PHYS12 CH6: Gravitation and Keplar's Laws

Sections 6.5-6.6

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Learning Objectives

By the end of this lesson, you will be able to:

- Understand and explain Earth's gravitational force
- Describe the mathematical form of Newton's Universal Law of Gravitation
- Calculate gravitational forces between masses
- Explain the significance of the gravitational constant G
- Discuss the historical development of gravitational theory

Historical Development

- Newton (1687): First precise definition of gravitational force
- Showed it explains both:
  - Falling objects on Earth
  - Astronomical motions
- du Châtelet's contributions:
  - Translation and augmentation
  - Use of calculus to explain gravity

(a) (b)

Video Media

- https://www.youtube.com/watch?v=7gf6YpdvtE0

Newton's Universal Law of Gravitation

- Every particle in the universe attracts every other particle with a force along a line joining them
- Force is:

$$F = G\frac{m_1 m_2}{r^2}$$

  where:
  - $G = 6.674 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$
  - m_1, m_2 are the masses of the objects
  - r is the distance between their centers
- The force is always attractive
- It follows the inverse square law

Gravity and Circular Motion

- For objects in circular orbit:

$$F_g = F_c$$

- This means:

$$G\frac{mM}{r^2} = m\frac{v^2}{r}$$

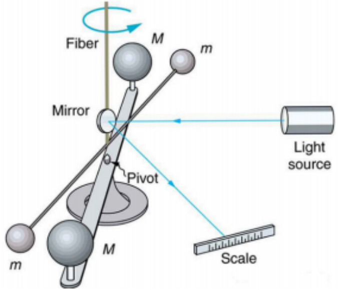
- Solving for orbital velocity:

$$v = \sqrt{\frac{GM}{r}}$$

- Applications:
    - Planetary orbits
    - Artificial satellites
    - Space stations

The Cavendish Experiment

- First accurate measurement of G (1798)
- Measured tiny gravitational attraction between lead spheres
- Led to first calculation of Earth's mass
- Modern version still used today



Example: Earth's Gravitational Force

Problem

Calculate the gravitational force between Earth ($M = 5.97 \times 10^{24}$ kg) and a 70 kg person at Earth's surface ($R = 6.37 \times 10^{6}$ m).

Solution

$$
\begin{aligned}
F &= G\frac{Mm}{r^2} \\
&= (6.67 \times 10^{-11})\frac{(5.97 \times 10^{24})(70)}{(6.37 \times 10^{6})^2} \\
&= ???\ N
\end{aligned}
$$

Kepler's First Law: Elliptical Orbits

Statement:

- All planets orbit the Sun in elliptical paths
- The Sun is located at one focus of the ellipse

Properties of Elliptical Orbits:

- **Semi-major axis** (a): half the longest diameter
- **Eccentricity** (e): measures deviation from circular orbit
  - $e = 0$: perfect circle
  - $0 < e < 1$: ellipse
  - Most planetary orbits have small e
- **Perihelion**: closest approach to Sun
- **Aphelion**: farthest point from Sun

Implications:

- Distance from Sun varies during orbit
- Orbital speed varies (connects to Second Law)
- True for all orbiting bodies under gravity

Video Media

- https://www.youtube.com/watch?v=Dvoe8Ib5D1o

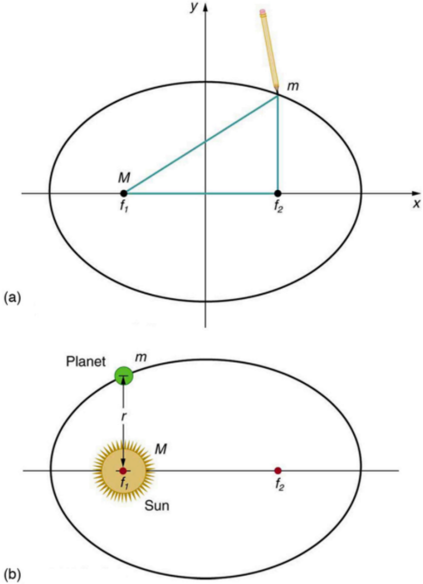


FIGURE 6.26 (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is a constant. You can draw an ellipse as shown by putting a pin at each focus, and then placing a string around a pencil and the pins and tracing a line on paper. A circle is a special case of an ellipse in which the two foci coincide (thus any point on the circle is the same distance from the center). (b) For any closed gravitational orbit, m follows an elliptical path with M at one focus. Kepler's first law states this fact for planets orbiting the Sun.

Kepler's Laws: Equal Areas (Second Law)

Kepler's Second Law:

- A line from the Sun to a planet sweeps out equal areas in equal times
- The shaded regions (A_1, A_2, A_3) have equal areas
- Important implications:
  - Planet moves fastest when closest to Sun
  - Planet moves slowest when farthest from Sun
  - Angular momentum is conserved

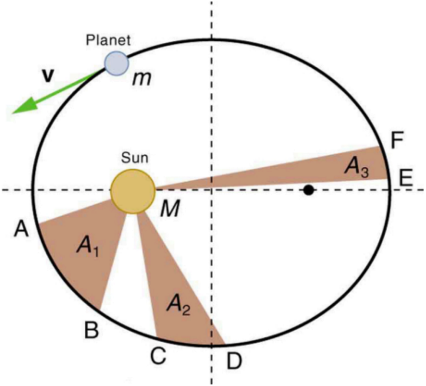


FIGURE 6.27 The shaded regions have equal areas. It takes equal times for m to go from A to B, from C to D, and from E to F. The mass m moves fastest when it is closest to M. Kepler's second law was originally devised for planets orbiting the Sun, but it has broader validity.

Kepler's Third Law of Planetary Motion

Mathematical Statement:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

where:

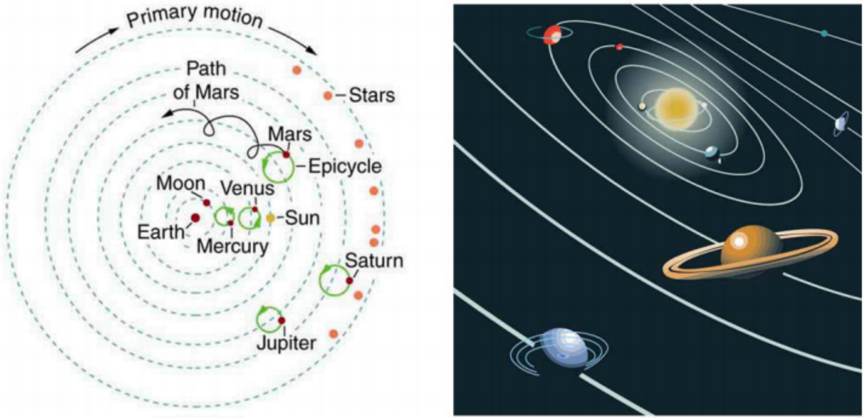
- T = orbital period
- r = average orbital radius
- Subscripts 1,2 refer to different planets

Key Points:

- Relates orbital period to orbital radius
- Squared period proportional to cubed radius
- Valid for all objects orbiting same central mass
- Can be derived from Newton's laws and universal gravitation

Example: If Planet 1 has period 1 year at 1 AU, a planet at 4 AU would have period:

$$T_2 = \sqrt{4^3} = 8 \text{ years}$$



Ptolemaic view
(a)

Copernican view
(b)

Video Media

- https://www.youtube.com/watch?v=yC74lhJX9Ck

What is a Planet? IAU Definition (2006)

Official IAU Definition

A planet in our solar system is a celestial body that:

1. Is in orbit around the Sun
   - Regular, elliptical orbit
   - Primary gravitational relationship with Sun
2. Has sufficient mass for hydrostatic equilibrium
   - Strong enough gravity to become spherical
   - Overcomes rigid body forces
3. Has cleared its orbital neighborhood
   - Gravitationally dominant in its orbit
   - No similar-sized objects in its orbital path

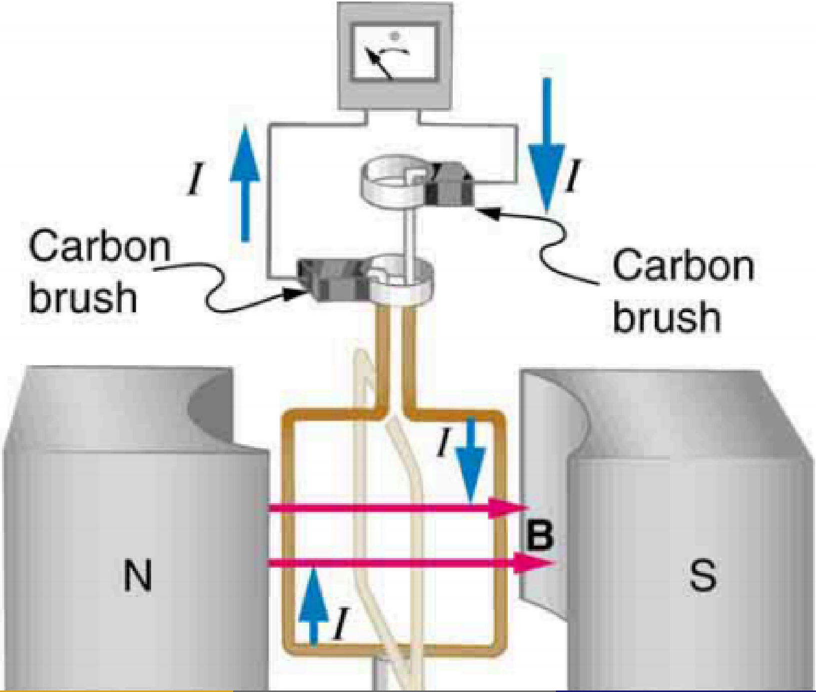
Dwarf Planets and the Case of Pluto

Dwarf Planet Definition

A celestial body that:

- Orbits the Sun
- Has hydrostatic equilibrium
- Has NOT cleared its orbital neighborhood

- Pluto was reclassified in 2006
- Reasons for reclassification:
  - Shares its orbit with many Kuiper Belt objects
  - Not gravitationally dominant in its region
  - Similar to other objects in its orbital zone
- Other recognized dwarf planets:
  - Ceres (in asteroid belt)
  - Eris (beyond Pluto)
  - Haumea and Makemake (Kuiper Belt)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Summary

Universal Gravitation:

- **Newton's Law:** $F_g = G\frac{m_1 m_2}{r^2}$
- **Gravitational Constant:** $G = 6.674 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$
- **Historical Development:** Newton's theory and du Châtelet's contributions
- **Cavendish Experiment:** First measurement of G

Kepler's Laws:

- **First Law:** Planets follow elliptical orbits with Sun at one focus
- **Second Law:** Equal areas in equal times
- **Third Law:** $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

Orbital Motion:

- **Orbital Velocity:** $v = \sqrt{\frac{GM}{r}}$
- **Gravitational Force = Centripetal Force:** $F_g = F_c$
- **Applications:** Planets, satellites, space stations, astroid mining