Section Summary

10.1 Angular Acceleration

- Uniform circular motion is the motion with a constant angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$.
- In non-uniform circular motion, the velocity changes with time and the rate of change of angular velocity (i.e. angular acceleration) is $\alpha = \frac{\Delta\omega}{\Delta t}$.
- Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction, given as $a_{\mathrm{t}} = \frac{\Delta v}{\Delta t}$.
- For circular motion, note that $v = r\omega$, so that
- $a_{\mathrm{t}} = \frac{\Delta(r\omega)}{\Delta t}$.
- The radius r is constant for circular motion, and so $\Delta(r\omega) = r\Delta\omega$. Thus,
- $a_{\mathrm{t}} = r\frac{\Delta\omega}{\Delta t}$.
- By definition, $\Delta\omega/\Delta t = \alpha$. Thus,
- $a_{\mathrm{t}} = r\alpha$

  or

  $\alpha = \frac{a_{\mathrm{t}}}{r}$.

10.2 Kinematics of Rotational Motion

- Kinematics is the description of motion.
- The kinematics of rotational motion describes the relationships among rotation angle, angular velocity, angular acceleration, and time.
- Starting with the four kinematic equations we developed in the One-Dimensional Kinematics, we can derive the four rotational kinematic equations (presented together with their translational counterparts) seen in Table 10.2.
- In these equations, the subscript 0 denotes initial values (x_0 and t_0 are initial values), and the average angular velocity $\bar{\omega}$ and average velocity $\bar{v}$ are defined as follows:
- $\bar{\omega} = \frac{\omega_0+\omega}{2}$ and $\bar{v} = \frac{v_0+v}{2}$.

10.3 Dynamics of Rotational Motion: Rotational Inertia

- The farther the force is applied from the pivot, the greater is the angular acceleration; angular acceleration is inversely proportional to mass.
- If we exert a force F on a point mass m that is at a distance r from a pivot point and because the force is perpendicular to r, an acceleration a = F/m is obtained in the direction of F. We can rearrange this equation such that

- $F = ma$,

  and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = ma$, yielding

  $F = mr\alpha$

- Torque is the turning effectiveness of a force. In this case, because F is perpendicular to r, torque is simply $\tau = rF$. If we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,
- $\mathrm{rF} = \mathrm{mr}^2\alpha$

  or

  $\tau = \mathrm{mr}^2\alpha$.

- The moment of inertia I of an object is the sum of MR^2 for all the point masses of which it is composed. That is,
- $I = \sum \mathrm{mr}^2$.
- The general relationship among torque, moment of inertia, and angular acceleration is
- $\tau = I\alpha$

  or

  $\alpha = \frac{\text{net }\tau}{I}$.

10.4 Rotational Kinetic Energy: Work and Energy Revisited

- The rotational kinetic energy $\mathrm{KE}_{\mathrm{rot}}$ for an object with a moment of inertia I and an angular velocity ω is given by
- $\mathrm{KE}_{\mathrm{rot}} = \frac{1}{2}I\omega^2$.
- Helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades.
- Work and energy in rotational motion are completely analogous to work and energy in translational motion.
- The equation for the work-energy theorem for rotational motion is,
- net $W = \frac{1}{2}I\omega^2 - \frac{1}{2}I{\omega_0}^2$.

10.5 Angular Momentum and Its Conservation

- Every rotational phenomenon has a direct translational analog , likewise angular momentum L can be defined as $L = I\omega$.
- This equation is an analog to the definition of linear momentum as $p = \mathrm{mv}$. The relationship between torque and angular momentum is net $\tau = \frac{\Delta L}{\Delta t}$.
- Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

10.6 Collisions of Extended Bodies in Two Dimensions

- Angular momentum L is analogous to linear momentum and is given by $L = I\omega$.
- Angular momentum is changed by torque, following the relationship net $\tau = \frac{\Delta L}{\Delta t}$.
- Angular momentum is conserved if the net torque is zero $L = \text{constant} \quad (\text{net } \tau = 0)$ or $L = L' \quad (\text{net } \tau = 0)$. This equation is known as the law of conservation of angular momentum, which may be conserved in collisions.

10.7 Gyroscopic Effects: Vector Aspects of Angular Momentum

- Torque is perpendicular to the plane formed by r and F and is the direction your right thumb would point if you curled the fingers of your right hand in the direction of F. The direction of the torque is thus the same as that of the angular momentum it produces.
- The gyroscope precesses around a vertical axis, since the torque is always horizontal and perpendicular to L. If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque (L = ΔL), and it rotates about a horizontal axis, falling over just as we would expect.
- Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star.