PHYS11 CH18-19: Electrostatics & Circuits

From Electric Forces to Current Flow

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Outline

Learning Objectives

By the end of this presentation, you will be able to:

- Apply Coulomb's law to calculate electrostatic forces between charges
- Describe electric fields and their relationship to force
- Explain electric potential energy and electric potential
- Understand how capacitors store energy and the effect of dielectrics
- Apply Ohm's law to calculate voltage, current, or resistance
- Analyze series and parallel circuit configurations
- Calculate electric power in circuit components

Outline

Coulomb's Law

Definition

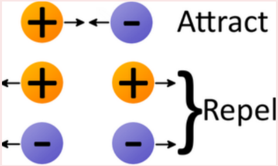
Coulomb's law describes the electrostatic force between charged particles.

Mathematical Form

$$F = k\frac{q_1 q_2}{r^2} \tag{1}$$

where:

- F = electrostatic force (newtons, N)
- k = Coulomb constant ($8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$)
- q_1, q_2 = electric charges (coulombs, C)
- r = distance between charges (meters, m)



Coulomb's Law - Key Properties

Inverse Square Law

The electrostatic force is inversely proportional to the square of the distance between charges.

Force Magnitude

- Proportional to each charge magnitude
- Inversely proportional to distance squared

Force Direction

- If $F < 0$ (negative result): attractive force
- If $F > 0$ (positive result): repulsive force
- Like charges repel, unlike charges attract

Electric Field

Definition

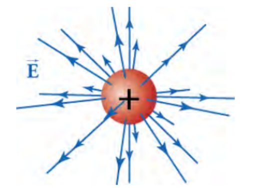
The electric field defines the force per unit charge in the space around a charge distribution.

Mathematical Form

$$\vec{E} = \frac{\vec{F}}{q} \tag{2}$$

For a point charge or uniform sphere:

$$E = k\frac{q}{r^2} \tag{3}$$



Electric Field Properties

Electric Field Lines

- Electric field lines never cross each other
- More force is applied in regions with many field lines
- Field lines start at positive charges and point away
- Field lines end at negative charges and point toward them

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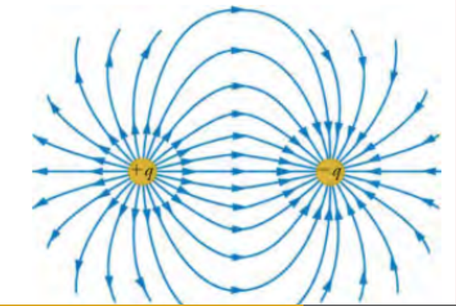

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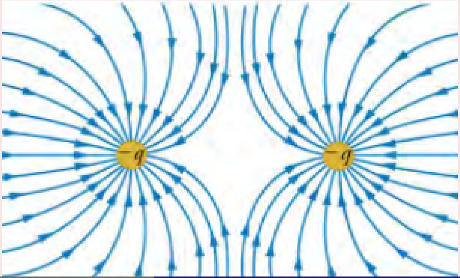


Electric Potential

Electric Potential Energy

Similar to gravitational potential energy: the potential that charges have to do work by virtue of their positions relative to each other.

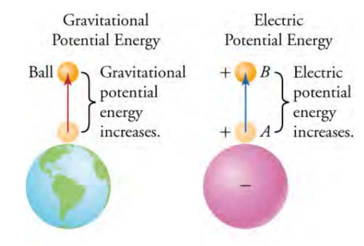
Electric Potential

Electric potential is the electric potential energy per unit charge:

$$V = \frac{U}{q} \tag{4}$$

For a point charge:

$$V = k\frac{q}{r} \tag{5}$$



Electric Potential - Key Properties

Measurement

- Potential is always measured between two points
- One point may be at infinity (reference point)
- Measured in volts (V)

Charge Movement

- Positive charges move from high potential to low potential
- Negative charges move from low potential to high potential
- Charges move spontaneously to minimize potential energy

Relationship to Electric Field

$$\vec{E} = -\nabla V \tag{6}$$

The electric field points in the direction of decreasing potential.

Outline

Capacitors

Definition

A capacitor is a device that stores electric charge and energy in an electric field.

Capacitance

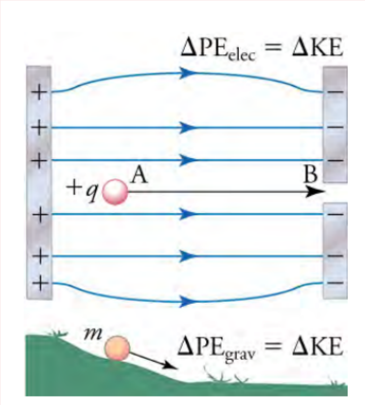
Capacitance is the ratio of charge to voltage:

$$C = \frac{Q}{V} \tag{7}$$

For a parallel plate capacitor:

$$C = \varepsilon_0 \frac{A}{d} \tag{8}$$

where A is plate area and d is separation distance.



Capacitors - Key Properties

Capacitance Properties

- Depends only on geometry and materials
- Does not depend on voltage across the capacitor
- Measured in farads (F)

Energy Storage

Capacitors store energy in the electric field between plates:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV \tag{9}$$

Electric Field in a Capacitor

$$E = \frac{V}{d} \tag{10}$$

where d is the separation between plates.

Dielectrics

Definition

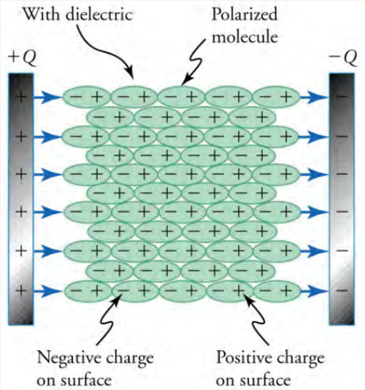
A dielectric material is an insulator that becomes polarized in an electric field.

Effect on Capacitance

Inserting a dielectric between capacitor plates increases capacitance:

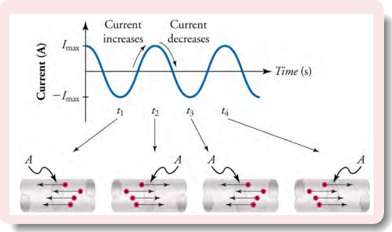
$$C = \kappa \varepsilon_0 \frac{A}{d} \tag{11}$$

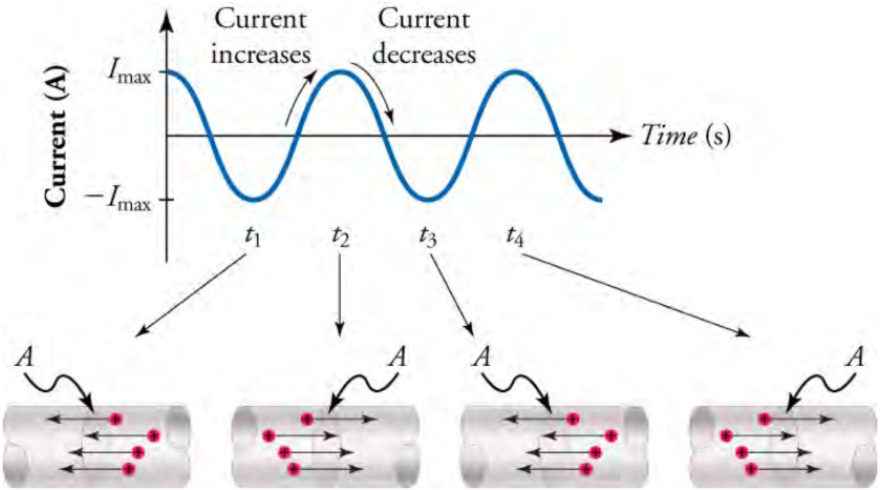
where κ is the dielectric constant (relative permittivity).



Outline

- **Direct Current (DC)**: Constant over time
- **Alternating Current (AC)**: Alternates smoothly back and forth over time
- **Electrical Resistance**: Causes materials to extract work from current flowing through them





Current increases
Current decreases
I_{max}
$-I_{max}$
Current (A)
Time (s)
t_1
t_2
t_3
t_4
A
A
A
A

Definition

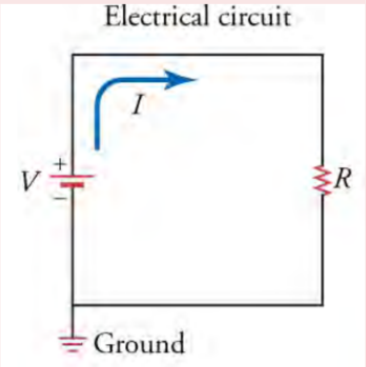
In ohmic materials, voltage drop along a path is proportional to the current that runs through the path.

Mathematical Form

$$V = IR \tag{12}$$

where:

- V = voltage (volts, V)
- I = current (amperes, A)
- R = resistance (ohms, Ω)



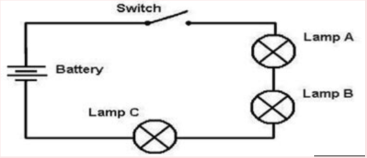
Series Circuits

Definition

Resistors in series are connected head to tail, with the same current flowing through each resistor.

Key Properties

- The same current flows through all resistors
- Voltage drops across each resistor can be different
- Voltage is the same at every point in a given wire



Equivalent Resistance in Series

Formula

For resistors in series, the total resistance is the sum of the individual resistances:

$$R_{\text{total}} = R_1 + R_2 + R_3 + \ldots + R_n \tag{13}$$

Voltage Drops

The voltage drops across each resistor follow Ohm's law:

$$V_1 = IR_1, \quad V_2 = IR_2, \quad \ldots \tag{14}$$

The total voltage across the circuit is the sum of the individual voltage drops:

$$V_{\text{total}} = V_1 + V_2 + V_3 + \ldots + V_n \tag{15}$$

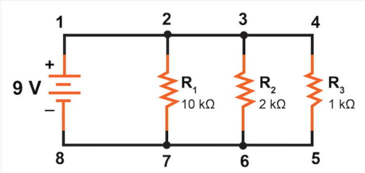
Parallel Circuits

Definition

Resistors in parallel are connected across the same two points in a circuit.

Key Properties

- Same voltage drop occurs across all resistors
- Current through each resistor can differ
- More paths for current means lower equivalent resistance



Equivalent Resistance in Parallel

Formula

For resistors in parallel, the reciprocal of the total resistance equals the sum of the reciprocals of the individual resistances:

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \tag{16}$$

Important Note

The equivalent resistance of parallel resistors is always less than the smallest individual resistance.

Electric Power

Definition

Electric power is the rate at which electrical energy is converted to other forms of energy (heat, light, mechanical work, etc.).

Key Concept

Electric power is dissipated in the resistances of a circuit. Capacitors do not dissipate electric power.

Power Formula

Electric power is proportional to voltage and current:

$$P = VI \tag{17}$$

where:

- P = power (watts, W)
- V = voltage (volts, V)

Power Calculations

Using Ohm's Law

Substituting $V = IR$ into $P = VI$:

$$P = VI = I(IR) = I^2R \tag{18}$$

Alternatively, substituting $I = \frac{V}{R}$ into $P = VI$:

$$P = VI = V\left(\frac{V}{R}\right) = \frac{V^2}{R} \tag{19}$$

Three Forms of Power Equation

$$P = VI \quad \text{or} \quad P = I^2R \quad \text{or} \quad P = \frac{V^2}{R} \tag{20}$$

Outline

Important Equations - Electrostatics

Coulomb's Law

$$F = k\frac{q_1 q_2}{r^2} \tag{21}$$

Electric Field

$$E = k\frac{q}{r^2} \quad \text{and} \quad \vec{E} = \frac{\vec{F}}{q} \tag{22}$$

Electric Potential

$$V = k\frac{q}{r} \quad \text{and} \quad V = \frac{U}{q} \tag{23}$$

Capacitance

$$C = \frac{Q}{V} \quad \text{and} \quad C = \kappa\varepsilon_0\frac{A}{d} \tag{24}$$

Important Equations - Circuits

Ohm's Law

$$V = IR \tag{25}$$

Series Circuits

$$R_{\text{total}} = R_1 + R_2 + R_3 + \ldots + R_n \tag{26}$$

Parallel Circuits

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \tag{27}$$

Electric Power

$$P = VI = I^2R = \frac{V^2}{R} \tag{28}$$

Outline

I Do: Coulomb's Law Application

Problem

Two point charges, $q_1 = 3\,\mu\text{C}$ and $q_2 = -2\,\mu\text{C}$, are separated by a distance of 15 cm. Calculate the electrostatic force between them.

Solution

Using Coulomb's law: $F = k\frac{q_1 q_2}{r^2}$

Given:

- $q_1 = 3 \times 10^{-6}\,\mathrm{C}$
- $q_2 = -2 \times 10^{-6}\,\mathrm{C}$
- $r = 0.15\,\mathrm{m}$
- $k = 8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}$

Substituting:

$$F = (8.99 \times 10^9)\frac{(3 \times 10^{-6})(-2 \times 10^{-6})}{(0.15)^2} \tag{29}$$

$$F = (8.99 \times 10^9)\frac{-6 \times 10^{-12}}{0.0225} \tag{30}$$

$$F = -2.4\,\mathrm{N} \tag{31}$$

The negative sign indicates an attractive force.

We Do: Electric Potential Problem

Problem

A point charge of $5\,\mu C$ creates an electric potential. Calculate the electric potential at points 10 cm and 20 cm from the charge.

Solution - Let's solve together

Using the formula for electric potential due to a point charge:

$$V = k\frac{q}{r} \tag{32}$$

At $r_1 = 10\,\text{cm} = 0.1\,\text{m}$:

$$V_1 = k\frac{q}{r_1} \tag{33}$$

$$V_1 = (8.99 \times 10^9)\frac{5 \times 10^{-6}}{0.1} \tag{34}$$

$$V_1 = ? \tag{35}$$

At $r_2 = 20\,\text{cm} = 0.2\,\text{m}$:

$$V_2 = k\frac{q}{r_2} \tag{36}$$

$$V_2 = (8.99 \times 10^9)\frac{5 \times 10^{-6}}{0.2} = ? \tag{37}$$

I Do: Series Circuit Analysis

Problem

Three resistors of 2 Ω, 4 Ω, and 6 Ω are connected in series with a 12 V battery. Calculate:

1. The total resistance
2. The current in the circuit
3. The voltage drop across each resistor

Solution

1. Total resistance:

$$R_{\text{total}} = R_1 + R_2 + R_3 \tag{38}$$

$$R_{\text{total}} = 2\,\Omega + 4\,\Omega + 6\,\Omega = 12\,\Omega \tag{39}$$

2. Current (using Ohm's law):

$$I = \frac{V}{R_{\text{total}}} = \frac{12\,\text{V}}{12\,\Omega} = 1\,\text{A} \tag{40}$$

3. Voltage drops:

$$V_1 = IR_1 = (1\,\text{A})(2\,\Omega) = 2\,\text{V} \tag{41}$$

$$V_2 = IR_2 = (1\,\text{A})(4\,\Omega) = 4\,\text{V} \tag{42}$$

$$V_3 = IR_3 = (1\,\text{A})(6\,\Omega) = 6\,\text{V} \tag{43}$$

Note that $V_1 + V_2 + V_3 = 2\,\text{V} + 4\,\text{V} + 6\,\text{V} = 12\,\text{V}$, which equals the battery voltage.

We Do: Parallel Circuit Problem

Problem

Three resistors of 3 Ω, 6 Ω, and 9 Ω are connected in parallel. Calculate:

1. The equivalent resistance
2. If this circuit is connected to a 12 V battery, find the total current drawn from the battery

Solution - Let's solve together

1. Equivalent resistance:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{44}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{3\,\Omega} + \frac{1}{6\,\Omega} + \frac{1}{9\,\Omega} \tag{45}$$

$$\frac{1}{R_{\text{eq}}} = \frac{3 + 1.5 + 1}{9\,\Omega} = \frac{5.5}{9\,\Omega} \tag{46}$$

$$R_{\text{eq}} = \frac{9\,\Omega}{5.5} = ?\,\Omega \tag{47}$$

2. Total current:

$$I_{\text{total}} = \frac{V}{R_{\text{eq}}} = \frac{12\,\text{V}}{R_{\text{eq}}} = ? \tag{48}$$

You Do: Combined Problem

Problem

A 12 V battery is connected to a parallel plate capacitor with a separation of 2 mm. The capacitor has a capacitance of 5 μF.

1. Calculate the electric field between the plates
2. Calculate the electric energy stored in the capacitor
3. If a dielectric with $\kappa = 4$ is inserted between the plates, what is the new capacitance?
4. Calculate the capacitor charge before and after inserting the dielectric

Hints

- Electric field in a parallel plate capacitor: $E = \frac{V}{d}$
- Energy stored in a capacitor: $U = \frac{1}{2}CV^2$
- Capacitance with dielectric: $C = \kappa C_0$
- Charge on a capacitor: $Q = CV$

Outline

Summary - Electrostatics

Coulomb's Law

- Inverse square law
- $F = k\frac{q_1 q_2}{r^2}$
- Like charges repel, unlike attract

Electric Field

- Force per unit charge
- $E = k\frac{q}{r^2}$
- Field lines show direction of force

Electric Potential

- Potential energy per unit charge
- $V = k\frac{q}{r}$
- Charges move to minimize potential energy

Capacitors

- Store energy in electric field
- $C = \frac{Q}{V}$
- Dielectrics increase capacitance

Summary - Electric Circuits

Ohm's Law

- $V = IR$
- Direct vs. alternating current
- Ohmic materials

Parallel Circuits

- Same voltage across all resistors
- $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$
- Currents can differ

Series Circuits

- Same current through all resistors
- $R_{\text{total}} = R_1 + R_2 + \dots$
- Voltage drops can differ

Electric Power

- $P = VI = I^2R = \frac{V^2}{R}$
- Power dissipated in resistances
- Measured in watts (W)

Thank You!

Any questions?

For additional practice problems, refer to your textbook Chapters 18-19