EEG/MEG 2:

(Linear) Source Estimation

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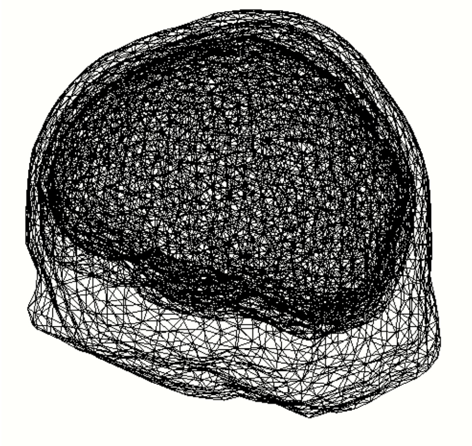
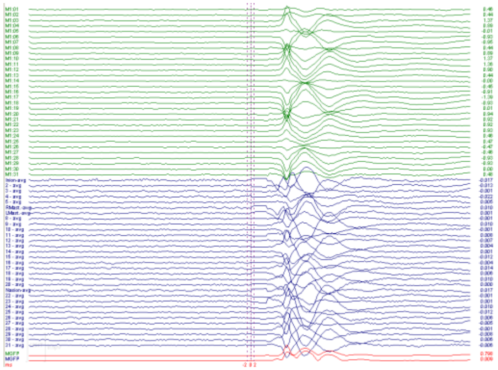
Ingredients for Source Estimation

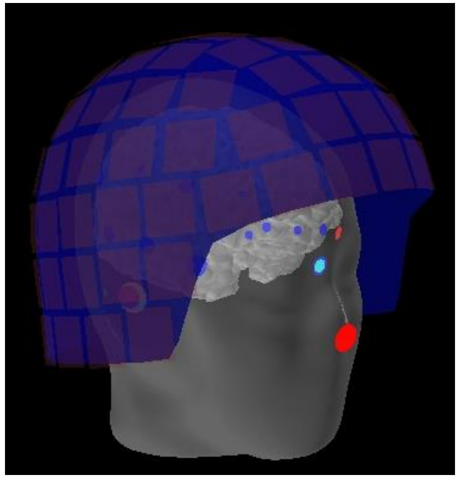
Volume Conductor/
Head Model



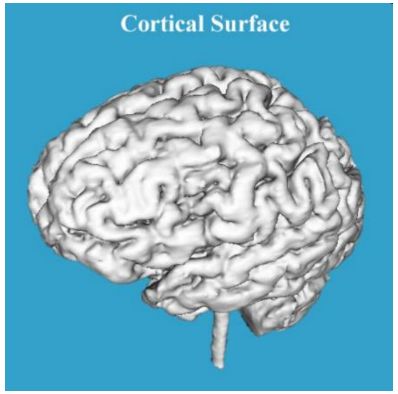
MEG data



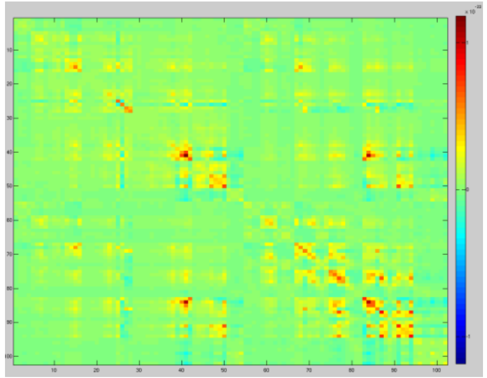
Coordinate
Transformation



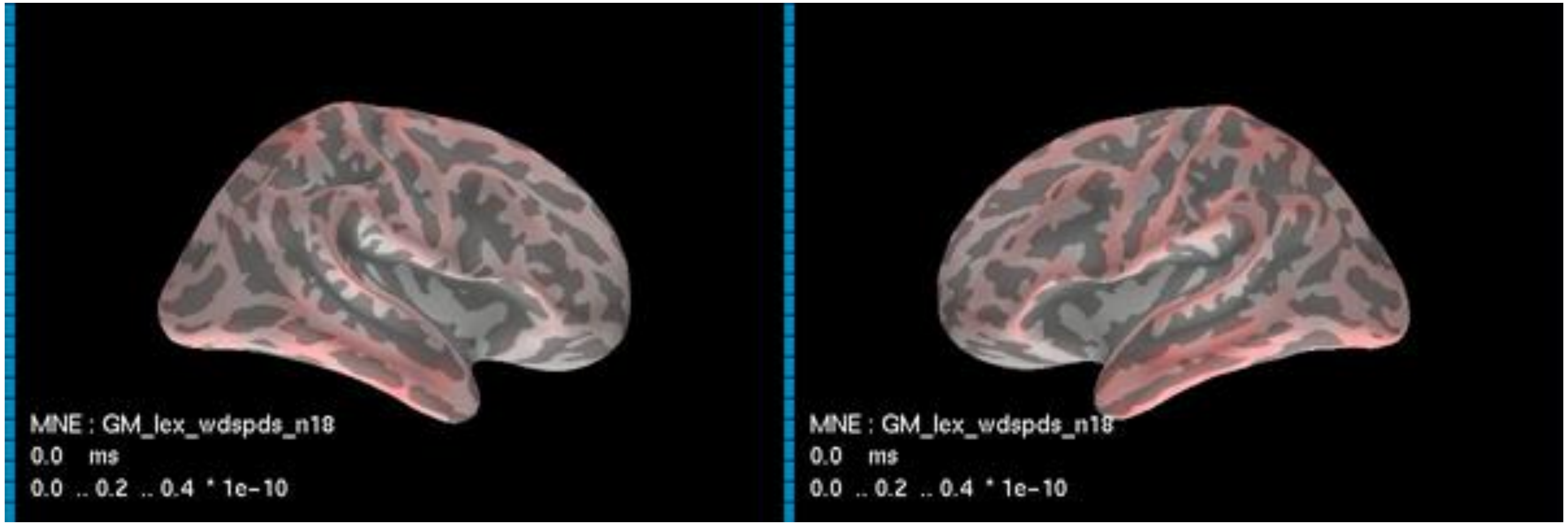
Source Space



Noise/Covariance Matrix

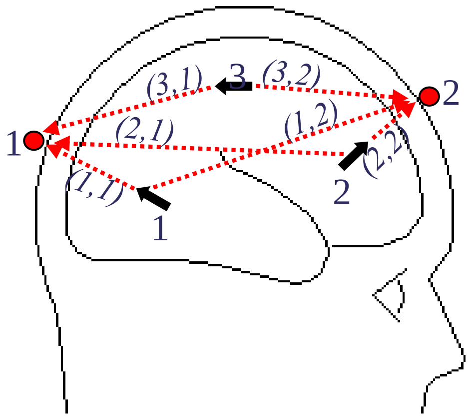


Our Goal: Spatio-Temporal Brain Dynamics
“Brain Movies”



The EEG/MEG Inverse Problem

The EEG/MEG Forward Problem



data "leadfield" dipoles

$$\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 0.5 & 0 & 0.3 \\ 0 & 1 & -0.3 \end{pmatrix} \begin{pmatrix} j_1 \\ j_2 \\ j_3 \end{pmatrix}$$

? inversion

dipoles inverse data

$$\begin{pmatrix} j_1 \\ j_2 \\ j_3 \end{pmatrix} = \begin{pmatrix} 1.5034 & 0.1241 \\ 0.2483 & 0.9379 \\ 0.8276 & -0.2069 \end{pmatrix} * \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

$j_1 + j_2 = 1$

under-determined problem, no unique solution

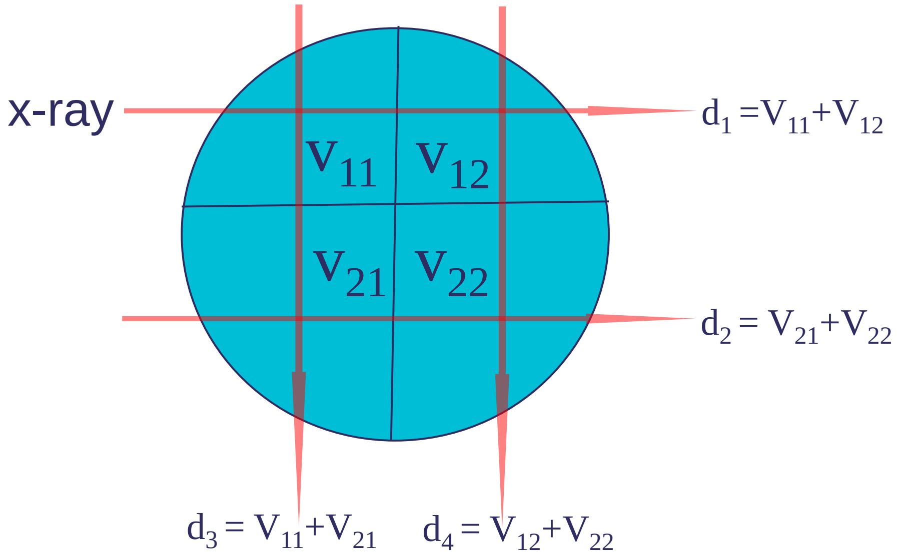
d=Lj

d: data (n_sensors x 1) **L**: "leadfield" (n_sensors x n_dipoles), **j**: dipoles (n_dipoles x 1)

Usually n_dipoles >> n_sensors.

EEG/MEG "Scanning" is not "Tomography"

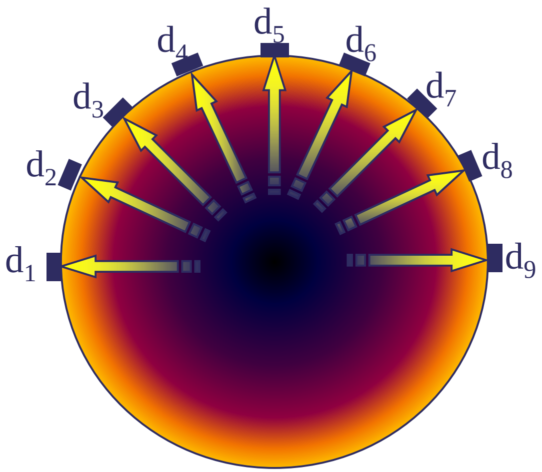
Tomography (CT, fMRI…)



$$d_1 = V_{11}+V_{12}$$
$$d_2 = V_{21}+V_{22}$$
$$d_3 = V_{11}+V_{21}$$
$$d_4 = V_{12}+V_{22}$$

Available information is determined by the equipment/experimenter

EEG/MEG



$$d_1 = V_{11}+V_{12}+V_{13}+V_{14} \ldots$$
$$d_2 = V_{21}+V_{22}+V_{23}+V_{24} \ldots$$

Information is lost during measurement

Cannot be retrieved by mathematics

Inherently limits spatial resolution

Why Inverse "Problem"?

Without additional constraints the solution is non-unique, i.e. there are infinitely many solutions

What is the solution to

$x_1 + x_2 = 1$

Maybe

$x_1 = 0 ; x_2 = 1$?

$x_1 = 1 ; x_2 = 0$?

$x_1 = 1000 ; x_2 = -999$?

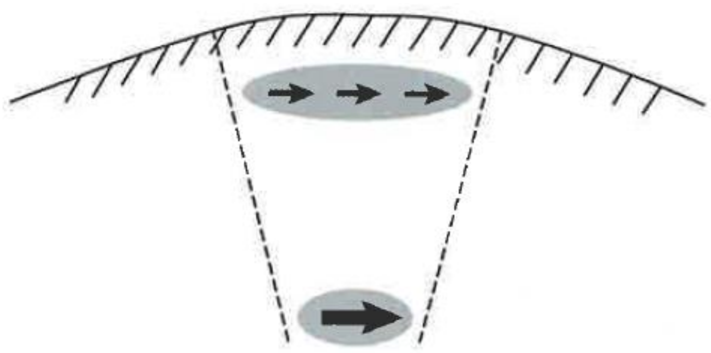
$x_1 = \pi ; x_2 = (1-\pi)$?

The "minimum norm solution" is:

$x_1 = 0.5 ; x_2 = 0.5$

with $(0.5^2 + 0.5^2)=0.5$ the minimum norm among all possible solutions.

Examples for Non-Uniqueness



A distributed superficial distribution may be indistinguishable from a focal deep source.

Jensen & Hesse, chap. 7 in “MEG”, OUP 2010, Hansen/Kringelbach/Salmelin (edts.)
See also Krishnaswamy et al. PNAS 2017 for approaches for deep sources,
https://www.pnas.org/content/114/48/E10465

Examples for Non-Uniqueness

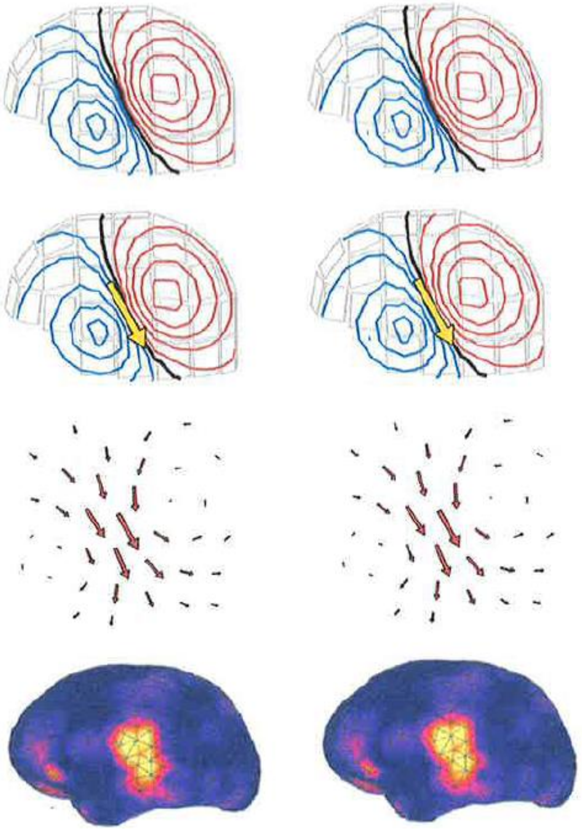
Field Patterns

Same Field Patterns

Dipole Model

Same Source Estimates

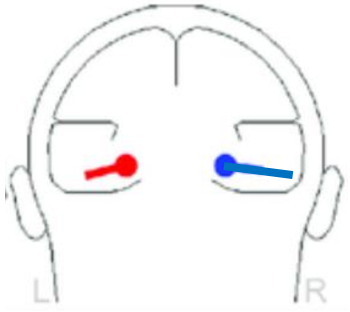
Minimum Norm Estimates

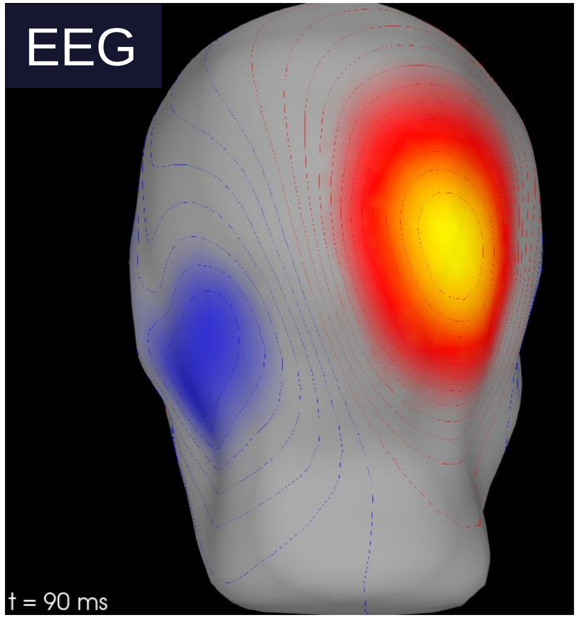


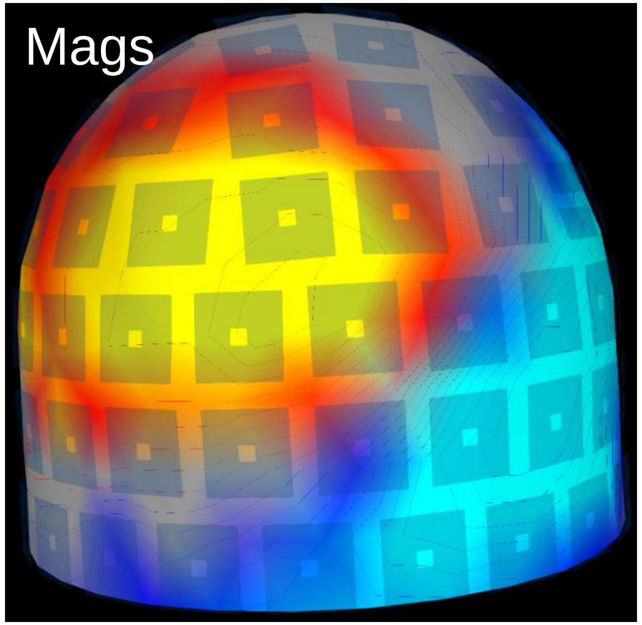
Hämäläinen & Hari, in Brain Mapping: The Methods (2nd), Elsevier 2002

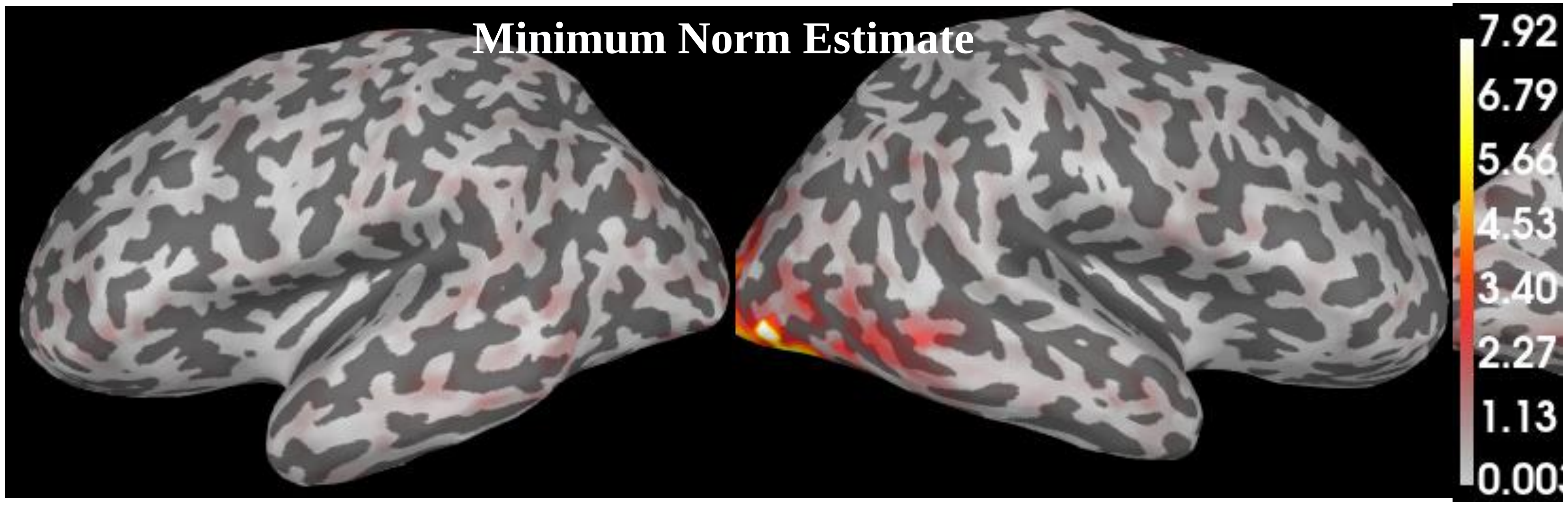
Example: Visually Evoked Activity ~100 ms

Checkerboard to left visual field



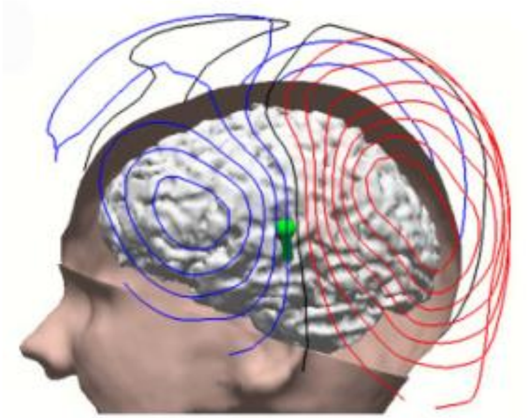


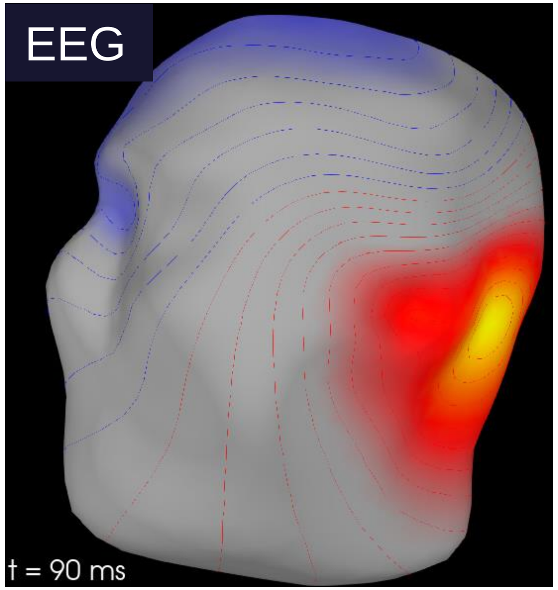


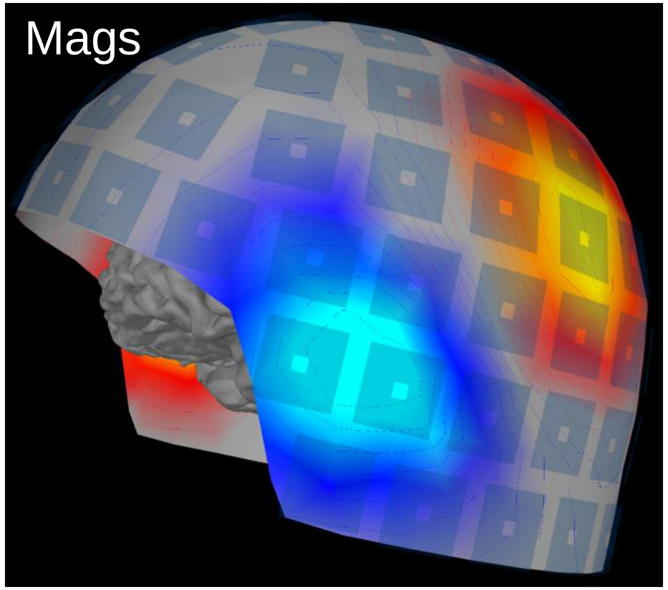


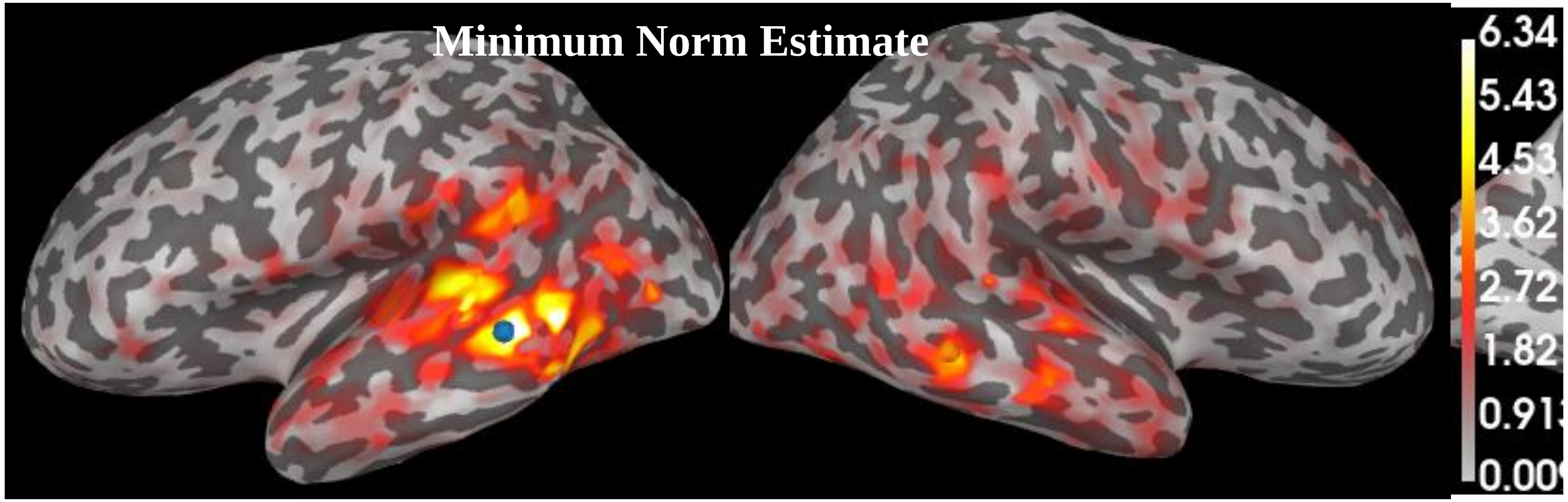
Example: Auditorily Evoked Activity

Tone to right ear







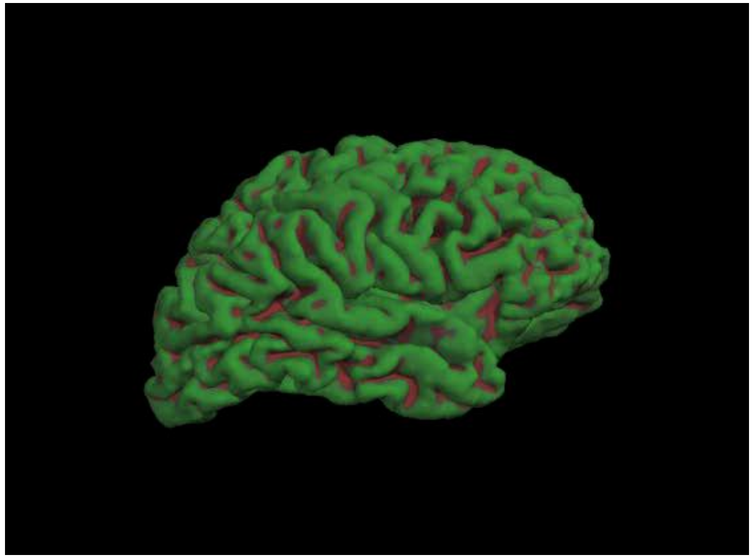


The Forward Problem and Head Modelling

Source Space and Head Model

Source Space

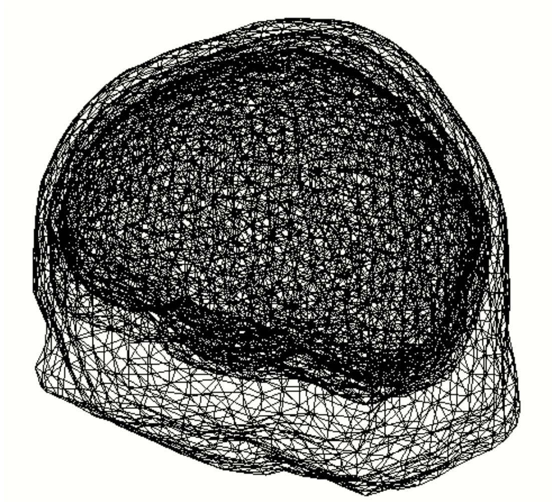
Where active sources may be located,
e.g. grey matter, 3D volume



http://www.cogsci.ucsd.edu/~sereno/movies.html

Volume Conductor/Head Model

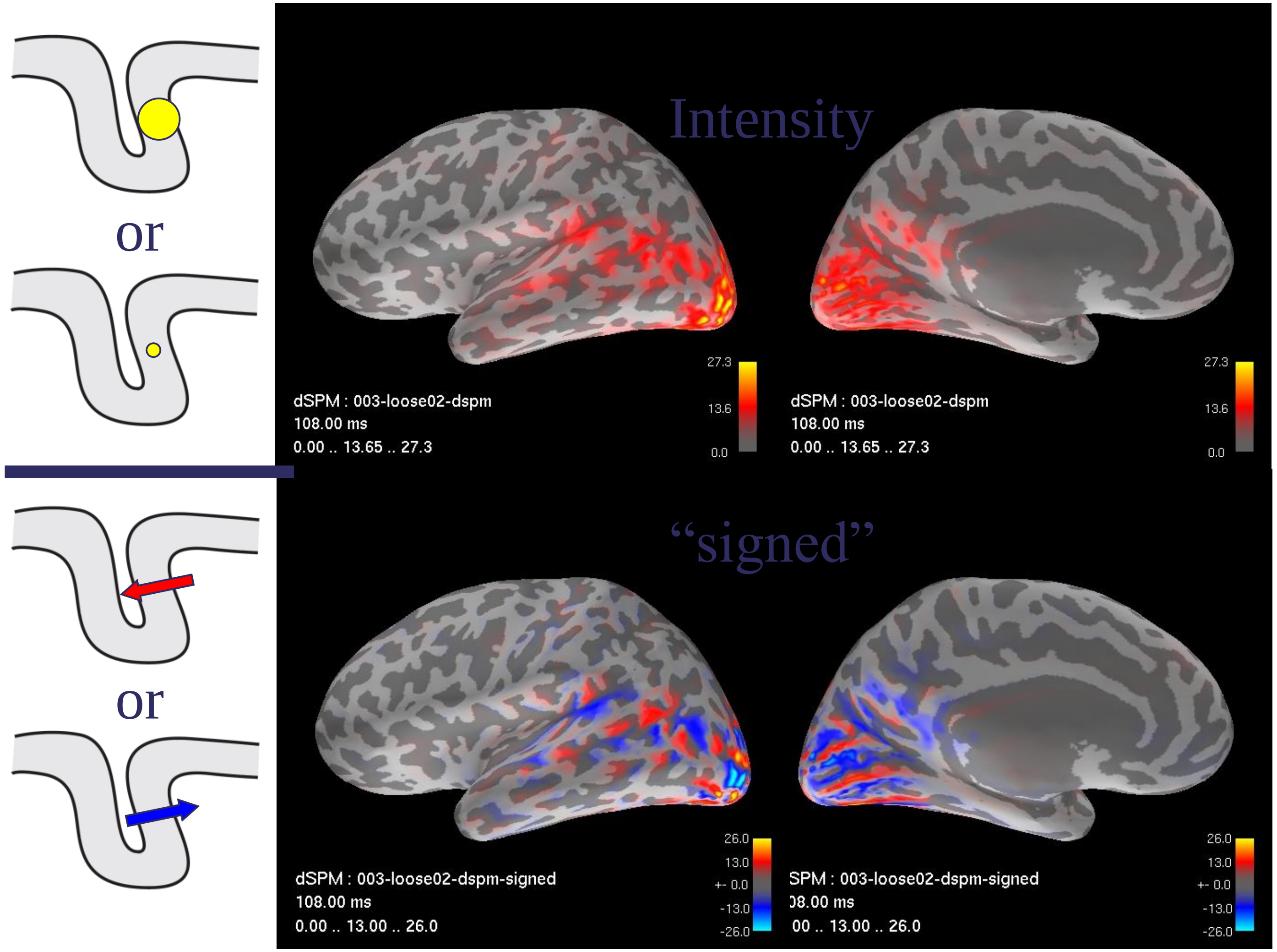
How we model conductivities/currents/potentials/fields in the head
e.g. sphere or realistic 1- or 3-compartments from MRI



Sometimes “standard head models” are used, when no individual MRIs available.

SPM uses the same “canonical mesh” as source space for every subjects, but adjusts it individually.

Direction of Current Flow



Solutions To The Inverse Problem – Source Estimation

Paths To Uniqueness

Dipole Fitting/Scanning

1. Assume there are only a few distinct sources

2. Iteratively adjust the location, orientation and strength of a few dipoles…

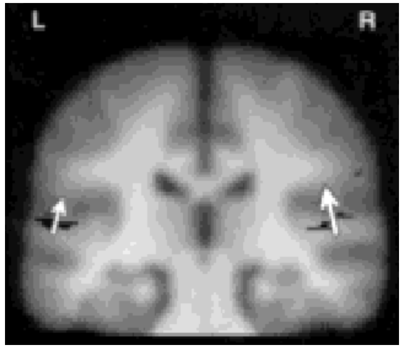
3. …until the result best fits the data

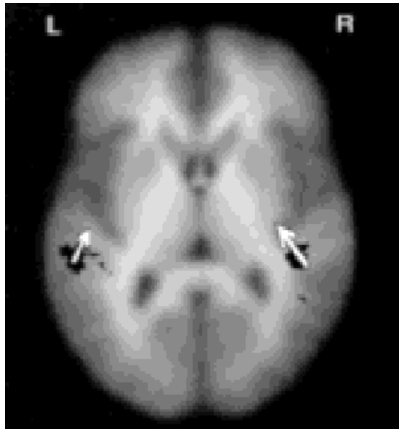
Distributed Sources

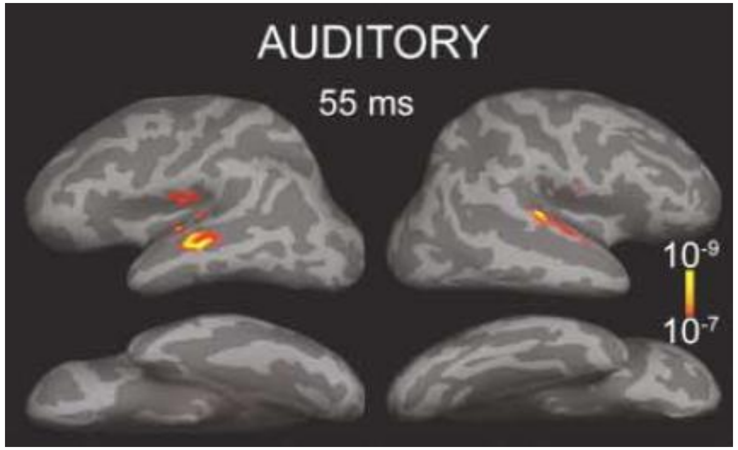
1. Assume sources are everywhere (e.g. distributed across the whole cortex)

2. Find the distribution of source strengths that explains the data…

3. …AND fulfils other constraints

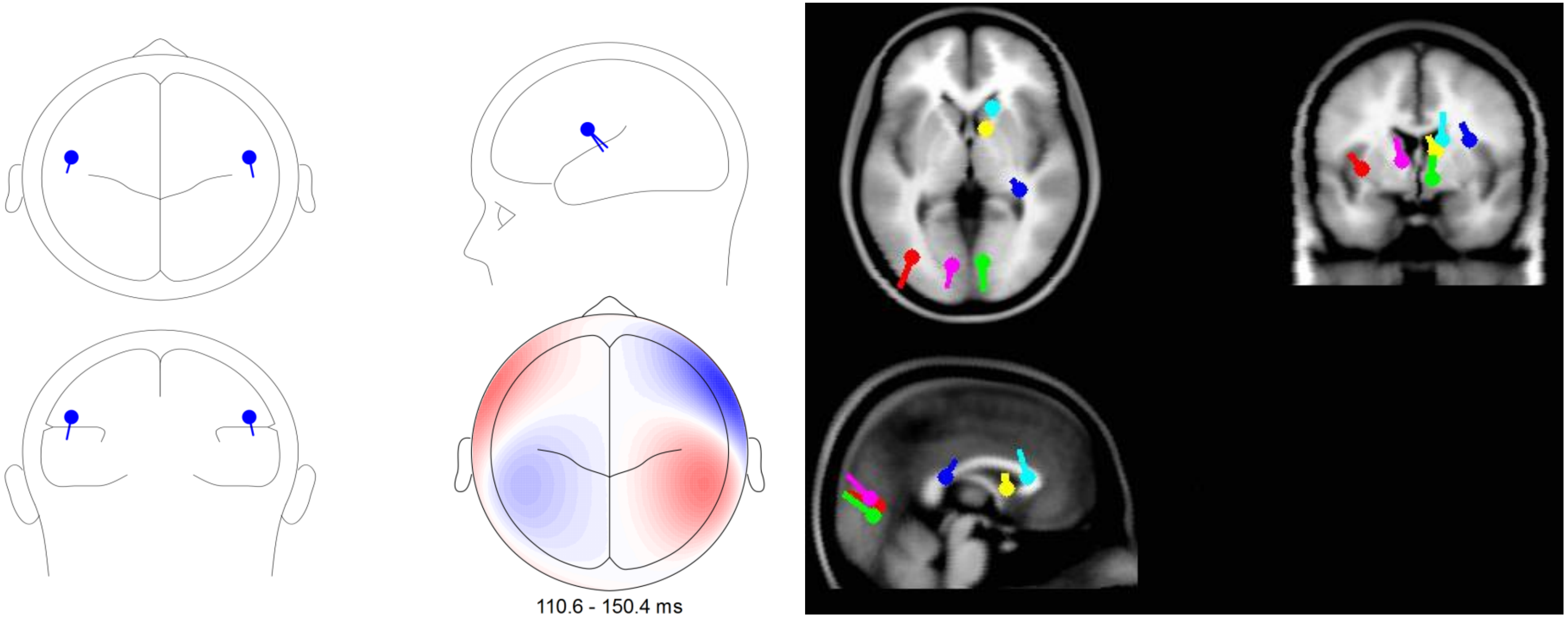






Hypothesis Testing - Dipole Fitting

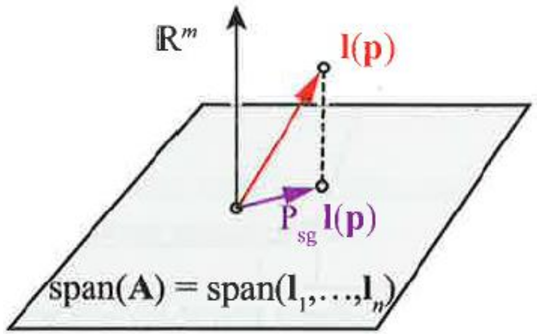
Explicit assumptions about the number of **focal sources (dipoles)** are tested by fitting dipole models to the data. The common criterion for the selection of models is the **goodness-of-fit**.



It can be hard to choose the appropriate number of dipoles – a priori knowledge is required.
Solutions for several/many dipoles can get stuck in local minima, and may not be robust to noise.

Multi-Dipole Scan: MUSIC
(Multiple Source Signal Classification)

Data and Noise Subspaces



Ilmoniemi & Sarvas, “Brain Signals”, MIT 2019

Classical MUSIC

1) Obtain a spatio-temporal data matrix $\boldsymbol{F}$, comprising information from m sensors and n time slices. Decompose $\boldsymbol{F}$ or $\boldsymbol{F}\boldsymbol{F}^T$ and select the rank of the signal subspace to obtain $\hat{\Phi}_s$. Overspecifying the true rank by a couple of dimensions usually has little effect on performance. Underspecifying the rank can dramatically reduce the performance.
2) Create a relatively dense grid of dipolar source locations. At each grid point, form the gain matrix $\boldsymbol{G}$ for the dipole. At each grid point, calculate the subspace correlations $\text{subcorr}\{\boldsymbol{G}, \hat{\Phi}_s\}$.
3) As a graphical aid, plot the inverse of $\sqrt{1-c_1^2}$, where c_1 is the maximum subspace correlation. Correlations close to unity will exhibit sharp peaks. Locate r or fewer peaks in the grid. At each peak, refine the search grid to improve the location accuracy, and check the second subspace correlation. A large second subspace correlation is an indication of a “rotating dipole.”

Mosher & Leahy, IEEE-TBME 1998

Recursively Applied (RAP) MUSIC

1) Estimate number of dipoles, e.g. using PCA/SVD.
2) Run MUSIC for one dipole.
3) Run MUSIC for 2^{nd} dipole, partialling out dipole 1.
4) Repeat for estimated number of dipoles.

See e.g. for overview and recent updates of MUSIC algorithms:
Ilmoniemi & Sarvas, “Brain Signals”, MIT 2019; Mäkelä et al., NI 2018 (“TRAP MUSIC”, https://pubmed.ncbi.nlm.nih.gov/29128542/)

One problem with MUSIC algorithms: They don’t give you source time courses.

“Spatial Filters”: Beamformers

Assumptions:

- All sources captured in data covariance matrix C (signal and noise)
- We are interested in one source *i* in many sources

Aim:

Design a spatial filter $\mathbf{w}_i$ which projects maximally on the source of interest and minimally on noise sources.

Project on source of interest: $\mathbf{w}_i^T \mathbf{f}_i$

Suppress noise: $\boldsymbol{min}(\mathbf{w}_i^T \mathbf{C} \mathbf{w}_i)$

$$\mathbf{w}_i = \frac{\mathbf{f}_i^T \mathbf{C}^{-1}}{\mathbf{f}_i^T \mathbf{C}^{-1} \mathbf{f}_i}$$

Linearly-Constrained Minimum-Variance (LCMV) Beamformer

Van Veen et al., 1997, https://pubmed.ncbi.nlm.nih.gov/9282479/

Create and apply these spatial filters vertex-by-vertex (dipole-by-dipole) and plot the distribution (possibly normalised by noise variance).

Spatial filters can also produce time courses for every source.

But note: The “spatial filter” interpretation applies to all linear methods, including MNE-type methods.

e.g. Hauk et al., Neuroimage 2022, https://pubmed.ncbi.nlm.nih.gov/35390459/
Hauk&Stenroos, HBM 2013, https://pubmed.ncbi.nlm.nih.gov/23616402/ ,

Minimum Norm Estimation Of Distributed Sources

$$\mathbf{L}\boldsymbol{s} = \mathbf{d} \quad \Rightarrow ||\mathbf{L}\boldsymbol{s} - \mathbf{d}||^2 = \mathbf{0}$$

(ignore noise for now)

subject to constraint

$$||\boldsymbol{s}||_{\mathbf{2}} = \textit{min}$$

yields the Minimum-Norm Least-Squares solution (“L2”)

$$\hat{\boldsymbol{s}} = \boldsymbol{G}_{\boldsymbol{MN}}\mathbf{d}$$

with

$$\boldsymbol{G}_{\boldsymbol{MN}} = \mathbf{L}^T(\mathbf{L}\mathbf{L}^T)^{-1}$$

But this is the result of mathematical desperation, and not based on physiology or what we want to know (e.g. localisation of multiple sources).

Noise and Regularisation in EEG/MEG Source Estimation

Noise and Regularization

Explaining the data 100% may not be desirable – some of the measured activity is not produced by sources in the model.

Explaining noise may require larger amplitudes in source space than the signal of interest:
Overfitting may seriously distort the solution
(“variance amplification” in statistics/regression).

"Whitening" and Choice of Regularisation Parameter

Whitened data have a noise covariance that is the identity matrix – i.e. noise is "white" (uncorrelated) noise.

$$\boldsymbol{G_{MN}} = \mathbf{L}^T\left(\mathbf{L}\mathbf{L}^T + \lambda\mathbf{C}^{-1}\right)^{-1}$$

can also be written as

$$\boldsymbol{G_{\widetilde{MN}}} = \tilde{\mathbf{L}}^T(\tilde{\mathbf{L}}\tilde{\mathbf{L}}^T + \lambda\mathbf{I})^{-1}$$

where $\tilde{\mathbf{L}}$ is the "whitened" leadfield $\mathbf{C}^{-1/2}\mathbf{L}$, and scaled such that trace($\tilde{\mathbf{L}}\tilde{\mathbf{L}}^T$)=trace($\mathbf{I}$).

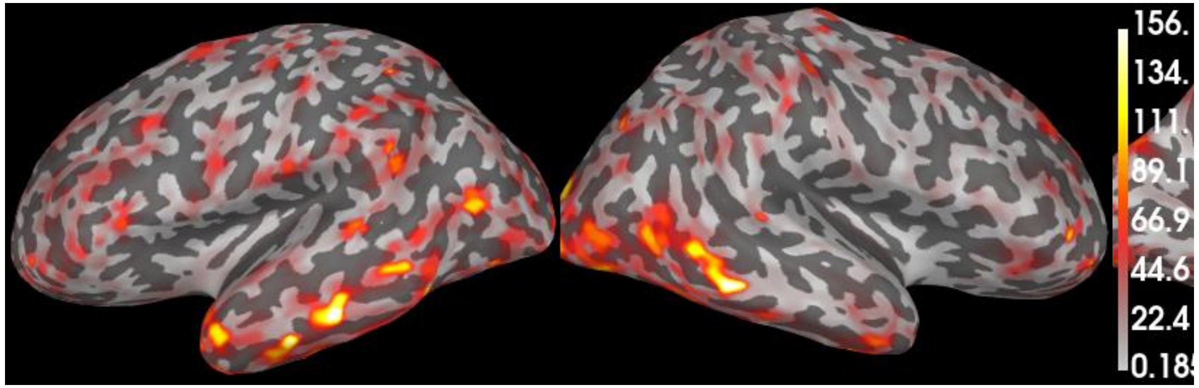
$\tilde{\mathbf{L}}$ and λ can now be interpreted in terms of signal-to-noise ratios.

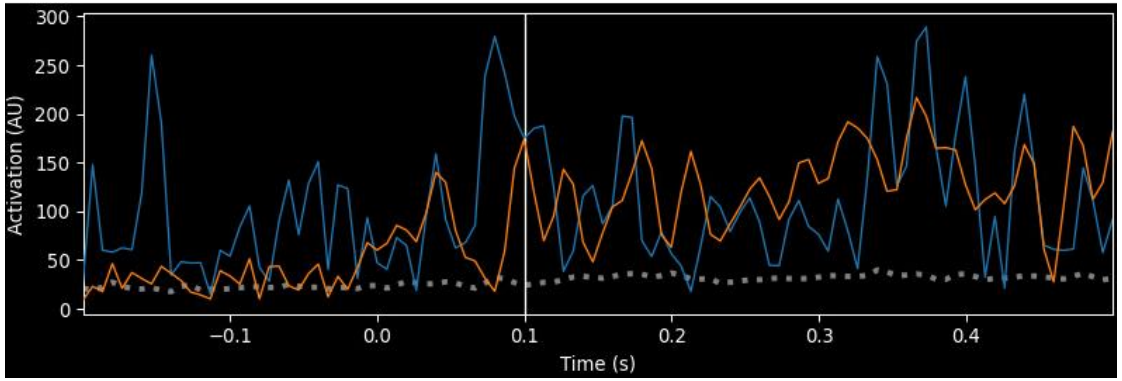
A reasonable choice for λ is then the approximate SNR of the data (e.g. in MNE software) –

usually heuristically chosen to be 3 (evoked) or 1 (raw/continuous).

The Effect of Regularisation ~ Over- and Under-Fitting

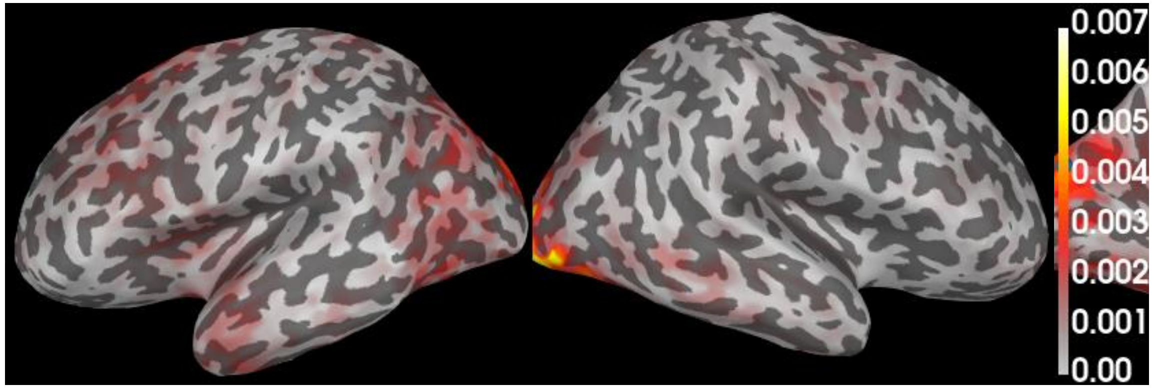
Over-fitting
“Under-smoothing”
SNR=300

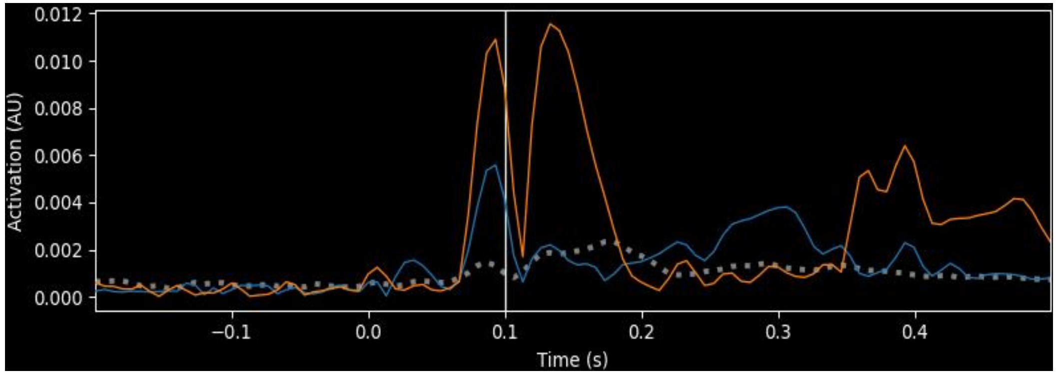




var=95.2%

Under-fitting
Over-smoothing
SNR=0.03





var=0.4%

Thank you