

# PHYS12 CH:28.1-28.3

## The Geometry of Spacetime

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- State Einstein's two postulates of special relativity.
- Explain why simultaneity is relative to the observer.
- Calculate time dilation effects for moving objects using the Lorentz factor.
- Calculate length contraction for objects moving at relativistic speeds.
- Distinguish between proper time/length and relativistic time/length.

# 28.1 Einstein's First Postulate

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- An inertial frame is one that is not accelerating (constant velocity).
- There is no "preferred" or "absolute" frame of reference.
- You cannot perform an experiment inside a smooth-moving train to determine if you are moving or standing still.

# 28.1 Einstein's Second Postulate

## The Constancy of the Speed of Light

The speed of light in a vacuum ( $c$ ) has the same value ( $c = 3.00 \times 10^8$  m/s) in all inertial frames of reference, regardless of the motion of the light source or the observer.

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- This contradicts our everyday experience with relative velocities (like throwing a ball from a moving car).
- $c$  is the cosmic speed limit.

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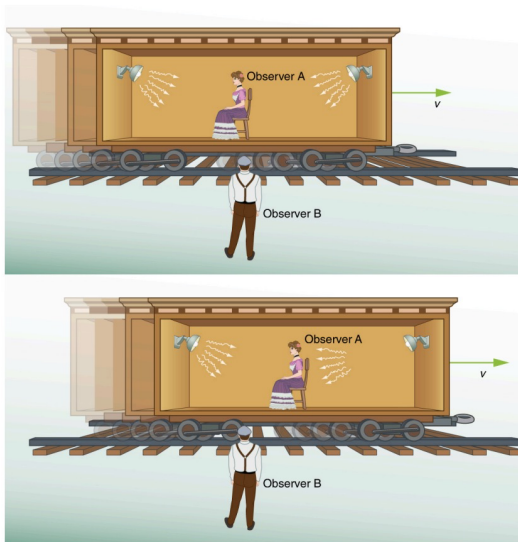
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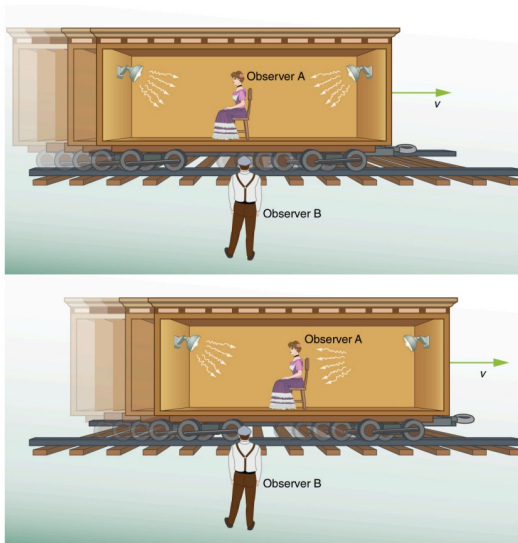
Two events that are simultaneous in one frame of reference are **not necessarily simultaneous** in another frame that is moving relative to the first.

- This is not an optical illusion; it is a fundamental property of time.
- If observers cannot agree on "when" things happen, time itself must be relative.

# Concept Visualization: Simultaneity

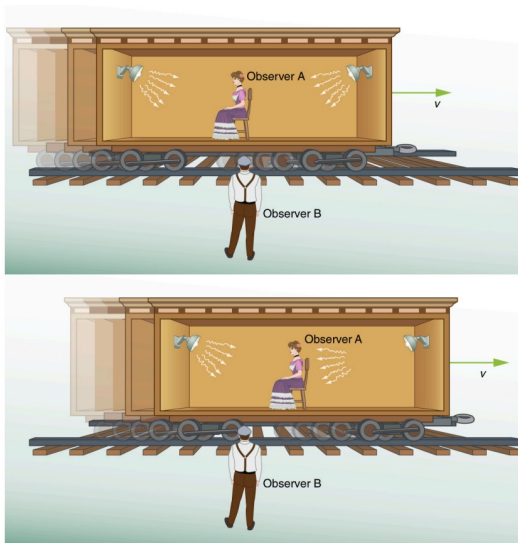


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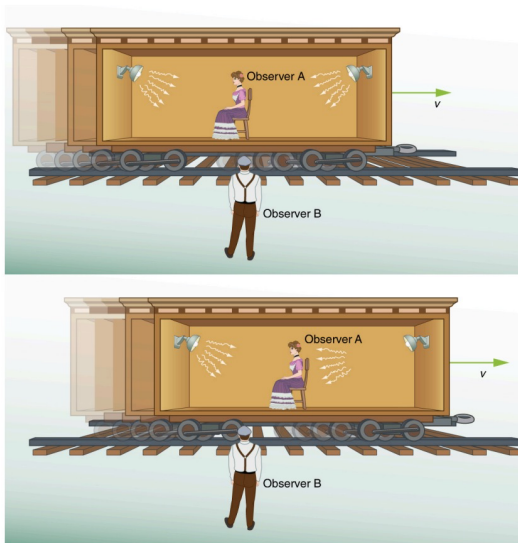
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- $\Delta t$ : Time interval measured by stationary observer (dilated time).
- $\Delta t_0$ : **Proper time** (measured by observer moving with the event).
- $\gamma$ : Lorentz factor (always  $\geq 1$ ).

# Essential Equation: Time Dilation

## Time Dilation Formula

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or

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- As  $v \rightarrow c$ ,  $\gamma \rightarrow \infty$  (Relativistic effects dominate).
- Calculating  $\gamma$  first often simplifies problems.

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- Contraction happens **only** in the direction of motion.
- Width and height (perpendicular to motion) remain unchanged.

## Example: I Do - Time Dilation

**Problem:** A spaceship travels at  $0.95c$  relative to Earth. An astronaut on board measures a trip to take 2.0 years. How long does the trip take according to Mission Control on Earth?

# I Do: Time Dilation - G & U

## G - Givens

- $v = 0.95c$
- $\Delta t_0 = 2.0$  years (Proper time, measured on ship)
- Frame: Earth (stationary relative to motion)

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- $\Delta t_0 = 2.0$  years (Proper time, measured on ship)
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## U - Unknown

- $\Delta t = ?$  (Dilated time on Earth)

# I Do: Time Dilation - Equation

## E - Equation

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- First, calculate  $\gamma$ :

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- Then use time dilation:

$$\Delta t = \gamma \Delta t_0$$

# I Do: Time Dilation - Substitute & Solve

## S - Substitute

- $\gamma = \frac{1}{\sqrt{1-0.95^2}} = \frac{1}{\sqrt{1-0.9025}} = \frac{1}{\sqrt{0.0975}}$
- $\gamma \approx 3.20$
- $\Delta t = (3.20)(2.0 \text{ years})$

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## S - Solve

- $\Delta t = 6.4 \text{ years}$
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- *Earth observers wait longer than the astronaut ages.*

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- 2 Who measures  $L$ ?
- 3 Will the answer be less than or greater than 100 m?

# You Do: Practice

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- 1 Calculate the Lorentz factor  $\gamma$ .
- 2 How long does the muon live as measured by a scientist in the lab?
- 3 Hint: Expect a longer time ( $\Delta t > \Delta t_0$ ).

# Reading Homework

Before the next lecture on Part 2, please read:

- Section 28.4: Relativistic Addition of Velocities
- Section 28.5: Relativistic Momentum
- Section 28.6: Relativistic Energy

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- $\gamma$  becomes significant only as  $v \rightarrow c$ .