

# PHYS11 CH:5.1-5.3

## Vector Analysis and Applications

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# Introduction to Vector Analysis

- Vectors are essential in physics for describing quantities with magnitude and direction
- Key topics:
  - Vector addition and subtraction
  - Resolving vectors into components
  - Vector applications in real-world problems

# Table of Contents

- 1 Vector Operations
- 2 Vector Applications
- 3 Projectile Motion

# Trig Review

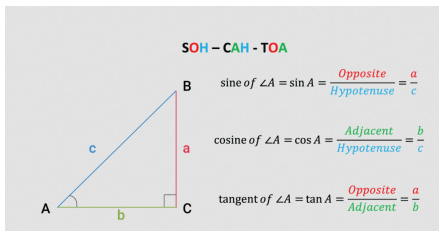


Figure: SOHCAHTOA mnemonic

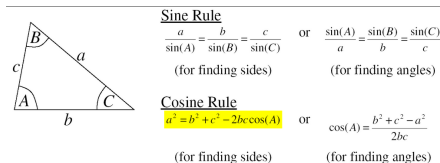


Figure: Sine and Cosine Laws

# Vector Addition and Subtraction

- Vector addition: - Tip-to-tail method - Parallelogram method
- Vector subtraction: - Add the negative of the vector:  
$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$
- Resultant vector:  $\vec{R} = \vec{A} + \vec{B}$ 
  - For n vectors:  $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \dots + \vec{N}$

# Vector Addition and Subtraction

- Magnitude of resultant:

- General case:  $|\vec{R}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$
- For perpendicular vectors:  $|\vec{R}| = \sqrt{A^2 + B^2}$

- Direction of resultant:

- Using magnitudes and angle:  $\tan \phi = \frac{B \sin \theta}{A + B \cos \theta}$
- Using components:  $\tan \phi = \frac{A_y + B_y}{A_x + B_x}$ ,

# Resolving Vectors into Components

- Any vector can be resolved into x and y components
- $A_x = A \cos \theta$ ,  $A_y = A \sin \theta$
- Magnitude:  $A = \sqrt{A_x^2 + A_y^2}$
- Direction:  $\tan \theta = \frac{A_y}{A_x}$

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# Example 1: River Crossing Problem

Problem: A river flows SW to NE at  $7.1 \text{ m/s}$ . A boat moving at  $13 \text{ m/s}$  wants to reach a point due east.

# Example 1: River Crossing Problem

Problem: A river flows SW to NE at 7.1 m/s. A boat moving at 13 m/s wants to reach a point due east.

Solution:

- River velocity components:  $v_{rx} = v_{ry} = 7.1 \cos 45 = 5.02 \text{ m/s}$
- Boat must counteract river's y-component:  $v_{by} = -5.02 \text{ m/s}$
- Angle of boat's heading:  $\theta = \arcsin(-5.02/13) \approx -22.6$

Answer: The boat should head  $22.6^\circ$  south of east.

## Example 2: Displacement Problem

Problem: A person walks 10.0 m north, then 2.0 m east. Find the resultant displacement.

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Problem: A person walks 10.0 m north, then 2.0 m east. Find the resultant displacement.

Solution:

- Use Pythagorean theorem:  $R = \sqrt{10.0^2 + 2.0^2} \approx 10.2$  m
- Angle:  $\theta = \arctan(10.0/2.0) \approx 78.7$  north of east

Answer:  $\vec{R} = 10.2$  m,  $78.7^\circ$  north of east

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# Projectile Motion Basics

- Combination of horizontal and vertical motion
- Horizontal motion: constant velocity (neglecting air resistance)
- Vertical motion: constant acceleration due to gravity
- Key equations:
  - Range:  $R = \frac{v_0^2 \sin(2\theta)}{g}$
  - Maximum height:  $h_{\max} = \frac{v_0^2 \sin^2 \theta}{2g}$
  - Time of flight:  $t = \frac{2v_0 \sin \theta}{g}$

## Example 3: Projectile Motion Problem

Problem: A water-balloon cannon fires at  $30 \text{ m/s}$ ,  $50^\circ$  above horizontal. Find the range.

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Problem: A water-balloon cannon fires at 30 m/s,  $50^\circ$  above horizontal. Find the range.

Solution:

- Use range equation:  $R = \frac{v_0^2 \sin(2\theta)}{g}$
- $R = \frac{(30 \text{ m/s})^2 \sin(2*50)}{9.8 \text{ m/s}^2} \approx 90.44 \text{ m}$

Answer: The water balloon will fall approximately 90.44 m away.



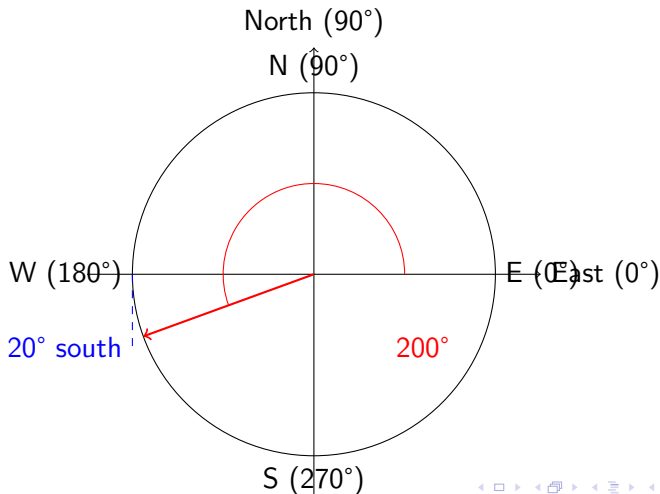
# Conclusion

- Vector analysis is crucial for understanding motion in physics
- Key concepts covered:
  - Vector addition and subtraction
  - Resolving vectors into components
  - Applications in real-world problems
  - Basics of projectile motion
- These concepts help analyze complex motions and solve practical problems
- Practice with various examples to master vector analysis

# Global Angles

## Problem

What is the global angle of  $20^\circ$  south of west?

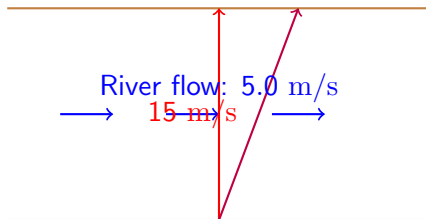


# River Crossing Problem

## Problem

A person attempts to cross a river in a straight line by navigating a boat at  $15 \text{ m/s}$ . If the river flows at  $5.0 \text{ m/s}$  from left to right, what would be:

- The magnitude of the boat's resultant velocity?
- The direction relative to the straight line across the river?



# River Crossing Solution

The correct solution:

- Resultant velocity: 15.8 m/s
- Direction:  $18.4^\circ$  to the right

Using the Pythagorean theorem:

$$v_{\text{resultant}} = \sqrt{(5.0)^2 + (15.0)^2} = 15.8 \text{ m/s}$$

The angle is given by:

$$\theta = \tan^{-1} \left( \frac{5.0}{15.0} \right) = 18.4^\circ$$