PHYS12 CH:5 Motion in Two Dimensions

Vectors and Projectiles

Mr. Gullo

December 2025

Outline

How do you describe motion
when objects move in two directions at once?

The Mystery

How do you describe motion

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A football spiraling downfield...

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A drone navigating through space...

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when objects move in two directions at once?

A football spiraling downfield...

A drone navigating through space...

Motion happens in multiple dimensions simultaneously.

Learning Objectives

By the end of this section, you will be able to:

- **5.1:** Describe the graphical method of vector addition and subtraction

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By the end of this section, you will be able to:

- **5.1:** Describe the graphical method of vector addition and subtraction
- **5.1:** Use the graphical method to solve physics problems

5.1 What Is a Vector?

The Dual Nature

A vector is a quantity with both magnitude AND direction.

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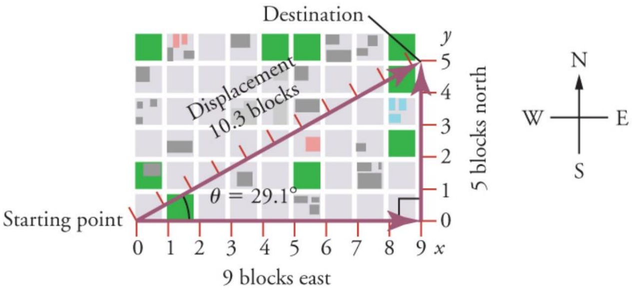
The Mental Model

Velocity is like a speedometer with a compass attached. Speed tells how fast. Velocity adds where.

5.1 The Journey to the Destination



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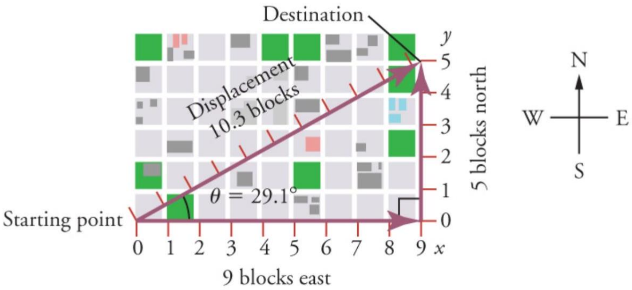
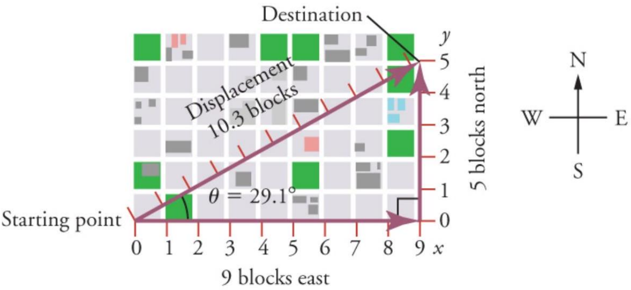


Path: 9 blocks east + 5 blocks north = 14 blocks walked
Displacement: 10.3 blocks at 29.1° north of east



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Displacement: 10.3 blocks at 29.1° north of east

Civilian View vs. Reality

Civilian: "I walked 14 blocks."
Physicist: "Displacement was 10.3 blocks."

5.1 The Head-to-Tail Method

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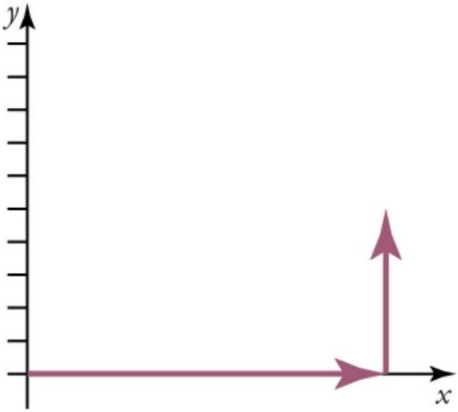
Key insight: Order doesn't matter! $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

5.1 Building the Resultant



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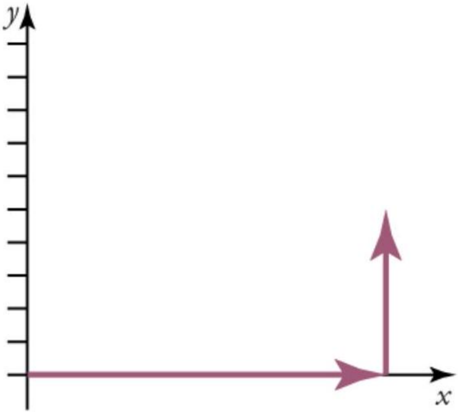

The resultant vector connects start to finish.

5.1 Vector Subtraction

To subtract: Add the negative vector

$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$

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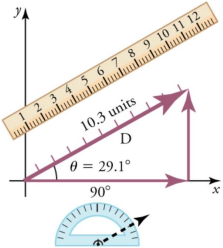

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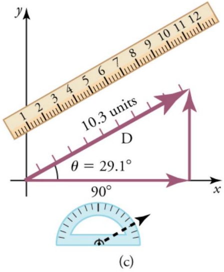
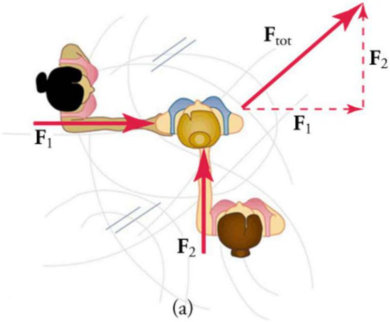
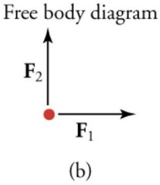


The negative vector: Same magnitude, opposite direction

5.1 Forces on Ice





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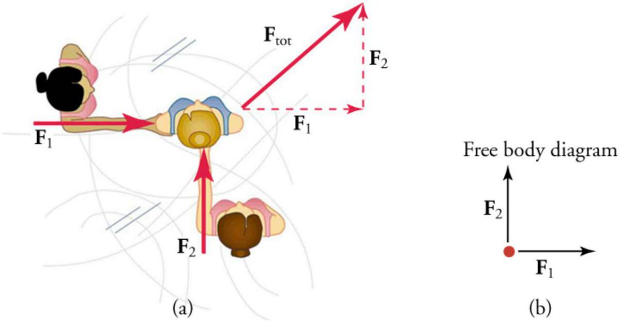
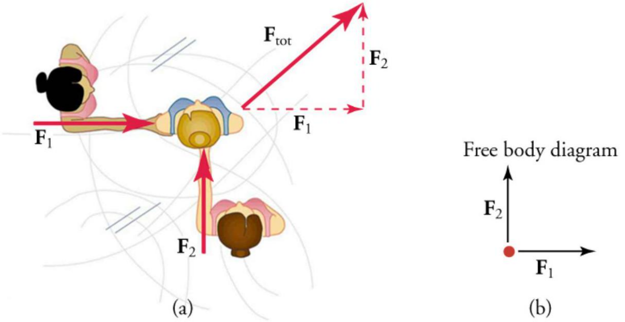
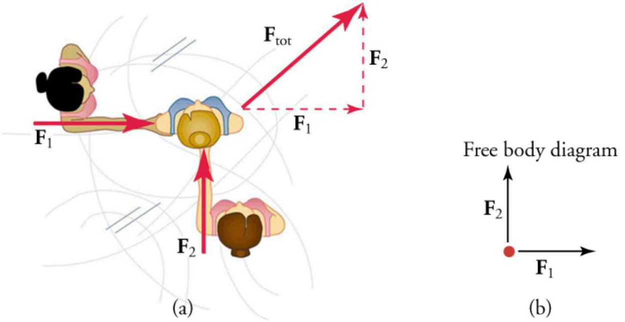


Two skaters push with 400 N each at right angles.



Two skaters push with 400 N each at right angles.
Resultant force: $F_{tot} = \sqrt{(400)^2 + (400)^2} = 566$ N



Two skaters push with 400 N each at right angles.
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Pythagorean theorem works when vectors are perpendicular!

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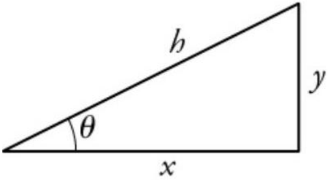
5.2 The Power of Trigonometry



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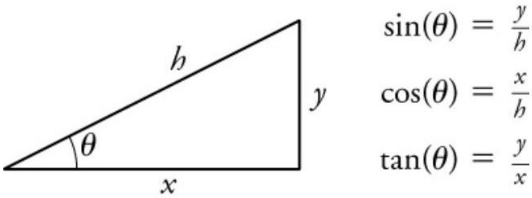
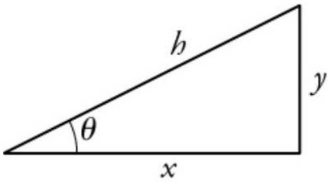


Refresher:

- $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$



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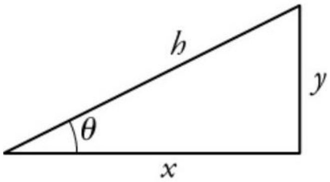

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The Source Code

$$\boxed{A_x = A\cos\theta}$$
$$\boxed{A_y = A\sin\theta}$$

Every 2D vector can be split into x and y components.

5.2 Breaking Vectors Into Components

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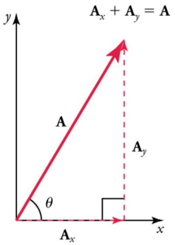

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Components match the original path: 9 east + 5 north!

5.2 Reverse: From Components to Vector

Reconstruction Formulas

$$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)}$$

Pythagorean theorem gives magnitude, inverse tangent gives direction.

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The Paradox

Students confuse: $\mathbf{A}_x + \mathbf{A}_y = \mathbf{A}$ (vector addition)
with $A = \sqrt{A_x^2 + A_y^2}$ (magnitude calculation)

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More accurate than graphical method - not limited by drawing precision!

Attempt: Two-Leg Journey

The Challenge (3 min, silent)

A person walks 53.0 m at 20.0° north of east, then 34.0 m at 63.0° north of east.

Given:

- $A = 53.0$ m, $\theta_A = 20.0^\circ$
- $B = 34.0$ m, $\theta_B = 63.0^\circ$

Find: Total displacement magnitude and direction

Can you decode this journey? Work silently.

Compare: Vector Addition Strategy

Turn and talk (2 min):

1. What's the first step - find components or add vectors?
2. How many components do you need to calculate?
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Name wheel: One pair share your approach (not your answer).

Self-correct in a different color:
Step 1: Find components of **A**

$$A_x = (53.0)(\cos 20.0^\circ) = 49.8 \text{ m}$$
$$A_y = (53.0)(\sin 20.0^\circ) = 18.1 \text{ m}$$

Reveal: The Navigation Solution

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Step 2: Find components of **B**

$$B_x = (34.0)(\cos 63.0^\circ) = 15.4 \text{ m}$$
$$B_y = (34.0)(\sin 63.0^\circ) = 30.3 \text{ m}$$

Reveal: Combining Components

Step 3: Add components

$$R_x = A_x + B_x = 49.8 + 15.4 = 65.2 \text{ m}$$
$$R_y = A_y + B_y = 18.1 + 30.3 = 48.4 \text{ m}$$

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Step 4: Find magnitude

$$R = \sqrt{(65.2)^2 + (48.4)^2} = \boxed{81.2 \text{ m}}$$

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$$R = \sqrt{(65.2)^2 + (48.4)^2} = \boxed{81.2 \text{ m}}$$

Step 5: Find direction

$$\theta = \tan^{-1}(48.4/65.2) = \boxed{36.6^\circ \text{ north of east}}$$

Learning Objectives

By the end of this section, you will be able to:

- **5.3:** Describe the properties of projectile motion

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By the end of this section, you will be able to:

- **5.3:** Describe the properties of projectile motion
- **5.3:** Apply kinematic equations and vectors to solve projectile problems

5.3 The Great Separation

Nature's Rule for Projectiles

Horizontal and vertical motions

are independent

They don't influence each other.

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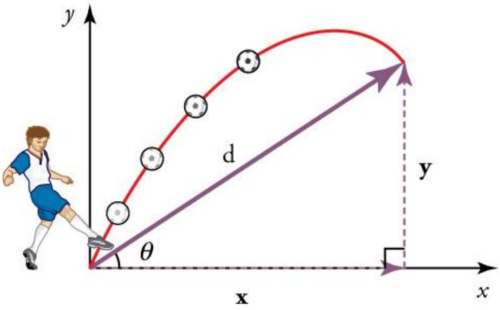

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We ignore air resistance in introductory physics.

5.3 The Independence Principle

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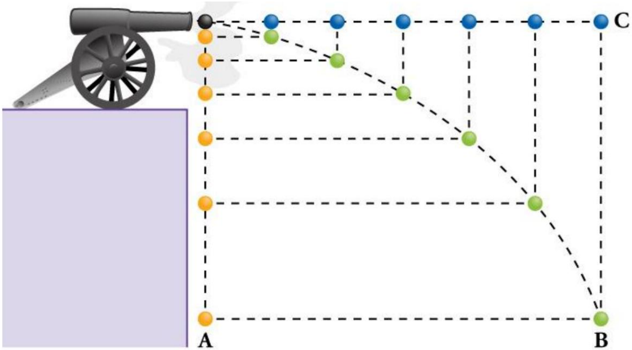
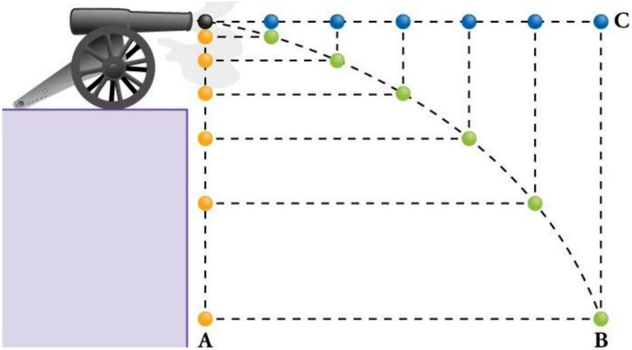
Time t is the only variable connecting the two motions!

5.3 The Trajectory





Ball kicked at angle θ follows parabolic path.

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2. Treat as two independent 1D motions
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4. Recombine to find total displacement and velocity

Recombination formulas:

$$d = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}(y/x)$$

$$v = \sqrt{v_x^2 + v_y^2}, \quad \theta_v = \tan^{-1}(v_y/v_x)$$

5.3 Maximum Height Formula

The Peak

$$\boxed{h = \frac{v_{0y}^2}{2g}}$$

Maximum height depends only on initial vertical velocity.

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Key insight: At maximum height, $v_y = 0$

What Your Brain Gets Wrong

Intuition: "Faster launch means higher peak."
Reality: "Only the *vertical* component matters."

Attempt: Fireworks Launch

The Challenge (3 min, silent)

A fireworks shell is launched at 70.0 m/s at 75° above horizontal.

Given:

- $v_0 = 70.0$ m/s
- $\theta = 75^\circ$

Find: Maximum height

Can you predict where it explodes? Work silently.

Compare: Projectile Strategy

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Reveal: Reaching the Sky

Self-correct in a different color:
Step 1: Find vertical velocity component

$$\begin{aligned} v_{0y} &= v_0 \sin\theta \\ &= (70.0)(\sin 75^\circ) = 67.6 \text{ m/s} \end{aligned}$$

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Step 1: Find vertical velocity component

$$\begin{aligned} v_{0y} &= v_0 \sin\theta \\ &= (70.0)(\sin 75^\circ) = 67.6 \text{ m/s} \end{aligned}$$

Step 2: Use maximum height formula

$$\begin{aligned} h &= \frac{v_{0y}^2}{2g} \\ &= \frac{(67.6)^2}{2(9.8)} = \boxed{233 \text{ m}} \end{aligned}$$

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\begin{aligned}
v_{0y} &= v_0 \sin\theta \\
&= (70.0)(\sin 75^\circ) = 67.6 \text{ m/s}
\end{aligned}
$$

Step 2: Use maximum height formula

$$
\begin{aligned}
h &= \frac{v_{0y}^2}{2g} \\
&= \frac{(67.6)^2}{2(9.8)} = \boxed{233 \text{ m}}
\end{aligned}
$$

Check: About 765 feet - reasonable for large fireworks!

The Distance Formula

$$\boxed{R = \frac{v_0^2 \sin 2\theta}{g}}$$

Range depends on initial speed and launch angle.

5.3 The Range Equation

The Distance Formula

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Key insights:

- Maximum range at $\theta = 45^\circ$

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Key insights:

- Maximum range at $\theta = 45^\circ$
- Same range for complementary angles: 30° and 60°
- Only valid when initial and final heights are equal

Attempt: Volcanic Rock

The Challenge (3 min, silent)

A rock is ejected from a volcano at 25.0 m/s at 35° above horizontal. It lands 20.0 m below its starting point.

Given:

- $v_0 = 25.0$ m/s, $\theta = 35^\circ$
- $y = -20.0$ m

Find: Time of flight

Can you predict the flight time? Work silently.

Compare: Quadratic Strategy

Turn and talk (2 min):

1. Which kinematic equation has both y and t?
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Name wheel: One pair share your approach (not your answer).

Reveal: Time of Flight

Self-correct in a different color:
Step 1: Find $v_{0y} = (25.0)(\sin 35^\circ) = 14.3$ m/s

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Step 4: Quadratic formula

$$t = \frac{14.3 \pm \sqrt{(14.3)^2 - 4(4.90)(-20.0)}}{2(4.90)}$$

Reveal: Solving the Quadratic

Step 5: Calculate discriminant

$$\sqrt{(14.3)^2 + 4(4.90)(20.0)} = \sqrt{596.5} = 24.4$$

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$$t = \frac{14.3 + 24.4}{9.8} = 3.96 \text{ s} \quad \text{(physical)}$$

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Negative time means before launch - physically impossible!

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1. Vectors have magnitude AND direction
2. Graphical addition: head-to-tail method
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4. Projectile motion: horizontal and vertical are independent
5. Time connects the two dimensions
6. Maximum height depends only on v_{0y}

Key Equations

Vector Components:

$$A_x = A\cos\theta \tag{1}$$
$$A_y = A\sin\theta \tag{2}$$

Magnitude and Direction:

$$A = \sqrt{A_x^2 + A_y^2} \tag{3}$$
$$\theta = \tan^{-1}(A_y/A_x) \tag{4}$$

Projectile Motion:

$$\text{Horizontal: } x = x_0 + v_x t \tag{5}$$
$$\text{Vertical: } y = y_0 + v_{0y} t - \frac{1}{2}gt^2 \tag{6}$$
$$\text{Max height: } h = \frac{v_{0y}^2}{2g} \tag{7}$$

Homework

Complete the assigned problems

posted on the LMS