1.3 The Language of Physics: Physical Quantities and Units

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Associate physical quantities with their International System of Units (SI)and perform conversions among SI units using scientific notation
- Relate measurement uncertainty to significant figures and apply the rules for using significant figures in calculations
- Correctly create, label, and identify relationships in graphs using mathematical relationships (e.g., slope, y-intercept, inverse, quadratic and logarithmic)

Teacher Support

Teacher Support The learning objectives in this section will help your students master the following standards:

- (2) Scientific processes. The student uses a systematic approach to answer scientific laboratory and field investigative questions. The student is expected to
  - (H) make measurements with accuracy and precision and record data using scientific notation and International System (SI) units;
  - (L) express and manipulate relationships among physical variables quantitatively, including the use of graphs, charts, and equations.

In addition, the High School Physics Laboratory Manual addresses content in this section in the lab titled: Measurement, Precision and Accuracy, as well as the following standards:

- (2) Scientific processes. The student uses a systematic approach to answer scientific laboratory and field investigative questions. The student is expected to:
  - (H) make measurements with accuracy and precision and record data using scientific notation and International System (SI) units;
  - (I) identify and quantify causes and effects of uncertainties in measured data;
  - (J) organize and evaluate data and make inferences from data, including the use of tables, charts, and graphs.

Section Key Terms

Teacher Support

Teacher Support [OL]Pre-assessment for this section could involve asking students what experience they have had with the four fundamental units in their daily lives. One could also poll the class for what they think accuracy, precision, and uncertainty refer to. For graphing, students could make a quick graph of some data and then edit their graph after reading to note ways they could improve the clarity of their graph.

The Role of Units

Physicists, like other scientists, make observations and ask basic questions. For example, how big is an object? How much mass does it have? How far did it travel? To answer these questions, they make measurements with various instruments (e.g., meter stick, balance, stopwatch, etc.).

The measurements of physical quantities are expressed in terms of units, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in meters (for sprinters) or kilometers (for long distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way (Figure 1.13).

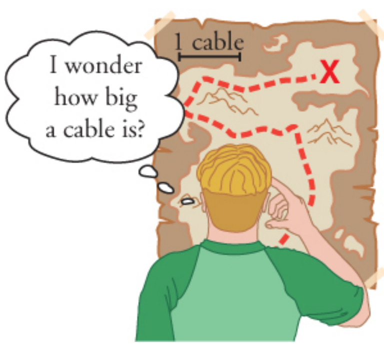


Figure 1.13 Distances given in unknown units are maddeningly useless.

All physical quantities in the International System of Units (SI) are expressed in terms of combinations of seven fundamental physical units, which are units for: length, mass, time, electric current, temperature, amount of a substance, and luminous intensity.

SI Units: Fundamental and Derived Units

In any system of units, the units for some physical quantities must be defined through a measurement process. These are called the base quantities for that system and their units are the system's base units. All other physical quantities can then be expressed as algebraic combinations of the base quantities. Each of these physical quantities is then known as a derived quantity and each unit is called a derived unit. The choice of base quantities is somewhat arbitrary, as long as they are independent of each other and all other quantities can be derived from them. Typically, the goal is to choose physical quantities that can be measured accurately to a high precision as the base quantities. The reason for this is simple. Since the derived units can be expressed as algebraic combinations of the base units, they can only be as accurate and precise as the base units from which they are derived.

Teacher Support

Teacher Support [OL]As a clarification, certain countries use the British system for a few of their measurements. For example, Britain still uses the pint to measure beer, miles to measure road distances, and pounds to measure body weight (although weight must be reported in kg in British medical records). The British people still use the British system extensively in their everyday lives, but the metric system is the official standard for the government. Likewise, many oil-producing countries measure oil in British gallons.

Based on such considerations, the International Standards Organization recommends using seven base quantities, which form the International System of Quantities (ISQ). These are the base quantities used to define the SI base units. (Table 1.1) lists these seven ISQ base quantities and the corresponding SI base units.

Table 1.1 SI Base Units

The Meter The SI unit for length is the **meter** (m). The definition of the meter has changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar. (The bar is now housed at the International Bureau of Weights and Meaures, near Paris). By 1960, some distances could be measured more precisely by comparing them to wavelengths of light. The meter was redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition as the distance light travels in a vacuum in 1/ 299,792,458 of a second (Figure 1.14).

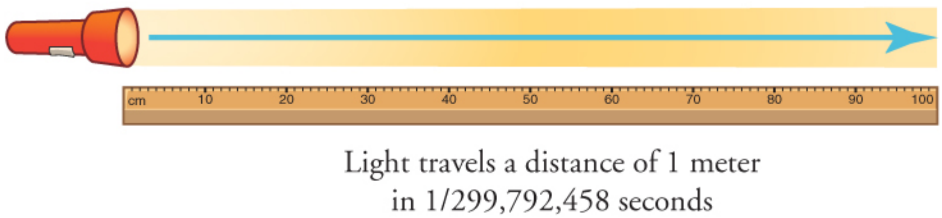


Figure 1.14 The meter is defined to be the distance light travels in 1/299,792,458 of a second through a vacuum. Distance traveled is speed multiplied by time.

The Kilogram The SI unit for mass is the **kilogram** (abbreviated kg); it was previously defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the previously defined kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses could be ultimately traced to a comparison with the standard mass. Even though the platinum-iridium cylinder was resistant to corrosion, airborne contaminants were able to adhere to its surface, slightly changing its mass over time. In May 2019, the scientific community adopted a more stable definition of the kilogram. The kilogram is now defined in terms of the second, the meter, and Planck's constant, h (a quantum mechanical value that relates a photon's energy to its frequency).

The Second The SI unit for time, the **second** (s) also has a long history. For many years it was defined as 1/86,400 of an average solar day. However, the average solar day is actually very gradually getting longer due to gradual slowing of Earth's rotation. Accuracy in the fundamental units is essential, since all other measurements are derived from them. Therefore, a new standard was adopted to define the second in terms of a non-varying, or constant, physical phenomenon. One constant phenomenon is the very steady vibration of Cesium atoms, which can be observed and counted. This vibration forms the basis of the cesium atomic clock. In 1967, the second was redefined as the time required for 9,192,631,770 Cesium atom vibrations (Figure 1.15).

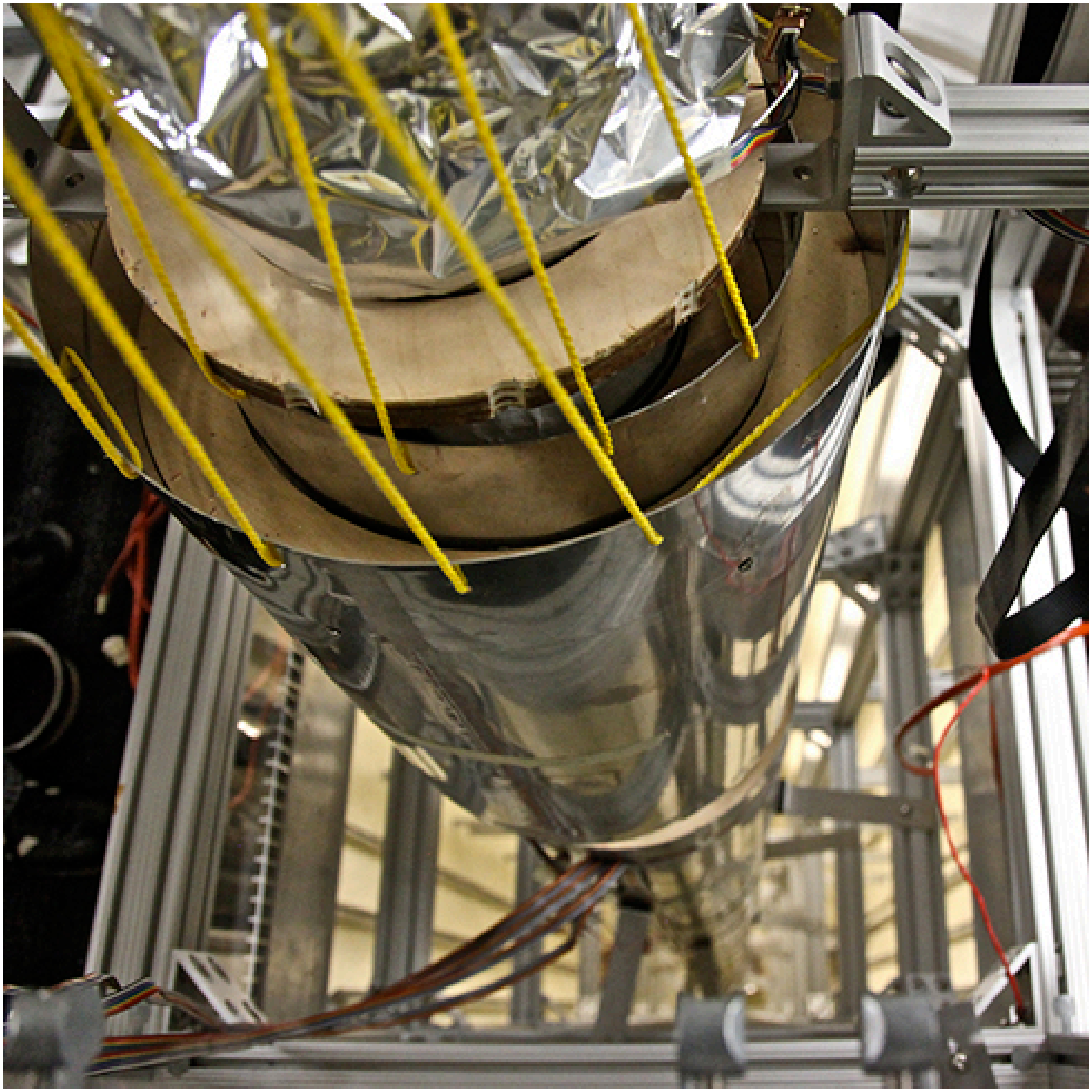


Figure 1.15 An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of one microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic clock. (Steve Jurvetson/Flickr)

Teacher Support

Teacher Support [BL]An average solar day was used to originally define the second because the length of a solar day varies throughout the year due to Earth's tilt of its axis as well as its elliptical orbit. The accumulation of these variations could result in a day length difference of up to 16 minutes during different seasons. Using an average solar day resolves these variations in day length.

The Ampere Electric current is measured in the **ampere** (A), named after Andre Ampere. You have probably heard of amperes, or *amps*, when people discuss electrical currents or electrical devices. Understanding an ampere requires a basic understanding of electricity and magnetism, something that will be explored in depth in later chapters of this book. Basically, two parallel wires with an electric current running through them will produce an attractive force on each other. One ampere is defined as the amount of electric current that will produce an attractive force of 2.7×10^{-7} newton per meter of separation between the two wires (the newton is the derived unit of force).

Teacher Support

Teacher Support [BL]Some students may not know that a vacuum is a region of space that contains no air.

Kelvins The SI unit of temperature is the kelvin (or kelvins, but not degrees kelvin). This scale is named after physicist William Thomson, Lord Kelvin, who was the first to call for an absolute temperature scale. The Kelvin scale is based on absolute zero. This is the point at which all thermal energy has been removed from all atoms or molecules in a system. This temperature, 0 K, is equal to −273.15 °C and −459.67 °F. Conveniently, the Kelvin scale actually changes in the same way as the Celsius scale. For example, the freezing point (0 °C) and boiling points of water (100 °C) are 100 degrees apart on the Celsius scale. These two temperatures are also 100 kelvins apart (freezing point = 273.15 K; boiling point = 373.15 K).

Metric Prefixes Physical objects or phenomena may vary widely. For example, the size of objects varies from something very small (like an atom) to something very large (like a star). Yet the standard metric unit of length is the meter. So, the metric system includes many prefixes that can be attached to a unit. Each prefix is based on factors of 10 (10, 100, 1,000, etc., as well as 0.1, 0.01, 0.001, etc.). Table 1.2 gives the metric prefixes and symbols used to denote the different various factors of 10 in the metric system.

Table 1.2 Metric Prefixes for Powers of 10 and Their Symbols [1]See Appendix A for a discussion of powers of 10.
Note—Some examples are approximate.

The metric system is convenient because conversions between metric units can be done simply by moving the decimal place of a number. This is because the metric prefixes are sequential powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as U.S. customary units, the relationships are less simple—there are 12 inches in a foot, 5,280 feet in a mile, 4 quarts in a gallon, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by switching to the most-appropriate metric prefix. For example, distances in meters are suitable for building construction, but kilometers are used to describe road construction. Therefore, with the metric system, there is no need to invent new units when measuring very small or very large objects—you just have to move the decimal point (and use the appropriate prefix).

Known Ranges of Length, Mass, and Time Table 1.3 lists known lengths, masses, and time measurements. You can see that scientists use a range of measurement units. This wide range demonstrates the vastness and complexity of the universe, as well as the breadth of phenomena physicists study. As you examine this table, note how the metric system allows us to discuss and compare an enormous range of phenomena, using one system of measurement (Figure 1.16 and Figure 1.17).

Table 1.3 Approximate Values of Length, Mass, and Time [1] More precise values are in parentheses.

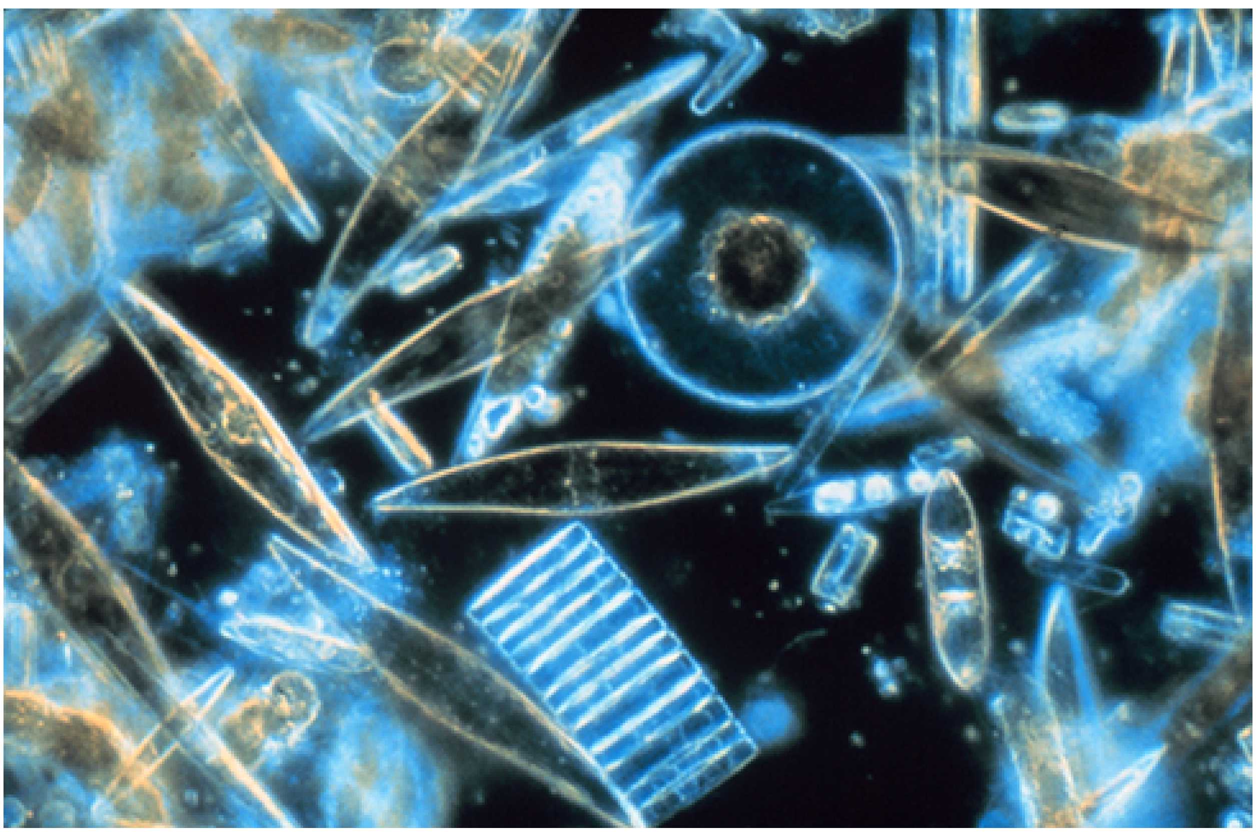


Figure 1.16 Tiny phytoplankton float among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (Prof.

Gordon T. Taylor, Stony Brook University; NOAA Corps Collections)

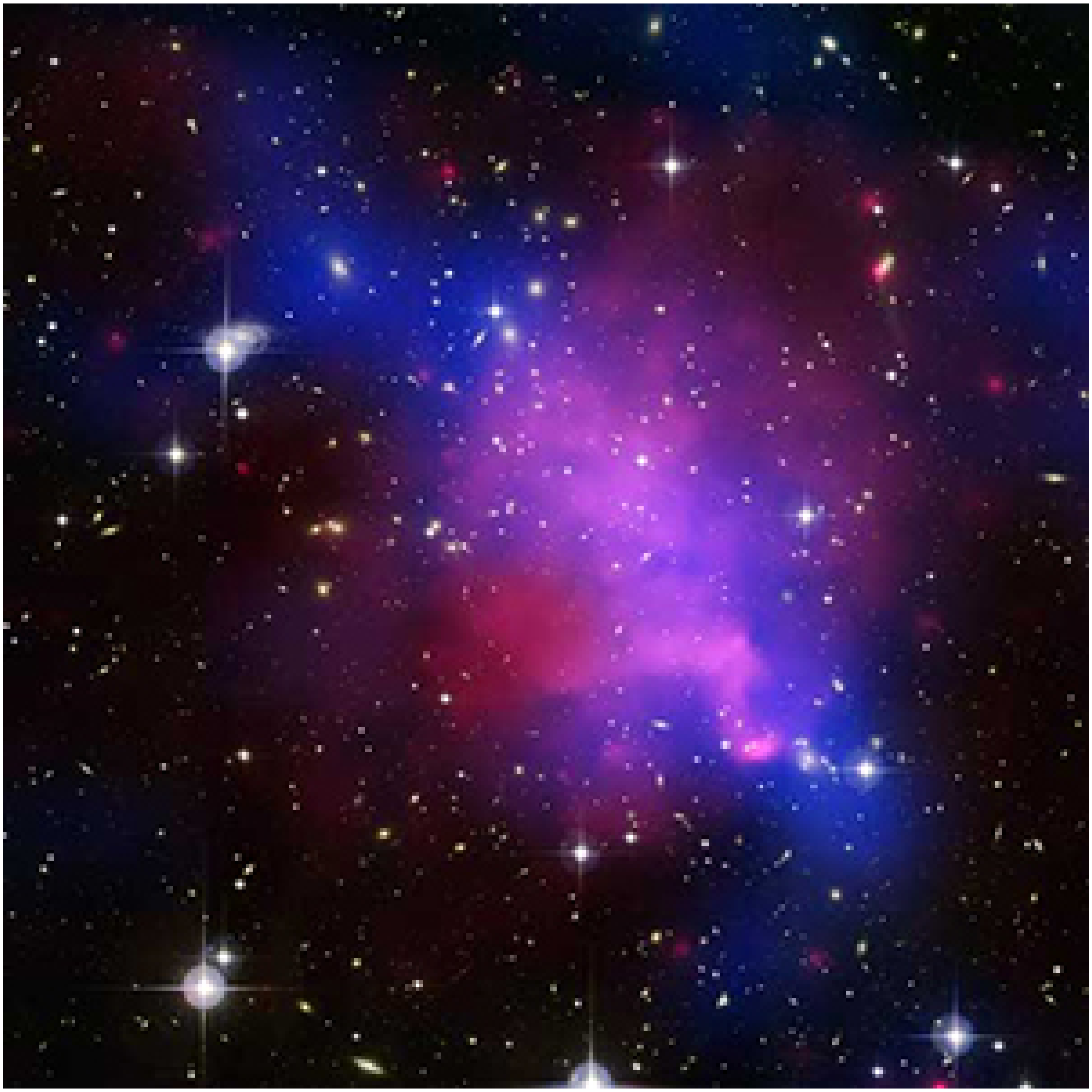


Figure 1.17 Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.)

Using Scientific Notation with Physical Measurements

Scientific notation is a way of writing numbers that are too large or small to be conveniently written as a decimal. For example, consider the number 840,000,000,000,000. It's a rather large number to write out. The scientific notation for this number is 8.40×10^{14}. Scientific notation follows this general format

$x \quad \times \quad 10^y$.

In this format x is the value of the measurement with all placeholder zeros removed. In the example above, x is 8.4. The x is multiplied by a factor, 10^y, which indicates the number of placeholder zeros in the measurement. Placeholder zeros are those at the end of a number that is 10 or greater, and at the beginning of a decimal number that is less than 1. In the example above, the factor is 10^{14}. This tells you that you should move the decimal point 14 positions to the right, filling in placeholder zeros as you go. In this case, moving the decimal point 14 places creates only 13 placeholder zeros, indicating that the actual measurement value is 840,000,000,000,000.

Numbers that are fractions can be indicated by scientific notation as well. Consider the number 0.0000045. Its scientific notation is 4.5×10^{-6}. Its scientific notation has the same format

$x \quad \times \quad 10^y$.

Here, x is 4.5. However, the value of y in the 10^y factor is negative, which indicates that the measurement is a fraction of 1. Therefore, we move the decimal place to the left, for a negative y. In our example of 4.5×10^{-6}, the decimal point would be moved to the left six times to yield the original number, which would be 0.0000045.

The term order of magnitude refers to the power of 10 when numbers are expressed in scientific notation. Quantities that have the same power of 10 when expressed in scientific notation, or come close to it, are said to be of the same order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Both numbers have the same value for y. Therefore, 800 and 450 are of the same order of magnitude. Similarly, 101 and 99 would be regarded as the same order of magnitude, 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of 10^{-9} m, while the diameter of the sun is on the order of 10^9 m. These two values are 18 orders of magnitude apart.

Scientists make frequent use of scientific notation because of the vast range of physical measurements possible in the universe, such as the distance from Earth to the moon (Figure 1.18), or to the nearest star.

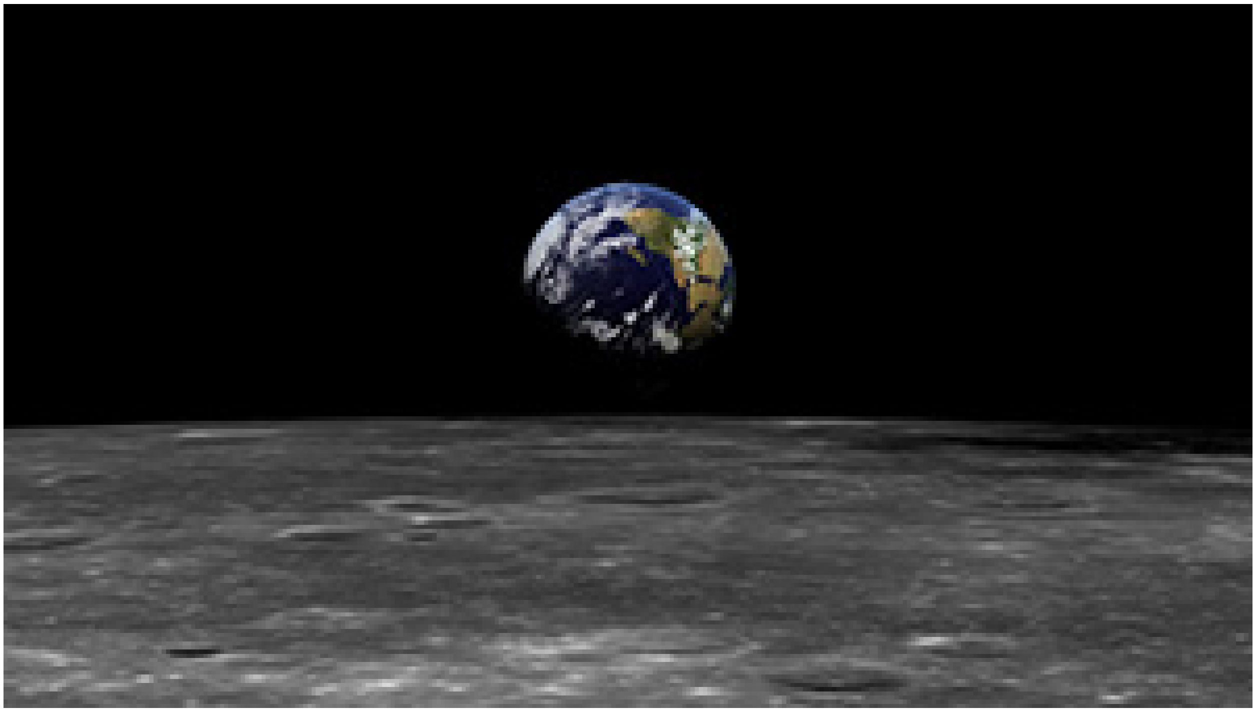


Figure 1.18 The distance from Earth to the moon may seem immense, but it is just a tiny fraction of the distance from Earth to our closest neighboring star. (NASA)

Unit Conversion and Dimensional Analysis It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook in the United States, some quantities may be expressed in liters and you need to convert them to cups. A Canadian tourist driving through the United States might want to convert miles to kilometers, to have a sense of how far away his next destination is. A doctor in the United States might convert a patient's weight in pounds to kilograms.

Let's consider a simple example of how to convert units within the metric system. How can we convert 1 hour to seconds?

First, we need to determine a conversion factor. A conversion factor is a ratio expressing how many of one unit are equal to another unit. A conversion factor is simply a fraction which equals 1. You can multiply any number by 1 and get the same value. When you multiply a number by a conversion factor, you are simply multiplying it by one. For example, the following are conversion factors: (1 foot)/(12 inches) = 1 to convert inches to feet, (1 meter)/(100 centimeters) = 1 to convert centimeters to meters, (1 minute)/(60 seconds) = 1 to convert seconds to minutes.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor (1 km/1,000m) = 1, so we are simply multiplying 80m by 1:

$1\ \text{h} \times \frac{60\ \text{min}}{1\ \text{h}} \times \frac{60\ \text{s}}{1\ \text{min}}$=3600 s = 3.6×10^3 s

1.1

When there is a unit in the original number, and a unit in the denominator (bottom) of the conversion factor, the units cancel. In this case, hours and minutes cancel and the value in seconds remains.

You can use this method to convert between any types of unit, including between the U.S. customary system and metric system. Notice also that, although you can multiply and divide units algebraically, you cannot add or subtract different units. An expression like *10 km + 5 kg* makes no sense. Even adding two lengths in different units, such as *10 km + 20 m* does not make sense. You express both lengths in the same unit. See Appendix C for a more complete list of conversion factors.

Worked Example

Unit Conversions: A Short Drive Home Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note—Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

1. Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

- average speed = $\frac{\text{distance}}{\text{time}}$.

2. Substitute the given values for distance and time.

- average speed = $\frac{10.0\ \text{km}}{20.0\ \text{min}} = 0.500\ \frac{\text{km}}{\text{min}}$

3. Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/1h . Thus,

- average speed = $0.500\ \frac{\text{km}}{\text{min}} \times \frac{60\ \text{min}}{1\ \text{h}} = 30.0\ \frac{\text{km}}{\text{h}}$.

Discussion for (a)

To check your answer, consider the following:

1. Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows

- $\frac{\text{km}}{\text{min}} \times \frac{1\text{ hr}}{60\text{ min}} = \frac{1}{60}\frac{\text{km} \cdot \text{h}}{\text{min}^2}$,

  which are obviously not the desired units of km/h.

2. Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.
3. Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/h does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 min, so the precision of the conversion factor is perfect.
4. Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution (b)

There are several ways to convert the average speed into meters per second.

1. Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

2. Multiplying by these yields

- $\text{Averagespeed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1\text{ h}}{3{,}600\text{ s}} \times \frac{1{,}000\text{ m}}{1\text{ km}}$

  $\text{Averagespeed} = 8.33\frac{\text{m}}{\text{s}}$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces?

Worked Example

Using Physics to Evaluate Promotional Materials A commemorative coin that is 2 in diameter is advertised to be plated with 15 mg of gold. If the density of gold is 19.3 g/cc, and the amount of gold around the edge of the coin can be ignored, what is the thickness of the gold on the top and bottom faces of the coin?

Strategy

To solve this problem, the volume of the gold needs to be determined using the gold's mass and density. Half of that volume is distributed on each face of the coin, and, for each face, the gold can be represented as a cylinder that is 2 in diameter with a height equal to the thickness. Use the volume formula for a cylinder to determine the thickness.

Solution

The mass of the gold is given by the formula

$m = \rho V = 15 \times 10^{-3}$ g,

where

$\rho = 19.3$ g/cc

and V is the volume. Solving for the volume gives

$V = \frac{m}{\rho} = \frac{15 \times 10^{-3}\text{ g}}{19.3\text{ g/cc}} \cong 7.8 \times 10^{-4}$ cc.

If t is the thickness, the volume corresponding to half the gold is

$\frac{1}{2}\left(7.8 \times 10^{-4}\right) = \pi r^2 t = \pi (2.54)^2 t,$

where the 1 radius has been converted to cm. Solving for the thickness gives

$t = \frac{(3.9 \times 10^{-4})}{\pi (2.54)^2} \cong 1.9 \times 10^{-5}$ cm $= 0.00019\ mm.$

Discussion

The amount of gold used is stated to be 15 mg, which is equivalent to a thickness of about 0.00019 mm. The mass figure may make the amount of gold sound larger, both because the number is much bigger (15 versus 0.00019), and because people may have a more intuitive feel for how much a millimeter is than for how much a milligram is. A simple analysis of this sort can clarify the significance of claims made by advertisers.

Teacher Support

Teacher Support Ask students to find other promotional materials that make claims that can be analyzed using physics principles. Compile any items that come in for later use at appropriate points in the course. For example, after covering power consumption in electric circuits, compare the performance of electric fireplaces advertised as *revolutionary* to the performance of standard space heaters.

Accuracy, Precision and Significant Figures

Science is based on experimentation that requires good measurements. The validity of a measurement can be described in terms of its accuracy and its precision (see Figure 1.19 and Figure 1.20). Accuracy is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard piece of printer paper. The packaging in which you purchased the paper states that it is 11 inches long, and suppose this stated value is correct. You measure the length of the paper three times and obtain the following measurements: 11.1 inches, 11.2 inches, and 10.9 inches. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate. This is why measuring instruments are calibrated based on a known measurement. If the instrument consistently returns the correct value of the known measurement, it is safe for use in finding unknown values.

Figure 1.19 A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects

of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The known masses are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (Serge Melki)

Figure 1.20 Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, some digital scales can measure the mass of an object up to the nearest thousandth of a gram. As in other measuring devices, the precision of a scale is limited to the last measured figures. This is the hundredths place in the scale pictured here. (Splarka, Wikimedia Commons)

Precision states how well repeated measurements of something generate the same or similar results. Therefore, the precision of measurements refers to how close together the measurements are when you measure the same thing several times. One way to analyze the precision of measurements would be to determine the range, or difference between the lowest and the highest measured values. In the case of the printer paper measurements, the lowest value was 10.9 inches and the highest value was 11.2 inches. Thus, the measured values deviated from each other by, at most, 0.3 inches. These measurements were reasonably precise because they varied by only a fraction of an inch. However, if the measured values had been 10.9 inches, 11.1 inches, and 11.9 inches, then the measurements would not be very precise because there is a lot of variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target. Then think of each GPS attempt to locate the restaurant as a black dot on the bull's eye.

In Figure 1.21, you can see that the GPS measurements are spread far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 1.22, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system. Finally, in Figure 1.23, the GPS is both precise and accurate, allowing the restaurant to be located.

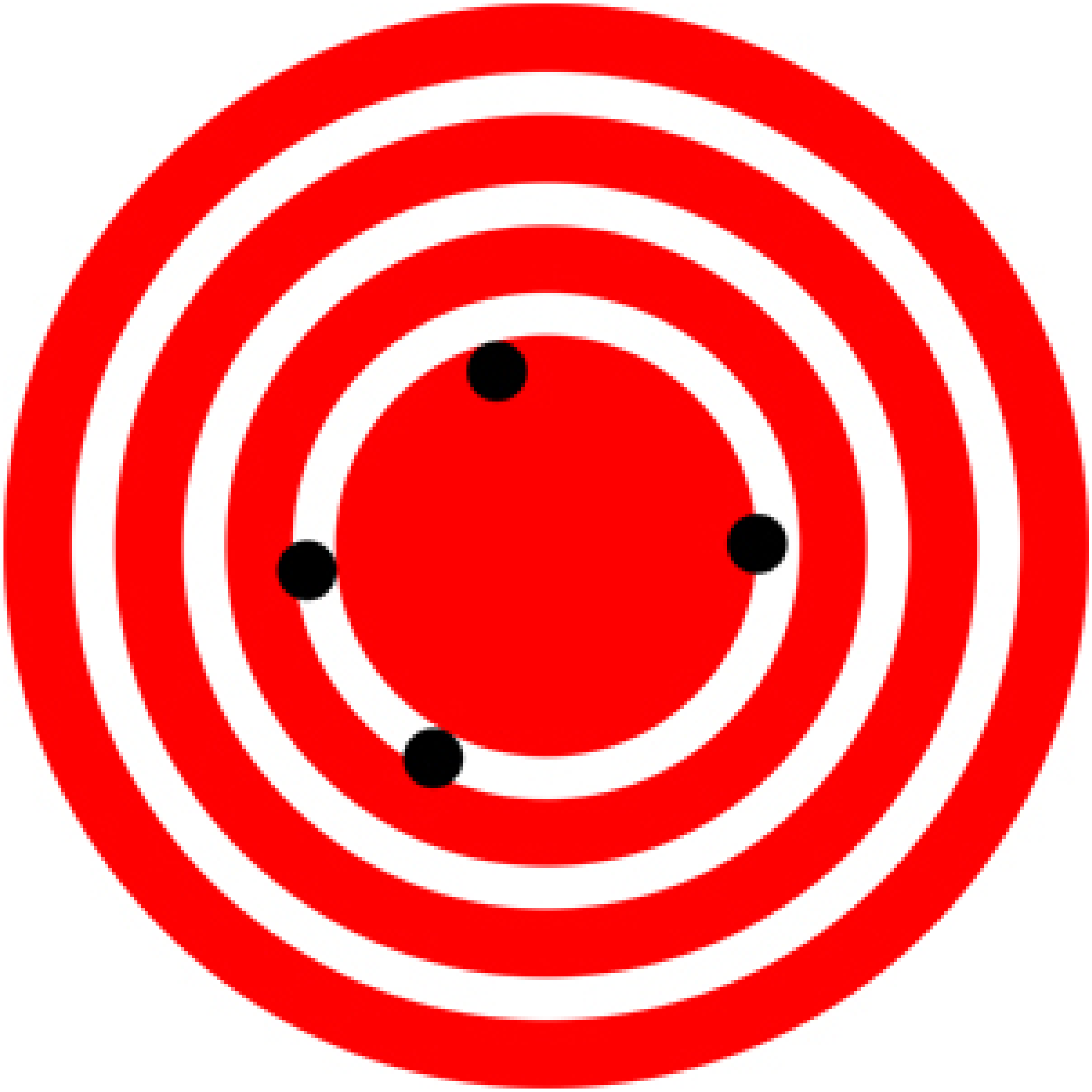


Figure 1.21 A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (Dark Evil)

Figure 1.22 In this figure, the dots are concentrated close to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (Dark Evil)

Figure 1.23 In this figure, the dots are concentrated close to one another, indicating high precision, and they are very close to the actual location of the restaurant, indicating high accuracy. (Dark Evil)

Uncertainty The accuracy and precision of a measuring system determine the uncertainty of its measurements. Uncertainty is a way to describe how much your measured value deviates from the actual value that the object has. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 inches plus or minus 0.2 inches or 11.0 ± 0.2 inches. The uncertainty in a measurement, A, is often denoted as A (”delta A”),

The factors contributing to uncertainty in a measurement include the following:

1. Limitations of the measuring device
2. The skill of the person making the measurement
3. Irregularities in the object being measured
4. Any other factors that affect the outcome (highly dependent on the situation)

In the printer paper example uncertainty could be caused by: the fact that the smallest division on the ruler is 0.1 inches, the person using the ruler has bad eyesight, or uncertainty caused by the paper cutting machine (e.g., one side of the paper is slightly longer than the other.) It is good practice to carefully consider all possible sources of uncertainty in a measurement and reduce or eliminate them,

Percent Uncertainty One method of expressing uncertainty is as a percent of the measured value. If a measurement, A, is expressed with uncertainty, A, the percent uncertainty is

$\%$ uncertainty $= \frac{\delta \mathrm{A}}{\mathrm{A}} \times 100\%$.

1.2

Worked Example

Calculating Percent Uncertainty: A Bag of Apples A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: 4. 8lb
- Week 2 weight: 5.3 lb
- Week 3 weight: 4. 9lb
- Week 4 weight: 5.4 lb

You determine that the weight of the 5 lb bag has an uncertainty of ± 0.4 lb. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight

$\%$ uncertainty $= \frac{\delta \mathrm{A}}{\mathrm{A}} \times 100\%$.

Solution

Plug the known values into the equation

$\%$ uncertainty $= \frac{0.4 \ \mathrm{lb}}{5 \ \mathrm{lb}} \times 100\% = 8\%$.

Discussion

We can conclude that the weight of the apple bag is 5 lb $\pm$ 8 percent. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100 percent. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainty in Calculations There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the both the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements in the calculation have small uncertainties (a few percent or less), then the method of adding percents can be used. This method says that the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2 percent and 1 percent, respectively, then the area of the floor is 12.0 m^2 and has an uncertainty of 3 percent (expressed as an area this is 0.36 m^2, which we round to 0.4 m^2 since the area of the floor is given to a tenth of a square meter).

For more information on the accuracy, precision, and uncertainty of measurements based upon the units of measurement, visit this website.

Precision of Measuring Tools and Significant Figures An important factor in the accuracy and precision of measurements is the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, consider measuring the thickness of a coin. A standard ruler can measure thickness to the nearest millimeter, while a micrometer can measure the thickness to the nearest 0.005 millimeter. The micrometer is a more precise measuring tool because it can measure extremely small differences in thickness. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool (such as the rulers shown in Figure 1.24). For example, if you use a standard ruler to measure the length of a stick, you may measure it with a decimeter ruler as 3.6 cm. You could not express this value as 3.65 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36 mm and 37 mm. He or she must estimate the value of the last digit. The rule is that the last digit written down in a measurement is the first digit with some uncertainty. For example, the last measured value 36.5 mm has three digits, or three significant figures. The number of significant figures in a measurement indicates the precision of the measuring tool. The more precise a measuring tool is, the greater the number of significant figures it can report.

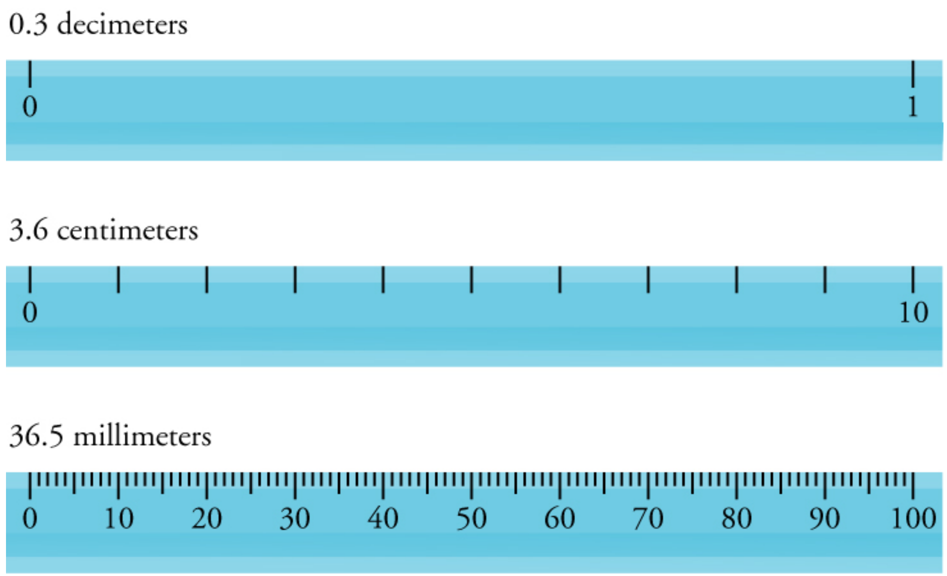


Figure 1.24 Three metric rulers are shown. The first ruler is in decimeters and can measure point three decimeters. The second ruler is in centimeters long and can measure three point six centimeters. The last ruler is in millimeters and can measure thirty-six point five millimeters.

Zeros Special consideration is given to zeros when counting significant figures. For example, the zeros in 0.053 are not significant because they are only placeholders that locate the decimal point. There are two significant figures in 0.053—the 5 and the 3. However, if the zero occurs between other significant figures, the zeros are significant. For example, both zeros in 10.053 are significant, as these zeros were actually measured. Therefore, the 10.053 placeholder has five significant figures. The zeros in 1300 may or may not be significant, depending on the style of writing numbers. They could mean the number is known to the last zero, or the zeros could be placeholders. So 1300 could have two, three, or four significant figures. To avoid this ambiguity, write 1300 in scientific notation as 1.3×10^3. Only significant figures are given in the x factor for a number in scientific notation (in the form $x \times 10^y$). Therefore, we know that 1 and 3 are the only significant digits in this number. In summary, zeros are significant except when they serve only as placeholders. Table 1.4 provides examples of the number of significant figures in various numbers.

Table 1.4

Significant Figures in Calculations When combining measurements with different degrees of accuracy and precision, the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value. There are two different rules, one for multiplication and division and another rule for addition and subtraction, as discussed below.

1. **For multiplication and division:** The answer should have the same number of significant figures as the starting value with the fewest significant figures. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area will have if the radius has only two significant figures, for example, $r =$ 2.0 m. Then, using a calculator that keeps eight significant figures, you would get

- $A = \ r^2 = \ (3.1415927...) \ \times \ (2.0 \ \ \text{m})^2 = \ 4.5238934 \text{ m}^2$.

  But because the radius has only two significant figures, the area calculated is meaningful only to two significant figures or

  $A = \ 4.5 \text{ m}^2$

  even though the value of π is meaningful to at least eight digits.

2. **For addition and subtraction**: The answer should have the same number places (e.g. tens place, ones place, tenths place, etc.) as the least-precise starting value. Suppose that you buy 7.56 kg of potatoes in a

grocery store as measured with a scale having a precision of 0.01 kg. Then you drop off 6.052 kg of potatoes at your laboratory as measured by a scale with a precision of 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with a precision of 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$\begin{array}{rl} 7.56 & \text{kg} \\ -6.052 & \text{kg} \\ \underline{+13.7} & \underline{\text{kg}} \end{array}$$

- $15.208 \quad \text{kg}$

The least precise measurement is 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer should be rounded to the tenths place, giving 15.2 kg. The same is true for non-decimal numbers. For example, $6527.23 + 2 = 6529.23 = 6529$.
We cannot report the decimal places in the answer because *2* has no decimal places that would be significant. Therefore, we can only report to the ones place.
It is a good idea to keep extra significant figures while calculating, and to round off to the correct number of significant figures only in the final answers. The reason is that small errors from rounding while calculating can sometimes produce significant errors in the final answer. As an example, try calculating $5,098 - (5.000) \times (1,010)$ to obtain a final answer to only two significant figures. Keeping all significant during the calculation gives 48. Rounding to two significant figures in the middle of the calculation changes it to 5,100 – (5.000) × (1,000) = 100, which is way off. You would similarly avoid rounding in the middle of the calculation in counting and in doing accounting, where many small numbers need to be added and subtracted accurately to give possibly much larger final numbers.

Teacher Support

Teacher Support Remind students that they will be expected to report the proper number of significant figures on assignment and test problems.

Significant Figures in this Text In this textbook, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, such as optics, more than three significant figures will be used. Finally, if a number is exact, such as the *2* in the formula, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

Worked Example

Approximating Vast Numbers: a Trillion Dollars The U.S. federal deficit in the 2008 fiscal year was a little greater than $10 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks, like that shown in Figure 1.25, and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Figure 1.25 A bank stack contains one hundred $100 bills, and is worth $10,000. How many bank stacks make up a trillion dollars? (Andrew Magill)

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

1. Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is

$$\text{volume of stack} = \text{length} \times \text{width} \times \text{height},$$

$$\text{volume of stack} = 6 \text{ in.} \times 3 \text{ in.} \times 0.5 \text{ in.},$$

- $$\text{volume of stack} = 9 \text{ in.}^3.$$

2. Calculate the number of stacks. Note that a trillion dollars is equal to $\$1 \times 10^{12}$, and a stack of one-hundred $100 bills is equal to $\$10,000$, or $\$1 \times 10^4$. The number of stacks you will have is

- $\$1 \times 10^{12}$ (a trillion dollars) / $\$1 \times 10^4$ per stack $= 1 \times 10^8$ stacks.

1.3

3. Calculate the area of a football field in square inches. The area of a football field is $100 \text{ yd} \times 50\text{yd}$, which gives $5,000 \text{ yd}^2$. Because we are working in inches, we need to convert square yards to square inches

$$\text{Area} = 5{,}000 \text{ yd}^2 \times \frac{3 \text{ ft}}{1\text{yd}} \times \frac{3 \text{ ft}}{1\text{yd}} \times \frac{12 \text{ in.}}{1 \text{ foot}} \times \frac{12 \text{ in.}}{1 \text{ foot}} = 6{,}480{,}000 \text{ in.}^2,$$

- $$\text{Area} \approx 6 \times 10^6 \text{ in.}^2.$$

This conversion gives us $6 \times 10^6 \text{ in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

4. Calculate the total volume of the bills. The volume of all the $100-bill stacks is

- 9 in.^3 / stack $\times 10^8$ stacks $= 9 \times 10^8 \text{ in.}^3$

5. Calculate the height. To determine the height of the bills, use the following equation

$$\begin{aligned} \text{volume of bills} &= \text{area of field} \times \text{height of money} \\ \text{Height of money} &= \frac{\text{volume of bills}}{\text{area of field}} \\ \text{Height of money} &= \frac{9 \times 10^8 \text{ in.}^3}{6 \times 10^6 \text{ in.}^2} = 1.33 \times 10^2 \text{ in.} \end{aligned}$$

- $$\text{Height of money} = 1 \times 10^2 \text{ in.} = 100 \text{ in.}$$

The height of the money will be about 100 in. high. Converting this value to feet gives
$100 \text{ in.} \times \frac{1\text{ft}}{12 \text{ in.}} = 8.33 \text{ ft} \approx 8 \text{ ft.}$

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough *guesstimates* versus carefully calculated approximations?

In the example above, the final approximate value is much higher than the first friend's early estimate of 3 in. However, the other friend's early estimate of 10 ft. (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise suggest about the value of rough *guesstimates* versus carefully calculated approximations?

Teacher Support

Teacher Support In [link], point out to students the importance of precision in their measurements. Greater precision allows measurements to be less uncertain, and therefore, a close approximation rather than a *guesstimate.*

Graphing in Physics

Most results in science are presented in scientific journal articles using graphs. Graphs present data in a way that is easy to visualize for humans in general, especially someone unfamiliar with what is being studied. They are also useful for presenting large amounts of data or data with complicated trends in an easily-readable way.

One commonly-used graph in physics and other sciences is the line graph, probably because it is the best graph for showing how one quantity changes in response to the other. Let's build a line graph based on the data in Table 1.5, which shows the measured distance that a train travels from its station versus time. Our two variables, or things that change along the graph, are time in minutes, and distance from the station, in kilometers. Remember that measured data may not have perfect accuracy.

Table 1.5

1. Draw the two axes. The horizontal axis, or x-axis, shows the independent variable, which is the variable that is controlled or manipulated. The vertical axis, or y-axis, shows the dependent variable, the non-manipulated variable that changes with (or is dependent on) the value of the independent variable. In the data above, time is the independent variable and should be plotted on the x-axis. Distance from the station is the dependent variable and should be plotted on the y-axis.

2. Label each axes on the graph with the name of each variable, followed by the symbol for its units in parentheses. Be sure to leave room so that you can number each axis. In this example, use *Time (min)* as the label for the x-axis.

3. Next, you must determine the best scale to use for numbering each axis. Because the time values on the x-axis are taken every 10 minutes, we could easily number the x-axis from 0 to 70 minutes with a tick mark every 10 minutes. Likewise, the y-axis scale should start low enough and continue high enough to include all of the *distance from station* values. A scale from 0 km to 160 km should suffice, perhaps with a tick mark every 10 km.

- In general, you want to pick a scale for both axes that 1) shows all of your data, and 2) makes it easy to identify trends in your data. If you make your scale too large, it will be harder to see how your data change. Likewise, the smaller and more fine you make your scale, the more space you will need to make the graph. The number of significant figures in the axis values should be coarser than the number of significant figures in the measurements.

4. Now that your axes are ready, you can begin plotting your data. For the first data point, count along the x-axis until you find the 10 min tick mark. Then, count up from that point to the 10 km tick mark on the y-axis, and approximate where 22 km is along the y-axis. Place a dot at this location. Repeat for the other six data points (Figure 1.26).

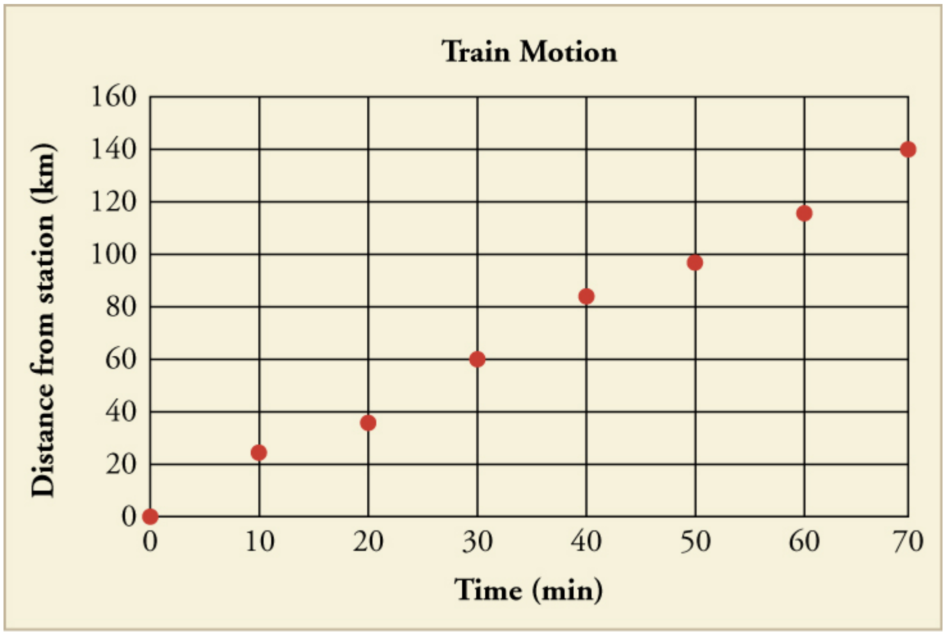
- 

  Figure 1.26 The graph of the train's distance from the station versus time from the exercise above.

5. Add a title to the top of the graph to state what the graph is describing, such as the y-axis parameter vs. the x-axis parameter. In the graph shown here, the title is *train motion*. It could also be titled distance of the train from the station vs. time.
6. Finally, with data points now on the graph, you should draw a trend line (Figure 1.27). The trend line represents the dependence you think the graph represents, so that the person who looks at your graph can see how close it is to the real data. In the present case, since the data points look like they ought to fall on a straight line, you would draw a straight line as the trend line. Draw it to come closest to all the points. Real data may have some inaccuracies, and the plotted points may not all fall on the trend line. In some cases, none of the data points fall exactly on the trend line.

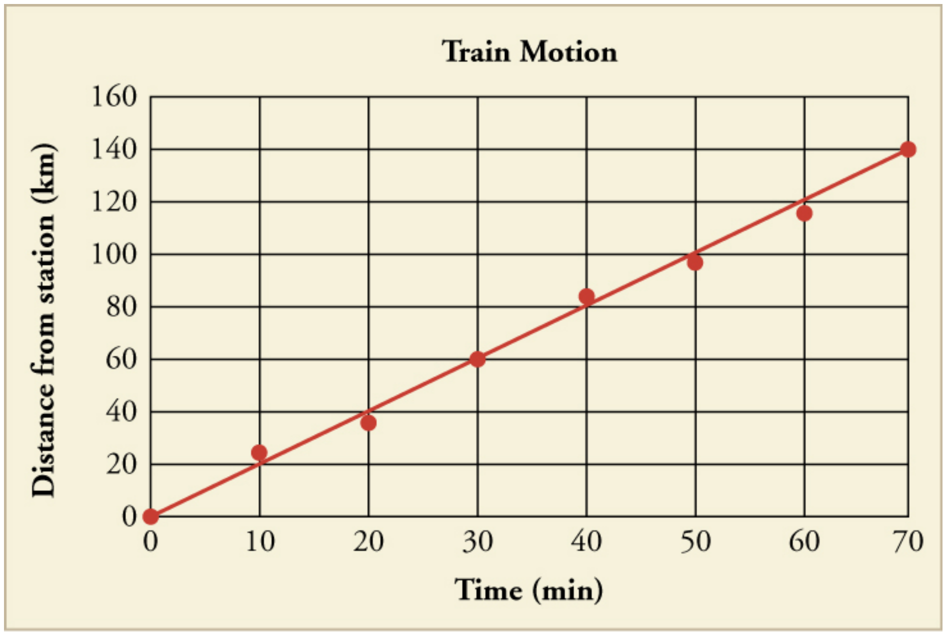
- 

Figure 1.27 The completed graph with the trend line included.

Teacher Support

Teacher Support [OL]The importance of bar graphs should also be mentioned as a useful way to show data relations when one variable is not continuous, such as in a frequency histogram, which compares how many data points fall into discrete categories.

[OL]If students have difficulty understanding the difference between dependent and independent variables in the train example, explain that time is independent because it will continue to move forward at the same rate whether the train leaves the station or not.

Analyzing a Graph Using Its Equation One way to get a quick snapshot of a dataset is to look at the equation of its trend line. If the graph produces a straight line, the equation of the trend line takes the form

$y = mx + b$.

The b in the equation is the y-intercept while the m in the equation is the slope. The y-intercept tells you at what y value the line intersects the y-axis. In the case of the graph above, the y-intercept occurs at 0, at the very beginning of the graph. The y-intercept, therefore, lets you know immediately where on the y-axis the plot line begins.

The m in the equation is the slope. This value describes how much the line on the graph moves up or down on the y-axis along the line's length. The slope is found using the following equation

$m = \frac{Y_2 - Y_1}{X_2 - X_1}$.

In order to solve this equation, you need to pick two points on the line (preferably far apart on the line so the slope you calculate describes the line accurately). The quantities Y_2 and Y_1 represent the y-values from the two points on the line (not data points) that you picked, while X_2 and X_1 represent the two x-values of the those points.

What can the slope value tell you about the graph? The slope of a perfectly horizontal line will equal zero, while the slope of a perfectly vertical line will be undefined because you cannot divide by zero. A positive slope indicates that the line moves up the y-axis as the x-value increases while a negative slope means that the line moves down the y-axis. The more negative or positive the slope is, the steeper the line moves up or down, respectively. The slope of our graph in Figure 1.26 is calculated below based on the two endpoints of the line

$$
\begin{aligned}
m &= \frac{Y_2 - Y_1}{X_2 - X_1} \\
m &= \frac{(80\text{ km}) - (20\text{ km})}{(40\text{ min}) - (10\text{ min})} \\
m &= \frac{60\text{ km}}{30\text{ min}} \\
m &= 2.0\text{ km/min.}
\end{aligned}
$$

Equation of line: $y = (2.0\ \ \text{km/min})\,x + 0$

Because the x axis is time in minutes, we would actually be more likely to use the time t as the independent (x-axis) variable and write the equation as

$y = (2.0\ \ \text{km/min})\,t + 0$.

1.4

The formula $y = mx + b$ only applies to linear relationships, or ones that produce a straight line. Another common type of line in physics is the quadratic relationship, which occurs when one of the variables is squared. One quadratic relationship in physics is the relation between the speed of an object its centripetal acceleration, which is used to determine the force needed to keep an object moving in a circle. Another common relationship in physics is the inverse relationship, in which one variable decreases whenever the other variable increases. An example in physics is Coulomb's law. As the distance between two charged objects increases, the electrical force between the two charged objects decreases. Inverse proportionality, such the relation between x and y in the equation

$y = k/x$,

for some number k, is one particular kind of inverse relationship. A third commonly-seen relationship is the exponential relationship, in which a change in the independent variable produces a proportional change in the dependent variable. As the value of the dependent variable gets larger, its rate of growth also increases. For example, bacteria often reproduce at an exponential rate when grown under ideal conditions. As each generation passes, there are more and more bacteria to reproduce. As a result, the growth rate of the bacterial population increases every generation (Figure 1.28).

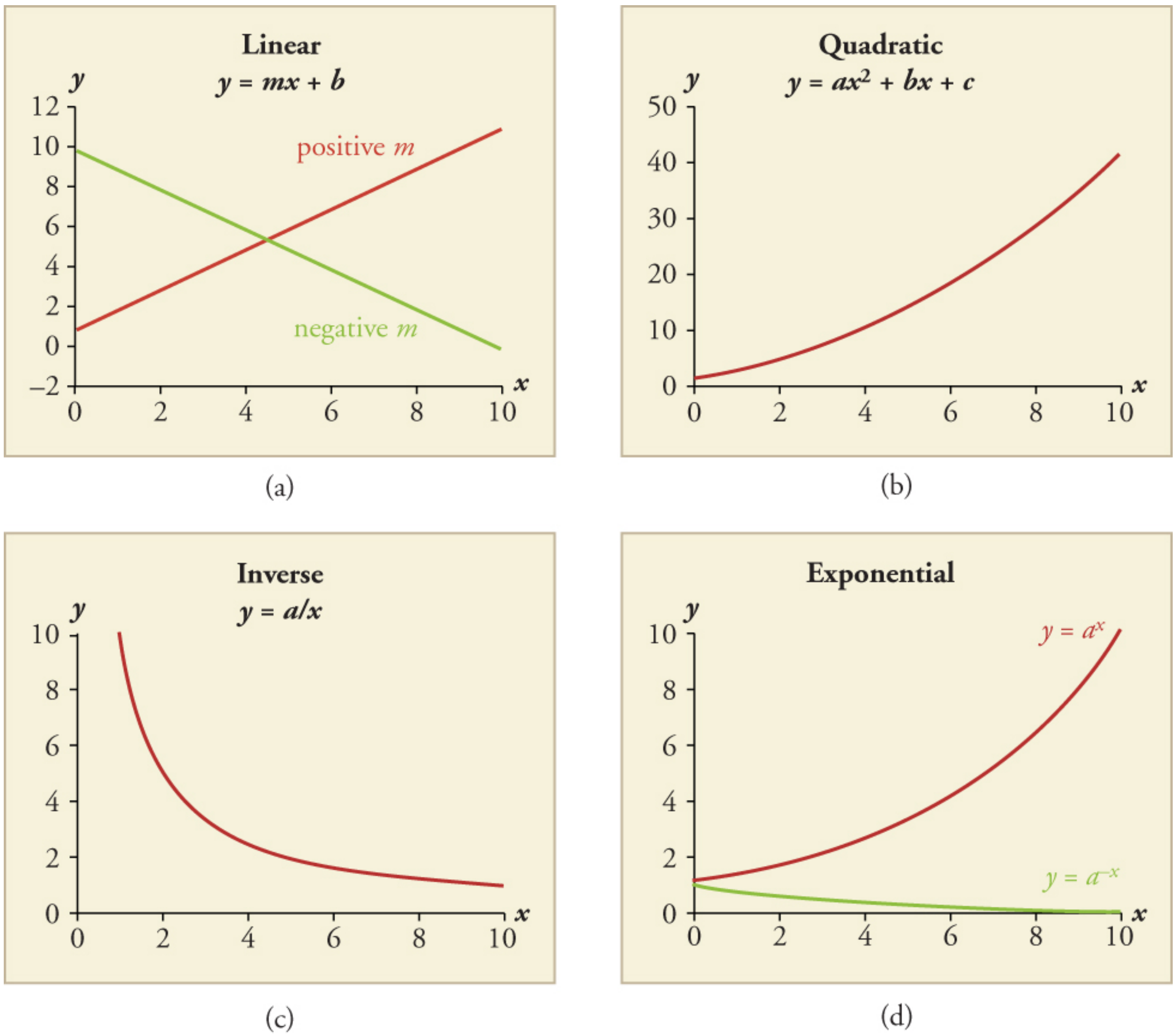
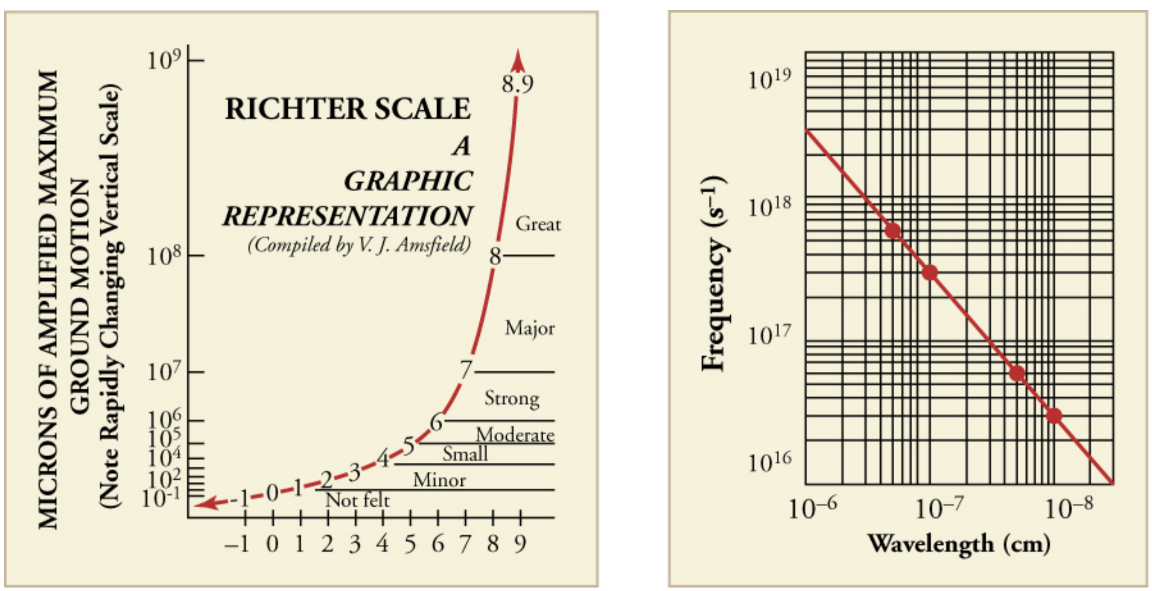


Figure 1.28 Examples of (a) linear, (b) quadratic, (c) inverse, and (d) exponential relationship graphs.

Using Logarithmic Scales in Graphing Sometimes a variable can have a very large range of values. This presents a problem when you're trying to figure out the best scale to use for your graph's axes. One option is to use a logarithmic (log) scale. In a logarithmic scale, the value each mark labels is the previous mark's value multiplied by some constant. For a log base 10 scale, each mark labels a value that is 10 times the value of the mark before it. Therefore, a base 10 logarithmic scale would be numbered: 0, 10, 100, 1,000, etc. You can see how the logarithmic scale covers a much larger range of values than the corresponding linear scale, in which the marks would label the values 0, 10, 20, 30, and so on.

If you use a logarithmic scale on one axis of the graph and a linear scale on the other axis, you are using a semi-log plot. The Richter scale, which measures the strength of earthquakes, uses a semi-log plot. The degree of ground movement is plotted on a logarithmic scale against the assigned intensity level of the earthquake, which ranges linearly from 1-10 (Figure 1.29 **(a)**).

If a graph has both axes in a logarithmic scale, then it is referred to as a log-log plot. The relationship between the wavelength and frequency of electromagnetic radiation such as light is usually shown as a log-log plot (Figure 1.29 **(b)**). Log-log plots are also commonly used to describe exponential functions, such as radioactive decay.



(a) (b)

Figure 1.29 (a) The Richter scale uses a log base 10 scale on its *y-axis* (microns of amplified maximum ground motion). (b) The relationship between the frequency and wavelength of electromagnetic radiation can be plotted as a straight line if a log-log plot is used.

Worked Example

Method of Adding Percents: Shingling Your Roof A series of shingles are used to protect the roof of a home. Using a measuring tape, you measure one shingle and find its dimensions to be 44 cm by 100 cm. Knowing that your measurements are not perfect, you estimate an uncertainty of ± 0.5 cm. Following the method of adding percents, what is the area of the shingle, including uncertainty?

Strategy

While calculating the area of the shingle is straightforward (44 cm x 100 cm = 4400 cm^2), determining the percent uncertainty is more challenging. In order to use the method of adding percents, you must first calculate the percent uncertainty of each measurement.

Solution

Length % Uncertainty: A/A x 100% = 0.5/44 x 100% = 1.1%
Width % Uncertainty: A/A x 100% = 0.5/100 x 100% = 0.5%

Adding Percents: 1.1% + 0.5% = 1.6% uncertainty

Area of the Shingle: 4400 cm^2 $\pm$ 1.6%

Note that this uncertainty can also be expressed in metric terms.

1.6% x 4400 cm^2 = 70.4 cm^2

Area of the Shingle: 4400 $\pm$ 70.4 cm^2

Discussion

Knowing the percent uncertainty of a shingle can help a contractor determine the number of shingles needed, and therefore the cost, of roofing a new home. Consider how using smaller shingles would affect this uncertainty, and what role this would play in the cost estimation process.

Virtual Physics

Graphing Lines In this simulation you will examine how changing the slope and y-intercept of an equation changes the appearance of a plotted line. Select slope-intercept form and drag the blue circles along the line to change the line's characteristics. Then, play the line game and see if you can determine the slope or y-intercept of a given line.

Click to view content

Grasp Check

How would the following changes affect a line that is neither horizontal nor vertical and has a positive slope?

1. increase the slope but keeping the y-intercept constant
2. increase the y-intercept but keeping the slope constant
    a. Increasing the slope will cause the line to rotate clockwise around the y-intercept. Increasing the y-intercept will cause the line to move vertically up on the graph without changing the line's slope.
    b. Increasing the slope will cause the line to rotate counter-clockwise around the y-intercept. Increasing the y-intercept will cause the line to move vertically up on the graph without changing the line's slope.
    c. Increasing the slope will cause the line to rotate clockwise around the y-intercept. Increasing the y-intercept will cause the line to move horizontally right on the graph without changing the line's slope.
    d. Increasing the slope will cause the line to rotate counter-clockwise around the y-intercept. Increasing the y-intercept will cause the line to move horizontally right on the graph without changing the line's slope.

Check Your Understanding

12.

Identify some advantages of metric units.

a. Conversion between units is easier in metric units.
b. Comparison of physical quantities is easy in metric units.
c. Metric units are more modern than English units.
d. Metric units are based on powers of 2.

13.

The length of an American football field is $100\,\text{yd}$, excluding the end zones. How long is the field in meters? Round to the nearest $0.1\,\text{m}$.

a. $10.2\,\text{m}$
b. $91.4\,\text{m}$
c. $109.4\,\text{m}$
d. $328.1\,\text{m}$

14.

The speed limit on some interstate highways is roughly $100\,\text{km/h}$. How many miles per hour is this if $1.0\,\text{mile}$ is about $1.609\,\text{km}$?

a. 0.1 mi/h
b. 27.8 mi/h
c. 62 mi/h
d. 160 mi/h

15.

Briefly describe the target patterns for accuracy and precision and explain the differences between the two.

a. Precision states how much repeated measurements generate the same or closely similar results, while accuracy states how close a measurement is to the true value of the measurement.
b. Precision states how close a measurement is to the true value of the measurement, while accuracy states how much repeated measurements generate the same or closely similar result.
c. Precision and accuracy are the same thing. They state how much repeated measurements generate the same or closely similar results.
d. Precision and accuracy are the same thing. They state how close a measurement is to the true value of the measurement.

Teacher Support

Teacher Support Use the Check Your Understanding questions to assess students' achievement of the sections learning objectives. If students are struggling with a specific objective, the Check Your Understanding will help identify which and direct students to the relevant content.