CS12: Introduction to Big O Notation

Understanding Algorithm Efficiency

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Key Terms for This Lesson

Vocabulary

Algorithm: A step-by-step set of instructions to solve a problem

Efficiency: How fast and how little memory an algorithm uses

Input size (n): The amount of data the algorithm works with

Complexity: How the time or space grows as n gets bigger

Learning Objectives

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- Identify and explain common Big O notations
- Analyze simple algorithms to determine their time complexity
- Compare different algorithms based on their efficiency

What is Big O Notation?

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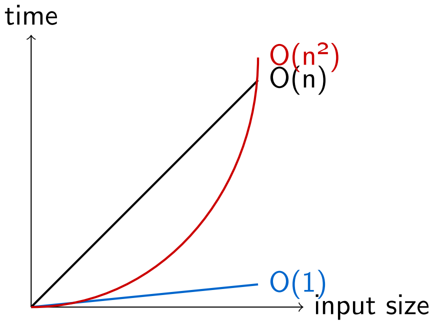
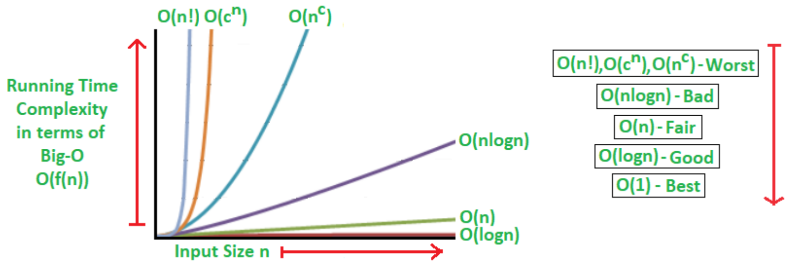
- Think of it as a way to measure an algorithm's speed
- Helps us compare different solutions
- Focuses on the slowest possible situation
- Ignores smaller details and focuses on the main pattern

How Different Algorithms Grow



As input grows...

Big O Complexity Comparison



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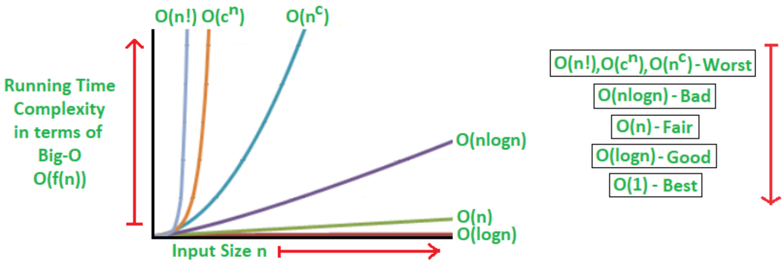

Key insight: As input grows, the gap between O(1) and O(n!) becomes enormous.

Understanding Through Real Examples

O(1) - Constant Time

- Finding a book on your desk
- Looking up array element by index

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O(n^2) - Quadratic Time

- Comparing every book with others
- Bubble sort algorithm

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O(log n) - Logarithmic Time

- Finding word in dictionary
- Binary search

I Do: Analyzing Linear Search

Problem

Let's analyze this linear search algorithm:

```
int linearSearch(int arr[], int n, int x) {
    for(int i = 0; i < n; i++) {
        if(arr[i] == x) {
            return i;  // Found it!
        }
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- Time Complexity: O(n)
- Why? In worst case, we check every element

We Do: Let's Analyze Together

What's the time complexity?

```
void printPairs(int arr[], int n) {
    for(int i = 0; i < n; i++) {
        for(int j = 0; j < n; j++) {
            cout << arr[i] <<    ,
                 << arr[j] << endl;
        }
    }
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- Let's count the operations...

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- Let's count the operations...
- Outer loop runs n times
- For each outer loop, inner loop runs n times
- Total operations: $n \times n = n^2$

You Do: Practice Time!

Analyze These Operations

Determine the Big O notation for:

1. Getting the first element of an array
2. Finding the maximum value in an unsorted array
3. Checking if a number is even or odd

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Solutions

1. O(1) - Direct access, no matter the size
2. O(n) - Must check every element once
3. O(1) - Single operation, size independent

Big O Quick Reference

Notation	**Name**	**If n = 1000**
O(1)	Constant	1 operation
O(log n)	Logarithmic	∼10 operations
O(n)	Linear	1,000 operations
O(n log n)	Linearithmic	∼10,000 operations
O(n^2)	Quadratic	1,000,000 operations
O(2^n)	Exponential	More than atoms in universe!

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Rule of thumb: Anything slower than $O(n^2)$ is usually too slow for large data.

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Remember These Points

- Big O notation helps us measure efficiency
- Most common notations: $O(1)$, $O(n)$, $O(n^2)$
- Consider how performance changes with input size
- Different problems require different solutions

Practice Makes Perfect

Try analyzing algorithms you write in your own code!