

## 3.2 Representing Acceleration with Equations and Graphs

### Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the kinematic equations related to acceleration and illustrate them with graphs
- Apply the kinematic equations and related graphs to problems involving acceleration

### Teacher Support

**Teacher Support** The Learning Objectives in this section will help your students master the following standards:

- (4) Science concepts. The student knows and applies the laws governing motion in a variety of situations. The student is expected to:
  - (A) generate and interpret graphs and charts describing different types of motion, including the use of real-time technology such as motion detectors or photogates;
  - (B) describe and analyze motion in one dimension using equations with the concepts of distance, displacement, speed, average velocity, instantaneous velocity, and acceleration.

In addition, the High School Physics Laboratory Manual addresses content in this section in the lab titled: Acceleration, as well as the following standards:

- (4) Science concepts. The student knows and applies the laws governing motion in a variety of situations. The student is expected to:
  - (A) generate and interpret graphs and charts describing different types of motion, including the use of real-time technology such as motion detectors or photogates.

### Section Key Terms

### Teacher Support

**Teacher Support** [BL] Briefly review displacement, time, velocity, and acceleration; their variables, and their units.

[OL][AL] Explain that this section introduces five equations that allow us to solve a wider range of problems than just finding acceleration from time and velocity. Review graphical analysis, including axes, algebraic signs, how to designate points on a coordinate plane, i.e.,  $(x, y)$ , slopes, and intercepts. Explain that these equations can also be represented graphically.

### How the Kinematic Equations are Related to Acceleration

We are studying concepts related to motion: time, displacement, velocity, and especially acceleration. We are only concerned with motion in one dimension. The kinematic equations apply to conditions of constant acceleration and show how these concepts are related. Constant acceleration is acceleration that does not change over time. The first kinematic equation relates displacement  $d$ , average velocity  $\bar{v}$ , and time  $t$ .

$$d = d_0 + \bar{v}t$$

3.4

The initial displacement  $d_0$  is often 0, in which case the equation can be written as  $\bar{v} = \frac{d}{t}$

This equation says that average velocity is displacement per unit time. We will express velocity in meters per second. If we graph displacement versus time, as in Figure 3.6, the slope will be the velocity. Whenever a rate, such as velocity, is represented graphically, time is usually taken to be the independent variable and is plotted along the  $x$  axis.

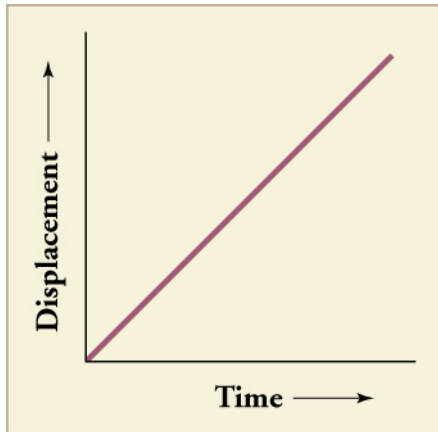


Figure 3.6 The slope of displacement versus time is velocity.

The second kinematic equation, another expression for average velocity  $\bar{v}$ , is simply the initial velocity plus the final velocity divided by two.

$$\bar{v} = \frac{v_0 + v_f}{2}$$

3.5

Now we come to our main focus of this chapter; namely, the kinematic equations that describe motion with constant acceleration. In the third kinematic equation, acceleration is the rate at which velocity increases, so velocity at any point equals initial velocity plus acceleration multiplied by time

$v = v_0 + at$  Also, if we start from rest ( $v_0 = 0$ ), we can write  $a = \frac{v}{t}$

3.6

Note that this third kinematic equation does not have displacement in it. Therefore, if you do not know the displacement and are not trying to solve for a displacement, this equation might be a good one to use.

The third kinematic equation is also represented by the graph in Figure 3.7.

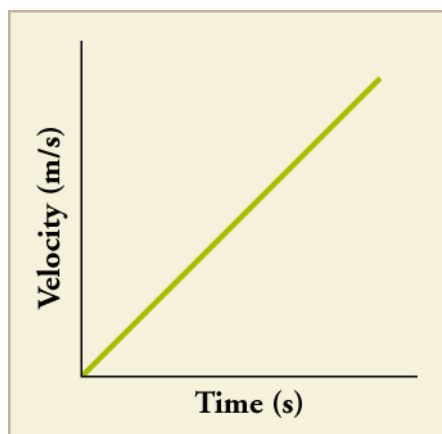


Figure 3.7 The slope of velocity versus time is acceleration.

The fourth kinematic equation shows how displacement is related to acceleration

$$d = d_0 + v_0 t + \frac{1}{2}at^2.$$

3.7

When starting at the origin,  $d_0 = 0$  and, when starting from rest,  $v_0 = 0$ , in which case the equation can be written as

$$a = \frac{2d}{t^2}.$$

This equation tells us that, for constant acceleration, the slope of a plot of  $2d$  versus  $t^2$  is acceleration, as shown in Figure 3.8.

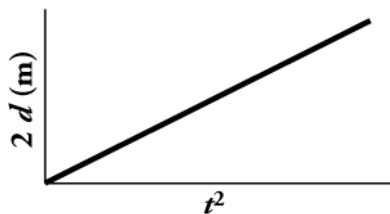


Figure 3.8 When acceleration is constant, the slope of  $2d$  versus  $t^2$  gives the acceleration.

The fifth kinematic equation relates velocity, acceleration, and displacement

$$v^2 = v_0^2 + 2a(d - d_0).$$

3.8

This equation is useful for when we do not know, or do not need to know, the time.

When starting from rest, the fifth equation simplifies to

$$a = \frac{v^2}{2d}.$$

According to this equation, a graph of velocity squared versus twice the displacement will have a slope equal to acceleration.

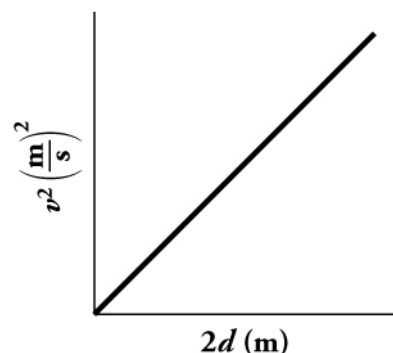


Figure 3.9

Note that, in reality, knowns and unknowns will vary. Sometimes you will want to rearrange a kinematic equation so that the knowns are the values on the axes and the unknown is the slope. Sometimes the intercept will not be at the origin (0,0). This will happen when  $v_0$  or  $d_0$  is not zero. This will be the case when the object of interest is already in motion, or the motion begins at some point other than at the origin of the coordinate system.

### Teacher Support

**Teacher Support** [BL] Be sure everyone is completely comfortable with the idea that velocity is displacement divided by the time during which the displacement occurs. Use everyday examples.

[OL] Remind students that they studied velocity in earlier chapters. Relate the simplified equation

$$\bar{v} = \frac{d}{t}$$

to a graph of displacement versus time. Point out that the plot starts at (0,0) because the initial velocity is zero. Pick a segment on the graph and explain how to find the slope at this segment. Explain that Figure 3.6 would show a straight line if velocity were changing; that is, if the object were accelerating. Show how the graph would change for both negative and positive acceleration.

[OL] Build on the understanding of velocity to introduce acceleration. Contrast constant velocity with changing velocity. Use everyday examples from transportation, sports, or amusement park rides. Explain that Figure 3.7 represents constant acceleration because it is a straight line. Give examples of increasing, decreasing, and constant acceleration, and explain how each affects the shape of plots of velocity versus time.

[OL][AL] Ask students why all three of the plots are straight lines. Refer to the graph of  $2d$  vs  $t^2$  and explain why this is a straight line, whereas  $d$  vs  $t$  would be nonlinear.

## Virtual Physics

**The Moving Man (Part 2)** Look at the Moving Man simulation again and this time use the *Charts* view. Again, vary the velocity and acceleration by sliding the red and green markers along the scales. Keeping the velocity marker near zero will make the effect of acceleration more obvious. Observe how the graphs of position, velocity, and acceleration vary with time. Note which are linear plots and which are not.

[Click to view content](#)

## Teacher Support

**Teacher Support** Ask the students to click the *Charts* option and adjust the settings for the animation to reproduce the plots found in Figure 3.6 and Figure 3.7.

## Grasp Check

On a velocity versus time plot, what does the slope represent?

- a. Acceleration
- b. Displacement
- c. Distance covered
- d. Instantaneous velocity

## Grasp Check

On a position versus time plot, what does the slope represent?

- a. Acceleration
- b. Displacement
- c. Distance covered
- d. Instantaneous velocity

The kinematic equations are applicable when you have constant acceleration.

1.  $d = d_0 + vt$ , or  $v = \frac{d}{t}$  when  $d_0 = 0$
2.  $v = \frac{v_0 + v_f}{2}$
3.  $v = v_0 + at$ , or  $a = \frac{v}{t}$  when  $v_0 = 0$
4.  $d = d_0 + v_0t + \frac{1}{2}at^2$ , or  $a = \frac{2d}{t^2}$  when  $d_0 = 0$  and  $v_0 = 0$
5.  $v^2 = v_0^2 + 2a(d - d_0)$ , or  $a = \frac{2d}{t^2}$  when  $d_0 = 0$  and  $v_0 = 0$

### Teacher Support

**Teacher Support** [OL] Go through the algebra to show how the kinematic equations can be simplified when some of the values are zero. Note that any motion starting or ending in a motionless state will simplify the equations.

### Applying Kinematic Equations to Situations of Constant Acceleration

Problem-solving skills are essential to success in a science and life in general. The ability to apply broad physical principles, which are often represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Essential analytical skills will be developed by solving problems in this text and will be useful for understanding physics and science in general throughout your life.

**Problem-Solving Steps** While no single step-by-step method works for every problem, the following general procedures facilitate problem solving and make the answers more meaningful. A certain amount of creativity and insight are required as well.

1. *Examine the situation to determine which physical principles are involved. It is vital to draw a simple sketch at the outset.* Decide which direction is positive and note that on your sketch.
2. *Identify the knowns: Make a list of what information is given or can be inferred from the problem statement.* Remember, not all given information will be in the form of a number with units in the problem. If something starts *from rest*, we know the initial velocity is zero. If something *stops*, we know the final velocity is zero.
3. *Identify the unknowns: Decide exactly what needs to be determined in the problem.*
4. *Find an equation or set of equations that can help you solve the problem.* Your list of knowns and unknowns can help here. For example, if time is not needed or not given, then the fifth kinematic equation, which does not include time, could be useful.
5. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.* This step produces the numerical answer; it also provides a check on units that can help you find

errors. If the units of the answer are incorrect, then an error has been made.

6. *Check the answer to see if it is reasonable: Does it make sense?* This final step is extremely important because the goal of physics is to accurately describe nature. To see if the answer is reasonable, check its magnitude, its sign, and its units. Are the significant figures correct?

### Summary of Problem Solving

- Determine the knowns and unknowns.
- Find an equation that expresses the unknown in terms of the knowns. More than one unknown means more than one equation is needed.
- Solve the equation or equations.
- Be sure units and significant figures are correct.
- Check whether the answer is reasonable.

### Teacher Support

**Teacher Support** [BL][OL][AL] Consider starting with the simplified summary of problem solving and then expand it to the more wordy description above. Stress that this is *not* a strict recipe that can solve all problems. Applying analytical skills is still required. If the preliminary analysis is correct and the knowns and unknowns are correctly sorted, the rest should be correct. Try applying the method to some non-mathematical everyday problems. Ask for suggestions.

## Fun In Physics



### Drag Racing

Figure 3.10 Smoke rises from the tires of a dragster at the beginning of a drag race. (Lt. Col. William Thurmond. Photo courtesy of U.S. Army.)

The object of the sport of drag racing is acceleration. Period! The races take place from a standing start on a straight one-quarter-mile (402 m) track. Usually two cars race side by side, and the winner is the driver who gets the car past the quarter-mile point first. At the finish line, the cars may be going more than 300 miles per hour (134 m/s). The driver then deploys a parachute to bring the car to a stop because it is unsafe to brake at such high speeds. The cars, called dragsters, are capable of accelerating at  $26 \text{ m/s}^2$ . By comparison, a typical sports car that is available to the general public can accelerate at about  $5 \text{ m/s}^2$ .

Several measurements are taken during each drag race:



- Reaction time is the time between the starting signal and when the front of the car crosses the starting line.
- Elapsed time is the time from when the vehicle crosses the starting line to when it crosses the finish line. The record is a little over 3 s.
- Speed is the average speed during the last 20 m before the finish line. The record is a little under 400 mph.

The video shows a race between two dragsters powered by jet engines. The actual race lasts about four seconds and is near the end of the video.

### Teacher Support

**Teacher Support** Explain the direction and magnitude of acceleration and velocity vectors before and after the braking chute is deployed. Explain that  $\frac{1}{4}$  mile is 440 yards. If any students are on the track team, you might ask them to describe this distance. Compare record times for this track event to 4 seconds.

A dragster crosses the finish line with a velocity of  $140 \frac{\text{m}}{\text{s}}$ . Assuming the vehicle maintained a constant acceleration from start to finish, what was its average velocity for the race?

- $0 \frac{\text{m}}{\text{s}}$
- $35 \frac{\text{m}}{\text{s}}$
- $70 \frac{\text{m}}{\text{s}}$
- $140 \frac{\text{m}}{\text{s}}$

### Worked Example

**Acceleration of a Dragster** The time it takes for a dragster to cross the finish line is unknown. The dragster accelerates from rest at  $26 \text{ m/s}^2$  for a quarter mile (0.250 mi). What is the final velocity of the dragster?

### Strategy

The equation  $v^2 = v_0^2 + 2a(d - d_0)$  is ideally suited to this task because it gives the velocity from acceleration and displacement, without involving the time.

Solution

1. Convert miles to meters.
  - $(0.250 \text{ mi}) \times \frac{1609 \text{ m}}{1 \text{ mi}} = 402 \text{ m}$
2. Identify the known values. We know that  $v_0 = 0$  since the dragster starts from rest, and we know that the distance traveled,  $d - d_0$  is 402 m. Finally, the acceleration is constant at  $a = 26.0 \text{ m/s}^2$ .
3. Insert the knowns into the equation  $v^2 = v_0^2 + 2a(d - d_0)$  and solve for  $v$ .

- $v^2 = 0 + 2 \left( 26.0 \frac{\text{m}}{\text{s}^2} \right) (402 \text{ m}) = 2.09 \times 10^4 \frac{\text{m}^2}{\text{s}^2}$

3.10

Taking the square root gives us  $v = \sqrt{2.09 \times 10^4 \frac{\text{m}^2}{\text{s}^2}} = 145 \frac{\text{m}}{\text{s}}$ .

Discussion

145 m/s is about 522 km/hour or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values. We took the positive value because we know that the velocity must be in the same direction as the acceleration for the answer to make physical sense.

An examination of the equation  $v^2 = v_0^2 + 2a(d - d_0)$  can produce further insights into the general relationships among physical quantities:

- The final velocity depends on the magnitude of the acceleration and the distance over which it applies.
- For a given acceleration, a car that is going twice as fast does not stop in twice the distance—it goes much further before it stops. This is why, for example, we have reduced speed zones near schools.

### Teacher Support

**Teacher Support** [OL] Work through the problem-solving steps with the student.

1. What is the goal?
2. What is known and unknown?
3. Which equation expresses the unknown in terms of the knowns?
4. Solve for the unknown.
5. Insert known values.
6. Calculate.
7. Checking that answer is reasonable and has the correct units, sign, and significant figures.

Repeat for the second sample problem.

[AL] Initiate a discussion on variation in gravity. Compare gravity on Earth to gravity on the moon. Explain the difference between  $g$  and constants that are the same everywhere in the universe, such as the speed of light.

### Practice Problems

6.

Dragsters can reach a top speed of 145 m/s in only 4.45 s. Calculate the average acceleration for such a dragster.

- a.  $-32.6 \text{ m/s}^2$
- b.  $0 \text{ m/s}^2$

- c.  $32.6 \text{ m/s}^2$
- d.  $145 \text{ m/s}^2$

7.

An Olympic-class sprinter starts a race with an acceleration of  $4.50 \text{ m/s}^2$ . Assuming she can maintain that acceleration, what is her speed  $2.40 \text{ s}$  later?

- a.  $4.50 \text{ m/s}$
- b.  $10.8 \text{ m/s}$
- c.  $19.6 \text{ m/s}$
- d.  $44.1 \text{ m/s}$

### Constant Acceleration

In many cases, acceleration is not uniform because the force acting on the accelerating object is not constant over time. A situation that gives constant acceleration is the acceleration of falling objects. When air resistance is not a factor, all objects near Earth's surface fall with an acceleration of about  $9.80 \text{ m/s}^2$ . Although this value decreases slightly with increasing altitude, it may be assumed to be essentially constant. The value of  $9.80 \text{ m/s}^2$  is labeled  $g$  and is referred to as acceleration due to gravity. Gravity is the force that causes nonsupported objects to accelerate downward—or, more precisely, toward the center of Earth. The magnitude of this force is called the weight of the object and is given by  $mg$  where  $m$  is the mass of the object (in kg). In places other than on Earth, such as the Moon or on other planets,  $g$  is not  $9.80 \text{ m/s}^2$ , but takes on other values. When using  $g$  for the acceleration  $a$  in a kinematic equation, it is usually given a negative sign because the acceleration due to gravity is downward.

## Work In Physics



### Effects of Rapid Acceleration

Figure 3.11 Astronauts train using G Force Simulators. (NASA)

When in a vehicle that accelerates rapidly, you experience a force on your entire body that accelerates your body. You feel this force in automobiles and slightly more on amusement park rides. For example, when you ride in a car that turns, the car applies a force on your body to make you accelerate in the direction in which the car is turning. If enough force is applied, you will accelerate at  $9.80 \text{ m/s}^2$ . This is the same as the acceleration due to gravity, so this force is called one G.

One G is the force required to accelerate an object at the acceleration due to gravity at Earth's surface. Thus, one G for a paper cup is much less than one G for an elephant, because the elephant is much more massive and requires a

greater force to make it accelerate at  $9.80 \text{ m/s}^2$ . For a person, a  $G$  of about 4 is so strong that his or her face will distort as the bones accelerate forward through the loose flesh. Other symptoms at extremely high  $G$ s include changes in vision, loss of consciousness, and even death. The space shuttle produces about 3  $G$ s during takeoff and reentry. Some roller coasters and dragsters produce forces of around 4  $G$ s for their occupants. A fighter jet can produce up to 12  $G$ s during a sharp turn.

Astronauts and fighter pilots must undergo  $G$ -force training in simulators. The video shows the experience of several people undergoing this training.

People, such as astronauts, who work with  $G$  forces must also be trained to experience zero  $G$ —also called free fall or weightlessness—which can cause queasiness. NASA has an aircraft that allows its occupants to experience about 25 s of free fall. The aircraft is nicknamed the *Vomit Comet*.

### Teacher Support

**Teacher Support** Mention that, later in this course, students will encounter some interesting concepts related to gravity and acceleration when studying the general theory of relativity. In part, the theory is based on the idea that gravity cannot be distinguished from acceleration, either experientially or mathematically. When in an upward-bound elevator, can you really tell whether you are accelerating or whether gravity has suddenly become stronger?

### Grasp Check

A common way to describe acceleration is to express it in multiples of  $g$ , Earth's gravitational acceleration. If a dragster accelerates at a rate of  $39.2 \text{ m/s}^2$ , how many  $g$ 's does the driver experience?

- a.  $1.5 g$
- b.  $4.0 g$
- c.  $10.5 g$
- d.  $24.5 g$

### Worked Example

**Falling Objects** A person standing on the edge of a high cliff throws a rock straight up with an initial velocity  $v_0$  of  $13 \text{ m/s}$ .

(a) Calculate the position and velocity of the rock at 1.00, 2.00, and 3.00 seconds after it is thrown. Ignore the effect of air resistance.

### Strategy

Sketch the initial velocity and acceleration vectors and the axes.

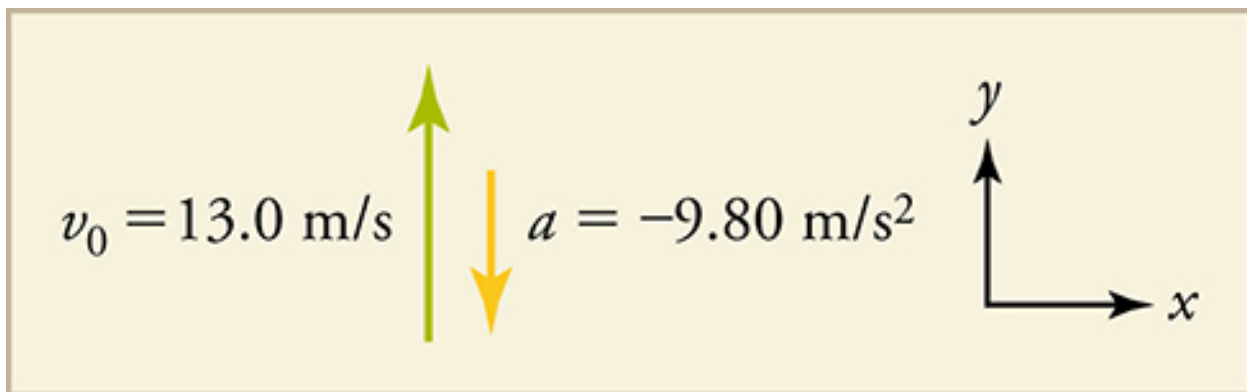


Figure 3.12 Initial conditions for rock thrown straight up.

List the knowns: time  $t = 1.00$  s,  $2.00$  s, and  $3.00$  s; initial velocity  $v_0 = 13$  m/s; acceleration  $a = g = -9.80$  m/s<sup>2</sup>; and position  $y_0 = 0$  m

List the unknowns:  $y_1$ ,  $y_2$ , and  $y_3$ ;  $v_1$ ,  $v_2$ , and  $v_3$ —where 1, 2, 3 refer to times 1.00 s, 2.00 s, and 3.00 s

Choose the equations.

$$d = d_0 + v_0t + \frac{1}{2}at^2 \text{ becomes } y = y_0 + v_0t - \frac{1}{2}gt^2$$

3.11

$$v = v_0 + at \text{ becomes } v = v_0 - gt$$

3.12

These equations describe the unknowns in terms of knowns only.

Solution

$$y_1 = 0 + (13.0 \text{ m/s})(1.00 \text{ s}) + \frac{(-9.80 \text{ m/s}^2)(1.00 \text{ s})^2}{2} = 8.10 \text{ m}$$

$$y_2 = 0 + (13.0 \text{ m/s})(2.00 \text{ s}) + \frac{(-9.80 \text{ m/s}^2)(2.00 \text{ s})^2}{2} = 6.40 \text{ m}$$

$$y_3 = 0 + (13.0 \text{ m/s})(3.00 \text{ s}) + \frac{(-9.80 \text{ m/s}^2)(3.00 \text{ s})^2}{2} = -5.10 \text{ m}$$

$$v_1 = 13.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(1.00 \text{ s}) = 3.20 \text{ m/s}$$

$$v_2 = 13.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(2.00 \text{ s}) = -6.60 \text{ m/s}$$

$$v_3 = 13.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(3.00 \text{ s}) = -16.4 \text{ m/s}$$

Discussion

The first two positive values for  $y$  show that the rock is still above the edge of the cliff, and the third negative  $y$  value shows that it has passed the starting point and is below the cliff. Remember that we set  $up$  to be positive. Any position with a positive value is above the cliff, and any velocity with a positive value is an upward velocity. The first value for  $v$  is positive, so the rock is still on the way up. The second and third values for  $v$  are negative, so the rock is on its way down.

(b) Make graphs of position versus time, velocity versus time, and acceleration versus time. Use increments of 0.5 s in your graphs.

### Strategy

Time is customarily plotted on the  $x$ -axis because it is the independent variable. Position versus time will not be linear, so calculate points for 0.50 s, 1.50 s, and 2.50 s. This will give a curve closer to the true, smooth shape.

Solution

The three graphs are shown in Figure 3.13.

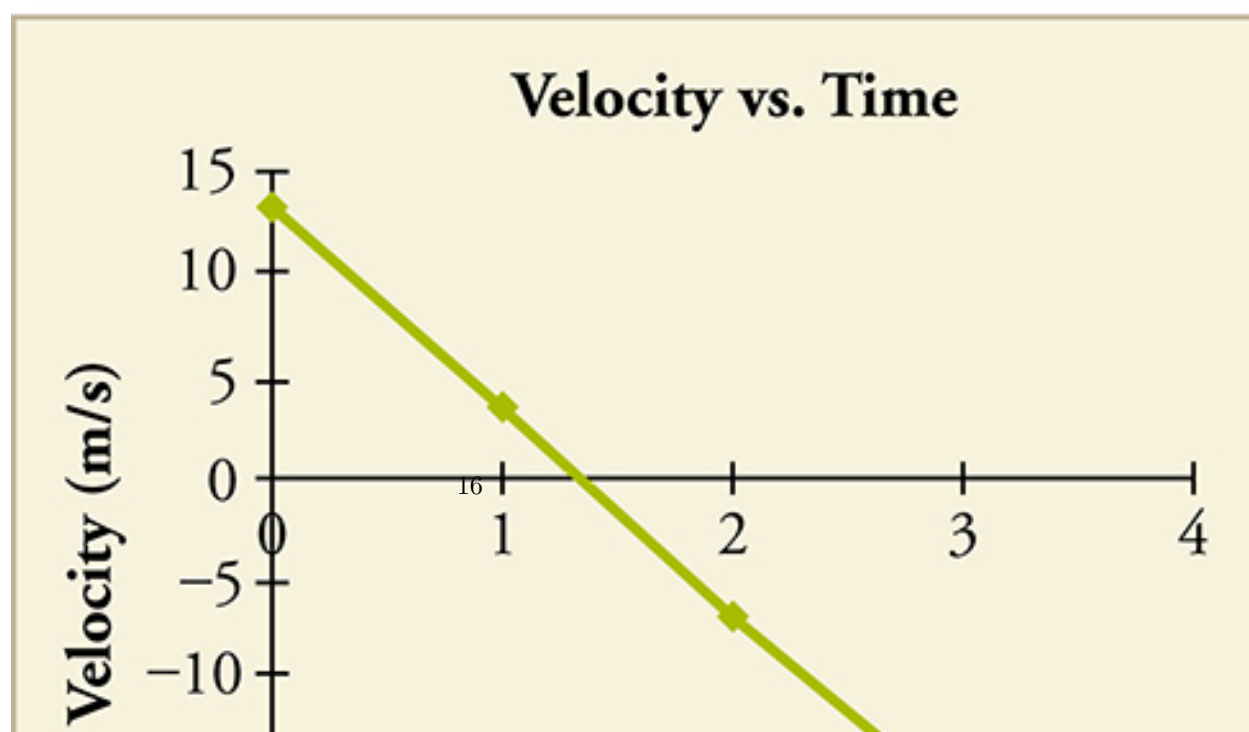
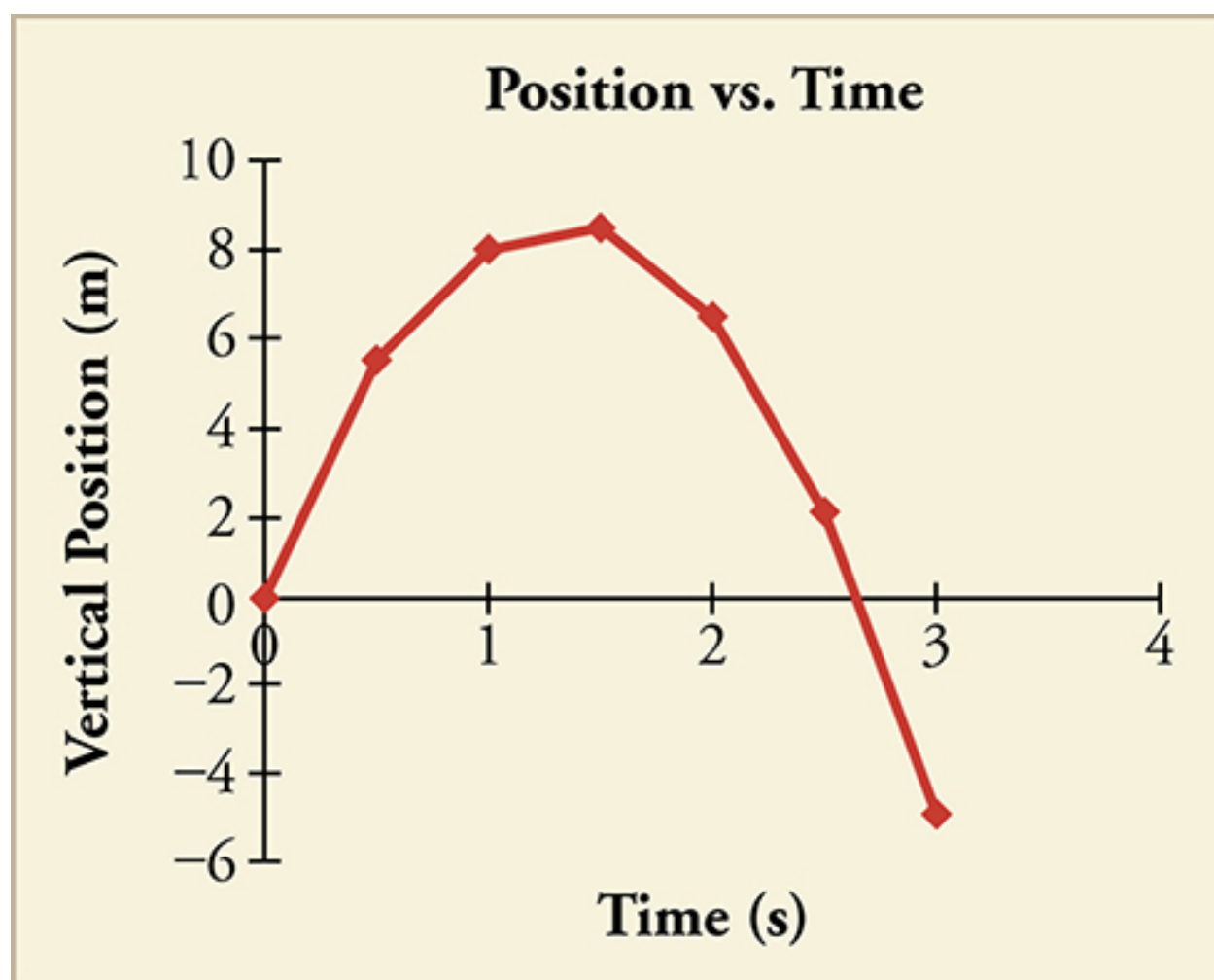




Figure 3.13

#### Discussion

- $y$  vs.  $t$  does *not* represent the two-dimensional parabolic path of a trajectory. The path of the rock is straight up and straight down. The slope of a line tangent to the curve at any point on the curve equals the velocity at that point—i.e., the instantaneous velocity.
- Note that the  $v$  vs.  $t$  line crosses the vertical axis at the initial velocity and crosses the horizontal axis at the time when the rock changes direction and begins to fall back to Earth. This plot is linear because acceleration is constant.
- The  $a$  vs.  $t$  plot also shows that acceleration is constant; that is, it does not change with time.

#### Teacher Support

**Teacher Support** Prior to solving the problem, have students consider the following questions:

1. What is the goal?
2. What is known and unknown?
3. Which equation expresses the unknown in terms of the knowns?

After solving the problem, have students check that the answer is reasonable and has the correct units, sign, and significant figures.

#### Practice Problems

8.

A cliff diver pushes off horizontally from a cliff and lands in the ocean  $2.00\text{ s}$  later. How fast was he going when he entered the water?

- a.  $0\text{ m/s}$
- b.  $19.0\text{ m/s}$
- c.  $19.6\text{ m/s}$
- d.  $20.0\text{ m/s}$

9.

A girl drops a pebble from a high cliff into a lake far below. She sees the splash of the pebble hitting the water  $2.00\text{ s}$  later. How fast was the pebble going when it hit the water?

- a.  $9.80\text{ m/s}$
- b.  $10.0\text{ m/s}$
- c.  $19.6\text{ m/s}$
- d.  $20.0\text{ m/s}$

## Check Your Understanding

### Teacher Support

**Teacher Support** Use these questions to assess students' achievement of the sections Learning Objectives. If students are struggling with a specific objective, the formative assessment will help identify which objective is the problem so that you can direct students to the relevant content.

10.

Identify the four variables found in the kinematic equations.

- a. Displacement, Force, Mass, and Time
- b. Acceleration, Displacement, Time, and Velocity
- c. Final Velocity, Force, Initial Velocity, and Mass
- d. Acceleration, Final Velocity, Force, and Initial Velocity

11.

Which of the following steps is always required to solve a kinematics problem?

- a. Find the force acting on the body.
- b. Find the acceleration of a body.
- c. Find the initial velocity of a body.
- d. Find a suitable kinematic equation and then solve for the unknown quantity.

12.

Which of the following provides a correct answer for a problem that can be solved using the kinematic equations?

- a. A body starts from rest and accelerates at  $4\text{ m/s}^2$  for  $2\text{ s}$ . The body's final velocity is  $8\text{ m/s}$ .
- b. A body starts from rest and accelerates at  $4\text{ m/s}^2$  for  $2\text{ s}$ . The body's final velocity is  $16\text{ m/s}$ .
- c. A body with a mass of  $2\text{ kg}$  is acted upon by a force of  $4\text{ N}$ . The acceleration of the body is  $2\text{ m/s}^2$ .
- d. A body with a mass of  $2\text{ kg}$  is acted upon by a force of  $4\text{ N}$ . The acceleration of the body is  $0.5\text{ m/s}^2$ .