

# PHYS12 CH:6 The Art of Falling Forever

## Circular Motion and Rotation

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# Outline

- 1 Introduction
- 2 Angle of Rotation and Angular Velocity
- 3 Uniform Circular Motion
- 4 Rotational Motion
- 5 Summary

How do you move forward  
*while constantly turning?*

# The Mystery

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How do you move forward  
*while constantly turning?*

From Formula 1 cars screaming around curves to the Moon circling Earth...

All require a force toward the center.

# Falling Forever



Figure: Formula 1 car in circular motion

# Falling Forever



Figure: Formula 1 car in circular motion

## The Mental Model

A satellite in orbit is falling toward Earth but moving fast enough sideways to keep missing it.

# Learning Objectives

By the end of this section, you will be able to:

- **6.1:** Describe the angle of rotation and relate it to its linear counterpart



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- **6.1:** Describe the angle of rotation and relate it to its linear counterpart
- **6.1:** Describe angular velocity and relate it to its linear counterpart
- **6.1:** Solve problems involving angle of rotation and angular velocity

## 6.1 Two Kinds of Rotation

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**Spin:** Object rotates about its own axis (Earth spinning)

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## Real-World Examples

- Earth spins on its axis (spin) AND orbits the Sun (circular motion)
- Your car tire spins (spin) while the car follows a curve (circular motion)

## 6.1 Angle of Rotation

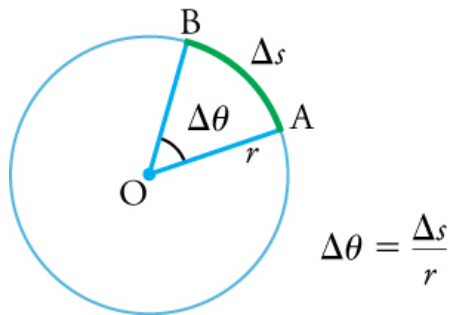
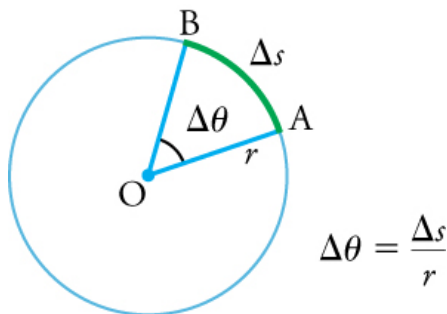


Figure: Arc length and radius

## 6.1 Angle of Rotation



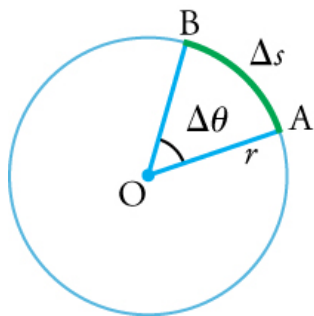
Universal Law: Angle of Rotation

$$\Delta\angle\theta = \frac{\Delta s}{r}$$

*Angle equals arc length divided by radius*

Figure: Arc length and radius

## 6.1 Angle of Rotation



$$\Delta\theta = \frac{\Delta s}{r}$$

Universal Law: Angle of Rotation

$$\Delta\angle\theta = \frac{\Delta s}{r}$$

Angle equals arc length divided by radius

Measured in **radians** (rad)

Figure: Arc length and radius



# 6.1 Radians vs Degrees

## The Conversion

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- $1 \text{ rad} \approx 57.3^\circ$

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### Common conversions:

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- $1 \text{ rad} \approx 57.3^\circ$

## Why Radians?

Radians simplify equations in physics. Degrees are arbitrary - radians are natural.

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Universal Law: Angular Velocity

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Angular velocity equals change in  $\angle$  divided by change in time

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**Units:** radians per second (rad/s)

**Direction:**

- Counterclockwise: positive (out of page toward you)
- Clockwise: negative (into page away from you)



## 6.1 Connecting Spinning to Moving

### The Bridge Equation

$$v = r\omega$$

Tangential **velocity** equals **radius** times **angular velocity**

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Points farther from the center move faster linearly, but all points have the same **angular velocity**.

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### The Bridge Equation

$$v = r\omega$$

Tangential **velocity** equals **radius** times **angular velocity**

### The Mental Model

Points farther from the center move faster linearly, but all points have the same **angular velocity**.

**Example:** CD spinning - outer edge moves faster than inner part, but both complete one revolution in same **time**.

## 6.1 Why Car Tires Matter

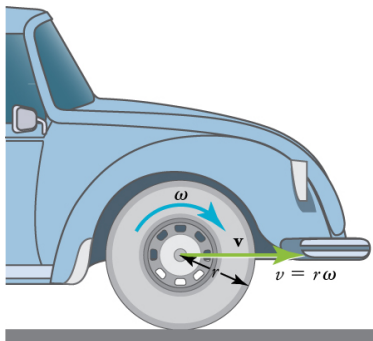


Figure: Car tire rolling

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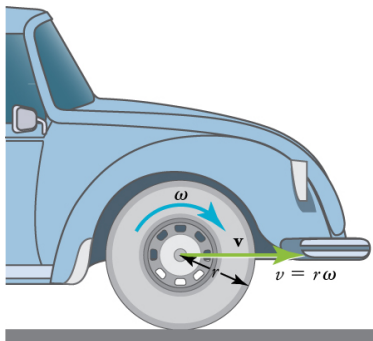


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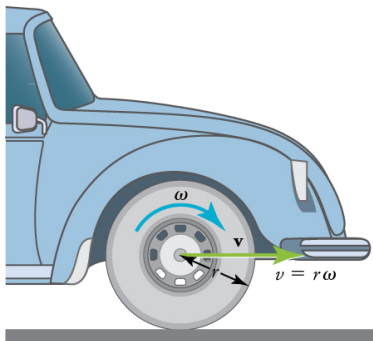


Figure: Car tire rolling

Large  $\omega$  means large  $v$  because  $v = r\omega$

Larger **radius** tire at same  $\omega$  produces greater  $v$

# Attempt: Clock Tower Angle

## The Challenge (3 min, silent)

A clock tower has a radius of 1.0 m. The hour hand moves from 12 p.m. to 3 p.m.

### Given:

- Radius  $r = 1.0$  m
- Time: 12 to 3 (quarter rotation)

### Find:

- 1  $\angle$  Angle of rotation in radians
- 1 Arc length along outer edge

*Can you decode this rotation? Work silently.*

# Compare: Clock Tower

## Turn and talk (2 min):

- 1 What fraction of a full rotation does the hour hand make from 12 to 3?
- 2 How many radians in a full circle?
- 3 What equation connects arc length to angle?



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**Name wheel:** One pair share your approach (not your answer).

# Reveal: The Geometry of Time

**Self-correct in a different color:**

**Part (a):** From 12 to 3 is  $\frac{1}{4}$  of full rotation

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Full rotation =  $2\pi$  rad, so  $\angle angle = 1 \times \frac{2\pi}{4} = \frac{\pi}{2}$  rad

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$$\Delta s = (1.0 \text{ m}) \left( \frac{\pi}{2} \text{ rad} \right) = 1.6 \text{ m}$$

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**Part (b):** Use  $\Delta s = r\Delta\theta$

$$\Delta s = (1.0 \text{ m}) \left( \frac{\pi}{2} \text{ rad} \right) = 1.6 \text{ m}$$

**Check:** Arc length is less than circumference ( $2\pi r \approx 6.3 \text{ m}$ ). Reasonable!

# Attempt: Spinning Car Tire

## The Challenge (3 min, silent)

A car tire has radius 0.300 m and the car travels at 15.0 m/s (about 54 km/h).

### Given:

- Radius  $r = 0.300$  m
- Tangential velocity  $v = 15.0$  m/s

**Find:** Angular velocity  $\omega$  of the tire in rad/s

*How fast is the tire spinning?*

# Compare: Tire Speed

## Turn and talk (2 min):

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**Name wheel:** One pair share your rearrangement strategy.

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**Substitute:**

$$\omega = \frac{15.0 \text{ m/s}}{0.300 \text{ m}}$$

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$$\omega = 50.0 \text{ rad/s}$$

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**Substitute:**

$$\omega = \frac{15.0 \text{ m/s}}{0.300 \text{ m}}$$

$$\omega = 50.0 \text{ rad/s}$$

**Check:** About 8 revolutions per second (since  $2\pi \text{ rad} = 1 \text{ rev}$ ). Fast but reasonable for highway speed!

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## 6.2 The Paradox of Constant Speed

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### Civilian View vs. Reality

**Civilian:** "Constant **speed** means no **acceleration**."

**Physicist:** "**Velocity** is changing direction, so there IS **acceleration**."

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**Civilian:** "Constant **speed** means no **acceleration**."

**Physicist:** "**Velocity** is changing direction, so there IS **acceleration**."

**Acceleration** is a change in **velocity** - magnitude OR direction!

## 6.2 The Illusion of Being Flung

### The Mental Model

When you turn in a car, you feel pushed outward. But no **force** pushes you out - your body wants to go straight (Newton's first law) while the car turns.

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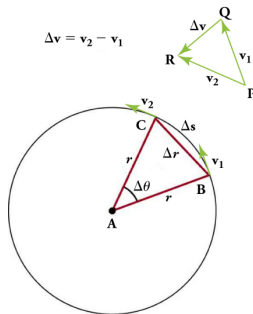
When you turn in a car, you feel pushed outward. But no **force** pushes you out - your body wants to go straight (Newton's first law) while the car turns.

### The Fictional Force

**Centrifugal force** is not real - it's the illusion created by your inertia resisting the turn.

The real **force** is **centripetal** - pulling you inward toward the center!

## 6.2 Centripetal Acceleration



**Figure:** Velocity changes direction, acceleration points toward center

## 6.2 Centripetal Acceleration

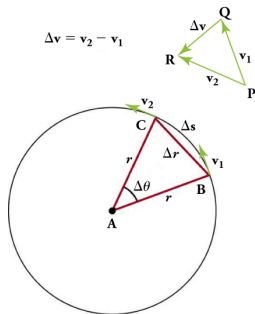


Figure: Velocity changes direction, acceleration points toward center

### Universal Law: Centripetal Acceleration

$$a_c = \frac{v^2}{r}$$

or

$$a_c = r\omega^2$$



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**Example:**

- Curve at 50 km/h: moderate acceleration
- Same curve at 100 km/h: 4 times the acceleration

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**Example:**

- Curve at 50 km/h: moderate **acceleration**
- Same curve at 100 km/h: **4 times** the **acceleration**

### The Warning

This is why **speed** limits are lower on curves - small **speed** increase creates huge **acceleration** increase.

## 6.2 Centripetal Force

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**Direction:** Always toward the center of rotation



## 6.2 Sources of Centripetal Force

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### The Mental Model

Centripetal **force** isn't a new kind of **force** - it's whatever **force** points toward the center and causes circular motion.

# Attempt: Car on Curve

## The Challenge (3 min, silent)

A 900 kg car rounds a curve with radius 600 m at speed 25.0 m/s.

### Given:

- Mass  $m = 900$  kg
- Radius  $r = 600$  m
- Speed  $v = 25.0$  m/s

**Find:** Centripetal force required to keep car on curve

*How much force do the tires provide?*

# Compare: Car Force

**Turn and talk (2 min):**

- 1 What equation did you use for centripetal **force**?
- 2 Did you remember to square the **velocity**?
- 3 What **force** provides the centripetal **force** for a car?

# Compare: Car Force

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- 3 What **force** provides the centripetal **force** for a car?

**Name wheel:** One pair share your equation and reasoning.



# Reveal: The Force That Turns

Self-correct in a different color:

Equation:  $F_c = m \frac{v^2}{r}$

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Equation:  $F_c = m \frac{v^2}{r}$

Substitute:

$$F_c = \frac{(900 \text{ kg})(25.0 \text{ m/s})^2}{600 \text{ m}}$$

# Reveal: The Force That Turns

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Equation:  $F_c = m \frac{v^2}{r}$

Substitute:

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$$F_c = \frac{(900)(625)}{600} = \boxed{938 \text{ N}}$$

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$$F_c = \frac{(900)(625)}{600} = \boxed{938 \text{ N}}$$

**Check:** About 940 N - this is the friction **force** between tires and road. Without it, car slides straight!

# Attempt: Acceleration Comparison

## The Challenge (3 min, silent)

A car follows a curve of radius 500 m at speed 25.0 m/s.

### Given:

- Radius  $r = 500$  m
- Speed  $v = 25.0$  m/s
- $g = 9.80$  m/s<sup>2</sup>

### Find:

- 1 Centripetal acceleration
- 2 Express as fraction of  $g$

*How does turning compare to falling?*

# Compare: Acceleration Scale

**Turn and talk (2 min):**

- 1 What equation did you use for centripetal **acceleration**?
- 2 How did you express it as a fraction of  $g$ ?
- 3 Is the **acceleration** large or small compared to gravity?

# Compare: Acceleration Scale

## Turn and talk (2 min):

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- 2 How did you express it as a fraction of  $g$ ?
- 3 Is the **acceleration** large or small compared to gravity?

**Name wheel:** One pair share your comparison method.

# Reveal: Comparing to Gravity

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Calculate  $a_c$ :

$$a_c = \frac{v^2}{r} = \frac{(25.0 \text{ m/s})^2}{500 \text{ m}}$$



# Reveal: Comparing to Gravity

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Calculate  $a_c$ :

$$a_c = \frac{v^2}{r} = \frac{(25.0 \text{ m/s})^2}{500 \text{ m}}$$

$$a_c = \frac{625}{500} = \boxed{1.25 \text{ m/s}^2}$$

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Compare to  $g$ :

$$\frac{a_c}{g} = \frac{1.25}{9.80} = 0.128 \Rightarrow \boxed{a_c = 0.13g}$$

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Compare to  $g$ :

$$\frac{a_c}{g} = \frac{1.25}{9.80} = 0.128 \quad \Rightarrow \quad \boxed{a_c = 0.13g}$$

**Revelation:** Gentle highway curve at moderate speed produces about 1/10th the acceleration of gravity!

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- **6.3:** Describe rotational kinematic variables and relate them to linear counterparts
- **6.3:** Describe torque and lever arm
- **6.3:** Solve problems involving torque and rotational kinematics

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- Figure skater pulls arms in - spins faster



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- CD player stops - disc slows to halt

Universal Law: **Angular Acceleration**

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Rate of change of **angular velocity**

## 6.3 Connecting Linear and Angular Acceleration

### The Bridge

$$a = r\alpha$$

or

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### The Bridge

$$a = r\alpha \quad \text{or} \quad \alpha = \frac{a}{r}$$

**Tangential acceleration:** Linear acceleration along the circle's edge

### The Mental Model

Greater angular acceleration means greater tangential acceleration. Points farther from center have larger tangential acceleration for same  $\alpha$ .

## 6.3 Rotational Kinematics Equations

### Linear

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

### Rotational

$$\omega = \omega_0 + \alpha t$$

$$\angle\theta = \angle\theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\angle\theta$$

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### Rotational

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$$\angle\theta = \angle\theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\angle\theta$$

### The Pattern

Every linear kinematics equation has a rotational analog. Just swap

$x \rightarrow \angle\theta$ ,  $v \rightarrow \omega$ ,  $a \rightarrow \alpha$ !



## 6.3 The Rotational Version of Force

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What causes **angular acceleration**?

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Universal Law: **Torque**

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**Torque** equals lever arm times **force** times sine of *angle*

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What causes **angular acceleration**?

Universal Law: **Torque**

$$\tau = rF \sin \angle \theta$$

**Torque** equals lever arm times **force** times sine of *angle*

**Units:** N·m (Newton-meters)

**Direction:** Same as the **angular acceleration** it produces

## 6.3 Maximizing Torque

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To maximize **torque**:

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- Apply **force** perpendicular to lever arm ( $\angle \theta = 90^\circ$ , so  $\sin \angle \theta = 1$ )

## 6.3 Maximizing Torque

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To maximize **torque**:

- Apply **force** far from pivot (large  $r$ )
- Apply **force** perpendicular to lever arm ( $\angle \theta = 90^\circ$ , so  $\sin \angle \theta = 1$ )
- Apply larger **force** (large  $F$ )

## 6.3 Maximizing Torque

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### Real-World Applications

- Door handle placed far from hinges
- Wrench with long handle
- Teeter-totter balanced by distance and weight



# Attempt: Fishing Reel

## The Challenge (3 min, silent)

A fishing reel spins at  $\omega_0 = 220 \text{ rad/s}$ . Fisherman applies brake creating angular acceleration  $\alpha = -300 \text{ rad/s}^2$ .

### Given:

- Initial  $\omega_0 = 220 \text{ rad/s}$
- Final  $\omega = 0$  (stops)
- $\alpha = -300 \text{ rad/s}^2$

**Find:** Time  $t$  for reel to stop

*How long does it take?*

# Compare: Fishing Reel

## Turn and talk (2 min):

- 1 Which rotational kinematics equation did you choose?
- 2 How did you solve for time  $t$ ?
- 3 Why is  $\alpha$  negative?

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- 1 Which rotational kinematics equation did you choose?
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**Name wheel:** One pair share your equation choice and reasoning.

# Reveal: Stopping the Spin

Self-correct in a different color:

Equation:  $\omega = \omega_0 + \alpha t$

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**Insight:** Less than one second because the angular acceleration is quite large!



# Attempt: Merry-Go-Round Torque

## The Challenge (3 min, silent)

A man pushes a merry-go-round with force 250 N at the edge, perpendicular to the radius of 1.50 m.

### Given:

- Force  $F = 250$  N
- Lever arm  $r = 1.50$  m
- $\angle \text{Angle } \theta = 90^\circ$  (perpendicular)

**Find:** Torque  $\tau$  produced

*How effective is his push?*

# Compare: Torque Calculation

## Turn and talk (2 min):

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**Name wheel:** One pair explain why perpendicular force maximizes torque.

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**Equation:**  $\tau = rF \sin \angle \theta$

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$$\tau = (1.50 \text{ m})(250 \text{ N})(1)$$

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$$\tau = 375 \text{ N} \cdot \text{m}$$

**Strategy:** Man maximized **torque** by pushing perpendicular at the outer edge!



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- 4 Rotational kinematics mirrors linear kinematics
- 5 **Torque**  $\tau = rF \sin \angle \theta$  is the rotational **force**

# Key Equations

$$\Delta\angle\theta = \frac{\Delta s}{r} \quad (\text{angle of rotation}) \quad (1)$$

$$\omega = \frac{\Delta\angle\theta}{\Delta t} \quad (\text{angular velocity}) \quad (2)$$

$$v = r\omega \quad (\text{tangential velocity}) \quad (3)$$

$$a_c = \frac{v^2}{r} = r\omega^2 \quad (\text{centripetal acceleration}) \quad (4)$$

$$F_c = m\frac{v^2}{r} = mr\omega^2 \quad (\text{centripetal force}) \quad (5)$$

$$\alpha = \frac{\Delta\omega}{\Delta t} \quad (\text{angular acceleration}) \quad (6)$$

$$a = r\alpha \quad (\text{tangential acceleration}) \quad (7)$$

$$\tau = rF \sin \angle\theta \quad (\text{torque}) \quad (8)$$

Complete the assigned problems  
posted on the LMS