PHYS11 CH:2 Reading the Story of Motion

Position and Velocity Graphs

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December 2025

Outline

The Mystery

How can a single line

tell the complete story of motion?

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Position, velocity, acceleration—all encoded in curves and slopes.

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tell the complete story of motion?

Position, velocity, acceleration—all encoded in curves and slopes.

A graph is worth a thousand equations.

Learning Objectives

By the end of this lesson, you will be able to:

- **2.3:** Explain the meaning of slope in position vs. time graphs

By the end of this lesson, you will be able to:

- **2.3:** Explain the meaning of slope in position vs. time graphs
- **2.3:** Solve problems using position vs. time graphs

2.3 The Language of Graphs

The Mental Model

A graph is like a picture—worth a thousand words. It reveals relationships between physical quantities.

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- Horizontal axis = independent variable (usually time)
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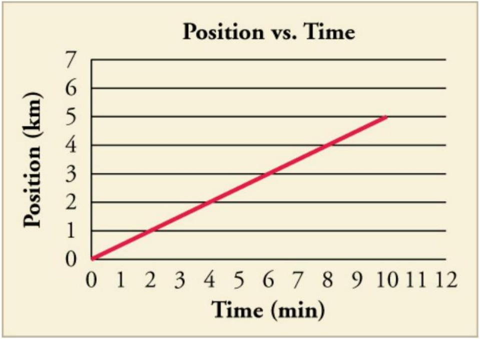
The Mental Model

A graph is like a picture—worth a thousand words. It reveals relationships between physical quantities.

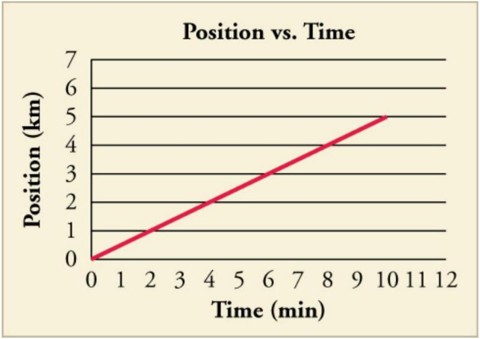
Standard graph anatomy:

- Horizontal axis = independent variable (usually time)
- Vertical axis = dependent variable (position, velocity)
- Straight line: $y = mx + b$ where m = slope, b = y-intercept

2.3 Drive to School



Graph of position vs. time for 5 km drive to school



Graph of position vs. time for 5 km drive to school

What does this line tell us?

- Starts at home ($d_0 = 0$)
- Ends at school ($d_f = 5$ km)
- Takes 10 minutes

2.3 Reading the Slope

Universal Law: Slope is Velocity

In a position vs. time graph:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta d}{\Delta t} = v_{\text{avg}}$$

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For the drive to school:

$$v_{\text{avg}} = \frac{5 \text{ km}}{10 \text{ min}} = 0.5 \text{ km/min}$$

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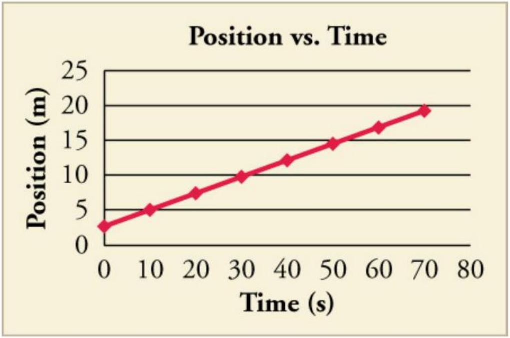
The Anchor

Steeper slope = faster motion. Flat line = at rest.

2.3 Round Trip



What does the graph look like with the return trip?

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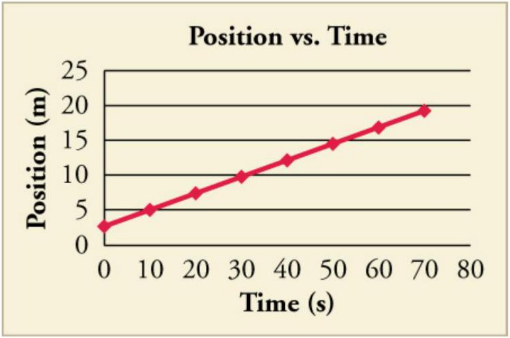

What does the graph look like with the return trip?

Second leg:

- Negative slope = moving backward
- Returns to $d = 0$ (back home)
- Net displacement = 0 km

2.3 Jet Car on Salt Flats

Position vs. time for jet-powered car

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Position vs. time for jet-powered car

Reading the graph:

- At $t = 0$ s: $d = 400$ m
- At $t = 1$ s: $d = 650$ m
- Slope = velocity = 250 m/s

2.3 The Position Equation

Universal Law: Linear Motion

From the graph equation $y = mx + b$, we get:

$$\boxed{d = vt + d_0}$$

or equivalently

$$\boxed{d = d_0 + vt}$$

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Where:

- m (slope) $=$ velocity v
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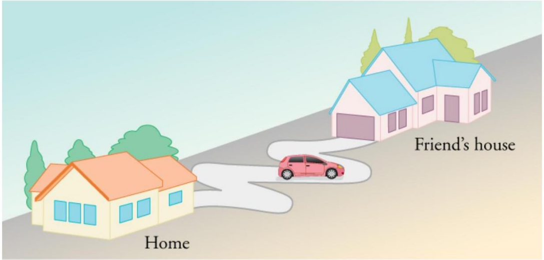
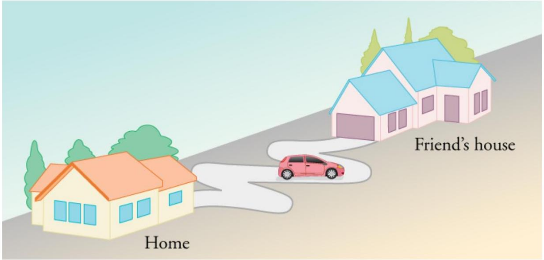


Jet car speeding up - curved graph

2.3 Curved Position Graphs



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When the graph curves, velocity is changing. Slope is not constant!

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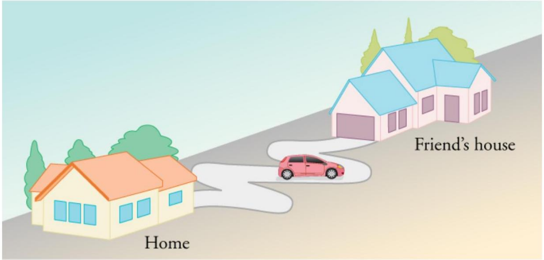

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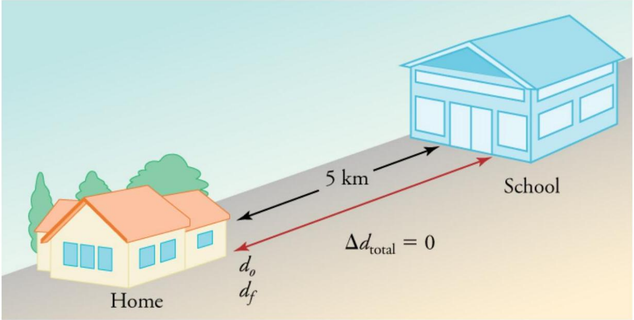
When the graph curves, velocity is changing. Slope is not constant!

Solution: Use tangent line to find instantaneous velocity at any point.

2.3 Instantaneous Velocity from Tangent



Slope of tangent line = instantaneous velocity

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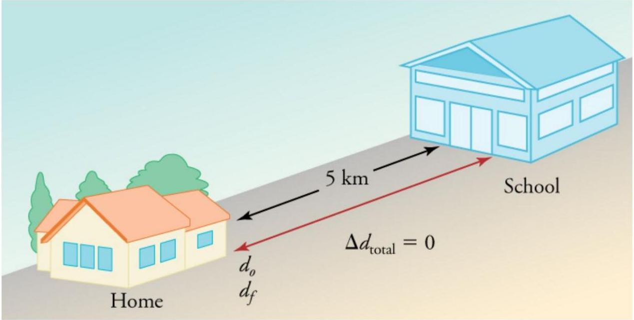


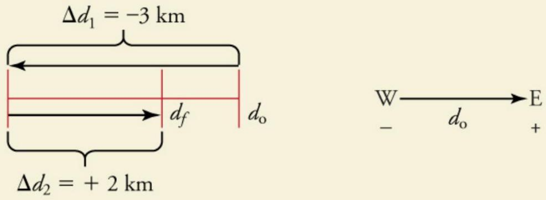
Slope of tangent line = instantaneous velocity

At point Q ($t = 25$ s):

$$v_Q = \frac{3120 - 1300 \text{ m}}{32 - 19 \text{ s}} = \frac{1820 \text{ m}}{13 \text{ s}} = 140 \text{ m/s}$$

Attempt: Reading a Position Graph

The Challenge (3 min, silent)



Given: The graph above

Find: Average velocity over entire time interval (0 to 16 s)

Can you decode this motion? Work silently.

Compare: Graph Reading Strategy

Turn and talk (2 min):

1. What two points did you choose?
2. How did you calculate the slope?
3. What units did you get?

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Reveal: Slope Equals Velocity

Self-correct in a different color:

Step 1: Identify endpoints: $(t_0, d_0) = (0, 0)$ m and $(t_f, d_f) = (16, 5)$ m

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$$v_{\text{avg}} = \frac{\Delta d}{\Delta t} = \frac{d_f - d_0}{t_f - t_0}$$

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Step 3: Substitute

$$v_{\text{avg}} = \frac{5 - 0 \text{ m}}{16 - 0 \text{ s}} = \boxed{0.31 \text{ m/s}}$$

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Check: About 1 km/h - a slow walk. Reasonable!

Learning Objectives

By the end of this lesson, you will be able to:

- **2.4:** Explain the meaning of slope and area in velocity vs. time graphs

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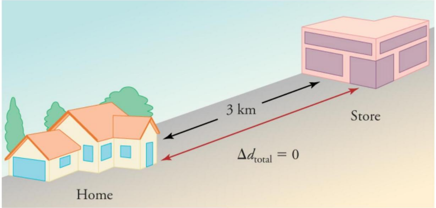
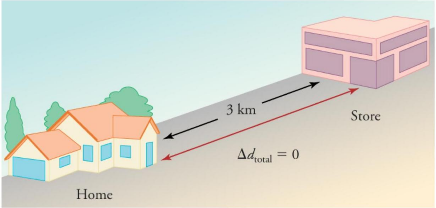
2.4 From Position to Velocity Graph



Position graph: drive to and from school

2.4 From Position to Velocity Graph



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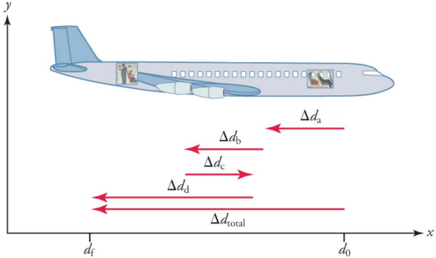
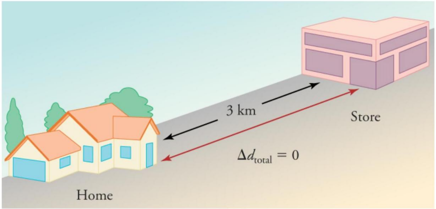


Velocity graph: two constant velocities



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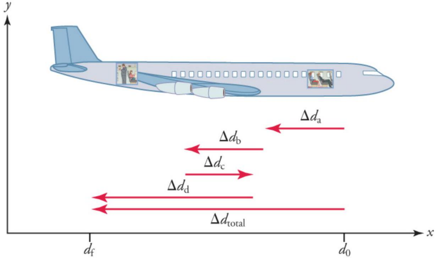

Velocity graph: two constant velocities

Key insight: Slope of position graph becomes height of velocity graph!

2.4 Reading Velocity Graphs

Universal Law: The Dual Nature

In a velocity vs. time graph:

1. **Slope** = acceleration (rate of velocity change)
2. **Area under curve** = displacement

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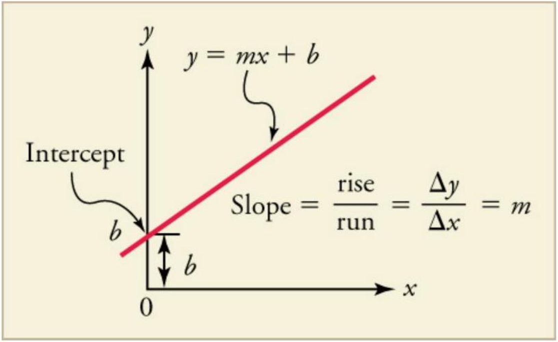
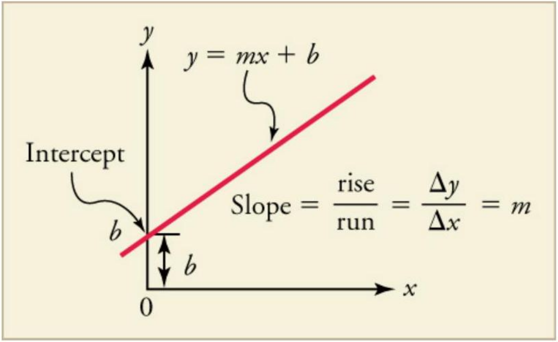
The Anchor

Position graphs give velocity. Velocity graphs give acceleration AND displacement.

2.4 Area Equals Displacement



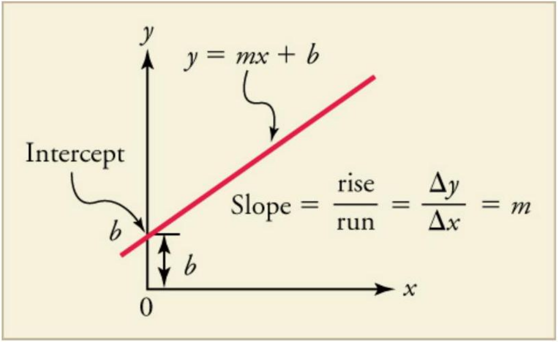
Velocity graph for drive to school



Velocity graph for drive to school

Calculate displacement:

$$d = v \times t = 0.5 \text{ km/min} \times 10 \text{ min} = 5 \text{ km}$$



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Return trip:

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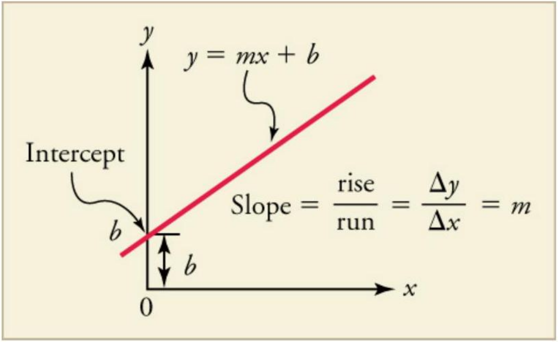

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Where:

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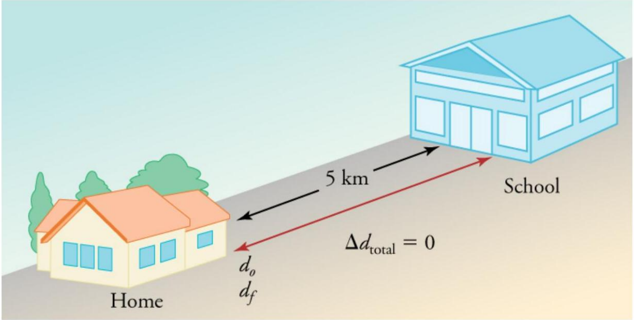
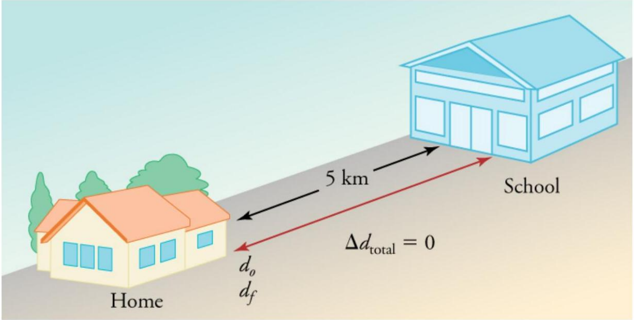
And from area:

$$\boxed{d = vt} \quad \text{(for constant velocity)}$$

2.4 Jet Car Velocity Graph



Jet car speeding up - straight line with positive slope

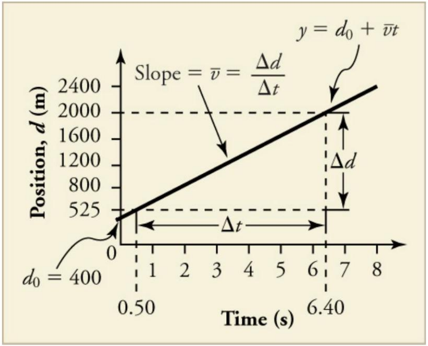


Jet car speeding up - straight line with positive slope

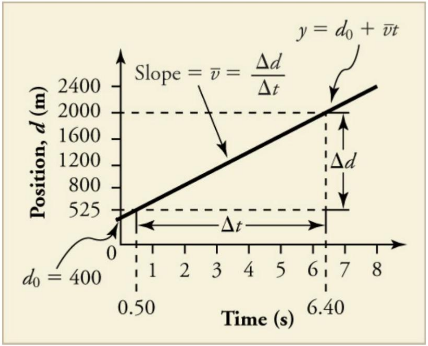
What we can read:

- Starts at $v_0 = 20$ m/s at $t = 0$
- Ends at $v_f = 160$ m/s at $t = 30$ s
- Slope = acceleration (constant)

2.4 Zero Slope Means Constant Velocity



Horizontal line in velocity graph



Horizontal line in velocity graph

Key Insight

Slope $=$ 0 means acceleration $=$ 0. Object moves at constant velocity.

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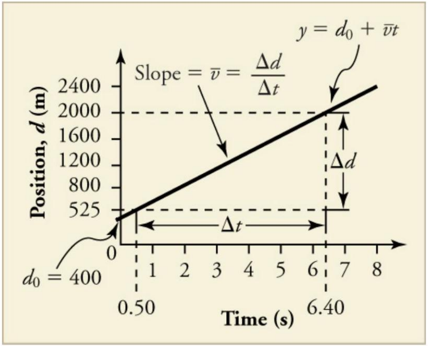


Horizontal line in velocity graph

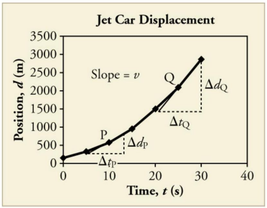
Key Insight

Slope = 0 means acceleration = 0. Object moves at constant velocity.

This is what we saw in the drive to school example!

Attempt: Calculating from Velocity Graph

The Challenge (3 min, silent)



Given: Velocity graph above (jet car from 0 to 30 s)
Find:

- (a) Displacement
- (b) Acceleration

Use both slope and area. Work silently.

Compare: Dual Extraction

Turn and talk (2 min):

1. How did you find displacement? (Hint: area)
2. How did you find acceleration? (Hint: slope)
3. Did you break the area into shapes?

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(a) Displacement = Area under curve

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Rectangle: $20 \text{ m/s} \times 30 \text{ s} = 600 \text{ m}$

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Rectangle: $20\ \text{m/s} \times 30\ \text{s} = 600\ \text{m}$

Triangle: $\frac{1}{2} \times 30\ \text{s} \times 140\ \text{m/s} = 2100\ \text{m}$

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Total: $\boxed{d = 2700 \text{ m}}$

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(b) Acceleration = Slope

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(b) Acceleration = Slope

$$a = \frac{\Delta v}{\Delta t} = \frac{140 \text{ m/s}}{30 \text{ s}} = \boxed{4.67 \text{ m/s}^2}$$

2.4 Curved Velocity Graphs

More realistic jet car - curved velocity graph

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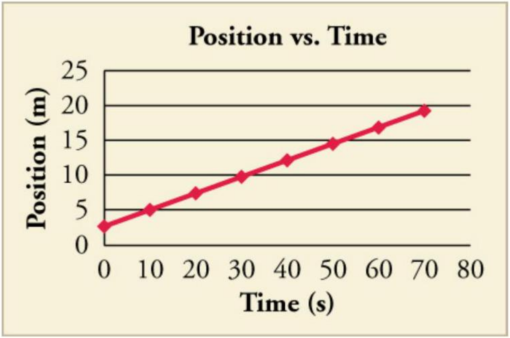
More realistic jet car - curved velocity graph

The Complication

When velocity graph curves, acceleration is changing! Use tangent for instantaneous acceleration.

2.4 Negative Velocity



Velocity graph going below zero

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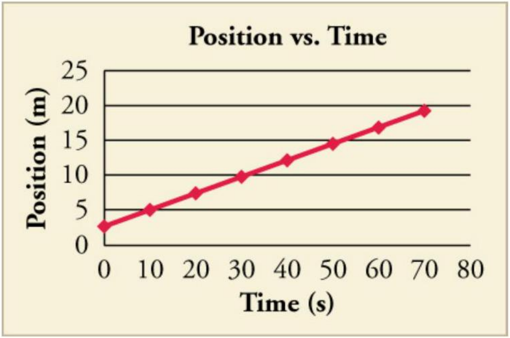

Velocity graph going below zero

Interpretation:

- Positive velocity = moving forward
- Negative velocity = moving backward
- Zero crossing = turning point (changes direction)

2.4 Position from Velocity Graph

The Connection

From position graph: slope $\rightarrow$ velocity

From velocity graph: area $\rightarrow$ displacement

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From position graph: slope $\rightarrow$ velocity
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Circular Relationship

$$\text{Position} \xrightarrow{\text{slope}} \text{Velocity} \xrightarrow{\text{slope}} \text{Acceleration}$$

$$\text{Position} \xleftarrow{\text{area}} \text{Velocity} \xleftarrow{?} \text{Acceleration}$$

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From position graph: slope $\rightarrow$ velocity
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$$\text{Position} \xrightarrow{\text{slope}} \text{Velocity} \xrightarrow{\text{slope}} \text{Acceleration}$$

$$\text{Position} \xleftarrow{\text{area}} \text{Velocity} \xleftarrow{?} \text{Acceleration}$$

We'll learn about acceleration graphs in the next chapter!

What You Now Know

The Revelations

1. Graphs are the visual language of motion

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What You Now Know

The Revelations

1. Graphs are the visual language of motion
2. Position graph: slope = velocity
3. Velocity graph: slope = acceleration, area = displacement
4. Tangent lines extract instantaneous values
5. Negative slopes/areas show direction
6. One graph encodes multiple quantities

Key Equations

$$\text{Position graph slope} = \frac{\Delta d}{\Delta t} = v_{\text{avg}} \tag{1}$$

$$\text{Position equation} = d = d_0 + vt \tag{2}$$

$$\text{Velocity graph slope} = \frac{\Delta v}{\Delta t} = a \tag{3}$$

$$\text{Velocity equation} = v = v_0 + at \tag{4}$$

$$\text{Displacement from velocity} = \text{area under } v\text{-}t \text{ curve} \tag{5}$$

Homework

Complete the assigned problems

posted on the LMS