PHYS12 CH459:

Test Prep

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Table of Contents

Problem 1: Circus Performance

Problem Statement

During a circus act:

- One performer hangs upside down from trapeze
- Holds another performer by the legs
- Upward force is three times the lower performer's weight
- Calculate the stretch in the upper legs (femurs)

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Problem 1: Given Information

- Mass of performer $= 60.0$ kg
- Each femur:
  - Length $(L_0) = 35.0$ cm $= 0.350$ m
  - Radius $= 1.80$ cm $= 0.0180$ m
- Young's modulus $(Y) = 1.6 \times 10^{10}$ N/m²
- Force $=$ 3 times weight

Problem 1: Solution Approach

Key equation for elastic deformation:

$$\Delta L = \frac{1}{Y}\frac{F}{A}L_0$$

where:

- ΔL = change in length
- Y = Young's modulus
- F = applied force
- A = cross-sectional area
- L_0 = original length

Problem 1: Calculations

1. Calculate total force:

$$F_{\text{tot}} = 3mg = 3(60.0\ \text{kg})(9.80\ \text{m/s}^2) = 1764\ \text{N}$$

2. Force per leg:

$$F_{\text{leg}} = F_{\text{tot}}/2 = 882\ \text{N}$$

3. Cross-sectional area:

$$A = \pi r^2 = \pi(0.0180\ \text{m})^2 = 1.018 \times 10^{-3}\ \text{m}^2$$

Problem 1: Final Result

Substituting into the equation:

$$\Delta L = \frac{1}{1.6 \times 10^{10}\ \text{N/m}^2} \frac{882\ \text{N}}{1.018 \times 10^{-3}\ \text{m}^2} (0.350\ \text{m})$$

$$\Delta L = 1.90 \times 10^{-5}\ \text{m}$$

Or in centimeters:

$$\Delta L = 1.90 \times 10^{-3}\ \text{cm}$$

Table of Contents

Problem 2: Aircraft Towing

Problem Statement

A tractor pushes an airplane:

- Tractor mass = 1800 kg
- Force on pavement = 1.75×10^4 N backward
- Total resistance = 2400 N
- Acceleration = 0.150 m/s^2

Find: (a) airplane mass, (b) force on airplane

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Problem 2: Solution Part (a)

Using Newton's Second Law:

$$\text{net } F = Ma = (m_a + m_t)a = F - f$$

$$m_a = \frac{F - f}{a} - m_t$$

$$m_a = \frac{1.75 \times 10^4 \text{ N} - 2400 \text{ N}}{0.150 \text{ m/s}^2} - 1800 \text{ kg}$$

$$m_a = 9.89 \times 10^4 \text{ kg}$$

Problem 2: Solution Part (b)

Force on airplane:

$$\begin{aligned} \text{net } F = F' - f' &= m_a a \\ F' &= m_a a + f' \\ F' &= (9.89 \times 10^4 \text{ kg})(0.150 \text{ m/s}^2) + 2200 \text{ N} \\ F' &= 1.70 \times 10^4 \text{ N} \end{aligned}$$

Table of Contents

Problem 3: Car Acceleration

Problem Statement

A car accelerates forward:

- Wheels exert 2100 N backward on road
- Friction force = 250 N
- Acceleration = 1.80 m/s^2

Find the mass of car plus occupants

Problem 3: Solution

Using Newton's Second Law:

$$\text{net } F = F - f = ma$$

$$m = \frac{F - f}{a}$$

$$m = \frac{2100 \text{ N} - 250 \text{ N}}{1.80 \text{ m/s}^2}$$

$$m = 1.03 \times 10^3 \text{ kg}$$

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Key Takeaways

- Elastic deformation depends on material properties and geometry
- Newton's laws apply to complex systems like aircraft-tractor combinations
- Free-body diagrams help organize force analysis
- Always check units and magnitudes for reasonable results

Problem: Forces on Head at Drafting Board

Problem Statement

A person working at a drafting board holds her head at an angle:

- Three major forces act on the head:
  - Weight (w) $= 50.0$ N
  - Muscle force (F_M) $= 60.0$ N at 33°
  - Vertebrae force (F_V) at unknown angle θ
- All forces act through center of mass
- Head is stationary (in equilibrium)

Find direction (θ) and magnitude of vertebrae force (F_V).

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Solution Approach

- Since head is stationary, sum of forces $= 0$
- Break forces into x and y components:

$$\hat{x} : F_M \cos 33 = F_V \cos \theta$$

$$\hat{y} : w + F_M \sin 33 = F_V \sin \theta$$

- Use these equations to find θ and F_V

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Solution: Finding the Angle

- Divide y-equation by x-equation to find θ:

$$\tan\theta = \frac{w + F_M \sin 33}{F_M \cos 33}$$

- Substitute known values:

$$\tan\theta = \frac{50.0\ \mathrm{N} + (60.0\ \mathrm{N})\sin 33}{(60.0\ \mathrm{N})\cos 33} = 1.643$$

- Solve for θ:

$$\theta = \tan^{-1}(1.643) = 58.7 \approx 59$$

Solution: Finding the Force Magnitude

- Use x-component equation:

$$F_V = \frac{F_M \cos 33}{\cos 58.7}$$

- Substitute values:

$$F_V = \frac{(60.0\ \text{N}) \cos 33}{\cos 58.7} = 97\ \text{N}$$

- Therefore:
  - Direction: $\theta = 59$ from horizontal
  - Magnitude: $F_V = 97$ N

Physical Interpretation

- The vertebrae must supply a significant force (97 N) to maintain head position
- Force is nearly 2 times the weight of the head
- Angle is determined by:
    - Head position (33° muscle angle)
    - Need to balance both vertical and horizontal components
    - Requirement that force passes through center of mass
- Demonstrates why poor posture can lead to muscle strain

Problem: Leg Exercise Device

Problem Statement

An exercise device for the upper leg muscle:

- Mass of 10.0 kg attached via pulleys
- System maintained at constant speed
- Lever arm distances:
  - $r_\perp = 35.0$ cm (perpendicular distance to tension force)
  - $r'_\perp = 2.00$ cm (perpendicular distance to muscle force)

Calculate the force exerted by the upper leg muscle.

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Analysis Strategy

Key physics concepts:

- Constant speed implies:
  - Acceleration $a = 0$
  - Net force $= 0$
  - Net torque $\tau = 0$
- Torque relationship:
  - $\tau = Fr_{\perp}$ (force $\times$ perpendicular distance)
  - Sum of torques $= 0$ for equilibrium

Solution Steps

1. Calculate tension force from weight:

$$T = w = (10.0\ \text{kg})(9.80\ \text{m/s}^2) = 98.0\ \text{N}$$

2. Use torque equilibrium about pivot:

$$F_m r'_\perp - T r_\perp = 0$$

3. Solve for muscle force:

$$F_m = T \frac{r_\perp}{r'_\perp}$$

Final Calculation

Substituting values:

$$
\begin{aligned}
F_m &= 98.0\ \text{N} \times \frac{35.0\ \text{cm}}{2.00\ \text{cm}} \\
&= 98.0\ \text{N} \times 17.5 \\
&= 1715\ \text{N} \\
&= 1.72 \times 10^3\ \text{N}
\end{aligned}
$$

Physical Interpretation

- The muscle must exert a force of 1715 N
- This large force results from mechanical disadvantage:
  - Muscle attachment point (2.00 cm) much closer to pivot
  - Than weight attachment point (35.0 cm)
  - Creates a force multiplier of 17.5
- Demonstrates why leg muscles need to be very strong
- Shows importance of lever arms in biomechanics