

# National Institute of Technology Patna

## Department of Mathematics

Mid Semester Examination : Oct 2024

### MA14102 : Engineering Mathematics I

Branch: CSE III (1<sup>st</sup>-Semester)

Maximum Marks: 30

Time: 02.00 hours

**Answer all questions**

1. For what value of  $\lambda$ , the equations

$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 4z &= \lambda \\ x + 4y + 10z &= \lambda^2 \end{aligned}$$

1, 2

have a solution. Solve them completely in each case.

[5 Marks]

2. Reduce the matrix  $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & -2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$  to Reduced Row Echelon Form (RREF), and find its

rank.  $\rightarrow$  3

[5 Marks]

3. Find the inverse of the matrix, using Gauss-Jordan method:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & 0 \\ a^2 & a & 1 & 0 \\ a^3 & a^2 & a & 1 \end{bmatrix}$$

[5 Marks]

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -a & 1 & 0 & 0 \\ 0 & -a & 1 & 0 \\ 0 & 0 & -a & 1 \end{pmatrix}$$

4. Find the determinant of the following matrix,

$$\begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & 2 & 0 & 5 \\ 6 & -7 & 1 & 0 \\ 3 & 2 & 0 & 6 \end{bmatrix}$$

0

[5 Marks]

5. Show that following vectors in the polynomial space  $P_2[x]$  are linearly independent,  $p_1(x) = x^2 - 2x + 3$ ,  $p_2(x) = 2x^2 + x + 8$  and  $p_3(x) = x^2 + 8x + 7$ .

[5 Marks]

6. Determine a basis and dimension of  $W = \{[x, y, z, w]^t \in \mathbb{R}^4 \mid x + y - z + w = 0\}$ . Also extend the basis to form the basis for  $\mathbb{R}^4$ .

$$\begin{array}{cccc} x & y & z & w \\ 1 & 1 & -1 & 1 \\ 0 & 6 & -6 & 3 \\ \hline 0 & 0 & 0 & 1 \\ 0 & -9 & 9 & 0 \end{array}$$

\*\*\*\*\*ALL THE BEST\*\*\*\*\*

$$\begin{array}{l} 2x^2 - 4x + 6 \\ 2x^2 + x + 8 \\ \hline -5x - 2 \end{array}$$

$$\begin{array}{l} 2x^2 + 16x + 14 \\ 2x^2 + x + 8 \\ \hline 15x + 6 \end{array}$$

$$s_1 v_1 + s_2 v_2 + s_3 v_3 - s_4 v_4 = 0$$

$$s_1 = s_2 = s_3 = 0$$

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$$\begin{pmatrix} 1 & 1 & -1 & 1 \\ 3 & 3 & -3 & 3 \\ 1 & 1 & -1 & 1 \end{pmatrix}$$

$$1 - 9$$

$$2x^2$$

(8)

$$2x + 1 + 9$$

(11)

$$1 - 2$$

$$\lambda^2 - 1 - 3(\lambda - 1)$$

$$\lambda^2 - 1 - 3\lambda + 3$$

$$\lambda^2 - 3\lambda + 2$$

$$\lambda^3 + \lambda^2 + \lambda + 1$$

$$-\lambda^2 + \lambda + 1$$