

Work and Energy

Chapter 7.1-7.3

Mr. Gullo

November 2024

Table of Contents

Work: Definition

- Work is the transfer of energy by a force acting on an object as it is displaced
- Mathematical definition: $W = Fd \cos \theta$
- Where:
 - W is work (measured in joules, J)
 - F is force (in newtons, N)
 - d is displacement (in meters, m)
 - θ is angle between force and displacement

Work: Important Points

- Work is zero when:
 - Force and displacement are perpendicular ($\theta = 90$)
 - There is no displacement ($d = 0$)
- Work is positive when:
 - Force and displacement are in same direction ($\theta < 90$)
- Work is negative when:
 - Force and displacement are in opposite directions ($90^\circ < \theta \leq 180$)

Table of Contents

Kinetic Energy

- Kinetic energy is the energy of motion
- Formula: $KE = \frac{1}{2}mv^2$
- Where:
 - m is mass (kg)
 - v is velocity (m/s)
- Work-Energy Theorem: $W_{net} = \Delta KE$
- Net work equals change in kinetic energy

Table of Contents

Gravitational Potential Energy

- Energy due to position in a gravitational field
- Formula: $PE_g = mgh$
- Where:
 - m is mass (kg)
 - g is acceleration due to gravity (9.8 m/s^2)
 - h is height (m)
- Depends only on vertical height change
- Reference level can be chosen arbitrarily



Table of Contents

Example 7.1: Calculating Work You Do to Push a Lawn Mower

A person pushing a lawn mower exerts a constant force of 75.0 N at an angle 35° below the horizontal. The lawn mower is pushed 25.0 m on level ground.

Solution:

$$W = Fd \cos \theta$$

$$W = (75.0 \text{ N})(25.0 \text{ m}) \cos(35.0)$$

$$W = 1536 \text{ J} = 1.54 \times 10^3 \text{ J}$$

$$\text{Convert to kcal:} = 0.367 \text{ kcal}$$

$$\text{Ratio to daily intake:} = 1.53 \times 10^{-4}$$

Example 7.2: Calculating the Kinetic Energy of a Package

A 30.0-kg package on a roller belt conveyor system moves at 0.500 m/s.

Solution Steps:

① $KE = \frac{1}{2}mv^2$

② $KE = 0.5(30.0 \text{ kg})(0.500 \text{ m/s})^2$

③ $KE = 3.75 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 3.75 \text{ J}$

Example 7.7: Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in the text if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible?

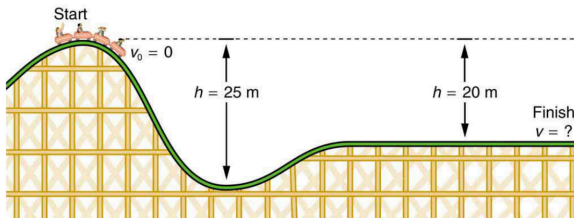


FIGURE 7.8 The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all ΔPE_g is converted to KE.

Try this on your own, then we'll discuss the solution!

(b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?

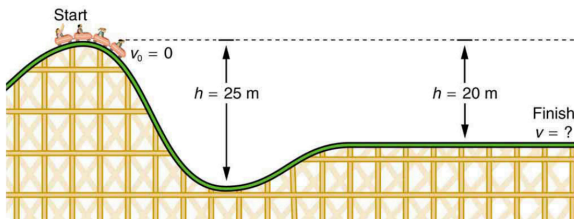


FIGURE 7.8 The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all ΔPE_g is converted to KE.

Try this on your own, then we'll discuss the solution!

You Do: Solution

Solution:

For (a):

$$v = \sqrt{2g|h|}$$

$$v = \sqrt{2(9.80 \text{ m/s}^2)(20.0 \text{ m})}$$

$$v = 19.8 \text{ m/s}$$

For (b):

$$v = \sqrt{2g|h| + v_0^2}$$

$$v = \sqrt{2(9.80)(20.0) + (5.00)^2}$$

$$v = 20.4 \text{ m/s}$$

Example 7.7: Discussion

- Mass cancels out - consistent with all objects falling at same rate
- Speed depends only on initial speed and height
- Path taken doesn't matter - only initial and final heights
- Final speed in (b) greater but by less than the initial 5.00 m/s

Key Principles

Key Principle

In a system with only conservative forces:

$$E_{\text{total}} = KE + PE = \text{constant}$$

Mathematical Expression

$$KE_i + PE_i = KE_f + PE_f$$

Where:

- $KE = \frac{1}{2}mv^2$ (Kinetic Energy)
- $PE = mgh$ (Gravitational Potential Energy)

Important Notes

- Energy can transform between KE and PE
- Total mechanical energy is conserved
- Valid only for conservative forces