3.1 Acceleration

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain acceleration and determine the direction and magnitude of acceleration in one dimension
- Analyze motion in one dimension using kinematic equations and graphic representations

Teacher Support

Teacher Support   The Learning Objectives in this section will help your students master the following standards:

- (4) Science concepts. The student knows and applies the laws governing motion in a variety of situations. The student is expected to:
    - (A) generate and interpret graphs and charts describing different types of motion, including the use of real-time technology such as motion detectors or photogates;
    - (B) describe and analyze motion in one dimension using equations with the concepts of distance, displacement, speed, average velocity, instantaneous velocity, and acceleration.

In addition, the High School Physics Laboratory Manual addresses content in this section in the lab titled: *Position and Speed of an Object*, as well as the following standards:

- (4) Science concepts. The student knows and applies the laws governing motion in a variety of situations. The student is expected to:
    - (B) describe and analyze motion in one dimension using equations with the concepts of distance, displacement, speed, average velocity, instantaneous velocity, and acceleration.

Section Key Terms

Teacher Support

Teacher Support   [BL][OL] Begin a general discussion about acceleration and deceleration. Ask for examples of both. Explain that deceleration is not used in physics because acceleration is either positive or negative. Lead students to their topics of interest, such as motor vehicles or sports. Explain that the capital Greek letter delta always means *final minus initial* and that the net change may be zero, positive, or negative.

[AL] See how much students remember about vectors. What does a vector arrow represent? Ask them to name some quantities that are vectors and some that are scalars.

Defining Acceleration

Throughout this chapter we will use the following terms: *time*, *displacement*, *velocity*, and *acceleration*. Recall that each of these terms has a designated variable and SI unit of measurement as follows:

- Time: t, measured in seconds (s)
- Displacement: Δd, measured in meters (m)
- Velocity: v, measured in meters per second (m/s)
- Acceleration: a, measured in meters per second per second (m/s^2, also called meters per second squared)
- Also note the following:
  - Δ means *change in*
  - The subscript 0 refers to an initial value; sometimes subscript i is instead used to refer to initial value.
  - The subscript f refers to final value
  - A bar over a symbol, such as $\bar{a}$, means *average*

Teacher Support

Teacher Support [BL] Review definitions of the terms: *time*, *displacement*, *velocity*, and *acceleration*. Point out that the variables commonly used to represent these quantities are the first letters of the corresponding term.

[OL] Verify that the students know the SI units in which time, displacement, velocity, and acceleration are expressed. Note that these are some of the seven *base units* of the metric system. Explain that converting to base units is a good first step when calculating these quantities. Explain the meaning of *seconds squared* in the denominator of the units of acceleration.

[AL] Review all the base units of the metric system. Explain how these units are interrelated. For example, show how length is defined by time.

[BL][OL] Use the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$ to emphasize the relationship between Δ and the subscripts f and 0. Distinguish between constant and variable acceleration. There could be confusion here, especially in the case of increasing acceleration. Be sure students understand that the word *deceleration* is not used in physics and that *acceleration* may be either positive or negative.

[AL] See if students can use the concept of acceleration to understand confusing statements such as "a decrease in the rate of increase." For example, use the concept of acceleration to analyze the statement "the rate of increase in the cost of health care is decreasing." If the increase in the cost is defined as positive, then the *acceleration* in the cost of health care would be negative.

[OL] The arrow for acceleration that points opposite to the arrow for velocity may be confusing. Explain that the acceleration arrow points in the direction opposite the velocity because the velocity is getting smaller, i.e., the velocity arrow is getting shorter.

Acceleration is the change in velocity divided by a period of time during which the change occurs. The SI units of velocity are m/s and the SI units for time are s, so the SI units for acceleration are m/s^2. Average acceleration is given by

$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Average acceleration is distinguished from instantaneous acceleration, which is acceleration at a specific instant in time. The magnitude of acceleration is often not constant over time. For example, runners in a race accelerate at a greater rate in the first second of a race than during the following seconds. You do not need to know all the instantaneous accelerations at all times to calculate average acceleration. All you need to know is the change in velocity (i.e., the final velocity minus the initial velocity) and the change in time (i.e., the final time minus the initial time), as shown in the formula. Note that the average acceleration can be positive, negative, or zero. A negative acceleration is simply an acceleration in the negative direction.

Keep in mind that although acceleration points in the same direction as the *change* in velocity, it is not always in the direction of the velocity itself. When an object slows down, its acceleration is opposite to the direction of its velocity. In everyday language, this is called deceleration; but in physics, it is acceleration—whose direction happens to be opposite that of the velocity. For now, let us assume that motion to the right along the *x*-axis is *positive* and motion to the left is *negative.*

Figure 3.2 shows a car with positive acceleration in (a) and negative acceleration in (b). The arrows represent vectors showing both direction and magnitude of velocity and acceleration.
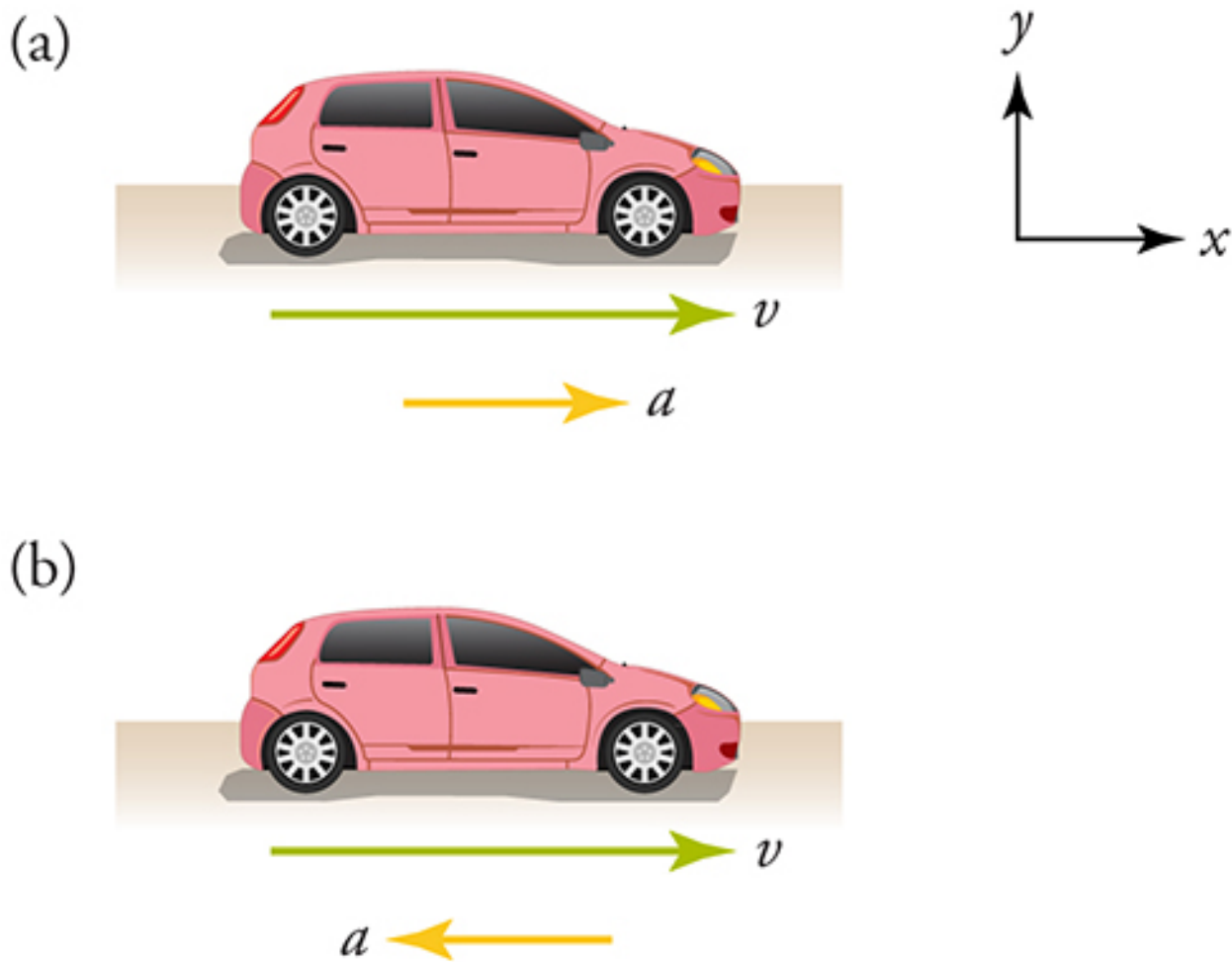
(a)

v

a

y

x

(b)

v

a

Figure 3.2 The car is speeding up in (a) and slowing down in (b).

Velocity and acceleration are both vector quantities. Recall that vectors have both magnitude and direction. An object traveling at a constant velocity—therefore having no acceleration—does accelerate if it changes direction. So, turning the steering wheel of a moving car makes the car accelerate because the velocity changes direction.

Virtual Physics

The Moving Man   With this animation, you can produce both variations of acceleration and velocity shown in Figure 3.2, plus a few more variations. Vary the velocity and acceleration by sliding the red and green markers along the scales. Keeping the velocity marker near zero will make the effect of acceleration

more obvious. Try changing acceleration from positive to negative while the man is moving. We will come back to this animation and look at the *Charts* view when we study graphical representation of motion.
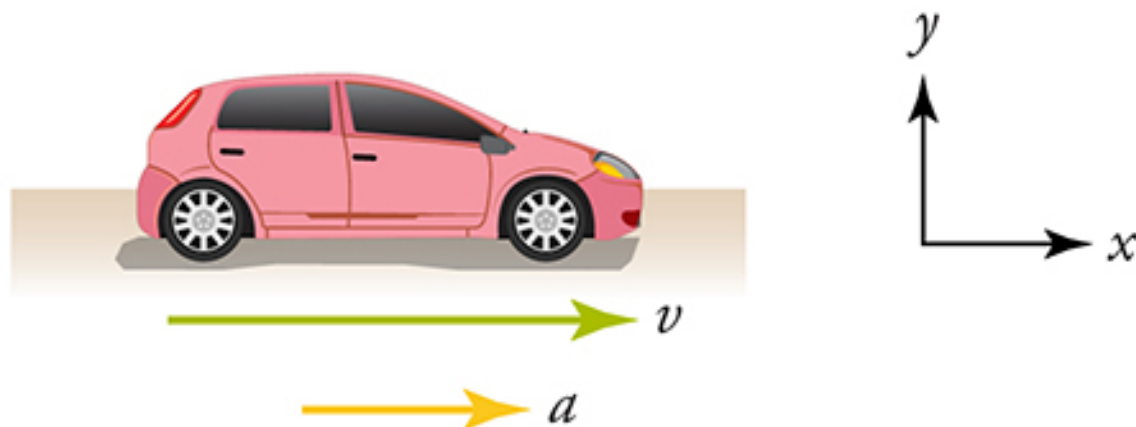
Click to view content

Teacher Support

Teacher Support   Have students use a very low setting for velocity and acceleration because it is easier to see how the motion changes. Show students how setting velocity as positive and acceleration as negative creates the motion that resembles that of an object thrown into the air.

Grasp Check

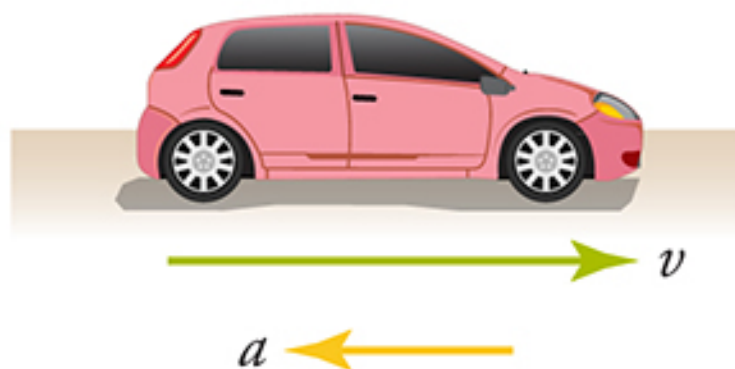(a)

v

a

y

x

(b)

v

a

Figure 3.3

Which part, (a) or (b), is represented when the velocity vector is on the positive side of the scale and the acceleration vector is set on the negative side of the scale? What does the car's motion look like for the given scenario?

a. Part (a). The car is slowing down because the acceleration and the velocity vectors are acting in the opposite direction.
b. Part (a). The car is speeding up because the acceleration and the velocity vectors are acting in the same direction.
c. Part (b). The car is slowing down because the acceleration and velocity vectors are acting in the opposite directions.
d. Part (b). The car is speeding up because the acceleration and the velocity vectors are acting in the same direction.

Calculating Average Acceleration

Look back at the equation for average acceleration. You can see that the calculation of average acceleration involves three values: change in time, (Δt); change in velocity, (Δv); and acceleration (a).

Change in time is often stated as a time interval, and change in velocity can often be calculated by subtracting the initial velocity from the final velocity. Average acceleration is then simply change in velocity divided by change in time. Before you begin calculating, be sure that all distances and times have been converted to meters and seconds. Look at these examples of acceleration of a subway train.

Teacher Support

Teacher Support [BL][OL] Before beginning the calculations, verify that students understand the equation for acceleration. Do they understand what it means when quantities have a plus or minus sign? Do they understand the units for each variable?

Worked Example

An Accelerating Subway Train A subway train accelerates from rest to 30.0 km/h in 20.0 s. What is the average acceleration during that time interval?

Strategy
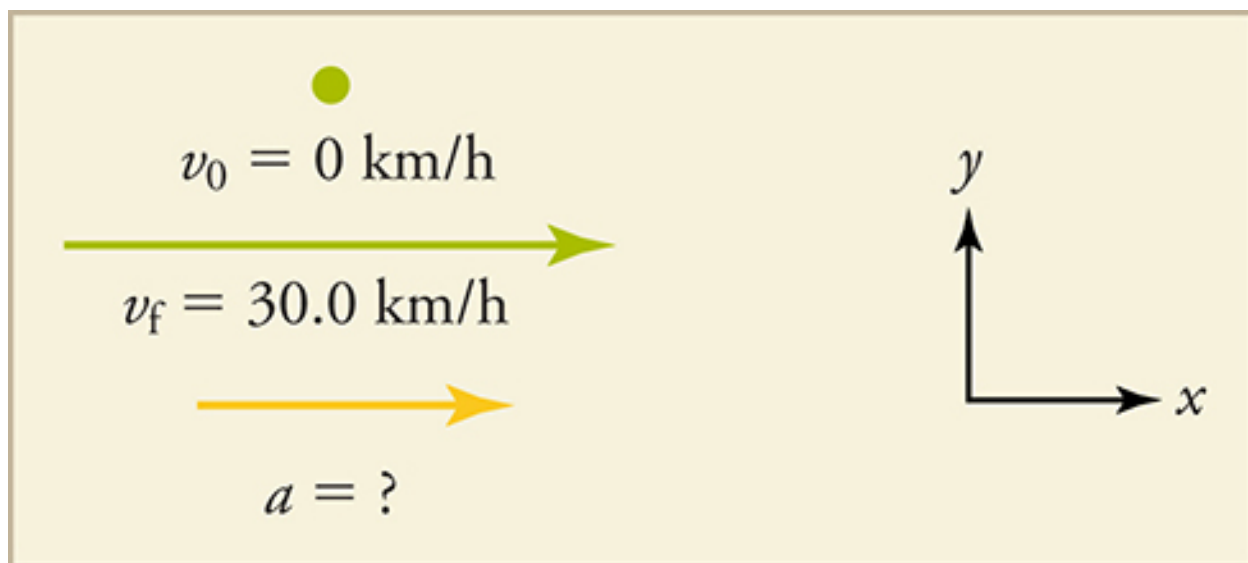
Start by making a simple sketch.

Figure 3.4

This problem involves four steps:

1. Convert to units of meters and seconds.
2. Determine the change in velocity.
3. Determine the change in time.
4. Use these values to calculate the average acceleration.

Solution

1. Identify the knowns. Be sure to read the problem for given information, which may not *look* like numbers. When the problem states that the train starts from rest, you can write down that the initial velocity is 0 m/s. Therefore, $v_0 = 0$; $v_f = 30.0$ km/h; and $\Delta t = 20.0$ s.

2. Convert the units.

- $\frac{30.0 \text{ km}}{\text{h}} \times \frac{10^3 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 8.333 \frac{\text{m}}{\text{s}}$

   3.1

3. Calculate change in velocity, $\Delta v = v_f - v_0 = 8.333$ m/s $- 0 = + 8.333$ m/s, where the plus sign means the change in velocity is to the right.

4. We know Δt, so all we have to do is insert the known values into the formula for average acceleration.

- $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{8.333 \text{ m/s}}{20.00 \text{ s}} = +0.417 \frac{\text{m}}{\text{s}^2}$

   3.2

Discussion

The plus sign in the answer means that acceleration is to the right. This is a reasonable conclusion because the train starts from rest and ends up with a velocity directed to the right (i.e., positive). So, acceleration is in the same direction as the *change* in velocity, as it should be.

Teacher Support

Teacher Support   Note that extra digits were carried along and rounding off to the correct number of significant figures, 3, was not done until the final answer was calculated.

Worked Example

An Accelerating Subway Train   Now, suppose that at the end of its trip, the train slows to a stop in 8.00 s from a speed of 30.0 km/h. What is its average acceleration during this time?

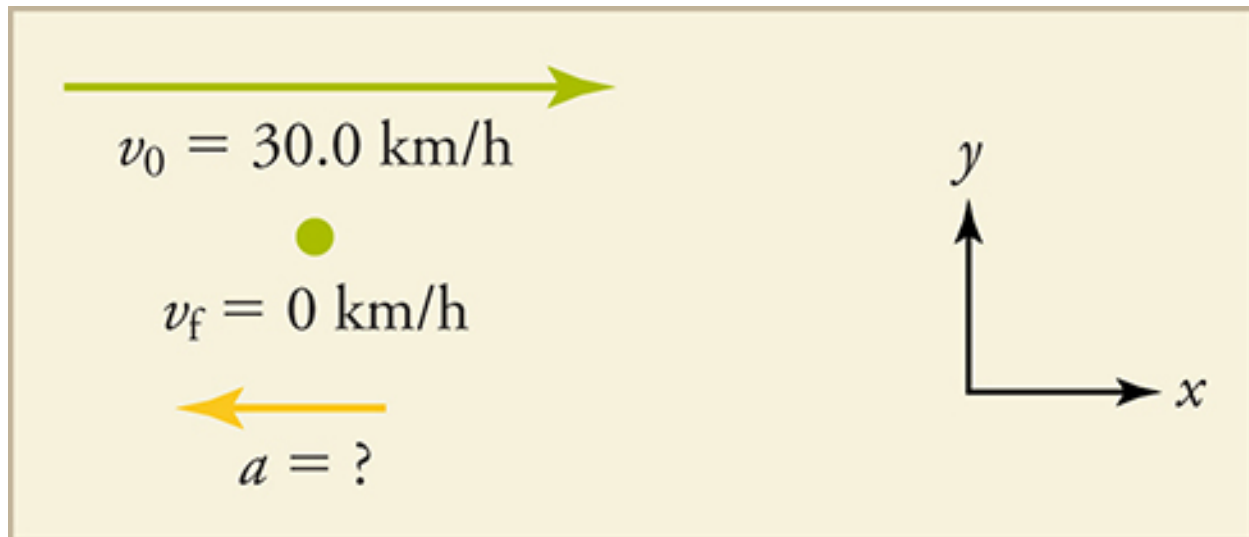**Strategy**

Again, make a simple sketch.

Figure 3.5

In this case, the train is decelerating and its acceleration is negative because it is pointing to the left. As in the previous example, we must find the change in velocity and change in time, then solve for acceleration.

Solution

1. Identify the knowns: $v_0 = 30.0$ km/h; $v_f = 0$; and $\Delta t = 8.00$ s.

2. Convert the units. From the first problem, we know that 30.0 km/h $= 8.333$ m/s.

3. Calculate change in velocity, $\Delta v = v_f - v_0 = 0 - 8.333$ m/s $= -8.333$ m/s, where the minus sign means that the change in velocity points to the left.

4. We know $\Delta t = 8.00$ s, so all we have to do is insert the known values into the equation for average acceleration.

- $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-8.333 \text{ m/s}}{8.00 \text{ s}} = -1.04\frac{\text{m}}{\text{s}^2}$

   3.3

Discussion

The minus sign indicates that acceleration is to the left. This is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would reduce the velocity. Again, acceleration is in the same direction as the *change* in velocity, which is negative in this case. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

Teacher Support

Teacher Support   Help students see the relationship between the direction of the vector arrows and the plus and minus signs. Explain that one indication of the sign for acceleration is that it is in the direction opposite that of the velocity. Also point out that correctly identifying the initial and final speeds will result in the correct sign for acceleration.

Tips For Success

- It is easier to get plus and minus signs correct if you always assume that motion is away from zero and toward positive values on the x-axis. This way v always starts off being positive and points to the right. If speed is increasing, then acceleration is positive and also points to the right. If speed is decreasing, then acceleration is negative and points to the left.
- It is a good idea to carry two extra significant figures from step-to-step when making calculations. Do not round off with each step. When you arrive at the final answer, apply the rules of significant figures for the operations you carried out and round to the correct number of digits. Sometimes this will make your answer slightly more accurate.

Practice Problems

1.

A cheetah can accelerate from rest to a speed of $30.0\,\text{m/s}$ in $7.00\,\text{s}$. What is its acceleration?

 a. $\{-0.23\}\,\text{m/s}^2$
 b. $\{-4.29\}\,\text{m/s}^2$
 c. $0.23\,\text{m/s}^2$
 d. $4.29\,\text{m/s}^2$

2.

A women backs her car out of her garage with an acceleration of $1.40\,\text{m/s}^2\!$. How long does it take her to reach a speed of $2.00\,\text{m/s}$?

 a. $0.70\,\text{s}$
 b. $1.43\,\text{s}$
 c. $2.80\,\text{s}$
 d. $3.40\,\text{s}$

Watch Physics

Acceleration This video shows the basic calculation of acceleration and some useful unit conversions.

Click to view content

Teacher Support

Teacher Support Ask students to note the explanation of units and the identification of the vector quantities. Tell them the calculations demonstrated in the video are fairly straightforward and that the definitions given for displacement, elapsed time, velocity, and acceleration should be clear.

Why is acceleration a vector quantity?

 a. It is a vector quantity because it has magnitude as well as direction.
 b. It is a vector quantity because it has magnitude but no direction.
 c. It is a vector quantity because it is calculated from distance and time.
 d. It is a vector quantity because it is calculated from speed and time.

What will be the change in velocity each second if acceleration is 10 m/s/s?

 a. An acceleration of $10\,\text{m/s/s}$ means that every second, the velocity increases by $10\,\text{m/s}$.
 b. An acceleration of $10\,\text{m/s/s}$ means that every second, the velocity decreases by $10\,\text{m/s}$.
 c. An acceleration of $10\,\text{m/s/s}$ means that every $10\,\text{seconds}$, the velocity increases by $10\,\text{m/s}$.
 d. An acceleration of $10\,\text{m/s/s}$ means that every $10\,\text{seconds}$, the velocity decreases by $10\,\text{m/s}$.

Snap Lab

Measure the Acceleration of a Bicycle on a Slope   In this lab you will take measurements to determine if the acceleration of a moving bicycle is constant. If the acceleration is constant, then the following relationships hold:
$\bar{v} = \frac{\Delta d}{\Delta t} = \frac{v_0 + v_f}{2}$ If $v_0 = 0$, then $v_f = 2\bar{v}$ and $\bar{a} = \frac{v_f}{\Delta t}$

You will work in pairs to measure and record data for a bicycle coasting down an incline on a smooth, gentle slope. The data will consist of distances traveled and elapsed times.

- Find an open area to minimize the risk of injury during this lab.

- stopwatch
- measuring tape
- bicycle

1. Find a gentle, paved slope, such as an incline on a bike path. The more gentle the slope, the more accurate your data will likely be.
2. Mark uniform distances along the slope, such as 5 m, 10 m, etc.
3. Determine the following roles: the bike rider, the timer, and the recorder. The recorder should create a data table to collect the distance and time data.
4. Have the rider at the starting point at rest on the bike. When the timer calls *Start*, the timer starts the stopwatch and the rider begins coasting down the slope on the bike without pedaling.
5. Have the timer call out the elapsed times as the bike passes each marked point. The recorder should record the times in the data table. It may be necessary to repeat the process to practice roles and make necessary adjustments.
6. Once acceptable data has been recorded, switch roles. Repeat Steps 3–5 to collect a second set of data.
7. Switch roles again to collect a third set of data.
8. Calculate average acceleration for each set of distance-time data. If your result for $\bar{a}$ is not the same for different pairs of Δv and Δt, then acceleration is not constant.
9. Interpret your results.

Teacher Support

Teacher Support   Explain that two factors that could prevent uniform acceleration are (i) friction between the tires and the pavement and friction in the bicycle axles, and (ii) air resistance. Discuss methods for minimizing these factors—e.g., selecting a smoother surface for the bike to coast, greasing the axles, etc. Explain that friction will only decrease acceleration, but air resistance to a tail wind would increase acceleration. Discuss why it would be difficult to study constant acceleration if students were to pedal the bicycle. Note that the given kinematic equation that is valid for constant acceleration, which

is presented at the start of the Snap Lab, will be presented in further detail in the following section.

Prior to the lab, investigate appropriate areas around the school that have gentle, uniform slopes. Should the number of bicycles be limited, consider conducting the lab as a whole class or in larger clusters. Ensure that the planned paths of student groups do not cross and that there is adequate space for riders to stop without risk of injury.

Grasp Check

If you graph the average velocity (y-axis) vs. the elapsed time (x-axis), what would the graph look like if acceleration is uniform?

    a. a horizontal line on the graph
    b. a diagonal line on the graph
    c. an upward-facing parabola on the graph
    d. a downward-facing parabola on the graph

Check Your Understanding

Teacher Support

Teacher Support   Use these questions to assess student achievement of the section's Learning Objectives. If students are struggling with a specific objective, these questions will help identify any gaps and direct students to the relevant content.

3.

What are three ways an object can accelerate?

    a. By speeding up, maintaining constant velocity, or changing direction
    b. By speeding up, slowing down, or changing direction
    c. By maintaining constant velocity, slowing down, or changing direction
    d. By speeding up, slowing down, or maintaining constant velocity

4.

What is the difference between average acceleration and instantaneous acceleration?

    a. Average acceleration is the change in displacement divided by the elapsed time; instantaneous acceleration is the acceleration at a given point in time.
    b. Average acceleration is acceleration at a given point in time; instantaneous acceleration is the change in displacement divided by the elapsed time.
    c. Average acceleration is the change in velocity divided by the elapsed time; instantaneous acceleration is acceleration at a given point in time.
    d. Average acceleration is acceleration at a given point in time; instantaneous acceleration is the change in velocity divided by the elapsed time.

5.

What is the rate of change of velocity called?

    a. time
    b. displacement
    c. velocity
    d. acceleration