

# PHYS12 CH:28.1-28.3

## The Geometry of Spacetime

Mr. Gullo

November 21, 2025

# Learning Objectives

By the end of this lesson, you will be able to:

# Learning Objectives

By the end of this lesson, you will be able to:

- State Einstein's two postulates of special relativity.

# Learning Objectives

By the end of this lesson, you will be able to:

- State Einstein's two postulates of special relativity.
- Explain why simultaneity is relative to the observer.

## Learning Objectives

By the end of this lesson, you will be able to:

- State Einstein's two postulates of special relativity.
  - Explain why simultaneity is relative to the observer.
  - Calculate time dilation effects for moving objects using the Lorentz factor.

## Learning Objectives

By the end of this lesson, you will be able to:

- State Einstein's two postulates of special relativity.
  - Explain why simultaneity is relative to the observer.
  - Calculate time dilation effects for moving objects using the Lorentz factor.
  - Calculate length contraction for objects moving at relativistic speeds.

# Learning Objectives

By the end of this lesson, you will be able to:

- State Einstein's two postulates of special relativity.
- Explain why simultaneity is relative to the observer.
- Calculate time dilation effects for moving objects using the Lorentz factor.
- Calculate length contraction for objects moving at relativistic speeds.
- Distinguish between proper time/length and relativistic time/length.

## 28.1 Einstein's First Postulate

### The Principle of Relativity

The laws of physics are the same in all **inertial frames of reference**.

## 28.1 Einstein's First Postulate

## The Principle of Relativity

The laws of physics are the same in all **inertial frames of reference**.

- An inertial frame is one that is not accelerating (constant velocity).

## 28.1 Einstein's First Postulate

### The Principle of Relativity

The laws of physics are the same in all **inertial frames of reference**.

- An inertial frame is one that is not accelerating (constant velocity).
- There is no "preferred" or "absolute" frame of reference.

## 28.1 Einstein's First Postulate

## The Principle of Relativity

The laws of physics are the same in all **inertial frames of reference**.

- An inertial frame is one that is not accelerating (constant velocity).
  - There is no "preferred" or "absolute" frame of reference.
  - You cannot perform an experiment inside a smooth-moving train to determine if you are moving or standing still.

## 28.1 Einstein's Second Postulate

# The Constancy of the Speed of Light

The speed of light in a vacuum ( $c$ ) has the same value ( $c = 3.00 \times 10^8$  m/s) in all inertial frames of reference, regardless of the motion of the light source or the observer.

## 28.1 Einstein's Second Postulate

### The Constancy of the Speed of Light

The speed of light in a vacuum ( $c$ ) has the same value ( $c = 3.00 \times 10^8$  m/s) in all inertial frames of reference, regardless of the motion of the light source or the observer.

- This contradicts our everyday experience with relative velocities (like throwing a ball from a moving car).

## 28.1 Einstein's Second Postulate

### The Constancy of the Speed of Light

The speed of light in a vacuum ( $c$ ) has the same value ( $c = 3.00 \times 10^8$  m/s) in all inertial frames of reference, regardless of the motion of the light source or the observer.

- This contradicts our everyday experience with relative velocities (like throwing a ball from a moving car).
- $c$  is the cosmic speed limit.

## 28.2 The Relativity of Simultaneity

**Simultaneity** refers to two events happening at the same time.

## 28.2 The Relativity of Simultaneity

**Simultaneity** refers to two events happening at the same time.

## Key Insight

Two events that are simultaneous in one frame of reference are **not necessarily simultaneous** in another frame that is moving relative to the first.

## 28.2 The Relativity of Simultaneity

**Simultaneity** refers to two events happening at the same time.

### Key Insight

Two events that are simultaneous in one frame of reference are **not necessarily simultaneous** in another frame that is moving relative to the first.

- This is not an optical illusion; it is a fundamental property of time.

## 28.2 The Relativity of Simultaneity

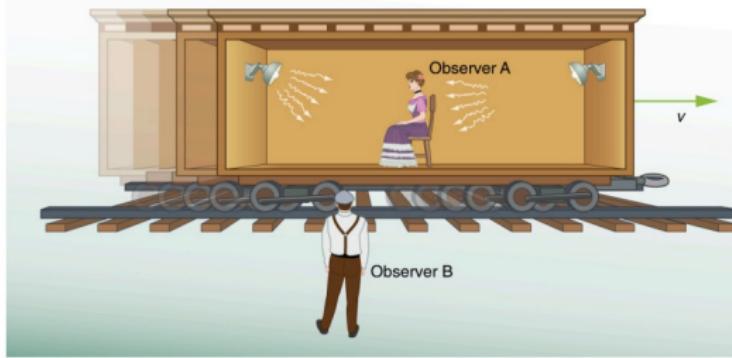
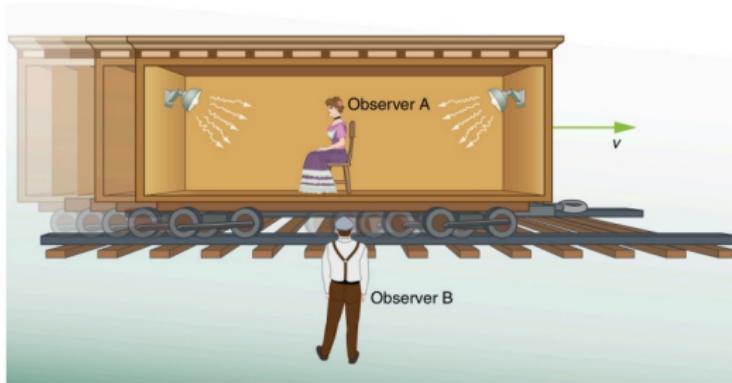
**Simultaneity** refers to two events happening at the same time.

### Key Insight

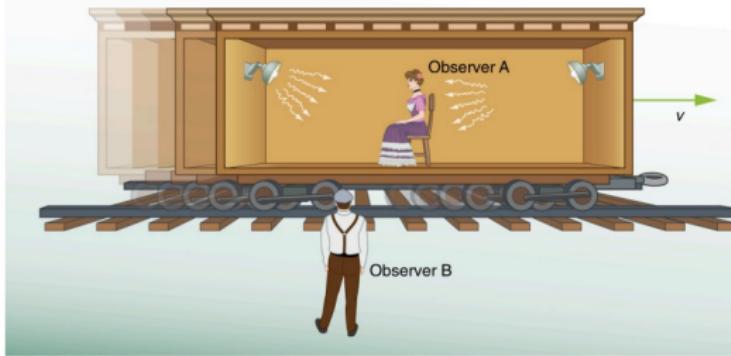
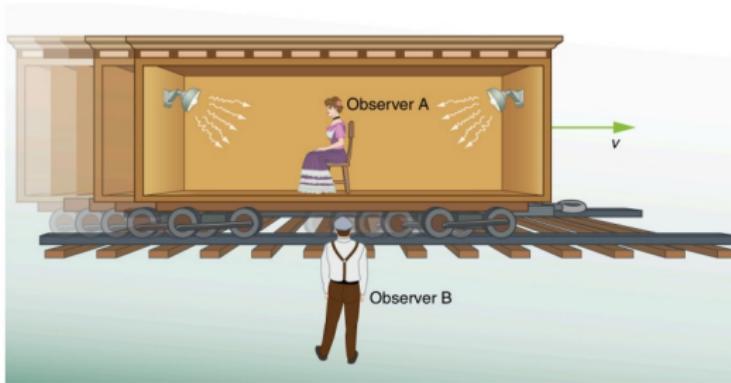
Two events that are simultaneous in one frame of reference are **not necessarily simultaneous** in another frame that is moving relative to the first.

- This is not an optical illusion; it is a fundamental property of time.
- If observers cannot agree on "when" things happen, time itself must be relative.

# Concept Visualization: Simultaneity

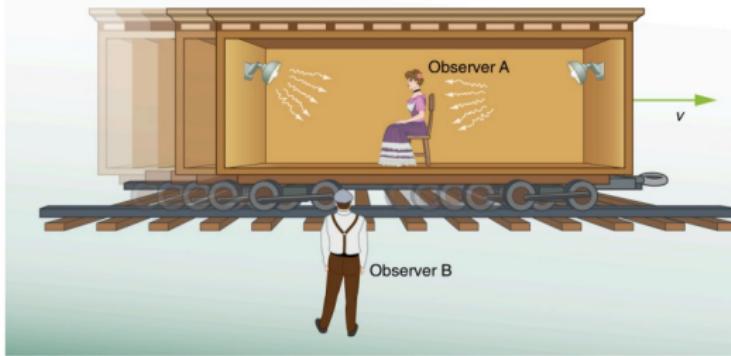
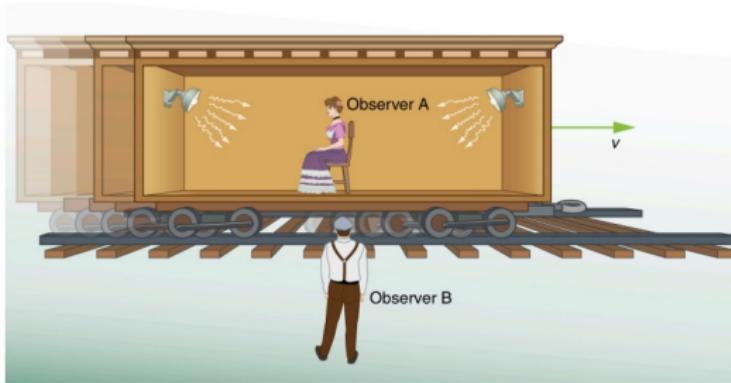


# Concept Visualization: Simultaneity



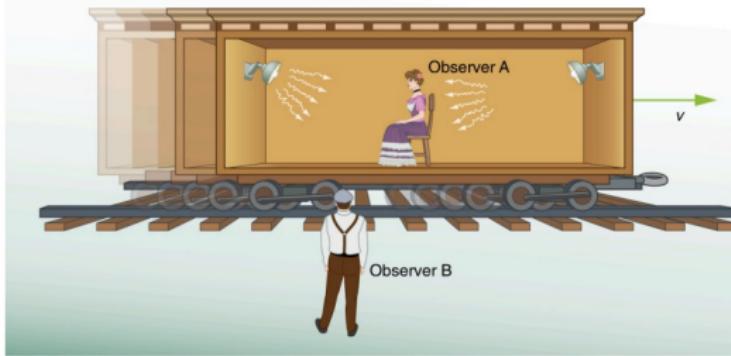
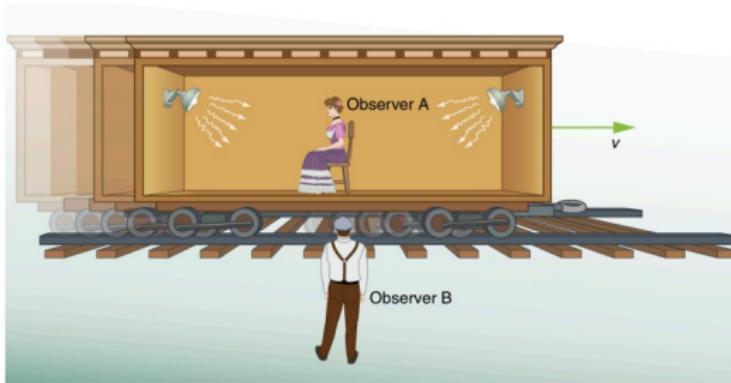
A thought experiment with a train car:

# Concept Visualization: Simultaneity



A thought experiment with a train car:

# Concept Visualization: Simultaneity



A thought experiment with a train car:

## 28.2 Time Dilation

**Time Dilation:** Moving clocks run slower as measured by an observer at rest.

## 28.2 Time Dilation

**Time Dilation:** Moving clocks run slower as measured by an observer at rest.

- $\Delta t$ : Time interval measured by stationary observer (dilated time).

## 28.2 Time Dilation

**Time Dilation:** Moving clocks run slower as measured by an observer at rest.

- $\Delta t$ : Time interval measured by stationary observer (dilated time).
- $\Delta t_0$ : **Proper time** (measured by observer moving with the event).

## 28.2 Time Dilation

**Time Dilation:** Moving clocks run slower as measured by an observer at rest.

- $\Delta t$ : Time interval measured by stationary observer (dilated time).
- $\Delta t_0$ : **Proper time** (measured by observer moving with the event).
- $\gamma$ : Lorentz factor (always  $\geq 1$ ).

# Essential Equation: Time Dilation

## Time Dilation Formula

$$\Delta t = \gamma \Delta t_0$$

or

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

# Essential Equation: Time Dilation

## Time Dilation Formula

$$\Delta t = \gamma \Delta t_0$$

or

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- $\Delta t > \Delta t_0$  (Moving clocks run slow).

# Essential Equation: Time Dilation

## Time Dilation Formula

$$\Delta t = \gamma \Delta t_0$$

or

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- $\Delta t > \Delta t_0$  (Moving clocks run slow).
- $v$ : Relative velocity (m/s).

# Essential Equation: Time Dilation

## Time Dilation Formula

$$\Delta t = \gamma \Delta t_0$$

or

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- $\Delta t > \Delta t_0$  (Moving clocks run slow).
- $v$ : Relative velocity (m/s).
- $c$ : Speed of light ( $3.00 \times 10^8$  m/s).

# The Lorentz Factor ( $\gamma$ )

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

# The Lorentz Factor ( $\gamma$ )

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- As  $v \rightarrow 0$ ,  $\gamma \rightarrow 1$  (Newtonian mechanics).

# The Lorentz Factor ( $\gamma$ )

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- As  $v \rightarrow 0$ ,  $\gamma \rightarrow 1$  (Newtonian mechanics).
- As  $v \rightarrow c$ ,  $\gamma \rightarrow \infty$  (Relativistic effects dominate).

# The Lorentz Factor ( $\gamma$ )

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- As  $v \rightarrow 0$ ,  $\gamma \rightarrow 1$  (Newtonian mechanics).
- As  $v \rightarrow c$ ,  $\gamma \rightarrow \infty$  (Relativistic effects dominate).
- Calculating  $\gamma$  first often simplifies problems.

## 28.3 Length Contraction

**Length Contraction:** Moving objects appear shorter in the direction of motion.

## 28.3 Length Contraction

**Length Contraction:** Moving objects appear shorter in the direction of motion.

- $L$ : Length measured by stationary observer (contracted length).

## 28.3 Length Contraction

**Length Contraction:** Moving objects appear shorter in the direction of motion.

- $L$ : Length measured by stationary observer (contracted length).
- $L_0$ : **Proper length** (measured by observer at rest relative to the object).

# Essential Equation: Length Contraction

## Length Contraction Formula

$$L = \frac{L_0}{\gamma}$$

or

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

# Essential Equation: Length Contraction

## Length Contraction Formula

$$L = \frac{L_0}{\gamma}$$

or

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

- $L < L_0$  (Moving objects shrink).

# Essential Equation: Length Contraction

## Length Contraction Formula

$$L = \frac{L_0}{\gamma}$$

or

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

- $L < L_0$  (Moving objects shrink).
- Contraction happens **only** in the direction of motion.

# Essential Equation: Length Contraction

## Length Contraction Formula

$$L = \frac{L_0}{\gamma}$$

or

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

- $L < L_0$  (Moving objects shrink).
- Contraction happens **only** in the direction of motion.
- Width and height (perpendicular to motion) remain unchanged.

## Example: I Do - Time Dilation

**Problem:** A spaceship travels at  $0.95c$  relative to Earth. An astronaut on board measures a trip to take 2.0 years. How long does the trip take according to Mission Control on Earth?

# I Do: Time Dilation - G & U

## G - Givens

- $v = 0.95c$
- $\Delta t_0 = 2.0$  years (Proper time, measured on ship)
- Frame: Earth (stationary relative to motion)

# I Do: Time Dilation - G & U

## G - Givens

- $v = 0.95c$
- $\Delta t_0 = 2.0$  years (Proper time, measured on ship)
- Frame: Earth (stationary relative to motion)

## U - Unknown

- $\Delta t = ?$  (Dilated time on Earth)

# I Do: Time Dilation - Equation

## E - Equation

- First, calculate  $\gamma$ :

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

# I Do: Time Dilation - Equation

## E - Equation

- First, calculate  $\gamma$ :

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Then use time dilation:

$$\Delta t = \gamma \Delta t_0$$

# I Do: Time Dilation - Substitute & Solve

## S - Substitute

- $\gamma = \frac{1}{\sqrt{1-0.95^2}} = \frac{1}{\sqrt{1-0.9025}} = \frac{1}{\sqrt{0.0975}}$
- $\gamma \approx 3.20$
- $\Delta t = (3.20)(2.0 \text{ years})$

# I Do: Time Dilation - Substitute & Solve

## S - Substitute

- $\gamma = \frac{1}{\sqrt{1-0.95^2}} = \frac{1}{\sqrt{1-0.9025}} = \frac{1}{\sqrt{0.0975}}$
- $\gamma \approx 3.20$
- $\Delta t = (3.20)(2.0 \text{ years})$

## S - Solve

- $\Delta t = 6.4 \text{ years}$
- $\boxed{\Delta t = 6.4 \text{ years}}$
- *Earth observers wait longer than the astronaut ages.*

# We Do: Length Contraction

**Problem:** A 100 m long spaceship (proper length) moves past a space station at  $0.80c$ . How long does the spaceship appear to an observer on the space station?

# We Do: Length Contraction

**Problem:** A 100 m long spaceship (proper length) moves past a space station at  $0.80c$ . How long does the spaceship appear to an observer on the space station? **Class Discussion:**

- ① Who measures  $L_0$ ? (The pilot or the station observer?)

# We Do: Length Contraction

**Problem:** A 100 m long spaceship (proper length) moves past a space station at  $0.80c$ . How long does the spaceship appear to an observer on the space station? **Class Discussion:**

- ① Who measures  $L_0$ ? (The pilot or the station observer?)
- ② Who measures  $L$ ?

# We Do: Length Contraction

**Problem:** A 100 m long spaceship (proper length) moves past a space station at  $0.80c$ . How long does the spaceship appear to an observer on the space station? **Class Discussion:**

- ① Who measures  $L_0$ ? (The pilot or the station observer?)
- ② Who measures  $L$ ?
- ③ Will the answer be less than or greater than 100 m?

# You Do: Practice

**Problem:** Muons are unstable particles with a proper lifetime of  $2.2 \times 10^{-6}$  s. If a muon travels at  $0.99c$  relative to the lab:

# You Do: Practice

**Problem:** Muons are unstable particles with a proper lifetime of  $2.2 \times 10^{-6}$  s. If a muon travels at  $0.99c$  relative to the lab:

- ① Calculate the Lorentz factor  $\gamma$ .

# You Do: Practice

**Problem:** Muons are unstable particles with a proper lifetime of  $2.2 \times 10^{-6}$  s. If a muon travels at  $0.99c$  relative to the lab:

- ① Calculate the Lorentz factor  $\gamma$ .
- ② How long does the muon live as measured by a scientist in the lab?

# You Do: Practice

**Problem:** Muons are unstable particles with a proper lifetime of  $2.2 \times 10^{-6}$  s. If a muon travels at  $0.99c$  relative to the lab:

- ① Calculate the Lorentz factor  $\gamma$ .
- ② How long does the muon live as measured by a scientist in the lab?
- ③ Hint: Expect a longer time ( $\Delta t > \Delta t_0$ ).

# Reading Homework

Before the next lecture on Part 2, please read:

- Section 28.4: Relativistic Addition of Velocities
- Section 28.5: Relativistic Momentum
- Section 28.6: Relativistic Energy

# Summary

# Summary

- **Postulate 1:** Laws of physics are invariant in inertial frames.

# Summary

- **Postulate 1:** Laws of physics are invariant in inertial frames.
- **Postulate 2:** Speed of light  $c$  is constant for all observers.

# Summary

- **Postulate 1:** Laws of physics are invariant in inertial frames.
- **Postulate 2:** Speed of light  $c$  is constant for all observers.
- **Simultaneity:** Relative to the observer's motion.

# Summary

- **Postulate 1:** Laws of physics are invariant in inertial frames.
- **Postulate 2:** Speed of light  $c$  is constant for all observers.
- **Simultaneity:** Relative to the observer's motion.
- **Time Dilation:** Moving clocks run slow ( $\Delta t = \gamma \Delta t_0$ ).

# Summary

- **Postulate 1:** Laws of physics are invariant in inertial frames.
- **Postulate 2:** Speed of light  $c$  is constant for all observers.
- **Simultaneity:** Relative to the observer's motion.
- **Time Dilation:** Moving clocks run slow ( $\Delta t = \gamma \Delta t_0$ ).
- **Length Contraction:** Moving objects shorten ( $L = L_0/\gamma$ ).

# Summary

- **Postulate 1:** Laws of physics are invariant in inertial frames.
- **Postulate 2:** Speed of light  $c$  is constant for all observers.
- **Simultaneity:** Relative to the observer's motion.
- **Time Dilation:** Moving clocks run slow ( $\Delta t = \gamma \Delta t_0$ ).
- **Length Contraction:** Moving objects shorten ( $L = L_0/\gamma$ ).
- $\gamma$  becomes significant only as  $v \rightarrow c$ .