Only questions LEC (5-1) /Discrete Mathematics

LEC 2

EX_1: The set V of all vowels in the English alphabet can be written as V = {a, e, i, o, u}.

EX_2: The set O of odd positive integers less than 10 can be expressed by O = {1, 3, 5, 7, 9}.

EX_ 3: Although sets are usually used to group together elements with common properties, there is nothing that prevents a set from having seemingly unrelated elements. For instance, {a, 2, Fred, New Jersey} is the set containing the four elements a, 2, Fred, and New Jersey. Sometimes the roster method is used to describe a set without listing all its members. Some members of the set are listed, and then ellipses (. . .) are used when the general pattern of the elements is obvious.

EX_4 The set of positive integers less than 100 can be denoted by {1, 2, 3, . . ., 99}.

EX_5: The set {N, Z, Q, R} is a set containing four elements, each of which is a set. The four elements of this set are N, the set of natural numbers; Z, the set of integers; Q, the set of rational numbers; and R, the set of real numbers.

EX_6: The sets {1, 3, 5} and {3, 5, 1} are equal, because they have the same elements.

EX_7: Draw a Venn diagram that represents V, the set of vowels in the English alphabet.

EX_10: Let A be the set of odd positive integers less than 10. Then |A| =?

EX_11: Let S be the set of letters in the English alphabet. Then |S| =?

EX_12: Because the null set has no elements, it follows that |∅| =?

EX_13: The set of positive integers is infinite.

EX_14: What is the power set of the set {0, 1, 2}?

EX_15: What is the power set of the empty set? What is the power set of the set {∅}?

EX_16: Let A represent the set of all students at a university, and let B represent the set of all courses offered at the university. What is the Cartesian product A × B and how can it be used?

EX_17: What is the Cartesian product of A = {1, 2} and B = {a, b, c}?

EX_18: Show that the Cartesian product B × A is not equal to the Cartesian product A × B, where A and B sets are as in Example 17.

EX_19: The union of the sets {1, 3, 5} and {1, 2, 3} is the set {1, 2, 3, 5}; that is, {1, 3, 5} ∪ {1, 2, 3} =

EX_20: The intersection of the sets {1, 3, 5} and {1, 2, 3} is the set {1, 3}; that is, {1, 3, 5} ∩ {1, 2, 3} =

EX_21: Let A = {1, 3, 5, 7, 9} and B = {2, 4, 6, 8, 10}. Because A ∩ B = ∅, A and B are disjoint.

EX_22: The difference of {1, 3, 5} and {1, 2, 3} is the set {5}; that is, {1, 3, 5} − {1, 2, 3} = {5}. This is different from the difference of {1, 2, 3} and {1, 3, 5}, which is the set {2}.

EX_23: Let A = {a, e, i, o, u} (where the universal set is the set of letters of the English alphabet). Then $\boldsymbol{A}$ = ?

EX_24: Let A be the set of positive integers greater than 10 (with universal set the set of all positive integers). Then $\boldsymbol{A}$ = ?

EX_25: Let A = {0, 2, 4, 6, 8}, B = {0, 1, 2, 3, 4}, and C = {0, 3, 6, 9}. What are A ∪ B ∪ C and A ∩ B ∩ C?

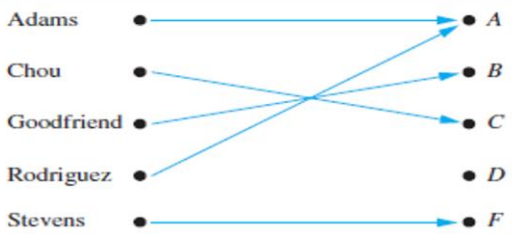
EX_26: For i = 1, 2, . . ., let Ai = {i, i + 1, i + 2, . . .}. Then,

$$\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} \{i, i+1, i+2, \dots\} = \{1, 2, 3, \dots\},$$

and

$$\bigcap_{i=1}^{n} A_i = \bigcap_{i=1}^{n} \{i, i+1, i+2, \dots\} = \{n, n+1, n+2, \dots\} = A_n.$$

EX_27: Suppose that each student in a discrete mathematics class is assigned a letter grade from the set {A, B, C, D, F}. And suppose that the grades are A for Adams, C for Chou, B for Goodfriend, A for Rodriguez, and F for Stevens. This assignment of grades is illustrated the Figure below:



Assignment of Grades in a Discrete Mathematics Class.

EX_28: What are the domain, codomain, and range of the function that assigns grades to students in EX_27?

EX_29: Let R be the relation with ordered pairs (Abdul, 22), (Brenda, 24), (Carla, 21), (Desire, 22), (Eddie, 24), and (Felicia, 22). Here each pair consists of a graduate student and this student's age. Specify a function determined by this relation.

EX_30 Let f: Z → Z assign the square of an integer to this integer. Then, f (x) =x2, where the domain of f is the set of all integers, the codomain off is the set of all integers, and the range of f is the set of all integers that are perfect squares, namely, {0, 1, 4, 9, . . .}.

EX_1: Let f1 and f2 be functions from R to R such that f1(x) = x2and f2(x) = x − x2. What are the functions f1 + f2 and f1f2?

EX_2: Determine whether the function f from {a, b, c, d} to {1, 2, 3, 4, 5} with f (a) = 4, f(b) = 5, f (c) = 1, and f (d) = 3 is one-to-one.

EX_3: Determine whether the function f (x) =$X$2 from the set of integers to the set of integers is one-to-one.

EX_4: Let f be the function from {a, b, c, d} to {1, 2, 3} defined by f (a) = 3, f (b) = 2, f (c) = 1 , and f (d) = 3. Is f an onto function?

EX_5 : Is the function f (x) =$X$2 from the set of integers to the set of integers onto?

EX_6: Let f be the function from {a, b, c} to {1, 2, 3} such that f (a) = 2, f (b) = 3, and f (c) = 1. is f invertible, and if it is, what is its inverse?

EX_7: Let f be the function from R to R with f (x) =$X$2. Is f invertible?

EX_9: Let A = {0, 1, 2} and B = {a, b}. Then {(0, a), (0, b), (1, a), (2, b)} is a relation from A to B. This means, for instance, that 0 R a, but that 1 b.

EX_10: Let A be the set {1, 2, 3, 4}. Which ordered pairs are in the relation R = {(a, b) | a divides b}?

EX_11: Consider these relations on the set of integers:

R1 = {(a, b) | a ≤ b},

R2 = {(a, b) | a > b},

R3 = {(a, b) | a = b or a = −b},

R4 = {(a, b) | a = b},

R5 = {(a, b) | a = b + 1},

R6 = {(a, b) | a + b ≤ 3}.

EX_12: Consider the following relations on {1, 2, 3, 4}:

R1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)},

R2 = {(1, 1), (1, 2), (2, 1)},

R3 = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)},

R4 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)},

R5 = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)},

R6 = {(3, 4)}.

Which of these relations are reflexive?

EX_13: Which of the relations from Ex_11 are reflexive?

R1 = {(a, b) | a ≤ b},

R2 = {(a, b) | a > b},

R3 = {(a, b) | a = b or a = −b},

R4 = {(a, b) | a = b},

R5 = {(a, b) | a = b + 1},

R6 = {(a, b) | a + b ≤ 3}.

EX_14: Let A = {1, 2, 3} and B = {1, 2, 3, 4}. The relations R1 = {(1, 1), (2, 2), (3, 3)} and R2 = {(1, 1), (1, 2), (1, 3), (1, 4)} can be combined to obtain:

R1 ∪ R2 =

R1 ∩ R2 =

R1 − R2 =

R2 − R1 =

LEC 4

EX_1: Consider the following four statements:

(i) Ice floats in water and $2 + 2 = 4$. (iii) China is in Europe and $2 + 2 = 4$.

(ii) Ice floats in water and $2 + 2 = 5$. (iv) China is in Europe and $2 + 2 = 5$.

EX_2: Consider the following four statements

(i) Ice floats in water or $2 + 2 = 4$. (ii) Ice floats in water or $2 + 2 = 5$.

(iii) China is in Europe or $2 + 2 = 4$. (iv) China is in Europe or $2 + 2 = 5$.

EX_3 Consider the following six statements: (a1) Ice floats in water. (a2) It is false that ice floats in water. (a3) Ice does not float in water. (b1) $2 + 2 = 5$ (b2) It is false that $2 + 2 = 5$. (b3) $2 + 2 \neq 5$

P_1: Let p be "It is cold" and let q be "It is raining". Give a simple verbal sentence which describes each of the following statements: (a) $\neg p$; (b) $p \land q$; (c) $p \lor q$; (d) $q \lor \neg p$.

P_2: Find the truth table of $\neg p \land q$.

P_3: Show that the propositions $\neg(p \land q)$ and $\neg p \lor \neg q$ are logically equivalent.

EX_5: Show that $p \lor (q \land r)$ and $(p \lor q) \land (p \lor r)$ are logically equivalent

EX_4: Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

EX_6: What is the value of the variable x after the statement if $2 + 2 = 4$ then $x := x + 1$ if $x = 0$ before this statement is encountered? (The symbol: = stands for assignment. The statement $x := x + 1$ means the assignment of the value of $x + 1$ to x.)

LEC 5

EX_1: Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \to q$ as a statement in English.

EX_2: A. the statement , "If Juan has a smartphone, then 2 + 3 = 5" is true from the definition of a conditional statement, because its conclusion is true. B. "If Juan has a smartphone, then 2 + 3 = 6" is False C. "If Juan has a smartphone, then 2 + 3 = 6" is true if Juan does not have a smartphone, even though 2 + 3 = 6 is false.

EX_3: What is the value of the variable x after the statement if 2 + 2 = 4 then x := x + 1 if x = 0 before this statement is encountered? (The symbol := stands for assignment. The statement x := x + 1 means the assignment of the value of x + 1 to x.)

EX_4: State the converse, contrapositive, and inverse of each of these conditional statements. a) If it snows tonight, then I will stay at home. b) I go to the beach whenever it is a sunny summer day. c) When I stay up late, it is necessary that I sleep until noon.

EX_5: Consider the conditional proposition $p \to q$. The simple propositions $q \to p$, $\neg p \to \neg q$ and $\neg q \to \neg p$ are called, respectively, the converse, inverse, and contrapositive of the conditional $p \to q$. Which if any of these propositions are logically equivalent to $p \to q$?

EX_6: Let p be the statement "You can take the flight," and let q be the statement "You buy a ticket." Then $p \leftrightarrow q$ is the statement: "You can take the flight if and only if you buy a ticket."

EX_7: Construct the truth table of the compound proposition: $(p \vee \neg q) \to (p \wedge q)$.

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