



ALJABAR LINEAR

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Pertemuan 5

Prodi Informatika

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UNIVERSITAS SULAWESI BARAT

Sub-CPMK

Mampu menyelesaikan masalah ruang vector



REVIEW

Operasi Vektor

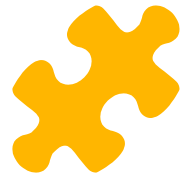
DEFINITION 3 If $\mathbf{v} = (v_1, v_2, \dots, v_n)$ and $\mathbf{w} = (w_1, w_2, \dots, w_n)$ are vectors in R^n , and if k is any scalar, then we define

$$\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2, \dots, v_n + w_n) \quad (10)$$

$$k\mathbf{v} = (kv_1, kv_2, \dots, kv_n) \quad (11)$$

$$-\mathbf{v} = (-v_1, -v_2, \dots, -v_n) \quad (12)$$

$$\mathbf{w} - \mathbf{v} = \mathbf{w} + (-\mathbf{v}) = (w_1 - v_1, w_2 - v_2, \dots, w_n - v_n) \quad (13)$$

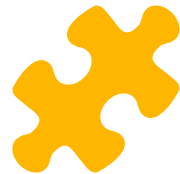


REVIEW

Operasi Vektor

THEOREM 3.1.1 *If \mathbf{u} , \mathbf{v} , and \mathbf{w} are vectors in R^n , and if k and m are scalars, then:*

- (a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
- (b) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
- (c) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$
- (d) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
- (e) $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$
- (f) $(k + m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}$
- (g) $k(m\mathbf{u}) = (km)\mathbf{u}$
- (h) $1\mathbf{u} = \mathbf{u}$



REVIEW

Norm of **Vektor**

THEOREM 3.2.1 *If \mathbf{v} is a vector in R^n , and if k is any scalar, then:*

- (a) $\|\mathbf{v}\| \geq 0$
- (b) $\|\mathbf{v}\| = 0$ if and only if $\mathbf{v} = \mathbf{0}$
- (c) $\|k\mathbf{v}\| = |k|\|\mathbf{v}\|$



REVIEW

Standar Unit Vektor

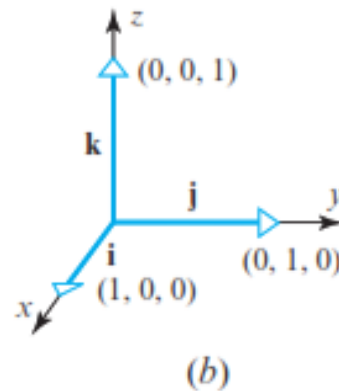
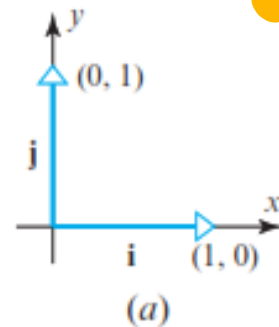
Standar unit vektor = satuan vektor dalam arah positif sumbu koordinat.

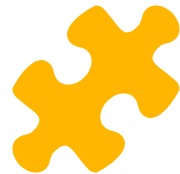
Notasi dalam ruang 2-d:

$$\mathbf{i} = (1, 0) \quad \text{and} \quad \mathbf{j} = (0, 1)$$

Notasi dalam ruang 3-d:

$$\mathbf{i} = (1, 0, 0), \quad \mathbf{j} = (0, 1, 0), \quad \text{and} \quad \mathbf{k} = (0, 0, 1)$$





REVIEW

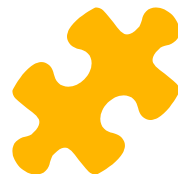
Standar Unit **Vektor**

Dengan menggeneralisasi standar unit vektor dalam ruang berdimensi-n diperoleh

$$\mathbf{e}_1 = (1, 0, 0, \dots, 0), \quad \mathbf{e}_2 = (0, 1, 0, \dots, 0), \dots, \quad \mathbf{e}_n = (0, 0, 0, \dots, 1)$$

Yangmana vektornya dapat di ekspresikan seperti berikut:

$$\mathbf{v} = (v_1, v_2, \dots, v_n) = v_1\mathbf{e}_1 + v_2\mathbf{e}_2 + \dots + v_n\mathbf{e}_n$$



REVIEW

Dot Product

DEFINITION 3 If \mathbf{u} and \mathbf{v} are nonzero vectors in R^2 or R^3 , and if θ is the angle between \mathbf{u} and \mathbf{v} , then the *dot product* (also called the *Euclidean inner product*) of \mathbf{u} and \mathbf{v} is denoted by $\mathbf{u} \cdot \mathbf{v}$ and is defined as

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta \quad (12)$$

If $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$, then we define $\mathbf{u} \cdot \mathbf{v}$ to be 0.

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}$$

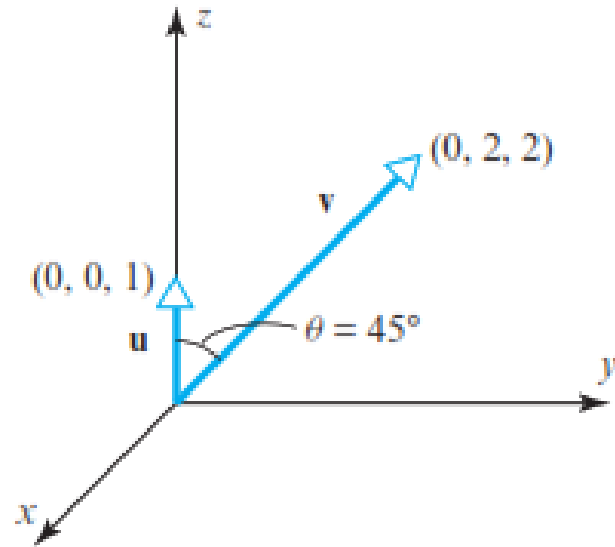


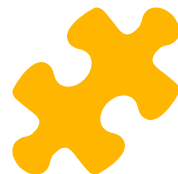
REVIEW

Dot Product

Contoh:

Tentukan hasil perkalian dot $\mathbf{u} \cdot \mathbf{v}$ berikut!





REVIEW

Dot Product

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

The companion formula for vectors in 2-space is

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2$$

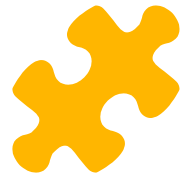


REVIEW

Dot Product

THEOREM 3.2.2 *If \mathbf{u} , \mathbf{v} , and \mathbf{w} are vectors in R^n , and if k is a scalar, then:*

- (a) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$ [Symmetry property]
- (b) $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$ [Distributive property]
- (c) $k(\mathbf{u} \cdot \mathbf{v}) = (k\mathbf{u}) \cdot \mathbf{v}$ [Homogeneity property]
- (d) $\mathbf{v} \cdot \mathbf{v} \geq 0$ and $\mathbf{v} \cdot \mathbf{v} = 0$ if and only if $\mathbf{v} = \mathbf{0}$ [Positivity property]



REVIEW

Dot Product

THEOREM 3.2.3 *If \mathbf{u} , \mathbf{v} , and \mathbf{w} are vectors in R^n , and if k is a scalar, then:*

(a) $\mathbf{0} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{0} = 0$

(b) $(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$

(c) $\mathbf{u} \cdot (\mathbf{v} - \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} - \mathbf{u} \cdot \mathbf{w}$

(d) $(\mathbf{u} - \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} - \mathbf{v} \cdot \mathbf{w}$

(e) $k(\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot (k\mathbf{v})$



Orthogonality (Ortogonalitas)

Sudut θ antara 2 vektor yang tidak nol \mathbf{u} dan \mathbf{v} adalah:

$$\theta = \cos^{-1} \left(\frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} \right)$$

Maka Ortogonalitas dapat di definisikan sebagai berikut:

DEFINITION 1 Two nonzero vectors \mathbf{u} and \mathbf{v} in R^n are said to be *orthogonal* (or *perpendicular*) if $\mathbf{u} \cdot \mathbf{v} = 0$. We will also agree that the zero vector in R^n is orthogonal to every vector in R^n .



Orthogonality (Ortogonalitas)

Contoh 1:

Show that $\mathbf{u} = (-2, 3, 1, 4)$ and $\mathbf{v} = (1, 2, 0, -1)$ are orthogonal vectors in R^4 .



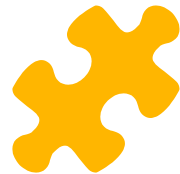
Cross Product

DEFINITION 1 If $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$ are vectors in 3-space, then the *cross product* $\mathbf{u} \times \mathbf{v}$ is the vector defined by

$$\mathbf{u} \times \mathbf{v} = (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$$

or, in determinant notation,

$$\mathbf{u} \times \mathbf{v} = \left(\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}, - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \right) \quad (1)$$



Cross Product

Contoh 2:

Find $\mathbf{u} \times \mathbf{v}$, where $\mathbf{u} = (1, 2, -2)$ and $\mathbf{v} = (3, 0, 1)$.

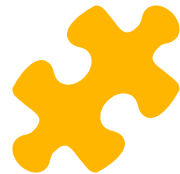


Cross Product

THEOREM 3.5.2 Properties of Cross Product

If \mathbf{u} , \mathbf{v} , and \mathbf{w} are any vectors in 3-space and k is any scalar, then:

- (a) $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$
- (b) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$
- (c) $(\mathbf{u} + \mathbf{v}) \times \mathbf{w} = (\mathbf{u} \times \mathbf{w}) + (\mathbf{v} \times \mathbf{w})$
- (d) $k(\mathbf{u} \times \mathbf{v}) = (k\mathbf{u}) \times \mathbf{v} = \mathbf{u} \times (k\mathbf{v})$
- (e) $\mathbf{u} \times \mathbf{0} = \mathbf{0} \times \mathbf{u} = \mathbf{0}$
- (f) $\mathbf{u} \times \mathbf{u} = \mathbf{0}$

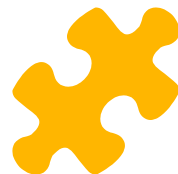


Cross Product

DEFINITION 2 If \mathbf{u} , \mathbf{v} , and \mathbf{w} are vectors in 3-space, then

$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$$

is called the *scalar triple product* of \mathbf{u} , \mathbf{v} , and \mathbf{w} .



Cross Product

Contoh 3:

Calculate the scalar triple product $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ of the vectors

$$\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}, \quad \mathbf{v} = \mathbf{i} + 4\mathbf{j} - 4\mathbf{k}, \quad \mathbf{w} = 3\mathbf{j} + 2\mathbf{k}$$



Cross Product

Contoh 4:

let $\mathbf{u} = (3, 2, -1)$, $\mathbf{v} = (0, 2, -3)$, and $\mathbf{w} = (2, 6, 7)$.

Compute the indicated vectors.

(a) $\mathbf{v} \times \mathbf{w}$

(b) $\mathbf{w} \times \mathbf{v}$

(c) $(\mathbf{u} + \mathbf{v}) \times \mathbf{w}$

(d) $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{w})$

(e) $\mathbf{v} \times \mathbf{v}$

(f) $(\mathbf{u} - 3\mathbf{w}) \times (\mathbf{u} - 3\mathbf{w})$



Thanks!

Any questions?

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