PHYS12 CH19: Electric Potential and Electric Field

Electric Potential, Fields, and Capacitors

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Outline

Learning Objectives

By the end of this presentation, you will be able to:

- Define electric potential and explain how it relates to potential energy
- Calculate potential difference between points in an electric field
- Relate electric field strength to potential gradient
- Calculate the electric potential due to a point charge
- Understand the concept of equipotential lines
- Explain how capacitors work and calculate capacitance
- Determine the equivalent capacitance of series and parallel combinations
- Calculate the energy stored in a capacitor

Electric Potential Energy vs. Electric Potential

Electric Potential Energy

- Energy possessed by a charge in an electric field
- Depends on both the field and the amount of charge
- Measured in joules (J)

Electric Potential

- Electric potential energy per unit charge
- Independent of the test charge being used
- Measured in volts (V), where $1\ \text{V} = 1\ \text{J/C}$
- A property of the electric field at a point

Potential Difference and Voltage

Potential Difference

- The change in potential energy per unit charge when moving from point A to point B
- Commonly called voltage

$$\Delta V = \frac{\Delta PE}{q} \tag{1}$$

$$\Delta PE = q\Delta V \tag{2}$$

- Work must be done to move a positive charge from a low potential to a high potential
- A positive charge naturally moves from high potential to low potential (releasing energy)
- A negative charge naturally moves from low potential to high potential

The Electron Volt

Definition

The electron volt (eV) is the energy given to a fundamental charge accelerated through a potential difference of 1 V.

$$
\begin{aligned}
1\ \text{eV} &= (1.60 \times 10^{-19}\ \text{C})(1\ \text{V}) && (3)\\
&= (1.60 \times 10^{-19}\ \text{C})(1\ \text{J/C}) && (4)\\
&= 1.60 \times 10^{-19}\ \text{J} && (5)
\end{aligned}
$$

- Useful unit for atomic and nuclear physics
- Common multiples: keV, MeV, GeV
- Example: A 12 V battery can give an electron 12 eV of energy

Conservation of Energy in Electric Fields

Mechanical Energy

- The sum of kinetic energy and potential energy of a system
- $E_{mechanical} = \text{KE} + \text{PE}$
- This sum is constant in a conservative field

Applications

- When a charge moves in an electric field, energy is converted between kinetic and potential forms
- $\Delta\text{KE} = -\Delta\text{PE} = -q\Delta V$
- Can find the final speed of a charged particle accelerated through a potential difference

Electric Potential in a Uniform Field

In a uniform electric field (like between parallel plates):

$$V_{AB} = Ed \tag{6}$$

$$E = \frac{V_{AB}}{d} \tag{7}$$

where:

- E is the electric field strength (V/m or N/C)
- d is the distance from A to B (m)
- V_{AB} is the potential difference (V)

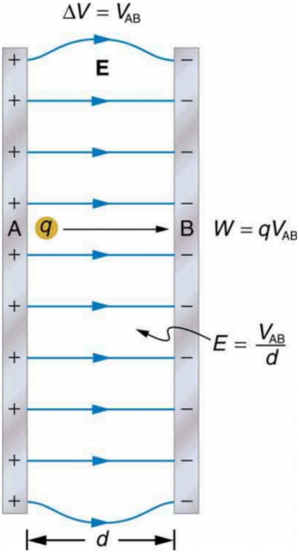


Figure: Fig 19.5

Relationship Between Electric Field and Potential

General Relationship

$$E = -\frac{\Delta V}{\Delta s} \tag{8}$$

where Δs is the distance over which the change in potential ΔV takes place

- The negative sign indicates that E points in the direction of decreasing potential
- Electric field is the gradient (slope) of the electric potential
- Units check: $\mathrm{V/m} = \mathrm{N/C}$
- Stronger electric fields create steeper potential gradients

Electric Potential Due to a Point Charge

Point Charge Potential

The electric potential at a distance r from a point charge Q is:

$$V = k\frac{Q}{r} \tag{9}$$

where $k = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2}$

- Potential is positive for positive charges and negative for negative charges
- Decreases with distance r from the charge
- Reference: $V = 0$ at $r = \infty$
- Note: $E = k\frac{Q}{r^2}$, so $E = -\frac{dV}{dr}$

Superposition of Potentials

Key Concept

Electric potential is a scalar quantity, so potentials from multiple charges add algebraically:

$$V_{total} = V_1 + V_2 + V_3 + \cdots = \sum_i V_i = k \sum_i \frac{Q_i}{r_i} \tag{10}$$

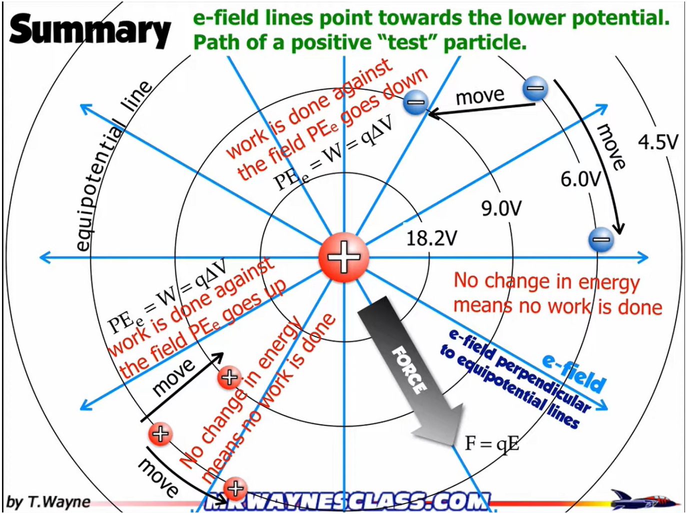
- This is simpler than adding electric fields (which are vectors)
- Makes it easier to solve some complex electrostatic problems
- Calculate the total potential, then find the electric field by taking the gradient

Equipotential Lines and Surfaces

Definition

An equipotential line is a line along which the electric potential is constant.

- An equipotential surface is the 3D version of equipotential lines
- No work is done moving a charge along an equipotential
- Equipotential lines are always perpendicular to electric field lines
- For a point charge, equipotentials are concentric spheres
- For a uniform field, equipotentials are parallel planes



Summary
e-field lines point towards the lower potential.
Path of a positive "test" particle.
equipotential line
work is done against the field PEe goes down
PEe = W = qΔV
move
move
4.5V
6.0V
9.0V
18.2V
No change in energy means no work is done
PEe = W = qΔV
work is done against the field PEe goes up
move
No change in energy means no work is done
FORCE
e-field perpendicular to equipotential lines
e-field
F = qE
move
by T.Wayne
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Equipotential Lines and Grounding

Properties of Equipotentials

- All points on a conductor in electrostatic equilibrium are at the same potential
- The surface of a conductor is an equipotential surface
- No electric field exists inside a conductor in electrostatic equilibrium

Grounding

- Grounding is the process of connecting a conductor to the Earth with a good conductor
- This fixes the conductor at zero volts (Earth's reference potential)
- Important for safety in electrical systems

What is a Capacitor?

Definition

A capacitor is a device used to store electric charge and energy in an electric field.

- Typically consists of two conductors (plates) separated by an insulator (dielectric)
- When connected to a voltage source, equal and opposite charges appear on the conductors
- The electric field is confined mostly to the region between the conductors
- Common forms: parallel plate, cylindrical, spherical

Capacitors

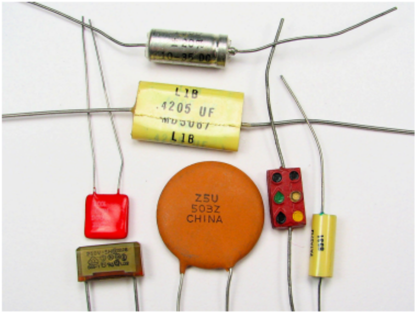


FIGURE 19.14 Some typical capacitors. Size and value of capacitance are not necessarily related. (credit: Windell Oskay)

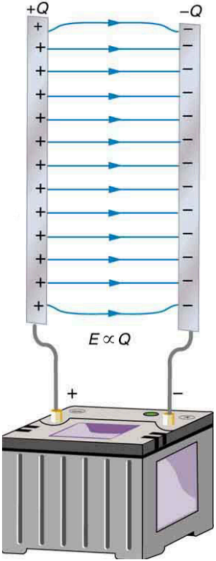


Figure: fig 19.13

Capacitance

Capacitance Definition

Capacitance (C) is the amount of charge stored per volt of potential difference:

$$C = \frac{Q}{V} \tag{11}$$

- Unit: Farad (F), where 1 F = 1 C/V
- Typical values: pF to F range
- Depends only on physical characteristics (geometry, materials)
- Does NOT depend on charge or voltage (for linear capacitors)

Parallel Plate Capacitor

Capacitance Formula

The capacitance of a parallel plate capacitor in a vacuum or air:

$$C = \epsilon_0 \frac{A}{d} \tag{12}$$

where:

- $\epsilon_0 = 8.85 \times 10^{-12}$ F/m (permittivity of free space)
- A = area of plates
- d = separation distance between plates

- Larger plate area $\rightarrow$ greater capacitance
- Smaller separation $\rightarrow$ greater capacitance

Dielectrics in Capacitors

Effect of Dielectrics

Inserting a dielectric material between capacitor plates:

$$C = \kappa \epsilon_0 \frac{A}{d} \tag{13}$$

where κ is the dielectric constant of the material

Common Dielectric Constants

- Air: $\kappa \approx 1.00059$
- Paper: $\kappa \approx 2 - 4$
- Glass: $\kappa \approx 4 - 10$
- Teflon: $\kappa \approx 2.1$
- Water: $\kappa \approx 80$

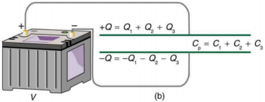
Capacitors in Series

Series Combination

When capacitors are connected in series:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{14}$$

- All capacitors in series have the same charge magnitude
- The total voltage is divided among the capacitors
- Equivalent capacitance is less than any individual capacitance
- Similar to resistors in parallel



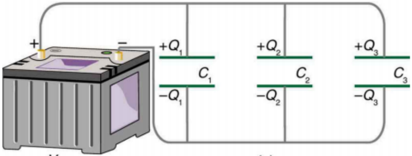
Capacitors in Parallel

Parallel Combination

When capacitors are connected in parallel:

$$C_P = C_1 + C_2 + C_3 + \cdots \tag{15}$$

- All capacitors in parallel have the same voltage
- The total charge is divided among the capacitors
- Equivalent capacitance is greater than any individual capacitance
- Similar to resistors in series

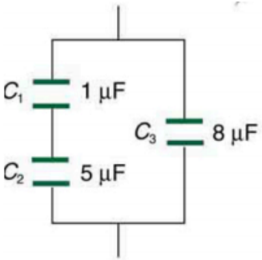


(a)

Combined Series-Parallel Networks

Strategy for Solving Combined Networks

1. Identify series and parallel parts in the circuit
2. Compute the equivalent capacitance for each part
3. Combine the results to find the total capacitance



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Energy Storage in Capacitors

Energy Formula

The energy stored in a capacitor can be expressed in three equivalent ways:

$$E_{cap} = \frac{QV}{2} \tag{16}$$

$$E_{cap} = \frac{CV^2}{2} \tag{17}$$

$$E_{cap} = \frac{Q^2}{2C} \tag{18}$$

- Energy is stored in the electric field between the plates
- Units: joules (J)
- Work must be done to charge a capacitor
- The stored energy can be recovered when the capacitor discharges

Applications of Capacitors

Common Applications

- Energy storage (backup power supplies)
- Filtering in power supplies
- Timing circuits
- Coupling and decoupling in electronic circuits
- Flash lamps in cameras
- Defibrillators in medical equipment
- Touch screens and sensors

Energy Density

The energy density in a capacitor's electric field is:

$$u = \frac{1}{2}\epsilon_0 E^2 \quad (\text{J/m}^3) \tag{19}$$

"I Do" Example - Electron Acceleration

Problem

An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce X-rays. What would be the maximum speed of these electrons? (Non-relativistic calculation)

Solution

Use energy conservation: electrical potential energy is converted to KE

$$qV = \frac{1}{2}mv^2 \tag{20}$$

$$v = \sqrt{\frac{2qV}{m}} \tag{21}$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19}\ \mathrm{C})(4.0 \times 10^{4}\ \mathrm{V})}{9.11 \times 10^{-31}\ \mathrm{kg}}} \tag{22}$$

$$v = 1.17 \times 10^{8}\ \mathrm{m/s} \tag{23}$$

"We Do" Example - Capacitor with Dielectric

Problem

A parallel plate capacitor has plates of area 5.00 m² separated by 0.100 mm of Teflon (dielectric constant = 2.1). What is the capacitance?

Solution

Use the formula for a parallel plate capacitor with a dielectric:

$$C = \kappa\epsilon_0 \frac{A}{d} \tag{24}$$

$$C = (2.1)(8.85 \times 10^{-12}\ \text{F/m}) \frac{5.00\ \text{m}^2}{0.100 \times 10^{-3}\ \text{m}} \tag{25}$$

$$C = (2.1)(8.85 \times 10^{-12}\ \text{F/m})(5.00 \times 10^{4}\ \text{m}) \tag{26}$$

$$C = 9.29 \times 10^{-7}\ \text{F} = 0.929\ \text{F} \tag{27}$$

"You Do" Example - Energy in a Capacitor

Problem

A 180 F capacitor is charged to 120 V.

1. How much charge is stored in the capacitor?
2. How much energy is stored in the capacitor?

Hints

- Use $Q = CV$ to find the charge
- Use $E_{cap} = \frac{1}{2}CV^2$ to find the energy
- Pay attention to the units $(F = 10^{-6}F)$

Try solving this problem on your own!

Summary

Key Concepts

- Electric potential: $V = \frac{PE}{q}$
- Potential difference: $\Delta V = \frac{\Delta PE}{q}$
- Electric field in a uniform field: $E = \frac{V}{d}$
- Electric field and potential relationship: $E = -\frac{\Delta V}{\Delta s}$
- Point charge potential: $V = k\frac{Q}{r}$
- Capacitance: $C = \frac{Q}{V}$
- Parallel plate capacitor: $C = \kappa\epsilon_0\frac{A}{d}$
- Series capacitors: $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$
- Parallel capacitors: $C_P = C_1 + C_2 + \cdots$
- Energy stored: $E_{cap} = \frac{1}{2}CV^2$