Discrete Mathematics
Basics of Counting

7th Lecture

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Class: 1st stage.
Time:8:30AM-10:30AM

Basics of Counting

Introduction:

Counting problems arise throughout mathematics and computer science. For example, we must count the successful outcomes of experiments and all the possible outcomes of these experiments to determine probabilities of discrete events. We need to count the number of operations used by an algorithm to study its time complexity.

Basic Counting Principles

We first present two basic counting principles, the **product rule** and the **sum rule**. Then we will show how they can be used to solve many different counting problems.

THE PRODUCT RULE

❖ Suppose that a procedure can be broken down into a sequence of two tasks.
❖ If there are $n1$ ways to do the first task and for each of these ways of doing the first task, there are $n2$ ways to do the second task, then there are $n1n2$ ways to do the procedure.
❖ The product rule applies when a procedure is made up of separate tasks.

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EX_1 A new company with just two employees, Sanchez and Patel, rents a floor of a building with 12 offices. How many ways are there to assign different offices to these two employees?

Sol: The procedure of assigning offices to these two employees consists of assigning an office to Sanchez, which can be done in 12 ways, then assigning an office to Patel different from the office assigned to Sanchez, which can be done in 11 ways. By the product rule, there are $12 \cdot 11 = 132$ ways to assign offices to these two employees.

EX_2 The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?

Sol : The procedure of labeling a chair consists of two tasks, namely, assigning to the seat one of the 26 uppercase English letters, and then assigning to it one of the 100 possible integers. The product rule shows that there are $26 \cdot 100 = 2600$ different ways that a chair can be labeled. Therefore, the largest number of chairs that can be labeled differently is 2600.

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EX_3: How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)?

Sol: There are 26 choices for each of the three uppercase English letters and ten choices for each of the three digits. Hence, by the product rule there are a total of $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17{,}576{,}000$ possible license plates.

Counting Functions

EX_4 How many functions are there from a set with m elements to a set with n elements?

Sol: A function corresponds to a choice of one of the n elements in the codomain for each of the m elements in the domain. Hence, by the product rule there are $n \cdot n \cdots \cdots n = n^m$ functions from a set with m elements to one with n elements.

For example, there are $5^3 = 125$ different functions from a set with three elements to a set with five elements.

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EX_5: Counting One-to-One Functions How many one-to-one functions are there from a set with m elements to one with n elements?

Sol: First note that when $m > n$ there are no one-to-one functions from a set with m elements to a set with n elements.

Now let $m \leq n$. Suppose the elements in the domain are $a_1, a_2, \ldots, a_m$. There are n ways to choose the value of the function at a_1. Because the function is one-to-one, the value of the function at a_2 can be picked in $n - 1$ ways (because the value used for a_1 cannot be used again).
In general, the value of the function at a_k can be chosen in $n - k + 1$ ways. By the product rule, there are $n(n - 1)(n - 2) \cdots (n - m + 1)$ one-to-one functions from a set with m elements to one with n elements.

For example, there are $5 \cdot 4 \cdot 3 = 60$ one-to-one functions from a set with three elements to a set with five elements.

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THE SUM RULE

❖ If a task can be done either in one of n_1 ways or in one of n_2 ways
❖ none of the set of n_1 ways is the same as any of the set of n_2 ways,
❖ then there are $n_1 + n_2$ ways to do the task.

EX_6: A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

Sol: The student can choose a project by selecting a project from the first list, the second list, or the third list. Because no project is on more than one list, by the sum rule there are $23 + 15 + 19 = 57$ ways to choose a project.

EX_7: Suppose that either a member of the mathematics faculty or a student who is a mathematics major is chosen as a representative to a university committee. How many different choices are there for this representative if there are 37 members of the mathematics faculty and 83 mathematics majors and no one is both a faculty member and a student?

Sol: There are 37 ways to choose a member of the mathematics faculty and there are 83 ways to choose a student who is a mathematics major. Choosing a member of the mathematics faculty is never the same as choosing a student who is a mathematics major because no one is both a faculty member and a student. By the sum rule it follows that there are $37 + 83 = 120$ possible ways to pick this representative.

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More Complex Counting Problems

EX_8 Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

Sol: Let P be the total number of possible passwords, and let $P6$, $P7$, and $P8$ denote the number of possible passwords of length 6, 7, and 8, respectively. By the sum rule, $P = P6 + P7 + P8$. We will now find $P6$, $P7$, and $P8$. Finding $P6$ directly is difficult. To find $P6$ it is easier to find the number of strings of uppercase letters and digits that are six characters long, including those with no digits, and subtract from this the number of strings with no digits. By the product rule, the number of strings of six characters is 36^6, and the number of strings with no digits is 26^6. Hence,

$P6 = 36^6 - 26^6 = 2,176,782,336 - 308,915,776 = 1,867,866,560.$

Similarly, we have

$P7 = 36^7 - 26^7 = 78,364,164,096 - 8,031,810,176 = 70,332,353,920$

and

$P8 = 36^8 - 26^8 = 2,821,109,907,456 - 208,827,064,576$
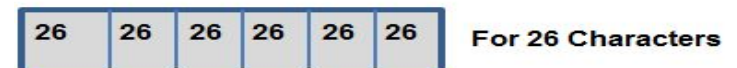$= 2,612,282,842,880.$

Consequently,

$P = P6 + P7 + P8 = 2,684,483,063,360$

Note:
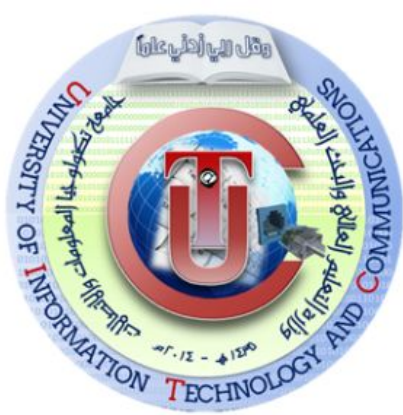36 characters:
(26 : A-Z + 10 : 0-9) to
choose from.

| 36 | 36 | 36 | 36 | 36 | 36 | For 36 Characters |

For P6 only

| 26 | 26 | 26 | 26 | 26 | 26 | For 26 Characters |

THANK YOU