

## 27.5 Single Slit Diffraction

### Learning Objectives

By the end of this section, you will be able to:

- Discuss the single slit diffraction pattern.

Light passing through a single slit forms a diffraction pattern somewhat different from those formed by double slits or diffraction gratings. Figure 27.21 shows a single slit diffraction pattern. Note that the central maximum is larger than those on either side, and that the intensity decreases rapidly on either side. In contrast, a diffraction grating produces evenly spaced lines that dim slowly on either side of center.

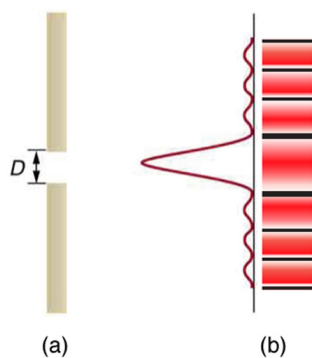


Figure 27.21 (a) Single slit diffraction pattern. Monochromatic light passing through a single slit has a central maximum and many smaller and dimmer maxima on either side. The central maximum is six times higher than shown. (b) The drawing shows the bright central maximum and dimmer and thinner maxima on either side.

The analysis of single slit diffraction is illustrated in Figure 27.22. Here we consider light coming from different parts of the *same* slit. According to Huygens's principle, every part of the wavefront in the slit emits wavelets. These are like rays that start out in phase and head in all directions. (Each ray is perpendicular to the wavefront of a wavelet.) Assuming the screen is very far away compared with the size of the slit, rays heading toward a common destination are nearly parallel. When they travel straight ahead, as in Figure 27.22(a), they remain in phase, and a central maximum is obtained. However, when rays travel at an angle  $\theta$  relative to the original direction of the beam, each travels a different distance to a common location, and they can arrive in or out of phase. In Figure 27.22(b), the ray from the bottom travels a distance of one wavelength  $\lambda$  farther than the ray from the top. Thus a ray from the center travels a distance  $\lambda/2$  farther than the one on the left, arrives out of phase, and interferes destructively. A ray from slightly above the center and one from slightly above the bottom will also cancel one another. In fact, each ray from the slit will have another to interfere destructively, and a minimum in intensity will occur at this

angle. There will be another minimum at the same angle to the right of the incident direction of the light.

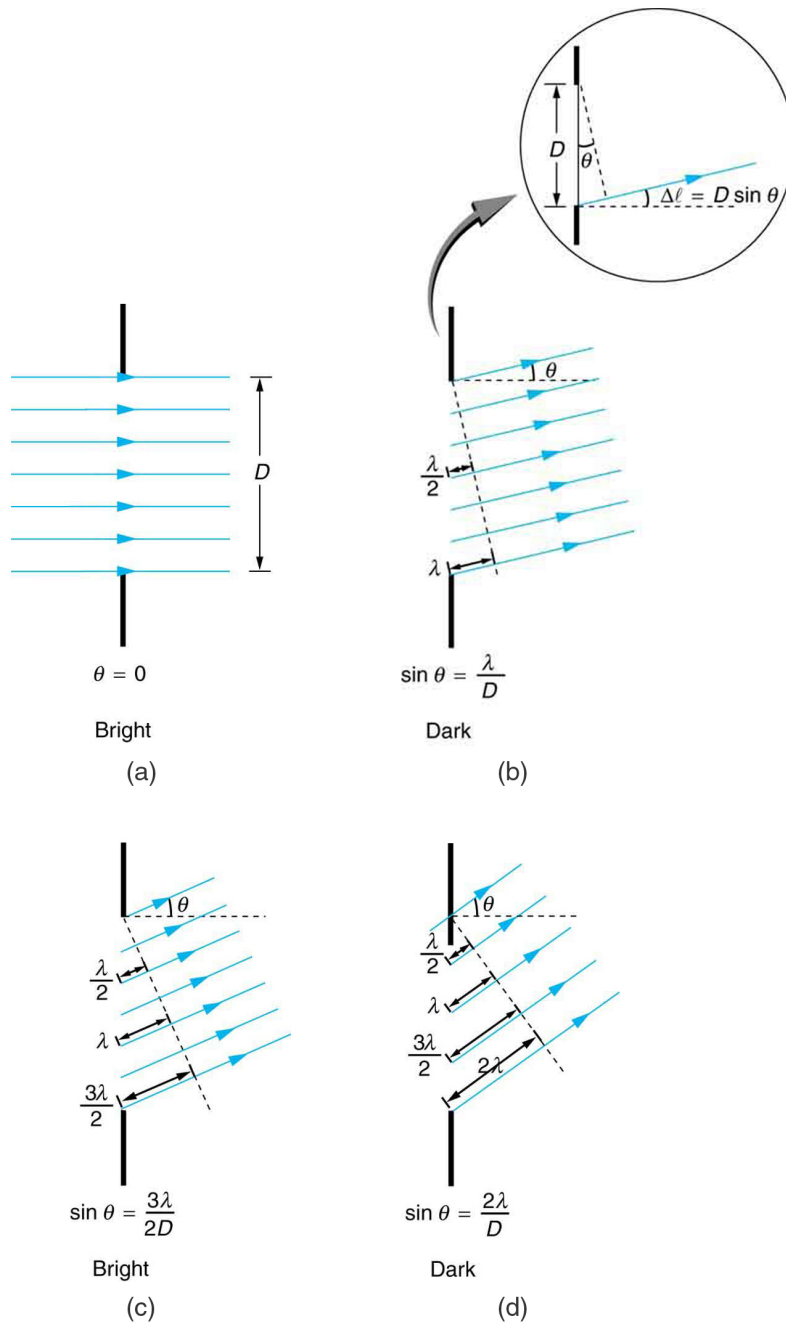


Figure 27.22 Light passing through a single slit is diffracted in all directions

and may interfere constructively or destructively, depending on the angle. The difference in path length for rays from either side of the slit is seen to be  $D \sin \theta$ .

At the larger angle shown in Figure 27.22(c), the path lengths differ by  $3\lambda/2$  for rays from the top and bottom of the slit. One ray travels a distance  $\lambda$  different from the ray from the bottom and arrives in phase, interfering constructively. Two rays, each from slightly above those two, will also add constructively. Most rays from the slit will have another to interfere with constructively, and a maximum in intensity will occur at this angle. However, all rays do not interfere constructively for this situation, and so the maximum is not as intense as the central maximum. Finally, in Figure 27.22(d), the angle shown is large enough to produce a second minimum. As seen in the figure, the difference in path length for rays from either side of the slit is  $D \sin \theta$ , and we see that a destructive minimum is obtained when this distance is an integral multiple of the wavelength.

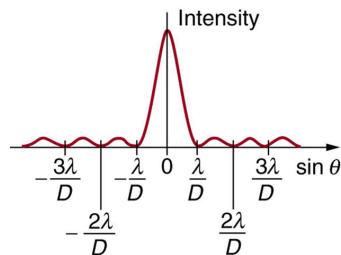


Figure 27.23 A graph of single slit diffraction intensity showing the central maximum to be wider and much more intense than those to the sides. In fact the central maximum is six times higher than shown here.

Thus, to obtain destructive interference for a single slit,

$$D \sin \theta = m\lambda, \text{ for } m = 1, -1, 2, -2, 3, \dots \text{ (destructive),}$$

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where  $D$  is the slit width,  $\lambda$  is the light's wavelength,  $\theta$  is the angle relative to the original direction of the light, and  $m$  is the order of the minimum. Figure 27.23 shows a graph of intensity for single slit interference, and it is apparent that the maxima on either side of the central maximum are much less intense and not as wide. This is consistent with the illustration in Figure 27.21(b).

#### Example 27.4

**Calculating Single Slit Diffraction** Visible light of wavelength 550 nm falls on a single slit and produces its second diffraction minimum at an angle of  $45.0^\circ$  relative to the incident direction of the light. (a) What is the width of the slit? (b) At what angle is the first minimum produced?

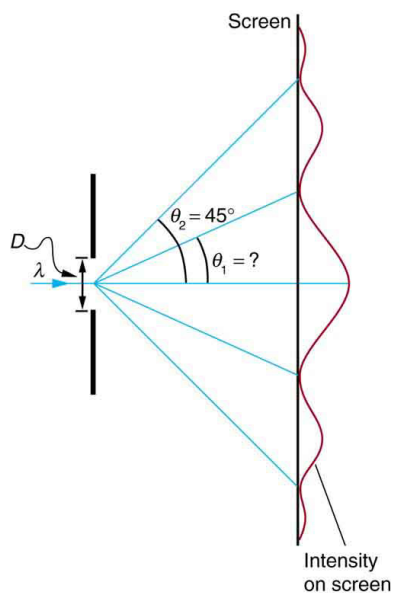


Figure 27.24 A graph of the single slit diffraction pattern is analyzed in this example.

**Strategy** From the given information, and assuming the screen is far away from the slit, we can use the equation  $D \sin \theta = m \lambda$  first to find  $D$ , and again to find the angle for the first minimum  $\theta_1$ .

**Solution for (a)** We are given that  $\lambda = 550 \text{ nm}$ ,  $m = 2$ , and  $\theta_2 = 45.0^\circ$ . Solving the equation  $D \sin \theta = m \lambda$  for  $D$  and substituting known values gives

$$\begin{aligned} D &= \frac{m\lambda}{\sin \theta_2} = \frac{2(550 \text{ nm})}{\sin 45.0^\circ} \\ &= \frac{1100 \times 10^{-9}}{0.707} \\ &= 1.56 \times 10^{-6}. \end{aligned}$$

27.22

**Solution for (b)** Solving the equation  $D \sin \theta = m \lambda$  for  $\sin \theta_1$  and substituting the known values gives

$$\sin \theta_1 = \frac{m\lambda}{D} = \frac{1(550 \times 10^{-9} \text{ m})}{1.56 \times 10^{-6} \text{ m}}.$$

27.23

Thus the angle  $\theta_1$  is

$$\theta_1 = \sin^{-1} 0.354 = 20.7^\circ.$$

27.24

**Discussion** We see that the slit is narrow (it is only a few times greater than the wavelength of light). This is consistent with the fact that light must interact with an object comparable in size to its wavelength in order to exhibit significant wave effects such as this single slit diffraction pattern. We also see that the central maximum extends  $20.7^\circ$  on either side of the original beam, for a width of about  $41^\circ$ . The angle between the first and second minima is only about  $24^\circ$  ( $45.0^\circ - 20.7^\circ$ ). Thus the second maximum is only about half as wide as the central maximum.