7.1 Kepler's Laws of Planetary Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain Kepler's three laws of planetary motion
- Apply Kepler's laws to calculate characteristics of orbits

Teacher Support

Teacher Support The learning objectives in this section will help your students master the following standards:

- (4) Science concepts. The student knows and applies the laws governing motion in a variety of situations. The student is expected to:
  - (C) analyze and describe accelerated motion in two dimensions using equations, including projectile and circular examples.

In this section students will apply Kepler's laws of planetary motion to objects in the solar system.

[BL][OL] Discuss the historical setting in which Kepler worked. Most people still thought Earth was the center of the universe, and yet Kepler not only knew that the planets circled the sun, he found patterns in the paths they followed. What would it be like to be that far ahead of almost everyone? A fascinating description of this is given in the program *Cosmos* with Carl Sagan (Episode 3, Harmony of the Worlds).

[AL] Explain that Kepler's laws were laws and not theories. Laws describe patterns in nature that always repeat themselves under the same set of conditions. Theories provide an explanation for the patterns. Kepler provided no explanation.

Section Key Terms

Concepts Related to Kepler's Laws of Planetary Motion

Examples of orbits abound. Hundreds of artificial satellites orbit Earth together with thousands of pieces of debris. The moon's orbit around Earth has intrigued humans from time immemorial. The orbits of planets, asteroids, meteors, and comets around the sun are no less interesting. If we look farther, we see almost unimaginable numbers of stars, galaxies, and other celestial objects orbiting one another and interacting through gravity.

All these motions are governed by gravitational force. The orbital motions of objects in our own solar system are simple enough to describe with a few fairly simple laws. The orbits of planets and moons satisfy the following two conditions:

- The mass of the orbiting object, m, is small compared to the mass of the object it orbits, M.
- The system is isolated from other massive objects.

Teacher Support

Teacher Support [OL] Ask the students to explain the criteria to see if they understand relative mass and isolated systems.

Based on the motion of the planets about the sun, Kepler devised a set of three classical laws, called Kepler's laws of planetary motion, that describe the orbits of all bodies satisfying these two conditions:

1. The orbit of each planet around the sun is an ellipse with the sun at one focus.
2. Each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal times.
3. The ratio of the squares of the periods of any two planets about the sun is equal to the ratio of the cubes of their average distances from the sun.

These descriptive laws are named for the German astronomer Johannes Kepler (1571–1630). He devised them after careful study (over some 20 years) of a large amount of meticulously recorded observations of planetary motion done by Tycho Brahe (1546–1601). Such careful collection and detailed recording of methods and data are hallmarks of good science. Data constitute the evidence from which new interpretations and meanings can be constructed. Let's look closer at each of these laws.

Teacher Support

Teacher Support [BL] Relate orbit to year and rotation to day. Be sure that students know that an object rotates on its axis and revolves around a parent body as it follows its orbit.

[OL] See how many levels of orbital motion the students know and fill in the ones they don't. For example, moons orbit around planets; planets around stars; stars around the center of the galaxy, etc.

[AL] From the point of view of Earth, which objects appear (incorrectly) to be orbiting Earth (stars, the sun, galaxies) and which can be seen to be orbiting parent bodies (the moon, moons of other planets, stars in other galaxies)?

Kepler's First Law The orbit of each planet about the sun is an ellipse with the sun at one focus, as shown in Figure 7.2. The planet's closest approach to the sun is called perihelion and its farthest distance from the sun is called aphelion.

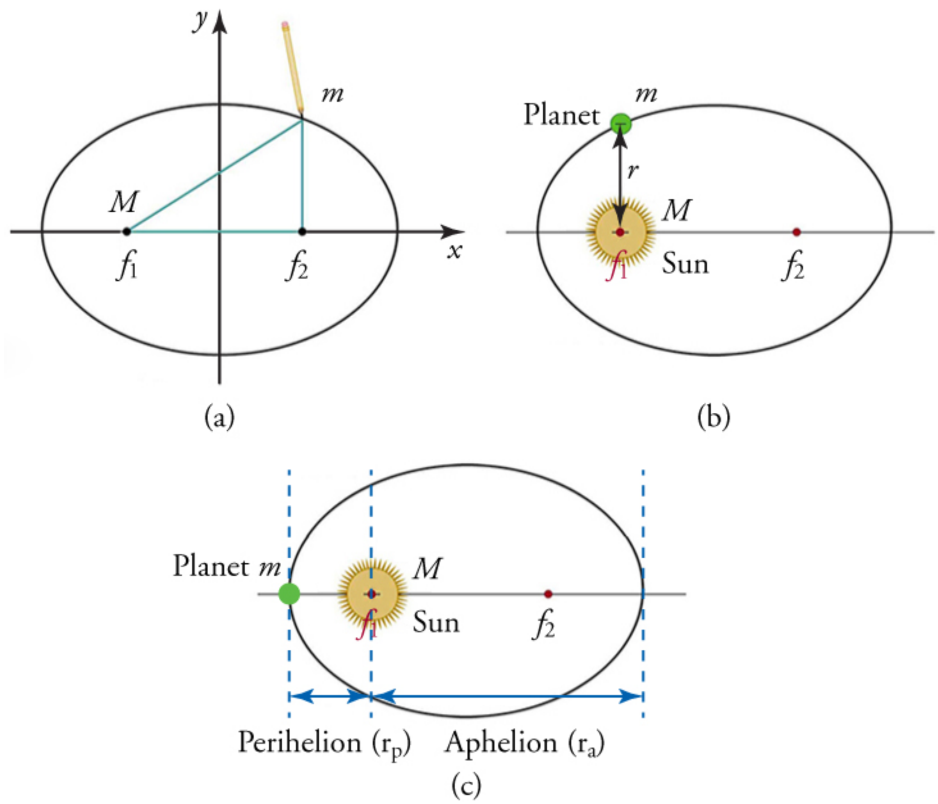

Figure 7.2 (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is constant. (b) For any closed orbit, m follows an elliptical path with M at one focus. (c) The aphelion (ra) is the furthest distance between the planet and the sun, while the perihelion (rp) is the closest distance from the sun.

Teacher Support

Teacher Support [AL] Ask for a definition of planet. Prepare to discuss Pluto's demotion if it comes up. Discuss the first criterion in terms of center of rotation of a moon-planet system. Explain that for all planet-moon systems in the solar system, the center of rotation is within the planet. This is not true for Pluto and its largest moon, Charon, because their masses are similar enough that they rotate around a point in space between them.

If you know the aphelion (r_{a}) and perihelion (r_{p}) distances, then you can calculate the semi-major axis (a) and semi-minor axis (b).

$a = \frac{(r_a + r_p)}{2}$
$b = \sqrt{r_a r_p}$

Teacher Support

Teacher Support [AL] If any students are interested and proficient in algebra and geometry, ask them to derive a formula that relates the length of the string and the distance between pins to the major and minor axes of an ellipse. Explain that this is a real world problem for workers who design elliptical tabletops and mirrors.

[BL][OL] Impress upon the students that Kepler had to crunch an enormous amount of data and that all his calculations had to be done by hand. Ask students to think of similar projects where scientists found order in a daunting amount of data (the periodic table, DNA structure, climate models, etc.).

Teacher Demonstration

Demonstrate the pins and string method of drawing an ellipse, as shown in Figure 7.3, or have the students try it at home or in class.

Ask students: Why does the string and pin method create a shape that conforms to Kepler's second law? That is, why is the shape an ellipse?

Teacher Support

Teacher Support Explain that the pins are the foci and explain what each of the three sections of string represents. Note that the pencil represents a planet and one of the pins represents the sun.

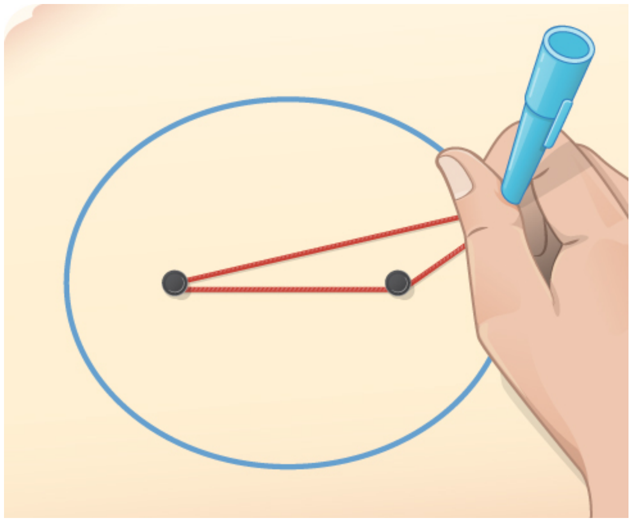

Figure 7.3 You can draw an ellipse as shown by putting a pin at each focus, and then placing a loop of string around a pen and the pins and tracing a line on the paper.

Kepler's Second Law Each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal times, as shown in Figure 7.4.

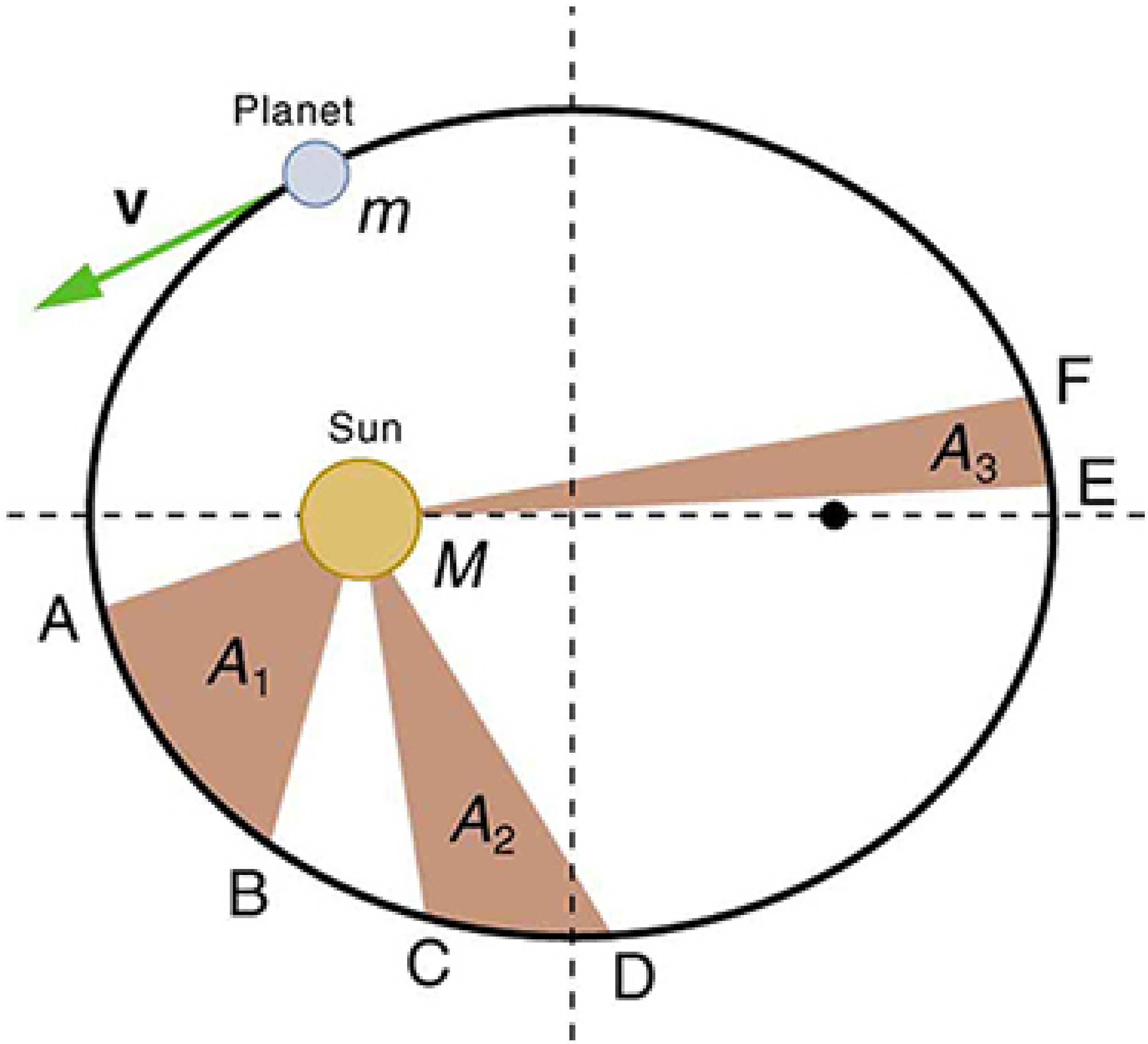

Figure 7.4 The shaded regions have equal areas. The time for m to go from A to B is the same as the time to go from C to D and from E to F. The mass m moves fastest when it is closest to M. Kepler's second law was originally devised for planets orbiting the sun, but it has broader validity.

Teacher Support

Teacher Support Ask the students to imagine how complicated it would be to describe the motion of the planets mathematically, if it is assumed that Earth is stationary. And yet, people tried to do this for hundreds of years, while overlooking the simple explanation that all planets circle the sun.

[OL] Ask students to use this figure to understand why planets and comets travel faster when they are closer to the sun. Explain that time intervals and areas are constant, but both velocity and distance from the sun vary.

Tips For Success

Note that while, for historical reasons, Kepler's laws are stated for planets orbiting the sun, they are actually valid for all bodies satisfying the two previously stated conditions.

Kepler's Third Law The ratio of the periods squared of any two planets around the sun is equal to the ratio of their average distances from the sun cubed. In equation form, this is

$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$,

where T is the period (time for one orbit) and r is the average distance (also called orbital radius). This equation is valid only for comparing two small masses orbiting a single large mass. Most importantly, this is only a descriptive equation; it gives no information about the cause of the equality.

Teacher Support

Teacher Support [BL] See if students can rearrange this equation to solve for any one of the variables when the other three are known.

[AL] Show a solution for one of the periods T or radii r and ask students to interpret the fractional powers on the right hand side of the equation.

[OL] Emphasize that this approach only works for two satellites orbiting the same parent body. The parent body must be the same because $r^2/T^2 = GM/(4\pi^2)$ and M is the mass of the parent body. If M changes, the ratio r^3/T^2 also changes.

Links To Physics

History: Ptolemy vs. Copernicus Before the discoveries of Kepler, Copernicus, Galileo, Newton, and others, the solar system was thought to revolve around Earth as shown in Figure 7.5 **(a)**. This is called the Ptolemaic model, named for the Greek philosopher Ptolemy who lived in the second century AD. The Ptolemaic model is characterized by a list of facts for the motions of planets, with no explanation of cause and effect. There tended to be a different rule for each heavenly body and a general lack of simplicity.

Figure 7.5 **(b)** represents the modern or Copernican model. In this model, a small set of rules and a single underlying force explain not only all planetary motion in the solar system, but also all other situations involving gravity. The breadth and simplicity of the laws of physics are compelling.

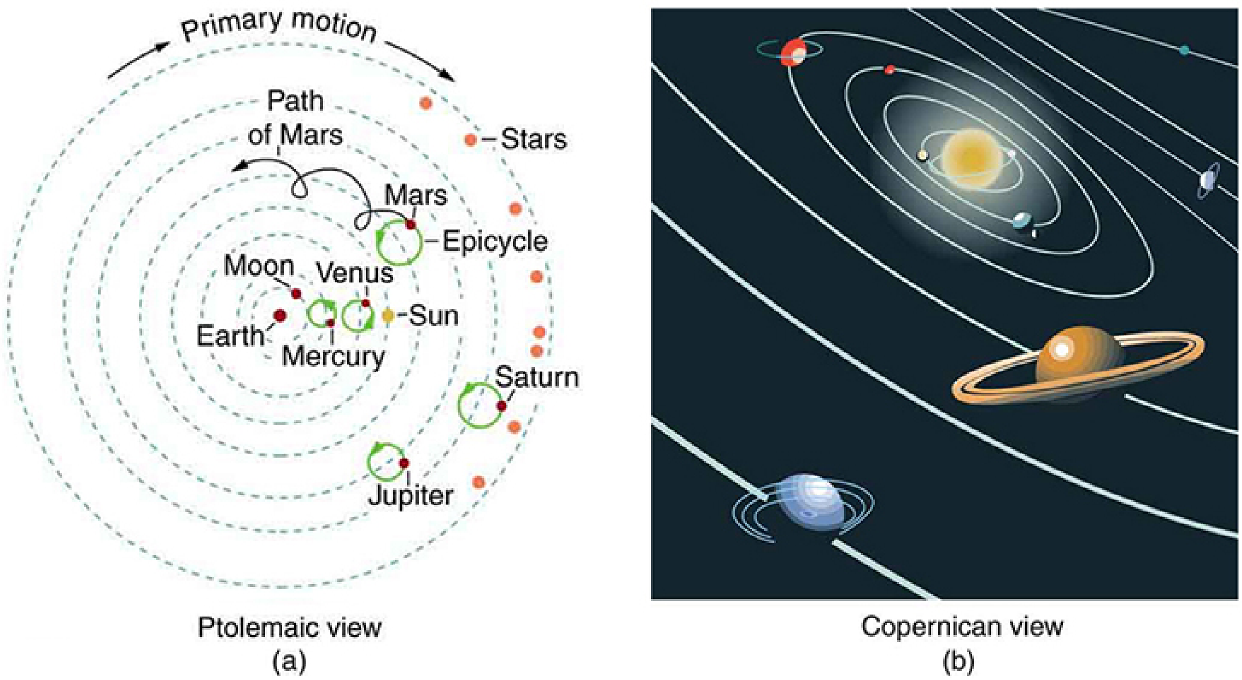

Figure 7.5 (a) The Ptolemaic model of the universe has Earth at the center with the moon, the planets, the sun, and the stars revolving about it in complex circular paths. This geocentric (Earth-centered) model, which can be made progressively more accurate by adding more circles, is purely descriptive, containing no hints about the causes of these motions. (b) The Copernican heliocentric (sun-centered) model is a simpler and more accurate model.

Nicolaus Copernicus (1473–1543) first had the idea that the planets circle the sun, in about 1514. It took him almost 20 years to work out the mathematical details for his model. He waited another 10 years or so to publish his work. It is thought he hesitated because he was afraid people would make fun of his theory. Actually, the reaction of many people was more one of fear and anger. Many people felt the Copernican model threatened their basic belief system. About 100 years later, the astronomer Galileo was put under house arrest for providing evidence that planets, including Earth, orbited the sun. In all, it took almost 300 years for everyone to admit that Copernicus had been right all along.

Grasp Check

Explain why Earth does actually appear to be the center of the solar system.

a. Earth appears to be the center of the solar system because Earth is at the center of the universe, and everything revolves around it in a circular orbit.
b. Earth appears to be the center of the solar system because, in the reference frame of Earth, the sun, moon, and planets all appear to move across the sky as if they were circling Earth.
c. Earth appears to be at the center of the solar system because Earth is at the center of the solar system and all the heavenly bodies revolve around it.
d. Earth appears to be at the center of the solar system because Earth is located at one of the foci of the elliptical orbit of the sun, moon, and other planets.

Teacher Support

Teacher Support Introduce the historical debate around the geocentric versus the heliocentric view of the universe. Stress how controversial this debate was at the time. Explain that this was important to people because their world view and cultural beliefs were at stake.

Virtual Physics

Acceleration This simulation allows you to create your own solar system so that you can see how changing distances and masses determines the orbits of planets. Click *Help* for instructions.

Click to view content

When the central object is off center, how does the speed of the orbiting object vary?

a. The orbiting object moves fastest when it is closest to the central object and slowest when it is farthest away.
b. The orbiting object moves slowest when it is closest to the central object and fastest when it is farthest away.
c. The orbiting object moves with the same speed at every point on the circumference of the elliptical orbit.
d. There is no relationship between the speed of the object and the location of the planet on the circumference of the orbit.

Teacher Support

Teacher Support Give the students ample time to manipulate this animation. It may take some time to get the parameters adjusted so that they can see how mass and eccentricity affect the orbit. Initially, the planet is likely to disappear off the screen or crash into the sun.

Calculations Related to Kepler's Laws of Planetary Motion

Kepler's First Law Refer back to Figure 7.2 **(a)**. Notice which distances are constant. The foci are fixed, so distance $\bar{f_1 f_2}$ is a constant. The definition of an ellipse states that the sum of the distances $\bar{f_1 m} + \bar{m f_2}$ is also constant. These two facts taken together mean that the perimeter of triangle $\Delta f_1 m f_2$ must also be constant. Knowledge of these constants will help you determine positions and distances of objects in a system that includes one object orbiting another.

Kepler's Second Law Refer back to Figure 7.4. The second law says that the segments have equal area and that it takes equal time to sweep through each segment. That is, the time it takes to travel from A to B equals the time it takes to travel from C to D, and so forth. Velocity **v** equals distance d divided by time t: $\mathbf{v} = d/t$. Then, $t = d/\mathbf{v}$, so distance divided by velocity is also a constant. For example, if we know the average velocity of Earth on June 21 and December 21, we can compare the distance Earth travels on those days.

The degree of elongation of an elliptical orbit is called its eccentricity (e). Eccentricity is calculated by dividing the distance f from the center of an ellipse to one of the foci by half the long axis a.

$(e) = f/a$

7.1

When $e = 0$, the ellipse is a circle.

The area of an ellipse is given by $A = \pi ab$, where b is half the short axis. If you know the axes of Earth's orbit and the area Earth sweeps out in a given period of time, you can calculate the fraction of the year that has elapsed.

Teacher Support

Teacher Support [OL] Review the definitions of major and minor axes, semi-major and semi-minor axes, and distance *f*. The major axis is the length of the ellipse and passes through both foci. The minor axis is the width of the ellipse and is perpendicular to the major axis. The semi-major and semi-minor axes are half of the major and minor axes, respectively.

Worked Example

Kepler's First Law At its closest approach, a moon comes within 200,000 km of the planet it orbits. At that point, the moon is 300,000 km from the other focus of its orbit, f_2. The planet is focus f_1 of the moon's elliptical orbit. How far is the moon from the planet when it is 260,000 km from f_2?

Strategy

Show and label the ellipse that is the orbit in your solution. Picture the triangle f_1mf_2 collapsed along the major axis and add up the lengths of the three sides. Find the length of the unknown side of the triangle when the moon is 260,000 km from f_2.

Solution

Perimeter of $f_1mf_2 = 200,000\text{ km} + 100,000\text{ km} + 300,000\text{ km} = 600,000\text{ km}$.

$mf_1 = 600,000\text{ km} - (100,000\text{ km} + 260,000\text{ km}) = 240,000\text{ km}$.

Discussion

The perimeter of triangle f_1mf_2 must be constant because the distance between the foci does not change and Kepler's first law says the orbit is an ellipse. For any ellipse, the sum of the two sides of the triangle, which are f_1m and mf_2, is constant.

Teacher Support

Teacher Support Walk the students through the process of mentally collapsing the f_1mf_2 at the end of the major axis to reveal what the three sides of the triangle f_1mf_2 are equal to. Picture the sections of the string as the pencil approaches the major axis. This distance f_1f_2 remains constant, f_1m is the distance from f_1 to the end of the major axis, and mf_2 is $f_1m + f_1f_2$.

[OL] Have students relate eccentricity, distance between foci, and shape of orbit.

[AL] Ask for examples of orbits with high eccentricity (comets, Pluto) and low eccentricity (moon, Earth).

Worked Example

Kepler's Second Law Figure 7.6 shows the major and minor axes of an ellipse. The semi-major and semi-minor axes are half of these, respectively.

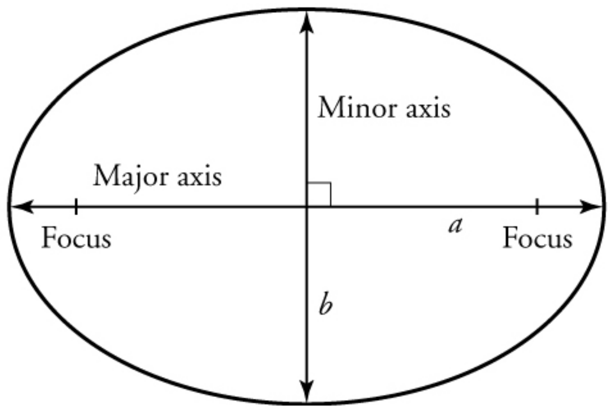

Figure 7.6 The major axis is the length of the ellipse, and the minor axis is the width of the ellipse. The semi-major axis is half the major axis, and the semi-minor axis is half the minor axis.

Earth's orbit is very slightly elliptical, with a semi-major axis of 1.49598×10^8 km and a semi-minor axis of 1.49577×10^8 km. If Earth's period is 365.26 days, what area does an Earth-to-sun line sweep past in one day?

Strategy

Each day, Earth sweeps past an equal-sized area, so we divide the total area by the number of days in a year to find the area swept past in one day. For total area use $A = \pi ab$. Calculate A, the area inside Earth's orbit and divide by the number of days in a year (i.e., its period).

Solution

$$\begin{aligned}\text{area per day} &= \frac{\text{total area}}{\text{total number of days}}\\ &= \frac{\pi ab}{365\ \text{d}}\\ &= \frac{\pi\left(1.496\times10^{8}\ \text{km}\right)\left(1.496\times10^{3}\ \text{km}\right)}{365\ \text{d}}\\ &= \mathbf{1.93 \times 10^{14}\ km^{2}/d}\end{aligned}$$

7.2

The area swept out in one day is thus $1.93 \times 10^{14}\text{km}^2$.

Discussion

The answer is based on Kepler's law, which states that a line from a planet to the sun sweeps out equal areas in equal times.

Teacher Support

Teacher Support Explain that this formula is easy to remember because it is similar to $A = \pi r^2$. Discuss Earth's eccentricity. Compare it with that of other planets, asteroids, or comets to further emphasize what defines a planet. Note that Earth has one of the least eccentric orbits and Mercury has the most eccentric orbit of the planets.

[BL]Have the students memorized the value of π?

[OL][AL]What is the formula when $a = b$? Is the formula familiar?

[OL]Can the student verify this statement by rearranging the equation?

Kepler's Third Law Kepler's third law states that the ratio of the squares of the periods of any two planets $(T_1,\ T_2)$ is equal to the ratio of the cubes of their average orbital distance from the sun $(r_1,\ r_2)$. Mathematically, this is represented by

$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$.

From this equation, it follows that the ratio r^3/T^2 is the same for all planets in the solar system. Later we will see how the work of Newton leads to a value for this constant.

Worked Example

Kepler's Third Law Given that the moon orbits Earth each 27.3 days and that it is an average distance of 3.84×10^8m from the center of Earth, calculate the period of an artificial satellite orbiting at an average altitude of 1,500 km above Earth's surface.

Strategy

The period, or time for one orbit, is related to the radius of the orbit by Kepler's third law, given in mathematical form by $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$. Let us use the subscript 1 for the moon and the subscript 2 for the satellite. We are asked to find T_2. The given information tells us that the orbital radius of the moon is $r_1 = 3.84 \times 10^8$m, and that the period of the moon is $T_1 = 27.3$ days . The height of the artificial satellite above Earth's surface is given, so to get the distance r_2 from the center of Earth we must add the height to the radius of Earth (6380 km). This gives $r_2 = 1500 \text{ km} + 6380 \text{ km} = 7880 \text{ km}$. Now all quantities are known, so T_2 can be found.

Solution

To solve for T_2, we cross-multiply and take the square root, yielding

$$T_2^2 = T_1^2\left(\frac{r_2}{r_1}\right)^3;\ T_2 = T_1\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$$

$$T_2 = (27.3\ \text{d})\left(\frac{24.0\ \text{h}}{\text{d}}\right)\left(\frac{7880\ \text{km}}{3.84\times10^5\ \text{km}}\right)^{\frac{3}{2}} = 1.93\ \text{h}.$$

7.3

Discussion

This is a reasonable period for a satellite in a fairly low orbit. It is interesting that any satellite at this altitude will complete one orbit in the same amount of time.

Teacher Support

Teacher Support Remind the students that this only works when the satellites are small compared to the parent object and when both satellites orbit the same parent object.

Practice Problems

1.

A planet with no axial tilt is located in another solar system. It circles its sun in a very elliptical orbit so that the temperature varies greatly throughout the year. If the year there has 612 days and the inhabitants celebrate the coldest day on day 1 of their calendar, when is the warmest day?

a. Day 1
b. Day 153
c. Day 306
d. Day 459

2.

A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). The ratio \frac{r^3}{T^2} for the moon is 1.01\times10^{18}\,\frac{\text{km}^3}{y^2}. Calculate the radius of the orbit of such a satellite.

a. 2.75\times10^3\,\text{km}
b. 1.96\times10^4\,\text{km}
c. 1.40\times10^5\,\text{km}
d. 1.00\times10^6\,\text{km}

Check Your Understanding

3.

Are Kepler's laws purely descriptive, or do they contain causal information?

a. Kepler's laws are purely descriptive.
b. Kepler's laws are purely causal.
c. Kepler's laws are descriptive as well as causal.
d. Kepler's laws are neither descriptive nor causal.

4.

True or false—According to Kepler's laws of planetary motion, a satellite increases its speed as it approaches its parent body and decreases its speed as it moves away from the parent body.

a. True
b. False

5.

Identify the locations of the foci of an elliptical orbit.

a. One focus is the parent body, and the other is located at the opposite end of the ellipse, at the same distance from the center as the parent body.
b. One focus is the parent body, and the other is located at the opposite end of the ellipse, at half the distance from the center as the parent body.
c. One focus is the parent body and the other is located outside of the elliptical orbit, on the line on which is the semi-major axis of the ellipse.
d. One focus is on the line containing the semi-major axis of the ellipse, and the other is located anywhere on the elliptical orbit of the satellite.

Teacher Support

Teacher Support Use the *Check Your Answers* questions to assess whether students master the learning objectives for this section. If students are struggling with a specific objective, the *Check Your Answers* will help identify which objective is causing the problem and direct students to the relevant content.