27.3 Young's Double Slit Experiment

Learning Objectives

By the end of this section, you will be able to:

- Explain the phenomena of interference.
- Define constructive interference for a double slit and destructive interference for a double slit.

Although Christiaan Huygens thought that light was a wave, Isaac Newton did not. Newton felt that there were other explanations for color, and for the interference and diffraction effects that were observable at the time. Owing to Newton's tremendous stature, his view generally prevailed. The fact that Huygens's principle worked was not considered evidence that was direct enough to prove that light is a wave. The acceptance of the wave character of light came many years later when, in 1801, the English physicist and physician Thomas Young (1773–1829) did his now-classic double slit experiment (see Figure 27.10).
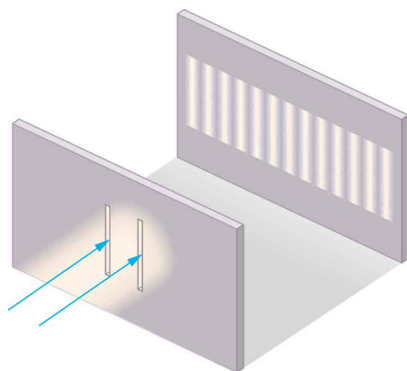

Figure 27.10 Young's double slit experiment. Here pure-wavelength light sent through a pair of vertical slits is diffracted into a pattern on the screen of numerous vertical lines spread out horizontally. Without diffraction and interference, the light would simply make two lines on the screen.

Why do we not ordinarily observe wave behavior for light, such as observed in Young's double slit experiment? First, light must interact with something small, such as the closely spaced slits used by Young, to show pronounced wave effects. Furthermore, Young first passed light from a single source (the Sun) through a single slit to make the light somewhat coherent. By coherent, we mean waves are in phase or have a definite phase relationship. Incoherent means the waves have random phase relationships. Why did Young then pass the light through a double slit? The answer to this question is that two slits provide two coherent light sources that then interfere constructively or destructively. Young used sunlight, where each wavelength forms its own pattern, making the effect more difficult to see. We illustrate the double slit experiment with monochromatic (single λ) light to clarify the effect. Figure 27.11 shows the pure constructive and

destructive interference of two waves having the same wavelength and amplitude.
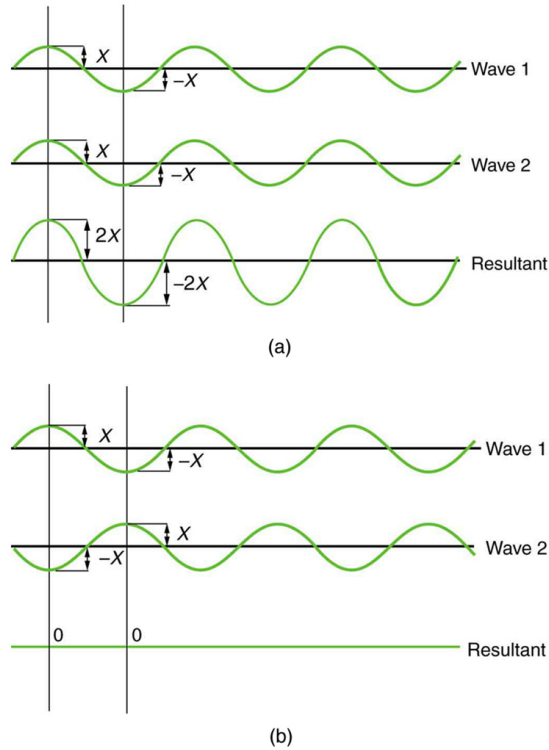

(a)

(b)

Figure 27.11 The amplitudes of waves add. (a) Pure constructive interference is obtained when identical waves are in phase. (b) Pure destructive interference occurs when identical waves are exactly out of phase, or shifted by half a wavelength.

When light passes through narrow slits, it is diffracted into semicircular waves, as shown in Figure 27.12(a). Pure constructive interference occurs where the waves are crest to crest or trough to trough. Pure destructive interference occurs where they are crest to trough. The light must fall on a screen and be scattered into our eyes for us to see the pattern. An analogous pattern for water waves is shown in Figure 27.12(b). Note that regions of constructive and destructive interference move out from the slits at well-defined angles to the original beam. These angles depend on wavelength and the distance between the slits, as we shall see below.
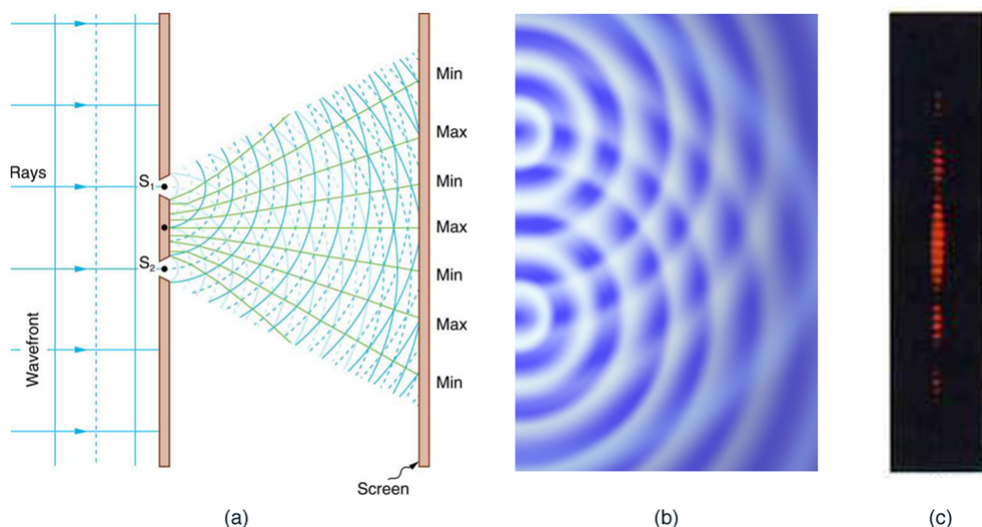
Figure 27.12 Double slits produce two coherent sources of waves that interfere. (a) Light spreads out (diffracts) from each slit, because the slits are narrow. These waves overlap and interfere constructively (bright lines) and destructively (dark regions). We can only see this if the light falls onto a screen and is scattered into our eyes. (b) Double slit interference pattern for water waves are nearly identical to that for light. Wave action is greatest in regions of constructive interference and least in regions of destructive interference. (c) When light that has passed through double slits falls on a screen, we see a pattern such as this. (credit: PASCO)

To understand the double slit interference pattern, we consider how two waves travel from the slits to the screen, as illustrated in Figure 27.13. Each slit is a different distance from a given point on the screen. Thus different numbers of wavelengths fit into each path. Waves start out from the slits in phase (crest to crest), but they may end up out of phase (crest to trough) at the screen if the paths differ in length by half a wavelength, interfering destructively as shown in Figure 27.13(a). If the paths differ by a whole wavelength, then the waves arrive in phase (crest to crest) at the screen, interfering constructively as shown in Figure 27.13(b). More generally, if the paths taken by the two waves differ by any half-integral number of wavelengths $[(1/2)\lambda, (3/2)\lambda, (5/2)\lambda,$ etc.], then destructive interference occurs. Similarly, if the paths taken by the two waves differ by any integral number of wavelengths (λ, 2λ, 3λ, etc.), then constructive interference occurs.

Take-Home Experiment: Using Fingers as Slits

Look at a light, such as a street lamp or incandescent bulb, through the narrow gap between two fingers held close together. What type of pattern do you see? How does it change when you allow the fingers to move a little farther apart?

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Is it more distinct for a monochromatic source, such as the yellow light from a sodium vapor lamp, than for an incandescent bulb?
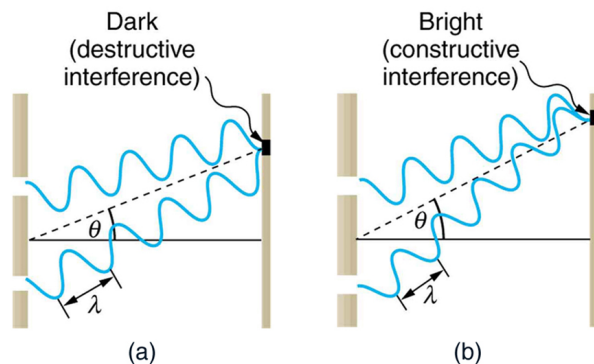

Figure 27.13 Waves follow different paths from the slits to a common point on a screen. (a) Destructive interference occurs here, because one path is a half wavelength longer than the other. The waves start in phase but arrive out of phase. (b) Constructive interference occurs here because one path is a whole wavelength longer than the other. The waves start out and arrive in phase.

Figure 27.14 shows how to determine the path length difference for waves traveling from two slits to a common point on a screen. If the screen is a large distance away compared with the distance between the slits, then the angle θ between the path and a line from the slits to the screen (see the figure) is nearly the same for each path. The difference between the paths is shown in the figure; simple trigonometry shows it to be $d \sin \theta$, where d is the distance between the slits. To obtain constructive interference for a double slit, the path length difference must be an integral multiple of the wavelength, or

$$d \sin \theta = m\lambda, \text{ for } m = 0, \ 1, \ -1, \ 2, \ -2, \ ... \quad \text{(constructive)}.$$

27.3

Similarly, to obtain destructive interference for a double slit, the path length difference must be a half-integral multiple of the wavelength, or

$$d \sin \theta = \left(m + \tfrac{1}{2}\right)\lambda, \text{ for } m = 0, \ 1, \ -1, \ 2, \ -2, \ ... \quad \text{(destructive)},$$

27.4

where λ is the wavelength of the light, d is the distance between slits, and θ is the angle from the original direction of the beam as discussed above. We call m the order of the interference. For example, $m = 4$ is fourth-order interference.
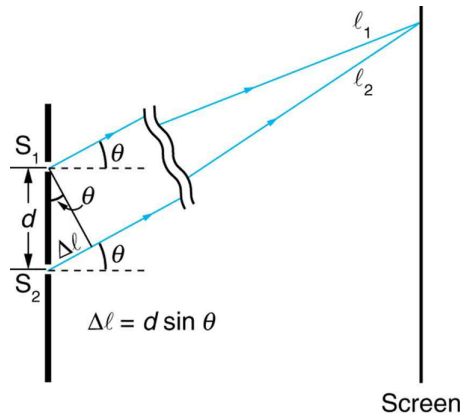
Figure 27.14 The paths from each slit to a common point on the screen differ by an amount $d \sin \theta$, assuming the distance to the screen is much greater than the distance between slits (not to scale here).

The equations for double slit interference imply that a series of bright and dark lines are formed. For vertical slits, the light spreads out horizontally on either side of the incident beam into a pattern called interference fringes, illustrated in Figure 27.15. The intensity of the bright fringes falls off on either side, being brightest at the center. The closer the slits are, the more is the spreading of the bright fringes. We can see this by examining the equation

$d \sin \theta = m\lambda$, for $m = 0,\ 1,\ -1,\ 2,\ -2,\ \dots$.

27.5

For fixed λ and m, the smaller d is, the larger θ must be, since $\sin \theta = m\lambda/d$. This is consistent with our contention that wave effects are most noticeable when the object the wave encounters (here, slits a distance d apart) is small. Small d gives large θ, hence a large effect.
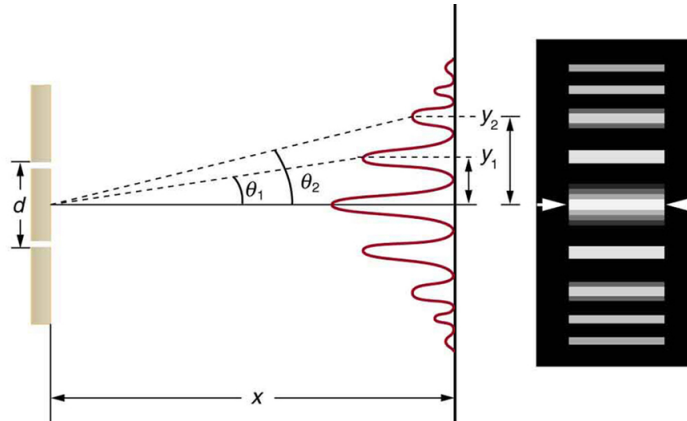

Figure 27.15 The interference pattern for a double slit has an intensity that falls

off with angle. The photograph shows multiple bright and dark lines, or fringes, formed by light passing through a double slit.

Example 27.1

Finding a Wavelength from an Interference Pattern   Suppose you pass light from a He-Ne laser through two slits separated by 0.0100 mm and find that the third bright line on a screen is formed at an angle of 10.95º relative to the incident beam. What is the wavelength of the light?

Strategy   The third bright line is due to third-order constructive interference, which means that $m = 3$. We are given $d = 0.0100$ mm and $\theta = 10.95º$. The wavelength can thus be found using the equation $d \sin \theta = m\lambda$ for constructive interference.

Solution   The equation is $d \sin \theta = m\lambda$. Solving for the wavelength λ gives

$$\lambda = \frac{d \sin \theta}{m}.$$

27.6

Substituting known values yields

$$\lambda \quad = \quad \frac{(0.0100 \text{ mm})(\sin 10.95º)}{3}$$
$$= \quad 6.33 \times 10^{-4} \text{ mm} = 633 \text{ nm}.$$

27.7

Discussion   To three digits, this is the wavelength of light emitted by the common He-Ne laser. Not by coincidence, this red color is similar to that emitted by neon lights. More important, however, is the fact that interference patterns can be used to measure wavelength. Young did this for visible wavelengths. This analytical technique is still widely used to measure electromagnetic spectra. For a given order, the angle for constructive interference increases with λ, so that spectra (measurements of intensity versus wavelength) can be obtained.

Example 27.2

Calculating Highest Order Possible   Interference patterns do not have an infinite number of lines, since there is a limit to how big m can be. What is the highest-order constructive interference possible with the system described in the preceding example?

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Strategy and Concept The equation $d \sin \theta = m\lambda$ (for $m = 0,\ 1,\ -1,\ 2,\ -2,\ ...$) describes constructive interference. For fixed values of d and λ, the larger m is, the larger $\sin \theta$ is. However, the maximum value that $\sin \theta$ can have is 1, for an angle of 90º. (Larger angles imply that light goes backward and does not reach the screen at all.) Let us find which m corresponds to this maximum diffraction angle.

Solution Solving the equation $d\sin \theta = m\lambda$ for m gives

$m = \frac{d \sin \theta}{\lambda}$.

27.8

Taking $\sin \theta = 1$ and substituting the values of d and λ from the preceding example gives

$m = \frac{(0.0100 \text{ mm})(1)}{633 \text{ nm}} \approx 15.8$.

27.9

Therefore, the largest integer m can be is 15, or

$m = 15$.

27.10

Discussion The number of fringes depends on the wavelength and slit separation. The number of fringes will be very large for large slit separations. However, if the slit separation becomes much greater than the wavelength, the intensity of the interference pattern changes so that the screen has two bright lines cast by the slits, as expected when light behaves like a ray. We also note that the fringes get fainter further away from the center. Consequently, not all 15 fringes may be observable.