PHYS12 CH:5 Motion in Two Dimensions

Vectors and Projectiles

Mr. Gullo

December 2025

Outline

The Mystery

How do you describe motion

when objects move in two directions at once?

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A football spiraling downfield...

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A drone navigating through space...

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How do you describe motion
when objects move in two directions at once?

A football spiraling downfield...

A drone navigating through space...

Motion happens in multiple dimensions simultaneously.

Learning Objectives

By the end of this section, you will be able to:

- **5.1:** Describe the graphical method of vector addition and subtraction

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By the end of this section, you will be able to:

- **5.1:** Describe the graphical method of vector addition and subtraction
- **5.1:** Use the graphical method to solve physics problems

5.1 What Is a Vector?

The Dual Nature

A vector is a quantity with both magnitude AND direction.

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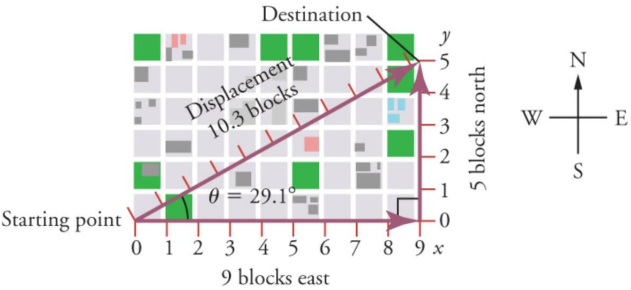
The Mental Model

Velocity is like a speedometer with a compass attached. Speed tells how fast. Velocity adds where.

5.1 The Journey to the Destination



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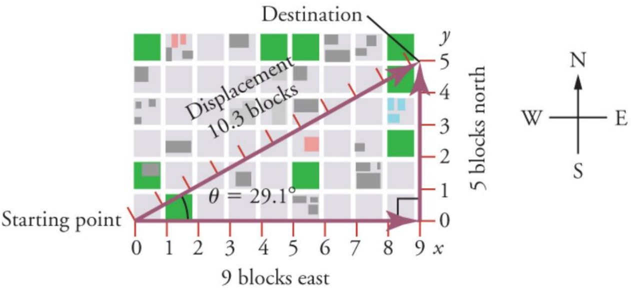

Path: 9 blocks east + 5 blocks north = 14 blocks walked
Displacement: 10.3 blocks at 29.1° north of east

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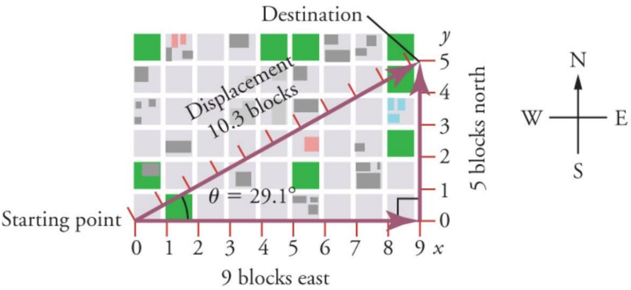

Path: 9 blocks east + 5 blocks north = 14 blocks walked
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Civilian View vs. Reality

Civilian: "I walked 14 blocks."
Physicist: "Displacement was 10.3 blocks."

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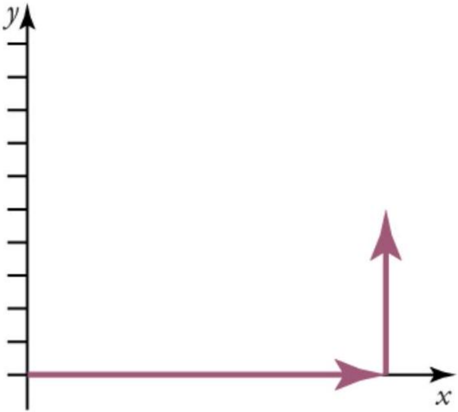
Key insight: Order doesn't matter! $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

5.1 Building the Resultant



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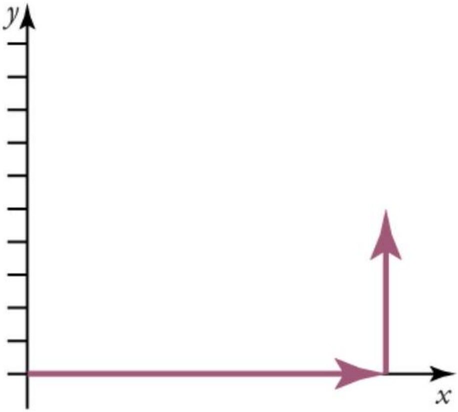

The resultant vector connects start to finish.

5.1 Vector Subtraction

To subtract: Add the negative vector

$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$

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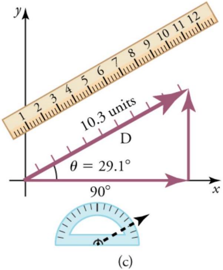

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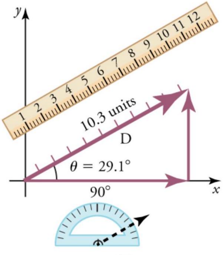
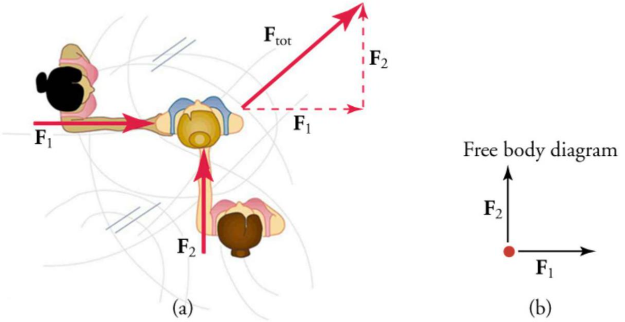
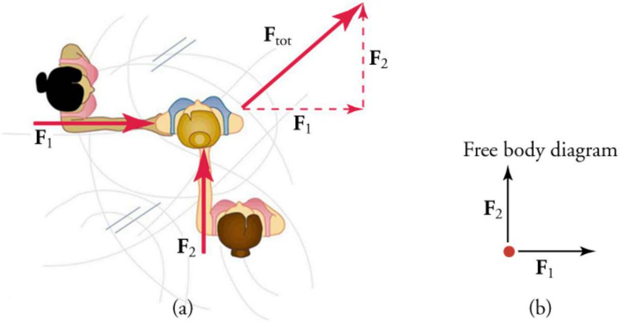


The negative vector: Same magnitude, opposite direction


$\mathbf{F}_{tot}$
$\mathbf{F}_2$
$\mathbf{F}_1$
$\mathbf{F}_1$
$\mathbf{F}_2$
(a)
Free body diagram
$\mathbf{F}_2$
$\mathbf{F}_1$
(b)

5.1 Forces on Ice



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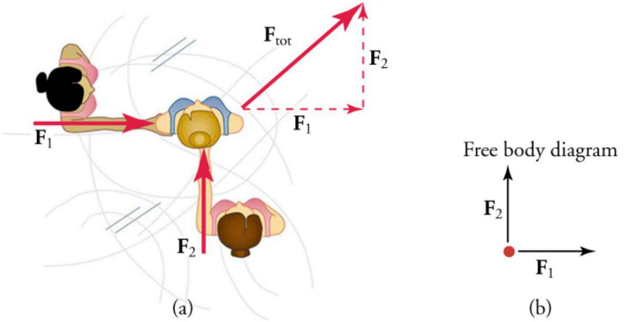

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Resultant force: $F_{tot} = \sqrt{(400)^2 + (400)^2} = 566$ N

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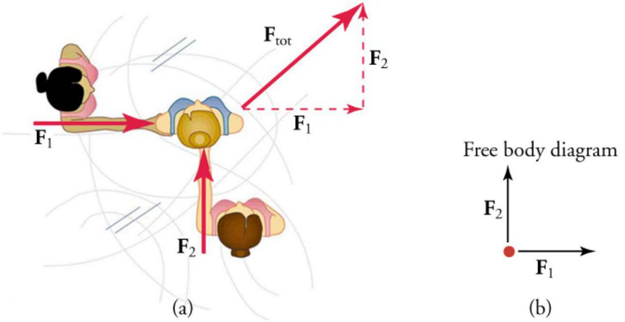

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Pythagorean theorem works when vectors are perpendicular!

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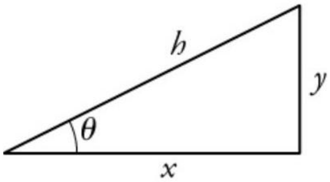
5.2 The Power of Trigonometry



$$\sin(\theta) = \frac{y}{h}$$

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$$\tan(\theta) = \frac{y}{x}$$

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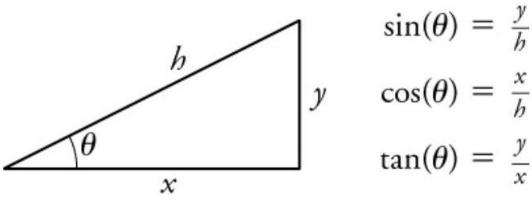
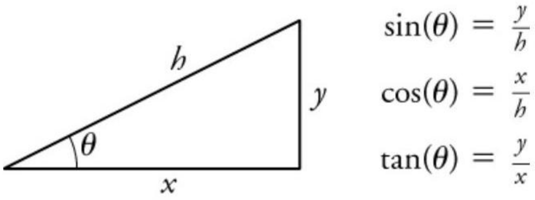


Refresher:

- $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$



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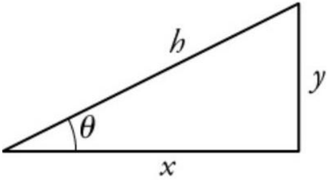

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The Source Code

$$\boxed{A_x = A\cos\theta}$$
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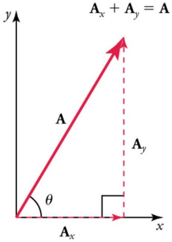

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Components match the original path: 9 east + 5 north!

5.2 Reverse: From Components to Vector

Reconstruction Formulas

$$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)}$$

Pythagorean theorem gives magnitude, inverse tangent gives direction.

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The Paradox

Students confuse: $\mathbf{A}_x + \mathbf{A}_y = \mathbf{A}$ (vector addition)
with $A = \sqrt{A_x^2 + A_y^2}$ (magnitude calculation)

To add vectors A and B analytically:

1. Find components: A_x, A_y, B_x, B_y

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More accurate than graphical method - not limited by drawing precision!

Attempt: Two-Leg Journey

The Challenge (3 min, silent)

A person walks 53.0 m at 20.0° north of east, then 34.0 m at 63.0° north of east.

Given:

- $A = 53.0$ m, $\theta_A = 20.0^\circ$
- $B = 34.0$ m, $\theta_B = 63.0^\circ$

Find: Total displacement magnitude and direction

Can you decode this journey? Work silently.

Compare: Vector Addition Strategy

Turn and talk (2 min):

1. What's the first step - find components or add vectors?
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Name wheel: One pair share your approach (not your answer).

Self-correct in a different color:

Step 1: Find components of **A**

$$A_x = (53.0)(\cos 20.0^\circ) = 49.8 \text{ m}$$
$$A_y = (53.0)(\sin 20.0^\circ) = 18.1 \text{ m}$$

Reveal: The Navigation Solution

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Step 2: Find components of **B**

$$B_x = (34.0)(\cos 63.0^\circ) = 15.4 \text{ m}$$
$$B_y = (34.0)(\sin 63.0^\circ) = 30.3 \text{ m}$$

Step 3: Add components

$$R_x = A_x + B_x = 49.8 + 15.4 = 65.2 \text{ m}$$
$$R_y = A_y + B_y = 18.1 + 30.3 = 48.4 \text{ m}$$

Reveal: Combining Components

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Step 4: Find magnitude

$$R = \sqrt{(65.2)^2 + (48.4)^2} = \boxed{81.2 \text{ m}}$$

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Step 5: Find direction

$$\theta = \tan^{-1}(48.4/65.2) = \boxed{36.6^\circ \text{ north of east}}$$

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- **5.3:** Describe the properties of projectile motion

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- **5.3:** Apply kinematic equations and vectors to solve projectile problems

5.3 The Great Separation

Nature's Rule for Projectiles

Horizontal and vertical motions

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They don't influence each other.

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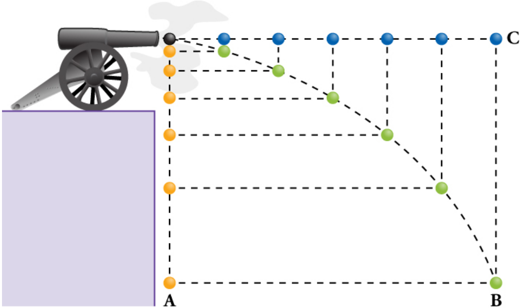

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- **Trajectory:** Its curved path
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We ignore air resistance in introductory physics.

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5.3 The Independence Principle

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- Constant acceleration: $a_y = -g = -9.8 \text{ m/s}^2$

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- Velocity: $v_y = v_{0y} - gt$
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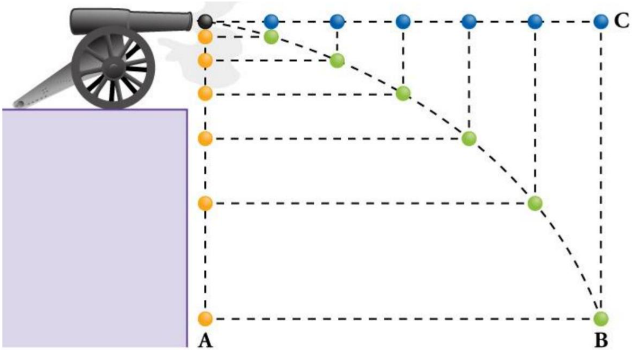
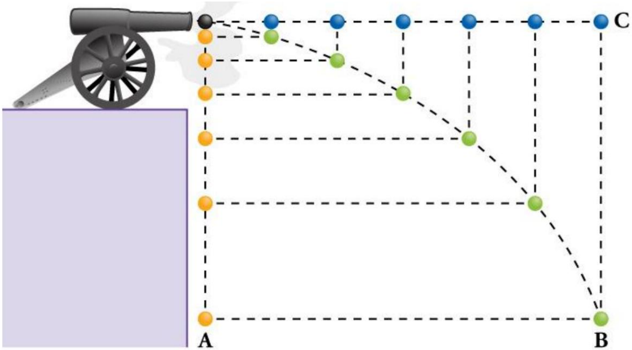
Time t is the only variable connecting the two motions!

5.3 The Trajectory





Ball kicked at angle θ follows parabolic path.

To solve projectile problems:

1. Separate into x and y components

5.3 The Four-Step Method

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2. Treat as two independent 1D motions

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Recombination formulas:

$$d = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}(y/x)$$

$$v = \sqrt{v_x^2 + v_y^2}, \quad \theta_v = \tan^{-1}(v_y/v_x)$$

5.3 Maximum Height Formula

The Peak

$$\boxed{h = \frac{v_{0y}^2}{2g}}$$

Maximum height depends only on initial vertical velocity.

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What Your Brain Gets Wrong

Intuition: "Faster launch means higher peak."
Reality: "Only the *vertical* component matters."

Attempt: Fireworks Launch

The Challenge (3 min, silent)

A fireworks shell is launched at 70.0 m/s at 75° above horizontal.

Given:

- $v_0 = 70.0$ m/s
- $\theta = 75^\circ$

Find: Maximum height

Can you predict where it explodes? Work silently.

Compare: Projectile Strategy

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Reveal: Reaching the Sky

Self-correct in a different color:
Step 1: Find vertical velocity component

$$\begin{aligned} v_{0y} &= v_0 \sin\theta \\ &= (70.0)(\sin 75^\circ) = 67.6 \text{ m/s} \end{aligned}$$

Self-correct in a different color:
Step 1: Find vertical velocity component

$$v_{0y} = v_0 \sin\theta$$
$$= (70.0)(\sin 75^\circ) = 67.6 \text{ m/s}$$

Step 2: Use maximum height formula

$$h = \frac{v_{0y}^2}{2g}$$
$$= \frac{(67.6)^2}{2(9.8)} = \boxed{233 \text{ m}}$$

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Step 2: Use maximum height formula

$$\begin{aligned} h &= \frac{v_{0y}^2}{2g} \\ &= \frac{(67.6)^2}{2(9.8)} = \boxed{233 \text{ m}} \end{aligned}$$

Check: About 765 feet - reasonable for large fireworks!

5.3 The Range Equation

The Distance Formula

$$\boxed{R = \frac{v_0^2 \sin 2\theta}{g}}$$

Range depends on initial speed and launch angle.

The Distance Formula

$$\boxed{R = \frac{v_0^2 \sin 2\theta}{g}}$$

Range depends on initial speed and launch angle.

Key insights:

- Maximum range at $\theta = 45^\circ$

5.3 The Range Equation

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- Same range for complementary angles: 30° and 60°
- Only valid when initial and final heights are equal

Attempt: Volcanic Rock

The Challenge (3 min, silent)

A rock is ejected from a volcano at 25.0 m/s at 35° above horizontal. It lands 20.0 m below its starting point.

Given:

- $v_0 = 25.0$ m/s, $\theta = 35^\circ$
- $y = -20.0$ m

Find: Time of flight

Can you predict the flight time? Work silently.

Compare: Quadratic Strategy

Turn and talk (2 min):

1. Which kinematic equation has both y and t?
2. What values do you substitute for y_0 and y?
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Name wheel: One pair share your approach (not your answer).

Reveal: Time of Flight

Self-correct in a different color:
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Step 4: Quadratic formula

$$t = \frac{14.3 \pm \sqrt{(14.3)^2 - 4(4.90)(-20.0)}}{2(4.90)}$$

Step 5: Calculate discriminant

$$\sqrt{(14.3)^2 + 4(4.90)(20.0)} = \sqrt{596.5} = 24.4$$

Reveal: Solving the Quadratic

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Negative time means before launch - physically impossible!

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1. Vectors have magnitude AND direction
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4. Projectile motion: horizontal and vertical are independent
5. Time connects the two dimensions
6. Maximum height depends only on v_{0y}

Key Equations

Vector Components:

$$A_x = A\cos\theta \tag{1}$$
$$A_y = A\sin\theta \tag{2}$$

Magnitude and Direction:

$$A = \sqrt{A_x^2 + A_y^2} \tag{3}$$
$$\theta = \tan^{-1}(A_y/A_x) \tag{4}$$

Projectile Motion:

$$\text{Horizontal: } x = x_0 + v_x t \tag{5}$$
$$\text{Vertical: } y = y_0 + v_{0y} t - \frac{1}{2} g t^2 \tag{6}$$
$$\text{Max height: } h = \frac{v_{0y}^2}{2g} \tag{7}$$

Homework

Complete the assigned problems

posted on the LMS