2.4 Velocity vs. Time Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the meaning of slope and area in velocity vs. time graphs
- Solve problems using velocity vs. time graphs

Teacher Support

Teacher Support The learning objectives in this section will help your students master the following standards:

- (4) Science concepts. The student knows and applies the laws governing motion in a variety of situations. The student is expected to:
  - (A) generate and interpret graphs and charts describing different types of motion, including the use of real-time technology such as motion detectors or photogates.

Section Key Terms

Teacher Support

Teacher Support Ask students to use their knowledge of position graphs to construct velocity vs. time graphs. Alternatively, provide an example of a velocity vs. time graph and ask students what information can be derived from the graph. Ask—Is it the same information as in a position vs. time graph? How is the information portrayed differently? Is there any new information in a velocity vs. time graph?

Graphing Velocity as a Function of Time

Earlier, we examined graphs of position versus time. Now, we are going to build on that information as we look at graphs of velocity vs. time. Velocity is the rate of change of displacement. Acceleration is the rate of change of velocity; we will discuss acceleration more in another chapter. These concepts are all very interrelated.

Virtual Physics

Maze Game In this simulation you will use a vector diagram to manipulate a ball into a certain location without hitting a wall. You can manipulate the ball directly with position or by changing its velocity. Explore how these factors change the motion. If you would like, you can put it on the *a* setting, as well. This is acceleration, which measures the rate of change of velocity. We will explore acceleration in more detail later, but it might be interesting to take a look at it here.

Click to view content

If a person takes 3 steps and ends up in the exact same place as their starting point, what must be true?

a. The three steps must have equal displacement
b. The displacement of the third step is larger than the displacement of the first two.
c. The average velocity must add up to zero.
d. The distance and average velocity must add up to zero.

What can we learn about motion by looking at velocity vs. time graphs? Let's return to our drive to school, and look at a graph of position versus time as shown in Figure 2.15.

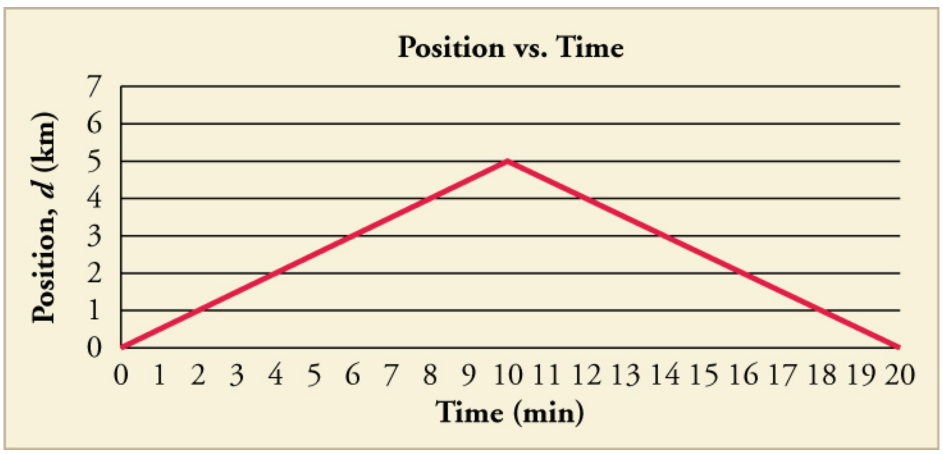

Figure 2.15 A graph of position versus time for the drive to and from school is shown.

We assumed for our original calculation that your parent drove with a constant velocity to and from school. We now know that the car could not have gone from rest to a constant velocity without speeding up. So the actual graph would be curved on either end, but let's make the same approximation as we did then, anyway.

Tips For Success

It is common in physics, especially at the early learning stages, for certain things to be *neglected*, as we see here. This is because it makes the concept clearer or the calculation easier. Practicing physicists use these kinds of short-cuts, as well.

It works out because usually the thing being *neglected* is small enough that it does not significantly affect the answer. In the earlier example, the amount of time it takes the car to speed up and reach its cruising velocity is very small compared to the total time traveled.

Looking at this graph, and given what we learned, we can see that there are two distinct periods to the car's motion—the way to school and the way back. The average velocity for the drive to school is 0.5 km/minute. We can see that the average velocity for the drive back is –0.5 km/minute. If we plot the data showing velocity versus time, we get another graph (Figure 2.16):

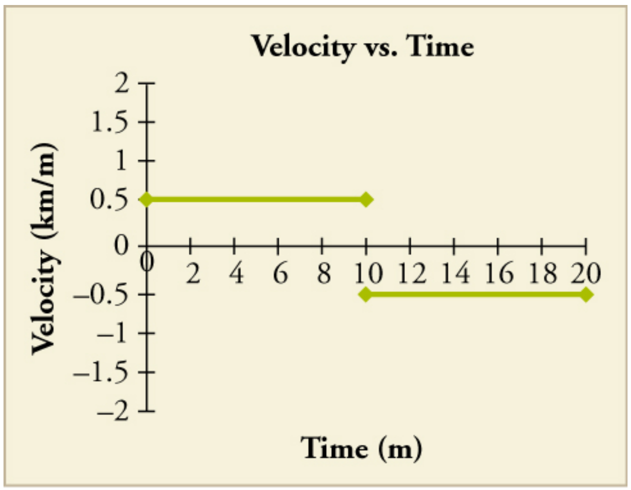

Figure 2.16 Graph of velocity versus time for the drive to and from school.

We can learn a few things. First, we can derive a $\mathbf{v}$ versus t graph from a $\mathbf{d}$ versus t graph. Second, if we have a straight-line position–time graph that is positively or negatively sloped, it will yield a horizontal velocity graph. There are a few other interesting things to note. Just as we could use a position vs. time graph to determine velocity, we can use a velocity vs. time graph to determine position. We know that $\mathbf{v} = \mathbf{d}/t$. If we use a little algebra to re-arrange the equation, we see that $\mathbf{d} = \mathbf{v} \times t$. In Figure 2.16, we have velocity on the y-axis and time along the x-axis. Let's take just the first half of the motion. We get 0.5 km/minute $\times$ 10 minutes. The units for *minutes* cancel each other, and we get 5 km, which is the displacement for the trip to school. If we calculate the same for the return trip, we get –5 km. If we add them together, we see that the net displacement for the whole trip is 0 km, which it should be because we started and ended at the same place.

Tips For Success

You can treat units just like you treat numbers, so a km/km=1 (or, we say, it cancels out). This is good because it can tell us whether or not we have calculated everything with the correct units. For instance, if we end up with m $\times$ s for velocity instead of m/s, we know that something has gone wrong, and we need to check our math. This process is called dimensional analysis, and it is one of the best ways to check if your math makes sense in physics.

The area under a velocity curve represents the displacement. The velocity curve also tells us whether the car is speeding up. In our earlier example, we stated that the velocity was constant. So, the car is not speeding up. Graphically, you can see that the slope of these two lines is 0. This slope tells us that the car is not speeding up, or accelerating. We will do more with this information in a later chapter. For now, just remember that the area under the graph and the slope are the two important parts of the graph. Just like we could define a linear equation for the motion in a position vs. time graph, we can also define one for a velocity vs. time graph. As we said, the slope equals the acceleration, $\mathbf{a}$. And in this graph, the y-intercept is $\mathbf{v}_0$. Thus, $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$.

But what if the velocity is not constant? Let's look back at our jet-car example. At the beginning of the motion, as the car is speeding up, we saw that its position is a curve, as shown in Figure 2.17.

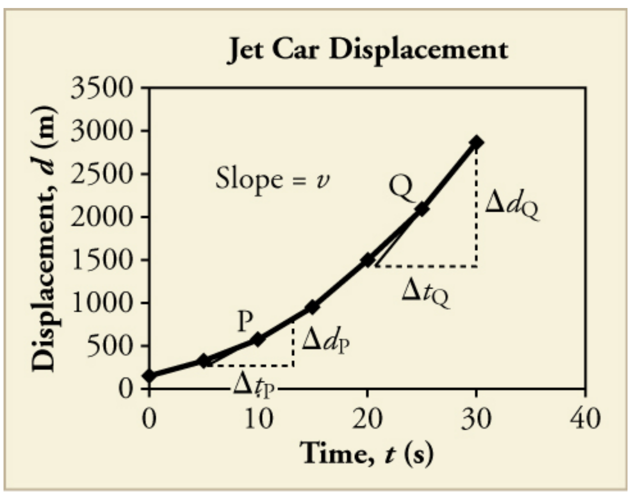

Figure 2.17 A graph is shown of the position of a jet-powered car during the time span when it is speeding up. The slope of a d vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

You do not have to do this, but you could, theoretically, take the instantaneous velocity at each point on this graph. If you did, you would get Figure 2.18, which is just a straight line with a positive slope.

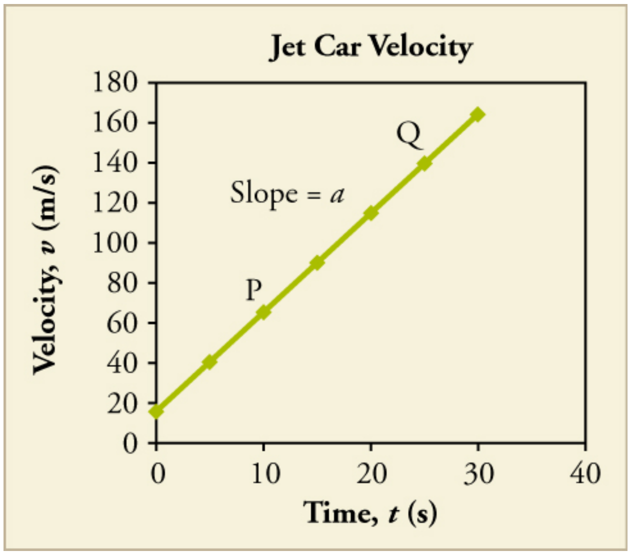

Figure 2.18 The graph shows the velocity of a jet-powered car during the time span when it is speeding up.

Again, if we take the slope of the velocity vs. time graph, we get the acceleration, the rate of change of the velocity. And, if we take the area under the slope, we get back to the displacement.

Teacher Support

Teacher Support

Teacher Demonstration

Return to the scenario of the drive to and from school. Re-draw the V-shaped position graph. Ask the students what the velocity is at different times on that graph. Students should then be able to see that the corresponding velocity graph is a horizontal line at 0.5km/minute and then a horizontal line at –0.5 km/minute. Then draw a few velocity graphs and see if they can get the corresponding position graph.

[OL][AL] Have students describe the relationship between the velocity and the position on these graphs. Ask—Can a velocity graph be used to find the position? Can a velocity graph be used to find anything else?

[AL] What is wrong with this graph? Ask students whether the velocity could actually be constant from rest or shift to negative so quickly. What would more realistic graphs look like? How inaccurate is it to ignore the non-constant portion of the motion?

[OL] Students should be able to see that if a position graph is a straight line, then the velocity graph will be a horizontal line. Also, the instantaneous velocity can be read off the velocity graph at any moment, but more steps are needed to calculate the average velocity.

[AL] Guide students in seeing that the area under the velocity curve is actually the position and the slope represents the rate of change of the velocity, just as the slope of the position line represents the rate of change of the position.

Solving Problems using Velocity–Time Graphs

Most velocity vs. time graphs will be straight lines. When this is the case, our calculations are fairly simple.

Worked Example

Using Velocity Graph to Calculate Some Stuff: Jet Car Use this figure to (a) find the displacement of the jet car over the time shown (b) calculate the rate of change (acceleration) of the velocity. (c) give the instantaneous velocity at 5 s, and (d) calculate the average velocity over the interval shown.

Strategy

a. The displacement is given by finding the area under the line in the velocity vs. time graph.
b. The acceleration is given by finding the slope of the velocity graph.
c. The instantaneous velocity can just be read off of the graph.
d. To find the average velocity, recall that $\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_{\text{f}} - \mathbf{d}_0}{t_{\text{f}} - t_0}$

Solution

a.
1. Analyze the shape of the area to be calculated. In this case, the area is made up of a rectangle between 0 and 20 m/s stretching to 30 s. The area of a rectangle is length × width. Therefore, the area of this piece is 600 m.
2. Above that is a triangle whose base is 30 s and height is 140 m/s. The area of a triangle is 0.5 × length × width. The area of this piece, therefore, is 2,100 m.
3. Add them together to get a net displacement of 2,700 m.

b.
1. Take two points on the velocity line. Say, t = 5 s and t = 25 s. At t = 5 s, the value of $\mathbf{v}$ = 40 m/s.
At t = 25 s, $\mathbf{v}$ = 140 m/s.
2. Find the slope.

$$\begin{aligned} \mathbf{a} &= \frac{\Delta \mathbf{v}}{\Delta t} \\ &= \frac{100 \text{ m/s}}{20 \text{ s}} \\ &= 5 \text{ m/s}^2 \end{aligned}$$

c. The instantaneous velocity at $t = 5$ s, as we found in part (b) is just 40 m/s.
d. 1. Find the net displacement, which we found in part (a) was 2,700 m.
   2. Find the total time which for this case is 30 s.
   3. Divide 2,700 m/30 s = 90 m/s.

Discussion

The average velocity we calculated here makes sense if we look at the graph. 100m/s falls about halfway across the graph and since it is a straight line, we would expect about half the velocity to be above and half below.

Teacher Support

Teacher Support The quantities solved for are slightly different in the different kinds of graphs, but students should begin to see that the process of analyzing or breaking down any of these graphs is similar. Ask—Where are the turning points in the motion? When is the object moving forward? What does a curve in the graph mean? Also, students should start to have an intuitive understanding of the relationship between position and velocity graphs.

Tips For Success

You can have negative position, velocity, and acceleration on a graph that describes the way the object is moving. You should never see a graph with negative time on an axis. Why?

Most of the velocity vs. time graphs we will look at will be simple to interpret. Occasionally, we will look at curved graphs of velocity vs. time. More often, these curved graphs occur when something is speeding up, often from rest. Let's look back at a more realistic velocity vs. time graph of the jet car's motion that takes this *speeding up* stage into account.

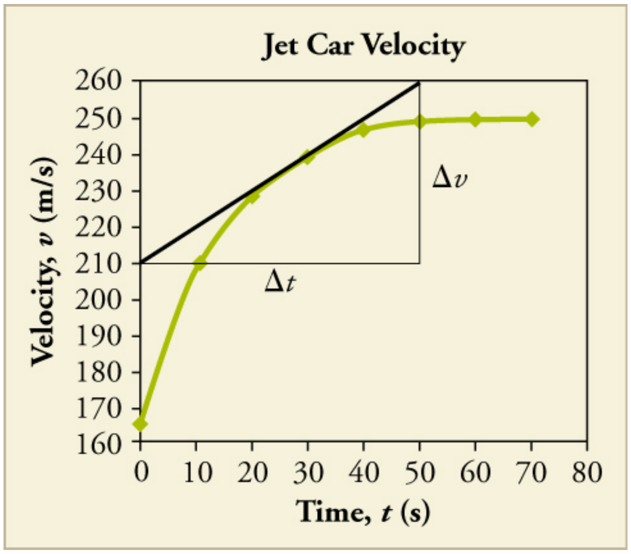

Figure 2.19 The graph shows a more accurate graph of the velocity of a jet-powered car during the time span when it is speeding up.

Worked Example

Using Curvy Velocity Graph to Calculate Some Stuff: Jet Car, Take Two Use Figure 2.19 to (a) find the approximate displacement of the jet car over the time shown, (b) calculate the instantaneous acceleration at $t = 30$ s, (c) find the instantaneous velocity at 30 s, and (d) calculate the approximate average velocity over the interval shown.

Strategy

a. Because this graph is an undefined curve, we have to estimate shapes over smaller intervals in order to find the areas.
b. Like when we were working with a curved displacement graph, we will need to take a tangent line at the instant we are interested and use that to calculate the instantaneous acceleration.
c. The instantaneous velocity can still be read off of the graph.
d. We will find the average velocity the same way we did in the previous example.

Solution

a.
   1. This problem is more complicated than the last example. To get a good estimate, we should probably break the curve into four sections. $0 \to 10$ s, $10 \to 20$ s, $20 \to 40$ s, and $40 \to 70$ s.
   2. Calculate the bottom rectangle (common to all pieces). 165 m/s $\times$ 70 s = 11,550 m.

3. Estimate a triangle at the top, and calculate the area for each section. Section 1 = 225 m; section 2 = 100 m + 450 m = 550 m; section 3 = 150 m + 1,300 m = 1,450 m; section 4 = 2,550 m.
4. Add them together to get a net displacement of 16,325 m.

b. Using the tangent line given, we find that the slope is 1 m/s^2.

c. The instantaneous velocity at $t = 30$ s, is 240 m/s.

d. 1. Find the net displacement, which we found in part (a), was 16,325 m.
2. Find the total time, which for this case is 70 s.
3. Divide $\frac{16{,}325\ \text{m}}{70\ \text{s}} \sim 233\ \text{m/s}$

Discussion

This is a much more complicated process than the first problem. If we were to use these estimates to come up with the average velocity over just the first 30 s we would get about 191 m/s. By approximating that curve with a line, we get an average velocity of 202.5 m/s. Depending on our purposes and how precise an answer we need, sometimes calling a curve a straight line is a worthwhile approximation.

Teacher Support

Teacher Support Finding the tangent line can be a challenging concept for high school students, and they need to understand it theoretically. If you drew a regular curve inside of the curve at the point you are interested in, you could draw a radius of that curve. The tangent line would be the line perpendicular to that radius.

[OL] Have the students compare this problem and the last one. Ask—What is the difference? When would you care about the more accurate picture of the motion? And when would it really not matter? Why would you ever want to look at a less accurate depiction of motion?

Practice Problems

20.

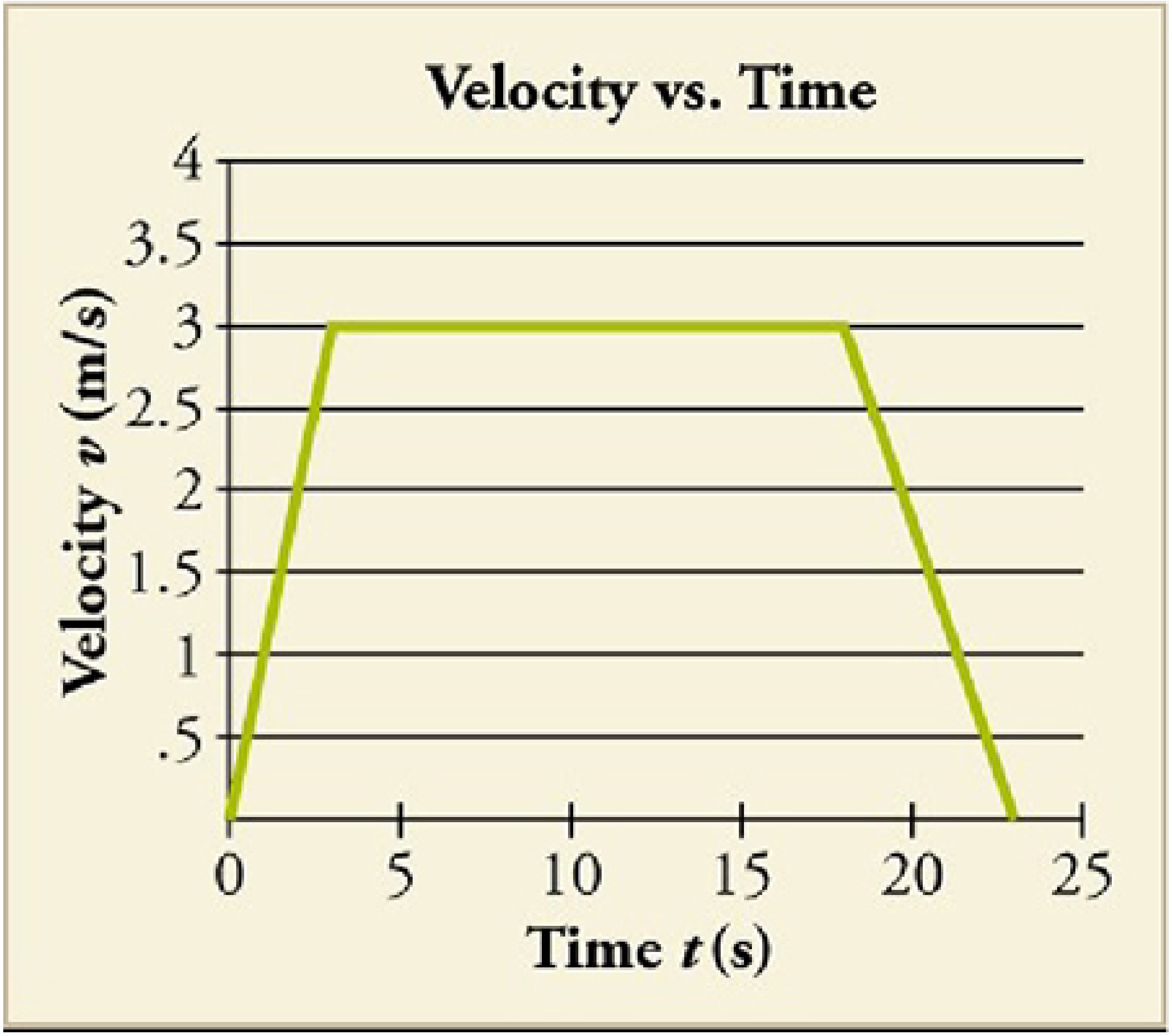

Figure 2.20

Consider the velocity vs. time graph shown below of a person in an elevator. Suppose the elevator is initially at rest. It then speeds up for 3 seconds, maintains that velocity for 15 seconds, then slows down for 5 seconds until it stops. Find the instantaneous velocity at $t = 10$ s and $t = 23$ s.

a. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 0 m/s and 0 m/s.
b. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 0 m/s and 3 m/s.
c. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 3 m/s and 0 m/s.
d. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 3 m/s and 1.5 m/s.

21.

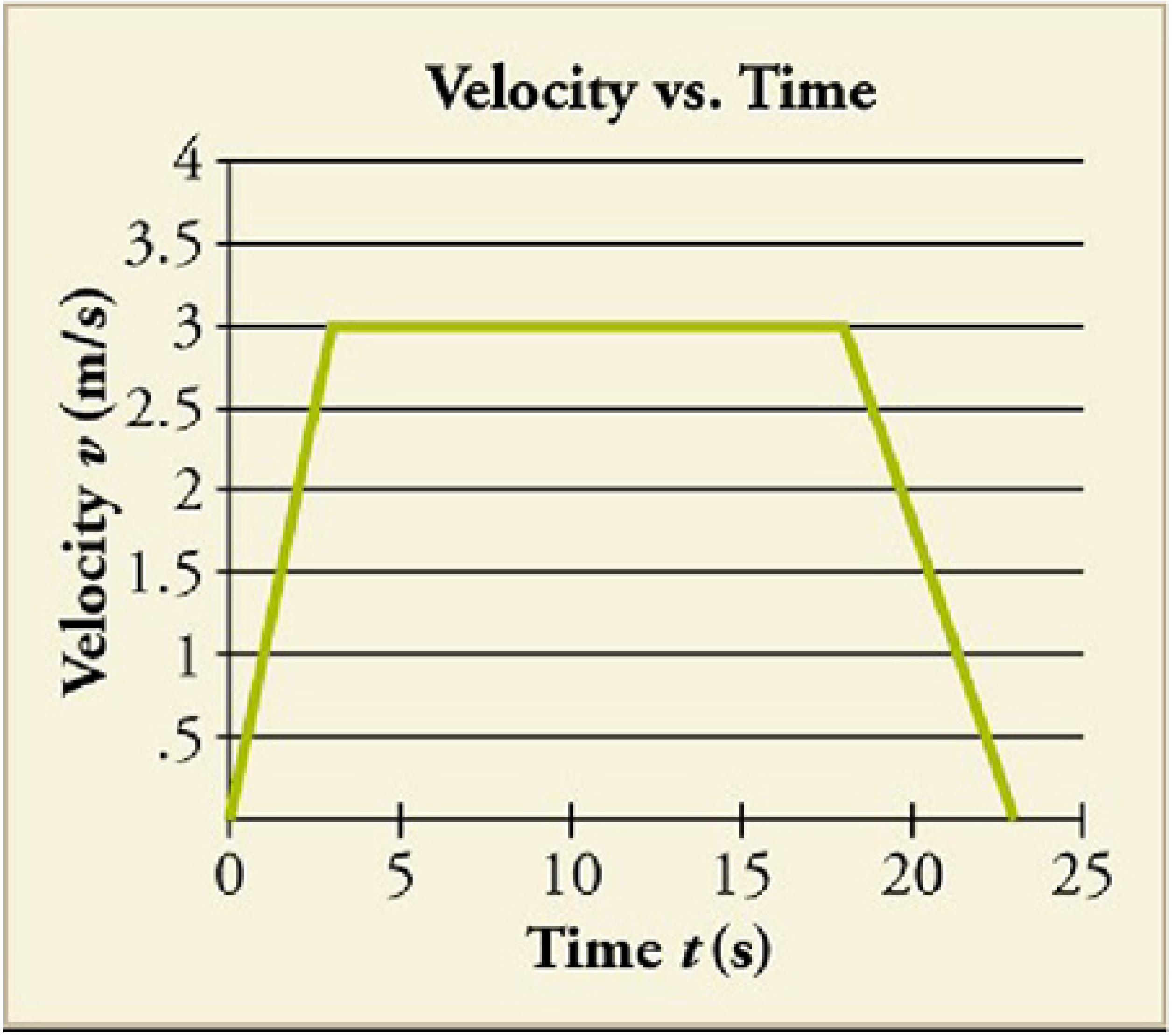

Figure 2.21

Calculate the net displacement and the average velocity of the elevator over the time interval shown.

a. Net displacement is 45 m and average velocity is 2.10 m/s.
b. Net displacement is 45 m and average velocity is 2.28 m/s.
c. Net displacement is 57 m and average velocity is 2.66 m/s.
d. Net displacement is 57 m and average velocity is 2.48 m/s.

Snap Lab

Graphing Motion, Take Two In this activity, you will graph a moving ball's velocity vs. time.

- your graph from the earlier Graphing Motion Snap Lab!
- 1 piece of graph paper
- 1 pencil

Procedure

1. Take your graph from the earlier Graphing Motion Snap Lab! and use it to create a graph of velocity vs. time.
2. Use your graph to calculate the displacement.

22.

Describe the graph and explain what it means in terms of velocity and acceleration.

a. The graph shows a horizontal line indicating that the ball moved with a constant velocity, that is, it was not accelerating.
b. The graph shows a horizontal line indicating that the ball moved with a constant velocity, that is, it was accelerating.
c. The graph shows a horizontal line indicating that the ball moved with a variable velocity, that is, it was not accelerating.
d. The graph shows a horizontal line indicating that the ball moved with a variable velocity, that is, it was accelerating.

Teacher Support

Teacher Support In this lab, students will use the displacement graph they drew in the last snap lab to create a velocity graph. If the rolling ball slowed down in the last snap lab, perhaps due to the ramp being too low, then the graph may not show constant velocity.

Check Your Understanding

23.

What information could you obtain by looking at a velocity vs. time graph?

a. acceleration
b. direction of motion
c. reference frame of the motion
d. shortest path

24.

How would you use a position vs. time graph to construct a velocity vs. time graph and vice versa?

a. The slope of a position vs. time curve is used to construct a velocity vs. time curve, and the slope of a velocity vs. time curve is used to construct a position vs. time curve.
b. The slope of a position vs. time curve is used to construct a velocity vs. time curve, and the area of a velocity vs. time curve is used to construct a position vs. time curve.
c. The area of a position vs. time curve is used to construct a velocity vs. time curve, and the slope of a velocity vs. time curve is used to construct a position vs. time curve.
d. The area of a position vs. time curve is used to construct a velocity vs. time curve, and the area of a velocity vs. time curve is used to construct a position vs. time curve.

Teacher Support

Teacher Support Use the *Check Your Understanding* questions to assess students' achievement of the section's learning objectives. If students are struggling with a specific objective, the *Check Your Understanding* will help direct students to the relevant content.