Discrete Mathematics

Discrete Probability

9th Lecture

Lecturer: Dr. Mustafa F. Mohammed
Class: 1st stage.
Time:8:30AM-10:30AM

Discrete Probability

Introduction:

Probability theory is a mathematical modeling of the phenomenon of chance or randomness. A probabilistic mathematical model of random phenomena is defined by assigning “probabilities” to all the possible outcomes of an experiment.

SAMPLE, SPACE AND EVENTS

- The set S of all possible outcomes of a given experiment is called the *sample space*.
- A particular outcome, i.e., an element in S, is called a *sample point*.
- An *event* A is a set of outcomes or, in other words, a subset of the sample space S.
- In particular, the set $\{a\}$ consisting of a single sample point $a \in S$ is called an *elementary event*.
- the empty set and S itself are subsets of S and so and S are also events; is sometimes called the *impossible event* or the *null event*.

Since an event is a set, we can combine events to form new events using the various set operations:

(i) $A \cup B$ is the event that occurs if A occurs *or* B occurs (or both).
(ii) $A \cap B$ is the event that occurs if A occurs *and* B occurs.
(iii) , the complement of A, also written & , is the event that occurs if A does *not* occur.

Two events A and B are called *mutually exclusive* if they are disjoint, that is, if $A \cap B =$.
In other words, A and B are mutually exclusive if they cannot occur simultaneously.

Discrete Probability

EX_1: Toss a coin three times and observe the sequence of heads *(H)* and tails *(T)* that appears. The sample space consists of the following eight elements:

$$S = \{\boldsymbol{HHH, HHT, HTH, HTT\ , T\ HH\ ,THT\ ,T\ T\ H\ ,T\ T\ T}\ \}$$

Let A be the event that two or more heads appear consecutively, and B that all the tosses are the same:

$$A = \{\boldsymbol{HHH,HHT,THH}\} \text{ and } B = \{\boldsymbol{HHH,T\ T\ T}\ \}$$

Then $A \cap B = \{\boldsymbol{HHH}\}$ is the elementary event that only heads appear. The event that five heads appears is the empty set .

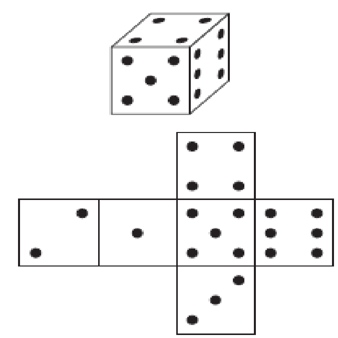
EX_2: Toss a (six-sided) die, pictured in the Fig. shown and observe the number (of dots) that appear on top. The sample space S consists of the six possible numbers, that is,

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Let A be the event that an even number appears, B that an odd number appears, and C that a prime number appears.

That is, let

$$A = \{2, 4, 6\},\ B= \{1, 3, 5\},\ C= \{2, 3, 5\}$$



Then

$A \cup C = \{2, 3, 4, 5, 6\}$ is the event that an even or a prime number occurs.

$B \cap C = \{3, 5\}$ is the event that an odd prime number occurs.

$= \{1, 4, 6\}$ is the event that a prime number does not occur.

Note: A prime number is a whole number greater than 1 whose only factors are 1 and itself

Note that A and B are mutually exclusive: $A \cap B =$ In other words, an even number and an odd number cannot occur simultaneously.

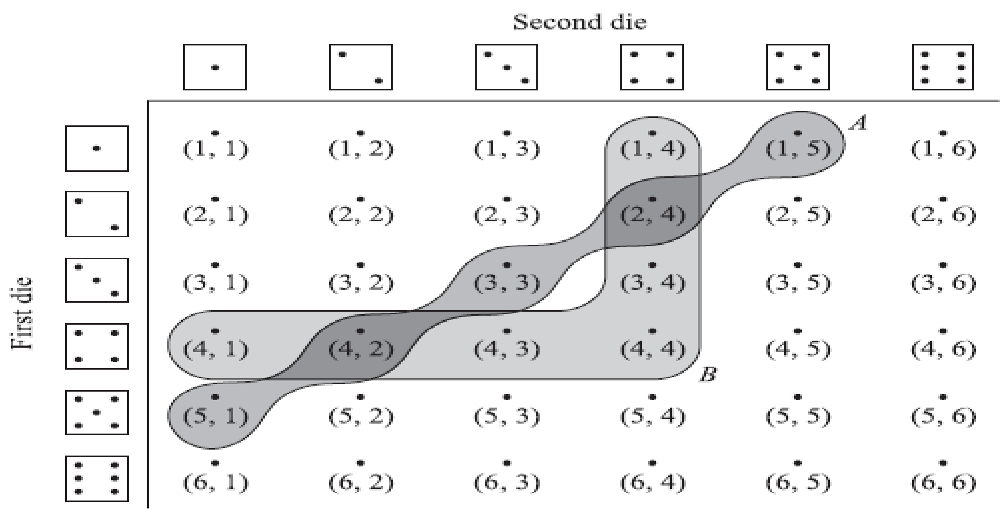
EX_3: (Pair of dice) Toss a pair of dice and record the two numbers on the top.
There are six possible numbers, 1, 2, . . . , 6, on each die. Thus S consists of the pairs of numbers from 1 to 6, and hence $n(S) = 36$. Figure below shows these 36 pairs of numbers arranged in an array where the rows are labeled by the first die and the columns by the second die.
Let A be the event that the sum of the two numbers is 6, and let B be the event that the largest of the two numbers is 4. That is, let

$A = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$, $B = \{(1, 4), (2, 4), (3, 4), (4, 4), (4, 3), (4, 2), (4, 1)\}$

Then the event "A and B" consists of those pairs of integers whose sum is 6 and whose largest number is 4 or, in other words, the intersection of A and B. Thus

$$A \cap B = \{(2, 4), (4, 2)\}$$

Similarly, "A or B," the sum is 6 or the largest is 4, shaded in Fig. shown is the union $A \cup B$.



FINITE PROBABILITY SPACES

- Let S be a finite sample space, say $S = \{a1, a2, \ldots, an\}$. A *finite probability space*, or *probability model*, is obtained by assigning to each point ai in S a real number pi , called the *probability* of ai satisfying the following properties:

(i) Each pi is nonnegative, that is, $pi \geq 0$.

(ii) The sum of the pi is 1, that is, is $p1 + p2 + \cdot \quad \cdot \quad \cdot \quad + pn = 1$.

- The *probability* of an event A written $P(A)$, is then defined to be the sum of the probabilities of the points in A.
- The singleton set $\{ai\}$ is called an *elementary* event and, for notational convenience, we write $P(ai)$ for $P(\{ai\})$.

EX_4: Suppose three coins are tossed, and the number of heads is recorded. (Compare with the above Ex_1. The sample space is $S = \{0, 1, 2, 3\}$. The following assignments on the elements of S define a probability space:

$$P(0) = 1/8\ ,\ P(1) = 3/8\ ,\ P(2) = 3/8\ ,\ P(3) = 1/8$$

That is, each probability is nonnegative, and the sum of the probabilities is 1.

Let A be the event that at least one head appears, and let B be the event that all heads or all tails appear; that is, let $A = \{1, 2, 3\}$ and $B = \{0, 3\}$.

Then, by definition,

$$P(A) = P(1) + P(2) + P(3) = 3/8 + 3/8+ 1/8 = 7/8$$

$$\text{and } P(B) = P(0) + P(3) = 1/8+ 1/8= 1/4$$

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More Problems

P.1. Let a coin and a die be tossed; and let the sample space S consists of the 12 elements:

$$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

Note: (*H & T for the coin; 1 -6 for the die*)

(*a*) Express explicitly the following events: A = {heads and an even number}, B = {prime number}, C = {tails and an odd number}

(*b*) Express explicitly the events: (i) A or B occurs; (ii) B and C occur: (iii) only B occurs.

(*c*) Which pair of the events A, B, and C are mutually exclusive?

Sol:

(*a*) The elements of A are those elements of S consisting of an H and an even number:

$$A = \{H2, H4, H6\}$$

The elements of B are those points in S whose second component is a prime number (2, 3, or 5):

$$B = \{H2, H3, H5, T2, T3, T5\}$$

The elements of C are those points in S consisting of a T and an odd number;

$$C = \{T1, T3, T5\}.$$

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(b)

(i) $A \cup B = \{H2, H4, H6, H3, H5, T2, T3, T5\}$

(ii) $B \cap C = \{T3, T5\}$

(iii) $B \cap \ \cap = \{H3, H5, T2\}$

Where:

$\mathbf{= \{\mathit{H}1, \mathit{H}3, \mathit{H}5, \mathit{T}1, \mathit{T}2, \mathit{T}3, \mathit{T}4, \mathit{T}5, \mathit{T}6\}}$

$\mathbf{= \{\mathit{H}1, \mathit{H}2, \mathit{H}3, \mathit{H}4, \mathit{H}5, \mathit{H}6, \mathit{T}2, \mathit{T}4, \mathit{T}6\}}$

$\boldsymbol{B} \mathbf{= \{\mathit{H}2, \mathit{H}3, \mathit{H}5, \mathit{T}2, \mathit{T}3, \mathit{T}5\}}$

$\cap$ **={H1,H3, H5, T2, T4, T6}**

$\boldsymbol{B} \cap \ \cap \mathbf{= \{\mathit{H}3, \mathit{H}5, \mathit{T}2\}}$

(c) A and C are mutually exclusive since $A \cap C =$

Discrete Probability

P.2: A pair of dice is tossed. (See Ex_3), Find the number of elements in each event:

(a) $A = \{\text{two numbers are equal}\}$

(b) $B = \{\text{sum is 10 or more}\}$

(c) $C = \{\text{5 appears on the first die}\}$

(d) $D = \{\text{5 appears on at least one die}\}$

Use Fig. of **EX_3** to help count the number of elements in the event.

Sol:

(*a*) $A = \{(1, 1), (2, 2), \ldots, (6, 6)\}$, so $n(A) = 6$.

(*b*) $B = \{(6, 4), (5, 5), (4, 6), (6, 5), (5, 6), (6, 6)\}$, so $n(B) = 6$.

(*c*) $C = \{(5, 1), (5, 2), \ldots, (5, 6)\}$, so $n(C) = 6$.

(*d*) There are six pairs with 5 as the first element, and six pairs with 5 as the second element. However, (5, 5) appears in both places. Hence

$$n(D) = 6 + 6 - 1 = 11$$

Alternately, count the pairs in Fig. below which are in D to get $n(D) = 11$.

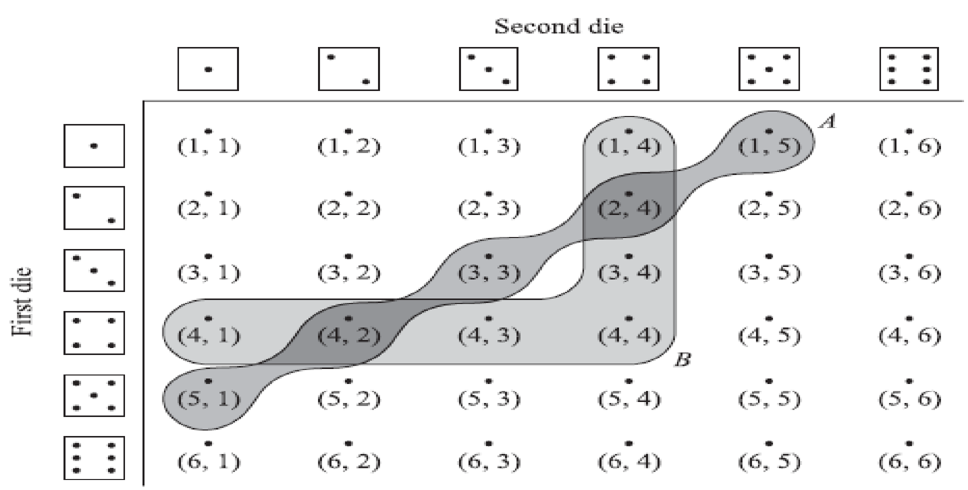

Fig. of EX_3

THANK YOU