PHYS12 CH: 9

Statics and Torque

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Equilibrium Conditions

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- Applies to both linear and rotational motion
- Required for absence of acceleration

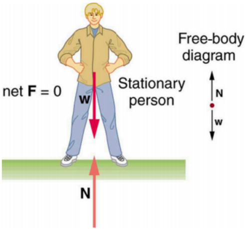

FIGURE 9.2 This motionless person is in static equilibrium. The forces acting on him add up to zero. Both forces are vertical in this case.

Torque and Rotational Equilibrium

Understanding Torque

Torque (τ) is the rotational equivalent of force:

- Measures effectiveness of force in changing angular velocity

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Torque (τ) is the rotational equivalent of force:

- Measures effectiveness of force in changing angular velocity
- Represents rotational acceleration capability
- Defined as: $\tau = rF \sin\theta$
- where:
  - r is distance from pivot to force application point
  - F is magnitude of force
  - θ is angle between force and position vector

Understanding the Angle θ in Torque

Definition of θ

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- θ is the angle between the force vector ($\vec{F}$) and the position vector ($\vec{r}$) from pivot to point of application
- As illustrated in Figure 9.6 and Figure 9.7
- **Tip:** Always identify the shortest rotation needed to align the force with the position vector

Key Insight

The torque magnitude depends on how effectively the force can produce rotation - maximum when $\theta = 90^\circ$, zero when $\theta = 0^\circ$ or 180°

Figure 9.6 - Torque Angle Visualization

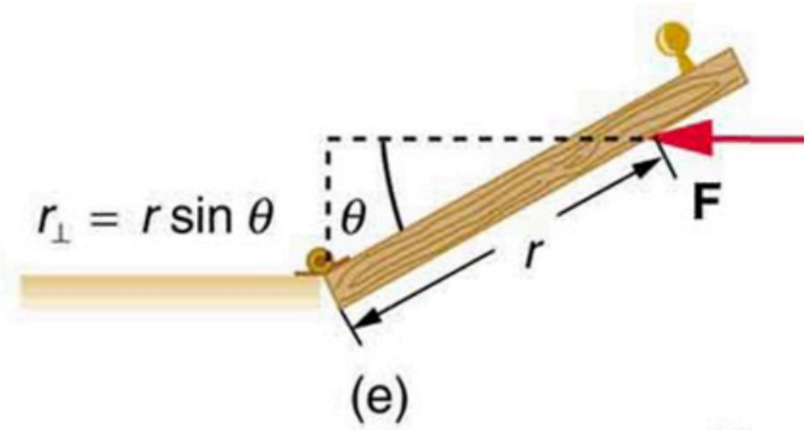
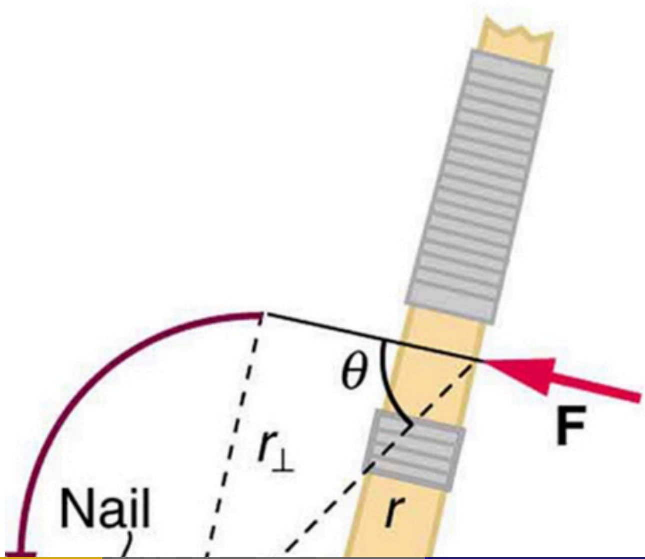


Figure: Figure 9.6: Visual representation of the angle θ between the force vector and position vector in torque calculations

Figure 9.7 - Torque Angle Examples



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- Where $\vec{r}$ is position vector from pivot point, $\vec{F}$ is force vector
- Perpendicular lever arm ($r_{\perp}$) is shortest distance from pivot to force line

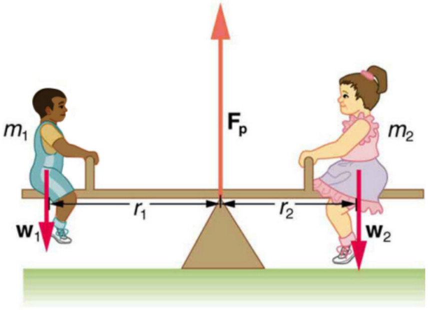

FIGURE 9.8 Two children balancing a seesaw satisfy both conditions for equilibrium. The lighter child sits farther from the pivot to create a torque equal in magnitude to that of the heavier child.

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- **Stable Equilibrium**
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- **Neutral Equilibrium**
  - Equilibrium independent of displacement
  - System remains in new position when displaced

Simple Machines

- **Basic Principles**
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  - Examples: lever, nail puller, wheelbarrow, crank

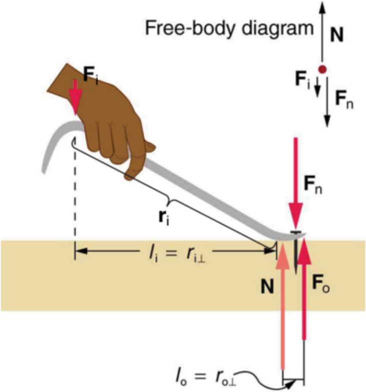

FIGURE 9.21 A nail puller is a lever with a large mechanical advantage. The external forces on the nail puller are represented by solid arrows. The force that the nail puller applies to the nail ($\mathbf{F}_o$) is not a force on the nail puller. The reaction force the nail exerts back on the puller ($\mathbf{F}_n$) is an external force and is equal and opposite to $\mathbf{F}_o$. The perpendicular lever arms of the input and output forces are l_i and l_o.

Mechanical Advantage

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  - Ratio of output force to input force
  - Key measure of machine effectiveness

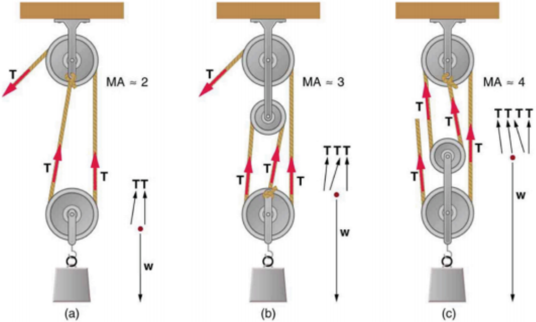

FIGURE 9.24 (a) The combination of pulleys is used to multiply force. The force is an integral multiple of tension if the pulleys are

Problem 2

When tightening a bolt, you push perpendicularly on a wrench with a force of 165 N at a distance of 0.140 m from the center of the bolt.

(a) How much torque are you exerting in newton $\times$ meters (relative to the center of the bolt)?

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(a) How much torque are you exerting in newton $\times$ meters (relative to the center of the bolt)?

(b) Convert this torque to foot-pounds.

Problem 2 - Solution

Solution:

(a) Using the torque equation $\tau = r_\perp F$:

- $\tau = 0.140 \text{ m} \times 165 \text{ N} = 23.1 \text{ N} \cdot \text{m}$

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(b) Converting to foot-pounds:

- $\tau = 23.1 \text{ N} \cdot \text{m} \times \frac{0.738 \text{ ft} \cdot \text{lb}}{1 \text{ N} \cdot \text{m}} = 17.0 \text{ ft} \cdot \text{lb}$

Problem 10

A 17.0-m-high and 11.0-m-long wall under construction and its bracing are shown in Figure 9.30. The wall is in stable equilibrium without the bracing but can pivot at its base. Calculate the force exerted by each of the 10 braces if a strong wind exerts a horizontal force of 650 N on each square meter of the wall. Assume that the net force from the wind acts at a height halfway up the wall and that all braces exert equal forces parallel to their lengths. Neglect the thickness of the wall.

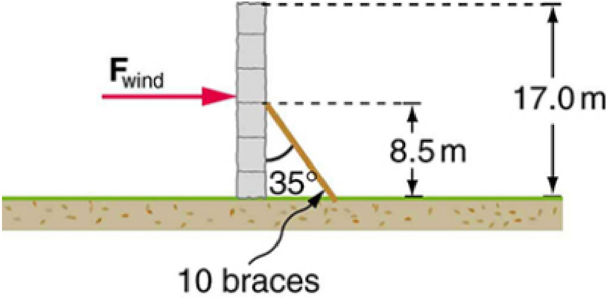

FIGURE 9.30

Problem Statement

A wall under construction with bracing:

- Wall dimensions:
  - Height: 17.0 m
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Goal: Calculate force exerted by each brace

Problem Analysis

- Key considerations:
  - Take pivot point at wall base

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    - Normal force (N)

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- Key considerations:
  - Take pivot point at wall base
  - Neglect wall thickness
  - Forces acting:
    - $$F_{brace} \times 10$$
    - Weight of wall (w)
    - Normal force (N)
- Using second condition for equilibrium:

$$\text{net}\tau = 0 \Rightarrow \text{net}\tau_{\text{cw}} = -\text{net}\tau_{\text{ccw}}$$

Mathematical Solution

$$
\begin{aligned}
\text{net}\tau_{\text{cw}} &= -\text{net}\tau_{\text{ccw}} \\
(8.5\ \text{m}) \times F_{\text{wind}} &= rF_b \times 10 = (8.5\ \text{m}) \sin 35^\circ \times F_b \times 10 \\
F_{\text{wind}} &= 10 \sin 35^\circ F_b \\
F_b &= \frac{F_{\text{wind}}}{10 \sin 35^\circ}
\end{aligned}
$$

Wind force calculation:

$$
\begin{aligned}
F_{\text{wind}} &= \frac{F}{A} \times A = 650\ \text{N}/\text{m}^2 \times 11.0\ \text{m} \times 17.0\ \text{m} \\
&= 121,550\ \text{N}
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\end{aligned}$$

Final Result

Therefore:

$$F_b = \frac{121,550 \text{ N}}{10 \times 0.5736} = 2.12 \times 10^4 \text{ N}$$

- Each brace must exert a force of 21.2 kN

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- This significant force demonstrates:
    - Importance of proper bracing in construction

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  - Impact of wind loads on tall structures

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Therefore:

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- Each brace must exert a force of 21.2 kN
- This significant force demonstrates:
  - Importance of proper bracing in construction
  - Impact of wind loads on tall structures
  - Need for careful engineering calculations

- https://www.youtube.com/watch?v=pK_oW62-zrc

- https://www.youtube.com/shorts/CxRw2n3lD7I

- https://www.youtube.com/shorts/sdC36VK_tY0

Problem 18

The center of gravity of a 5.0 kg pole held by a pole vaulter is 2.00 m from the left hand, and the hands are 0.700 m apart. Calculate the force exerted by:

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(a) his right hand

(b) his left hand

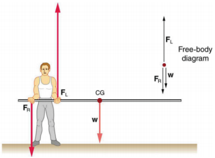

FIGURE 9.20 A pole vaulter is holding a pole horizontally with both hands. The center of gravity is to the left side of the vaulter.

Problem 18 - Solution

Solution: Using the center of gravity as reference:

(a) Taking pivot at left hand:

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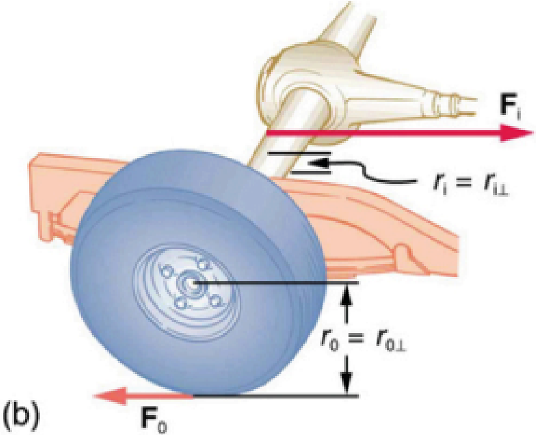
(b) Total weight must be supported:

- $F_L + F_R = (5.0\text{ kg})(9.80\text{ m/s}^2)$
- $F_L = 49\text{ N}$

Problem 22

A typical car has an axle with 2.0 cm radius driving a tire with a radius of 30.0 cm. What is its mechanical advantage assuming the very simplified model in Figure 9.23(b)?



Problem 22 - Solution

Solution: Mechanical advantage $= r_2/r_1 = 30.0\text{ cm}/2.0\text{ cm} = 15$

Step-by-step explanation:

1. Identify radii:
   - Inner radius $(r_1) = 2.0$ cm
   - Outer radius $(r_2) = 30.0$ cm

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Step-by-step explanation:

1. Identify radii:
   - Inner radius $(r_1) = 2.0$ cm
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2. Calculate mechanical advantage:
   - $\text{MA} = \frac{F_\text{o}}{F_\text{i}} = \frac{l_\text{i}}{l_\text{o}}$

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Problem 26

Verify that the force in the elbow joint in Example 9.4 is 407 N, as stated in the text.

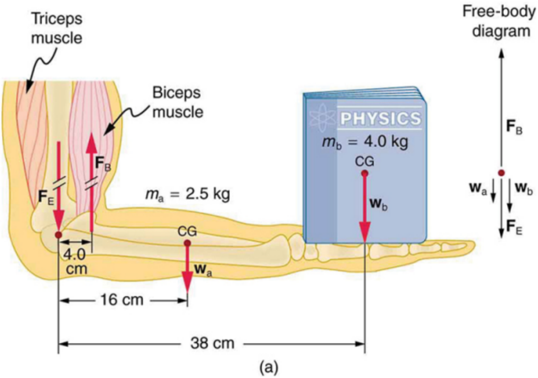

Figure: FIGURE 9.25 (a) The figure shows the forearm of a person holding a book. The biceps exert a force $\mathbf{F}_\mathrm{B}$ to support the weight of the forearm and the book. The triceps are assumed to be relaxed.

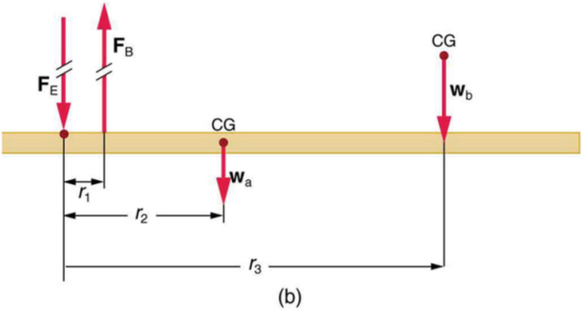

Figure: (b) Here, you can view an approximately equivalent mechanical system with the pivot at the elbow joint as seen in Example 9.4.

Problem Statement

Problem 26: Verify that the force in the elbow joint in Example 9.4 is 407 N.

Given Values

$$F_B = 470 \text{ N} \qquad r_1 = 4.00 \text{ cm}$$
$$m_a = 2.50 \text{ kg} \qquad r_2 = 16.0 \text{ cm}$$
$$m_b = 4.00 \text{ kg} \qquad r_3 = 38.0 \text{ cm}$$

Detailed Derivation

Starting from torque balance(second condition of equilibrium):

$$\tau_{\text{Bicep}} = \tau_{\text{arm}} + \tau_{\text{book}}$$

$$F_B r_1 = w_a r_2 + w_B r_3$$

Solving for F_B:

$$F_B = \frac{w_a r_2 + w_B r_3}{r_1}$$

For equilibrium of forces(first condition of equilibrium):

$$\begin{aligned} F_e &= w_a + w_B - F_B \\ &= w_a + w_B - \frac{w_a r_2 + w_B r_3}{r_1} \\ &= w_a \left(1 - \frac{r_2}{r_1}\right) + w_B \left(1 - \frac{r_3}{r_1}\right) \\ &= w_a \left(\frac{r_2}{r_1} - 1\right) + w_B \left(\frac{r_3}{r_1} - 1\right) \end{aligned}$$

Multiply both sides by r_1:

$$F_e \times r_1 = w_a \left(\frac{r_2}{r_1} - 1 \right) + w_B \left(\frac{r_3}{r_1} - 1 \right)$$

Calculation

Substituting the values:

$$F_E \times r_1 = (2.50\text{ kg})(9.80\text{ m/s}^2)\left(\frac{16.0\text{ cm}}{4.0\text{ cm}} - 1\right) + (4.00\text{ kg})(9.80\text{ m/s}^2)\left(\frac{38.0\text{ cm}}{4.00\text{ cm}} - 1\right)$$

Calculation

Substituting the values:

$$F_E \times r_1 = (2.50 \text{ kg})(9.80 \text{ m/s}^2)\left(\frac{16.0 \text{ cm}}{4.0 \text{ cm}} - 1\right) + (4.00 \text{ kg})(9.80 \text{ m/s}^2)\left(\frac{38.0 \text{ cm}}{4.00 \text{ cm}} - 1\right)$$

Final Result

Therefore:

$$F_E = 407 \text{ N}$$

This verifies the stated value in Example 9.4