

PHYS12 CH:6 The Art of Falling Forever

Circular Motion and Rotation

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Outline

- 1 Introduction
- 2 Angle of Rotation and Angular Velocity
- 3 Uniform Circular Motion
- 4 Rotational Motion
- 5 Summary

How do you move forward
while constantly turning?

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From Formula 1 cars screaming around curves to the Moon circling Earth...

The Mystery

How do you move forward
while constantly turning?

From Formula 1 cars screaming around curves to the Moon circling Earth...

All require a force toward the center.

Falling Forever



Figure: Formula 1 car in circular motion

Falling Forever



Figure: Formula 1 car in circular motion

The Mental Model

A satellite in orbit is falling toward Earth but moving fast enough sideways to keep missing it.

Learning Objectives

By the end of this section, you will be able to:

- **6.1:** Describe the angle of rotation and relate it to its linear counterpart

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- **6.1:** Describe the angle of rotation and relate it to its linear counterpart
- **6.1:** Describe angular velocity and relate it to its linear counterpart
- **6.1:** Solve problems involving angle of rotation and angular velocity

6.1 Two Kinds of Rotation

Circular motion: Object moves in a circular path (race car on track)

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Spin: Object rotates about its own axis (Earth spinning)

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Real-World Examples

- Earth spins on its axis (spin) AND orbits the Sun (circular motion)
- Your car tire spins (spin) while the car follows a curve (circular motion)

6.1 Angle of Rotation

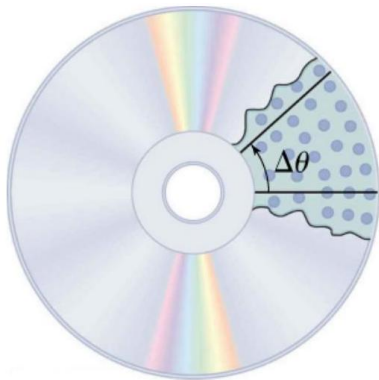


Figure: Arc length and radius

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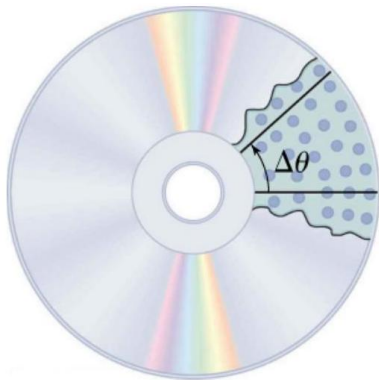


Figure: Arc length and radius

Universal Law: Angle of Rotation

$$\Delta\theta = \frac{\Delta s}{r}$$

Angle equals arc length divided by radius

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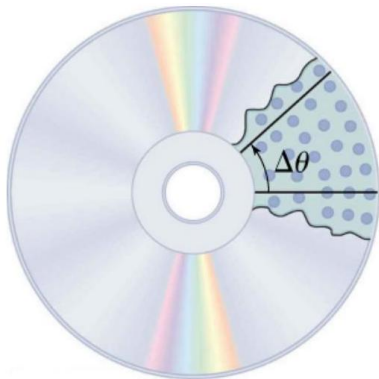


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Measured in **radians** (rad)

6.1 Radians vs Degrees

The Conversion

$$1 \text{ revolution} = 2\pi \text{ rad} = 360^\circ$$

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Why Radians?

Radians simplify equations in physics. Degrees are arbitrary - radians are natural.

6.1 Angular Velocity

Universal Law: Angular Velocity

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Angular velocity equals change in angle divided by change in time

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Direction:

- Counterclockwise: positive (out of page toward you)
- Clockwise: negative (into page away from you)

6.1 Connecting Spinning to Moving

The Bridge Equation

$$v = r\omega$$

Tangential velocity equals radius times angular velocity

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The Mental Model

Points farther from the center move faster linearly, but all points have the same angular velocity.

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Example: CD spinning - outer edge moves faster than inner part, but both complete one revolution in same time.

6.1 Why Car Tires Matter

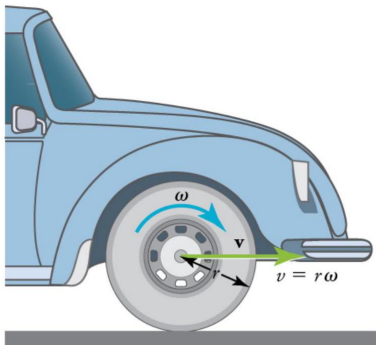


Figure: Car tire rolling

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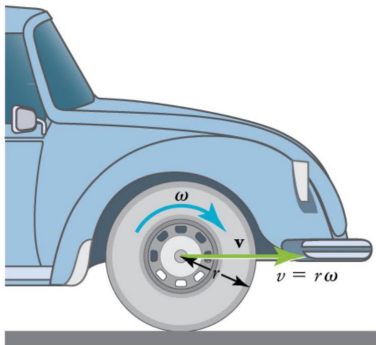


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Large ω means large v because $v = r\omega$

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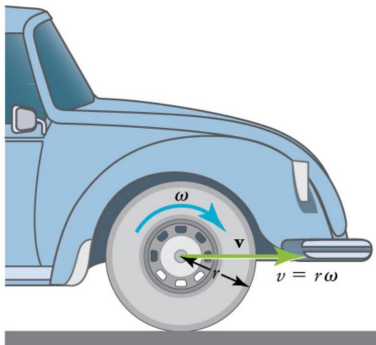


Figure: Car tire rolling

Large ω means large v because $v = r\omega$

Larger radius tire at same ω produces greater v

Attempt: Clock Tower Angle

The Challenge (3 min, silent)

A clock tower has a radius of 1.0 m. The hour hand moves from 12 p.m. to 3 p.m.

Given:

- Radius $r = 1.0$ m
- Time: 12 to 3 (quarter rotation)

Find:

- ① Angle of rotation in radians
- ② Arc length along outer edge

Can you decode this rotation? Work silently.

Compare: Clock Tower

Turn and talk (2 min):

- 1 What fraction of a full rotation does the hour hand make from 12 to 3?
- 2 How many radians in a full circle?
- 3 What equation connects arc length to angle?

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Turn and talk (2 min):

- 1 What fraction of a full rotation does the hour hand make from 12 to 3?
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Name wheel: One pair share your approach (not your answer).

Reveal: The Geometry of Time

Self-correct in a different color:

Part (a): From 12 to 3 is $\frac{1}{4}$ of full rotation

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Part (b): Use $\Delta s = r\Delta\theta$

$$\Delta s = (1.0 \text{ m}) \left(\frac{\pi}{2} \text{ rad} \right) = \boxed{1.6 \text{ m}}$$

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Check: Arc length is less than circumference ($2\pi r \approx 6.3 \text{ m}$). Reasonable!

Attempt: Spinning Car Tire

The Challenge (3 min, silent)

A car tire has radius 0.300 m and the car travels at 15.0 m/s (about 54 km/h).

Given:

- Radius $r = 0.300$ m
- Tangential velocity $v = 15.0$ m/s

Find: Angular velocity ω of the tire in rad/s

How fast is the tire spinning?

Compare: Tire Speed

Turn and talk (2 min):

- 1 What equation connects linear and angular velocity?
- 2 How did you rearrange it to solve for ω ?
- 3 What are the units of your answer?

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Name wheel: One pair share your rearrangement strategy.

Reveal: The Spinning Wheel

Self-correct in a different color:

Equation: $v = r\omega$, so $\omega = \frac{v}{r}$

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Substitute:

$$\omega = \frac{15.0 \text{ m/s}}{0.300 \text{ m}}$$

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Equation: $v = r\omega$, so $\omega = \frac{v}{r}$

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$$\omega = 50.0 \text{ rad/s}$$

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Equation: $v = r\omega$, so $\omega = \frac{v}{r}$

Substitute:

$$\omega = \frac{15.0 \text{ m/s}}{0.300 \text{ m}}$$

$$\omega = 50.0 \text{ rad/s}$$

Check: About 8 revolutions per second (since $2\pi \text{ rad} = 1 \text{ rev}$). Fast but reasonable for highway speed!

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- **6.2:** Describe centripetal acceleration and relate it to linear acceleration

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6.2 The Paradox of Constant Speed

Uniform circular motion: Object travels circular path at constant speed

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Civilian View vs. Reality

Civilian: "Constant speed means no acceleration."

Physicist: "Velocity is changing direction, so there IS acceleration."

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Uniform circular motion: Object travels circular path at constant speed

Civilian View vs. Reality

Civilian: "Constant speed means no acceleration."

Physicist: "Velocity is changing direction, so there IS acceleration."

Acceleration is a change in velocity - magnitude OR direction!

6.2 The Illusion of Being Flung

The Mental Model

When you turn in a car, you feel pushed outward. But no force pushes you out - your body wants to go straight (Newton's first law) while the car turns.

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The Fictional Force

Centrifugal force is not real - it's the illusion created by your inertia resisting the turn.

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The Mental Model

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The Fictional Force

Centrifugal force is not real - it's the illusion created by your inertia resisting the turn.

The real force is **centripetal** - pulling you inward toward the center!

6.2 Centripetal Acceleration

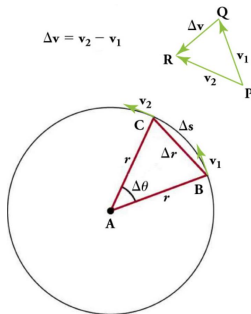


Figure: Velocity changes direction, acceleration points toward center

6.2 Centripetal Acceleration

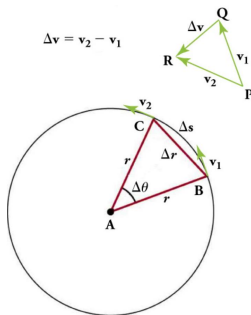


Figure: Velocity changes direction, acceleration points toward center

Universal Law: Centripetal Acceleration

$$a_c = \frac{v^2}{r} \quad \text{or} \quad a_c = r\omega^2$$

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Example:

- Curve at 50 km/h: moderate acceleration
- Same curve at 100 km/h: **4 times** the acceleration

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Example:

- Curve at 50 km/h: moderate acceleration
- Same curve at 100 km/h: **4 times** the acceleration

The Warning

This is why speed limits are lower on curves - small speed increase creates huge acceleration increase.

6.2 Centripetal Force

Newton's second law: $F_{\text{net}} = ma$

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Direction: Always toward the center of rotation

6.2 Sources of Centripetal Force

Centripetal force can be provided by:

- **Friction:** Car tires on road

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- **Tension:** String on tetherball
- **Gravity:** Moon orbiting Earth
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The Mental Model

Centripetal force isn't a new kind of force - it's whatever force points toward the center and causes circular motion.

Attempt: Car on Curve

The Challenge (3 min, silent)

A 900 kg car rounds a curve with radius 600 m at speed 25.0 m/s.

Given:

- Mass $m = 900$ kg
- Radius $r = 600$ m
- Speed $v = 25.0$ m/s

Find: Centripetal force required to keep car on curve

How much force do the tires provide?

Compare: Car Force

Turn and talk (2 min):

- 1 What equation did you use for centripetal force?
- 2 Did you remember to square the velocity?
- 3 What force provides the centripetal force for a car?

Compare: Car Force

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Name wheel: One pair share your equation and reasoning.

Reveal: The Force That Turns

Self-correct in a different color:

Equation: $F_c = m \frac{v^2}{r}$

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Self-correct in a different color:

Equation: $F_c = m \frac{v^2}{r}$

Substitute:

$$F_c = \frac{(900 \text{ kg})(25.0 \text{ m/s})^2}{600 \text{ m}}$$

Reveal: The Force That Turns

Self-correct in a different color:

Equation: $F_c = m \frac{v^2}{r}$

Substitute:

$$F_c = \frac{(900 \text{ kg})(25.0 \text{ m/s})^2}{600 \text{ m}}$$

$$F_c = \frac{(900)(625)}{600} = \boxed{938 \text{ N}}$$

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Check: About 940 N - this is the friction force between tires and road. Without it, car slides straight!

Attempt: Acceleration Comparison

The Challenge (3 min, silent)

A car follows a curve of radius 500 m at speed 25.0 m/s.

Given:

- Radius $r = 500$ m
- Speed $v = 25.0$ m/s
- $g = 9.80$ m/s²

Find:

- 1 Centripetal acceleration
- 2 Express as fraction of g

How does turning compare to falling?

Compare: Acceleration Scale

Turn and talk (2 min):

- 1 What equation did you use for centripetal acceleration?
- 2 How did you express it as a fraction of g ?
- 3 Is the acceleration large or small compared to gravity?

Compare: Acceleration Scale

Turn and talk (2 min):

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Name wheel: One pair share your comparison method.

Reveal: Comparing to Gravity

Self-correct in a different color:

Calculate a_c :

$$a_c = \frac{v^2}{r} = \frac{(25.0 \text{ m/s})^2}{500 \text{ m}}$$

Reveal: Comparing to Gravity

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Calculate a_c :

$$a_c = \frac{v^2}{r} = \frac{(25.0 \text{ m/s})^2}{500 \text{ m}}$$

$$a_c = \frac{625}{500} = \boxed{1.25 \text{ m/s}^2}$$

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$$a_c = \frac{625}{500} = \boxed{1.25 \text{ m/s}^2}$$

Compare to g :

$$\frac{a_c}{g} = \frac{1.25}{9.80} = 0.128 \quad \Rightarrow \quad \boxed{a_c = 0.13g}$$

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Revelation: Gentle highway curve at moderate speed produces about 1/10th the acceleration of gravity!

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- **6.3:** Describe torque and lever arm
- **6.3:** Solve problems involving torque and rotational kinematics

6.3 When Spin Changes

So far: constant angular velocity

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But what if spin changes?

- Figure skater pulls arms in - spins faster

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Universal Law: Angular Acceleration

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Rate of change of angular velocity

6.3 Connecting Linear and Angular Acceleration

The Bridge

$$a = r\alpha$$

or

$$\alpha = \frac{a}{r}$$

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The Bridge

$$a = r\alpha \quad \text{or} \quad \alpha = \frac{a}{r}$$

Tangential acceleration: Linear acceleration along the circle's edge

The Mental Model

Greater angular acceleration means greater tangential acceleration. Points farther from center have larger tangential acceleration for same α .

6.3 Rotational Kinematics Equations

Linear

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

Rotational

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

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$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

The Pattern

Every linear kinematics equation has a rotational analog. Just swap $x \rightarrow \theta$, $v \rightarrow \omega$, $a \rightarrow \alpha$!

6.3 The Rotational Version of Force

Force causes linear acceleration

What causes angular acceleration?

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Universal Law: Torque

$$\tau = rF \sin \theta$$

Torque equals lever arm times force times sine of angle

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Force causes linear acceleration

What causes angular acceleration?

Universal Law: Torque

$$\tau = rF \sin \theta$$

Torque equals lever arm times force times sine of angle

Units: N·m (Newton-meters)

Direction: Same as the angular acceleration it produces

6.3 Maximizing Torque

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To maximize torque:

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- Apply force far from pivot (large r)
- Apply force perpendicular to lever arm ($\theta = 90^\circ$, so $\sin \theta = 1$)
- Apply larger force (large F)

Real-World Applications

- Door handle placed far from hinges
- Wrench with long handle
- Teeter-totter balanced by distance and weight

Attempt: Fishing Reel

The Challenge (3 min, silent)

A fishing reel spins at $\omega_0 = 220 \text{ rad/s}$. Fisherman applies brake creating angular acceleration $\alpha = -300 \text{ rad/s}^2$.

Given:

- Initial $\omega_0 = 220 \text{ rad/s}$
- Final $\omega = 0$ (stops)
- $\alpha = -300 \text{ rad/s}^2$

Find: Time t for reel to stop

How long does it take?

Compare: Fishing Reel

Turn and talk (2 min):

- 1 Which rotational kinematics equation did you choose?
- 2 How did you solve for time t ?
- 3 Why is α negative?

Compare: Fishing Reel

Turn and talk (2 min):

- 1 Which rotational kinematics equation did you choose?
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Name wheel: One pair share your equation choice and reasoning.

Reveal: Stopping the Spin

Self-correct in a different color:

Equation: $\omega = \omega_0 + \alpha t$

Reveal: Stopping the Spin

Self-correct in a different color:

Equation: $\omega = \omega_0 + \alpha t$

Solve for t :

$$t = \frac{\omega - \omega_0}{\alpha}$$

Reveal: Stopping the Spin

Self-correct in a different color:

Equation: $\omega = \omega_0 + \alpha t$

Solve for t :

$$t = \frac{\omega - \omega_0}{\alpha}$$

Substitute:

$$t = \frac{0 - 220 \text{ rad/s}}{-300 \text{ rad/s}^2} = \frac{-220}{-300}$$

Reveal: Stopping the Spin

Self-correct in a different color:

Equation: $\omega = \omega_0 + \alpha t$

Solve for t :

$$t = \frac{\omega - \omega_0}{\alpha}$$

Substitute:

$$t = \frac{0 - 220 \text{ rad/s}}{-300 \text{ rad/s}^2} = \frac{-220}{-300}$$

$t = 0.733 \text{ s}$

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Insight: Less than one second because the angular acceleration is quite large!

Attempt: Merry-Go-Round Torque

The Challenge (3 min, silent)

A man pushes a merry-go-round with force 250 N at the edge, perpendicular to the radius of 1.50 m.

Given:

- Force $F = 250$ N
- Lever arm $r = 1.50$ m
- Angle $\theta = 90^\circ$ (perpendicular)

Find: Torque τ produced

How effective is his push?

Compare: Torque Calculation

Turn and talk (2 min):

- 1 What is the value of $\sin 90^\circ$?
- 2 How does this simplify the torque equation?
- 3 Why did the man push at the edge and perpendicular?

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Name wheel: One pair explain why perpendicular force maximizes torque.

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Strategy: Man maximized torque by pushing perpendicular at the outer edge!

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- 5 Rotational kinematics mirrors linear kinematics
- 6 Torque $\tau = rF \sin \theta$ is the rotational force

Key Equations

$$\Delta\theta = \frac{\Delta s}{r} \quad (\text{angle of rotation}) \quad (1)$$

$$\omega = \frac{\Delta\theta}{\Delta t} \quad (\text{angular velocity}) \quad (2)$$

$$v = r\omega \quad (\text{tangential velocity}) \quad (3)$$

$$a_c = \frac{v^2}{r} = r\omega^2 \quad (\text{centripetal acceleration}) \quad (4)$$

$$F_c = m\frac{v^2}{r} = mr\omega^2 \quad (\text{centripetal force}) \quad (5)$$

$$\alpha = \frac{\Delta\omega}{\Delta t} \quad (\text{angular acceleration}) \quad (6)$$

$$a = r\alpha \quad (\text{tangential acceleration}) \quad (7)$$

$$\tau = rF \sin \theta \quad (\text{torque}) \quad (8)$$

Complete the assigned problems
posted on the LMS