Physics Problem: Forces on a Ladder

Solution Guide

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Problem Statement

To get up on the roof, a person (mass 70.0 kg) places a 6.00 m aluminum ladder (mass 10.0 kg) against the house on a concrete pad with the base of the ladder 2.00 m from the house. The ladder rests against a plastic rain gutter, which we can assume to be frictionless. The center of mass of the ladder is 2 m from the bottom. The person is standing 3 m from the bottom. What are the magnitudes of the forces on the ladder at the top and bottom?

Solution

The forces involved are:

- The weight of the man (w)
- The weight of the ladder (W)
- The normal force of the ground on the ladder bottom (N)
- The normal force of the gutter on the ladder top (N')
- Friction between the ground and ladder bottom (f)

The condition of no net force horizontally leads to $f = N' \sin\theta$, where θ is the angle between the ladder and the ground:

$$\theta = \arccos\left(\frac{2}{6}\right) = 70.5^\circ \tag{1}$$

The condition of no net force vertically leads to $w + W = N + N' \cos\theta$, which combines with the previous condition to give $f = (w + W - N)\tan\theta$.

The condition of no torque about the ladder bottom leads to $3w\cos\theta + 2W\cos\theta = 6N'$, which combines with the first condition to give:

$$f = \left(\frac{1}{2}w + \frac{1}{3}W\right)\sin\theta\cos\theta \tag{2}$$

Combining these last two conditions, we can solve for N:

$$\begin{aligned}
f &= \left(\frac{1}{2}w + \frac{1}{3}W\right)\sin\theta\cos\theta = (w + W - N)\tan\theta \\
N &= \left(1 - \frac{\cos^2\theta}{2}\right)w + \left(1 - \frac{\cos^2\theta}{3}\right)W \\
&= (0.944)(9.80\,\mathrm{m\,s^{-2}})(70.0\,\mathrm{kg}) + (0.963)(9.80\,\mathrm{m\,s^{-2}})(10.0\,\mathrm{kg}) \\
&= 742\,\mathrm{N}
\end{aligned}$$

We can use this value to solve for f and N':

$$f = (w + W - N)\tan\theta = 119\,\mathrm{N}$$
$$N' = \frac{f}{\sin\theta} = 126\,\mathrm{N}$$

Final Answer

The magnitude of the force at the top is $N' = \boxed{126\,\mathrm{N}}$

The force at the bottom is the sum of friction and the normal force, with a magnitude of:

$$\sqrt{f^2 + N^2} = \boxed{751\,\mathrm{N}}$$