Learning Unit 2 – Integrals and Integration

Lesson 2 – Indefinite Integral Formulas

2.1 – Integral of a Constant

Definition:

Given the integrand C, wherein C is any constant, its integral, denoted by $\int C\,dx$, is equal to Cx + C. In symbols, that is, $\int \mathbf{C dx} = \mathbf{Cx + C}$.

Illustrative Examples:

1. $\int 5\,dx = 5x + C$
2. $\int -3\,dt = -3t + C$
3. $\int -\frac{1}{2}\,dx = -\frac{1}{2}x + C$
4. $\int 0.65\,du = 0.65u + C$
5. $\int \pi\,dy = \pi y + C$

2.2 – Integral of a Power

Definition:

Given the integrand x^n, wherein n is any real number, its integral, denoted by $\int x^n dx$, is equal to $\frac{x^{n+1}}{n+1}$ + C. In symbols, that is, $\int \mathbf{x^n dx} = \mathbf{\frac{x^{n+1}}{n+1} + C}$.

Illustrative Examples:

1. $\int x^{10} dx = \frac{x^{10+1}}{10+1} + C$
   $= \frac{x^{11}}{11} + C$

2. $\int x^{-3} dx = \frac{x^{-3+1}}{-3+1} + C$
   $= \frac{x^{-2}}{-2} + C$
   $= -\frac{1}{2x^2} + C$

3. $\int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$
   $= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C$
   $= \frac{\sqrt{x^3}}{\frac{3}{2}} + C$
   $= \sqrt{x^3} \cdot \frac{2}{3} + C$
   $= \frac{2\sqrt{x^3}}{3} + C$
   $= \frac{2x\sqrt{x}}{3} + C$

2.3 – Integral of a Constant and a Power

Definition:

Given the integrand $C \cdot x^n$, wherein n is any real number and C is any constant, its integral, denoted by $\int C \cdot x^n dx$, is equal to C multiplied by the integral of x^n. In symbols, that is, $\int \mathbf{C \cdot x^n dx} = \mathbf{C\int x^n dx + C}$.

Illustrative Examples:

1. $\int 3x^2\,dx = 3\int x^2\,dx$

   $= 3 \cdot \frac{x^3}{3} + C$

   $= x^3 + C$

2. $\int \frac{3}{5}x^5\,dx = \frac{3}{5}\int x^5\,dx$

   $= \frac{3}{5} \cdot \frac{x^6}{6} + C$

   $= \frac{x^6}{10} + C$

3. $\int -8x^{\frac{2}{5}}\,dx = -8\int x^{\frac{2}{5}}\,dx$

   $= -8 \cdot \frac{x^{\frac{7}{5}}}{\frac{7}{5}} + C$

   $= -8 \cdot \frac{5\sqrt[5]{x^7}}{7} + C$

   $= \frac{-40x\sqrt[5]{x^2}}{7} + C$

4. $\int 48x^{-7}\,dx = 48\int x^{-7}\,dx$

   $= 48 \cdot \frac{x^{-6}}{-6} + C$

   $= -\frac{8}{x^6} + C$

2.4 – Integral of Sum and Difference

Definition:

Given the integrand f(x) $\pm$ g(x), its integral is equal to the integral of each item. In symbols, that is, $\int[\mathbf{f(x)} \pm \mathbf{g(x)}]\,\mathbf{dx} = \int \mathbf{f(x)} \pm \int \mathbf{g(x)} + \mathbf{C}$.

Illustrative Examples:

1. $\int(3x^2 - 6x + 5)\,dx = \frac{3x^3}{3} - \frac{6x^2}{2} + 5x + C$

   $= x^3 - 3x^2 + 5x + C$

2. $\int(10x^4 - 9x^2 - 14x + 12)\,dx = \frac{10x^5}{5} - \frac{9x^3}{3} - \frac{14x^2}{2} + 12x + C$

   $= 2x^5 - 3x^3 - 7x^2 + 12x + C$

3. $\int(6x^3 + 3x^2 - 4x + 7)\,dx = \frac{6x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + 7x + C$

   $= \frac{3x^4}{2} + x^3 - 2x^2 + 7x + C$

   or $\frac{3x^4+2x^3-4x^2+14x+C_1}{2}$