

## 8.7 Introduction to Rocket Propulsion

### Learning Objectives

By the end of this section, you will be able to:

- State Newton’s third law of motion.
- Explain the principle involved in propulsion of rockets and jet engines.
- Derive an expression for the acceleration of the rocket and discuss the factors that affect the acceleration.
- Describe the function of a space shuttle.

Rockets range in size from fireworks so small that ordinary people use them to immense Saturn Vs that once propelled massive payloads toward the Moon. The propulsion of all rockets, jet engines, deflating balloons, and even squids and octopuses is explained by the same physical principle—Newton’s third law of motion. Matter is forcefully ejected from a system, producing an equal and opposite reaction on what remains. Another common example is the recoil of a gun. The gun exerts a force on a bullet to accelerate it and consequently experiences an equal and opposite force, causing the gun’s recoil or kick.

### Making Connections: Take-Home Experiment—Propulsion of a Balloon

Hold a balloon and fill it with air. Then, let the balloon go. In which direction does the air come out of the balloon and in which direction does the balloon get propelled? If you fill the balloon with water and then let the balloon go, does the balloon’s direction change? Explain your answer.

Figure 8.12 shows a rocket accelerating straight up. In part (a), the rocket has a mass  $m$  and a velocity  $v$  relative to Earth, and hence a momentum  $mv$ . In part (b), a time  $\Delta t$  has elapsed in which the rocket has ejected a mass  $\Delta m$  of hot gas at a velocity  $v_e$  relative to the rocket. The remainder of the mass ( $m - \Delta m$ ) now has a greater velocity ( $v + \Delta v$ ). The momentum of the entire system (rocket plus expelled gas) has actually decreased because the force of gravity has acted for a time  $\Delta t$ , producing a negative impulse  $\Delta p = -mg\Delta t$ . (Remember that impulse is the net external force on a system multiplied by the time it acts, and it equals the change in momentum of the system.) So, the center of mass of the system is in free fall but, by rapidly expelling mass, part of the system can accelerate upward. It is a commonly held misconception that the rocket exhaust pushes on the ground. If we consider thrust; that is, the force exerted on the rocket by the exhaust gases, then a rocket’s thrust is greater in outer space than in the atmosphere or on the launch pad. In fact, gases are easier to expel into a vacuum.

By calculating the change in momentum for the entire system over  $\Delta t$ , and equating this change to the impulse, the following expression can be shown to be a good approximation for the acceleration of the rocket.

$$a = \frac{v_e}{m} \frac{\Delta m}{\Delta t} - g$$

8.77

“The rocket” is that part of the system remaining after the gas is ejected, and  $g$  is the acceleration due to gravity.

### Acceleration of a Rocket

Acceleration of a rocket is

$$a = \frac{v_e}{m} \frac{\Delta m}{\Delta t} - g,$$

8.78

where  $a$  is the acceleration of the rocket,  $v_e$  is the exhaust velocity,  $m$  is the mass of the rocket,  $\Delta m$  is the mass of the ejected gas, and  $\Delta t$  is the time in which the gas is ejected.

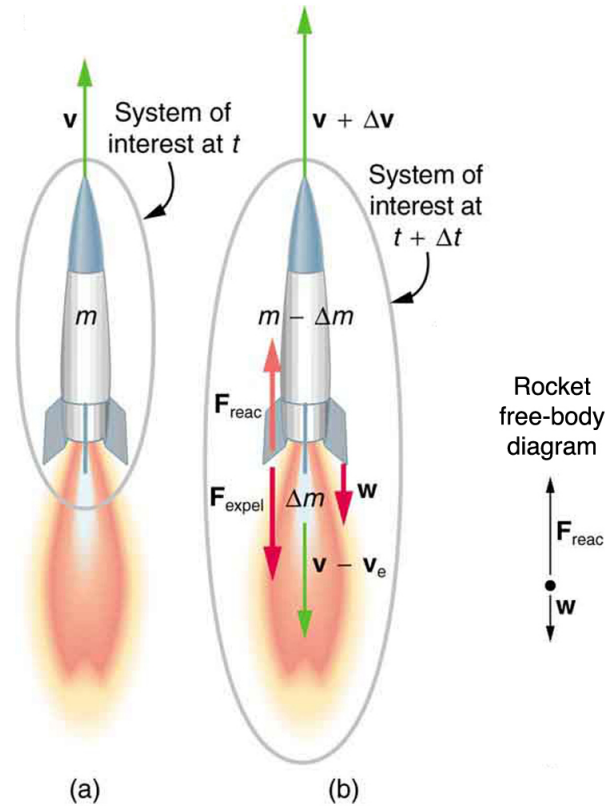


Figure 8.12 (a) This rocket has a mass  $m$  and an upward velocity  $v$ . The net external force on the system is  $-mg$ , if air resistance is neglected. (b) A time  $\Delta t$  later the system has two main parts, the ejected gas and the remainder of

the rocket. The reaction force on the rocket is what overcomes the gravitational force and accelerates it upward.

A rocket's acceleration depends on three major factors, consistent with the equation for acceleration of a rocket. First, the greater the exhaust velocity of the gases relative to the rocket,  $v_e$ , the greater the acceleration is. The practical limit for  $v_e$  is about  $2.5 \times 10^3$  m/s for conventional (non-nuclear) hot-gas propulsion systems. The second factor is the rate at which mass is ejected from the rocket. This is the factor  $\Delta m/\Delta t$  in the equation. The quantity  $(\Delta m/\Delta t)v_e$ , with units of newtons, is called "thrust." The faster the rocket burns its fuel, the greater its thrust, and the greater its acceleration. The third factor is the mass  $m$  of the rocket. The smaller the mass is (all other factors being the same), the greater the acceleration. The rocket mass  $m$  decreases dramatically during flight because most of the rocket is fuel to begin with, so that acceleration increases continuously, reaching a maximum just before the fuel is exhausted.

#### Factors Affecting a Rocket's Acceleration

- The greater the exhaust velocity  $v_e$  of the gases relative to the rocket, the greater the acceleration.
- The faster the rocket burns its fuel, the greater its acceleration.
- The smaller the rocket's mass (all other factors being the same), the greater the acceleration.

#### Example 8.8

**Calculating Acceleration: Initial Acceleration of a Moon Launch** A Saturn V's mass at liftoff was  $2.80 \times 10^6$  kg, its fuel-burn rate was  $1.40 \times 10^4$  kg/s, and the exhaust velocity was  $2.40 \times 10^3$  m/s. Calculate its initial acceleration.

**Strategy** This problem is a straightforward application of the expression for acceleration because  $a$  is the unknown and all of the terms on the right side of the equation are given.

**Solution** Substituting the given values into the equation for acceleration yields

$$\begin{aligned} a &= \frac{v_e}{m} \frac{\Delta m}{\Delta t} - g \\ &= \frac{2.40 \times 10^3 \text{ m/s}}{2.80 \times 10^6 \text{ kg}} (1.40 \times 10^4 \text{ kg/s}) - 9.80 \text{ m/s}^2 \\ &= 2.20 \text{ m/s}^2. \end{aligned}$$

8.79

**Discussion** This value is fairly small, even for an initial acceleration. The acceleration does increase steadily as the rocket burns fuel, because  $m$  decreases while  $v_e$  and  $\frac{\Delta m}{\Delta t}$  remain constant. Knowing this acceleration and the mass of the rocket, you can show that the thrust of the engines was  $3.36 \times 10^7$  N.

To achieve the high speeds needed to hop continents, obtain orbit, or escape Earth's gravity altogether, the mass of the rocket other than fuel must be as small as possible. It can be shown that, in the absence of air resistance and neglecting gravity, the final velocity of a one-stage rocket initially at rest is

$$v = v_e \ln \frac{m_0}{m_r},$$

8.80

where  $\ln(m_0/m_r)$  is the natural logarithm of the ratio of the initial mass of the rocket ( $m_0$ ) to what is left ( $m_r$ ) after all of the fuel is exhausted. (Note that  $v$  is actually the change in velocity, so the equation can be used for any segment of the flight. If we start from rest, the change in velocity equals the final velocity.) For example, let us calculate the mass ratio needed to escape Earth's gravity starting from rest, given that the escape velocity from Earth is about  $11.2 \times 10^3$  m/s, and assuming an exhaust velocity  $v_e = 2.5 \times 10^3$  m/s.

$$\ln \frac{m_0}{m_r} = \frac{v}{v_e} = \frac{11.2 \times 10^3 \text{ m/s}}{2.5 \times 10^3 \text{ m/s}} = 4.48$$

8.81

Solving for  $m_0/m_r$  gives

$$\frac{m_0}{m_r} = e^{4.48} = 88.$$

8.82

Thus, the mass of the rocket is

$$m_r = \frac{m_0}{88}.$$

8.83

This result means that only 1/88 of the mass is left when the fuel is burnt, and 87/88 of the initial mass was fuel. Expressed as percentages, 98.9% of the rocket is fuel, while payload, engines, fuel tanks, and other components make up only 1.10%. Taking air resistance and gravitational force into account, the mass  $m_r$  remaining can only be about  $m_0/180$ . It is difficult to build a rocket in which the fuel has a mass 180 times everything else. The solution is multistage rockets. Each stage only needs to achieve part of the final velocity and is discarded after it burns its fuel. The result is that each successive stage can have smaller engines and more payload relative to its fuel. Once out of the atmosphere, the ratio of payload to fuel becomes more favorable, too.

The space shuttle was an attempt at an economical vehicle with some reusable parts, such as the solid fuel boosters and the craft itself. (See Figure 8.13) The shuttle's need to be operated by humans, however, made it at least as costly

for launching satellites as expendable, unpiloted rockets. Ideally, the shuttle would only have been used when human activities were required for the success of a mission, such as the repair of the Hubble space telescope. Rockets with satellites can also be launched from airplanes. Using airplanes has the double advantage that the initial velocity is significantly above zero and a rocket can avoid most of the atmosphere's resistance.



Figure 8.13 The space shuttle had a number of reusable parts. Solid fuel boosters on either side were recovered and refueled after each flight, and the entire orbiter returned to Earth for use in subsequent flights. The large liquid fuel tank was expended. The space shuttle was a complex assemblage of technologies, employing both solid and liquid fuel and pioneering ceramic tiles as reentry heat shields. As a result, it permitted multiple launches as opposed to single-use rockets. (credit: NASA)