 \text{ m/s}\end{aligned}$$

Step 2: Use maximum height formula

$$\begin{aligned}h &= \frac{v_{0y}^2}{2g} \\&= \frac{(67.6)^2}{2(9.8)} = \boxed{233 \text{ m}}\end{aligned}$$

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Step 1: Find vertical **velocity** component

$$\begin{aligned}v_{0y} &= v_0 \sin \angle\theta \\ &= (70.0)(\sin 75^\circ) = 67.6 \text{ m/s}\end{aligned}$$

Step 2: Use maximum height formula

$$\begin{aligned}h &= \frac{v_{0y}^2}{2g} \\ &= \frac{(67.6)^2}{2(9.8)} = \boxed{233 \text{ m}}\end{aligned}$$

Check: About 765 feet - reasonable for large fireworks!

5.3 The Range Equation

The Distance Formula

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Range depends on initial speed and launch \angle angle.

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- Same range for complementary angles: 30° and 60°
- Only valid when initial and final heights are equal

Attempt: Volcanic Rock

The Challenge (3 min, silent)

A rock is ejected from a volcano at 25.0 m/s at 35° above horizontal. It lands 20.0 m below its starting point.

Given:

- $v_0 = 25.0 \text{ m/s}$, $\angle\theta = 35^\circ$
- $y = -20.0 \text{ m}$

Find: Time of flight

Can you predict the flight time? Work silently.

Compare: Quadratic Strategy

Turn and talk (2 min):

- 1 Which kinematic equation has both y and t ?
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Name wheel: One pair share your approach (not your answer).

Reveal: Time of Flight

Self-correct in a different color:

Step 1: Find $v_{0y} = (25.0)(\sin 35^\circ) = 14.3 \text{ m/s}$

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Step 4: Quadratic formula

$$t = \frac{14.3 \pm \sqrt{(14.3)^2 - 4(4.90)(-20.0)}}{2(4.90)}$$

Reveal: Solving the Quadratic

Step 5: Calculate discriminant

$$\sqrt{(14.3)^2 + 4(4.90)(20.0)} = \sqrt{596.5} = 24.4$$

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$$t = \frac{14.3 + 24.4}{9.8} = 3.96 \text{ s} \quad (\text{physical})$$

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Negative time means before launch - physically impossible!

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- 5 Time connects the two dimensions
- 6 Maximum height depends only on v_{0y}

Key Equations

Vector Components:

$$A_x = A \cos \angle \theta \quad (1)$$

$$A_y = A \sin \angle \theta \quad (2)$$

Magnitude and Direction:

$$A = \sqrt{A_x^2 + A_y^2} \quad (3)$$

$$\angle \theta = \tan^{-1}(A_y/A_x) \quad (4)$$

Projectile Motion:

$$\text{Horizontal: } x = x_0 + v_x t \quad (5)$$

$$\text{Vertical: } y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \quad (6)$$

$$\text{Max height: } h = \frac{v_{0y}^2}{2g} \quad (7)$$

Complete the assigned problems
posted on the LMS