



ALJABAR LINEAR

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Pertemuan 7

Prodi Informatika

Fakultas Teknik

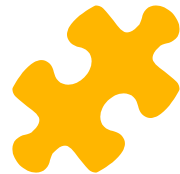
UNIVERSITAS SULAWESI BARAT

Sub-CPMK

Mampu memahami konsep matriks termasuk operasi-operasinya untuk menyelesaikan permasalahan matematik secara efektif dan efisien.

Indikator:

- Ketepatan memahami, menyelesaikan soal tentang kesamaan matriks
- Ketepatan memahami, menyelesaikan soal tentang operasi operasi matriks
- Ketepatan memahami, menyelesaikan soal tentang jenis jenis matriks




Definisi **Matriks**

DEFINITION 1 A *matrix* is a rectangular array of numbers. The numbers in the array are called the *entries* in the matrix.

Contoh:

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
Math	2	3	2	4	1	4	2
History	0	3	1	4	3	2	2
Language	4	1	3	1	0	0	2

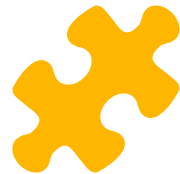

$$\begin{bmatrix} 2 & 3 & 2 & 4 & 1 & 4 & 2 \\ 0 & 3 & 1 & 4 & 3 & 2 & 2 \\ 4 & 1 & 3 & 1 & 0 & 0 & 2 \end{bmatrix} \quad 3$$



Pemanfaatan **Matriks**

Pemanfaatan matriks antara lain:

- Pada bidang keamanan komputer (Enkripsi data),
- Pemrograman yang membutuhkan array dalam Ilmu Komputer.



Notasi Matriks

- Huruf kapital untuk menotasikan matriks
- Huruf kecil untuk menotasikan kuantitas numerik(skalar) dalam matriks

$$A = \begin{bmatrix} 2 & 1 & 7 \\ 3 & 4 & 2 \end{bmatrix} \text{ or } C = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

- Entry nilai pada baris i dan kolom j matriks dinotasikan dengan a_{ij}

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} \\ a_{21} & a_{22} & \cdots & a_{2j} \\ \vdots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} \end{bmatrix}$$

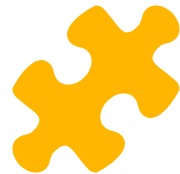
Diagram illustrating the notation for matrix entries a_{ij} . The matrix A is shown with rows labeled "baris 1", "baris 2", ..., "baris i " and columns labeled "kolom 1", "kolom 2", ..., "kolom j ". Arrows indicate the mapping from the row and column labels to the corresponding entry a_{ij} in the matrix.



Notasi Vektor dalam Matriks

- Notasi vektor menggunakan matriks direpresentasikan menggunakan **huruf kecil** yang **ditebalkan**
- $1 \times n$ vektor baris \mathbf{a} dan $m \times 1$ vektor kolom \mathbf{b} dapat ditulis sebagai berikut:

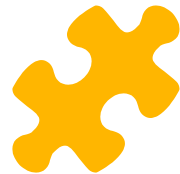
$$\mathbf{a} = [a_1 \quad a_2 \quad \cdots \quad a_n] \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$



Notasi Matriks

- Sebuah matriks A dengan baris = kolom = n disebut **square matrix of order n**
- Entry pada $a_{11}, a_{22}, \dots, a_{nn}$ disebut main dari A

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$



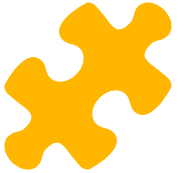
Operasi Matriks

DEFINITION 2 Two matrices are defined to be *equal* if they have the same size and their corresponding entries are equal.

Contoh:

Tentukan manakah matriks yang sama?

$$A = \begin{bmatrix} 2 & 1 \\ 3 & x \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \end{bmatrix}$$



Operasi Matriks

DEFINITION 3 If A and B are matrices of the same size, then the *sum* $A + B$ is the matrix obtained by adding the entries of B to the corresponding entries of A , and the *difference* $A - B$ is the matrix obtained by subtracting the entries of B from the corresponding entries of A . Matrices of different sizes cannot be added or subtracted.

Jika matriks A dan B memiliki ukuran yang sama

$$(A + B)_{ij} = (A)_{ij} + (B)_{ij} = a_{ij} + b_{ij} \quad \text{and} \quad (A - B)_{ij} = (A)_{ij} - (B)_{ij} = a_{ij} - b_{ij}$$

Operasi Penjumlahan-Pengurangan Matriks



Contoh: Jika diketahui

$$A = \begin{bmatrix} 2 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 4 & -2 & 7 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -4 & 3 & 5 & 1 \\ 2 & 2 & 0 & -1 \\ 3 & 2 & -4 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Tentukan:

1. $A + B$
2. $A - B$
3. $A + C$
4. $B + C$
5. $A - C$
6. $B - C$



Operasi Perkalian Skalar **Matriks**

Contoh: Jika diketahui

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 2 & 7 \\ -1 & 3 & -5 \end{bmatrix}, \quad C = \begin{bmatrix} 9 & -6 & 3 \\ 3 & 0 & 12 \end{bmatrix}$$

Tentukan:

1. $2A$
2. $(-1)B$
3. $1/3 C$



Operasi Perkalian Matriks

Jika diketahui $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$

Maka perkalian matriks A dan B

$$(1 \cdot 4) + (2 \cdot 0) + (4 \cdot 2) = 12$$

$$(1 \cdot 1) - (2 \cdot 1) + (4 \cdot 7) = 27$$

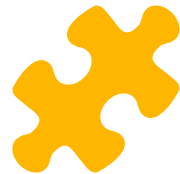
$$(1 \cdot 4) + (2 \cdot 3) + (4 \cdot 5) = 30$$

$$(2 \cdot 4) + (6 \cdot 0) + (0 \cdot 2) = 8$$

$$(2 \cdot 1) - (6 \cdot 1) + (0 \cdot 7) = -4$$

$$(2 \cdot 3) + (6 \cdot 1) + (0 \cdot 2) = 12$$

$$AB = \begin{bmatrix} 12 & 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix}$$



Partitioned **Matrices**

Sebuah matriks dapat dibagi(partisi) menjadi matriks-matriks yang lebih kecil dengan menggunakan aturan horisontal/vertikal di antara baris dan kolom terpilih.

Contoh:

Matriks A yang berukuran 3 x 4 dapat di partisi menjadi 3 kemungkinan

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$



Partitioned Matrices

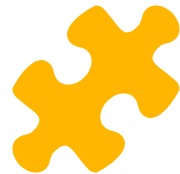
Contoh:

Partisi Matriks A yang berukuran 3 x 4

$$(1) \quad A = \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{array} \right] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$(2) \quad A = \left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{array} \right] = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \end{bmatrix}$$

$$(3) \quad A = \left[\begin{array}{c|c|c|c} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{array} \right] = [\mathbf{c}_1 \quad \mathbf{c}_2 \quad \mathbf{c}_3 \quad \mathbf{c}_4]$$



Partitioned Matrices

Contoh: Jika diketahui $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$

Maka vektor kolom ke-2 perkalian matriks A dan B

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix} = \begin{bmatrix} 27 \\ -4 \end{bmatrix}$$

↑↑
Second column of B Second column of AB

Transpose Matriks

Sistem Persamaan Linear

$$\mathbf{Ax} = \mathbf{b}$$

Augmentasi Matriks

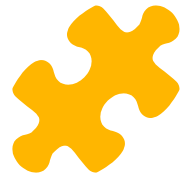
$$[A \mid \mathbf{b}] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array} \right]$$

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots & \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$





Transpose Matriks

DEFINITION 7 If A is any $m \times n$ matrix, then the *transpose of A* , denoted by A^T , is defined to be the $n \times m$ matrix that results by interchanging the rows and columns of A ; that is, the first column of A^T is the first row of A , the second column of A^T is the second row of A , and so forth.

Sifat-sifat Matriks

If A and B are matrices (with sizes such that the given matrix operations are defined) and c is a scalar, then the following properties are true.

1. $(A^T)^T = A$ Transpose of a transpose
2. $(A + B)^T = A^T + B^T$ Transpose of a sum
3. $(cA)^T = c(A^T)$ Transpose of a scalar multiple
4. $(AB)^T = B^T A^T$ Transpose of a product



Transpose Matriks

Contoh:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 5 & 6 \end{bmatrix}, \quad C = [1 \ 3 \ 5], \quad D = [4]$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \\ a_{14} & a_{24} & a_{34} \end{bmatrix}, \quad B^T = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 4 & 6 \end{bmatrix}, \quad C^T = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \quad D^T = [4]$$



Transpose Matriks

Contoh:

$$A = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 7 & 0 \\ -5 & 8 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 4 \\ 3 & 7 & 0 \\ -5 & 8 & 6 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 3 & -5 \\ -2 & 7 & 8 \\ 4 & 0 & 6 \end{bmatrix}$$

Interchange entries that are symmetrically positioned about the main diagonal.



Trace of Matrices

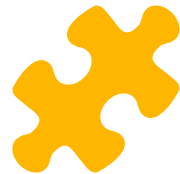
DEFINITION 8 If A is a square matrix, then the *trace of A* , denoted by $\text{tr}(A)$, is defined to be the sum of the entries on the main diagonal of A . The trace of A is undefined if A is not a square matrix.

Contoh:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 7 & 0 \\ 3 & 5 & -8 & 4 \\ 1 & 2 & 7 & -3 \\ 4 & -2 & 1 & 0 \end{bmatrix}$$

$$\text{tr}(A) = a_{11} + a_{22} + a_{33}$$

$$\text{tr}(B) = -1 + 5 + 7 + 0 = 11$$



Latihan

Diketahui: $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$,
 $D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$, $E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$ ◀

Tentukan:

(1) $D + E$

(4) BA

(7) $4 \operatorname{tr}(7B)$

(2) $-7C$

(5) $A(BC)$

(8) $\operatorname{tr}(4E^T - D)$

(3) $2B - C$

(6) $(DA)^T$



Latihan

Dengan menggunakan matriks berikut, tunjukkan bahwa $(AB)_T$ dan $B_T A_T$ adalah sama

$$A = \begin{bmatrix} 2 & 1 & -2 \\ -1 & 0 & 3 \\ 0 & -2 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$$



Thanks!

Any questions?

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