Probability and Statistics

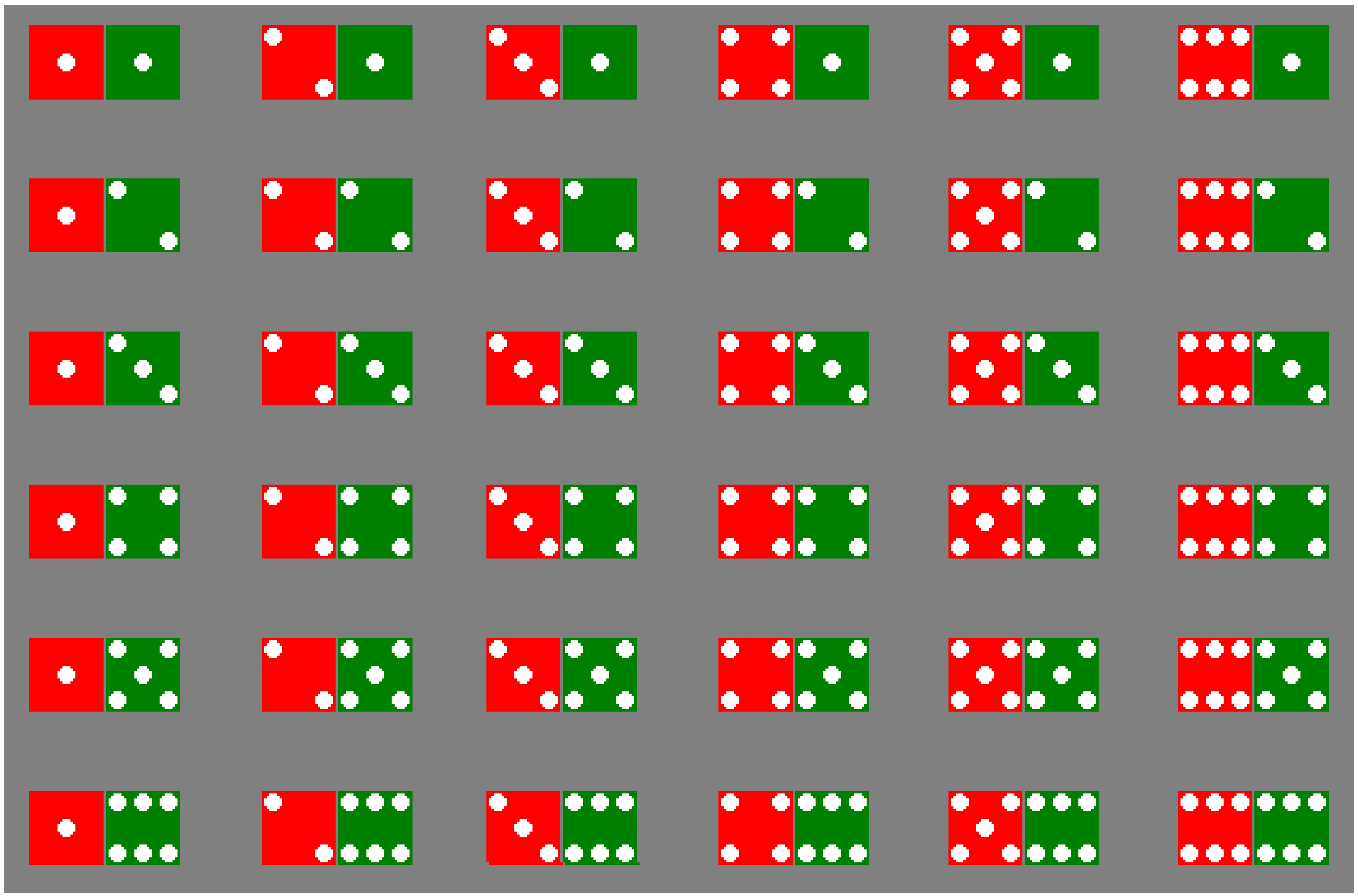
Review

Permutations & Combinations

Lecture 2.
Class 1.
Time: 8:30- 10:30
Department: BIT

Example 3

The sample space of throwing a pair of dice is



Example 3

Event	Simple events	Probability
Dice add to 3	(1,2),(2,1)	2/36
Dice add to 6	(1,5),(2,4),(3,3), (4,2),(5,1)	5/36
Red die show 1	(1,1),(1,2),(1,3), (1,4),(1,5),(1,6)	6/36
Green die show 1	(1,1),(2,1),(3,1), (4,1),(5,1),(6,1)	6/36

Counting Rules

- Sample space of throwing 3 dice has 216 entries, sample space of throwing 4 dice has 1296 entries, …
- At some point, we have to stop listing and start thinking …
- We need some counting rules

The *mn* Rule

- If an experiment is performed in two stages, with ***m*** ways to accomplish the first stage and ***n*** ways to accomplish the second stage, then there are ***mn*** ways to accomplish the experiment.
- This rule is easily extended to ***k*** stages, with the number of ways equal to

$$n_1 n_2 n_3 \ldots n_k$$

Example: Toss two coins. The total number of simple events is:

$$2 \times 2 = 4$$

Examples

Example: Toss three coins. The total number of simple events is:

$$2 \times 2 \times 2 = 8$$

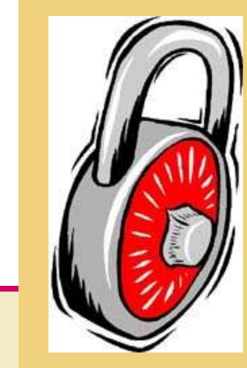
Example: Toss two dice. The total number of simple events is:

$$6 \times 6 = 36$$

Example: Toss three dice. The total number of simple events is:

$$6 \times 6 \times 6 = 216$$

Permutations التباديل



- The number of ways you can arrange ***n*** distinct مميزة objects, taking them ***r*** at a time is

$$P_r^n = \frac{n!}{(n-r)!}$$

where $n! = n(n-1)(n-2)...(2)(1)$ and $0! \equiv 1$.

Example: How many 3-digit lock combinations can we make from the numbers 1, 2, 3, and 4?

The order of the choice is important! →

$$P_3^4 = \frac{4!}{1!} = 4(3)(2) = 24$$

Examples

Example: A lock consists of five parts and can be assembled in any order. A quality control engineer wants to test each order for efficiency of assembly. How many orders are there?

The order of the choice is important!

$$P_5^5 = \frac{5!}{0!} = 5(4)(3)(2)(1) = 120$$

Combinations التوافيق

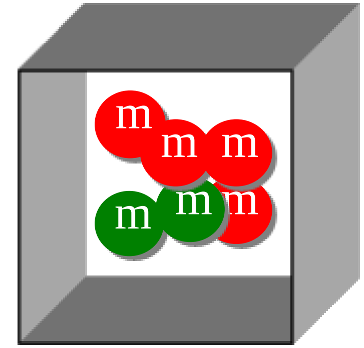
- The number of distinct combinations of ***n*** distinct objects that can be formed, taking them ***r*** at a time is $C_r^n = \dfrac{n!}{r!(n-r)!}$

Example: Three members of a 5-person committee must be chosen to form a subcommittee. How many different subcommittees could be formed?

The order of the choice is not important!

$$C_3^5 = \frac{5!}{3!(5-3)!} = \frac{5(4)(3)(2)1}{3(2)(1)(2)1} = \frac{5(4)}{(2)1} = 10$$

Example



- A box contains six M&Ms®, four red and two green. A child selects two M&Ms at random. What is the probability that exactly one is red?

The order of the choice is not important!

$$C_2^6 = \frac{6!}{2!4!} = \frac{6(5)}{2(1)} = 15$$

ways to choose 2 M & Ms.

$$C_1^2 = \frac{2!}{1!1!} = 2$$

ways to choose 1 green M & M.

$$C_1^4 = \frac{4!}{1!3!} = 4$$

ways to choose 1 red M & M.

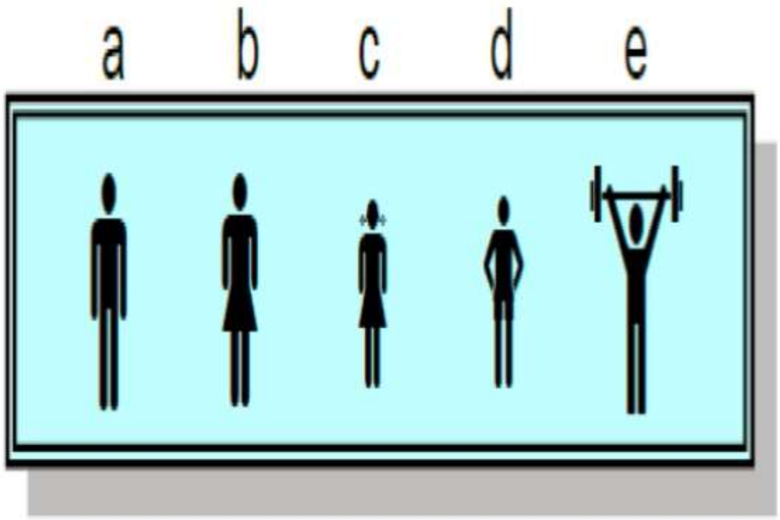
$4 \times 2 = 8$ ways to choose 1 red and 1 green M&M.

P(exactly one red) = 8/15

Homework 1

➢ **Question 1:** Suppose we wish to arrange $n = 5$ people {a, b, c, d, e}, standing side by side, for a portrait. How many such distinct portraits ("permutations") are possible?

Example:

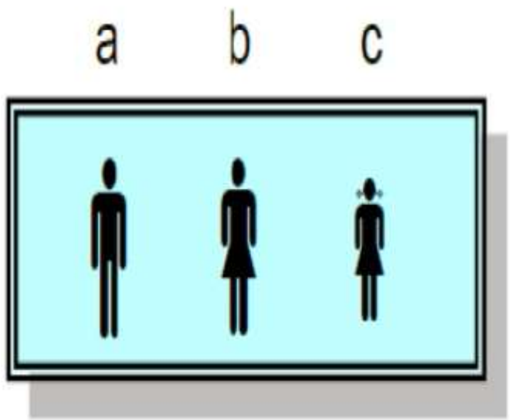


Here, every different ordering counts as a distinct permutation. For instance, the ordering (a,b,c,d,e) is distinct from (c,e,a,d,b), etc.

Homework 2

➢ **Question 2:** Now suppose we start with the same $n = 5$ people {a, b, c, d, e}, but we wish to make portraits *of only* $k = 3$ *of them at a time*. How many such distinct portraits are possible?

Example:

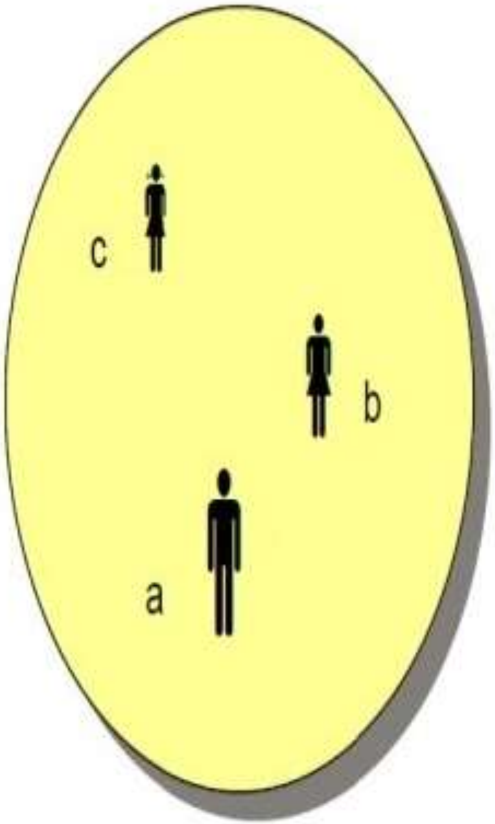


Again, as above, every different ordering counts as a distinct permutation. For instance, the ordering (a,b,c) is distinct from (c,a,b), etc.

Homework 3

➤ **Question 3:** Finally suppose that instead of portraits ("permutations"), we wish to form committees ("combinations") of $k = 3$ people from the original $n = 5$. How many such distinct committees are possible?

Example:



Now, every different ordering does NOT count as a distinct combination. For instance, the committee {a,b,c} is the <u>same</u> as the committee {c,a,b}, etc.

THANK YOU