

PHYS12 CH6: Gravitation and Kepler's Laws

Sections 6.5-6.6

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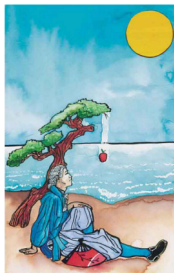
Learning Objectives

By the end of this lesson, you will be able to:

- Understand and explain Earth's gravitational force
- Describe the mathematical form of Newton's Universal Law of Gravitation
- Calculate gravitational forces between masses
- Explain the significance of the gravitational constant G
- Discuss the historical development of gravitational theory

Historical Development

- Newton (1687): First precise definition of gravitational force
- Showed it explains both:
 - Falling objects on Earth
 - Astronomical motions
- du Châtelet's contributions:
 - Translation and augmentation
 - Use of calculus to explain gravity



(a)



(b)

- <https://www.youtube.com/watch?v=7gf6YpdvtE0>

Newton's Universal Law of Gravitation

- Every particle in the universe attracts every other particle with a force along a line joining them
- Force is:

$$F = G \frac{m_1 m_2}{r^2}$$

where:

- $G = 6.674 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
- m_1, m_2 are the masses of the objects
- r is the distance between their centers
- The force is always attractive
- It follows the inverse square law

Gravity and Circular Motion

- For objects in circular orbit:

$$F_g = F_c$$

- This means:

$$G \frac{mM}{r^2} = m \frac{v^2}{r}$$

- Solving for orbital velocity:

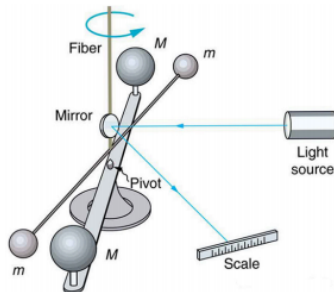
$$v = \sqrt{\frac{GM}{r}}$$

- Applications:

- Planetary orbits
- Artificial satellites
- Space stations

The Cavendish Experiment

- First accurate measurement of G (1798)
- Measured tiny gravitational attraction between lead spheres
- Led to first calculation of Earth's mass
- Modern version still used today



Example: Earth's Gravitational Force

Problem

Calculate the gravitational force between Earth ($M = 5.97 \times 10^{24}$ kg) and a 70 kg person at Earth's surface ($R = 6.37 \times 10^6$ m).

Solution

$$\begin{aligned} F &= G \frac{Mm}{r^2} \\ &= (6.67 \times 10^{-11}) \frac{(5.97 \times 10^{24})(70)}{(6.37 \times 10^6)^2} \\ &= ??? \text{ N} \end{aligned}$$

Kepler's First Law: Elliptical Orbits

Statement:

- All planets orbit the Sun in elliptical paths
- The Sun is located at one focus of the ellipse

Properties of Elliptical Orbits:

- **Semi-major axis** (a): half the longest diameter
- **Eccentricity** (e): measures deviation from circular orbit
 - $e = 0$: perfect circle
 - $0 < e < 1$: ellipse
 - Most planetary orbits have small e
- **Perihelion**: closest approach to Sun
- **Aphelion**: farthest point from Sun

Implications:

- Distance from Sun varies during orbit
- Orbital speed varies (connects to Second Law)
- True for all orbiting bodies under gravity

- <https://www.youtube.com/watch?v=Dvoe8lb5D1o>

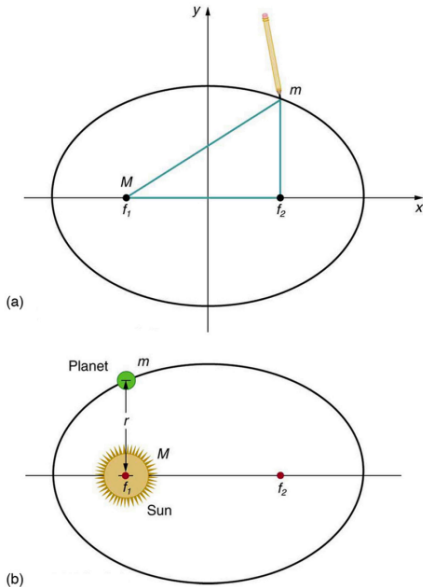


FIGURE 6.26 (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is a constant. You can draw an ellipse as shown by putting a pin at each focus, and then placing a string around a pencil and the pins and tracing a line on paper. A circle is a special case of an ellipse in which the two foci coincide (thus any point on the circle is the same distance from the center). (b) For any closed gravitational orbit, m follows an elliptical path with M at one focus. Kepler's first law states this fact for planets orbiting the Sun.

Kepler's Laws: Equal Areas (Second Law)

Kepler's Second Law:

- A line from the Sun to a planet sweeps out equal areas in equal times
- The shaded regions (A_1 , A_2 , A_3) have equal areas
- Important implications:
 - Planet moves fastest when closest to Sun
 - Planet moves slowest when farthest from Sun
 - Angular momentum is conserved

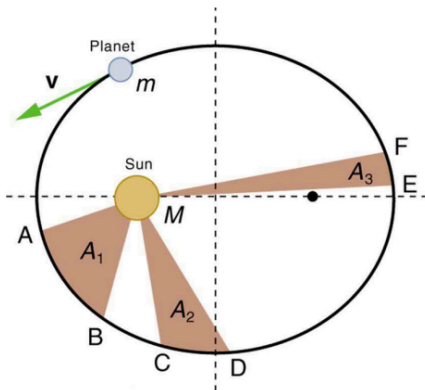


FIGURE 6.27 The shaded regions have equal areas. It takes equal times for m to go from A to B, from C to D, and from E to F. The mass m moves fastest when it is closest to M . Kepler's second law was originally devised for planets orbiting the Sun, but it has broader validity.

Kepler's Third Law of Planetary Motion

Mathematical Statement:

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

where:

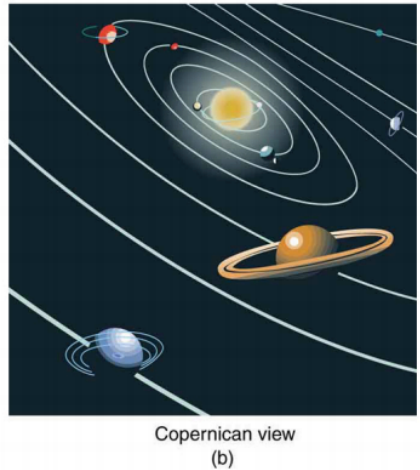
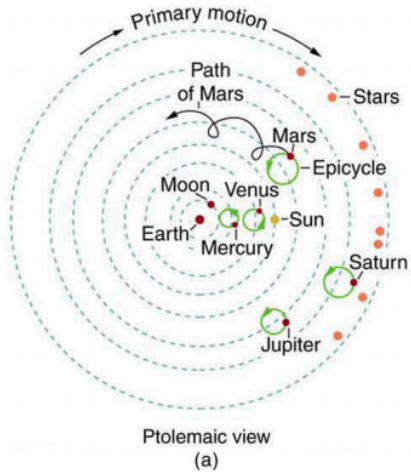
- T = orbital period
- r = average orbital radius
- Subscripts 1,2 refer to different planets

Key Points:

- Relates orbital period to orbital radius
- Squared period proportional to cubed radius
- Valid for all objects orbiting same central mass
- Can be derived from Newton's laws and universal gravitation

Example: If Planet 1 has period 1 year at 1 AU, a planet at 4 AU would have period:

$$T_2 = \sqrt{4^3} = 8 \text{ years}$$



- <https://www.youtube.com/watch?v=yC74lhJX9Ck>

What is a Planet? IAU Definition (2006)

Official IAU Definition

A planet in our solar system is a celestial body that:

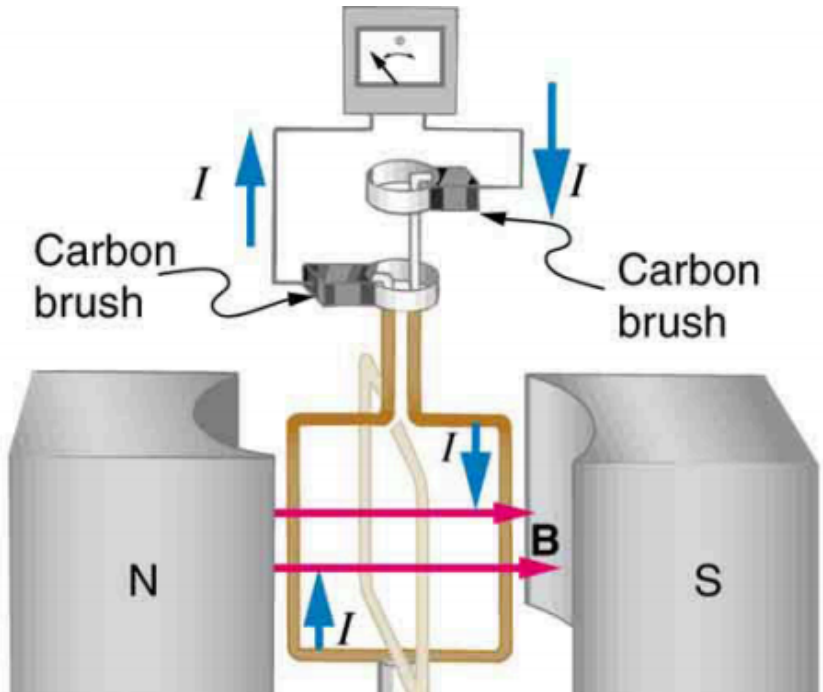
- ① Is in orbit around the Sun
 - Regular, elliptical orbit
 - Primary gravitational relationship with Sun
- ② Has sufficient mass for hydrostatic equilibrium
 - Strong enough gravity to become spherical
 - Overcomes rigid body forces
- ③ Has cleared its orbital neighborhood
 - Gravitationally dominant in its orbit
 - No similar-sized objects in its orbital path

Dwarf Planets and the Case of Pluto

Dwarf Planet Definition

A celestial body that:

- Orbits the Sun
 - Has hydrostatic equilibrium
 - Has NOT cleared its orbital neighborhood
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- Pluto was reclassified in 2006
 - Reasons for reclassification:
 - Shares its orbit with many Kuiper Belt objects
 - Not gravitationally dominant in its region
 - Similar to other objects in its orbital zone
 - Other recognized dwarf planets:
 - Ceres (in asteroid belt)
 - Eris (beyond Pluto)
 - Haumea and Makemake (Kuiper Belt)



Summary

Universal Gravitation:

- **Newton's Law:** $F_g = G \frac{m_1 m_2}{r^2}$
- **Gravitational Constant:** $G = 6.674 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
- **Historical Development:** Newton's theory and du Châtelet's contributions
- **Cavendish Experiment:** First measurement of G

Kepler's Laws:

- **First Law:** Planets follow elliptical orbits with Sun at one focus
- **Second Law:** Equal areas in equal times
- **Third Law:** $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

Orbital Motion:

- **Orbital Velocity:** $v = \sqrt{\frac{GM}{r}}$
- **Gravitational Force = Centripetal Force:** $F_g = F_c$
- **Applications:** Planets, satellites, space stations, asteroid mining