Probability and Statistics
Course Descriptions
Aim of the course
Basic Definitions

Lecture 1.
Class 1.
Time: 8:30- 10:30
Department: BIT

Descriptive and Inferential Statistics

Statistics can be Divided into two basic types:

- Descriptive Statistics:
- Inferential Statistics:

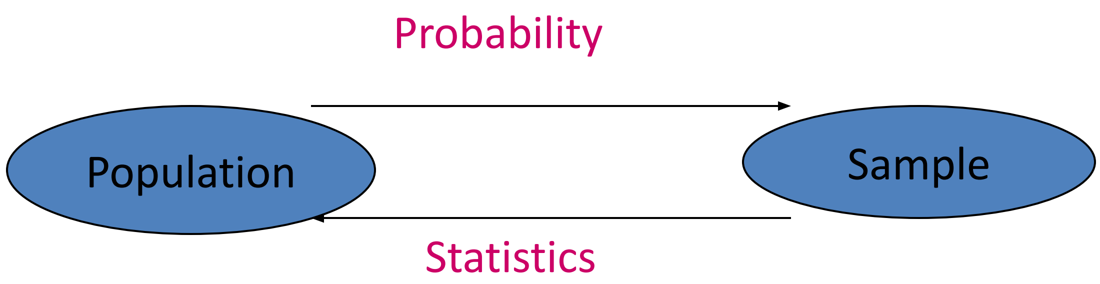
  Methods that making decisions or predictions about a population based on sampled data.

How is that?

By Probability

Why Learn Probability?

- Nothing in life is certain. In everything we do, we measure the chances of successful outcomes, from business to medicine to the weather
- A probability provides a quantitative description of the chances or likelihoods associated with various outcomes
- It provides a bridge between descriptive and inferential statistics



Probabilistic vs Statistical Reasoning

If I live in California

- Suppose I know exactly the proportions of car makes in California. Then I can find the probability that the first car I see in the street is a Ford. This is probabilistic reasoning as I know the population and predict the sample
- Now suppose that I do not know the proportions of car makes in California, but would like to estimate them. I observe a random sample of cars in the street and then I have an estimate of the proportions of the population. This is statistical reasoning

Basic Concepts

- An **experiment** is the process by which an observation (or measurement) is obtained.
- An **event** is an outcome of an experiment, usually denoted by a capital letter.

Experiments and Events

- **Experiment: Record an age**
  - A: person is 30 years old
  - B: person is older than 65
- **Experiment: Toss a die**
  - A: observe an odd number
  - B: observe a number greater than 2

Basic Concepts

- Two events are **mutually exclusive (يستبعد احدهما الاخر)** if, when one event occurs, the other cannot, and vice versa.

- **Experiment: Toss a die**
  - A: observe an odd number
  - B: observe a number greater than 2
  - C: observe a 6
  - D: observe a 3

Not Mutually Exclusive

Mutually Exclusive

B and C?
B and D?

Basic Concepts

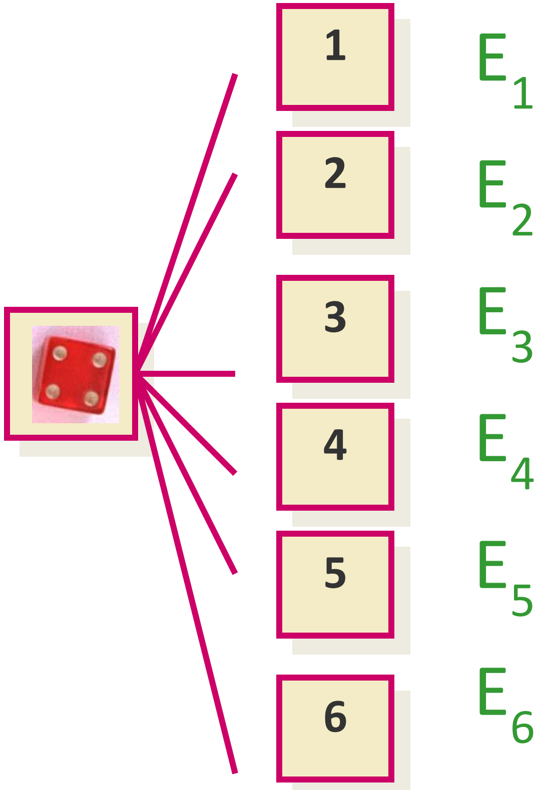
- An event that cannot be decomposed is called a **simple event.**

(الحدث الذي لا يمكن أن يتحلل يسمى حدث بسيط)

- Denoted by **E** with a subscript.
- Each simple event will be assigned a probability, measuring "how often" it occurs.
- The set of all simple events of an experiment is called the **sample space, S.**

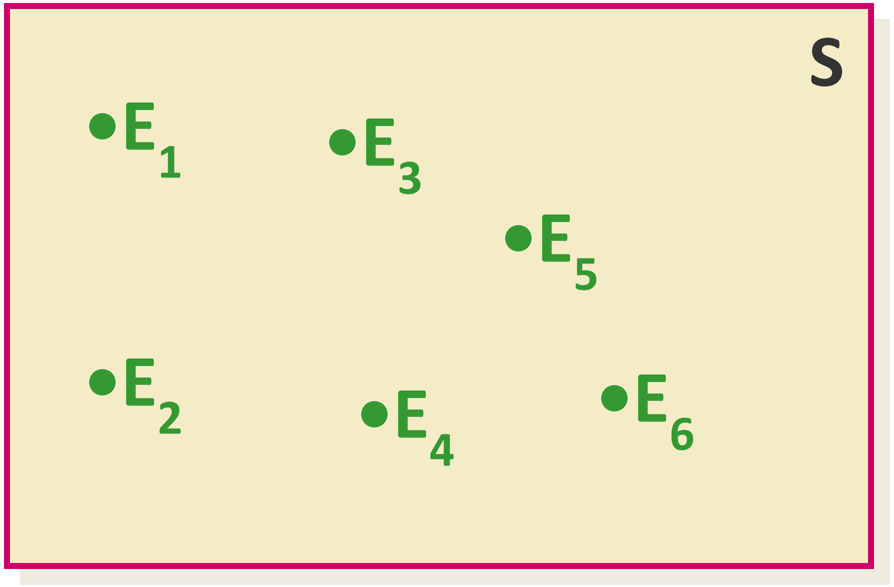
Example

- **The die toss:**
- Simple events:



Sample space:

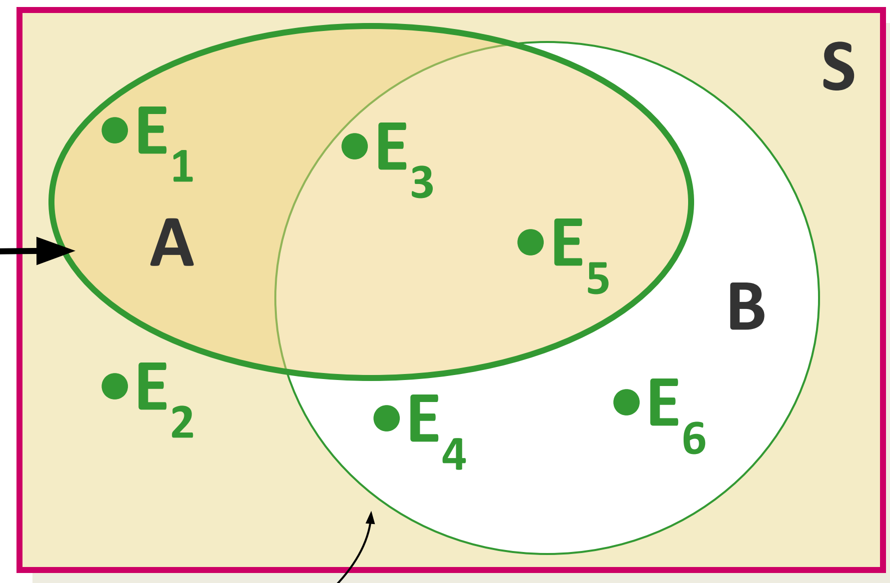
$$S = \{E_1, E_2, E_3, E_4, E_5, E_6\}$$



Basic Concepts

- An **event** is a collection of one or more **simple events.**

- **The die toss:**
  - A: an odd number
  - B: a number > 2



$A = \{E_1, E_3, E_5\}$

$B = \{E_3, E_4, E_5, E_6\}$

The Probability of an Event

- The probability of an event A measures "how often" A will occur. We write **P(A).**
- Suppose that an experiment is performed *n* times. The relative frequency for an event A is

$$\frac{\text{Number of times A occurs}}{n} = \frac{f}{n}$$

- If we let *n* get infinitely large,

$$\boxed{P(A) = \lim_{n \to \infty} \frac{f}{n}}$$

The Probability of an Event

- P(A) must be between 0 and 1.
  - If event A can never occur, P(A) = 0. If event A always occurs when the experiment is performed, P(A) =1.
- The sum of the probabilities for all simple events in S equals 1.

- The **probability of an event A** is found by adding the probabilities of all the simple events contained in A.

Finding Probabilities

- Probabilities can be found using
  - Estimates from empirical studies(تقديرات من الدراسات التجريبية)
  - Common sense estimates based on equally likely events. **(تقديرات الحس السليم على أساس الأحداث المحتملة على قدم المساواة)**

- **Examples:**
  - Toss a fair coin.

P(Head) = 1/2

  - Suppose that 10% of the Iraqi population has red hair. Then for a person selected at random,

P(Red hair) = .10

Using Simple Events

- The probability of an event A is equal to the sum of the probabilities of the simple events contained in A
- If the simple events in an experiment are **equally likely,** you can calculate

$$P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in A}}{\text{total number of simple events}}$$

Example 1

Toss a fair coin twice. What is the probability of observing at least one head?

1st Coin	2nd Coin	E_i	$P(E_i)$
H	H	HH	1/4
	T	HT	1/4
T	H	TH	1/4
	T	TT	1/4

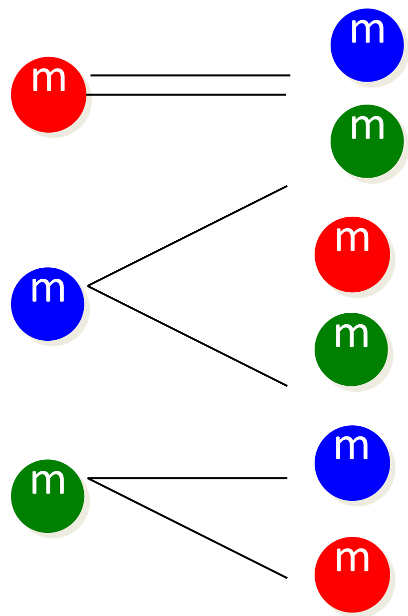
P(at least 1 head)

$= P(E_1) + P(E_2) + P(E_3)$

$= 1/4 + 1/4 + 1/4 = 3/4$

Example 2

A bowl contains three M&Ms®, one red, one blue and one green. A child selects two M&Ms (sequentially) at random. What is the probability that at least one is red?



1st M&M	2nd M&M	E_i	$P(E_i)$
		RB	1/6
		RG	1/6
		BR	1/6
		BG	1/6
		GB	1/6
		GR	1/6

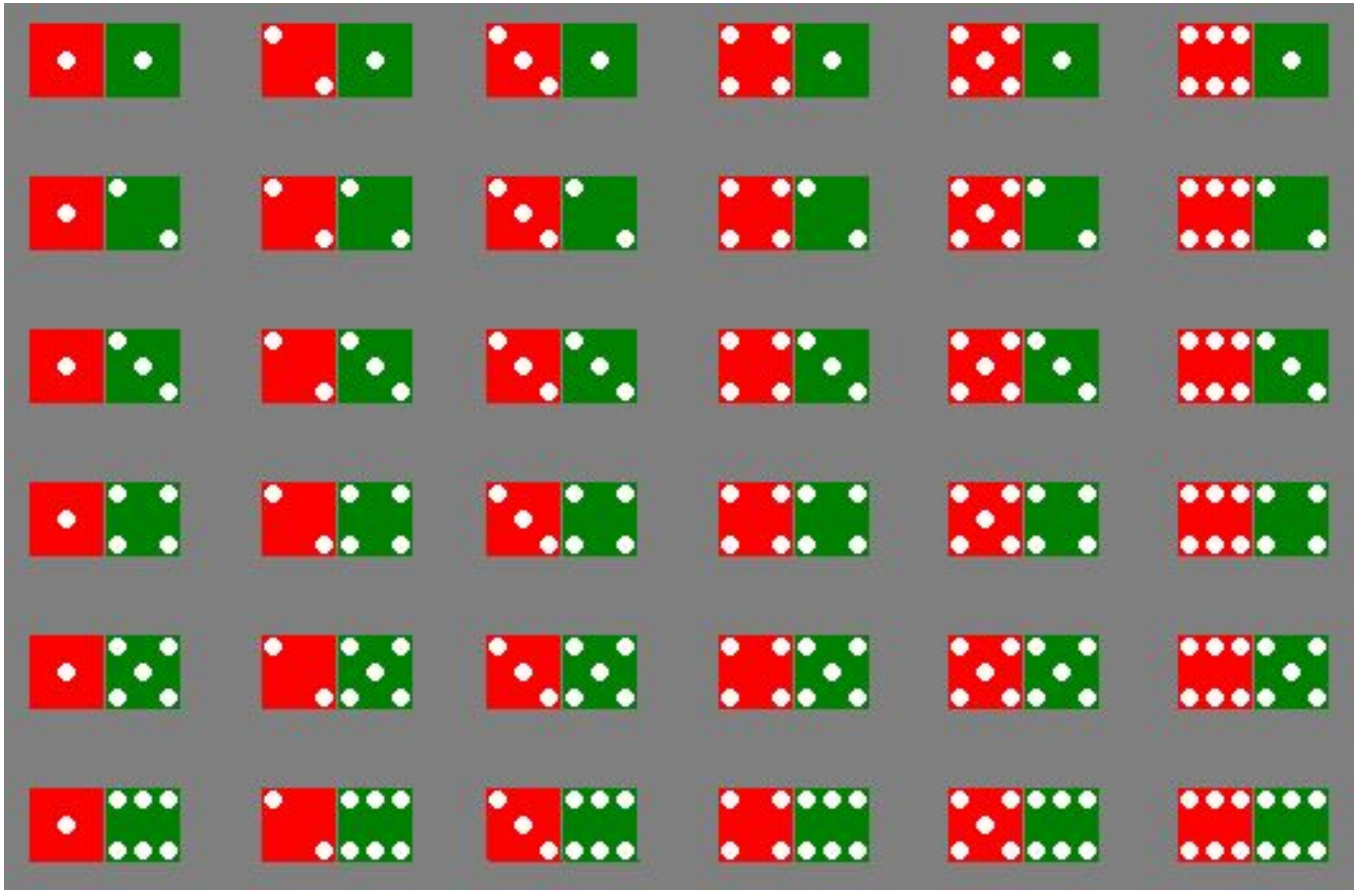
P(at least 1 red)

= P(RB) + P(BR)+ P(RG) + P(GR)

= 4/6 = 2/3

Example 3

The sample space of throwing a pair of dice is



Example 3

Event	Simple events	Probability
Dice add to 3	(1,2),(2,1)	2/36
Dice add to 6	(1,5),(2,4),(3,3), (4,2),(5,1)	5/36
Red die show 1	(1,1),(1,2),(1,3), (1,4),(1,5),(1,6)	6/36
Green die show 1	(1,1),(2,1),(3,1), (4,1),(5,1),(6,1)	6/36

Counting Rules

- Sample space of throwing 3 dice has 216 entries, sample space of throwing 4 dice has 1296 entries, …
- At some point, we have to stop listing and start thinking …
- We need some counting rules

The *mn* Rule

- If an experiment is performed in two stages, with ***m*** ways to accomplish the first stage and ***n*** ways to accomplish the second stage, then there are ***mn*** ways to accomplish the experiment.
- This rule is easily extended to ***k*** stages, with the number of ways equal to

$$n_1 n_2 n_3 \ldots n_k$$

Example: Toss two coins. The total number of simple events is:

$$2 \times 2 = 4$$

Examples

Example: Toss three coins. The total number of simple events is:

$$2 \times 2 \times 2 = 8$$

Example: Toss two dice. The total number of simple events is:

$$6 \times 6 = 36$$

Example: Toss three dice. The total number of simple events is:

$$6 \times 6 \times 6 = 216$$

THANK YOU