

# PHYS12 CH:5 Motion in Two Dimensions

## Vectors and Projectiles

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# Outline

- 1 Introduction
- 2 Vector Addition and Subtraction: Graphical Methods
- 3 Vector Addition and Subtraction: Analytical Methods
- 4 Projectile Motion
- 5 Summary

# The Mystery

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Motion happens in multiple dimensions simultaneously.

# Learning Objectives

By the end of this section, you will be able to:

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- **5.1:** Use the graphical method to solve physics problems

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### The Dual Nature

A vector is a quantity with both magnitude AND direction.

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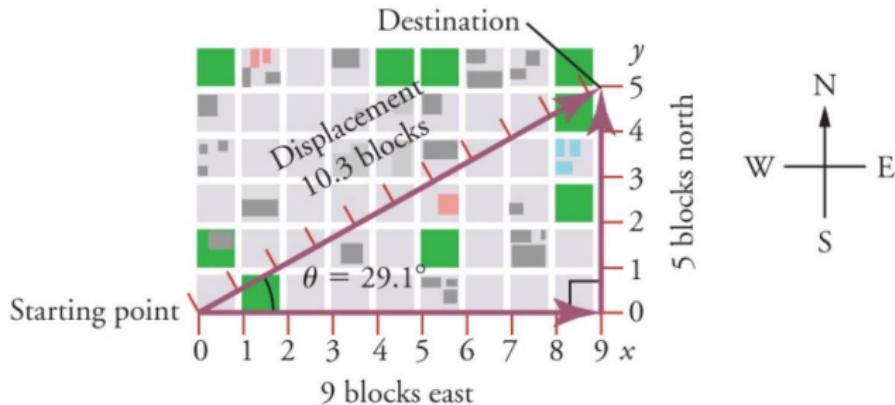
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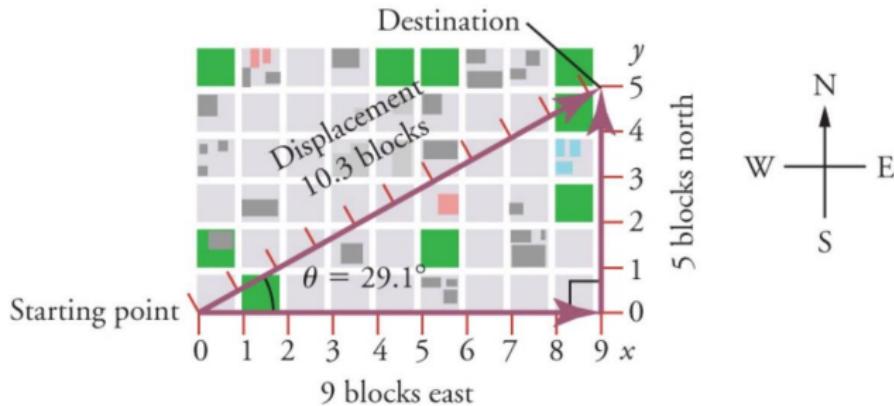
### The Mental Model

Velocity is like a speedometer with a compass attached. Speed tells how fast. Velocity adds where.

## 5.1 The Journey to the Destination



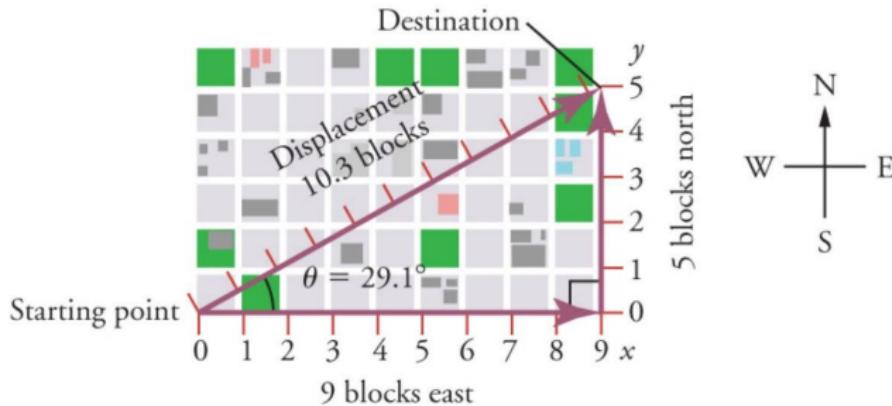
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### Civilian View vs. Reality

**Civilian:** "I walked 14 blocks."

**Physicist:** "Displacement was 10.3 blocks."

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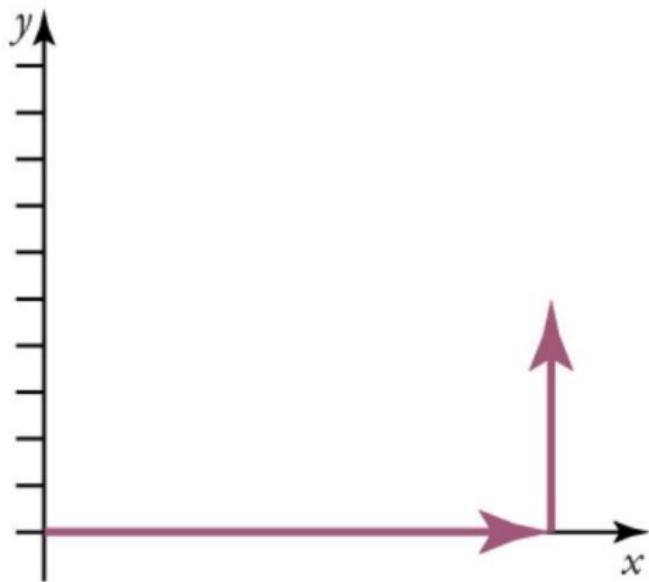
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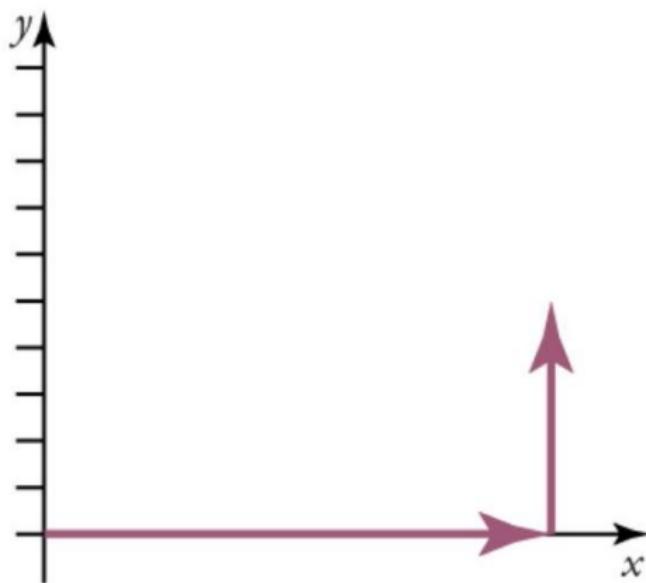
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**Key insight:** Order doesn't matter!  $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

## 5.1 Building the Resultant



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The resultant vector connects start to finish.

## 5.1 Vector Subtraction

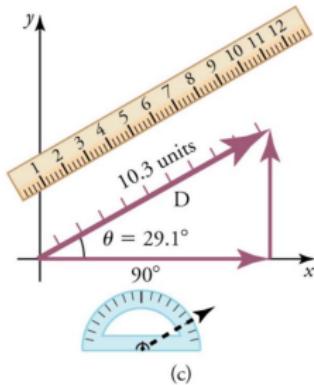
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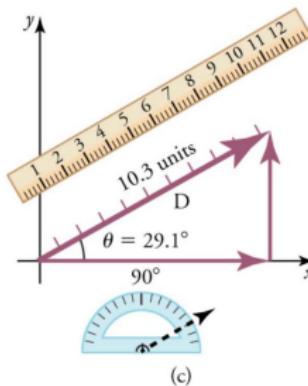
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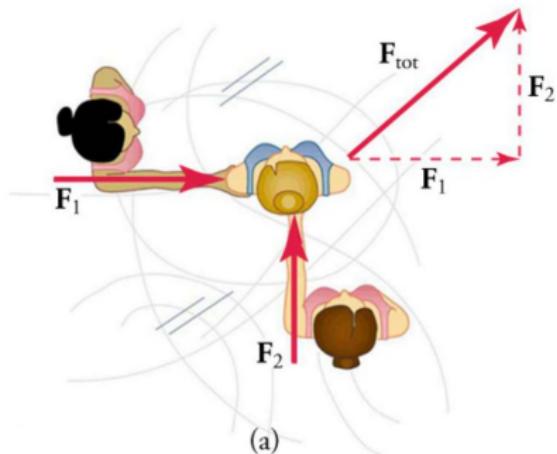
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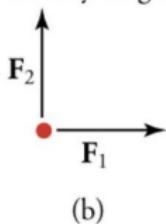


**The negative vector:** Same magnitude, opposite direction

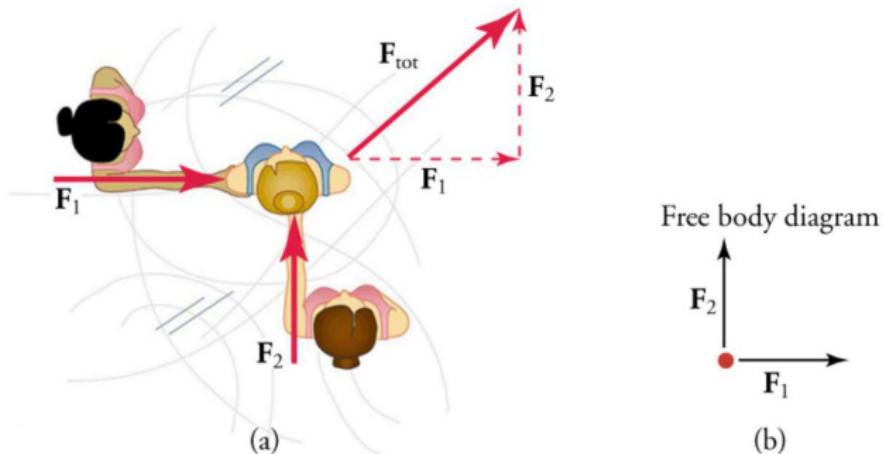
## 5.1 Forces on Ice



Free body diagram

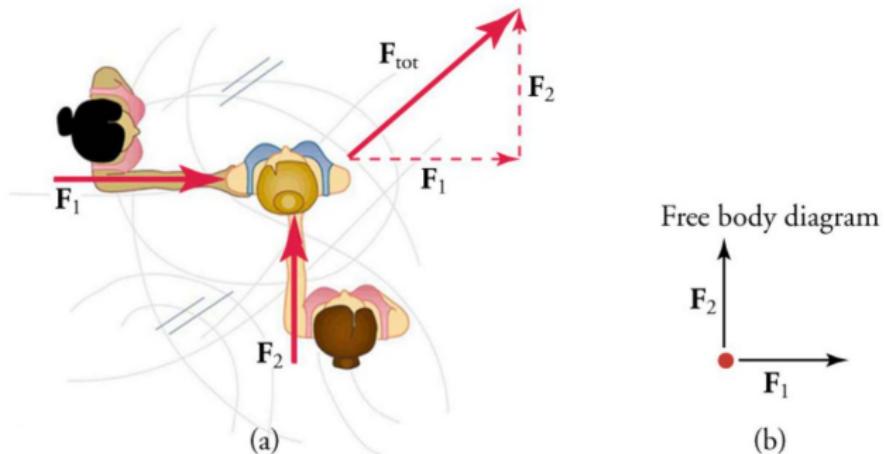


## 5.1 Forces on Ice



Two skaters push with 400 N each at right angles.

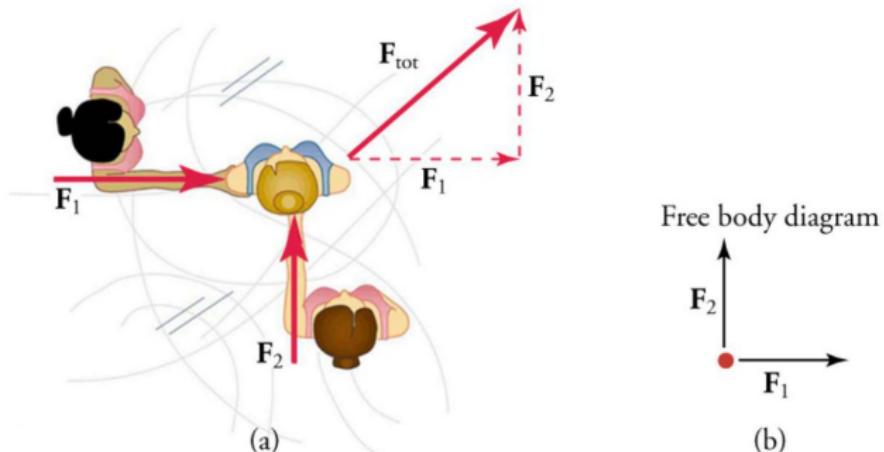
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Pythagorean theorem works when vectors are perpendicular!

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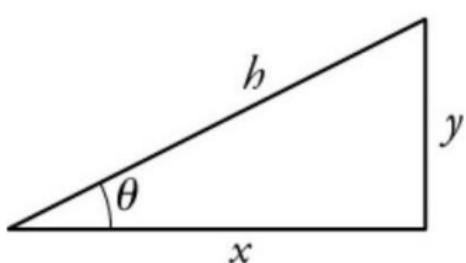
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By the end of this section, you will be able to:

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- **5.2:** Use the analytical method to solve problems

## 5.2 The Power of Trigonometry

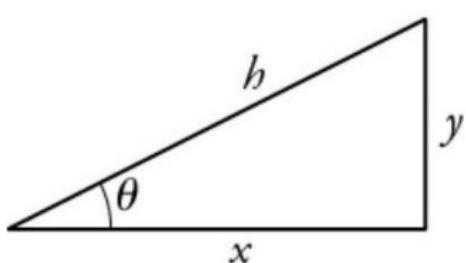


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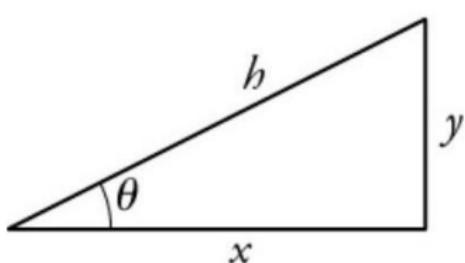
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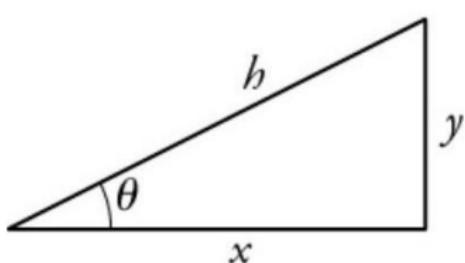
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## 5.2 Breaking Vectors Into Components

### The Source Code

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Every 2D vector can be split into x and y components.

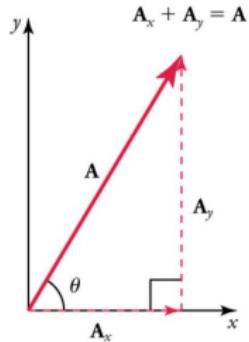
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Components match the original path: 9 east + 5 north!

## 5.2 Reverse: From Components to Vector

### Reconstruction Formulas

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1} \left( \frac{A_y}{A_x} \right)$$

Pythagorean theorem gives magnitude, inverse tangent gives direction.

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### The Paradox

**Students confuse:**  $\mathbf{A}_x + \mathbf{A}_y = \mathbf{A}$  (vector addition)

with  $A = \sqrt{A_x^2 + A_y^2}$  (magnitude calculation)

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More accurate than graphical method - not limited by drawing precision!

# Attempt: Two-Leg Journey

## The Challenge (3 min, silent)

A person walks 53.0 m at  $20.0^\circ$  north of east, then 34.0 m at  $63.0^\circ$  north of east.

### Given:

- $A = 53.0 \text{ m}$ ,  $\theta_A = 20.0^\circ$
- $B = 34.0 \text{ m}$ ,  $\theta_B = 63.0^\circ$

### Find: Total displacement magnitude and direction

*Can you decode this journey? Work silently.*

# Compare: Vector Addition Strategy

**Turn and talk (2 min):**

- ① What's the first step - find components or add vectors?
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**Name wheel:** One pair share your approach (not your answer).

# Reveal: The Navigation Solution

**Self-correct in a different color:**

**Step 1:** Find components of A

$$A_x = (53.0)(\cos 20.0^\circ) = 49.8 \text{ m}$$

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**Step 2:** Find components of B

$$B_x = (34.0)(\cos 63.0^\circ) = 15.4 \text{ m}$$

$$B_y = (34.0)(\sin 63.0^\circ) = 30.3 \text{ m}$$

# Reveal: Combining Components

## Step 3: Add components

$$R_x = A_x + B_x = 49.8 + 15.4 = 65.2 \text{ m}$$

$$R_y = A_y + B_y = 18.1 + 30.3 = 48.4 \text{ m}$$

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**Step 5:** Find direction

$$\theta = \tan^{-1}(48.4/65.2) = 36.6^\circ \text{ north of east}$$

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- **5.3:** Apply kinematic equations and vectors to solve projectile problems

## 5.3 The Great Separation

Nature's Rule for Projectiles

Horizontal and vertical motions  
are independent

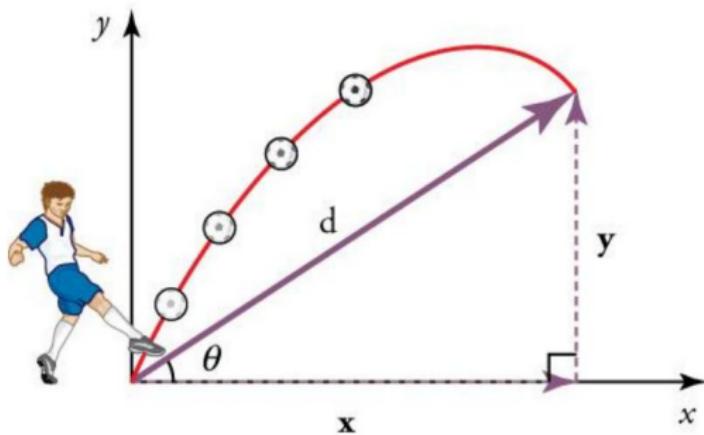
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We ignore air resistance in introductory physics.

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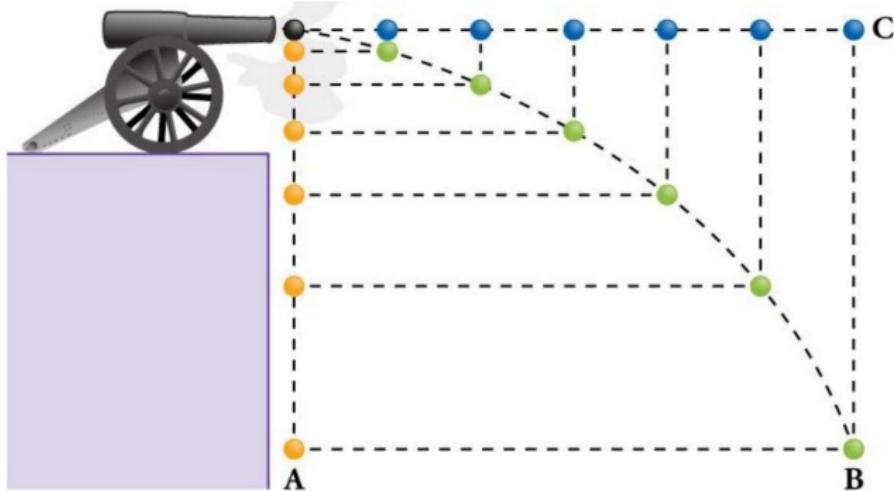
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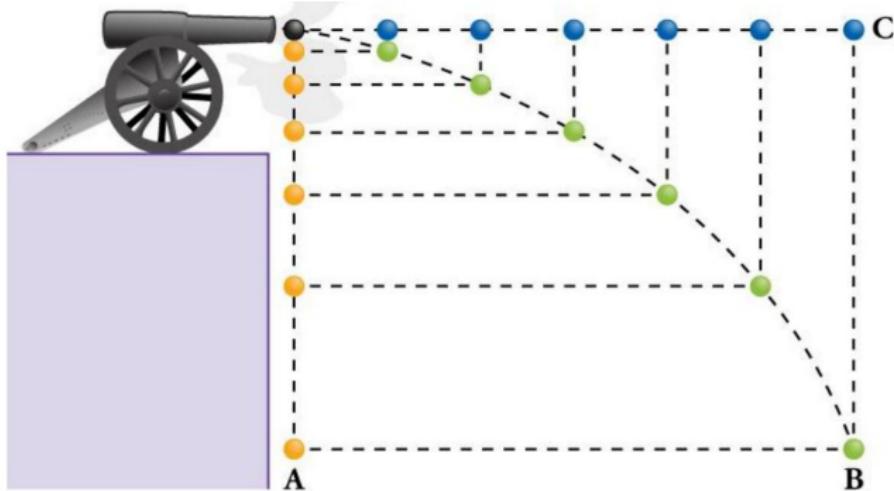
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- Velocity:  $v_y = v_{0y} - gt$
- Position:  $y = y_0 + v_{0y} t - \frac{1}{2}gt^2$

Time  $t$  is the only variable connecting the two motions!

## 5.3 The Trajectory



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Ball kicked at angle  $\theta$  follows parabolic path.

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Recombination formulas:

$$d = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}(y/x)$$

$$v = \sqrt{v_x^2 + v_y^2}, \quad \theta_v = \tan^{-1}(v_y/v_x)$$

## 5.3 Maximum Height Formula

The Peak

$$h = \frac{v_{0y}^2}{2g}$$

Maximum height depends only on initial vertical velocity.

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**Key insight:** At maximum height,  $v_y = 0$

### What Your Brain Gets Wrong

**Intuition:** "Faster launch means higher peak."

**Reality:** "Only the *vertical* component matters."

# Attempt: Fireworks Launch

## The Challenge (3 min, silent)

A fireworks shell is launched at 70.0 m/s at  $75^\circ$  above horizontal.

### Given:

- $v_0 = 70.0 \text{ m/s}$
- $\theta = 75^\circ$

### Find: Maximum height

*Can you predict where it explodes? Work silently.*

# Compare: Projectile Strategy

**Turn and talk (2 min):**

- ① What's the first step - find components or use a formula?
- ② Which velocity matters - total or vertical?
- ③ Which kinematic equation did you choose?

# Compare: Projectile Strategy

**Turn and talk (2 min):**

- ① What's the first step - find components or use a formula?
- ② Which velocity matters - total or vertical?
- ③ Which kinematic equation did you choose?

**Name wheel:** One pair share your approach (not your answer).

# Reveal: Reaching the Sky

**Self-correct in a different color:**

**Step 1:** Find vertical velocity component

$$\begin{aligned}v_{0y} &= v_0 \sin \theta \\&= (70.0)(\sin 75^\circ) = 67.6 \text{ m/s}\end{aligned}$$

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**Check:** About 765 feet - reasonable for large fireworks!

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The Distance Formula

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#### Key insights:

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### Key insights:

- Maximum range at  $\theta = 45^\circ$
- Same range for complementary angles:  $30^\circ$  and  $60^\circ$
- Only valid when initial and final heights are equal

# Attempt: Volcanic Rock

## The Challenge (3 min, silent)

A rock is ejected from a volcano at 25.0 m/s at  $35^\circ$  above horizontal. It lands 20.0 m below its starting point.

### Given:

- $v_0 = 25.0 \text{ m/s}$ ,  $\theta = 35^\circ$
- $y = -20.0 \text{ m}$

### Find: Time of flight

*Can you predict the flight time? Work silently.*

# Compare: Quadratic Strategy

**Turn and talk (2 min):**

- ① Which kinematic equation has both  $y$  and  $t$ ?
- ② What values do you substitute for  $y_0$  and  $y$ ?
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**Step 4:** Quadratic formula

$$t = \frac{14.3 \pm \sqrt{(14.3)^2 - 4(4.90)(-20.0)}}{2(4.90)}$$

## Reveal: Solving the Quadratic

### **Step 5: Calculate discriminant**

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$$t = \frac{14.3 + 24.4}{9.8} = 3.96 \text{ s} \quad (\text{physical})$$

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Negative time means before launch - physically impossible!

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- ⑤ Time connects the two dimensions
- ⑥ Maximum height depends only on  $v_{0y}$

# Key Equations

## Vector Components:

$$A_x = A \cos \theta \quad (1)$$

$$A_y = A \sin \theta \quad (2)$$

## Magnitude and Direction:

$$A = \sqrt{A_x^2 + A_y^2} \quad (3)$$

$$\theta = \tan^{-1}(A_y/A_x) \quad (4)$$

## Projectile Motion:

$$\text{Horizontal: } x = x_0 + v_x t \quad (5)$$

$$\text{Vertical: } y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \quad (6)$$

$$\text{Max height: } h = \frac{v_{0y}^2}{2g} \quad (7)$$

# Homework

Complete the assigned problems  
posted on the LMS