CS12: Introduction to Big O Notation

Understanding Algorithm Efficiency

Mr. Gullo

Key Terms for This Lesson

Vocabulary

Algorithm: A step-by-step set of instructions to solve a problem

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Algorithm: A step-by-step set of instructions to solve a problem

Efficiency: How fast and how little memory an algorithm uses

Input size (n): The amount of data the algorithm works with

Complexity: How the time or space grows as n gets bigger

Learning Objectives

By the end of this lesson, you will be able to:

- Define algorithm efficiency in your own words

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- Define algorithm efficiency in your own words
- Identify and explain common Big O notations
- Analyze simple algorithms to determine their time complexity
- Compare different algorithms based on their efficiency

What is Big O Notation?

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- Think of it as a way to measure an algorithm's speed
- Helps us compare different solutions
- Focuses on the slowest possible situation
- Ignores smaller details and focuses on the main pattern

What is Worst Case?

Why do we care about worst case?

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Example: Searching for 99 in this array:

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- **Worst case**: Target is at the END (or not there at all)
- We had to check **all 9 elements** = O(n) operations

Best Case vs Worst Case

12	5	23	7	42	15	8	31	99

Best Case: O(1)

- Target = 12 (first element)
- Found in 1 check!

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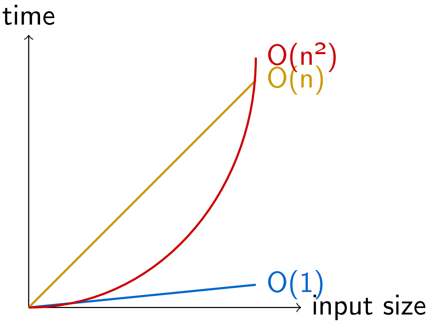
Why Worst Case?

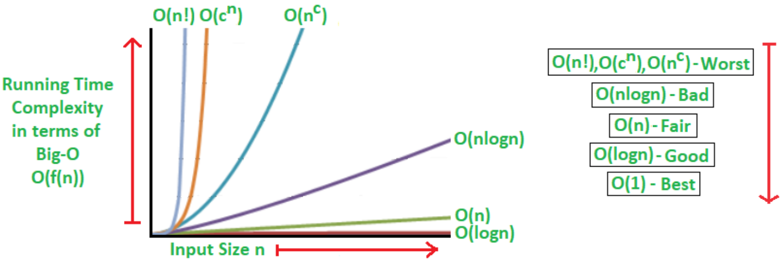
We plan for the worst so our program never surprises us with slow performance.

How Different Algorithms Grow

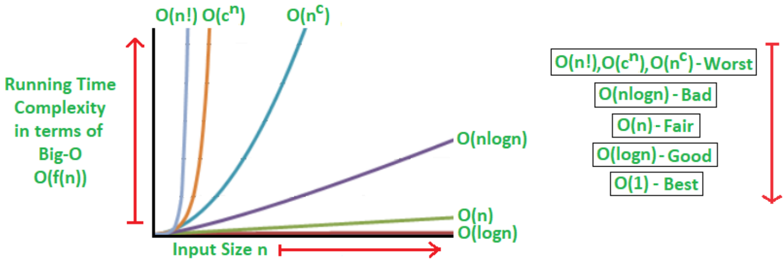


As input grows...

Big O Complexity Comparison



Big O Complexity Comparison



Key insight: As input grows, the gap between O(1) and O(n!) becomes enormous.

O(1) - Constant Time

- Finding a book on your desk
- Looking up array element by index

Understanding Through Real Examples

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O(n) - **Linear Time**

- Searching through a line of books
- Finding maximum in unsorted array

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- Bubble sort algorithm

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O(log n) - Logarithmic Time

- Guessing a number 1-100 by halving
- Binary search (sorted data only!)

Linear Search: Check Every Element

Searching for 42:

Found!

12	5	23	7	42	15	8	31	99

Linear Search: Check Every Element

Searching for 42:

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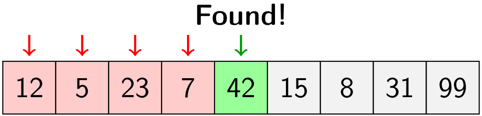
12	5	23	7	42	15	8	31	99

- Check index 0, 1, 2, 3... until we find 42

Linear Search: Check Every Element

Searching for 42:



- Check index 0, 1, 2, 3... until we find 42
- **5 checks** to find element at index 4

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- Check index 0, 1, 2, 3... until we find 42
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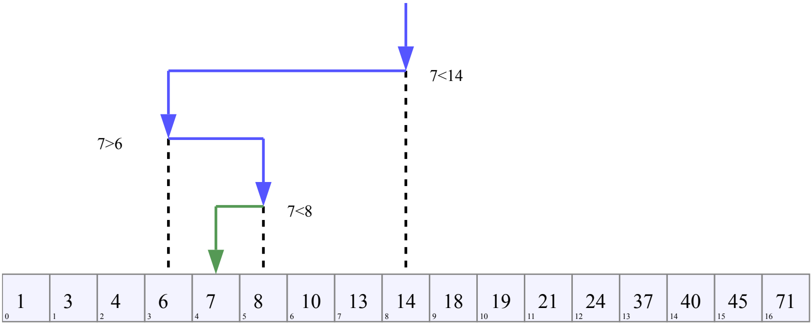
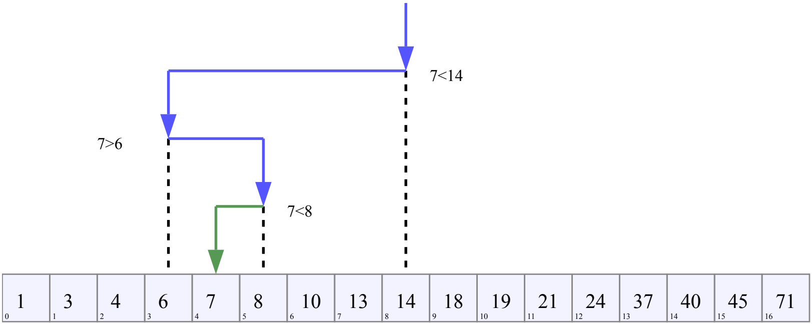
O(n): In worst case, we check every single element.

Binary Search: Divide and Conquer



Binary Search: Divide and Conquer



O(log n): Each step eliminates **half** the remaining elements.

Requirement

Binary search only works on **sorted** data!

Image: Wikimedia Commons

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Simple Definition

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$$\boxed{\text{8 items}} \xrightarrow[\nabla \cdot 2]{} \boxed{4} \xrightarrow[\nabla \cdot 2]{} \boxed{2} \xrightarrow[\nabla \cdot 2]{} \boxed{1}$$

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The Pattern

$$\begin{aligned} 2^3 &= 8 &\Leftrightarrow\quad& \log_2(8) = 3 \\ 2^4 &= 16 &\Leftrightarrow\quad& \log_2(16) = 4 \\ 2^{10} &= 1024 &\Leftrightarrow\quad& \log_2(1024) = 10 \end{aligned}$$

Why Logarithms are Amazing

n (items)	O(n) checks	O(log n) checks
8	8	3
16	16	4
256	256	8
1,024	1,024	10
1,000,000	1,000,000	20
1,000,000,000	1,000,000,000	30

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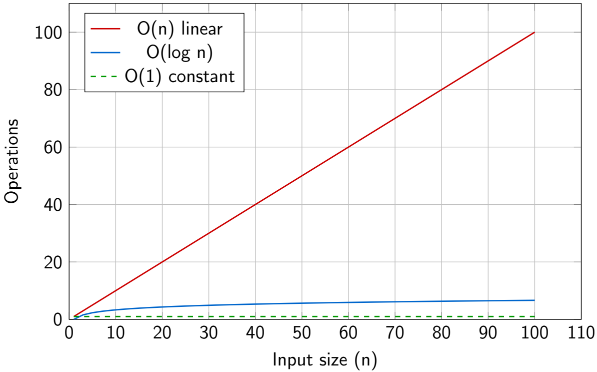
Mind-Blowing

1 billion items with binary search = only **30 checks!**

Graph: O(log n) vs O(n)



Binary Search: Halving in Action

Find 44 in sorted array of 16 elements:

Step 1:	3	7	12	19	23	31	38	42	44	51	56	62	71	78	85	99	check 42, go RIGHT

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check 42, go RIGHT

Step 2:

44	51	56	62	71	78	85	99

check 56, go LEFT

Step 3:

44	51	56	62

check 51, go LEFT

Binary Search: Halving in Action

Find 44 in sorted array of 16 elements:

Step 1:	3 7 12 19 23 31 38 42 44 51 56 62 71 78 85 99	check 42, go RIGHT
Step 2:	44 51 56 62 71 78 85 99	check 56, go LEFT
Step 3:	44 51 56 62	check 51, go LEFT
Step 4:	44	FOUND! Only 4 checks for 16 items

Binary Search: Halving in Action

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Step 1: 3 | 7 | 12 | 19 | 23 | 31 | 38 | 42 | 44 | 51 | 56 | 62 | 71 | 78 | 85 | 99 — check 42, go RIGHT

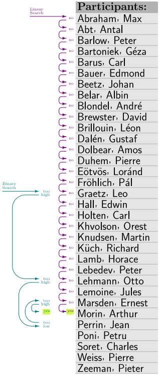
Step 2: 44 | 51 | 56 | 62 | 71 | 78 | 85 | 99 — check 56, go LEFT

Step 3: 44 | 51 | 56 | 62 — check 51, go LEFT

Step 4: 44 — FOUND! Only 4 checks for 16 items

$\log_2(16) = 4$ checks vs Linear: up to 16 checks

Linear vs Binary Search



Linear Search

- Works on any data
- O(n) time

Image: Wikimedia Commons

Binary Search

- Requires sorted data
- O(log n) time

I Do: Analyzing Linear Search

Problem

Let's analyze this linear search algorithm:

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```
int linearSearch(int arr[], int n, int x) {
    for(int i = 0; i < n; i++) {
        if(arr[i] == x) {
            return i;  // Found it!
        }
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- Time Complexity: O(n)
- Why? In worst case, we check every element

We Do: Let's Analyze Together

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void printPairs(int arr[], int n) {
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- Let's count the operations...
- Outer loop runs n times
- For each outer loop, inner loop runs n times
- Total operations: n times n = n squared

You Do: Practice Time!

Analyze These Operations

Determine the Big O notation for:

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1. O(1) - Direct access, no matter the size

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Solutions

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2. O(n) - Must check every element once
3. O(1) - Single operation, size independent

Big O Quick Reference

Notation	Name	If n = 1000
O(1)	Constant	1 operation
O(log n)	Logarithmic	$\sim$10 operations
O(n)	Linear	1,000 operations
O(n log n)	Linearithmic	$\sim$10,000 operations
O(n^2)	Quadratic	1,000,000 operations
O(2^n)	Exponential	More than atoms in universe!

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Rule of thumb: Anything slower than O(n^2) is usually too slow for large data.

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Practice Makes Perfect

Try analyzing algorithms you write in your own code!