PHYS11 CH:2 Reading the Story of Motion

Position and Velocity Graphs

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December 2025

Outline

How can a single line
tell the complete story of motion?

The Mystery

How can a single line
tell the complete story of motion?

Position, velocity, acceleration—all encoded in curves and slopes.

The Mystery

How can a single line
tell the complete story of motion?

Position, velocity, acceleration—all encoded in curves and slopes.

A graph is worth a thousand equations.

Learning Objectives

By the end of this lesson, you will be able to:

- **2.3:** Explain the meaning of slope in position vs. time graphs

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By the end of this lesson, you will be able to:

- **2.3:** Explain the meaning of slope in position vs. time graphs
- **2.3:** Solve problems using position vs. time graphs

2.3 The Language of Graphs

The Mental Model

A graph is like a picture—worth a thousand words. It reveals relationships between physical quantities.

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- Horizontal axis = independent variable (usually time)

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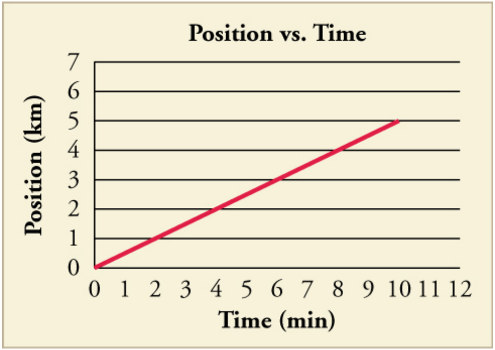
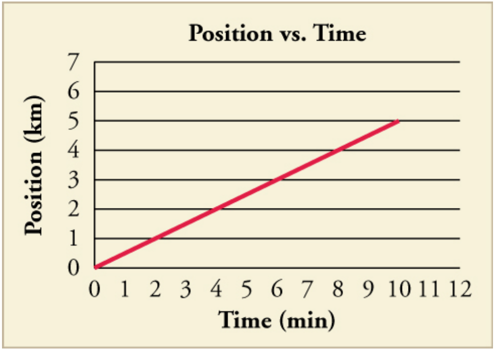
Standard graph anatomy:

- Horizontal axis = independent variable (usually time)
- Vertical axis = dependent variable (position, velocity)
- Straight line: $y = mx + b$ where m = slope, b = y-intercept

2.3 Drive to School



Graph of position vs. time for 5 km drive to school



Graph of position vs. time for 5 km drive to school

What does this line tell us?

- Starts at home ($d_0 = 0$)
- Ends at school ($d_f = 5$ km)
- Takes 10 minutes

2.3 Reading the Slope

Universal Law: Slope is Velocity

In a position vs. time graph:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta d}{\Delta t} = v_{\text{avg}}$$

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For the drive to school:

$$v_{\text{avg}} = \frac{5\ \text{km}}{10\ \text{min}} = 0.5\ \text{km/min}$$

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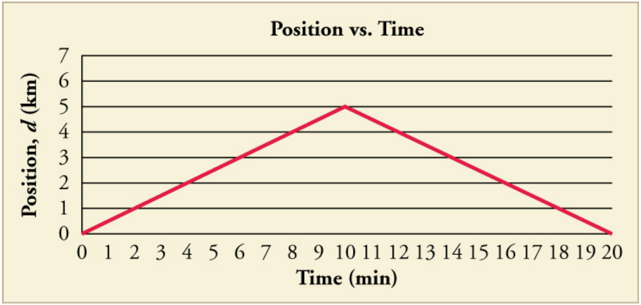
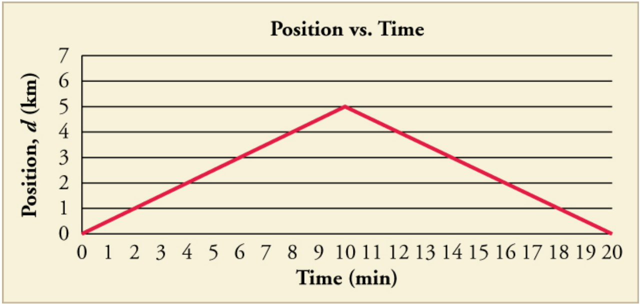
The Anchor

Steeper slope = faster motion. Flat line = at rest.

2.3 Round Trip



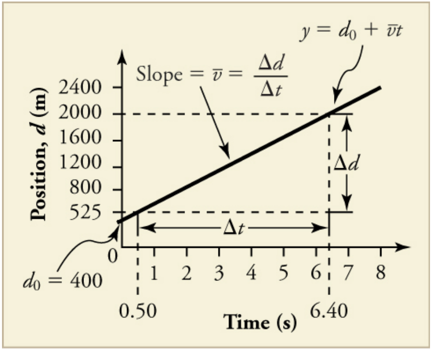
What does the graph look like with the return trip?



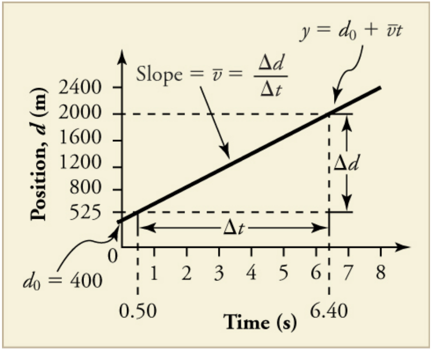
What does the graph look like with the return trip?

Second leg:

- Negative slope = moving backward
- Returns to $d = 0$ (back home)
- Net displacement = 0 km



Position vs. time for jet-powered car



Position vs. time for jet-powered car

Reading the graph:

- At $t = 0$ s: $d = 400$ m
- At $t = 1$ s: $d = 650$ m
- Slope = velocity = 250 m/s

2.3 The Position Equation

Universal Law: Linear Motion

From the graph equation $y = mx + b$, we get:

$$\boxed{d = vt + d_0}$$

or equivalently

$$\boxed{d = d_0 + vt}$$

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Where:

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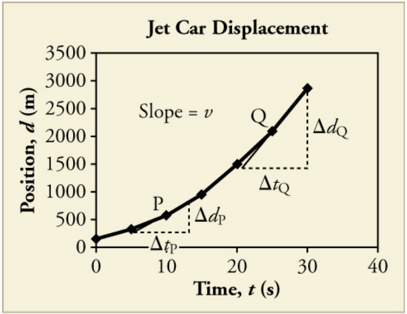
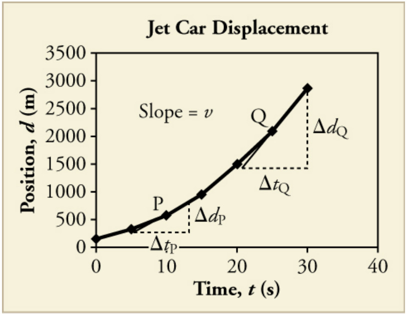
- b (y-intercept) = initial position d_0

2.3 Curved Position Graphs



Jet car speeding up - curved graph

2.3 Curved Position Graphs



Jet car speeding up - curved graph

The Conflict

When the graph curves, velocity is changing. Slope is not constant!

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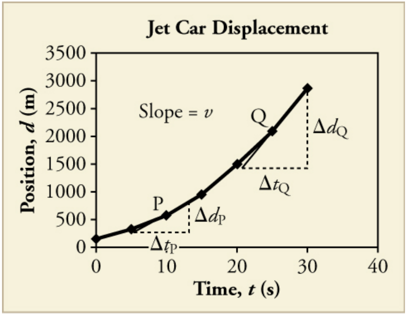


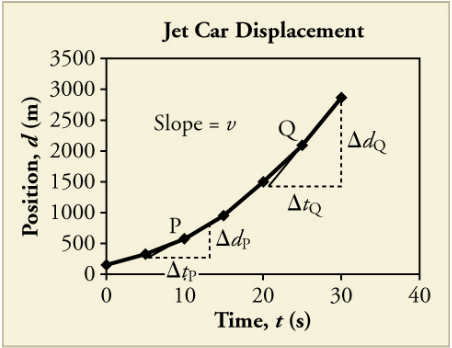
Jet car speeding up - curved graph

The Conflict

When the graph curves, velocity is changing. Slope is not constant!

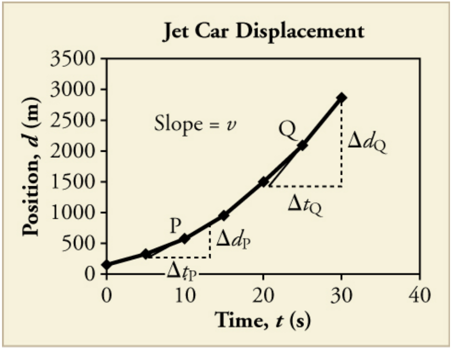
Solution: Use tangent line to find instantaneous velocity at any point.

2.3 Instantaneous Velocity from Tangent



Slope of tangent line = instantaneous velocity

2.3 Instantaneous Velocity from Tangent



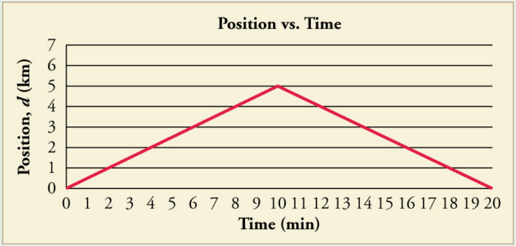
Slope of tangent line = instantaneous velocity

At point Q ($t = 25$ s):

$$v_Q = \frac{3120 - 1300 \text{ m}}{32 - 19 \text{ s}} = \frac{1820 \text{ m}}{13 \text{ s}} = 140 \text{ m/s}$$

Attempt: Reading a Position Graph

The Challenge (3 min, silent)



Position vs. time graph showing motion with direction change

Given: The graph above

Find: Average velocity over entire time interval (0 to 20 min)

Can you decode this motion? Work silently.

Compare: Graph Reading Strategy

Turn and talk (2 min):

1. What two points did you choose?
2. How did you calculate the slope?
3. What units did you get?

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Reveal: Slope Equals Velocity

Self-correct in a different color:

Step 1: Identify endpoints: $(t_0, d_0) = (0, 0)$ km and $(t_f, d_f) = (20, 0)$ km

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$$v_{\text{avg}} = \frac{\Delta d}{\Delta t} = \frac{d_f - d_0}{t_f - t_0}$$

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$$v_{\text{avg}} = \frac{\Delta d}{\Delta t} = \frac{d_f - d_0}{t_f - t_0}$$

Step 3: Substitute

$$v_{\text{avg}} = \frac{0 - 0 \text{ km}}{20 - 0 \text{ min}} = \boxed{0 \text{ km/min}}$$

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Check: Zero! Started and ended at same position - net displacement is zero!

Learning Objectives

By the end of this lesson, you will be able to:

- **2.4:** Explain the meaning of slope and area in velocity vs. time graphs

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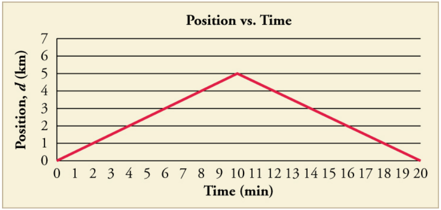
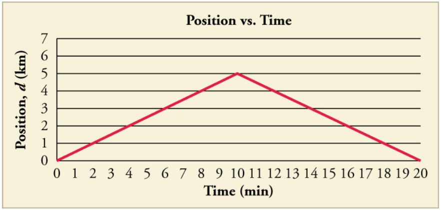
2.4 From Position to Velocity Graph



Position graph: drive to and from school

2.4 From Position to Velocity Graph



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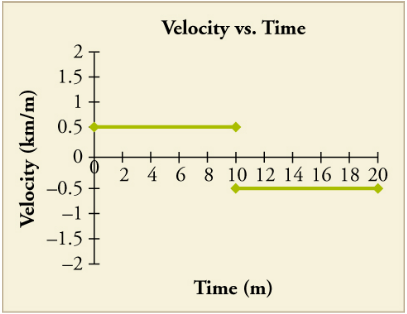

Velocity graph: two constant velocities

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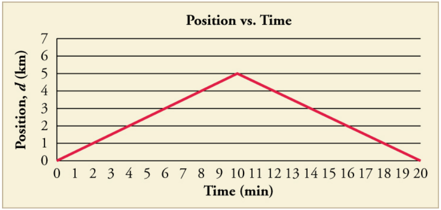

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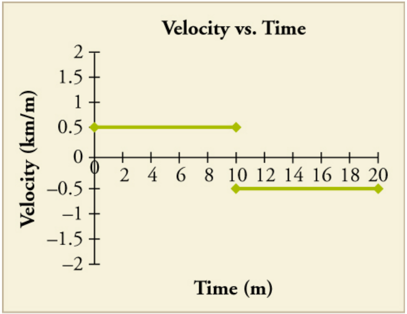

Velocity graph: two constant velocities

Key insight: Slope of position graph becomes height of velocity graph!

2.4 Reading Velocity Graphs

Universal Law: The Dual Nature

In a velocity vs. time graph:

1. **Slope** = acceleration (rate of velocity change)
2. **Area under curve** = displacement

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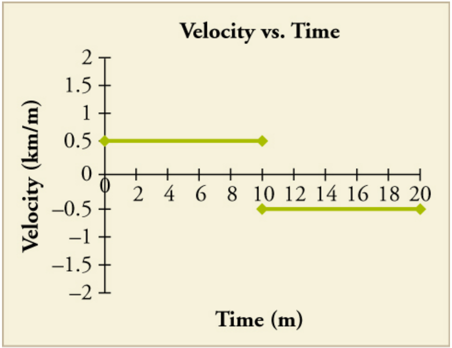
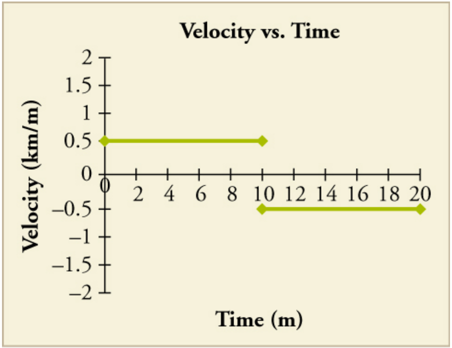
The Anchor

Position graphs give velocity. Velocity graphs give acceleration AND displacement.

2.4 Area Equals Displacement



Velocity graph for drive to school

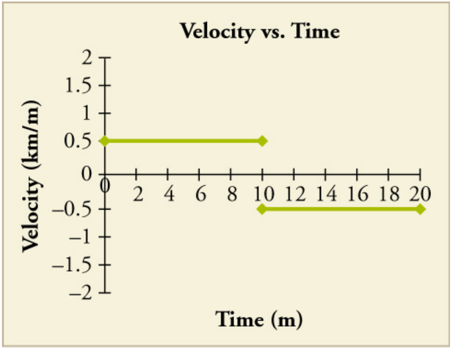


Velocity graph for drive to school

Calculate displacement:

$$d = v \times t = 0.5 \text{ km/min} \times 10 \text{ min} = 5 \text{ km}$$

2.4 Area Equals Displacement



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Return trip:

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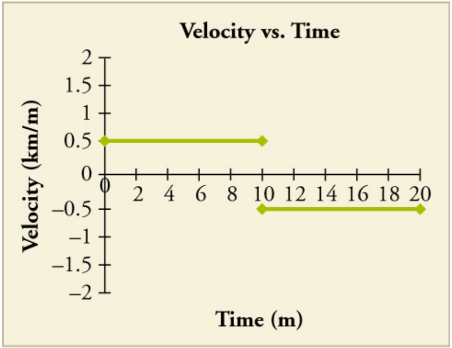

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Where:

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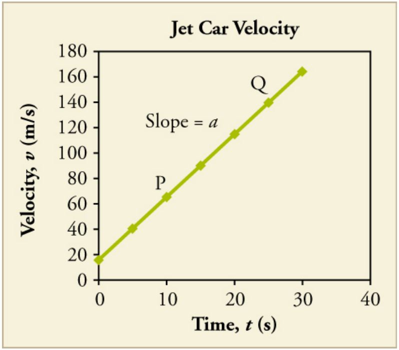
And from area:

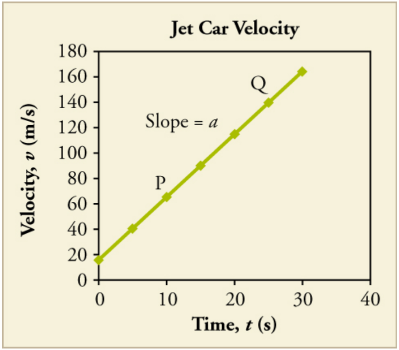
$$\boxed{d = vt} \quad \text{(for constant velocity)}$$

2.4 Jet Car Velocity Graph



Jet car speeding up - straight line with positive slope

2.4 Jet Car Velocity Graph

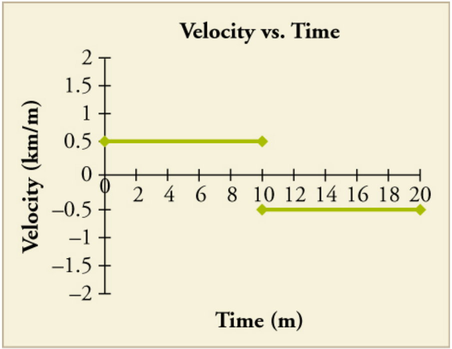


Jet car speeding up - straight line with positive slope

What we can read:

- Starts at $v_0 = 20$ m/s at $t = 0$
- Ends at $v_f = 160$ m/s at $t = 30$ s
- Slope = acceleration (constant)

2.4 Zero Slope Means Constant Velocity



Horizontal line in velocity graph

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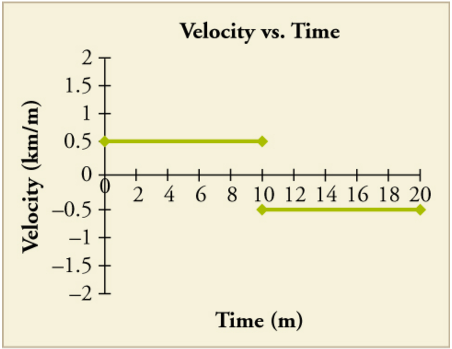

Horizontal line in velocity graph

Key Insight

Slope = 0 means acceleration = 0. Object moves at constant velocity.

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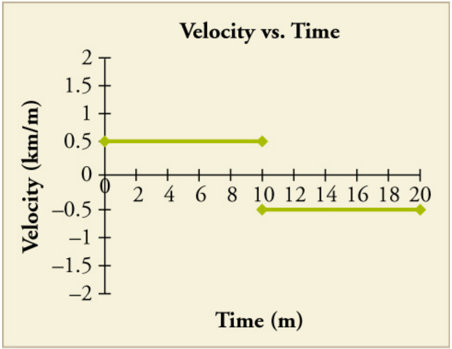


Horizontal line in velocity graph

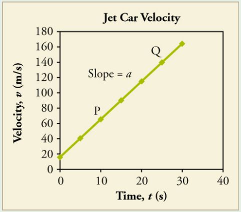
Key Insight

Slope $= 0$ means acceleration $= 0$. Object moves at constant velocity.

This is what we saw in the drive to school example!

Attempt: Calculating from Velocity Graph

The Challenge (3 min, silent)



Jet car velocity vs. time

Given: Velocity graph above (jet car from 0 to 30 s)
Find:

- (a) Displacement
- (b) Acceleration

Use both slope and area. Work silently.

Compare: Dual Extraction

Turn and talk (2 min):

1. How did you find displacement? (Hint: area)
2. How did you find acceleration? (Hint: slope)
3. Did you break the area into shapes?

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Reveal: Area and Slope

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(a) Displacement = Area under curve

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Rectangle: $20 \text{ m/s} \times 30 \text{ s} = 600 \text{ m}$

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Rectangle: $20 \text{ m/s} \times 30 \text{ s} = 600 \text{ m}$
Triangle: $\frac{1}{2} \times 30 \text{ s} \times 140 \text{ m/s} = 2100 \text{ m}$

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Total: $\boxed{d = 2700 \text{ m}}$

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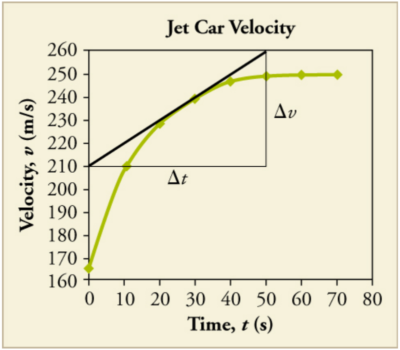
Total: $\boxed{d = 2700 \text{ m}}$

(b) Acceleration = Slope

$$a = \frac{\Delta v}{\Delta t} = \frac{140 \text{ m/s}}{30 \text{ s}} = \boxed{4.67 \text{ m/s}^2}$$

2.4 Curved Velocity Graphs



More realistic jet car - curved velocity graph

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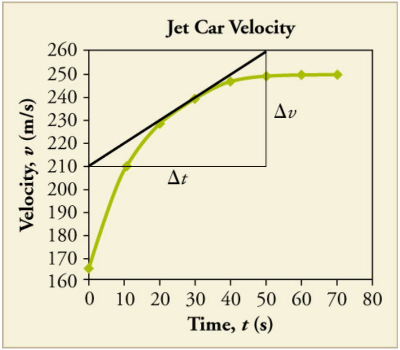
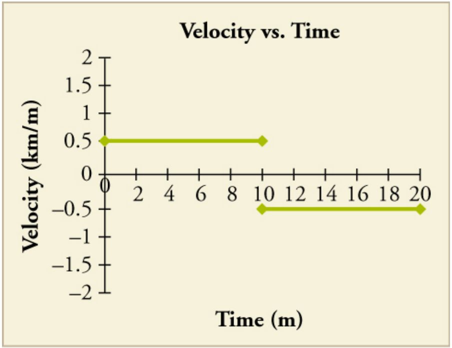


More realistic jet car - curved velocity graph

The Complication

When velocity graph curves, acceleration is changing! Use tangent for instantaneous acceleration.

2.4 Negative Velocity



Velocity graph going below zero

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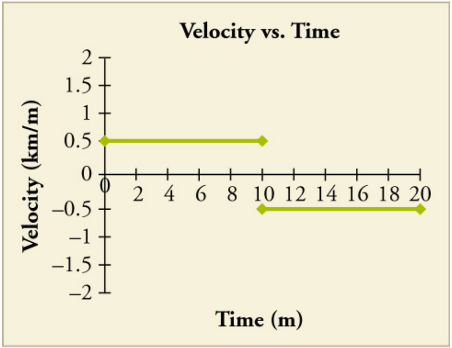

Velocity graph going below zero

Interpretation:

- Positive velocity = moving forward
- Negative velocity = moving backward
- Zero crossing = turning point (changes direction)

2.4 Position from Velocity Graph

The Connection

From position graph: slope → velocity

From velocity graph: area → displacement

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The Connection

From position graph: slope $\rightarrow$ velocity
From velocity graph: area $\rightarrow$ displacement

Circular Relationship

$$\text{Position} \xrightarrow{\text{slope}} \text{Velocity} \xrightarrow{\text{slope}} \text{Acceleration}$$

$$\text{Position} \xleftarrow{\text{area}} \text{Velocity} \xleftarrow{?} \text{Acceleration}$$

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From position graph: slope $\rightarrow$ velocity
From velocity graph: area $\rightarrow$ displacement

Circular Relationship

$$\text{Position} \xrightarrow{\text{slope}} \text{Velocity} \xrightarrow{\text{slope}} \text{Acceleration}$$

$$\text{Position} \xleftarrow{\text{area}} \text{Velocity} \xleftarrow{?} \text{Acceleration}$$

We'll learn about acceleration graphs in the next chapter!

What You Now Know

The Revelations

1. Graphs are the visual language of motion

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5. Negative slopes/areas show direction

What You Now Know

The Revelations

1. Graphs are the visual language of motion
2. Position graph: slope = velocity
3. Velocity graph: slope = acceleration, area = displacement
4. Tangent lines extract instantaneous values
5. Negative slopes/areas show direction
6. One graph encodes multiple quantities

Key Equations

$$\text{Position graph slope} = \frac{\Delta d}{\Delta t} = v_{\text{avg}} \tag{1}$$

$$\text{Position equation}: d = d_0 + vt \tag{2}$$

$$\text{Velocity graph slope} = \frac{\Delta v}{\Delta t} = a \tag{3}$$

$$\text{Velocity equation}: v = v_0 + at \tag{4}$$

$$\text{Displacement from velocity} = \text{area under } v\text{-}t \text{ curve} \tag{5}$$

Homework

Complete the assigned problems

posted on the LMS

Temporary page!

LaTeX was unable to guess the total number of pages correctly. A
was some unprocessed data that should have been added to the
this extra page has been added to receive it.
If you rerun the document (without altering it) this surplus page
away, because LaTeX now knows how many pages to expect for t
document.