ALJABAR LINEAR

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Pertemuan 4

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4

Sub-CPMK

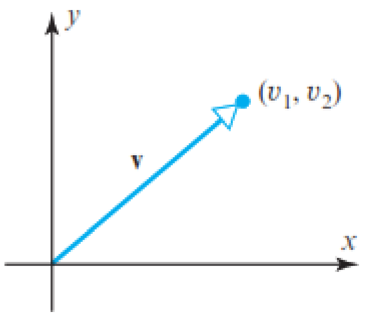
Mampu memahami konsep vector termasuk operasi-operasinya untuk menyelesaikan permasalahan matematik secara efektif dan efisien

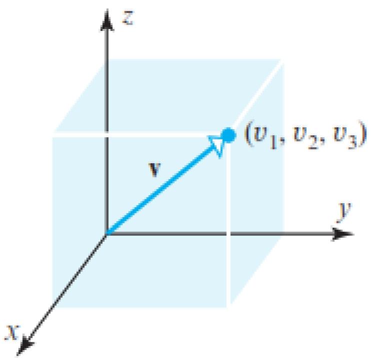
Indikator:

- Ketepatan memahami dan menyelesaikan soal tentang Dot product vector
- Memahami dan menyelesaikan soal tentang satuan vektor

REVIEW
Vektor dalam sistem koordinat

Vektor dalam ruang **2-dimensi** dan **3-dimensi**





v_1, v_2 adalah komponen dari **v**

Ditulis:

$\mathbf{v} = (v_1, v_2)$ atau

$\mathbf{v} = (v_1, v_2, v_3)$

REVIEW
Vektor dalam Sistem Koordinat

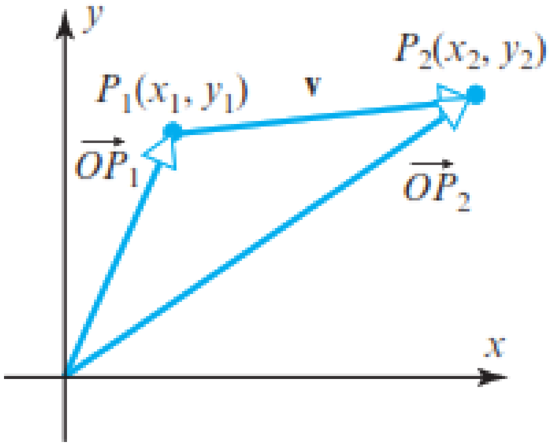
Jika terdapat dua vektor **v** dan **w** dalam ruang 3-dimensi, dimana:

$$\mathbf{v} = (v_1, v_2, v_3) \quad \text{and} \quad \mathbf{w} = (w_1, w_2, w_3)$$

Maka, kedua vektor ekuivalen jika:

$$v_1 = w_1, \quad v_2 = w_2, \quad v_3 = w_3$$

REVIEW
Vektor dalam Sistem Koordinat



$$\mathbf{v} = \overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$$

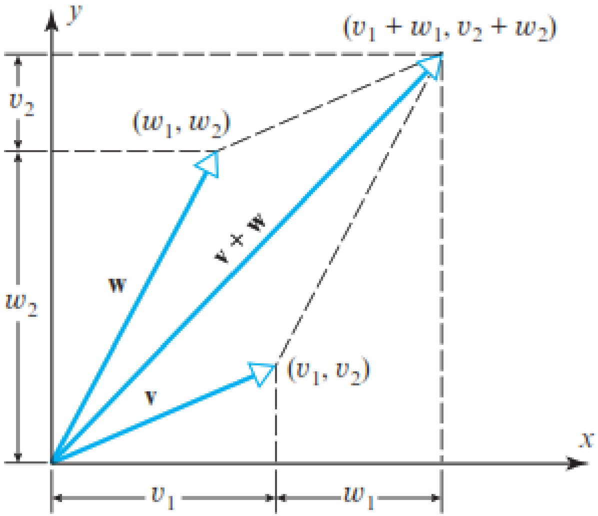
Jika suatu vektor memiliki titik pangkal yang tidak berada pada titik origin, sehingga $P_1(x_1, y_1)$ adalah titik pangkal dan $P_2(x_2, y_2)$ adalah titik ujung, maka:

$$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1} = (x_2, y_2) - (x_1, y_1) = (x_2 - x_1, y_2 - y_1)$$

$$\overrightarrow{P_1P_2} = (x_2 - x_1, y_2 - y_1)$$

Dalam ruang 3-d: $\overrightarrow{P_1P_2} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$

REVIEW
Operasi Penjumlahan Vektor



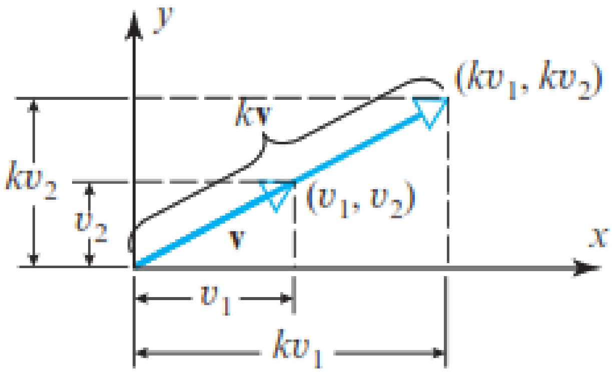
Jika diketahui vektor **v** dan **w**, maka penjumlahannya:

$$\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2)$$

REVIEW

Operasi Perkalian Vektor dengan Skalar

a



$$\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2)$$

$$k\mathbf{v} = (kv_1, kv_2)$$

REVIEW
Operasi Vektor

DEFINITION 3 If $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ and $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ are vectors in R^n, and if k is any scalar, then we define

$$\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2, \ldots, v_n + w_n) \tag{10}$$

$$k\mathbf{v} = (kv_1, kv_2, \ldots, kv_n) \tag{11}$$

$$-\mathbf{v} = (-v_1, -v_2, \ldots, -v_n) \tag{12}$$

$$\mathbf{w} - \mathbf{v} = \mathbf{w} + (-\mathbf{v}) = (w_1 - v_1, w_2 - v_2, \ldots, w_n - v_n) \tag{13}$$

REVIEW
Operasi Vektor

THEOREM 3.1.1 *If* $\mathbf{u}$, $\mathbf{v}$, *and* $\mathbf{w}$ *are vectors in* R^n, *and if* k *and* m *are scalars, then:*

(*a*) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(*b*) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

(*c*) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$

(*d*) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

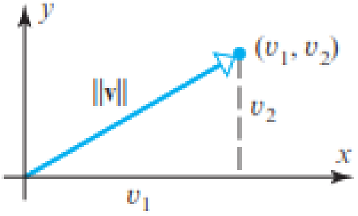
(*e*) $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$

(*f*) $(k + m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}$

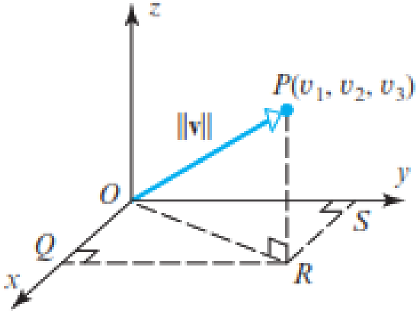
(*g*) $k(m\mathbf{u}) = (km)\mathbf{u}$

(*h*) $1\mathbf{u} = \mathbf{u}$

Norm of Vektor



(a)



(b)

Norma vektor = panjang dari suatu vektor.

Panjang vektor dalam ruang 2-d: $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$

DEFINITION 1 If $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ is a vector in R^n, then the ***norm*** of $\mathbf{v}$ (also called the ***length*** of $\mathbf{v}$ or the ***magnitude*** of $\mathbf{v}$) is denoted by $\|\mathbf{v}\|$, and is defined by the formula

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2} \tag{3}$$

Contoh 1

Tentukan nilai norma vektor **v** jika **v** = (-3, 2, 1)!

Norm of Vektor

THEOREM 3.2.1 *If* $\mathbf{v}$ *is a vector in* R^n, *and if* k *is any scalar, then:*

(*a*) $\|\mathbf{v}\| \geq 0$

(*b*) $\|\mathbf{v}\| = 0$ *if and only if* $\mathbf{v} = \mathbf{0}$

(*c*) $\|k\mathbf{v}\| = |k|\|\mathbf{v}\|$

Norm of Vektor

Vektor satuan = Suatu vektor bernorma 1.

$$\mathbf{u} = \frac{1}{\|\mathbf{v}\|}\mathbf{v}$$

Standar Unit Vektor

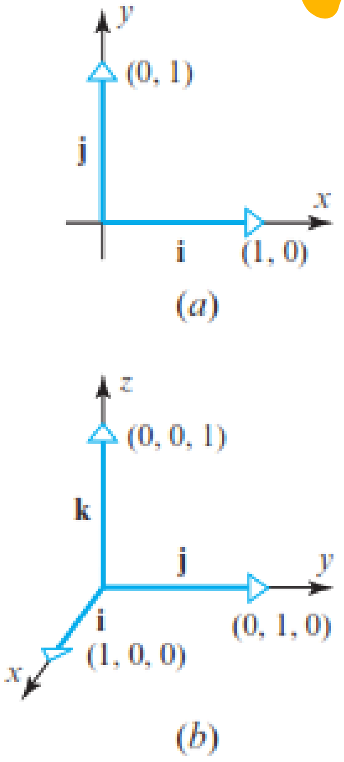
Standar unit vektor = satuan vektor dalam arah positif sumbu koordinat.

Notasi dalam ruang 2-d:

$$\mathbf{i} = (1, 0) \quad \text{and} \quad \mathbf{j} = (0, 1)$$

Notasi dalam ruang 3-d:

$$\mathbf{i} = (1, 0, 0), \quad \mathbf{j} = (0, 1, 0), \quad \text{and} \quad \mathbf{k} = (0, 0, 1)$$



Standar Unit Vektor

Setiapvektor dalam ruang 2-d dan 3-d dapat ditunjukkan sebagai sebuah kombinasi linear dari standar unit vektor sebagai berikut

$$\mathbf{v} = (v_1, v_2) = v_1(1, 0) + v_2(0, 1) = v_1\mathbf{i} + v_2\mathbf{j}$$

$$\mathbf{v} = (v_1, v_2, v_3) = v_1(1, 0, 0) + v_2(0, 1, 0) + v_3(0, 0, 1) = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$$

Standar Unit Vektor

Dengan menggeneralisasi standar unit vektor dalam ruang berdimensi-n diperoleh

$$\mathbf{e}_1 = (1, 0, 0, \ldots, 0), \quad \mathbf{e}_2 = (0, 1, 0, \ldots, 0), \ldots, \quad \mathbf{e}_n = (0, 0, 0, \ldots, 1)$$

Yangmana vektornya dapat di ekspresikan seperti berikut:

$$\mathbf{v} = (v_1, v_2, \ldots, v_n) = v_1\mathbf{e}_1 + v_2\mathbf{e}_2 + \cdots + v_n\mathbf{e}_n$$

Contoh 2

Tentukan kombinasi linear dari:

$(2, -3, 4)$

$(7, 3, -4, 5)$!

Jarak Vektor

Jika P_1 dan P_2 adalah titik dalam ruang 2-d dan 3-d maka jarak vektor:

dalam ruang 2-d

$$d = \|\overrightarrow{P_1 P_2}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

dalam ruang 3-d

$$d(\mathbf{u}, \mathbf{v}) = \|\overrightarrow{P_1 P_2}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

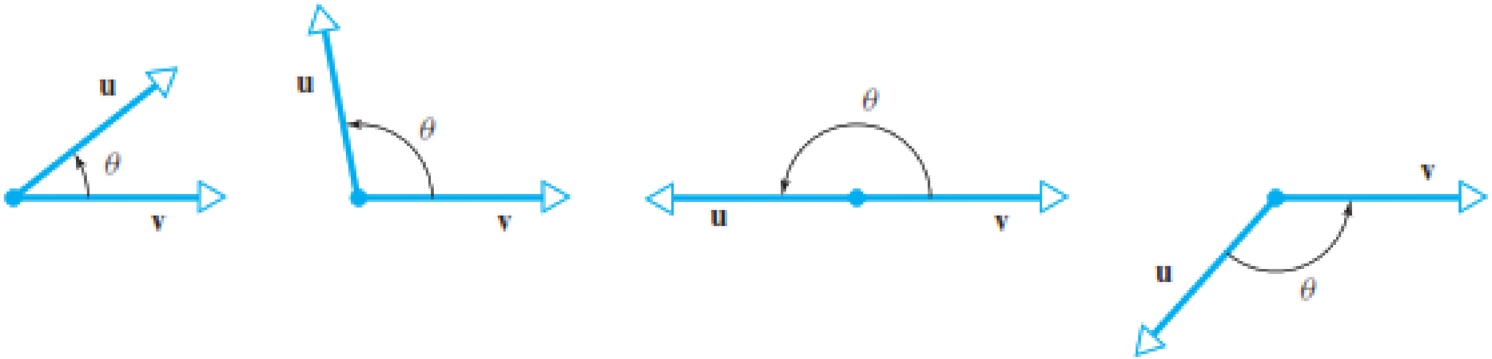
Contoh 3

Jika diketahui vektor: **u** = (1, 3, -2, 7) dan **v** = (0, 7, 2, 2)

Tentukan jarak(norma) **u** dan **v** !

Dot Product

Dalam operasi perkalian vektor diperlukan informasi terkait “sudut”.



The angle θ between $\mathbf{u}$ and $\mathbf{v}$ satisfies $0 \le \theta \le \pi$.

Dot **Product**

DEFINITION 3 If $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors in R^2 or R^3, and if θ is the angle between $\mathbf{u}$ and $\mathbf{v}$, then the ***dot product*** (also called the ***Euclidean inner product***) of $\mathbf{u}$ and $\mathbf{v}$ is denoted by $\mathbf{u} \cdot \mathbf{v}$ and is defined as

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos\theta \tag{12}$$

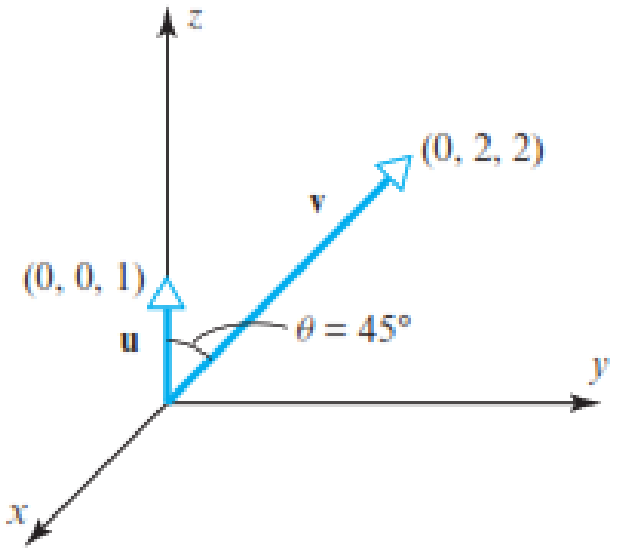
If $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$, then we define $\mathbf{u} \cdot \mathbf{v}$ to be 0.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

Dot Product

Contoh:

Tentukan hasil perkalian dot **u.v** berikut!



Dot Product

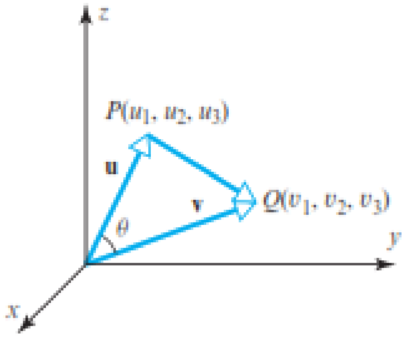


Figure 3.2.6

Let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ be two nonzero vectors. If, as shown in Figure 3.2.6, θ is the angle between $\mathbf{u}$ and $\mathbf{v}$, then the law of cosines yields

$$\|\overrightarrow{PQ}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - 2\|\mathbf{u}\|\|\mathbf{v}\|\cos\theta \tag{14}$$

Since $\overrightarrow{PQ} = \mathbf{v} - \mathbf{u}$, we can rewrite (14) as

$$\|\mathbf{u}\|\|\mathbf{v}\|\cos\theta = \tfrac{1}{2}(\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - \|\mathbf{v} - \mathbf{u}\|^2)$$

or

$$\mathbf{u} \cdot \mathbf{v} = \tfrac{1}{2}(\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - \|\mathbf{v} - \mathbf{u}\|^2)$$

Substituting

$$\|\mathbf{u}\|^2 = u_1^2 + u_2^2 + u_3^2, \qquad \|\mathbf{v}\|^2 = v_1^2 + v_2^2 + v_3^2$$

and

$$\|\mathbf{v} - \mathbf{u}\|^2 = (v_1 - u_1)^2 + (v_2 - u_2)^2 + (v_3 - u_3)^2$$

Dot Product

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

The companion formula for vectors in 2-space is

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2$$

Contoh 4

Tentukan hasil perkalian dot **u.v** dalam ruang 4-d berikut jika diketahui vektor:

$$\mathbf{u} = (-1, 3, 5, 7), \quad \mathbf{v} = (-3, -4, 1, 0)$$

Dot Product

THEOREM 3.2.2 *If* $\mathbf{u}$, $\mathbf{v}$, *and* $\mathbf{w}$ *are vectors in* R^n, *and if* k *is a scalar, then:*

(*a*) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$ **[Symmetry property]**

(*b*) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$ **[Distributive property]**

(*c*) $k(\mathbf{u} \cdot \mathbf{v}) = (k\mathbf{u}) \cdot \mathbf{v}$ **[Homogeneity property]**

(*d*) $\mathbf{v} \cdot \mathbf{v} \geq 0$ *and* $\mathbf{v} \cdot \mathbf{v} = 0$ *if and only if* $\mathbf{v} = \mathbf{0}$ **[Positivity property]**

Dot Product

THEOREM 3.2.3 *If* $\mathbf{u}$, $\mathbf{v}$, *and* $\mathbf{w}$ *are vectors in* R^n, *and if* k *is a scalar, then:*

(*a*) $\mathbf{0} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{0} = 0$

(*b*) $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$

(*c*) $\mathbf{u} \cdot (\mathbf{v} - \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} - \mathbf{u} \cdot \mathbf{w}$

(*d*) $(\mathbf{u} - \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} - \mathbf{v} \cdot \mathbf{w}$

(*e*) $k(\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot (k\mathbf{v})$

Contoh 5

Tentukan hasil perkalian dot vektor berikut!

$$(\mathbf{u} + 3\mathbf{v}) \cdot (2\mathbf{u} - 4\mathbf{v})$$

Dot **Product**

Vektor dapat dinotasikan dalam bentuk matriks seperti tabel di samping.

Form	Dot Product	Example	
$\mathbf{u}$ a column matrix and $\mathbf{v}$ a column matrix	$\mathbf{u}\cdot\mathbf{v}=\mathbf{u}^T\mathbf{v}=\mathbf{v}^T\mathbf{u}$	$\mathbf{u}=\begin{bmatrix}1\\-3\\5\end{bmatrix}$ $\mathbf{v}=\begin{bmatrix}5\\4\\0\end{bmatrix}$	$\mathbf{u}^T\mathbf{v}=[1\quad -3\quad 5]\begin{bmatrix}5\\4\\0\end{bmatrix}=-7$ $\mathbf{v}^T\mathbf{u}=[5\quad 4\quad 0]\begin{bmatrix}1\\-3\\5\end{bmatrix}=-7$
$\mathbf{u}$ a row matrix and $\mathbf{v}$ a column matrix	$\mathbf{u}\cdot\mathbf{v}=\mathbf{u}\mathbf{v}=\mathbf{v}^T\mathbf{u}^T$	$\mathbf{u}=[1\quad -3\quad 5]$ $\mathbf{v}=\begin{bmatrix}5\\4\\0\end{bmatrix}$	$\mathbf{u}\mathbf{v}=[1\quad -3\quad 5]\begin{bmatrix}5\\4\\0\end{bmatrix}=-7$ $\mathbf{v}^T\mathbf{u}^T=[5\quad 4\quad 0]\begin{bmatrix}1\\-3\\5\end{bmatrix}=-7$
$\mathbf{u}$ a column matrix and $\mathbf{v}$ a row matrix	$\mathbf{u}\cdot\mathbf{v}=\mathbf{v}\mathbf{u}=\mathbf{u}^T\mathbf{v}^T$	$\mathbf{u}=\begin{bmatrix}1\\-3\\5\end{bmatrix}$ $\mathbf{v}=[5\quad 4\quad 0]$	$\mathbf{v}\mathbf{u}=[5\quad 4\quad 0]\begin{bmatrix}1\\-3\\5\end{bmatrix}=-7$ $\mathbf{u}^T\mathbf{v}^T=[1\quad -3\quad 5]\begin{bmatrix}5\\4\\0\end{bmatrix}=-7$
$\mathbf{u}$ a row matrix and $\mathbf{v}$ a row matrix	$\mathbf{u}\cdot\mathbf{v}=\mathbf{u}\mathbf{v}^T=\mathbf{v}\mathbf{u}^T$	$\mathbf{u}=[1\quad -3\quad 5]$ $\mathbf{v}=[5\quad 4\quad 0]$	$\mathbf{u}\mathbf{v}^T=[1\quad -3\quad 5]\begin{bmatrix}5\\4\\0\end{bmatrix}=-7$ $\mathbf{v}\mathbf{u}^T=[5\quad 4\quad 0]\begin{bmatrix}1\\-3\\5\end{bmatrix}=-7$

Contoh 6

Jika diketahui $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & 1 \\ -1 & 0 & 1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$

Tunjukkan bahwa:

$$A\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot A^T\mathbf{v}$$

Thanks!

Any questions?

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