PHYS12 CH3

Kinematics in Two Dimensions

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Vectors and Scalars

Scalar Quantities

- Have only magnitude
- Examples: mass, temperature, time

Vector Quantities

- Have magnitude and direction
- Examples: displacement, velocity, acceleration

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Graphical Methods

Head-to-Tail Method

1. Draw the first vector
2. Draw the second vector from the head of the first
3. Resultant vector: from tail of first to head of second

Parallelogram Method

1. Draw both vectors from a common point
2. Complete the parallelogram
3. Resultant vector: diagonal of the parallelogram

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Trig Review

phys12-math-sohcahtoa-mnemonic.png

Figure: SOHCAHTOA mnemonic

phys12-math-sine-cosine-laws.p

Figure: Sine and Cosine Laws

Vector Components

Components of a vector $\vec{A}$:

$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

Where:

- A is the magnitude of the vector
- θ is the angle with the positive x-axis

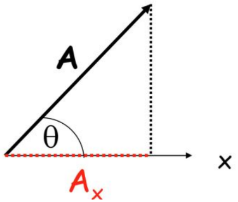

Figure: Vector Components

Adding Vectors Analytically

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Adding Vectors Analytically

1. Resolve vectors into x and y components
2. Sum x-components: $R_x = \sum A_x$
3. Sum y-components: $R_y = \sum A_y$
4. Find magnitude: $R = \sqrt{R_x^2 + R_y^2}$
5. Find direction: $\theta_R = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

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Projectile Motion

Assumptions

- Air resistance is negligible
- Acceleration due to gravity (g) is constant
- Horizontal motion is at constant velocity

Key Concepts

- Two-dimensional motion
- Vertical motion: accelerated
- Horizontal motion: constant velocity

Equations of Motion

Horizontal motion:

$$x = v_{0x} t$$

$$v_x = v_{0x}$$

Where:

- $v_{0x} = v_0 \cos\theta$
- $v_{0y} = v_0 \sin\theta$

Vertical motion:

$$y = v_{0y} t - \frac{1}{2} g t^2$$

$$v_y = v_{0y} - g t$$

Range of a Projectile

For a projectile launched and landing at the same vertical level:

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

Where:

- R is the range
- v_0 is the initial velocity
- θ is the launch angle
- g is the acceleration due to gravity

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Example 1: Vector Addition

Problem: Walk 18.0 m west, then 25.0 m north. Find distance and direction from start.

Solution steps:

1. Resolve vectors into components
2. Sum components
3. Find magnitude of resultant
4. Find direction

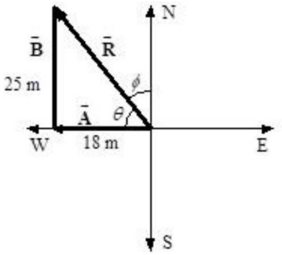

Figure: Vector Addition

Example 1: Solution

$$R_x = -18.0 \text{ m}$$
$$R_y = 25.0 \text{ m}$$
$$R = \sqrt{(-18.0 \text{ m})^2 + (25.0 \text{ m})^2} = 30.8 \text{ m}$$
$$\theta = \tan^{-1}\left(\frac{25.0}{-18.0}\right) = -54.25^\circ$$

Answer: 30.8 m at 35.8° west of north

Example 2: Vector Components in Rotated Axes

Problem: You fly 32.0 km in a straight line in still air in the direction 35.0° south of west.
(a) Find the distances you would have to fly straight south and then straight west to arrive at the same point.

$$D_W = 32.0 \cos 35^\circ = 26.2 \text{ km}$$
$$D_S = 32.0 \sin 35^\circ = 18.4 \text{ km}$$

Example 2: Solution (continued)

(b) Find the distances you would have to fly first in a direction 45.0° south of west and then in a direction 45.0° west of north. These are the components of the displacement along a different set of axes-one rotated 45°.

$$\theta' = 35^\circ - 45^\circ = -10^\circ$$
$$D_{SW} = 32.0 \cos 10^\circ = 31.5 \text{ km}$$
$$D_{NW} = 32.0 \sin 10^\circ = 5.56 \text{ km}$$

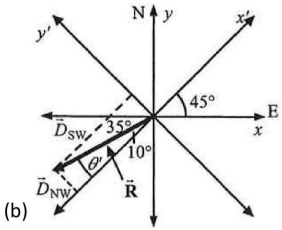

Figure: Rotated Axes

Example 3: Vector Addition Verification

Problem: Verify sum of vectors $\vec{A}$ (27.5 m at 66° North of East) and $\vec{B}$ (30.0 m at 112°North of East)

Steps:

1. Resolve vectors into components
2. Sum components
3. Find magnitude and direction of resultant

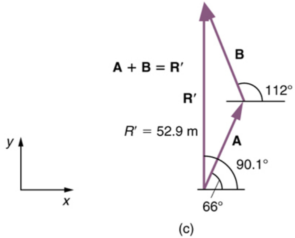

Figure: Vector Addition

Example 3: Solution

Trigonometry is left as an exercise.

$$R_x = 11.19 \text{ m} + (-11.25 \text{ m}) = -0.06 \text{ m}$$
$$R_y = 25.28 \text{ m} + 28.00 \text{ m} = 53.28 \text{ m}$$
$$R = \sqrt{(-0.06 \text{ m})^2 + (53.28 \text{ m})^2} = 53.28 \text{ m}$$
$$\theta_R = \tan^{-1}\left(\frac{53.28}{-0.06}\right) \approx -89.9^\circ$$

Result: 53.28 m almost due north, slightly west

Long Jump World Record Analysis

Problem: The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person with a take-off speed of 9.5 m/s?

Assumptions:

- Scenario 1: Motion is on level ground (take-off and landing at same height)
- Scenario 2: Person's center of mass is 1.0 m above the ground at take-off
- Optimal angle is 45°
- Air resistance is negligible

Scenario 1: Level Ground

Using the range formula:

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{(9.5 \text{ m/s})^2 \sin 90^\circ}{9.8 \text{ m/s}^2} = 9.21 \text{ m}$$

Scenario 2: 1.0 m Height Difference

1. Find time of flight:

$$y = -1.0 \text{ m} = (9.5 \text{ m/s})(\sin 45^\circ)t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

   Solving: $t = 1.51$ s

2. Calculate range:

$$R = v_0(\cos 45^\circ)t = (9.5 \text{ m/s})(\cos 45^\circ)(1.51 \text{ s}) = 10.44 \text{ m}$$

Conclusion

Maximum obtainable range:

- Scenario 1 (level ground): 9.21 m
- Scenario 2 (1.0 m height difference): 10.44 m

Both scenarios exceed the world record of 8.95 m, likely due to idealized assumptions.

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Conclusion

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- Vectors are essential for describing motion in two dimensions
- Both graphical and analytical methods are important
- Projectile motion combines horizontal and vertical components
- Practice problem-solving to master these concepts