

# Lesson CH:9

## Statics and Torque

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# Equilibrium Conditions

## First Condition for Equilibrium

- Net external force must be zero ( $\sum \vec{F} = \vec{0}$ )
- Applies to both linear and rotational motion
- Required for absence of acceleration

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# Torque and Rotational Equilibrium

## Understanding Torque

Torque ( $\tau$ ) is the rotational equivalent of force:

- Measures effectiveness of force in changing angular velocity
- Represents rotational acceleration capability
- Defined as:  $\tau = rF \sin \theta$
- where:
  - $r$  is distance from pivot to force application point
  - $F$  is magnitude of force
  - $\theta$  is angle between force and position vector

## Second Condition for Equilibrium

- Net external torque must be zero ( $\sum \vec{\tau} = \vec{0}$ )
- Torque ( $\vec{\tau}$ ) =  $rF \sin \theta$
- Where  $\vec{r}$  is position vector from pivot point,  $\vec{F}$  is force vector
- Perpendicular lever arm ( $r_{\perp}$ ) is shortest distance from pivot to force line

# Types of Equilibrium

- **Stable Equilibrium**

- When displaced, experiences force/torque opposing displacement
- System returns to original position

- **Unstable Equilibrium**

- When displaced, experiences force/torque in same direction as displacement
- System moves further from original position

- **Neutral Equilibrium**

- Equilibrium independent of displacement
- System remains in new position when displaced

# Simple Machines

## • Basic Principles

- Devices that multiply or augment applied forces
- Trade-off between force and distance
- Examples: lever, nail puller, wheelbarrow, crank

## Mechanical Advantage

- Ratio of output force to input force
- Key measure of machine effectiveness

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## Problem 2

When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt.

- (a) How much torque are you exerting in newton  $\times$  meters (relative to the center of the bolt)?
- (b) Convert this torque to foot-pounds.

## Problem 2 - Solution

### Solution:

(a) Using the torque equation  $\tau = r_{\perp} F$ :

- $\tau = 0.140 \text{ m} \times 165 \text{ N} = 23.1 \text{ N} \cdot \text{m}$

(b) Converting to foot-pounds:

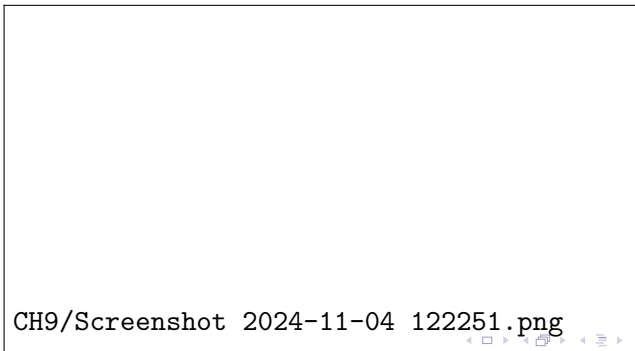
- $\tau = 23.1 \text{ N} \cdot \text{m} \times \frac{0.738 \text{ ft} \cdot \text{lb}}{1 \text{ N} \cdot \text{m}} = 17.0 \text{ ft} \cdot \text{lb}$

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## Problem 10

A 17.0-m-high and 11.0-m-long wall under construction and its bracing are shown in Figure 9.30. The wall is in stable equilibrium without the bracing but can pivot at its base. Calculate the force exerted by each of the 10 braces if a strong wind exerts a horizontal force of 650 N on each square meter of the wall. Assume that the net force from the wind acts at a height halfway up the wall and that all braces exert equal forces parallel to their lengths. Neglect the thickness of the wall.



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# Problem Statement

A wall under construction with bracing:

- Wall dimensions:
  - Height: 17.0 m
  - Length: 11.0 m
- Wind force: 650 N per square meter
- 10 braces at  $35^\circ$  angle
- Wind acts at half height
- Wall can pivot at base
- All braces exert equal forces

**Goal:** Calculate force exerted by each brace

# Problem Analysis

- Key considerations:
  - Take pivot point at wall base
  - Neglect wall thickness
  - Forces acting:



$$F_{brace} \times 10$$

- Weight of wall ( $w$ )
    - Normal force ( $N$ )
- Using second condition for equilibrium:

$$\text{net}\tau = 0 \Rightarrow \text{net}\tau_{\text{cw}} = -\text{net}\tau_{\text{ccw}}$$



# Mathematical Solution

$$\text{net}\tau_{\text{cw}} = -\text{net}\tau_{\text{ccw}}$$

$$(8.5 \text{ m}) \times F_{\text{wind}} = rF_b \times 10 = (8.5 \text{ m}) \sin 35^\circ \times F_b \times 10$$

$$F_{\text{wind}} = 10 \sin 35^\circ F_b$$

$$F_b = \frac{F_{\text{wind}}}{10 \sin 35^\circ}$$

Wind force calculation:

$$\begin{aligned} F_{\text{wind}} &= \frac{F}{A} \times A = 650 \text{ N/m}^2 \times 11.0 \text{ m} \times 17.0 \text{ m} \\ &= 121,550 \text{ N} \end{aligned}$$

Therefore:

$$F_b = \frac{121,550 \text{ N}}{10 \times 0.5736} = 2.12 \times 10^4 \text{ N}$$

- Each brace must exert a force of 21.2 kN
- This significant force demonstrates:
  - Importance of proper bracing in construction
  - Impact of wind loads on tall structures
  - Need for careful engineering calculations

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- [https://www.youtube.com/watch?v=pK\\_oW62-zrc](https://www.youtube.com/watch?v=pK_oW62-zrc)

- <https://www.youtube.com/shorts/CxRw2n3lD7I>

- [https://www.youtube.com/shorts/sdC36VK\\_tY0](https://www.youtube.com/shorts/sdC36VK_tY0)

## Problem 18

The center of gravity of a 5.0 kg pole held by a pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by:

- (a) his right hand
- (b) his left hand

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# Problem 18 - Solution

**Solution:** Using the center of gravity as reference:

(a) Taking pivot at left hand:

- net  $\tau = 0$
- $F_R(0.7 \text{ m}) = (5.0 \text{ kg})(9.80 \text{ m/s}^2)(2.0 \text{ m})$
- $F_R = 140 \text{ N}$

(b) Total weight must be supported:

- $F_L + F_R = (5.0 \text{ kg})(9.80 \text{ m/s}^2)$
- $F_L = 49 \text{ N}$

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## Problem 22

A typical car has an axle with 2.0 cm radius driving a tire with a radius of 30.0 cm. What is its mechanical advantage assuming the very simplified model in Figure 9.23(b)?

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# Problem 22 - Solution

**Solution:** Mechanical advantage =  $r_2/r_1 = 30.0 \text{ cm}/2.0 \text{ cm} = 15$

Step-by-step explanation:

① Identify radii:

- Inner radius ( $r_1$ ) = 2.0 cm
- Outer radius ( $r_2$ ) = 30.0 cm

② Calculate mechanical advantage:

- $MA = F_o \over F_i = \frac{l_i}{l_o}$
- $MA = r_2/r_1$
- $MA = 30.0/2.0 = 15$

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# Problem 26

Verify that the force in the elbow joint in Example 9.4 is 407 N, as stated in the text.

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# Problem Statement

**Problem 26:** Verify that the force in the elbow joint in Example 9.4 is 407 N.

## Given Values

$$F_B = 470 \text{ N}$$

$$m_a = 2.50 \text{ kg}$$

$$m_b = 4.00 \text{ kg}$$

$$r_1 = 4.00 \text{ cm}$$

$$r_2 = 16.0 \text{ cm}$$

$$r_3 = 38.0 \text{ cm}$$

# Detailed Derivation

Starting from torque balance(second condition of equilibrium):

$$\tau_{\text{Bicep}} = \tau_{\text{arm}} + \tau_{\text{book}}$$

$$F_B r_1 = w_a r_2 + w_B r_3$$

Solving for  $F_B$ :

$$F_B = \frac{w_a r_2 + w_B r_3}{r_1}$$

For equilibrium of forces(first condition of equilibrium):

$$F_e = w_a + w_B - F_B$$

$$= w_a + w_B - \frac{w_a r_2 + w_B r_3}{r_1}$$

$$= w_a \left(1 - \frac{r_2}{r_1}\right) + w_B \left(1 - \frac{r_3}{r_1}\right)$$

$$= w_a \left(\frac{r_2}{r_1} - 1\right) + w_B \left(\frac{r_3}{r_1} - 1\right)$$

Multiply both sides by  $r_1$ :

$$F_e \times r_1 = w_a \left( \frac{r_2}{r_1} - 1 \right) + w_B \left( \frac{r_3}{r_1} - 1 \right)$$

# Calculation

Substituting the values:

$$F_E \times r_1 = (2.50 \text{ kg})(9.80 \text{ m/s}^2) \left( \frac{16.0 \text{ cm}}{4.0 \text{ cm}} - 1 \right) \\ + (4.00 \text{ kg})(9.80 \text{ m/s}^2) \left( \frac{38.0 \text{ cm}}{4.00 \text{ cm}} - 1 \right)$$

## Final Result

Therefore:

$$F_E = 407 \text{ N}$$

This verifies the stated value in Example 9.4