## Section Summary

### 6.1 Rotation Angle and Angular Velocity

- Uniform circular motion is motion in a circle at constant speed. The rotation angle $\Delta\theta$ is defined as the ratio of the arc length to the radius of curvature:

  $$\Delta\theta = \frac{\Delta s}{r},$$

  where arc length $\Delta s$ is distance traveled along a circular path and $r$ is the radius of curvature of the circular path. The quantity $\Delta\theta$ is measured in units of radians (rad), for which

  $$2\pi \text{ rad} = 360^\circ = 1 \text{ revolution.}$$

- The conversion between radians and degrees is $1$ rad = $57.3^\circ$.
- Angular velocity $\omega$ is the rate of change of an angle,

  $$\omega = \frac{\Delta\theta}{\Delta t},$$

  where a rotation $\Delta\theta$ takes place in a time $\Delta t$. The units of angular velocity are radians per second (rad/s). Linear velocity $v$ and angular velocity $\omega$ are related by

  $$v = r\omega \text{ or } \omega = \frac{v}{r}.$$

### 6.2 Centripetal Acceleration

- Centripetal acceleration $a_c$ is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity $v$ and has the magnitude

  $$a_c = \frac{v^2}{r};\ a_c = r\omega^2 .$$

- The unit of centripetal acceleration is $m/s^2$.

### 6.3 Centripetal Force

- Centripetal force $F_c$ is any force causing uniform circular motion. It is a "center-seeking" force that always points toward the center of rotation. It is perpendicular to linear velocity $v$ and has magnitude

  $$F_c = ma_c ,$$

which can also be expressed as

$$\left.\begin{array}{c} F_c = m\frac{v^2}{r} \\ \text{or} \\ F_c = mr\omega^2 \end{array}\right\},$$

## 6.4 Fictitious Forces and Non-inertial Frames: The Coriolis Force

- Rotating and accelerated frames of reference are non-inertial.
- Fictitious forces, such as the Coriolis force, are needed to explain motion in such frames.

## 6.5 Newton's Universal Law of Gravitation

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is

  $$F = G\frac{\text{mM}}{r^2},$$

  where F is the magnitude of the gravitational force. $G$ is the gravitational constant, given by $G = 6.674 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$.

- Newton's law of gravitation applies universally.

## 6.6 Satellites and Kepler's Laws: An Argument for Simplicity

- Kepler's laws are stated for a small mass $m$ orbiting a larger mass $M$ in near-isolation. Kepler's laws of planetary motion are then as follows:

  Kepler's first law

  The orbit of each planet about the Sun is an ellipse with the Sun at one focus.

  Kepler's second law

  Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.

  Kepler's third law

  The ratio of the squares of the periods of any two planets about the Sun is equal to the ratio of the cubes of their average distances from the Sun:

  $$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

where $T$ is the period (time for one orbit) and $r$ is the average radius of the orbit.

- The period and radius of a satellite's orbit about a larger body $M$ are related by

$$T^2 = \frac{4\pi^2}{\mathrm{GM}} r^3$$

or

$$\frac{r^3}{T^2} = \frac{G}{4\pi^2} M.$$