Discrete Mathematics

Basic Logic

4th Lecture

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Class: 1st stage.
Time:8:30AM-10:30AM

Propositions

A ***proposition*** (or *statement*) is a declarative statement which is true or false, but not both. Consider, for example, the following six sentences:

(i) Ice floats in water.
(ii) China is in Europe.
(iii) $2 + 2 = 4$
(iv) $2 + 2 = 5$
(v) Where are you going?
(vi) Do your homework.

The first four are propositions, the last two are not. Also, (i) and (iii) are true, but (ii) and (iv) are false.

- Let p be a proposition. (also denoted by p), is the statement
- The *negation of p*, denoted by $\neg p$
- "It is not the case that p."
- The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

BASIC LOGICAL OPERATIONS

This section discusses the three basic logical operations of conjunction, disjunction, and negation which correspond, respectively, to the English words “and,” “or,” and “not.”

Conjunction, $p \wedge q$

- Any two propositions can be combined by the word “and” to form a compound proposition called the *conjunction* of the original propositions. Symbolically,
- $p \wedge q$
- read “*p* and *q*,” denotes the conjunction of *p* and *q*. Since $p \wedge q$ is a proposition it has a truth value, and this truth value depends only on the truth values of *p* and *q*.
- As a rule: If *p* and *q* are true, then $p \wedge q$ is true; otherwise $p \wedge q$ is false.

Truth table of $\wedge$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

“p and q”

T= True
F= False

BASIC LOGICAL OPERATIONS

EX_1: Consider the following four statements:
(i) Ice floats in water and $2 + 2 = 4$. (iii) China is in Europe and $2 + 2 = 4$.
(ii) Ice floats in water and $2 + 2 = 5$. (iv) China is in Europe and $2 + 2 = 5$.

Only the first statement is true. Each of the others is false since at least one of its substatements is false.

Disjunction, $p \vee q$

- Any two propositions can be combined by the word "or" to form a compound proposition called the *disjunction* of the original propositions. Symbolically,
- $p \vee q$
- read "p or q," denotes the disjunction of p and q. The truth value of $p \vee q$ depends only on the truth values of p and q
- *As a rule:* If p and q are false, then $p \vee q$ is false; otherwise $p \vee q$ is true.

Truth table of ∨

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

"p or q"

BASIC LOGICAL OPERATIONS

EX_2: Consider the following four statements:

(i) Ice floats in water or 2 + 2 = 4.
(ii) Ice floats in water or 2 + 2 = 5.
(iii) China is in Europe or 2 + 2 = 4.
(iv) China is in Europe or 2 + 2 = 5.

Only the last statement (iv) is false. Each of the others is true since at least one of its sub-statements is true.

Negation, $\neg p$

- Given any proposition *p*, another proposition, called the *negation* of *p*, can be formed by writing "It is not true that . . ." or "It is false that . . ." before *p* or, if possible, by inserting in *p* the word "not."
- Symbolically, the negation of *p*, read "not *p*," is denoted by $\neg p$
- *As a rule :* If *p* is true, then $\neg p$ is false; and if *p* is false, then $\neg p$ is true.

Truth table of $\neg$

p	$\neg p$
T	F
F	T

"not p"

BASIC LOGICAL OPERATIONS

EX_3 Consider the following six statements:
(a1) Ice floats in water.
(a2) It is false that ice floats in water.
(a3) Ice does not float in water.
(b1) $2 + 2 = 5$
(b2) It is false that $2 + 2 = 5$.
(b3) $2 + 2 \neq 5$

Then (a2) and (a3) are each the negation of (a1); and (b2) and (b3) are each the negation of (b1). Since (a1) is true, (a2) and (a3) are false; and since (b1) is false, (b2) and (b3) are true.

Exclusive Or "$\oplus$"

❖ Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

The Truth Table for the Exclusive Or of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

BASIC LOGICAL OPERATIONS

Solved Problems

P_1: Let p be "It is cold" and let q be "It is raining". Give a simple verbal sentence which describes each of the following statements: (a) $\neg p$; (b) $p \wedge q$; (c) $p \vee q$; (d) $q \vee \neg p$.

Sol:

In each case, translate $\wedge$, $\vee$, and $\sim$ to read "and," "or," and "It is false that" or "not," respectively, and then simplify the English sentence.

(a) It is not cold.

(b) It is cold and raining.

(c) It is cold or it is raining.

(d) It is raining or it is not cold.

BASIC LOGICAL OPERATIONS

Solved Problems

P_2: Find the truth table of $\neg p \wedge q$.

Sol:

p	q	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

BASIC LOGICAL OPERATIONS

Solved Problems

P_3: Show that the propositions $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent.

Sol:

p	q	$p \wedge q$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

$\neg(p \wedge q)$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

$\neg p \vee \neg q$

EX_5: Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

Solution: We construct the truth table for these compound propositions in the table below. Because the truth values of $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ agree, these compound propositions are logically equivalent.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

BASIC LOGICAL OPERATIONS

Conditional Statements

- Let p and q be propositions.
- The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise.
- In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

EX_4: Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

Sol: "If Maria learns discrete mathematics, then she will find a good job."

BASIC LOGICAL OPERATIONS

EX_6: What is the value of the variable x after the statement

if $2 + 2 = 4$ **then** $x := x + 1$

if $x = 0$ before this statement is encountered? (The symbol := stands for assignment. The statement $x := x + 1$ means the assignment of the value of $x + 1$ to x.)

Sol*:* Because $2 + 2 = 4$ is true, the assignment statement $x := x + 1$ is executed. Hence, x has the value $0 + 1 = 1$ after this statement is encountered.

THANK YOU