10.2 Consequences of Special Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the relativistic effects seen in time dilation, length contraction, and conservation of relativistic momentum
- Explain and perform calculations involving mass-energy equivalence

Teacher Support

Teacher Support The learning objectives in this section will help your students master the following student standards:

- (4) Science concepts. The student knows and applies the laws governing motion in a variety of situations. The student is expected to:
  - (F) Identify and describe motion relative to different frames of reference.
- (8) Science concepts. The student knows simple examples of atomic, nuclear, and quantum phenomena. The student is expected to:
  - (C) Describe the significance of mass-energy equivalence and apply it in explanations of phenomena such as nuclear stability, fission, and fusion.

Section Key Terms

Teacher Support

Teacher Support In this section, you will see how the postulates lead to the theory of special relativity and see how that theory predicts effects on time, distance, momentum, and energy at velocities approaching the speed of light.

[BL] Begin a discussion by asking if students have ever seen a science fiction movie where space travelers age more slowly than the people left behind on Earth. Tell them there is some basis in fact to these stories. Discuss nuclear power. Ask if they know the basic difference between the nuclear power and combustion power.

[OL] Explain that Newton's laws are valid for everyday mechanics but break down at speeds approaching the speed of light. Discuss the relationship between relativity theory and Newton's laws. Briefly describe the changes predicted for measurements of time, length, momentum, and energy. See how much they know about energy derived from nuclear reactions.

[AL] Ask what the students already know about relativity theory. See if they know that relative motion is an old idea and ask for examples of relative motion in everyday situations. Explain that special relativity is similar but describes unexpected results at speeds approaching the speed of light. Ask if anyone can explain why this statement is true: "The original source of all the energy we use is the conversion of matter into energy."

Relativistic Effects on Time, Distance, and Momentum

Consideration of the measurement of elapsed time and simultaneity leads to an important relativistic effect. Time dilation is the phenomenon of time passing more slowly for an observer who is moving relative to another observer.

For example, suppose an astronaut measures the time it takes for light to travel from the light source, cross her ship, bounce off a mirror, and return. (See Figure 10.5.) How does the elapsed time the astronaut measures compare with the elapsed time measured for the same event by a person on the earth? Asking this question (another thought experiment) produces a profound result. We find that the elapsed time for a process depends on who is measuring it. In this case, the time measured by the astronaut is smaller than the time measured by the earth bound observer. The passage of time is different for the two observers because the distance the light travels in the astronaut's frame is smaller than in the earth bound frame. Light travels at the same speed in each frame, and so it will take longer to travel the greater distance in the earth bound frame.

Teacher Support

Teacher Support [OL] Discuss the expression for the relativistic factor. Explain that this is involved in all relativistic effects. Show how to tell when relativistic effects are significant and when they are negligible by plugging in values of v and c.

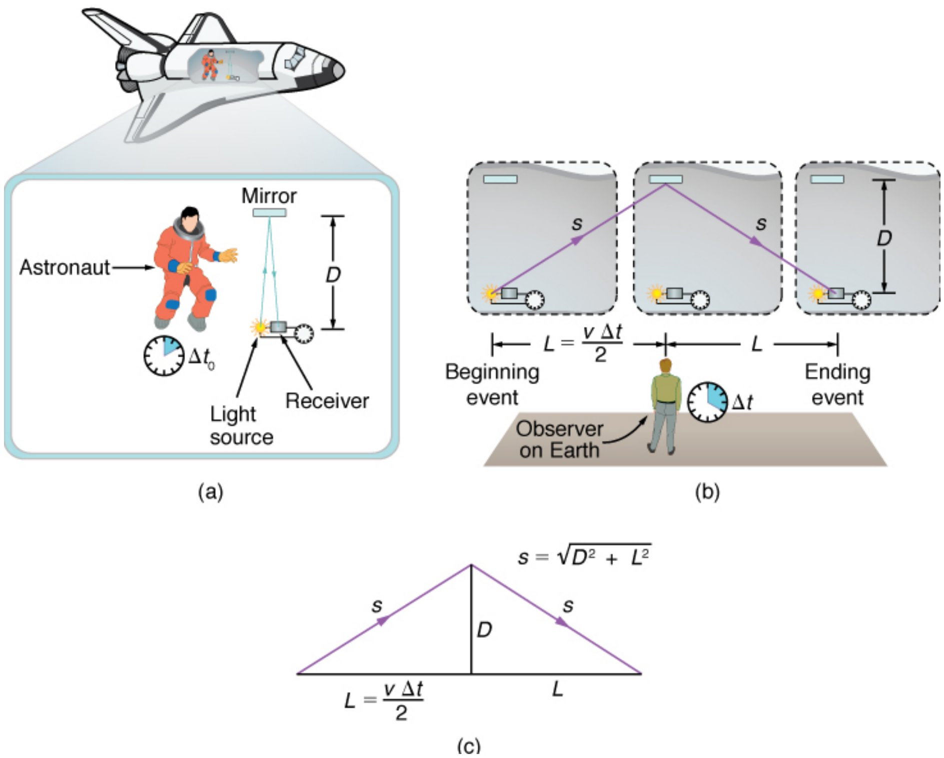


Figure 10.5 (a) An astronaut measures the time Δt_0 for light to cross her ship using an electronic timer. Light travels a distance $2D$ in the astronaut's frame. (b) A person on the earth sees the light follow the longer path $2s$ and take a longer time Δt.

Teacher Support

Teacher Support [AL]Figure 10.5, like Figure 10.4, may be hard for some students to grasp. Refer back to the previous figure. The animation in the discussion of length contraction further on should also be some help.

The relationship between Δt and Δt_o is given by

$\Delta t = \gamma \Delta t_0,$

where γ is the relativistic factor given by

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}},$

and v and c are the speeds of the moving observer and light, respectively.

Tips For Success

Try putting some values for v into the expression for the relativistic factor (γ). Observe at which speeds this factor will make a difference and when is so close to 1 that it can be ignored. Try 225 m/s, the speed of an airliner; 2.98×10^4 m/s, the speed of Earth in its orbit; and 2.990×10^8 m/s, the speed of a particle in an accelerator.

Teacher Support

Teacher Support Try putting some values for v into the expression for the relativistic factor. Observe at which speeds this factor will make a difference and when it is so close to 1 that it can be ignored.

Notice that when the velocity v is small compared to the speed of light c, then v/c becomes small, and γ becomes close to 1. When this happens, time measurements are the same in both frames of reference. Relativistic effects, meaning those that have to do with special relativity, usually become significant when speeds become comparable to the speed of light. This is seen to be the case for time dilation.

You may have seen science fiction movies in which space travelers return to Earth after a long trip to find that the planet and everyone on it has aged much more than they have. This type of scenario is a based on a thought experiment, known as the twin paradox, which imagines a pair of twins, one of whom goes on a trip into space while the other stays home. When the space traveler returns, she finds her twin has aged much more than she. This happens because the traveling twin has been in two frames of reference, one leaving Earth and one returning.

Time dilation has been confirmed by comparing the time recorded by an atomic clock sent into orbit to the time recorded by a clock that remained on Earth. GPS satellites must also be adjusted to compensate for time dilation in order to give accurate positioning.

Have you ever driven on a road, like that shown in Figure 10.6, that seems like it goes on forever? If you look ahead, you might say you have about 10 km left to go. Another traveler might say the road ahead looks like it is about 15 km long. If you both measured the road, however, you would agree. Traveling at everyday speeds, the distance you both measure would be the same. You will read in this section, however, that this is not true at relativistic speeds. Close to the speed of light, distances measured are not the same when measured by different observers moving with respect to one other.

Figure 10.6 People might describe distances differently, but at relativistic speeds, the distances really are different. (Corey Leopold, Flickr)

Teacher Support

Teacher Support [OL] Discuss the relationship between time dilation and length contraction. If observers agree on speed, but not on time, they must also disagree on length because $v = d/t$.

One thing all observers agree upon is their relative speed. When one observer is traveling away from another, they both see the other receding at the same speed, regardless of whose frame of reference is chosen. Remember that speed equals distance divided by time: $v = d/t$. If the observers experience a difference in elapsed time, they must also observe a difference in distance traversed. This is because the ratio d/t must be the same for both observers.

The shortening of distance experienced by an observer moving with respect to the points whose distance apart is measured is called length contraction. Proper length, L_0, is the distance between two points measured in the reference frame where the observer and the points are at rest. The observer in motion with respect to the points measures L. These two lengths are related by the equation

$L = \frac{L_0}{\gamma}$.

Because is the same expression used in the time dilation equation above, the equation becomes

$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$.

To see how length contraction is seen by a moving observer, go to this simulation. Here you can also see that simultaneity, time dilation, and length contraction are interrelated phenomena.

This link is to a simulation that illustrates the relativity of simultaneous events.

In classical physics, momentum is a simple product of mass and velocity. When special relativity is taken into account, objects that have mass have a speed limit. What effect do you think mass and velocity have on the momentum of objects moving at relativistic speeds; i.e., speeds close to the speed of light?

Momentum is one of the most important concepts in physics. The broadest form of Newton's second law is stated in terms of momentum. Momentum is conserved in classical mechanics whenever the net external force on a system is zero. This makes momentum conservation a fundamental tool for analyzing collisions. We will see that momentum has the same importance in modern physics. Relativistic momentum is conserved, and much of what we know about subatomic structure comes from the analysis of collisions of accelerator-produced relativistic particles.

One of the postulates of special relativity states that the laws of physics are the same in all inertial frames. Does the law of conservation of momentum survive this requirement at high velocities? The answer is yes, provided that the momentum is defined as follows.

Relativistic momentum, $\mathbf{p}$, is classical momentum multiplied by the relativistic factor γ.

$\mathbf{p} = \gamma m \mathbf{u}$,

10.3

where m is the rest mass of the object (that is, the mass measured at rest, without any γ factor involved), $\mathbf{u}$ is its velocity relative to an observer, and

γ,

as before, is the relativistic factor. We use the mass of the object as measured at rest because we cannot determine its mass while it is moving.

Note that we use $\mathbf{u}$ for velocity here to distinguish it from relative velocity $\mathbf{v}$ between observers. Only one observer is being considered here. With $\mathbf{p}$ defined in this way, $\mathbf{p}_{\text{tot}}$ is conserved whenever the net external force is zero, just as in classical physics. Again we see that the relativistic quantity becomes virtually the same as the classical at low velocities. That is, relativistic momentum $\gamma m \mathbf{u}$ becomes the classical $m\mathbf{u}$ at low velocities, because γ is very nearly equal to 1 at low velocities.

Relativistic momentum has the same intuitive feel as classical momentum. It is greatest for large masses moving at high velocities. Because of the factor γ, however, relativistic momentum behaves differently from classical momentum by approaching infinity as $\mathbf{u}$ approaches c. (See Figure 10.7.) This is another indication that an object with mass cannot reach the speed of light. If it did, its momentum would become infinite, which is an unreasonable value.

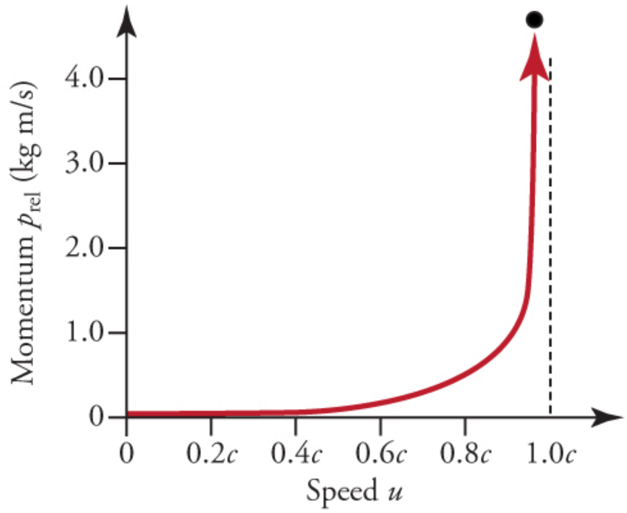


Figure 10.7 Relativistic momentum approaches infinity as the velocity of an object approaches the speed of light.

Teacher Support

Teacher Support [OL] Discuss the graph. Explain how it shows that objects that have mass cannot reach the speed of light. Have the students analyze the equation for relativistic momentum and see how this supports this conclusion. Explain that light can travel at the speed of light because it has no rest mass.

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This has been verified in numerous experiments.

Mass-Energy Equivalence

Let us summarize the calculation of relativistic effects on objects moving at speeds near the speed of light. In each case we will need to calculate the relativistic factor, given by

$\gamma = \frac{1}{\sqrt{1-\frac{\mathbf{v}^2}{c^2}}}$,

where $\mathbf{v}$ and c are as defined earlier. We use $\mathbf{u}$ as the velocity of a particle or an object in one frame of reference, and $\mathbf{v}$ for the velocity of one frame of reference with respect to another.

Time Dilation Elapsed time on a moving object, Δt_0, as seen by a stationary observer is given by $\Delta t = \gamma \Delta t_0$, where Δt_0 is the time observed on the moving object when it is taken to be the frame or reference.

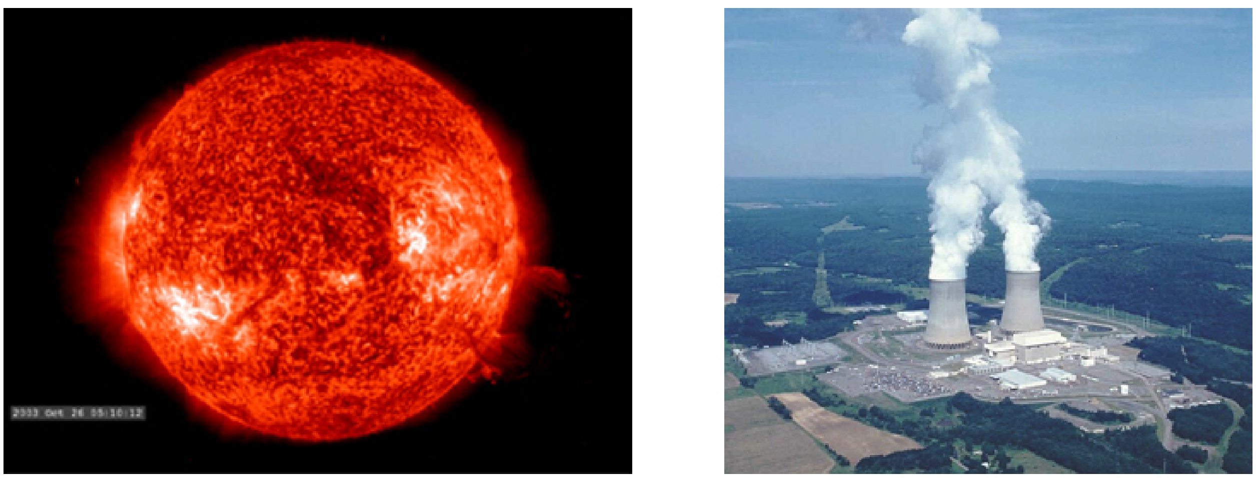
Length Contraction Length measured by a person at rest with respect to a moving object, L, is given by

$L = \frac{L_0}{\gamma}$,

where L_0 is the length measured on the moving object.

Relativistic Momentum Momentum, $\mathbf{p}$, of an object of mass, m, traveling at relativistic speeds is given by $\mathbf{p} = \gamma m \mathbf{u}$, where $\mathbf{u}$ is velocity of a moving object as seen by a stationary observer.

Relativistic Energy The original source of all the energy we use is the conversion of mass into energy. Most of this energy is generated by nuclear reactions in the sun and radiated to Earth in the form of electromagnetic radiation, where it is then transformed into all the forms with which we are familiar. The remaining energy from nuclear reactions is produced in nuclear power plants and in Earth's interior. In each of these cases, the source of the energy is the conversion of a small amount of mass into a large amount of energy. These sources are shown in Figure 10.8.



(a) (b)

Figure 10.8 The sun (a) and the Susquehanna Steam Electric Station (b) both convert mass into energy. ((a) NASA/Goddard Space Flight Center, Scientific Visualization Studio; (b) U.S. government)

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Einstein showed that the law of conservation of energy is valid relativistically, if we define energy to include a relativistic factor. The result of his analysis is that a particle or object of mass m moving at velocity $\mathbf{u}$ has relativistic energy given by

$E = \gamma mc^2.$

This is the expression for the total energy of an object of mass m at any speed $\mathbf{u}$ and includes both kinetic and potential energy. Look back at the equation for γ and you will see that it is equal to 1 when $\mathbf{u}$ is 0; that is, when an object is at rest. Then the rest energy, E_0, is simply

$E_0 = mc^2.$

This is the correct form of Einstein's famous equation.

This equation is very useful to nuclear physicists because it can be used to calculate the energy released by a nuclear reaction. This is done simply by subtracting the mass of the products of such a reaction from the mass of the reactants. The difference is the m in $E_0 = mc^2$. Here is a simple example:

A positron is a type of antimatter that is just like an electron, except that it has a positive charge. When a positron and an electron collide, their masses are completely annihilated and converted to energy in the form of gamma rays. Because both particles have a rest mass of 9.11×10^{-31} kg, we multiply the mc^2 term by 2. So the energy of the gamma rays is

$$\begin{aligned} E_0 &= 2(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^8 \tfrac{\text{m}}{\text{s}}\right)^2 \\ &= 1.64 \times 10^{-13} \tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \\ &= 1.64 \times 10^{-13}\text{J} \end{aligned}$$

10.4

where we have the expression for the joule (J) in terms of its SI base units of kg, m, and s. In general, the nuclei of stable isotopes have less mass then their constituent subatomic particles. The energy equivalent of this difference is called the binding energy of the nucleus. This energy is released during the formation of the isotope from its constituent particles because the product is more stable than the reactants. Expressed as mass, it is called the mass defect. For example, a helium nucleus is made of two neutrons and two protons and has a mass of 4.0003 atomic mass units (u). The sum of the masses of two protons and two neutrons is 4.0330 u. The mass defect then is 0.0327 u. Converted to kg, the mass defect is 5.0442×10^{-30} kg. Multiplying this mass times c^2 gives a binding energy of 4.540×10^{-12} J. This does not sound like much because it is only one atom. If you were to make one gram of helium out of neutrons and protons, it would release 683,000,000,000 J. By comparison, burning one gram of coal releases about 24 J.

Teacher Support

Teacher Support [BL] In regards to the change in the law of conservation of energy to the law of conservation of mass-energy, it may help to think of mass as simply a very concentrated form of energy.

[OL] Impress upon the students the enormous amount of energy derived from the conversion of a small amount of mass. Have them note that c^2 is a very large number. Students try to understand new concepts by using previous knowledge, and that may result in a misconception here. They are comfortable with chemical reactions and may try to relate this to the burning of a piece of wood. Tell them that burning the wood chemically might provide energy for a single room in a house, but converting the mass of the wood completely to energy according to $E = mc^2$ would provide power for thousands of houses.

[AL] Ask students if they know the difference between fission and fusion and where examples of each of these occur.

Boundless Physics

The RHIC Collider Figure 10.9 shows the Brookhaven National Laboratory in Upton, NY. The circular structure houses a particle accelerator called the RHIC, which stands for Relativistic Heavy Ion Collider. The heavy ions in the name are gold nuclei that have been stripped of their electrons. Streams of ions are accelerated in several stages before entering the big ring seen in the figure. Here, they are accelerated to their final speed, which is about 99.7 percent the speed of light. Such high speeds are called relativistic. All the relativistic phenomena we have been discussing in this chapter are very pronounced in this case. At this speed $\gamma = 12.9$, so that relativistic time dilates by a factor of about 13, and relativistic length contracts by the same factor.

Figure 10.9 Brookhaven National Laboratory. The circular structure houses the RHIC. (energy.gov, Wikimedia Commons)

Two ion beams circle the 2.4-mile long track around the big ring in opposite directions. The paths can then be made to cross, thereby causing ions to collide. The collision event is very short-lived but amazingly intense. The temperatures and pressures produced are greater than those in the hottest suns. At 4 trillion degrees Celsius, this is the hottest material ever created in a laboratory

But what is the point of creating such an extreme event? Under these conditions, the neutrons and protons that make up the gold nuclei are smashed apart into their components, which are called quarks and gluons. The goal is to recreate the conditions that theorists believe existed at the very beginning of the universe. It is thought that, at that time, matter was a sort of soup of quarks and gluons. When things cooled down after the initial bang, these particles condensed to form protons and neutrons.

Some of the results have been surprising and unexpected. It was thought the quark-gluon soup would resemble a gas or plasma. Instead, it behaves more like a liquid. It has been called a *perfect* liquid because it has virtually no viscosity, meaning that it has no resistance to flow.

Teacher Support

Teacher Support Discuss particle colliders such as the relatively new Large Hadron Collider built by CERN. Students may want to know more about this project and the *God particle*. Explain why there are so many applications of special relativity theory in the field of particle physics.

Grasp Check

Calculate the relativistic factor , for a particle traveling at 99.7 percent of the speed of light.

a. 0.08
b. 0.71
c. 1.41
d. 12.9

Worked Example

The Speed of Light One night you are out looking up at the stars and an extraterrestrial spaceship flashes across the sky. The ship is 50 meters long and is travelling at 95 percent of the speed of light. What would the ship's length be when measured from your earthbound frame of reference?

Strategy

List the knowns and unknowns.

Knowns: proper length of the ship, $L_0 = 50$ m; velocity, $\mathbf{v}$, $= 0.95c$

Unknowns: observed length of the ship accounting for relativistic length contraction, L.

Choose the relevant equation.

$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{\mathbf{u}^2}{c^2}}$

Solution

$L = \ 50 \text{ m}\sqrt{1 - \frac{(0.95)^2 c^2}{c^2}} = 50 \text{ m}\sqrt{1 - (0.95)^2} = 16 \ \ \text{m}$

Discussion

Calculations of can usually be simplified in this way when v is expressed as a percentage of c because the c^2 terms cancel. Be sure to also square the decimal representing the percentage before subtracting from 1. Note that the aliens will still see the length as L_0 because they are moving with the frame of reference that is the ship.

Teacher Support

Teacher Support Identify the variables, the knowns and unknowns, and the relevant equation. Understand clearly which length applies to your frame of reference and which applies to the ship's frame of reference; that is, which is proper length.

Practice Problems

7.

Calculate the relativistic factor, , for an object traveling at 2.00×10^8 m/s.

a. 0.74
b. 0.83
c. 1.2
d. 1.34

8.

The distance between two points, called the proper length, L0, is 1.00 km. An observer in motion with respect to the frame of reference of the two points measures 0.800 km, which is L. What is the relative speed of the frame of reference with respect to the observer?

a. 1.80×10^8 m/s
b. 2.34×10^8 m/s
c. 3.84×10^8 m/s
d. 5.00×10^8 m/s

9.

Consider the nuclear fission reaction n+ U 92 235 → C 55 137 s+ R 37 97 b+2n+E . If a neutron has a rest mass of 1.009u, U 92 235 has a rest mass of

235.044u, C 55 137 s has rest mass of 136.907u, and R 37 97 b has a rest mass of 96.937u, what is the value of E in joules?

a. 1.8×10^{-11} J
b. 2.9×10^{-11} J
c. 1.8×10^{-10} J
d. 2.9×10^{-10} J

Solution The correct answer is (b). The mass deficit in the reaction is $235.044\ \mathrm{u} - (136.907 + 96.937 + 1.009)\mathrm{u}$, or 0.191u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives $(0.191\ \mathrm{u})\left(1.66 \times 10^{-27}\mathrm{kg/u}\right)\left(3.00 \times 10^{8}\mathrm{m/s}\right)^2 \cong 2.85 \times 10^{-11}\mathrm{J}$.

10.

Consider the nuclear fusion reaction H 1 2 + H 1 2 → H 1 3 + H 1 1 +E . If H 1 2 has a rest mass of 2.014u, H 1 3 has a rest mass of 3.016u, and H 1 1 has a rest mass of 1.008u, what is the value of E in joules?

a. 6×10^{-13} J
b. 6×10^{-12} J
c. 6×10^{-11} J
d. 6×10^{-10} J

Solution The correct answer is (a). The mass deficit in the reaction is $2\,(2.014\ \mathrm{u}) - (3.016 + 1.008)\mathrm{u}$, or 0.004u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives $(0.004\ \mathrm{u})\left(1.66 \times 10^{-27}\mathrm{kg/u}\right)\left(3.00 \times 10^{8}\mathrm{m/s}\right)^2 \cong 5.98 \times 10^{-13}\mathrm{J}$.

Check Your Understanding

11.

Describe time dilation and state under what conditions it becomes significant.

a. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.
b. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.
c. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.
d. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same

location in one frame appears shorter when measured from the second frame.

12.

The equation used to calculate relativistic momentum is $p = \ \cdot m \cdot u$. Define the terms to the right of the equal sign and state how m and u are measured.

a. is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity of the frame.
b. is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity relative to an observer.
c. is the relativistic factor, m is the relativistic mass $\left(\text{i.e.,} \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity of the frame.
d. is the relativistic factor, m is the relativistic mass $\left(\text{i.e.,} \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity relative to an observer.

13.

Describe length contraction and state when it occurs.

a. When the speed of an object becomes the speed of light, its length appears to shorten when viewed by a stationary observer.
b. When the speed of an object approaches the speed of light, its length appears to shorten when viewed by a stationary observer.
c. When the speed of an object becomes the speed of light, its length appears to increase when viewed by a stationary observer.
d. When the speed of an object approaches the speed of light, its length appears to increase when viewed by a stationary observer.

Teacher Support

Teacher Support Use the Check Your Understanding questions to assess students' achievement of the section's learning objectives. If students are struggling with a specific objective, the Check Your Understanding will help identify which and direct students to the relevant content.