

PHYS11 CH6: Uniform Circular Motion

Sections 6.1-6.4: Rotational Motion and Forces

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Learning Objectives

By the end of this presentation, you will be able to:

- Define and calculate rotation angle and angular velocity
- Explain centripetal acceleration and its properties
- Analyze forces in circular motion
- Understand non-inertial frames and fictitious forces

Rotation Angle

Definition

The rotation angle $\Delta\theta$ is defined as:

$$\Delta\theta = \frac{\Delta s}{r}$$

where:

- Δs = arc length
- r = radius of curvature
- Measured in radians (rad)
- One complete revolution: $2\pi \text{ rad} = 360^\circ$

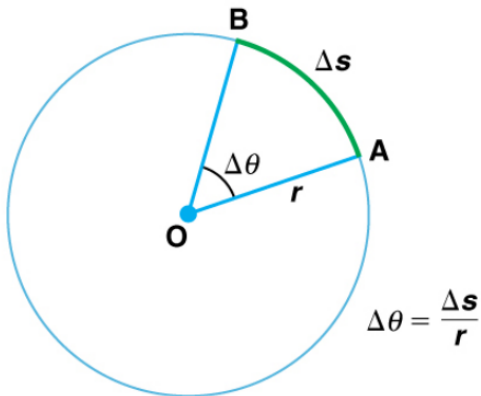


FIGURE 6.3 The radius of a circle is rotated through an angle $\Delta\theta$. The arc length Δs is described on the circumference.

Angular Velocity

Definition

Angular velocity ω is the rate of change of angle:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Relationship to Linear Velocity

$$v = r\omega$$

where:

- v = linear velocity
- r = radius
- ω = angular velocity

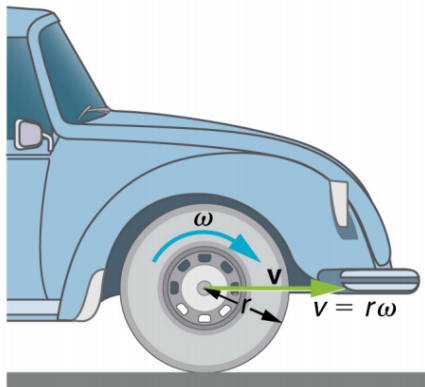


FIGURE 6.5 A car moving at a velocity v to the right has a tire rotating with an angular velocity ω . The speed of the tread of the tire relative to the axle is v , the same as if the car were jacked up. Thus the car moves forward at linear velocity $v = r\omega$, where r is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

- Centripetal Acceleration
- <https://www.youtube.com/watch?v=90rFibLktF4>
- Application
- https://youtu.be/im-JM0f_j7s?si=VO4FyEuT5SLf7Fzr

Centripetal Acceleration

Definition

Centripetal acceleration is the acceleration toward the center of circular motion:

$$a_c = \frac{v^2}{r} = r\omega^2$$

- Always points toward center of circle
- Magnitude depends on speed and radius
- Required for circular motion

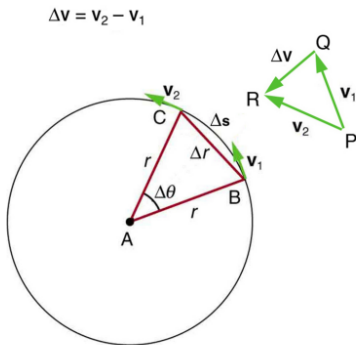


FIGURE 6.7 The directions of the velocity of an object at two different points are shown, and the change in velocity $\Delta \mathbf{v}$ is seen to point directly toward the center of curvature. (See small inset.) Because $\mathbf{a}_c = \Delta \mathbf{v} / \Delta t$, the acceleration is also toward the center; \mathbf{a}_c is called centripetal acceleration. (Because $\Delta \theta$ is very small, the arc length Δs is equal to the chord length Δr for small time differences.)

Example: Centripetal Acceleration

I Do: Car on Curved Path

A car travels around a curve of radius 100 m at 20 m/s. Calculate the centripetal acceleration.

$$\begin{aligned}a_c &= \frac{v^2}{r} \\&= \frac{(20 \text{ m/s})^2}{100 \text{ m}} \\&= 4 \text{ m/s}^2\end{aligned}$$

- Centripetal Force
- <https://www.youtube.com/watch?v=4bMawIIWi7w>

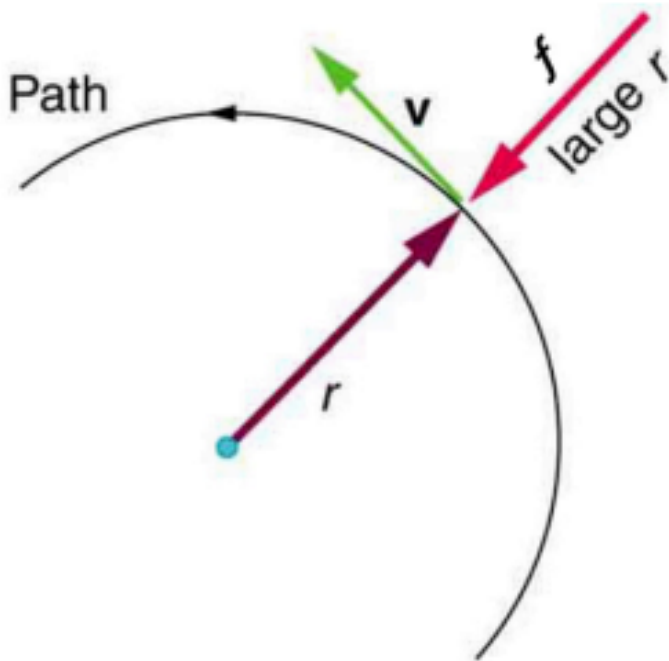
Centripetal Force

Definition

The centripetal force required for circular motion is:

$$F_c = ma_c = m\frac{v^2}{r} = mr\omega^2$$

- Net force must point toward center
- Can be provided by various forces:
 - Tension
 - Gravity
 - Friction
 - Normal force



We Do: Centripetal Force Problem

Problem

A 1000 kg car travels at 15 m/s around a curve of radius 50 m. What centripetal force is required?

$$\begin{aligned} F_c &= m \frac{v^2}{r} \\ &= (1000 \text{ kg}) \frac{(15 \text{ m/s})^2}{50 \text{ m}} \\ &= 4500 \text{ N} \end{aligned}$$

Key Points

- Appear in non-inertial (accelerating) frames
- Not "real" forces - arise from acceleration of reference frame
- Examples:
 - Centrifugal force
 - Coriolis force

- Centrifugal force
- <https://www.youtube.com/watch?v=gRVIWWJwzfY>

- Coriolis force
- <https://www.youtube.com/watch?v=rdGtcZSFRLk>

The Coriolis Effect

Properties

- Appears in rotating reference frames
- Affects motion on rotating Earth
- Causes deflection of:
 - Weather systems
 - Projectiles
 - Ocean currents

You Do: Practice Problem

Problem

A 0.5 kg ball is attached to a string and swung in a horizontal circle of radius 1.5 m. If the ball makes one complete revolution in 2 seconds:

- 1 Calculate the angular velocity
- 2 Find the centripetal acceleration
- 3 Determine the tension in the string

Summary

Key Concepts

- Angular quantities describe rotational motion
- Centripetal acceleration points to center
- Centripetal force causes circular motion
- Fictitious forces appear in non-inertial frames