PHYS12 CH:6 The Art of Falling Forever

Circular Motion and Rotation

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Outline

The Mystery

How do you move forward
while constantly turning?

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From Formula 1 cars screaming around curves to the Moon circling Earth...

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How do you move forward
while constantly turning?

From Formula 1 cars screaming around curves to the Moon circling Earth...

All require a force toward the center.

Falling Forever

Figure: Formula 1 car in circular motion

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The Mental Model

A satellite in orbit is falling toward Earth but moving fast enough sideways to keep missing it.

Learning Objectives

By the end of this section, you will be able to:

- **6.1:** Describe the angle of rotation and relate it to its linear counterpart

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- **6.1:** Describe the angle of rotation and relate it to its linear counterpart
- **6.1:** Describe angular velocity and relate it to its linear counterpart
- **6.1:** Solve problems involving angle of rotation and angular velocity

6.1 Two Kinds of Rotation

Circular motion: Object moves in a circular path (race car on track)

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Spin: Object rotates about its own axis (Earth spinning)

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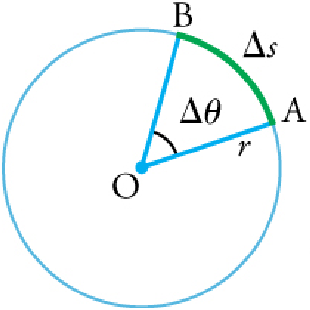
Real-World Examples

- Earth spins on its axis (spin) AND orbits the Sun (circular motion)
- Your car tire spins (spin) while the car follows a curve (circular motion)

6.1 Angle of Rotation



$$\Delta\theta = \frac{\Delta s}{r}$$

Figure: Arc length and radius

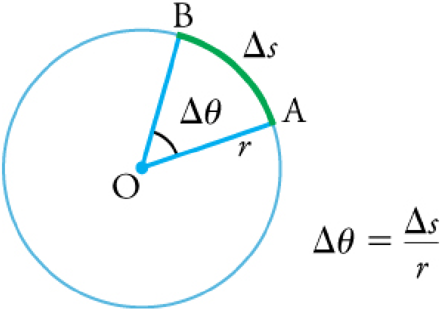


Figure: Arc length and radius

Universal Law: Angle of Rotation

$$\boxed{\Delta\theta = \frac{\Delta s}{r}}$$

Angle equals arc length divided by radius

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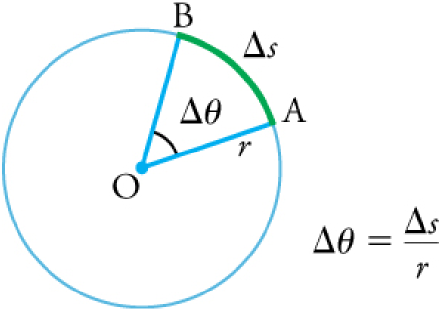

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Measured in **radians** (rad)

6.1 Radians vs Degrees

The Conversion

$$\boxed{1 \text{ revolution} = 2\pi \text{ rad} = 360^\circ}$$

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Why Radians?

Radians simplify equations in physics. Degrees are arbitrary - radians are natural.

6.1 Angular Velocity

Universal Law: Angular Velocity

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$$

Angular velocity equals change in angle divided by change in time

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Direction:

- Counterclockwise: positive (out of page toward you)
- Clockwise: negative (into page away from you)

6.1 Connecting Spinning to Moving

The Bridge Equation

$$\boxed{v = r\omega}$$

Tangential velocity equals radius times angular velocity

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Points farther from the center move faster linearly, but all points have the same angular velocity.

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Example: CD spinning - outer edge moves faster than inner part, but both complete one revolution in same time.

6.1 Why Car Tires Matter

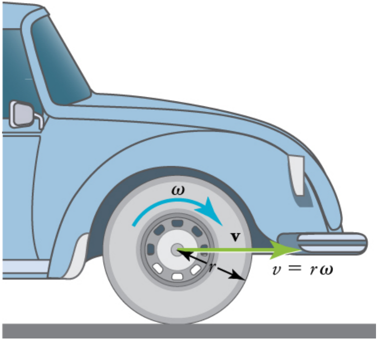

Figure: Car tire rolling

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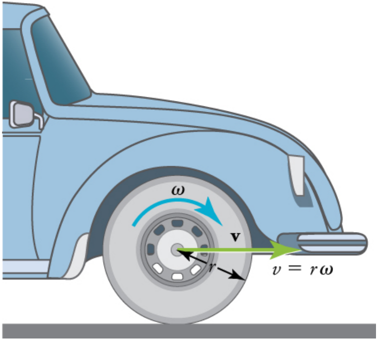

Figure: Car tire rolling

Large ω means large v because $v = r\omega$

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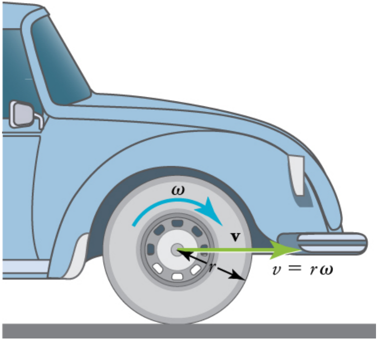

Figure: Car tire rolling

Large ω means large v because $v = r\omega$
Larger radius tire at same ω produces greater v

Attempt: Clock Tower Angle

The Challenge (3 min, silent)

A clock tower has a radius of 1.0 m. The hour hand moves from 12 p.m. to 3 p.m.

Given:

- Radius $r = 1.0$ m
- Time: 12 to 3 (quarter rotation)

Find:

1. Angle of rotation in radians
2. Arc length along outer edge

Can you decode this rotation? Work silently.

Compare: Clock Tower

Turn and talk (2 min):

1. What fraction of a full rotation does the hour hand make from 12 to 3?
2. How many radians in a full circle?
3. What equation connects arc length to angle?

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Name wheel: One pair share your approach (not your answer).

Self-correct in a different color:

Part (a): From 12 to 3 is $\frac{1}{4}$ of full rotation

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Full rotation $= 2\pi$ rad, so angle $= \frac{1}{4} \times 2\pi = \boxed{\dfrac{\pi}{2}\text{ rad}}$

Reveal: The Geometry of Time

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Part (b): Use $\Delta s = r\Delta\theta$

$$\Delta s = (1.0 \text{ m}) \left(\frac{\pi}{2} \text{ rad}\right) = \boxed{1.6 \text{ m}}$$

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Check: Arc length is less than circumference ($2\pi r \approx 6.3$ m). Reasonable!

Attempt: Spinning Car Tire

The Challenge (3 min, silent)

A car tire has radius 0.300 m and the car travels at 15.0 m/s (about 54 km/h).

Given:

- Radius $r = 0.300$ m
- Tangential velocity $v = 15.0$ m/s

Find: Angular velocity ω of the tire in rad/s

How fast is the tire spinning?

Compare: Tire Speed

Turn and talk (2 min):

1. What equation connects linear and angular velocity?
2. How did you rearrange it to solve for ω?
3. What are the units of your answer?

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Name wheel: One pair share your rearrangement strategy.

Reveal: The Spinning Wheel

Self-correct in a different color:

Equation: $v = r\omega$, so $\omega = \frac{v}{r}$

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Equation: $v = r\omega$, so $\omega = \frac{v}{r}$
Substitute:

$$\omega = \frac{15.0 \text{ m/s}}{0.300 \text{ m}}$$

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Equation: $v = r\omega$, so $\omega = \frac{v}{r}$
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$$\omega = \frac{15.0 \text{ m/s}}{0.300 \text{ m}}$$

$$\boxed{\omega = 50.0 \text{ rad/s}}$$

Reveal: The Spinning Wheel

Self-correct in a different color:

Equation: $v = r\omega$, so $\omega = \frac{v}{r}$
Substitute:

$$\omega = \frac{15.0\text{ m/s}}{0.300\text{ m}}$$

$$\boxed{\omega = 50.0\text{ rad/s}}$$

Check: About 8 revolutions per second (since 2π rad $= 1$ rev). Fast but reasonable for highway speed!

Learning Objectives

By the end of this section, you will be able to:

- **6.2:** Describe centripetal acceleration and relate it to linear acceleration

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Uniform circular motion: Object travels circular path at constant speed

6.2 The Paradox of Constant Speed

Uniform circular motion: Object travels circular path at constant speed

Civilian View vs. Reality

Civilian: "Constant speed means no acceleration."
Physicist: "Velocity is changing direction, so there IS acceleration."

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Uniform circular motion: Object travels circular path at constant speed

Civilian View vs. Reality

Civilian: "Constant speed means no acceleration."
Physicist: "Velocity is changing direction, so there IS acceleration."

Acceleration is a change in velocity - magnitude OR direction!

The Mental Model

When you turn in a car, you feel pushed outward. But no force pushes you out - your body wants to go straight (Newton's first law) while the car turns.

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The Fictional Force

Centrifugal force is not real - it's the illusion created by your inertia resisting the turn.

6.2 The Illusion of Being Flung

The Mental Model

When you turn in a car, you feel pushed outward. But no force pushes you out - your body wants to go straight (Newton's first law) while the car turns.

The Fictional Force

Centrifugal force is not real - it's the illusion created by your inertia resisting the turn.

The real force is **centripetal** - pulling you inward toward the center!

6.2 Centripetal Acceleration

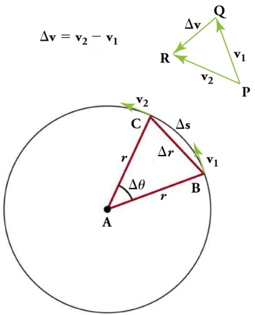

Figure: Velocity changes direction, acceleration points toward center

6.2 Centripetal Acceleration

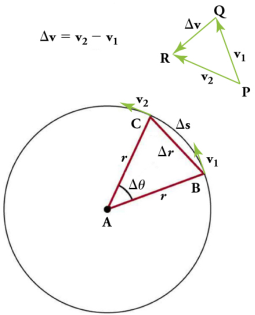

Figure: Velocity changes direction, acceleration points toward center

Universal Law: Centripetal Acceleration

$$\boxed{a_c = \frac{v^2}{r}} \quad \text{or} \quad \boxed{a_c = r\omega^2}$$

6.2 Why Speed Squared Matters

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Example:

- Curve at 50 km/h: moderate acceleration
- Same curve at 100 km/h: 4 times the acceleration

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Example:

- Curve at 50 km/h: moderate acceleration
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The Warning

This is why speed limits are lower on curves - small speed increase creates huge acceleration increase.

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For circular motion: $F_{\text{net}} = ma_c$

Universal Law: Centripetal Force

$$\boxed{F_c = m\frac{v^2}{r}} \quad \text{or} \quad \boxed{F_c = mr\omega^2}$$

Direction: Always toward the center of rotation

6.2 Sources of Centripetal Force

Centripetal force can be provided by:

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The Mental Model

Centripetal force isn't a new kind of force - it's whatever force points toward the center and causes circular motion.

Attempt: Car on Curve

The Challenge (3 min, silent)

A 900 kg car rounds a curve with radius 600 m at speed 25.0 m/s.

Given:

- Mass $m = 900$ kg
- Radius $r = 600$ m
- Speed $v = 25.0$ m/s

Find: Centripetal force required to keep car on curve

How much force do the tires provide?

Compare: Car Force

Turn and talk (2 min):

1. What equation did you use for centripetal force?
2. Did you remember to square the velocity?
3. What force provides the centripetal force for a car?

Compare: Car Force

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Name wheel: One pair share your equation and reasoning.

Reveal: The Force That Turns

Self-correct in a different color:

Equation: $F_c = m\frac{v^2}{r}$

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Self-correct in a different color:

Equation: $F_c = m\frac{v^2}{r}$
Substitute:

$$F_c = \frac{(900\ \text{kg})(25.0\ \text{m/s})^2}{600\ \text{m}}$$

Self-correct in a different color:

Equation: $F_c = m\frac{v^2}{r}$
Substitute:

$$F_c = \frac{(900 \text{ kg})(25.0 \text{ m/s})^2}{600 \text{ m}}$$

$$F_c = \frac{(900)(625)}{600} = \boxed{938 \text{ N}}$$

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$$F_c = \frac{(900)(625)}{600} = \boxed{938 \text{ N}}$$

Check: About 940 N - this is the friction force between tires and road. Without it, car slides straight!

Attempt: Acceleration Comparison

The Challenge (3 min, silent)

A car follows a curve of radius 500 m at speed 25.0 m/s.

Given:

- Radius $r = 500$ m
- Speed $v = 25.0$ m/s
- $g = 9.80$ m/s^2

Find:

1. Centripetal acceleration
2. Express as fraction of g

How does turning compare to falling?

Compare: Acceleration Scale

Turn and talk (2 min):

1. What equation did you use for centripetal acceleration?
2. How did you express it as a fraction of g?
3. Is the acceleration large or small compared to gravity?

Compare: Acceleration Scale

Turn and talk (2 min):

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Name wheel: One pair share your comparison method.

Reveal: Comparing to Gravity

Self-correct in a different color:

Calculate a_c:

$$a_c = \frac{v^2}{r} = \frac{(25.0 \text{ m/s})^2}{500 \text{ m}}$$

Reveal: Comparing to Gravity

Self-correct in a different color:

Calculate a_c:

$$a_c = \frac{v^2}{r} = \frac{(25.0\ \text{m/s})^2}{500\ \text{m}}$$

$$a_c = \frac{625}{500} = \boxed{1.25\ \text{m/s}^2}$$

Reveal: Comparing to Gravity

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$$a_c = \frac{v^2}{r} = \frac{(25.0 \text{ m/s})^2}{500 \text{ m}}$$

$$a_c = \frac{625}{500} = \boxed{1.25 \text{ m/s}^2}$$

Compare to g:

$$\frac{a_c}{g} = \frac{1.25}{9.80} = 0.128 \quad \Rightarrow \quad \boxed{a_c = 0.13g}$$

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Compare to g:

$$\frac{a_c}{g} = \frac{1.25}{9.80} = 0.128 \quad \Rightarrow \quad \boxed{a_c = 0.13g}$$

Revelation: Gentle highway curve at moderate speed produces about 1/10th the acceleration of gravity!

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- **6.3:** Describe torque and lever arm
- **6.3:** Solve problems involving torque and rotational kinematics

6.3 When Spin Changes

So far: constant angular velocity

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But what if spin changes?

- Figure skater pulls arms in - spins faster

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- Child pushes merry-go-round - starts rotating

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- CD player stops - disc slows to halt

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- CD player stops - disc slows to halt

Universal Law: Angular Acceleration

$$\boxed{\alpha = \frac{\Delta\omega}{\Delta t}}$$

Rate of change of angular velocity

6.3 Connecting Linear and Angular Acceleration

The Bridge

$$\boxed{a = r\alpha} \quad \text{or} \quad \boxed{\alpha = \frac{a}{r}}$$

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Tangential acceleration: Linear acceleration along the circle's edge

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The Bridge

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Tangential acceleration: Linear acceleration along the circle's edge

The Mental Model

Greater angular acceleration means greater tangential acceleration. Points farther from center have larger tangential acceleration for same α.

6.3 Rotational Kinematics Equations

Linear	Rotational
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2} at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

6.3 Rotational Kinematics Equations

Linear	**Rotational**
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2} a t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$
$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

The Pattern

Every linear kinematics equation has a rotational analog. Just swap $x \to \theta$, $v \to \omega$, $a \to \alpha$!

Force causes linear acceleration
What causes angular acceleration?

6.3 The Rotational Version of Force

Force causes linear acceleration

What causes angular acceleration?

Universal Law: Torque

$$\boxed{\tau = rF \sin\theta}$$

Torque equals lever arm times force times sine of angle

6.3 The Rotational Version of Force

Force causes linear acceleration
What causes angular acceleration?

Universal Law: Torque

$$\boxed{\tau = rF \sin\theta}$$

Torque equals lever arm times force times sine of angle

Units: N·m (Newton-meters)
Direction: Same as the angular acceleration it produces

6.3 Maximizing Torque

$$\tau = rF\sin\theta$$

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To maximize torque:

- Apply force far from pivot (large r)

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- Apply force perpendicular to lever arm ($\theta = 90^\circ$, so $\sin\theta = 1$)

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$$\tau = rF \sin\theta$$

To maximize torque:

- Apply force far from pivot (large r)
- Apply force perpendicular to lever arm ($\theta = 90^\circ$, so $\sin\theta = 1$)
- Apply larger force (large F)

Real-World Applications

- Door handle placed far from hinges
- Wrench with long handle
- Teeter-totter balanced by distance and weight

Attempt: Fishing Reel

The Challenge (3 min, silent)

A fishing reel spins at $\omega_0 = 220$ rad/s. Fisherman applies brake creating angular acceleration $\alpha = -300$ rad/s^2.

Given:

- Initial $\omega_0 = 220$ rad/s
- Final $\omega = 0$ (stops)
- $\alpha = -300$ rad/s^2

Find: Time t for reel to stop

How long does it take?

Compare: Fishing Reel

Turn and talk (2 min):

1. Which rotational kinematics equation did you choose?
2. How did you solve for time t?
3. Why is α negative?

Compare: Fishing Reel

Turn and talk (2 min):

1. Which rotational kinematics equation did you choose?
2. How did you solve for time t?
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Name wheel: One pair share your equation choice and reasoning.

Reveal: Stopping the Spin

Self-correct in a different color:

Equation: $\omega = \omega_0 + \alpha t$

Reveal: Stopping the Spin

Self-correct in a different color:

Equation: $\omega = \omega_0 + \alpha t$
Solve for t:

$$t = \frac{\omega - \omega_0}{\alpha}$$

Reveal: Stopping the Spin

Self-correct in a different color:

Equation: $\omega = \omega_0 + \alpha t$
Solve for t:

$$t = \frac{\omega - \omega_0}{\alpha}$$

Substitute:

$$t = \frac{0 - 220 \text{ rad/s}}{-300 \text{ rad/s}^2} = \frac{-220}{-300}$$

Reveal: Stopping the Spin

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Equation: $\omega = \omega_0 + \alpha t$
Solve for t**:**

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Insight: Less than one second because the angular acceleration is quite large!

Attempt: Merry-Go-Round Torque

The Challenge (3 min, silent)

A man pushes a merry-go-round with force 250 N at the edge, perpendicular to the radius of 1.50 m.

Given:

- Force $F = 250$ N
- Lever arm $r = 1.50$ m
- Angle $\theta = 90^\circ$ (perpendicular)

Find: Torque τ produced

How effective is his push?

Compare: Torque Calculation

Turn and talk (2 min):

1. What is the value of $\sin 90^\circ$?
2. How does this simplify the torque equation?
3. Why did the man push at the edge and perpendicular?

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Name wheel: One pair explain why perpendicular force maximizes torque.

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Strategy: Man maximized torque by pushing perpendicular at the outer edge!

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5. Rotational kinematics mirrors linear kinematics
6. Torque $\tau = rF \sin\theta$ is the rotational force

Key Equations

$$\Delta\theta = \frac{\Delta s}{r} \quad \text{(angle of rotation)} \tag{1}$$

$$\omega = \frac{\Delta\theta}{\Delta t} \quad \text{(angular velocity)} \tag{2}$$

$$v = r\omega \quad \text{(tangential velocity)} \tag{3}$$

$$a_c = \frac{v^2}{r} = r\omega^2 \quad \text{(centripetal acceleration)} \tag{4}$$

$$F_c = m\frac{v^2}{r} = mr\omega^2 \quad \text{(centripetal force)} \tag{5}$$

$$\alpha = \frac{\Delta\omega}{\Delta t} \quad \text{(angular acceleration)} \tag{6}$$

$$a = r\alpha \quad \text{(tangential acceleration)} \tag{7}$$

$$\tau = rF\sin\theta \quad \text{(torque)} \tag{8}$$

Complete the assigned problems

posted on the LMS