Discrete Mathematics

Basic Logic (cont.)

5th Lecture

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Class: 1st stage.
Time:8:30AM-10:30AM

BASIC LOGICAL OPERATIONS

Conditional Statements

- Let p and q be propositions.
- The *conditional statement* $\boldsymbol{p \to q}$ is the proposition "**if $\boldsymbol{p}$, then $\boldsymbol{q}$**." The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise.
- In the conditional statement $p \to q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

The Truth Table for the Conditional Statement $p \to q$.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

EX_1: Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \to q$ as a statement in English.

Sol: "If Maria learns discrete mathematics, then she will find a good job."

BASIC LOGICAL OPERATIONS

EX_2:

A. the statement , “If Juan has a smartphone, then 2 + 3 = 5” is true from the definition of a conditional statement, because its conclusion is true.
B. “If Juan has a smartphone, then 2 + 3 = 6” is False
C. “If Juan has a smartphone, then 2 + 3 = 6” is true if Juan does not have a smartphone, even though 2 + 3 = 6 is false.

EX_3: What is the value of the variable x after the statement

if $2 + 2 = 4$ **then** $x := x + 1$

if $x = 0$ before this statement is encountered? (The symbol := stands for assignment. The statement $x := x + 1$ means the assignment of the value of $x + 1$ to x.)

***Sol**:* Because $2 + 2 = 4$ is true, the assignment statement $x := x + 1$ is executed. Hence, x has the value $0 + 1 = 1$ after this statement is encountered.

CONVERSE المضاد, CONTRAPOSITIVE المانع , AND INVERSE العكس

- starting with a conditional statement $p \rightarrow q$.
- The proposition $q \rightarrow p$ is called the **converse** of $p \rightarrow q$.
- The **contrapositive** of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$.
- The proposition $\neg p \rightarrow \neg q$ is called the **inverse** of $p \rightarrow q$.

EX_3: What are the contrapositive, the converse, and the inverse of the conditional statement "The home team wins whenever it is raining?"
i.e., *P: it is raining (hypothesis), q: The home team wins (conclusion)*
Sol:
Because "*q* whenever *p*" is one of the ways to express the conditional statement $p \rightarrow q$, the original statement can be rewritten as
"If it is raining, then the home team wins."
The **converse** is **"If the home team wins, then it is raining."**
Consequently, the **contrapositive** of this conditional statement is
"If the home team does not win, then it is not raining."
The **inverse** is "**If it is not raining, then the home team does not win**."

Only the contrapositive is equivalent to the original statement

EX_4: State the converse, contrapositive, and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.

b) I go to the beach whenever it is a sunny summer day.

c) When I stay up late, it is necessary that I sleep until noon.

Sol:

a) **Converse**: If I stay home, then it will snow tonight.
Contrapositive: If I do not stay at home, then it will not snow tonight.
Inverse: If it does not snow tonight, then I will not stay home.

b) **Converse**: Whenever I go to the beach, it is a sunny summer day.
Contrapositive: Whenever I do not go to the beach, it is not a sunny summer day.
Inverse: Whenever it is not a sunny day, I do not go to the beach.

c) **Converse**: If I sleep until noon, then I stayed up late.
Contrapositive: If I do not sleep until noon, then I did not stay up late.
Inverse: If I don’t stay up late, then I don’t sleep until noon.

EX_ 5: Consider the conditional proposition $p \to q$. The simple propositions $q \to p$, $\neg p \to \neg q$ and $\neg q \to \neg p$ are called, respectively, the *converse*, *inverse*, and *contrapositive* of the conditional $p \to q$. Which if any of these propositions are logically equivalent to $p \to q$?

Sol:

Construct their truth tables below. Only the contrapositive $\neg q \to \neg p$ is logically equivalent to the original , conditional proposition $p \to q$.

				Conditional	Converse	Inverse	Contrapositive
p	q	$\neg p$	$\neg q$	$p \to q$	$q \to p$	$\neg p \to \neg q$	$\neg q \to \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

BICONDITIONALS

- Let p and q be propositions.
- The compound statement $(p \rightarrow q) \wedge (q \rightarrow p)$ is a conjunction of two conditional statements. In the first conditional, p is the hypothesis and q is the conclusion; in the second conditional, q is the hypothesis and p is the conclusion. Let's look at a truth table for this compound statement. The compound statement $(p \rightarrow q) \wedge (q \rightarrow p)$ is a conjunction of two conditional statements. In the first conditional, p is the hypothesis and q is the conclusion; in the second conditional, q is the hypothesis and p is the conclusion. Let's look at a truth table for this compound statement.

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

- The *Biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q."
- The Biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.
- Biconditional statements are also called *bi-implications* i.e. the conclusion that can be drawn from something although it is not explicitly stated.

The Truth Table for the Biconditional $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

BICONDITIONALS

EX_6: Let p be the statement "You can take the flight,"
and let q be the statement "You buy a ticket."
Then $p \leftrightarrow q$ is the statement:
"You can take the flight if and only if you buy a ticket."

Truth Tables of Compound Propositions

EX_7: Construct the truth table of the compound proposition:

$$(p \vee \neg q) \rightarrow (p \wedge q).$$

Sol:

The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

THANK YOU