

## Section Summary

### 23.1 Induced Emf and Magnetic Flux

- The crucial quantity in induction is magnetic flux  $\Phi$ , defined to be  $\Phi = BA \cos \theta$ , where  $B$  is the magnetic field strength over an area  $A$  at an angle  $\theta$  with the perpendicular to the area.
- Units of magnetic flux  $\Phi$  are  $\text{T} \cdot \text{m}^2$ .
- Any change in magnetic flux  $\Phi$  induces an emf—the process is defined to be electromagnetic induction.

### 23.2 Faraday's Law of Induction: Lenz's Law

- Faraday's law of induction states that the emf induced by a change in magnetic flux is
- $\text{emf} = -N \frac{\Delta \Phi}{\Delta t}$   
when flux changes by  $\Delta \Phi$  in a time  $\Delta t$ .
- If emf is induced in a coil,  $N$  is its number of turns.
- The minus sign means that the emf creates a current  $I$  and magnetic field  $B$  that *oppose the change in flux*  $\Delta \Phi$ —this opposition is known as Lenz's law.

### 23.3 Motional Emf

- An emf induced by motion relative to a magnetic field  $B$  is called a *motional emf* and is given by
- $\text{emf} = B\ell v$  ( $B$ ,  $\ell$ , and  $v$  perpendicular),  
where  $\ell$  is the length of the object moving at speed  $v$  relative to the field.

### 23.4 Eddy Currents and Magnetic Damping

- Current loops induced in moving conductors are called eddy currents.
- They can create significant drag, called magnetic damping.

### 23.5 Electric Generators

- An electric generator rotates a coil in a magnetic field, inducing an emf—given as a function of time by
- $\text{emf} = NAB\omega \sin \omega t$ ,  
where  $A$  is the area of an  $N$ -turn coil rotated at a constant angular velocity  $\omega$  in a uniform magnetic field  $B$ .
- The peak emf  $\text{emf}_0$  of a generator is
- $\text{emf}_0 = NAB\omega$ .

### 23.6 Back Emf

- Any rotating coil will have an induced emf—in motors, this is called back emf, since it opposes the emf input to the motor.

### 23.7 Transformers

- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils are related by
- $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ ,  
where  $V_p$  and  $V_s$  are the voltages across primary and secondary coils having  $N_p$  and  $N_s$  turns.
- The currents  $I_p$  and  $I_s$  in the primary and secondary coils are related by  $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ .
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

### 23.8 Electrical Safety: Systems and Devices

- Electrical safety systems and devices are employed to prevent thermal and shock hazards.
- Circuit breakers and fuses interrupt excessive currents to prevent thermal hazards.
- The three-wire system guards against thermal and shock hazards, utilizing live/hot, neutral, and earth/ground wires, and grounding the neutral wire and case of the appliance.
- A ground fault interrupter (GFI) prevents shock by detecting the loss of current to unintentional paths.
- An isolation transformer insulates the device being powered from the original source, also to prevent shock.
- Many of these devices use induction to perform their basic function.

### 23.9 Inductance

- Inductance is the property of a device that tells how effectively it induces an emf in another device.
- Mutual inductance is the effect of two devices in inducing emfs in each other.
- A change in current  $\Delta I_1/\Delta t$  in one induces an emf  $\text{emf}_2$  in the second:
- $\text{emf}_2 = -M \frac{\Delta I_1}{\Delta t}$ ,

where  $M$  is defined to be the mutual inductance between the two devices, and the minus sign is due to Lenz's law.

- Symmetrically, a change in current  $\Delta I_2/\Delta t$  through the second device induces an emf  $\text{emf}_1$  in the first:
- $\text{emf}_1 = -M \frac{\Delta I_2}{\Delta t}$ ,  
where  $M$  is the same mutual inductance as in the reverse process.
- Current changes in a device induce an emf in the device itself.
- Self-inductance is the effect of the device inducing emf in itself.
- The device is called an inductor, and the emf induced in it by a change in current through it is
- $\text{emf} = -L \frac{\Delta I}{\Delta t}$ ,  
where  $L$  is the self-inductance of the inductor, and  $\Delta I/\Delta t$  is the rate of change of current through it. The minus sign indicates that emf opposes the change in current, as required by Lenz's law.
- The unit of self- and mutual inductance is the henry (H), where  $1 \text{ H} = 1 \Omega \cdot \text{s}$ .
- The self-inductance  $L$  of an inductor is proportional to how much flux changes with current. For an  $N$ -turn inductor,
- $L = N \frac{\Delta \Phi}{\Delta I}$ .
- The self-inductance of a solenoid is
- $L = \frac{\mu_0 N^2 A}{\ell}$  (solenoid),  
where  $N$  is its number of turns in the solenoid,  $A$  is its cross-sectional area,  $\ell$  is its length, and  $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$  is the permeability of free space.
- The energy stored in an inductor  $E_{\text{ind}}$  is
- $E_{\text{ind}} = \frac{1}{2} LI^2$ .

### 23.10 RL Circuits

- When a series connection of a resistor and an inductor—an  $RL$  circuit—is connected to a voltage source, the time variation of the current is
- $I = I_0(1 - e^{-t/\tau})$  (turning on).  
where  $I_0 = V/R$  is the final current.
- The characteristic time constant  $\tau$  is  $\tau = \frac{L}{R}$ , where  $L$  is the inductance and  $R$  is the resistance.
- In the first time constant  $\tau$ , the current rises from zero to  $0.632I_0$ , and 0.632 of the remainder in every subsequent time interval  $\tau$ .
- When the inductor is shorted through a resistor, current decreases as
- $I = I_0 e^{-t/\tau}$  (turning off).

Here  $I_0$  is the initial current.

- Current falls to  $0.368I_0$  in the first time interval  $\tau$ , and 0.368 of the remainder toward zero in each subsequent time  $\tau$ .

### 23.11 Reactance, Inductive and Capacitive

- For inductors in AC circuits, we find that when a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle, or by a  $90^\circ$  phase angle.
- The opposition of an inductor to a change in current is expressed as a type of AC resistance.
- Ohm's law for an inductor is

$$I = \frac{V}{X_L},$$

where  $V$  is the rms voltage across the inductor.

- $X_L$  is defined to be the inductive reactance, given by
- $X_L = 2\pi fL$ ,

with  $f$  the frequency of the AC voltage source in hertz.

- Inductive reactance  $X_L$  has units of ohms and is greatest at high frequencies.
- For capacitors, we find that when a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle, or by a  $90^\circ$  phase angle.
- Since a capacitor can stop current when fully charged, it limits current and offers another form of AC resistance; Ohm's law for a capacitor is
- $I = \frac{V}{X_C}$ ,

where  $V$  is the rms voltage across the capacitor.

- $X_C$  is defined to be the capacitive reactance, given by
- $X_C = \frac{1}{2\pi fC}$ .
- $X_C$  has units of ohms and is greatest at low frequencies.

### 23.12 RLC Series AC Circuits

- The AC analogy to resistance is impedance  $Z$ , the combined effect of resistors, inductors, and capacitors, defined by the AC version of Ohm's law:
- $I_0 = \frac{V_0}{Z}$  or  $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$ ,  
where  $I_0$  is the peak current and  $V_0$  is the peak source voltage.
- Impedance has units of ohms and is given by  $Z = \sqrt{R^2 + (X_L - X_C)^2}$ .

- The resonant frequency  $f_0$ , at which  $X_L = X_C$ , is
  - $f_0 = \frac{1}{2\pi\sqrt{LC}}$ .
  - In an AC circuit, there is a phase angle  $\phi$  between source voltage  $V$  and the current  $I$ , which can be found from
  - $\cos \phi = \frac{R}{Z}$ ,
  - $\phi = 0$  for a purely resistive circuit or an  $RLC$  circuit at resonance.
  - The average power delivered to an  $RLC$  circuit is affected by the phase angle and is given by
  - $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos \phi$ ,
- $\cos \phi$  is called the power factor, which ranges from 0 to 1.