PHYS11 CH6: Uniform Circular Motion

Sections 6.1-6.4: Rotational Motion and Forces

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Learning Objectives

By the end of this presentation, you will be able to:

- Define and calculate rotation angle and angular velocity
- Explain centripetal acceleration and its properties
- Analyze forces in circular motion
- Understand non-inertial frames and fictitious forces

Rotation Angle

Definition

The rotation angle $\Delta\theta$ is defined as:

$$\Delta\theta = \frac{\Delta s}{r}$$

where:

- Δs = arc length
- r = radius of curvature

- Measured in radians (rad)
- One complete revolution: 2π rad $= 360^\circ$

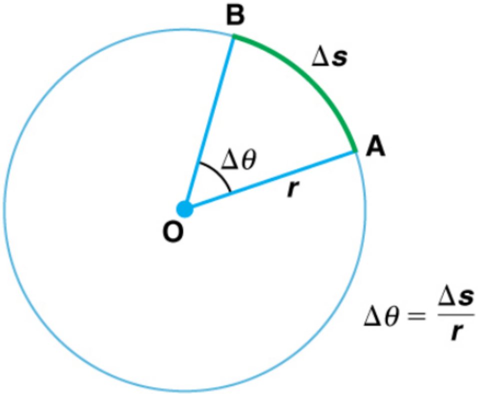


FIGURE 6.3 The radius of a circle is rotated through an angle $\Delta\theta$. The arc length Δs is described on the circumference.

Angular Velocity

Definition

Angular velocity ω is the rate of change of angle:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Relationship to Linear Velocity

$$v = r\omega$$

where:

- v = linear velocity
- r = radius
- ω = angular velocity

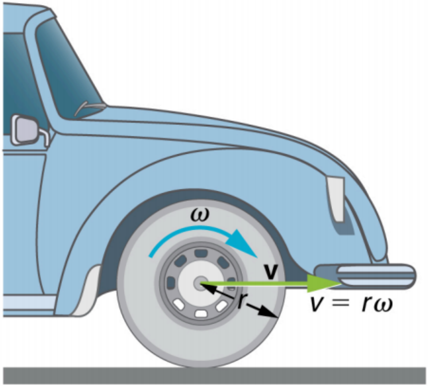


FIGURE 6.5 A car moving at a velocity v to the right has a tire rotating with an angular velocity ω.The speed of the tread of the tire relative to the axle is v, the same as if the car were jacked up. Thus the car moves forward at linear velocity $v = r\omega$, where r is the tire radius. A larger angular velocity for the tire means a greater velocity for the car.

Media

- Centripetal Acceleration
- https://www.youtube.com/watch?v=90rFibLktF4
- Application
- https://youtu.be/im-JM0f$_J7s?si = VO4FyEuT5SLf7Fzr$

Centripetal Acceleration

Definition

Centripetal acceleration is the acceleration toward the center of circular motion:

$$a_c = \frac{v^2}{r} = r\omega^2$$

- Always points toward center of circle
- Magnitude depends on speed and radius
- Required for circular motion

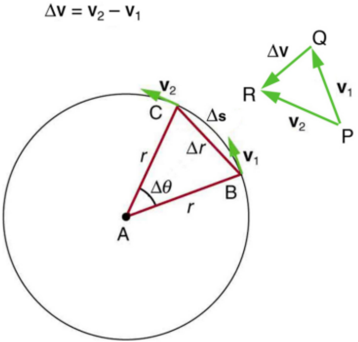


FIGURE 6.7 The directions of the velocity of an object at two different points are shown, and the change in velocity $\Delta \mathbf{v}$ is seen to point directly toward the center of curvature. (See small inset.) Because $\mathbf{a}_c = \Delta \mathbf{v}/\Delta t$, the acceleration is also toward the center; $\mathbf{a}_c$ is called centripetal acceleration. (Because $\Delta\theta$ is very small, the arc length Δs is equal to the chord length Δr for small time differences.)

Example: Centripetal Acceleration

I Do: Car on Curved Path

A car travels around a curve of radius 100 m at 20 m/s. Calculate the centripetal acceleration.

$$a_c = \frac{v^2}{r} = \frac{(20 \text{ m/s})^2}{100 \text{ m}} = 4 \text{ m/s}^2$$

Media

- Centripetal Force
- https://www.youtube.com/watch?v=4bMawIIWi7w

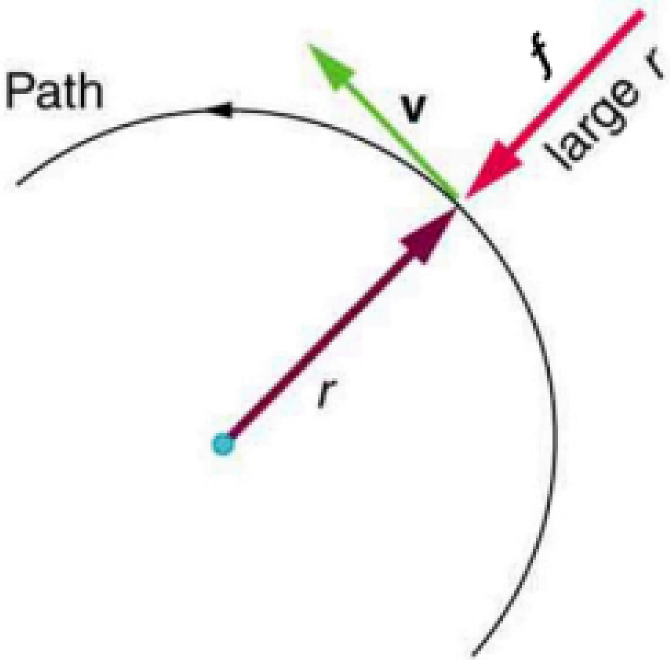
Centripetal Force

Definition

The centripetal force required for circular motion is:

$$F_c = ma_c = m\frac{v^2}{r} = mr\omega^2$$

- Net force must point toward center
- Can be provided by various forces:
  - Tension
  - Gravity
  - Friction
  - Normal force


Path
v
f
large r
r

We Do: Centripetal Force Problem

Problem

A 1000 kg car travels at 15 m/s around a curve of radius 50 m. What centripetal force is required?

$$
\begin{aligned}
F_c &= m\frac{v^2}{r} \\
&= (1000 \text{ kg})\frac{(15 \text{ m/s})^2}{50 \text{ m}} \\
&= 4500 \text{ N}
\end{aligned}
$$

Fictitious Forces

Key Points

- Appear in non-inertial (accelerating) frames
- Not "real" forces - arise from acceleration of reference frame
- Examples:
    - Centrifugal force
    - Coriolis force

Media

- Centrifugal force
- https://www.youtube.com/watch?v=gRVIWWJwzfY

Media

- Coriolis force
- https://www.youtube.com/watch?v=rdGtcZSFRLk

The Coriolis Effect

Properties

- Appears in rotating reference frames
- Affects motion on rotating Earth
- Causes deflection of:
  - Weather systems
  - Projectiles
  - Ocean currents

You Do: Practice Problem

Problem

A 0.5 kg ball is attached to a string and swung in a horizontal circle of radius 1.5 m. If the ball makes one complete revolution in 2 seconds:

1. Calculate the angular velocity
2. Find the centripetal acceleration
3. Determine the tension in the string

Summary

Key Concepts

- Angular quantities describe rotational motion
- Centripetal acceleration points to center
- Centripetal force causes circular motion
- Fictitious forces appear in non-inertial frames