

Section Summary

23.1 Induced Emf and Magnetic Flux

- The crucial quantity in induction is magnetic flux Φ , defined to be $\Phi = BA \cos \theta$, where B is the magnetic field strength over an area A at an angle θ with the perpendicular to the area.
- Units of magnetic flux Φ are $\text{T} \cdot \text{m}^2$.
- Any change in magnetic flux Φ induces an emf—the process is defined to be electromagnetic induction.

23.2 Faraday's Law of Induction: Lenz's Law

- Faraday's law of induction states that the emf induced by a change in magnetic flux is

$$\text{emf} = -N \frac{\Delta \Phi}{\Delta t}$$

when flux changes by $\Delta \Phi$ in a time Δt .

- If emf is induced in a coil, N is its number of turns.
- The minus sign means that the emf creates a current I and magnetic field B that *oppose the change in flux* $\Delta \Phi$ —this opposition is known as Lenz's law.

23.3 Motional Emf

- An emf induced by motion relative to a magnetic field B is called a *motional emf* and is given by

$$\text{emf} = B\ell v \quad (B, \ell, \text{ and } v \text{ perpendicular}),$$

where ℓ is the length of the object moving at speed v relative to the field.

23.4 Eddy Currents and Magnetic Damping

- Current loops induced in moving conductors are called eddy currents.
- They can create significant drag, called magnetic damping.

23.5 Electric Generators

- An electric generator rotates a coil in a magnetic field, inducing an emf given as a function of time by

$$\text{emf} = NAB\omega \sin \omega t,$$

where A is the area of an N -turn coil rotated at a constant angular velocity ω in a uniform magnetic field B .

- The peak emf emf_0 of a generator is $\text{emf}_0 = NAB\omega$.

23.6 Back Emf

- Any rotating coil will have an induced emf—in motors, this is called back emf, since it opposes the emf input to the motor.

23.7 Transformers

- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils are related by $\frac{V_s}{V_p} = \frac{N_s}{N_p}$,

where V_p and V_s are the voltages across primary and secondary coils having N_p and N_s turns.

- The currents I_p and I_s in the primary and secondary coils are related by $\frac{I_s}{I_p} = \frac{N_p}{N_s}$.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

23.8 Electrical Safety: Systems and Devices

- Electrical safety systems and devices are employed to prevent thermal and shock hazards.
- Circuit breakers and fuses interrupt excessive currents to prevent thermal hazards.
- The three-wire system guards against thermal and shock hazards, utilizing live/hot, neutral, and earth/ground wires, and grounding the neutral wire and case of the appliance.
- A ground fault interrupter (GFI) prevents shock by detecting the loss of current to unintentional paths.
- An isolation transformer insulates the device being powered from the original source, also to prevent shock.
- Many of these devices use induction to perform their basic function.

23.9 Inductance

- Inductance is the property of a device that tells how effectively it induces an emf in another device.

- Mutual inductance is the effect of two devices in inducing emfs in each other.
- A change in current $\Delta I_1/\Delta t$ in one induces an emf emf_2 in the second:

$$\text{emf}_2 = -M \frac{\Delta I_1}{\Delta t},$$

where M is defined to be the mutual inductance between the two devices, and the minus sign is due to Lenz's law.

- Symmetrically, a change in current $\Delta I_2/\Delta t$ through the second device induces an emf emf_1 in the first:

$$\text{emf}_1 = -M \frac{\Delta I_2}{\Delta t},$$

where M is the same mutual inductance as in the reverse process.

- Current changes in a device induce an emf in the device itself.
- Self-inductance is the effect of the device inducing emf in itself.
- The device is called an inductor, and the emf induced in it by a change in current through it is

$$\text{emf} = -L \frac{\Delta I}{\Delta t},$$

where L is the self-inductance of the inductor, and $\Delta I/\Delta t$ is the rate of change of current through it. The minus sign indicates that emf opposes the change in current, as required by Lenz's law.

- The unit of self- and mutual inductance is the henry (H), where $1 \text{ H} = 1 \Omega \cdot \text{s}$.
- The self-inductance L of an inductor is proportional to how much flux changes with current. For an N -turn inductor,

$$L = N \frac{\Delta \Phi}{\Delta I}.$$

- The self-inductance of a solenoid is

$$L = \frac{\mu_0 N^2 A}{\ell} (\text{solenoid}),$$

where N is its number of turns in the solenoid, A is its cross-sectional area, ℓ is its length, and $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space.

- The energy stored in an inductor E_{ind} is

$$E_{\text{ind}} = \frac{1}{2} LI^2.$$

23.10 RL Circuits

- When a series connection of a resistor and an inductor—an RL circuit—is connected to a voltage source, the time variation of the current is

$$I = I_0(1 - e^{-t/\tau}) \quad (\text{turning on}).$$

where $I_0 = V/R$ is the final current.

- The characteristic time constant τ is $\tau = \frac{L}{R}$, where L is the inductance and R is the resistance.
- In the first time constant τ , the current rises from zero to $0.632I_0$, and 0.632 of the remainder in every subsequent time interval τ .
- When the inductor is shorted through a resistor, current decreases as $I = I_0 e^{-t/\tau}$ (turning off).

Here I_0 is the initial current.

- Current falls to $0.368I_0$ in the first time interval τ , and 0.368 of the remainder toward zero in each subsequent time τ .

23.11 Reactance, Inductive and Capacitive

- For inductors in AC circuits, we find that when a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle, or by a 90° phase angle.
- The opposition of an inductor to a change in current is expressed as a type of AC resistance.
- Ohm's law for an inductor is

$$I = \frac{V}{X_L},$$

where V is the rms voltage across the inductor.

- X_L is defined to be the inductive reactance, given by

$$X_L = 2\pi fL,$$

with f the frequency of the AC voltage source in hertz.

- Inductive reactance X_L has units of ohms and is greatest at high frequencies.
- For capacitors, we find that when a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle, or by a 90° phase angle.
- Since a capacitor can stop current when fully charged, it limits current and offers another form of AC resistance; Ohm's law for a capacitor is

$$I = \frac{V}{X_C},$$

where V is the rms voltage across the capacitor.

- X_C is defined to be the capacitive reactance, given by

$$X_C = \frac{1}{2\pi fC}.$$

- X_C has units of ohms and is greatest at low frequencies.

23.12 RLC Series AC Circuits

- The AC analogy to resistance is impedance Z , the combined effect of resistors, inductors, and capacitors, defined by the AC version of Ohm's law:

$$I_0 = \frac{V_0}{Z} \text{ or } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z},$$

where I_0 is the peak current and V_0 is the peak source voltage.

- Impedance has units of ohms and is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
- The resonant frequency f_0 , at which $X_L = X_C$, is

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

- In an AC circuit, there is a phase angle ϕ between source voltage V and the current I , which can be found from

$$\cos \phi = \frac{R}{Z},$$

- $\phi = 0^\circ$ for a purely resistive circuit or an RLC circuit at resonance.
- The average power delivered to an RLC circuit is affected by the phase angle and is given by

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos \phi,$$

$\cos \phi$ is called the power factor, which ranges from 0 to 1.