National Institute of Technology Patna
Department of Mathematics

Mid Semester Examination : Oct 2024

MA14102 : Engineering Mathematics I

Branch: CSE III (1^{st}-Semester) **Maximum Marks:** $\boxed{30}$

Time: 02.00 hours

Answer all questions

1. For what value of λ, the equations

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y + 10z = \lambda^2$$

1, 2

have a solution. Solve them completely in each case. [5 Marks]

2. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & -2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ to Reduced Row Echelon Form(RREF), and find its rank. → 3 [5 Marks]

3. Find the inverse of the matrix, using Gauss-Jordan method: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ a & 1 & 0 & 0 \\ a^2 & a & 1 & 0 \\ a^3 & a^2 & a & 1 \end{bmatrix}$. [5 Marks]

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -a & 1 & 0 & 0 \\ 0 & -a & 1 & 0 \\ 0 & 0 & -a & 1 \end{pmatrix}$$

4. Find the determinant of the following matrix, $\begin{bmatrix} 3 & 0 & 0 & 1 \\ 0 & 2 & 0 & 5 \\ 6 & -7 & 1 & 0 \\ 3 & 2 & 0 & 6 \end{bmatrix}$. 0 [5 Marks]

5. Show that following vectors in the polynomial space $P_2[x]$ are linearly independent, $p_1(x) = x^2 - 2x + 3$, $p_2(x) = 2x^2 + x + 8$ and $p_3(x) = x^2 + 8x + 7$. [5 Marks]

6. Determine a basis and dimension of $\mathbb{W} = \{[x, y, z, w]^t \in \mathbb{R}^4 \mid x + y - z + w = 0\}$. Also extend the basis to form the basis for $\mathbb{R}^4$.

*****ALL THE BEST*****