PHYS12 CH:2.1-2.8 and 3.1-3.5

Kinematics in One and Two Dimensions

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Learning Objectives

1D Kinematics:

- Define position, displacement, distance, velocity, speed, and acceleration.
- Distinguish between scalar and vector quantities.
- Interpret graphs of position, velocity, and acceleration vs. time.
- Use kinematic equations to solve problems for objects with constant acceleration.
- Describe the motion of objects in free fall.

2D Kinematics:

- Understand the independence of horizontal and vertical motions.
- Add and subtract vectors graphically and analytically.
- Resolve vectors into perpendicular components.
- Apply kinematic equations to solve projectile motion problems.
- Use vector addition to solve relative velocity problems.

From Physics 11 to Physics 12

Building on Foundations

This course builds directly upon the concepts you learned in Physics 11. We'll quickly review Chapter 2 (1D kinematics) and extend to 2D motion.

From Chapter 2 (1D)

New in 2D

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- Scalars vs. vectors (+/- directions)

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- Vector components and trigonometry
- Independence of horizontal/vertical motion

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- Scalars vs. vectors (+/- directions)
- Kinematic equations for constant acceleration

New in 2D

- Vector components and trigonometry
- Independence of horizontal/vertical motion
- Projectile motion analysis

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From Chapter 2 (1D)

- Position, displacement, distance
- Scalars vs. vectors (+/- directions)
- Kinematic equations for constant acceleration
- Free fall and graphical analysis

New in 2D

- Vector components and trigonometry
- Independence of horizontal/vertical motion
- Projectile motion analysis
- Relative velocity problems

Reference to Chapter 2: 1D Kinematics

Foundation for 2D Motion

Quick Reference

For detailed explanations of 1D kinematics concepts, see **Chapter 2 slides**:

- Position, displacement, and distance
- Scalars vs. vectors with detailed examples
- Full derivation of kinematic equations
- Free fall motion and analysis
- Graphical analysis with worked examples
- GUESS method for problem solving

This Chapter

We'll focus on extending these concepts to 2D motion using vector components!

1D Kinematics: Essential Review

Building on Chapter 2 Foundations

Core Concepts (From Chapter 2)

- **Position vs. Displacement**: $\Delta x = x_f - x_0$ (vector)
- **Distance**: Total path length (scalar, always positive)
- **Velocity**: $\bar{v} = \Delta x / \Delta t$ (vector)
- **Speed**: Distance/time (scalar)
- **Acceleration**: $\bar{a} = \Delta v / \Delta t$ (vector)

Key 2D Extension: In 1D, direction was simple (+ or -). In 2D, we need full vector mathematics!

Vectors in 1D vs. 2D: Quick Review

From Simple Signs to Full Components

1D Motion (Chapter 2)

- Direction: + or - sign
- Example: $v = +5$ m/s or $v = -3$ m/s
- Vector addition: Simple algebra

2D Motion (This Chapter)

- Direction: Angle and magnitude
- Example: $\vec{v} = 5$ m/s at 30°
- Vector addition: Components needed

Why This Matters

In 2D, we can't just use + and - to represent all possible directions. We need trigonometry and vector components!

Acceleration: Quick Review

The Bridge to 2D Motion

From Chapter 2

- **Acceleration**: Rate of velocity change, $\bar{a} = \Delta v / \Delta t$
- **Key Insight**: In 2D, acceleration can change speed, direction, or both!
- **Free Fall**: $a = -g = -9.80 \text{ m/s}^2$ (constant downward)

2D Challenge

How do we handle acceleration that's not aligned with our motion direction? This is where vector components become essential!

The Four Equations (From Chapter 2)

For constant acceleration only:

$$v_f = v_0 + at$$
$$\Delta x = \frac{1}{2}(v_0 + v_f)t$$
$$\Delta x = v_0 t + \frac{1}{2}at^2$$
$$v_f^2 = v_0^2 + 2a\Delta x$$

2D Strategy

We'll apply these equations **separately** to x and y components!

Free Fall: Essential Concepts

Building on Chapter 2 for Projectile Motion

Key Review (From Chapter 2)

- **Free Fall**: Motion under gravity only ($a = -g = -9.80$ m/s²)
- **Sign Convention**: Up = positive, so $a_y = -9.80$ m/s²
- **Key Insight**: All objects fall at same rate (neglecting air resistance)

2D Application

In projectile motion, only the **vertical component** follows free fall. The **horizontal component** has no acceleration!

Free Fall Graphs: Quick Review

Essential Patterns for Projectile Motion

Position vs. Time

- Parabolic shape
- Slope = velocity

Velocity vs. Time

- Straight line
- Slope $= -g$
- Area $= \Delta y$

Acceleration vs. Time

- Constant: $-g$
- Always downward

2D Connection

These same patterns apply to the **vertical component** of projectile motion!

Motion Graphs: Key Relationships

Quick Review from Chapter 2

Position-Time Graph

- Slope = velocity
- Curved = acceleration
- Steeper = faster

Velocity-Time Graph

- Slope = acceleration
- Area = displacement
- Horizontal = constant velocity

2D Application

We'll analyze x and y components separately using these same principles!

Problem Solving: GUESS Method Review

From Chapter 2 to 2D Applications

The GUESS Method (Chapter 2)

- **G** - Givens: List known quantities, define coordinate system
- **U** - Unknown: Identify what to find
- **E** - Equation: Choose appropriate kinematic equation
- **S** - Substitute: Plug in values with units
- **S** - Solve: Calculate and check units/significant figures

2D Extension

For 2D problems: Apply GUESS **separately** to x and y components!

2D Motion: The Independence of Motion

The Most Important Concept in 2D Kinematics

The horizontal and vertical components of two-dimensional motion are **independent** of each other.

- Motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.
- This allows us to break complex 2D problems into two simpler 1D problems: one for the x-direction and one for the y-direction.
- The only variable that connects the two separate motions is time (t).

Concept Visualization: Independence of Motion (Context)

Scenario: The Two-Ball Drop

Imagine two identical balls at the same height.

- Ball 1 is dropped straight down.
- Ball 2 is launched horizontally at the exact same moment.

Question: Which ball hits the ground first?

Let's visualize their motion. The result demonstrates the independence of vertical and horizontal motion.

Concept Visualization: Independence of Motion

[Diagram based on Figure 3.6]

A composite image showing the motion of two balls.

- The red ball is dropped vertically from rest.
- The blue ball is projected horizontally with an initial velocity.
- Strobe flashes at equal time intervals show that both balls have the same vertical position at any given moment.
- This demonstrates that the horizontal motion of the blue ball does not affect its vertical motion due to gravity. They hit the ground at the same time.

Vector Addition: Head-to-Tail Method

Graphical Method for Adding Vectors

To add vectors graphically, we draw them one after another.

1. Draw the first vector to scale and in the correct direction.
2. Draw the second vector starting from the head (tip) of the first vector.
3. Continue for all vectors.
4. The **resultant vector** ($\vec{R}$) is the vector drawn from the tail (start) of the first vector to the head of the last vector.

[Diagram illustrating the head-to-tail method for adding vectors $\vec{A}$ and $\vec{B}$, showing the resultant vector $\vec{R}$. Based on Figure 3.10]

Analytical Method: Vector Components

Using Trigonometry for Precision

Any 2D vector can be broken down into two perpendicular components. We typically use x and y axes.

- For a vector $\vec{A}$ with magnitude A and at an angle θ (measured from the positive x-axis):
  - The x-component is $A_x = A\cos\theta$
  - The y-component is $A_y = A\sin\theta$
- This process is called **resolving the vector**.
- To add vectors $\vec{A}$ and $\vec{B}$ to get $\vec{R}$:
  - Add the x-components: $R_x = A_x + B_x$
  - Add the y-components: $R_y = A_y + B_y$
- Then, find the magnitude and direction of $\vec{R}$ using its components.

Concept Visualization: Vector Components (Context)

Breaking a Vector Apart

Let's visualize how a single vector $\vec{A}$ can be represented as the sum of its perpendicular components, $\vec{A}_x$ and $\vec{A}_y$.
This is the reverse of adding vectors and is a crucial first step for solving almost any 2D physics problem.

Concept Visualization: Vector Components

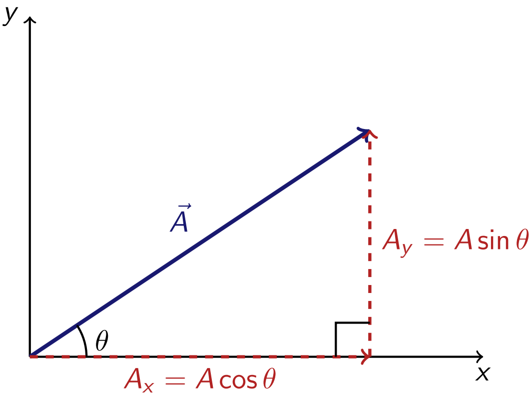

Figure: The vector $\vec{A}$ is the vector sum of its components: $\vec{A} = \vec{A}_x + \vec{A}_y$.

Key Concepts: Projectile Motion

Applying 2D Kinematics

A **projectile** is any object that is thrown or launched and then moves subject only to gravity.

Analysis Steps:

1. Set up a coordinate system (usually origin at launch, +y is up).
2. Resolve the initial velocity (v_0) into components:
   - $v_{0x} = v_0 \cos\theta_0$
   - $v_{0y} = v_0 \sin\theta_0$
3. Treat as two independent 1D motion problems:
   - **Horizontal (x):** Constant velocity ($a_x = 0$)
   - **Vertical (y):** Constant acceleration ($a_y = -g$)
4. Use the kinematic equations for each direction. Time (t) is the same for both.

Concept Visualization: Projectile Trajectory (Context)

Scenario: The Path of a Cannonball

Let's trace the path of a projectile, paying close attention to its velocity vector.

- How do the horizontal (v_x) and vertical (v_y) components of velocity change during the flight?
- What is the velocity at the highest point (the apex) of the trajectory?

This visualization is key to understanding why we separate the motion into two parts.

Concept Visualization: Projectile Trajectory

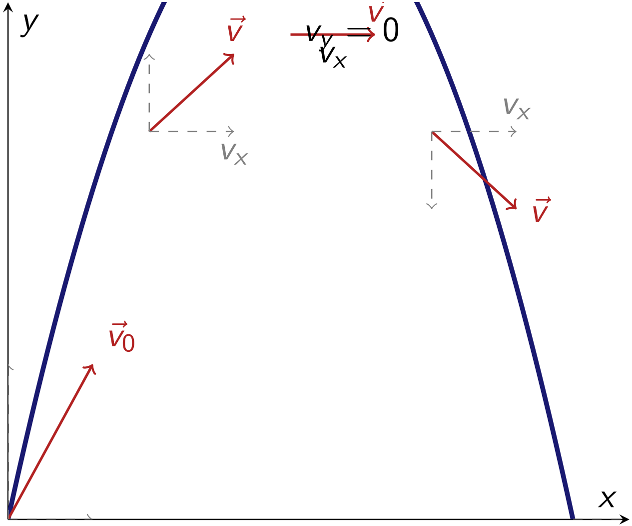

Figure: v_x is constant. v_y decreases, becomes zero at the apex, and then increases in the negative direction.

Key Concepts: Relative Velocity

Motion Depends on the Observer

Velocity is always measured *relative* to a frame of reference.

- The velocity of an object can have different values when measured by different observers.
- We use vector addition to find the velocity of an object relative to a stationary observer (e.g., the ground).
- **Subscript Notation** is very helpful:
  - $\vec{v}_{PG}$ = Velocity of the **P**lane relative to the **G**round.
  - $\vec{v}_{PA}$ = Velocity of the **P**lane relative to the **A**ir.
  - $\vec{v}_{AG}$ = Velocity of the **A**ir relative to the **G**round (i.e., the wind).
- **Relative Velocity Equation**: $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$

Concept Visualization: Boat in a River (Context)

Scenario: Crossing a Current

A boat tries to travel straight across a river. However, the river's current carries the boat downstream.

- $\vec{v}_{bw}$: Velocity of the **b**oat relative to the **w**ater.
- $\vec{v}_{wg}$: Velocity of the **w**ater relative to the **g**round (the current).
- $\vec{v}_{bg}$: Velocity of the **b**oat relative to the **g**round (its actual path).

The boat's actual velocity is the vector sum of its velocity in the water and the water's velocity.

Concept Visualization: Boat in a River

[Diagram based on Figure 3.40]

A diagram showing a river with a current flowing to the right.

- A vector labeled $\vec{v}_{bw}$ points straight across the river.
- A vector labeled $\vec{v}_{wg}$ points downstream, parallel to the banks.
- The resultant vector $\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg}$ points diagonally downstream, showing the boat's true path relative to the ground.

I do: Fireworks Projectile (Ex. 3.4)

Problem

A firework is shot with an initial speed of 70.0 m/s at an angle of 75.0° above the horizontal. (a) Calculate the height at which it explodes (its apex). (b) How much time passes until it explodes?

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```
G - Givens:
  v_0 = 70.0 m/s,  theta_0 = 75.0 deg
  y_0 = 0 m, x_0 = 0 m
  a_y = -9.80 m/s^2, a_x = 0 m/s^2
  At apex, v_fy = 0 m/s

U - Unknowns:  (a) Max height y_f (or h), (b) Time to apex t

E - Equations:
  (1) Resolve v_0: v_0y = v_0*sin(theta_0)
  (2) For height (a): v_fy^2 = v_0y^2 + 2*a_y*Delta_y
  (3) For time (b): v_fy = v_0y + a_y*t

S - Substitute & Solve:
```

We do: Hot Rock Projectile (Ex. 3.5)

Problem

A rock is ejected from a volcano with speed 25.0 m/s at 35.0° above the horizontal. It strikes the side of the volcano 20.0 m *lower* than its starting point. (a) Calculate the time it takes.

G - Givens:

- $v_0 = 25.0$ m/s, $\theta_0 = 35.0^\circ$
- $y_0 = 0$ m, so $\Delta y = y_f - y_0 = -20.0$ m
- $a_y = -9.80$ m/s^2

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U - Unknown: Time of flight, t. **E - Equation:** First, find initial vertical velocity: $v_{0y} = v_0 \sin\theta_0 = (25.0)\sin(35.0^\circ) = 14.3$ m/s.
Now, which y-direction kinematic equation involves Δy, v_{0y}, a_y, and t?

$$\Delta y = v_{0y} t + \frac{1}{2} a_y t^2$$

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S - Substitute:

$$-20.0 = (14.3)t + \frac{1}{2}(-9.80)t^2 \implies 4.90t^2 - 14.3t - 20.0 = 0$$

S - Solve: How do we solve this for t?

You do: Projectile Launch

Problem (Ch.3, Q.25)

A projectile is launched at ground level with an initial speed of 50.0 m/s at an angle of 30.0° above the horizontal. It strikes a target 3.00 seconds later.

1. What is the horizontal distance (x) to the target?
2. What is the vertical distance (y) to the target?

Use the GUESS method. Remember to break the problem into x and y components.

Hint

First, resolve the initial velocity into v_{0x} and v_{0y}. Then, solve the horizontal and vertical problems separately using $t = 3.00$ s.

Reading Homework

Practice and Deeper Understanding

To solidify your understanding, please work through the following sections in your textbook:

- **Chapter 2: 1D Kinematics**
  - Conceptual Questions (Page 73)
  - Problems & Exercises (Page 82)
- **Chapter 3: 2D Kinematics**
  - Conceptual Questions (Page 156)
  - Problems & Exercises (Page 163)

Summary of Key Concepts

- **1D Motion**: We describe motion using scalars (distance, speed) and vectors (displacement, velocity, acceleration). The kinematic equations are our primary tool for solving problems with constant acceleration.
- **Graphical Analysis**: The slope and area of motion graphs have physical meaning. (Slope of x-t is v, slope of v-t is a, area of v-t is Δx).
- **2D Motion**: The key is the independence of motion. We break 2D problems into two 1D problems (horizontal and vertical) connected by time.
- **Projectile Motion**: A classic case of 2D motion where $a_x = 0$ (constant velocity) and $a_y = -g$ (constant acceleration).
- **Relative Velocity**: All velocities are relative to a reference frame. We use vector addition to find resultant velocities.