PHYS12 CH:28.4-28.6

Relativistic Mechanics

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November 21, 2025

Learning Objectives

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- Apply the relativistic velocity addition formula.
- Calculate relativistic momentum and compare it to classical momentum.
- Define rest energy, total energy, and kinetic energy in relativistic terms.
- Solve problems involving mass-energy equivalence ($E = mc^2$).

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- What happens when we push objects near the speed of light?

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Today: Dynamics

- What happens when we push objects near the speed of light?
- Does $F = ma$ still work? (Spoiler: Not simply)
- How do we add velocities correctly?

28.4 The Limit of Galilean Relativity

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Relativistic Velocity Addition

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Notice: If $v \ll c$ and $u' \ll c$, the denominator ≈ 1, giving back $u \approx v + u'$.

Example: I Do - Velocity Addition

Problem: A spaceship travels away from Earth at $v = 0.5c$. It fires a missile forward at speed $u' = 0.5c$ relative to the ship. What is the missile's speed relative to Earth?

I Do: Velocity Addition - G & U

G - Givens

- $v = 0.5c$ (Ship relative to Earth)

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U - Unknown

- $u =?$ (Missile relative to Earth)

I Do: Velocity Addition - Equation

E - Equation

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- Use the relativistic addition formula:

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

I Do: Velocity Addition - Substitute & Solve

S - Substitute

- $u = \frac{0.5c + 0.5c}{1 + \frac{(0.5c)(0.5c)}{c^2}}$

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- $\boxed{u = 0.8c}$
- *Result is less than c, as required.*

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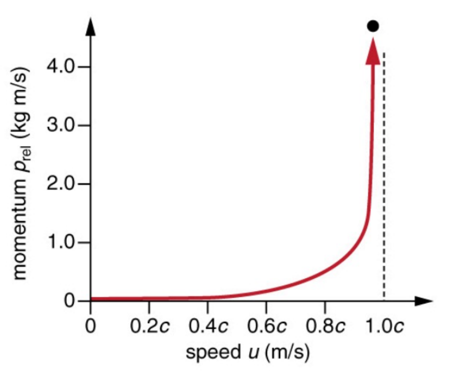
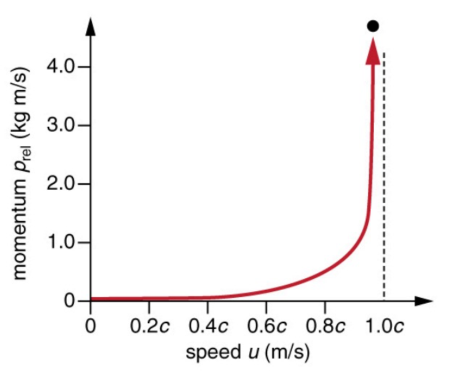
- As $v \to c$, $\gamma \to \infty$, so $p \to \infty$.
- This explains why a massive object cannot reach c: it would require infinite momentum (and infinite energy).


momentum p_{rel} (kg m/s)
4.0
3.0
2.0
1.0
0
0
0.2c
0.4c
0.6c
0.8c
1.0c
speed u (m/s)

Concept Visualization: Momentum Limit



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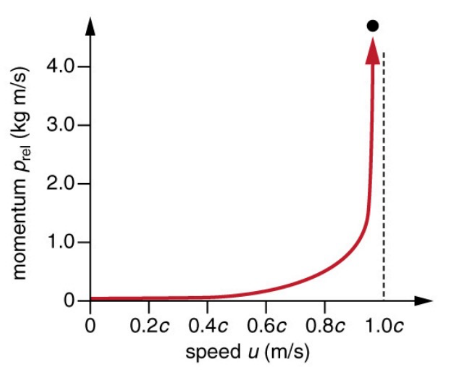

- **Classical line**: Straight line ($p = mv$).
- **Relativistic curve**: Follows classical line at low speeds, then curves upward sharply, approaching a vertical asymptote at $v = c$.

28.6 Rest Energy and Total Energy

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Total Energy (E)

The sum of rest energy and kinetic energy.

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic Kinetic Energy

Kinetic Energy is the "extra" energy due to motion.

$$E = E_0 + KE$$

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Note: At low speeds, this simplifies to $\frac{1}{2}mv^2$.

We Do: Relativistic Energy

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1. Calculate γ for $v = 0.999c$.
2. Use $E = \gamma mc^2$.
3. (Optional) Convert Joules to eV or MeV.

You Do: Practice

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1. What is its kinetic energy? ($KE = E - E_0$)
2. What is the value of γ?
3. How fast is it moving? (Solve γ for v).

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- **Total Energy**: $E = \gamma mc^2$.
- **Kinetic Energy**: $KE = (\gamma - 1)mc^2$.