14.4 Sound Interference and Resonance

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe resonance and beats
- Define fundamental frequency and harmonic series
- Contrast an open-pipe and closed-pipe resonator
- Solve problems involving harmonic series and beat frequency

Teacher Support

Teacher Support The learning objectives in this section will help your students master the following standards:

- (7) Science concepts. The student knows the characteristics and behavior of waves. The student is expected to:
  - (D) investigate behaviors of waves, including reflection, refraction, diffraction, interference, resonance, and the Doppler effect.

In addition, the High School Physics Laboratory Manual addresses content in this section in the lab titled: Sound Waves, as well as the following standards:

- (7) Science concepts. The student knows the characteristics and behavior of waves. The student is expected to:
  - (D) investigate behaviors of waves, including reflection, refraction, diffraction, interference, resonance, and the Doppler effect.

Section Key Terms

Teacher Support

Teacher Support [BL]Before the start of this section, it would be useful to review the properties of sound waves and how they are related to each other, standing waves, superposition and interference of waves.

Resonance and Beats

Sit in front of a piano sometime and sing a loud brief note at it while pushing down on the sustain pedal. It will sing the same note back at you—the strings that have the same frequencies as your voice, are resonating in response to the forces from the sound waves that you sent to them. This is a good example of the fact that objects—in this case, piano strings—can be forced to oscillate but oscillate best at their natural frequency.

A driving force (such as your voice in the example) puts energy into a system at a certain frequency, which is not necessarily the same as the natural frequency of the system. Over time the energy dissipates, and the amplitude gradually reduces to zero- this is called damping. The natural frequency is the frequency at which a system would oscillate if there were no driving and no damping force. The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance, and a system being driven at its natural frequency is said to resonate.

Most of us have played with toys where an object bobs up and down on an elastic band, something like the paddle ball suspended from a finger in Figure 14.18. At first you hold your finger steady, and the ball bounces up and down with a small amount of damping. If you move your finger up and down slowly, the ball will follow along without bouncing much on its own. As you increase the frequency at which you move your finger up and down, the ball will respond by oscillating with increasing amplitude. When you drive the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as you drive it. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller, until the oscillations nearly disappear and your finger simply moves up and down with little effect on the ball.

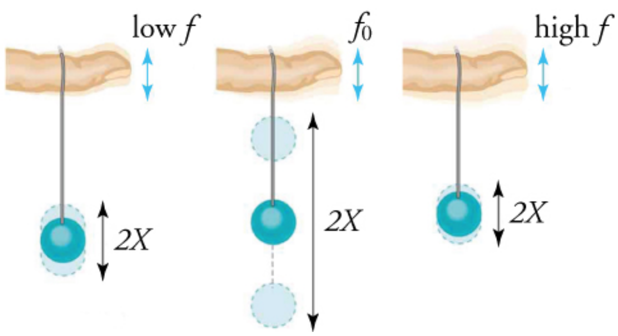

Figure 14.18 The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations.

Another example is that when you tune a radio, you adjust its resonant frequency so that it oscillates only at the desired station's broadcast (driving) frequency. Also, a child on a swing is driven (pushed) by a parent at the swing's natural frequency to reach the maximum amplitude (height). In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance.

Figure 14.19 Some types of headphones use the phenomena of constructive and destructive interference to cancel out outside noises.

Teacher Support

Teacher Support [BL][OL][AL] Tuning forks and pipes may be used to demonstrate the concept of resonance. Use any pipe or tube closed at one end. Fix it so that it stands upright with the open end on top. Choose a tuning fork and strike it to make it vibrate. Place it near the mouth of the pipe and hear the sound. Now fill the pipe with some water and repeat. The changing water level changes the length of the resonating air column. Continue doing this. When a certain length is obtained, the sound of the tuning fork will resonate through the column.

All sound resonances are due to constructive and destructive interference. Only the resonant frequencies interfere constructively to form standing waves, while others interfere destructively and are absent. From the toot made by blowing over a bottle to the recognizability of a great singer's voice, resonance and standing waves play a vital role in sound.

Interference happens to all types of waves, including sound waves. In fact, one way to support that something *is a wave* is to observe interference effects. Figure 14.19 shows a set of headphones that employs a clever use of sound interference to cancel noise. To get destructive interference, a fast electronic analysis is performed, and a second sound is introduced with its maxima and minima exactly reversed from the incoming noise.

In addition to resonance, superposition of waves can also create beats. Beats are produced by the superposition of two waves with slightly different frequencies but the same amplitude. The waves alternate in time between constructive interference and destructive interference, giving the resultant wave an amplitude that varies over time. (See the resultant wave in Figure 14.20).

This wave fluctuates in amplitude, or beats, with a frequency called the beat frequency. The equation for beat frequency is

$$f_B = |f_1 - f_2|,$$

14.13

where f_1 and f_2 are the frequencies of the two original waves. If the two frequencies of sound waves are similar, then what we hear is an average frequency that gets louder and softer at the beat frequency.

Tips For Success

Don't confuse the beat frequency with the regular frequency of a wave resulting from superposition. While the beat frequency is given by the formula above, and describes the frequency of the beats, the actual frequency of the wave resulting from superposition is the average of the frequencies of the two original waves.

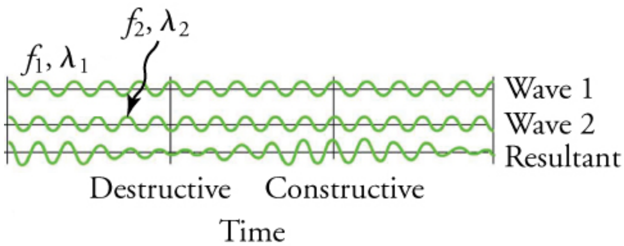

Figure 14.20 Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

Virtual Physics

Wave Interference Click to view content

For this activity, switch to the Sound tab. Turn on the Sound option, and experiment with changing the frequency and amplitude, and adding in a second speaker and a barrier.

According to the graph, what happens to the amplitude of pressure over time. What is this phenomenon called, and what causes it ?

a. The amplitude decreases over time. This phenomenon is called damping. It is caused by the dissipation of energy.
b. The amplitude increases over time. This phenomenon is called feedback. It is caused by the gathering of energy.
c. The amplitude oscillates over time. This phenomenon is called echoing. It is caused by fluctuations in energy.

Fundamental Frequency and Harmonics

Suppose we hold a tuning fork near the end of a tube that is closed at the other end, as shown in Figure 14.21, Figure 14.22, and Figure 14.23. If the tuning fork has just the right frequency, the air column in the tube resonates loudly, but at most frequencies it vibrates very little. This means that the air column has only certain natural frequencies. The figures show how a resonance at the lowest of these natural frequencies is formed. A disturbance travels down the tube at the speed of sound and bounces off the closed end. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly half a cycle later, and it interferes constructively with the continuing sound produced by the tuning fork. The incoming and reflected sounds form a standing wave in the tube as shown.

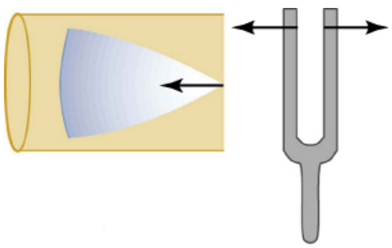

Figure 14.21 Resonance of air in a tube closed at one end, caused by a tuning fork. A disturbance moves down the tube.

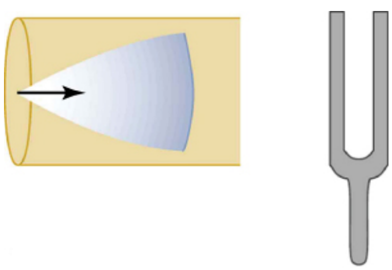

Figure 14.22 Resonance of air in a tube closed at one end, caused by a tuning fork. The disturbance reflects from the closed end of the tube.

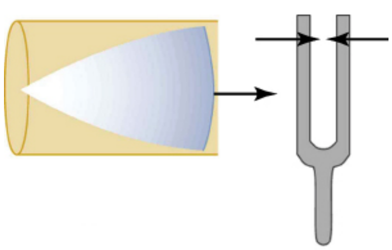

Figure 14.23 Resonance of air in a tube closed at one end, caused by a tuning fork. If the length of the tube L is just right, the disturbance gets back to the

tuning fork half a cycle later and interferes constructively with the continuing sound from the tuning fork. This interference forms a standing wave, and the air column resonates.

The standing wave formed in the tube has its maximum air displacement (an antinode) at the open end, and no displacement (a node) at the closed end. Recall from the last chapter on waves that motion is unconstrained at the antinode, and halted at the node. The distance from a node to an antinode is one-fourth of a wavelength, and this equals the length of the tube; therefore, $\lambda = 4L$. This same resonance can be produced by a vibration introduced at or near the closed end of the tube, as shown in Figure 14.24.

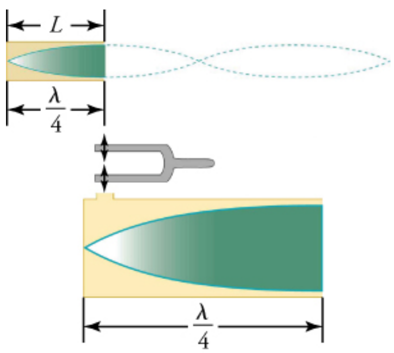

Figure 14.24 The same standing wave is created in the tube by a vibration introduced near its closed end.

Since maximum air displacements are possible at the open end and none at the closed end, there are other, shorter wavelengths that can resonate in the tube see Figure 14.25). Here the standing wave has three-fourths of its wavelength in the tube, or $L = (3/4)\lambda'$, so that $\lambda' = 4L/3$. There is a whole series of shorter-wavelength and higher-frequency sounds that resonate in the tube.

We use specific terms for the resonances in any system. The lowest resonant frequency is called the fundamental, while all higher resonant frequencies are called overtones. All resonant frequencies are multiples of the fundamental, and are called harmonics. The fundamental is the first harmonic, the first overtone is the second harmonic, and so on. Figure 14.26 shows the fundamental and the first three overtones (the first four harmonics) in a tube closed at one end.

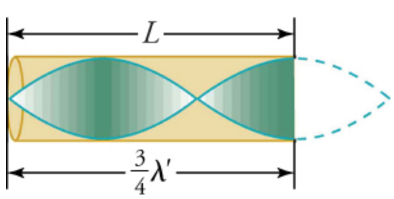

Figure 14.25 Another resonance for a tube closed at one end. This has maximum air displacements at the open end, and none at the closed end. The wavelength is

shorter, with three-fourths λ' equaling the length of the tube, so that $\lambda' = 4L/3$. This higher-frequency vibration is the first overtone.

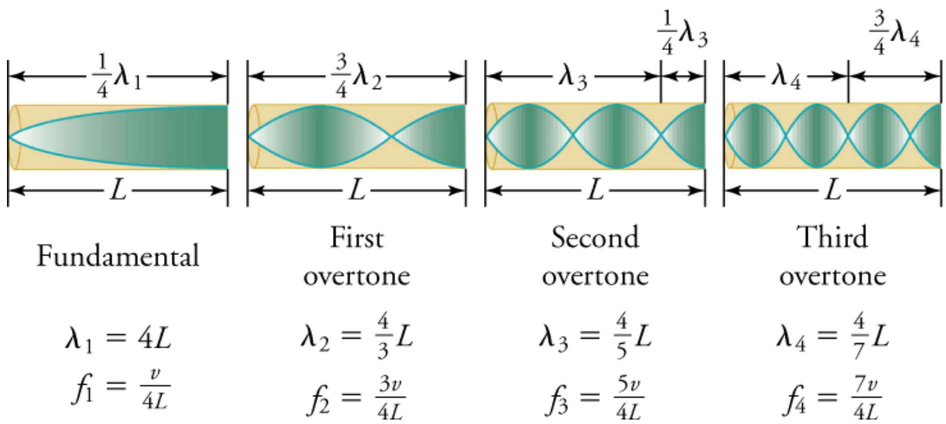

Figure 14.26 The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end.

The fundamental and overtones can be present at the same time in a variety of combinations. For example, the note middle C on a trumpet sounds very different from middle C on a clarinet, even though both instruments are basically modified versions of a tube closed at one end. The fundamental frequency is the same (and usually the most intense), but the overtones and their mix of intensities are different. This mix is what gives musical instruments (and human voices) their distinctive characteristics, whether they have air columns, strings, or drumheads. In fact, much of our speech is determined by shaping the cavity formed by the throat and mouth and positioning the tongue to adjust the fundamental and combination of overtones.

Open-Pipe and Closed-Pipe Resonators

The resonant frequencies of a tube closed at one end (known as a closed-pipe resonator) are

$f_n = n\frac{v}{4L},\ n = 1, 3, 5...,$

where f_1 is the fundamental, f_3 is the first overtone, and so on. Note that the resonant frequencies depend on the speed of sound v and on the length of the tube L.

Another type of tube is one that is *open* at both ends (known as an open-pipe resonator). Examples are some organ pipes, flutes, and oboes. The air columns in tubes open at both ends have maximum air displacements at both ends. (See Figure 14.27). Standing waves form as shown.

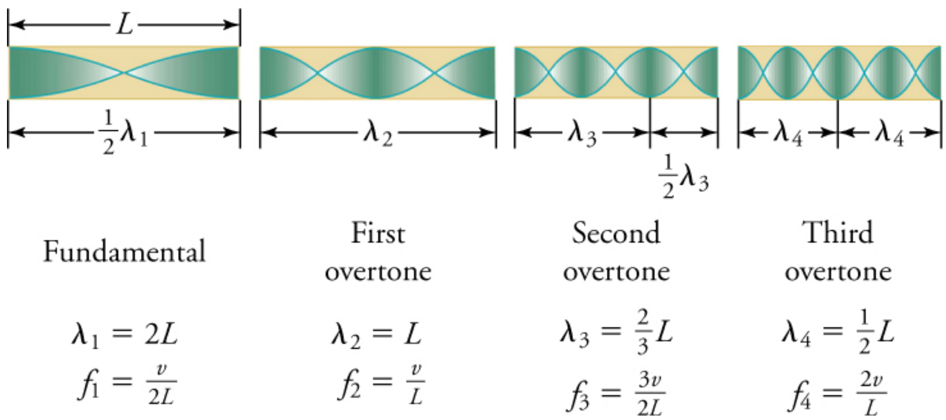

Figure 14.27 The resonant frequencies of a tube open at both ends are shown, including the fundamental and the first three overtones. In all cases the maximum air displacements occur at both ends of the tube, giving it different natural frequencies than a tube closed at one end.

The resonant frequencies of an open-pipe resonator are

$f_n = n\frac{v}{2L},\ n = 1, 2, 3...,$

where f_1 is the fundamental, f_2 is the first overtone, f_3 is the second overtone, and so on. Note that a tube open at both ends has a fundamental frequency twice what it would have if closed at one end. It also has a different spectrum of overtones than a tube closed at one end. So if you had two tubes with the same fundamental frequency but one was open at both ends and the other was closed at one end, they would sound different when played because they have different overtones.

Middle C, for example, would sound richer played on an open tube since it has more overtones. An open-pipe resonator has more overtones than a closed-pipe resonator because it has even multiples of the fundamental as well as odd, whereas a closed tube has only odd multiples.

In this section we have covered resonance and standing waves for wind instruments, but vibrating strings on stringed instruments also resonate and have fundamentals and overtones similar to those for wind instruments.

Teacher Support

Teacher Support [BL][OL][AL] Other instruments also use air resonance in different ways to amplify sound. For instance, a violin and a guitar both have sounding boxes but with different shapes, resulting in different overtone structures. The vibrating string creates a sound that resonates in the sounding box, greatly amplifying the sound and creating overtones that give the instrument its characteristic flavor. The more complex the shape of the sounding box, the greater its ability to resonate over a wide range of frequencies. The type and thickness of wood or other materials used to make the sounding box also affects the quality of sound. Ask students to give more examples of how different musical instruments use the phenomenon of resonance.

Solving Problems Involving Harmonic Series and Beat Frequency

Worked Example

Finding the Length of a Tube for a Closed-Pipe Resonator If sound travels through the air at a speed of 344 m/s, what should be the length of a tube closed at one end to have a fundamental frequency of 128 Hz?

Strategy

The length L can be found by rearranging the equation $f_n = n\frac{v}{4L}$.

Solution

(1) Identify knowns.

- The fundamental frequency is 128 Hz.
- The speed of sound is 344 m/s.

(2) Use $f_n = n\frac{v_w}{4L}$ to find the fundamental frequency ($n = 1$).

$f_1 = \frac{v}{4L}$

14.14

(3) Solve this equation for length.

$L = \frac{v}{4f_1}$

14.15

(4) Enter the values of the speed of sound and frequency into the expression for L.

$L = \frac{v}{4f_1} = \frac{344 \text{ m/s}}{4(128 \text{ Hz})} = 0.672 \text{ m}$

14.16

Discussion

Many wind instruments are modified tubes that have finger holes, valves, and other devices for changing the length of the resonating air column and therefore, the frequency of the note played. Horns producing very low frequencies, such as tubas, require tubes so long that they are coiled into loops.

Worked Example

Finding the Third Overtone in an Open-Pipe Resonator If a tube that's open at both ends has a fundamental frequency of 120 Hz, what is the frequency of its third overtone?

Strategy

Since we already know the value of the fundamental frequency (n = 1), we can solve for the third overtone (n = 4) using the equation $f_n = n\frac{v}{2L}$.

Solution

Since fundamental frequency (n = 1) is

$f_1 = \frac{v}{2L}$,

14.17

and

$f_4 = 4\frac{v}{2L}$,$f_4 = 4f_1 = 4$(120 Hz)=480 Hz.

14.18

Discussion

To solve this problem, it wasn't necessary to know the length of the tube or the speed of the air because of the relationship between the fundamental and the third overtone. This example was of an open-pipe resonator; note that for a closed-pipe resonator, the third overtone has a value of n = 7 (not n = 4).

Worked Example

Using Beat Frequency to Tune a Piano Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). If a piano tuner hears two beats per second, and the tuning fork has a frequency of 256 Hz, what are the possible frequencies of the piano?

Strategy

Since we already know that the beat frequency f_B is 2, and one of the frequencies (let's say f_2) is 256 Hz, we can use the equation $f_B = |f_1 - f_2|$ to solve for the frequency of the piano f_1.

Solution

Since $f_B = |f_1 - f_2|$,

we know that either $f_B = f_1 - f_2$ or $-f_B = f_1 - f_2$.

Solving for f_1,

$f_1 = f_B + f_2 \text{or} f_1 = -f_B + f_2$.

14.19

Substituting in values,

$f_1 = 2 + 256 \text{ Hz or } f_1 = -2 + 256 \text{ Hz}$

14.20

So,

$f_1 = 258 \text{ Hz or 254 Hz.}$

14.21

Discussion

The piano tuner might not initially be able to tell simply by listening whether the frequency of the piano is too high or too low and must tune it by trial and error, making an adjustment and then testing it again. If there are even more beats after the adjustment, then the tuner knows that he went in the wrong direction.

Practice Problems

21.

Two sound waves have frequencies 250\,\text{Hz} and 280\,\text{Hz}. What is the beat frequency produced by their superposition?

a. 290\,\text{Hz}
b. 265\,\text{Hz}
c. 60\,\text{Hz}
d. 30\,\text{Hz}

22.

What is the length of a pipe closed at one end with fundamental frequency 350\,\text{Hz}? (Assume the speed of sound in air is 331\,\text{m/s}.)

a. 26\,\text{cm}
b. 26\,\text{m}
c. 24\,\text{m}
d. 24\,\text{cm}

Check Your Understanding

Teacher Support

Teacher Support Use these questions to assess student achievement of the section's Learning Objectives. If students are struggling with a specific objective, these questions will help identify it and direct students to the relevant content.

23.

What is damping?

a. Over time the energy increases and the amplitude gradually reduces to zero. This is called damping.
b. Over time the energy dissipates and the amplitude gradually increases. This is called damping.
c. Over time the energy increases and the amplitude gradually increases. This is called damping.
d. Over time the energy dissipates and the amplitude gradually reduces to zero. This is called damping.

24.

What is resonance? When can you say that the system is resonating?

a. The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance, and a system being driven at its natural frequency is said to resonate.
b. The phenomenon of driving a system with a frequency higher than its natural frequency is called resonance, and a system being driven at its natural frequency does not resonate.
c. The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance, and a system being driven at its natural frequency does not resonate.
d. The phenomenon of driving a system with a frequency higher than its natural frequency is called resonance, and a system being driven at its natural frequency is said to resonate.

25.

In the tuning fork and tube experiment, in case a standing wave is formed, at what point on the tube is the maximum disturbance from the tuning fork observed? Recall that the tube has one open end and one closed end.

a. At the midpoint of the tube
b. Both ends of the tube
c. At the closed end of the tube
d. At the open end of the tube

26.

In the tuning fork and tube experiment, when will the air column produce the loudest sound?

a. If the tuning fork vibrates at a frequency twice that of the natural frequency of the air column.
b. If the tuning fork vibrates at a frequency lower than the natural frequency of the air column.

c. If the tuning fork vibrates at a frequency higher than the natural frequency of the air column.
d. If the tuning fork vibrates at a frequency equal to the natural frequency of the air column.

27.

What is a closed-pipe resonator?

a. A pipe or cylindrical air column closed at both ends
b. A pipe with an antinode at the closed end
c. A pipe with a node at the open end
d. A pipe or cylindrical air column closed at one end

28.

Give two examples of open-pipe resonators.

a. piano, violin
b. drum, tabla
c. rlectric guitar, acoustic guitar
d. flute, oboe