Discrete Mathematics

Basics of Counting Cont…

8th Lecture

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Class: 1st stage.
Time:8:30AM-10:30AM

Basics of Counting

THE SUBTRACTION RULE

- If a task can be done in either $n1$ ways or $n2$ ways, then
- The number of ways to do the task is $n1 + n2$ minus the number of ways to do the task that are common to the two different ways.
- The subtraction rule is also known as the **principle of inclusion–exclusion** especially when it is used to count the number of elements in the union of two sets.
- $|A1 \cup A2| = |A1| + |A2| - |A1 \cap A2|$

In other words, to find the number n(A∪B) of elements in the union of A and B, we add n(A) and n(B) and then we subtract n(A ∩ B); that is, we "include" n(A) and n(B), and we "exclude" n(A ∩ B). This follows from the fact that, when we add n(A) and n(B), we have counted the elements of (A ∩ B) twice. The above principle holds for any number of sets. We first state it for three sets.

Theorem: For any finite sets A, B, C we have

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

That is, we "include" $n(A), n(B), n(C)$, we "exclude" $n(A \cap B), n(A \cap C), n(B \cap C)$, and finally "include" $n(A \cap B \cap C)$.

Basics of Counting

EX_1: A computer company receives 350 applications from computer graduates for a job planning a line of new Web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

Sol: To find the number of these applicants who majored neither in computer science nor in business, we can subtract the number of students who majored either in computer science or in business (or both) from the total number of applicants. Let $A1$ be the set of students who majored in computer science and $A2$ the set of students who majored in business. Then $A1 \cup A2$ is the set of students who majored in computer science or business (or both), and $A1 \cap A2$ is the set of students who majored both in computer science and in business.

By the subtraction rule the number of students who majored either in computer science or in business (or both) equals
$|A1 \cup A2| = |A1| + |A2| - |A1 \cap A2| = 220 + 147 - 51 = 316.$
We conclude that $350 - 316 = 34$ of the applicants majored neither in computer science nor in business.

EX_2: Find the number of mathematics students at a college taking at least one of the languages French, German, and Russian, given the following data:

65 study French, 20 study French and German,

45 study German, 25 study French and Russian, 8 study all three languages.

42 study Russian, 15 study German and Russian,

We want to find $n(F \cup G \cup R)$ where F, G, and R denote the sets of students studying French, German, and Russian, respectively. By the Inclusion–Exclusion Principle,

$n(F \cup G \cup R) = n(F) + n(G) + n(R) - n(F \cap G) - n(F \cap R) - n(G \cap R) + n(F \cap G \cap R)$

$= 65 + 45 + 42 - 20 - 25 - 15 + 8 = 100$

Namely, 100 students study at least one of the three languages.

Basics of Counting

The Division Rule

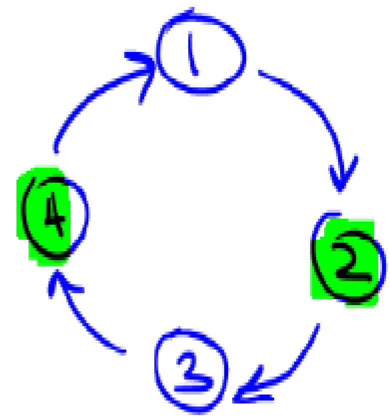
- There are n/d ways to do a task if it can be done using a procedure that can be carried out in n ways, and for every way w, exactly d of the n ways correspond to way w.
- We can restate the division rule in terms of sets: "If the finite set A is the union of n pairwise disjoint subsets each with d elements, then $n = |A|/d$."

EX_3: How many different ways are there to seat four people around a circular table, where two seatings are considered the same when each person has the same left neighbor and the same right neighbor?

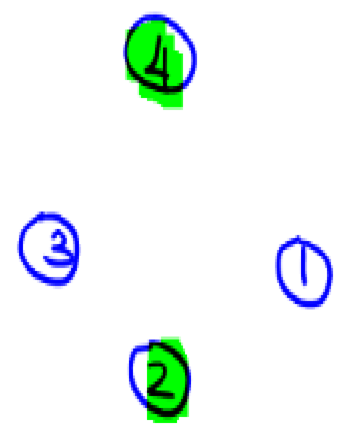
Sol: We arbitrarily select a seat at the table and label it seat 1. We number the rest of the seats in numerical order, proceeding clockwise around the table. Note that are four ways to select the person for seat 1, three ways to select the person for seat 2, two ways to select the person for seat 3, and one way to select the person for seat 4. Thus, there are $4! = 24$ ways to order the given four people for these seats. However, each of the four choices for seat 1 leads to the same arrangement, as we distinguish two arrangements only when one of the people has a different immediate left or immediate right neighbor. Because there are four ways to choose the person for seat 1, by the division rule there are $24/4 = 6$ different seating arrangements of four people around the circular table.

4 3 2 1

$$4(3)(2)(1) = 4! = 24$$



$$\frac{24}{4} = \boxed{6}$$

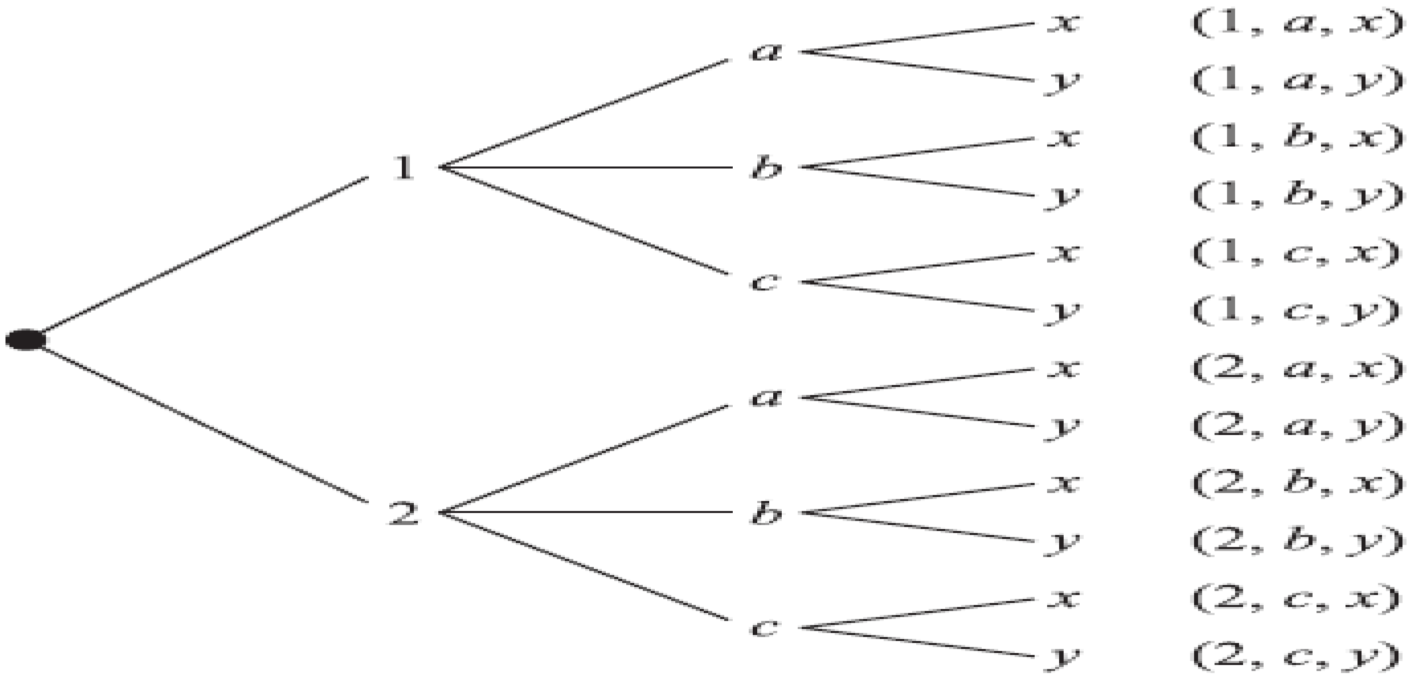


TREE DIAGRAMS

A tree diagram is a device used to enumerate all the possible outcomes of a sequence of events where each event can occur in a finite number of ways. The construction of tree diagrams is illustrated in the following example.

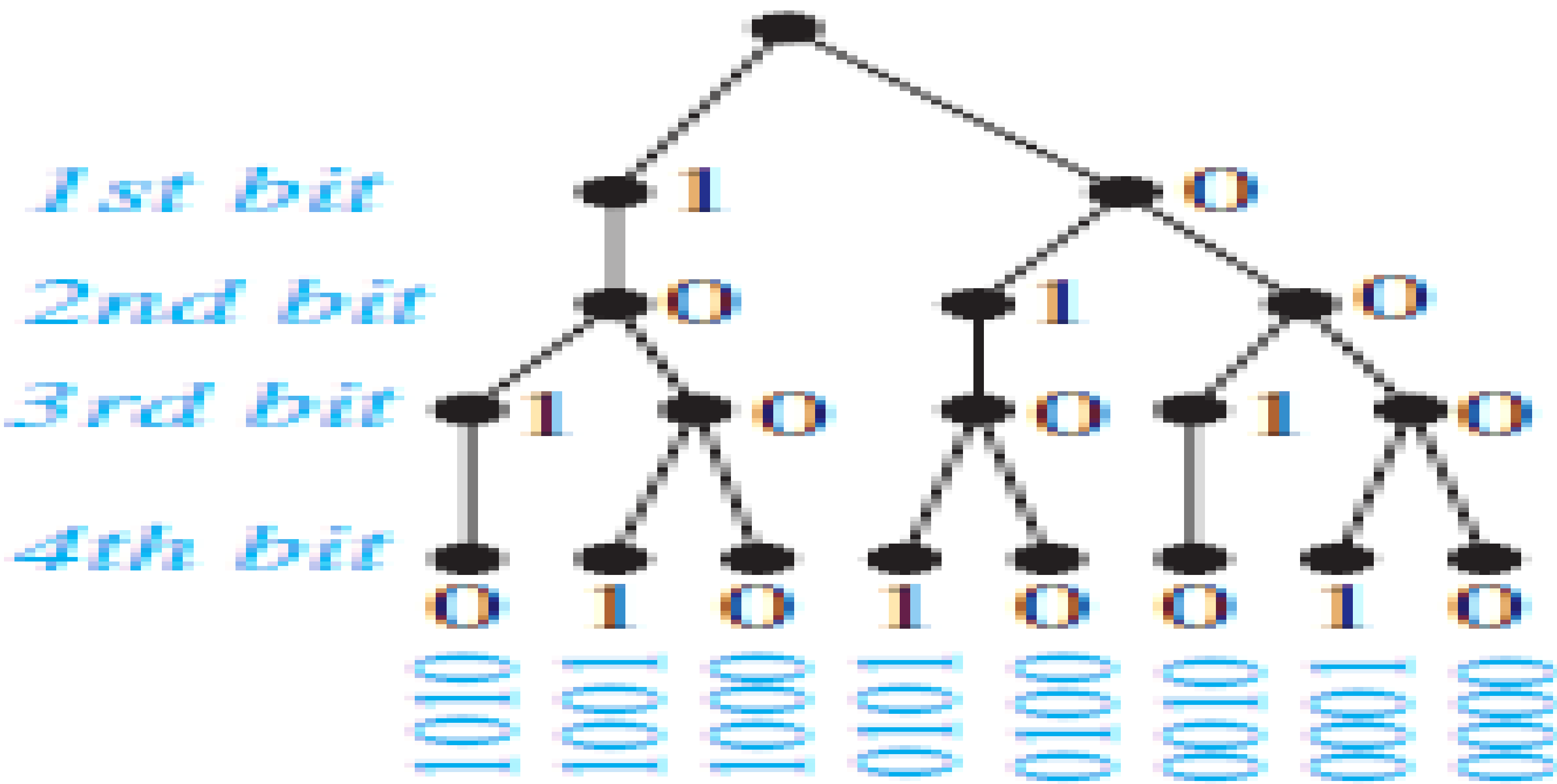
EX_4 Find the product set $A \times B \times C$, where $A = \{l, 2\}$, $B = \{a, b, c\}$, $C = \{x, y\}$.

The tree diagram for $A \times B \times C$ appears in Fig. below. Here the tree is constructed from left to right, and the number of branches at each point corresponds to the possible outcomes of the next event. Each endpoint (leaf) of the tree is labeled by the corresponding element of $A \times B \times C$. As noted previously, $A \times B \times C$ has $n = 2(3)(2) = 12$ elements.



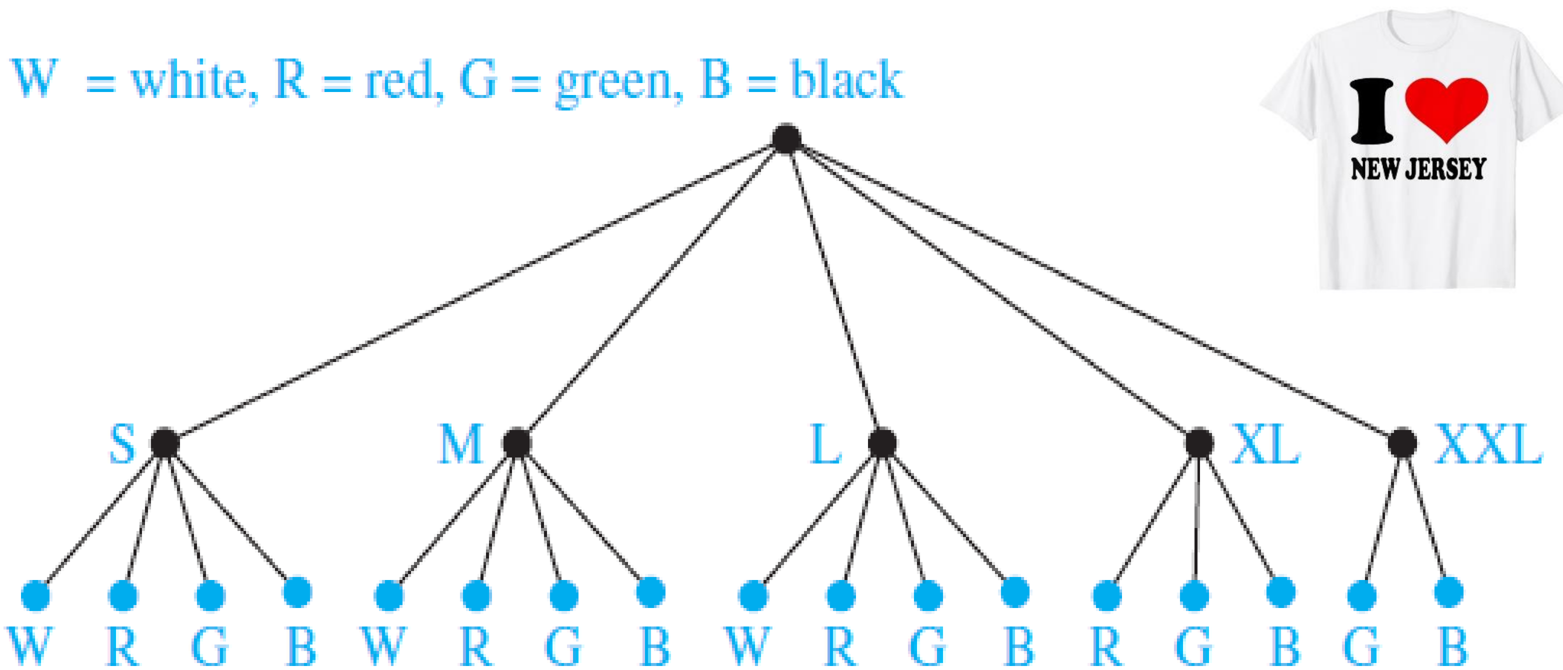
EX_5 How many bit strings of length four do not have two consecutive 1s (ones) ? (use tree diagram)

Sol.:: The tree diagram in Figure below displays all bit strings of length four without two consecutive 1s. We see that there are eight bit strings of length four without two consecutive 1s.



EX_ 6 Suppose that "I Love New Jersey" T-shirts come in five different sizes: S, M, L, XL, and XXL. Further suppose that each size comes in four colors, white, red, green, and black, except for XL, which comes only in red, green, and black, and XXL, which comes only in green and black. By using tree diagram, how many different shirts does a souvenir shop have to stock to have at least one of each available size and color of the T-shirt?

Sol : The tree diagram in Figure below displays all possible size and color pairs. It follows that the souvenir shop owner needs to stock 17 different T-shirts.



THANK YOU