Probability and Statistics
Probability Distributions for Discrete Random Variables
Conditional probability

Taxonomy of Probability Distributions

Discrete probability distributions

- Binomial distribution
- Multinomial distribution
- Poisson distribution
- Hypergeometric distribution

Continuous probability distributions

- Normal distribution
- Standard normal distribution
- Gamma distribution
- Exponential distribution
- Chi square distribution
- Lognormal distribution
- Weibull distribution
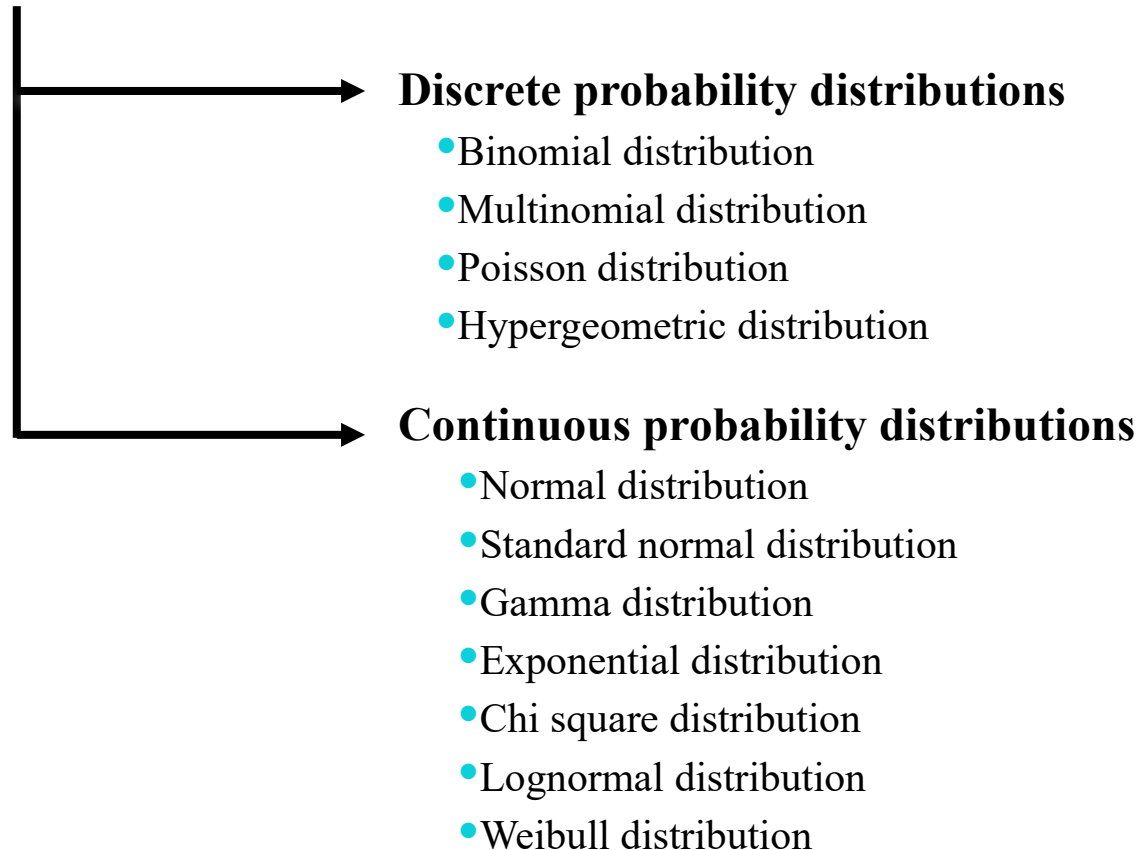
Probability Distributions for Discrete Random Variables

The **probability distribution for a discrete random variable** x is a graph, table or formula that gives the possible values of x and the probability $p(x)$ associated with each value.

We must have
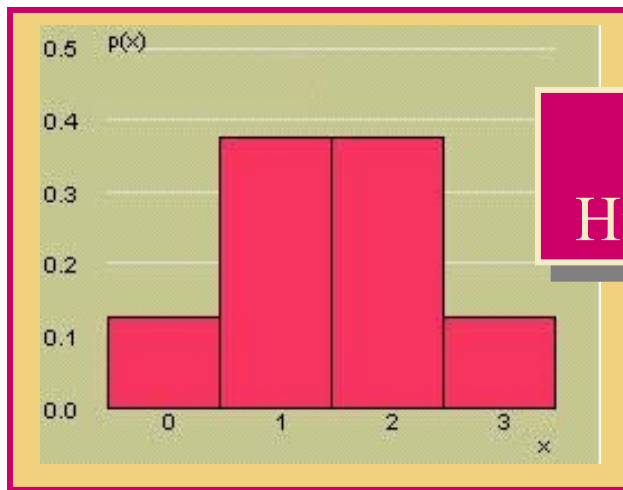
$$0 \le p(x) \le 1 \text{ and } \sum p(x) = 1$$

Example

Toss a fair coin three times and define *x* = **number of heads**.

		_x_
HHH		
	1/8	3
HHT		
	1/8	2
HTH		
	1/8	2
THH		
	1/8	2
HTT		
	1/8	1
THT		
	1/8	1
TTH		
	1/8	1
	1/8	0
TTT		

$P(x = 0) = 1/8$
$P(x = 1) = 3/8$
$P(x = 2) = 3/8$
$P(x = 3) = 1/8$

x	$p(x)$
0	1/8
1	3/8
2	3/8
3	1/8

Probability Histogram for *x*

Example

Toss two dice and define
x = sum of two dice.

probability histogram
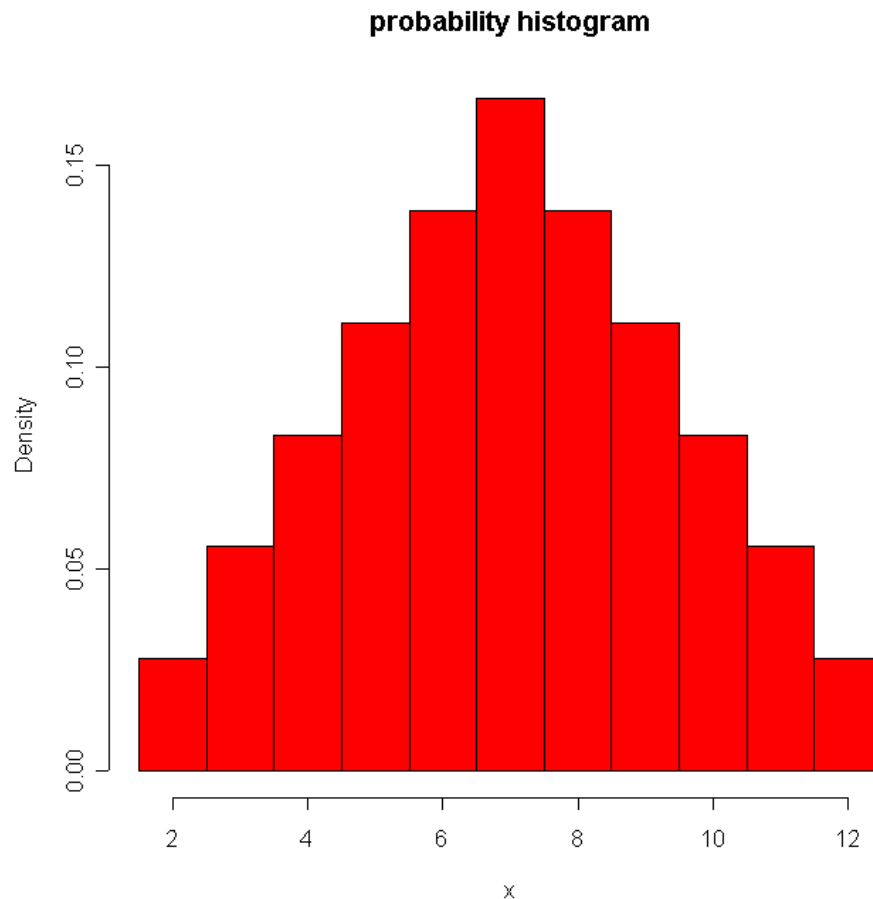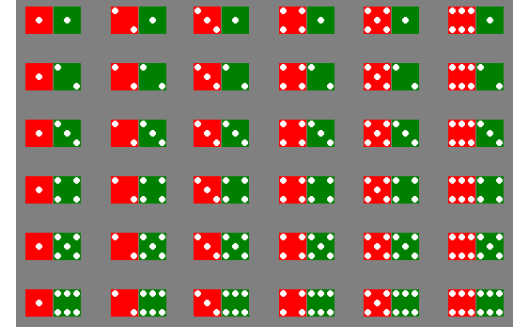
x	p(x)
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

Probability Distributions

Probability distributions can be used to describe the population.

- **Shape:** Symmetric, skewed, mound-shaped…
- **Outliers:** unusual or unlikely measurements
- **Center and spread:** mean and standard deviation. A population mean usually is called μ and a population standard deviation usually is called σ.

The Mean and Standard Deviation

Let x be a discrete random variable with probability distribution $p(x)$. Then the mean, variance and standard deviation of x are given as

$$\text{Mean}: \mu = \sum xp(x)$$

$$\text{Variance}: \sigma^2 = \sum (x - \mu)^2 p(x)$$

$$\text{Standard deviation}: \sigma = \sqrt{\sigma^2}$$

Example

Toss a fair coin 3 times and record x the number of heads.

x	$p(x)$	$xp(x)$	$(x-\mu)^2p(x)$
0	1/8	0	$(-1.5)^2(1/8)$
1	3/8	3/8	$(-0.5)^2(3/8)$
2	3/8	6/8	$(0.5)^2(3/8)$
3	1/8	3/8	$(1.5)^2(1/8)$

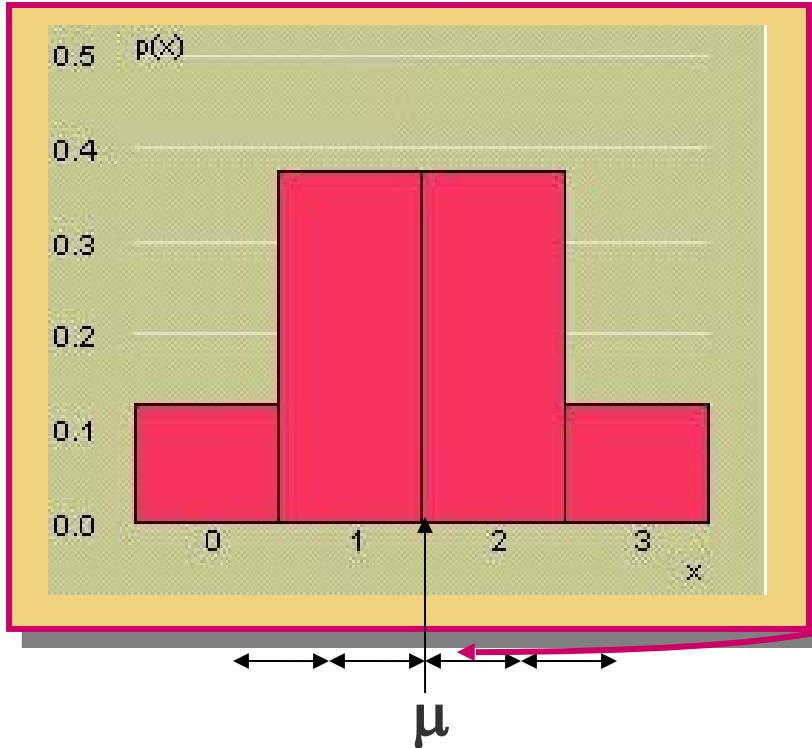$$\mu = \sum xp(x) = \frac{12}{8} = 1.5$$

$$\sigma^2 = \sum (x - \mu)^2 p(x)$$

$$\sigma^2 = .28125 + .09375 + .09375 + .28125 = .75$$

$$\sigma = \sqrt{.75} = .688$$

Example

The probability distribution for *x* the number of heads in tossing 3 fair coins.

- Shape?
- Outliers?
- Center?
- Spread?

Symmetric; mound-shaped

None

$\mu = 1.5$

$\sigma = .688$

μ

Key Concepts

1. Experiments and the Sample Space

    1. Experiments, events, mutually exclusive events, simple events

    2. The sample space

2. Probabilities

    1. Relative frequency definition of probability

    2. Properties of probabilities

            a. Each probability lies between 0 and 1.

            b. Sum of all simple-event probabilities equals 1.

    3. $P(A)$, the sum of the probabilities for all simple events in A

Key Concepts

3. Counting Rules

1. *mn* Rule; extended *mn* Rule

2. Permutations:
$$P_r^n = \frac{n!}{(n-r)!}$$

3. Combinations:
$$C_r^n = \frac{n!}{r!(n-r)!}$$

Key Concepts

4. Discrete Random Variables and Probability Distributions

1. Random variables, discrete and continuous

2. Properties of probability distributions

$$0 \le p(x) \le 1 \text{ and } \sum p(x) = 1$$

3. Mean or expected value of a discrete random variable:

$$\text{Mean} : \mu = \sum x p(x)$$

4. Variance and standard deviation of a discrete random variable:

$$\text{Variance} : \sigma^2 = \sum (x - \mu)^2 p(x)$$

$$\text{Standard deviation} : \sigma = \sqrt{\sigma^2}$$

Conditional probability

- Def : The prob event A given that B was happen is

- P(A/B) =   , P(B) ≠ 0

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الشرطي .

EX

- Three coins are tossed , Let
- A= first coin is head
- B= at least two heads . Find p(A/ B)
- SOL/ S = 2^3=8
- S = { HHH,TTT,THH,HTH,HHT,HTT,THT,TTH }
- A = { HHH,HTH,HHT,HTT }

B= $\{$ HHH, THH , HTH , HHT $\}$

A = AB = HHH, HTH ,HHT

P (A) = 4/8   P(B) = 4/ 8 ,

P (AB) = 3/8

P (A / B) =   = ¾

Are A and B independent

p(AB) = P(A) P(B)

3/ 8 ≠ 4/ 8 . 4/8

A, B not indep

THANK YOU