

PHYS12 CH19: Electric Potential and Electric Field

Electric Potential, Fields, and Capacitors

Mr. Gullo

Physics Department

February, 2025

Outline

- 1 Introduction
- 2 Electric Potential Energy and Potential Difference
- 3 Electric Potential in a Uniform Field
- 4 Electric Potential Due to a Point Charge
- 5 Equipotential Lines
- 6 Capacitors and Dielectrics
- 7 Capacitors in Series and Parallel
- 8 Energy Stored in Capacitors
- 9 Example Problems

Learning Objectives

By the end of this presentation, you will be able to:

- Define electric potential and explain how it relates to potential energy
- Calculate potential difference between points in an electric field
- Relate electric field strength to potential gradient
- Calculate the electric potential due to a point charge
- Understand the concept of equipotential lines
- Explain how capacitors work and calculate capacitance
- Determine the equivalent capacitance of series and parallel combinations
- Calculate the energy stored in a capacitor

Electric Potential Energy vs. Electric Potential

Electric Potential Energy

- Energy possessed by a charge in an electric field
- Depends on both the field and the amount of charge
- Measured in joules (J)

Electric Potential

- Electric potential energy per unit charge
- Independent of the test charge being used
- Measured in volts (V), where $1 \text{ V} = 1 \text{ J/C}$
- A property of the electric field at a point

Potential Difference and Voltage

Potential Difference

- The change in potential energy per unit charge when moving from point A to point B
- Commonly called voltage

$$\Delta V = \frac{\Delta PE}{q} \quad (1)$$

$$\Delta PE = q\Delta V \quad (2)$$

- Work must be done to move a positive charge from a low potential to a high potential
- A positive charge naturally moves from high potential to low potential (releasing energy)
- A negative charge naturally moves from low potential to high potential

The Electron Volt

Definition

The electron volt (eV) is the energy given to a fundamental charge accelerated through a potential difference of 1 V.

$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) \quad (3)$$

$$= (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) \quad (4)$$

$$= 1.60 \times 10^{-19} \text{ J} \quad (5)$$

- Useful unit for atomic and nuclear physics
- Common multiples: keV, MeV, GeV
- Example: A 12 V battery can give an electron 12 eV of energy

Conservation of Energy in Electric Fields

Mechanical Energy

- The sum of kinetic energy and potential energy of a system
- $E_{\text{mechanical}} = \text{KE} + \text{PE}$
- This sum is constant in a conservative field

Applications

- When a charge moves in an electric field, energy is converted between kinetic and potential forms
- $\Delta \text{KE} = -\Delta \text{PE} = -q\Delta V$
- Can find the final speed of a charged particle accelerated through a potential difference

Electric Potential in a Uniform Field

In a uniform electric field (like between parallel plates):

$$V_{AB} = Ed \quad (6)$$

$$E = \frac{V_{AB}}{d} \quad (7)$$

where:

- E is the electric field strength (V/m or N/C)
- d is the distance from A to B (m)
- V_{AB} is the potential difference (V)

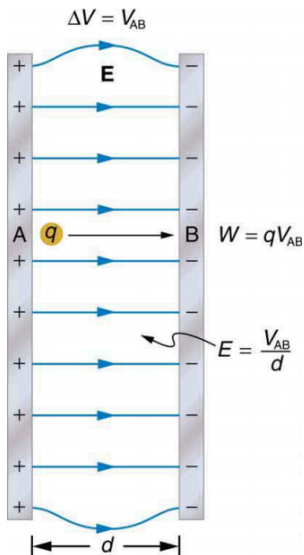


Figure: Fig 19.5

Relationship Between Electric Field and Potential

General Relationship

$$E = -\frac{\Delta V}{\Delta s} \quad (8)$$

where Δs is the distance over which the change in potential ΔV takes place

- The negative sign indicates that E points in the direction of decreasing potential
- Electric field is the gradient (slope) of the electric potential
- Units check: $V/m = N/C$
- Stronger electric fields create steeper potential gradients

Electric Potential Due to a Point Charge

Point Charge Potential

The electric potential at a distance r from a point charge Q is:

$$V = k \frac{Q}{r} \quad (9)$$

where $k = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$

- Potential is positive for positive charges and negative for negative charges
- Decreases with distance r from the charge
- Reference: $V = 0$ at $r = \infty$
- Note: $E = k \frac{Q}{r^2}$, so $E = -\frac{dV}{dr}$

Superposition of Potentials

Key Concept

Electric potential is a scalar quantity, so potentials from multiple charges add algebraically:

$$V_{total} = V_1 + V_2 + V_3 + \cdots = \sum_i V_i = k \sum_i \frac{Q_i}{r_i} \quad (10)$$

- This is simpler than adding electric fields (which are vectors)
- Makes it easier to solve some complex electrostatic problems
- Calculate the total potential, then find the electric field by taking the gradient

Equipotential Lines and Surfaces

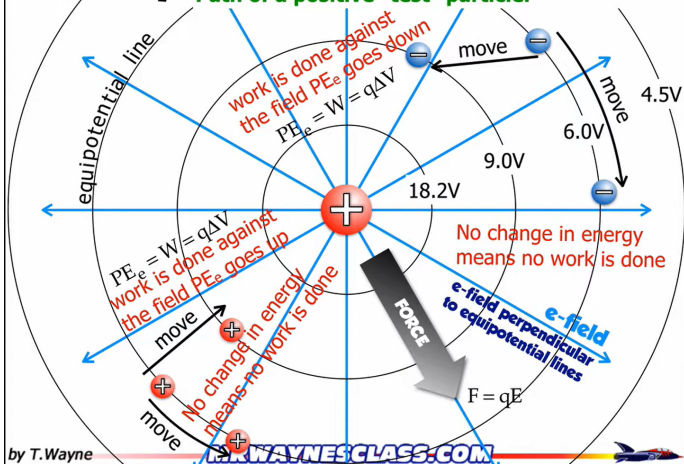
Definition

An equipotential line is a line along which the electric potential is constant.

- An equipotential surface is the 3D version of equipotential lines
- No work is done moving a charge along an equipotential
- Equipotential lines are always perpendicular to electric field lines
- For a point charge, equipotentials are concentric spheres
- For a uniform field, equipotentials are parallel planes

Summary

e-field lines point towards the lower potential.
Path of a positive "test" particle.



Equipotential Lines and Grounding

Properties of Equipotentials

- All points on a conductor in electrostatic equilibrium are at the same potential
- The surface of a conductor is an equipotential surface
- No electric field exists inside a conductor in electrostatic equilibrium

Grounding

- Grounding is the process of connecting a conductor to the Earth with a good conductor
- This fixes the conductor at zero volts (Earth's reference potential)
- Important for safety in electrical systems

What is a Capacitor?

Definition

A capacitor is a device used to store electric charge and energy in an electric field.

- Typically consists of two conductors (plates) separated by an insulator (dielectric)
- When connected to a voltage source, equal and opposite charges appear on the conductors
- The electric field is confined mostly to the region between the conductors
- Common forms: parallel plate, cylindrical, spherical

Capacitors

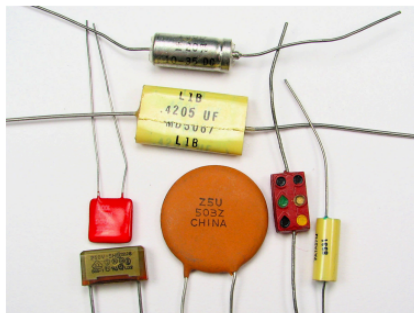


FIGURE 19.14 Some typical capacitors. Size and value of capacitance are not necessarily related. (credit: Windell Oskay)

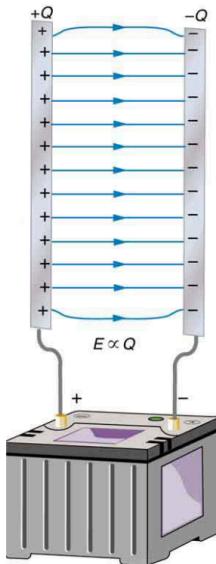


Figure: fig 19.13

Capacitance Definition

Capacitance (C) is the amount of charge stored per volt of potential difference:

$$C = \frac{Q}{V} \quad (11)$$

- Unit: Farad (F), where $1 \text{ F} = 1 \text{ C/V}$
- Typical values: pF to F range
- Depends only on physical characteristics (geometry, materials)
- Does NOT depend on charge or voltage (for linear capacitors)

Parallel Plate Capacitor

Capacitance Formula

The capacitance of a parallel plate capacitor in a vacuum or air:

$$C = \epsilon_0 \frac{A}{d} \quad (12)$$

where:

- $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ (permittivity of free space)
- A = area of plates
- d = separation distance between plates
- Larger plate area \rightarrow greater capacitance
- Smaller separation \rightarrow greater capacitance

Dielectrics in Capacitors

Effect of Dielectrics

Inserting a dielectric material between capacitor plates:

$$C = \kappa \epsilon_0 \frac{A}{d} \quad (13)$$

where κ is the dielectric constant of the material

Common Dielectric Constants

- Air: $\kappa \approx 1.00059$
- Paper: $\kappa \approx 2 - 4$
- Glass: $\kappa \approx 4 - 10$
- Teflon: $\kappa \approx 2.1$
- Water: $\kappa \approx 80$

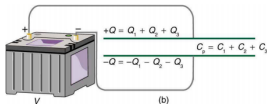
Capacitors in Series

Series Combination

When capacitors are connected in series:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \quad (14)$$

- All capacitors in series have the same charge magnitude
- The total voltage is divided among the capacitors
- Equivalent capacitance is less than any individual capacitance
- Similar to resistors in parallel



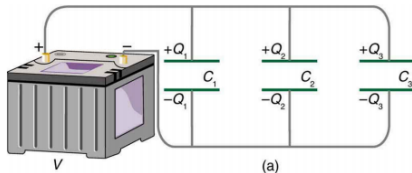
Capacitors in Parallel

Parallel Combination

When capacitors are connected in parallel:

$$C_P = C_1 + C_2 + C_3 + \dots \quad (15)$$

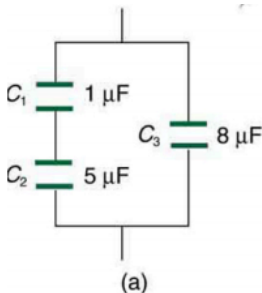
- All capacitors in parallel have the same voltage
- The total charge is divided among the capacitors
- Equivalent capacitance is greater than any individual capacitance
- Similar to resistors in series



Combined Series-Parallel Networks

Strategy for Solving Combined Networks

- 1 Identify series and parallel parts in the circuit
- 2 Compute the equivalent capacitance for each part
- 3 Combine the results to find the total capacitance



Energy Storage in Capacitors

Energy Formula

The energy stored in a capacitor can be expressed in three equivalent ways:

$$E_{cap} = \frac{QV}{2} \quad (16)$$

$$E_{cap} = \frac{CV^2}{2} \quad (17)$$

$$E_{cap} = \frac{Q^2}{2C} \quad (18)$$

- Energy is stored in the electric field between the plates
- Units: joules (J)
- Work must be done to charge a capacitor
- The stored energy can be recovered when the capacitor discharges

Applications of Capacitors

Common Applications

- Energy storage (backup power supplies)
- Filtering in power supplies
- Timing circuits
- Coupling and decoupling in electronic circuits
- Flash lamps in cameras
- Defibrillators in medical equipment
- Touch screens and sensors

Energy Density

The energy density in a capacitor's electric field is:

$$u = \frac{1}{2}\epsilon_0 E^2 \quad (\text{J/m}^3) \quad (19)$$

"I Do" Example - Electron Acceleration

Problem

An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce X-rays. What would be the maximum speed of these electrons? (Non-relativistic calculation)

Solution

Use energy conservation: electrical potential energy is converted to KE

$$qV = \frac{1}{2}mv^2 \quad (20)$$

$$v = \sqrt{\frac{2qV}{m}} \quad (21)$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19} \text{ C})(4.0 \times 10^4 \text{ V})}{9.11 \times 10^{-31} \text{ kg}}} \quad (22)$$

$$v = 1.17 \times 10^8 \text{ m/s} \quad (23)$$

"We Do" Example - Capacitor with Dielectric

Problem

A parallel plate capacitor has plates of area 5.00 m^2 separated by 0.100 mm of Teflon (dielectric constant $= 2.1$). What is the capacitance?

Solution

Use the formula for a parallel plate capacitor with a dielectric:

$$C = \kappa \epsilon_0 \frac{A}{d} \quad (24)$$

$$C = (2.1)(8.85 \times 10^{-12} \text{ F/m}) \frac{5.00 \text{ m}^2}{0.100 \times 10^{-3} \text{ m}} \quad (25)$$

$$C = (2.1)(8.85 \times 10^{-12} \text{ F/m})(5.00 \times 10^4 \text{ m}) \quad (26)$$

$$C = 9.29 \times 10^{-7} \text{ F} = 0.929 \text{ F} \quad (27)$$

"You Do" Example - Energy in a Capacitor

Problem

A 180 F capacitor is charged to 120 V.

- 1 How much charge is stored in the capacitor?
- 2 How much energy is stored in the capacitor?

Hints

- Use $Q = CV$ to find the charge
- Use $E_{cap} = \frac{1}{2}CV^2$ to find the energy
- Pay attention to the units ($F = 10^{-6}F$)

Try solving this problem on your own!

Summary

Key Concepts

- Electric potential: $V = \frac{PE}{q}$
- Potential difference: $\Delta V = \frac{\Delta PE}{q}$
- Electric field in a uniform field: $E = \frac{V}{d}$
- Electric field and potential relationship: $E = -\frac{\Delta V}{\Delta s}$
- Point charge potential: $V = k\frac{Q}{r}$
- Capacitance: $C = \frac{Q}{V}$
- Parallel plate capacitor: $C = \kappa\epsilon_0\frac{A}{d}$
- Series capacitors: $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$
- Parallel capacitors: $C_P = C_1 + C_2 + \dots$
- Energy stored: $E_{cap} = \frac{1}{2}CV^2$