

20.3 Electromagnetic Induction

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain how a changing magnetic field produces a current in a wire
- Calculate induced electromotive force and current

Teacher Support

Teacher Support The learning objectives in this section will help your students master the following standards:

- (5) The student knows the nature of forces in the physical world. The student is expected to:
 - (G) investigate and describe the relationship between electric and magnetic fields in applications such as generators, motors, and transformers.

In addition, the OSX High School Physics Laboratory Manual addresses content in this section in the lab titled: Magnetism, as well as the following standards:

- (5) Science concepts. The student knows the nature of forces in the physical world. The student is expected to:
 - (G) investigate and describe the relationship between electric and magnetic fields in applications such as generators, motors, and transformers.

Section Key Terms

Changing Magnetic Fields

In the preceding section, we learned that a current creates a magnetic field. If nature is symmetrical, then perhaps a magnetic field can create a current. In 1831, some 12 years after the discovery that an electric current generates a magnetic field, English scientist Michael Faraday (1791–1862) and American scientist Joseph Henry (1797–1878) independently demonstrated that magnetic fields can produce currents. The basic process of generating currents with magnetic fields is called induction; this process is also called magnetic induction to distinguish it from charging by induction, which uses the electrostatic Coulomb force.

When Faraday discovered what is now called Faraday’s law of induction, Queen Victoria asked him what possible use was electricity. “Madam,” he replied,

“What good is a baby?” Today, currents induced by magnetic fields are essential to our technological society. The electric generator—found in everything from automobiles to bicycles to nuclear power plants—uses magnetism to generate electric current. Other devices that use magnetism to induce currents include pickup coils in electric guitars, transformers of every size, certain microphones, airport security gates, and damping mechanisms on sensitive chemical balances.

One experiment Faraday did to demonstrate magnetic induction was to move a bar magnet through a wire coil and measure the resulting electric current through the wire. A schematic of this experiment is shown in Figure 20.33. He found that current is induced only when the magnet moves with respect to the coil. When the magnet is motionless with respect to the coil, no current is induced in the coil, as in Figure 20.33. In addition, moving the magnet in the opposite direction (compare Figure 20.33 with Figure 20.33) or reversing the poles of the magnet (compare Figure 20.33 with Figure 20.33) results in a current in the opposite direction.

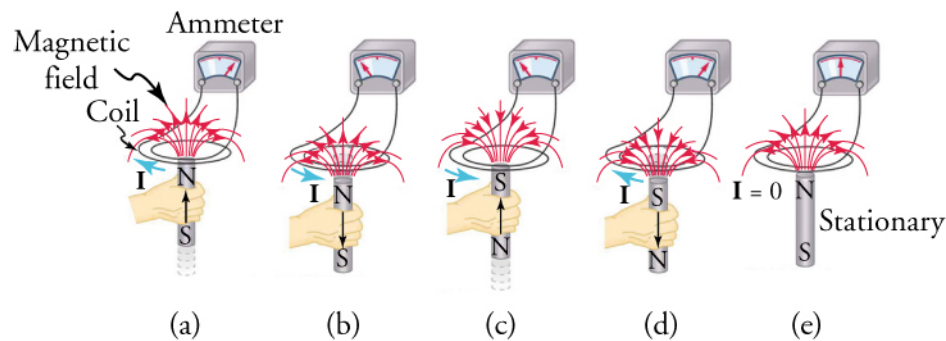


Figure 20.33 Movement of a magnet relative to a coil produces electric currents as shown. The same currents are produced if the coil is moved relative to the magnet. The greater the speed, the greater the magnitude of the current, and the current is zero when there is no motion. The current produced by moving the magnet upward is in the opposite direction as the current produced by moving the magnet downward.

Virtual Physics

Faraday's Law [Click to view content](#)

Try this simulation to see how moving a magnet creates a current in a circuit. A light bulb lights up to show when current is flowing, and a voltmeter shows the voltage drop across the light bulb. Try moving the magnet through a four-turn coil and through a two-turn coil. For the same magnet speed, which coil produces a higher voltage?

With the north pole to the left and moving the magnet from right to left, a positive voltage is produced as the magnet enters the coil. What sign voltage will be produced if the experiment is repeated with the south pole to the left?

- a. The sign of voltage will change because the direction of current flow will change by moving south pole of the magnet to the left.
- b. The sign of voltage will remain same because the direction of current flow will not change by moving south pole of the magnet to the left.
- c. The sign of voltage will change because the magnitude of current flow will change by moving south pole of the magnet to the left.
- d. The sign of voltage will remain same because the magnitude of current flow will not change by moving south pole of the magnet to the left.

Induced Electromotive Force

If a current is induced in the coil, Faraday reasoned that there must be what he called an *electromotive force* pushing the charges through the coil. This interpretation turned out to be incorrect; instead, the external source doing the work of moving the magnet adds energy to the charges in the coil. The energy added per unit charge has units of volts, so the electromotive force is actually a potential. Unfortunately, the name electromotive force stuck and with it the potential for confusing it with a real force. For this reason, we avoid the term *electromotive force* and just use the abbreviation *emf*, which has the mathematical symbol ε . The emf may be defined as the rate at which energy is drawn from a source per unit current flowing through a circuit. Thus, emf is the energy per unit charge *added* by a source, which contrasts with voltage, which is the energy per unit charge *released* as the charges flow through a circuit.

To understand why an emf is generated in a coil due to a moving magnet, consider Figure 20.34, which shows a bar magnet moving downward with respect to a wire loop. Initially, seven magnetic field lines are going through the loop (see left-hand image). Because the magnet is moving away from the coil, only five magnetic field lines are going through the loop after a short time Δt (see right-hand image). Thus, when a change occurs in the number of magnetic field lines going through the area defined by the wire loop, an emf is induced in the wire loop. Experiments such as this show that the induced emf is proportional to the *rate of change* of the magnetic field. Mathematically, we express this as

$$\varepsilon \propto \frac{\Delta B}{\Delta t},$$

20.24

where ΔB is the change in the magnitude in the magnetic field during time Δt and A is the area of the loop.

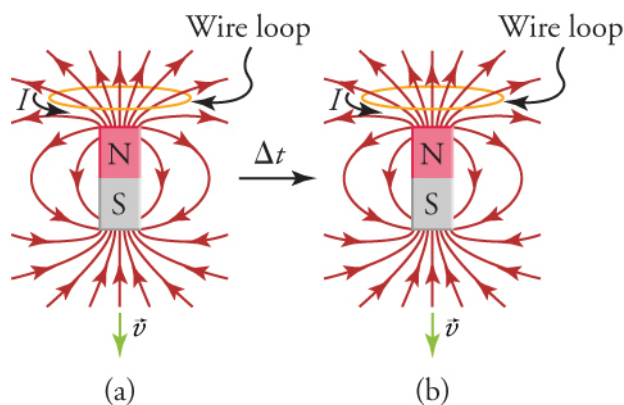


Figure 20.34 The bar magnet moves downward with respect to the wire loop, so that the number of magnetic field lines going through the loop decreases with time. This causes an emf to be induced in the loop, creating an electric current.

Note that magnetic field lines that lie in the plane of the wire loop do not actually pass through the loop, as shown by the left-most loop in Figure 20.35. In this figure, the arrow coming out of the loop is a vector whose magnitude is the area of the loop and whose direction is perpendicular to the plane of the loop. In Figure 20.35, as the loop is rotated from $\theta = 90^\circ$ to $\theta = 0^\circ$, the contribution of the magnetic field lines to the emf increases. Thus, what is important in generating an emf in the wire loop is the component of the magnetic field that is *perpendicular* to the plane of the loop, which is $B\cos\theta$.

This is analogous to a sail in the wind. Think of the conducting loop as the sail and the magnetic field as the wind. To maximize the force of the wind on the sail, the sail is oriented so that its surface vector points in the same direction as the winds, as in the right-most loop in Figure 20.35. When the sail is aligned so that its surface vector is perpendicular to the wind, as in the left-most loop in Figure 20.35, then the wind exerts no force on the sail.

Thus, taking into account the angle of the magnetic field with respect to the area, the proportionality $E \propto \Delta B/\Delta t$ becomes

$$E \propto \frac{\Delta B \cos \theta}{\Delta t}.$$

20.25

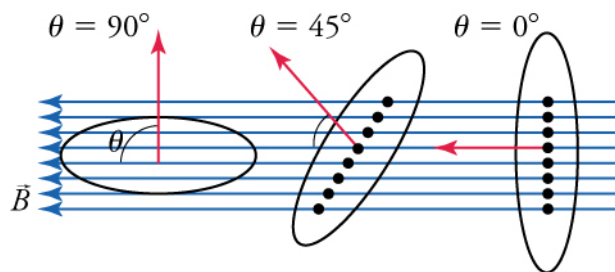


Figure 20.35 The magnetic field lies in the plane of the left-most loop, so it cannot generate an emf in this case. When the loop is rotated so that the angle of the magnetic field with the vector perpendicular to the area of the loop increases to 90° (see right-most loop), the magnetic field contributes maximally to the emf in the loop. The dots show where the magnetic field lines intersect the plane defined by the loop.

Another way to reduce the number of magnetic field lines that go through the conducting loop in Figure 20.35 is not to move the magnet but to make the loop smaller. Experiments show that changing the area of a conducting loop in a stable magnetic field induces an emf in the loop. Thus, the emf produced in a conducting loop is proportional to the rate of change of the *product* of the perpendicular magnetic field and the loop area

$$\varepsilon \propto \frac{\Delta[(B\cos\theta)A]}{\Delta t},$$

20.26

where $B\cos\theta$ is the perpendicular magnetic field and A is the area of the loop. The product $BA\cos\theta$ is very important. It is proportional to the number of magnetic field lines that pass perpendicularly through a surface of area A . Going back to our sail analogy, it would be proportional to the force of the wind on the sail. It is called the magnetic flux and is represented by Φ .

$$\Phi = BA\cos\theta$$

20.27

The unit of magnetic flux is the weber (Wb), which is magnetic field per unit area, or T/m^2 . The weber is also a volt second (Vs).

The induced emf is in fact proportional to the rate of change of the magnetic flux through a conducting loop.

$$\varepsilon \propto \frac{\Delta\Phi}{\Delta t}$$

20.28

Finally, for a coil made from N loops, the emf is N times stronger than for a single loop. Thus, the emf induced by a changing magnetic field in a coil of N loops is

$$\varepsilon \propto N \frac{\Delta B\cos\theta}{\Delta t} A.$$

The last question to answer before we can change the proportionality into an equation is “In what direction does the current flow?” The Russian scientist Heinrich Lenz (1804–1865) explained that the current flows in the direction that creates a magnetic field that tries to keep the flux constant in the loop. For example, consider again Figure 20.34. The motion of the bar magnet causes the number of upward-pointing magnetic field lines that go through the loop to decrease. Therefore, an emf is generated in the loop that drives a current in the direction that creates more upward-pointing magnetic field lines. By using

the right-hand rule, we see that this current must flow in the direction shown in the figure. To express the fact that the induced emf acts to counter the change in the magnetic flux through a wire loop, a minus sign is introduced into the proportionality $\varepsilon \propto \Delta\Phi/\Delta t$, which gives Faraday's law of induction.

$$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$$

20.29

Lenz's law is very important. To better understand it, consider Figure 20.36, which shows a magnet moving with respect to a wire coil and the direction of the resulting current in the coil. In the top row, the north pole of the magnet approaches the coil, so the magnetic field lines from the magnet point toward the coil. Thus, the magnetic field $\vec{B}_{\text{mag}} = B_{\text{mag}}(\hat{x})$ pointing to the right increases in the coil. According to Lenz's law, the emf produced in the coil will drive a current in the direction that creates a magnetic field $\vec{B}_{\text{coil}} = B_{\text{coil}}(-\hat{x})$ inside the coil pointing to the left. This will counter the increase in magnetic flux pointing to the right. To see which way the current must flow, point your right thumb in the desired direction of the magnetic field \vec{B}_{coil} , and the current will flow in the direction indicated by curling your right fingers. This is shown by the image of the right hand in the top row of Figure 20.36. Thus, the current must flow in the direction shown in Figure 4(a).

In Figure 4(b), the direction in which the magnet moves is reversed. In the coil, the right-pointing magnetic field \vec{B}_{mag} due to the moving magnet decreases. Lenz's law says that, to counter this decrease, the emf will drive a current that creates an additional right-pointing magnetic field \vec{B}_{coil} in the coil. Again, point your right thumb in the desired direction of the magnetic field, and the current will flow in the direction indicated by curling your right fingers (Figure 4(b)).

Finally, in Figure 4(c), the magnet is reversed so that the south pole is nearest the coil. Now the magnetic field \vec{B}_{mag} points toward the magnet instead of toward the coil. As the magnet approaches the coil, it causes the left-pointing magnetic field in the coil to increase. Lenz's law tells us that the emf induced in the coil will drive a current in the direction that creates a magnetic field pointing to the right. This will counter the increasing magnetic flux pointing to the left due to the magnet. Using the right-hand rule again, as indicated in the figure, shows that the current must flow in the direction shown in Figure 4(c).

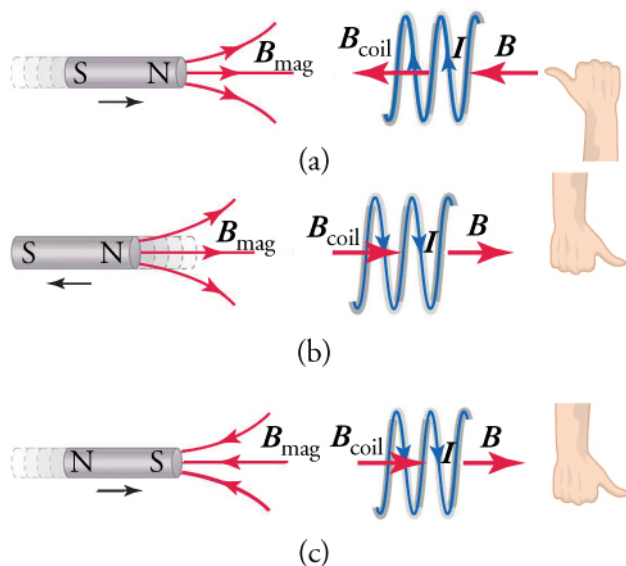


Figure 20.36 Lenz's law tells us that the magnetically induced emf will drive a current that resists the change in the magnetic flux through a circuit. This is shown in panels (a)–(c) for various magnet orientations and velocities. The right hands at right show how to apply the right-hand rule to find in which direction the induced current flows around the coil.

Virtual Physics

Faraday's Electromagnetic Lab [Click to view content](#)

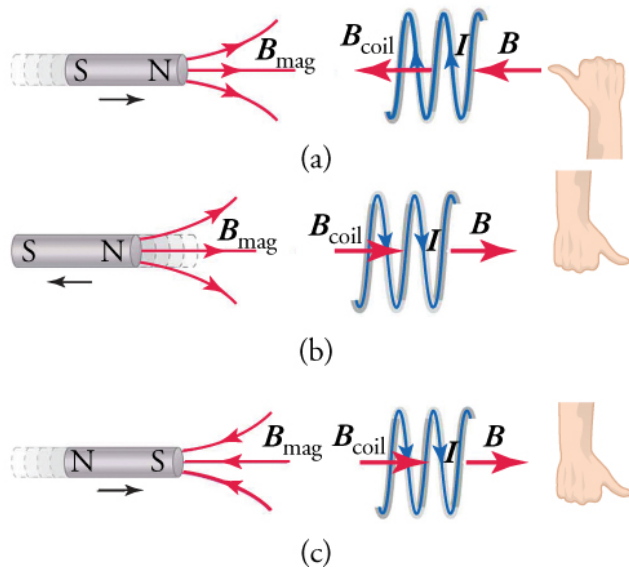
This simulation proposes several activities. For now, click on the tab Pickup Coil, which presents a bar magnet that you can move through a coil. As you do so, you can see the electrons move in the coil and a light bulb will light up or a voltmeter will indicate the voltage across a resistor. Note that the voltmeter allows you to see the sign of the voltage as you move the magnet about. You can also leave the bar magnet at rest and move the coil, although it is more difficult to observe the results.

PhET Explorations: Faraday's Electromagnetic Lab Play with a bar magnet and coils to learn about Faraday's law. Move a bar magnet near one or two coils to make a light bulb glow. View the magnetic field lines. A meter shows the direction and magnitude of the current. View the magnetic field lines or use a meter to show the direction and magnitude of the current. You can also play with electromagnets, generators and transformers!

[Click to view content](#)

Orient the bar magnet with the north pole facing to the right and place the pickup coil to the right of the bar magnet. Now move the bar mag-

net toward the coil and observe in which way the electrons move. This is the same situation as depicted below. Does the current in the simulation flow in the same direction as shown below? Explain why or why not.



- Yes, the current in the simulation flows as shown because the direction of current is opposite to the direction of flow of electrons.
- No, current in the simulation flows in the opposite direction because the direction of current is same to the direction of flow of electrons.

Watch Physics

Induced Current in a Wire This video explains how a current can be induced in a straight wire by moving it through a magnetic field. The lecturer uses the *cross product*, which is a type of vector multiplication. Don't worry if you are not familiar with this, it basically combines the right-hand rule for determining the force on the charges in the wire with the equation $F = qvB\sin\theta$.

[Click to view content](#)

Grasp Check

What emf is produced across a straight wire 0.50 m long moving at a velocity of $(1.5 \text{ m/s}) \hat{x}$ through a uniform magnetic field $(0.30 \text{ T}) \hat{z}$? The wire lies in the \hat{y} -direction. Also, which end of the wire is at the higher potential—let the lower end of the wire be at $y = 0$ and the upper end at $y = 0.5 \text{ m}$?

- 0.15 V and the lower end of the wire will be at higher potential
- 0.15 V and the upper end of the wire will be at higher potential

- c. 0.075 V and the lower end of the wire will be at higher potential
- d. 0.075 V and the upper end of the wire will be at higher potential

Worked Example

EMF Induced in Conducing Coil by Moving Magnet Imagine a magnetic field goes through a coil in the direction indicated in Figure 20.37. The coil diameter is 2.0 cm. If the magnetic field goes from 0.020 T to 0.010 T in 34 s, what is the direction and magnitude of the induced current? Assume the coil has a resistance of 0.1 Ω .

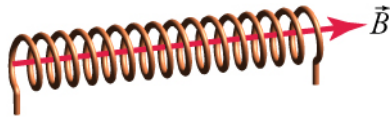


Figure 20.37 A coil through which passes a magnetic field B .

Strategy

Use the equation

$$\varepsilon = -N\Delta\Phi/\Delta t$$

to find the induced emf in the coil, where $\Delta t = 34$ s. Counting the number of loops in the solenoid, we find it has 16 loops, so $N = 16$. Use the equation

$$\Phi = BA\cos\theta$$

to calculate the magnetic flux

$$\Phi = BA\cos\theta = B\pi\left(\frac{d}{2}\right)^2,$$

20.30

where d is the diameter of the solenoid and we have used $\cos 0^\circ = 1$. Because the area of the solenoid does not vary, the change in the magnetic of the flux through the solenoid is

$$\Delta\Phi = \Delta B\pi\left(\frac{d}{2}\right)^2.$$

20.31

Once we find the emf, we can use Ohm's law, $\varepsilon = IR$, to find the current.

Finally, Lenz's law tells us that the current should produce a magnetic field that acts to oppose the decrease in the applied magnetic field. Thus, the current should produce a magnetic field to the right.

Solution

Combining equations $\varepsilon = -N\Delta\Phi/\Delta t$ and $\Phi = BA\cos\theta$ gives

$$\varepsilon = -N \frac{\Delta \Phi}{\Delta t} = -N \frac{\Delta B \pi d^2}{4 \Delta t}.$$

20.32

Solving Ohm's law for the current and using this result gives

$$\begin{aligned} I &= \frac{\varepsilon}{R} = -N \frac{\Delta B \pi d^2}{4 R \Delta t} \\ &= -16 \frac{(-0.010 \text{ T}) \pi (0.020 \text{ m})^2}{4 (0.10 \text{ } \Omega) (34 \text{ s})}. \\ &= 15 \text{ } \mu\text{A} \end{aligned}$$

20.33

Lenz's law tells us that the current must produce a magnetic field to the right. Thus, we point our right thumb to the right and curl our right fingers around the solenoid. The current must flow in the direction in which our fingers are pointing, so it enters at the left end of the solenoid and exits at the right end.

Discussion

Let's see if the minus sign makes sense in Faraday's law of induction. Define the direction of the magnetic field to be the positive direction. This means the change in the magnetic field is negative, as we found above. The minus sign in Faraday's law of induction negates the negative change in the magnetic field, leaving us with a positive current. Therefore, the current must flow in the direction of the magnetic field, which is what we found.

Now try defining the positive direction to be the direction opposite that of the magnetic field, that is positive is to the left in Figure 20.37. In this case, you will find a negative current. But since the positive direction is to the left, a negative current must flow to the right, which again agrees with what we found by using Lenz's law.

Worked Example

Magnetic Induction due to Changing Circuit Size The circuit shown in Figure 20.38 consists of a U-shaped wire with a resistor and with the ends connected by a sliding conducting rod. The magnetic field filling the area enclosed by the circuit is constant at 0.01 T. If the rod is pulled to the right at speed $v = 0.50 \text{ m/s}$, what current is induced in the circuit and in what direction does the current flow?

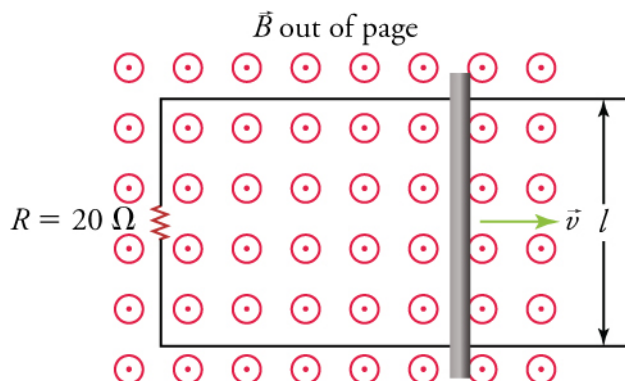


Figure 20.38 A slider circuit. The magnetic field is constant and the rod is pulled to the right at speed v . The changing area enclosed by the circuit induces an emf in the circuit.

Strategy

We again use Faraday's law of induction, $E = -N \frac{\Delta\Phi}{\Delta t}$, although this time the magnetic field is constant and the area enclosed by the circuit changes. The circuit contains a single loop, so $N = 1$. The rate of change of the area is $\frac{\Delta A}{\Delta t} = v\ell$. Thus the rate of change of the magnetic flux is

$$\frac{\Delta\Phi}{\Delta t} = \frac{\Delta(BA\cos\theta)}{\Delta t} = B \frac{\Delta A}{\Delta t} = Bv\ell,$$

20.34

where we have used the fact that the angle θ between the area vector and the magnetic field is 0° . Once we know the emf, we can find the current by using Ohm's law. To find the direction of the current, we apply Lenz's law.

Solution

Faraday's law of induction gives

$$E = -N \frac{\Delta\Phi}{\Delta t} = -Bv\ell.$$

20.35

Solving Ohm's law for the current and using the previous result for emf gives

$$I = \frac{E}{R} = \frac{-Bv\ell}{R} = \frac{-(0.010 \text{ T})(0.50 \text{ m/s})(0.10 \text{ m})}{20 \Omega} = 25 \text{ A}.$$

20.36

As the rod slides to the right, the magnetic flux passing through the circuit increases. Lenz's law tells us that the current induced will create a magnetic field that will counter this increase. Thus, the magnetic field created by the induced current must be into the page. Curling your right-hand fingers around the loop in the clockwise direction makes your right thumb point into the page,

which is the desired direction of the magnetic field. Thus, the current must flow in the clockwise direction around the circuit.

Discussion

Is energy conserved in this circuit? An external agent must pull on the rod with sufficient force to just balance the force on a current-carrying wire in a magnetic field—recall that $F = I\ell B \sin\theta$. The rate at which this force does work on the rod should be balanced by the rate at which the circuit dissipates power. Using $F = I\ell B \sin\theta$, the force required to pull the wire at a constant speed v is

$$F_{\text{pull}} = I\ell B \sin\theta = I\ell B,$$

20.37

where we used the fact that the angle θ between the current and the magnetic field is 90° . Inserting our expression above for the current into this equation gives

$$F_{\text{pull}} = I\ell B = -\frac{Bv\ell}{R}(\ell B) = -\frac{B^2 v \ell^2}{R}.$$

20.38

The power contributed by the agent pulling the rod is $F_{\text{pull}}v$, or

$$P_{\text{pull}} = F_{\text{pull}}v = -\frac{B^2 v^2 \ell^2}{R}.$$

20.39

The power dissipated by the circuit is

$$P_{\text{dissipated}} = I^2 R = \left(\frac{-Bv\ell}{R}\right)^2 R = \frac{B^2 v^2 \ell^2}{R}.$$

20.40

We thus see that $P_{\text{pull}} + P_{\text{dissipated}} = 0$, which means that power is conserved in the system consisting of the circuit and the agent that pulls the rod. Thus, energy is conserved in this system.

Practice Problems

11.

The magnetic flux through a single wire loop changes from 3.5 Wb to 1.5 Wb in 2.0 s. What emf is induced in the loop?

- a. -2.0 V
- b. -1.0 V
- c. +1.0 V
- d. +2.0 V

12.

What is the emf for a 10-turn coil through which the flux changes at 10 Wb/s?

- a. -100 V
- b. -10 V
- c. $+10\text{ V}$
- d. $+100\text{ V}$

Check Your Understanding

13.

Given a bar magnet, how can you induce an electric current in a wire loop?

- a. An electric current is induced if a bar magnet is placed near the wire loop.
- b. An electric current is induced if a wire loop is wound around the bar magnet.
- c. An electric current is induced if a bar magnet is moved through the wire loop.
- d. An electric current is induced if a bar magnet is placed in contact with the wire loop.

14.

What factors can cause an induced current in a wire loop through which a magnetic field passes?

- a. Induced current can be created by changing the size of the wire loop only.
- b. Induced current can be created by changing the orientation of the wire loop only.
- c. Induced current can be created by changing the strength of the magnetic field only.
- d. Induced current can be created by changing the strength of the magnetic field, changing the size of the wire loop, or changing the orientation of the wire loop.