EEG/MEG 3:

Time-Frequency Analysis

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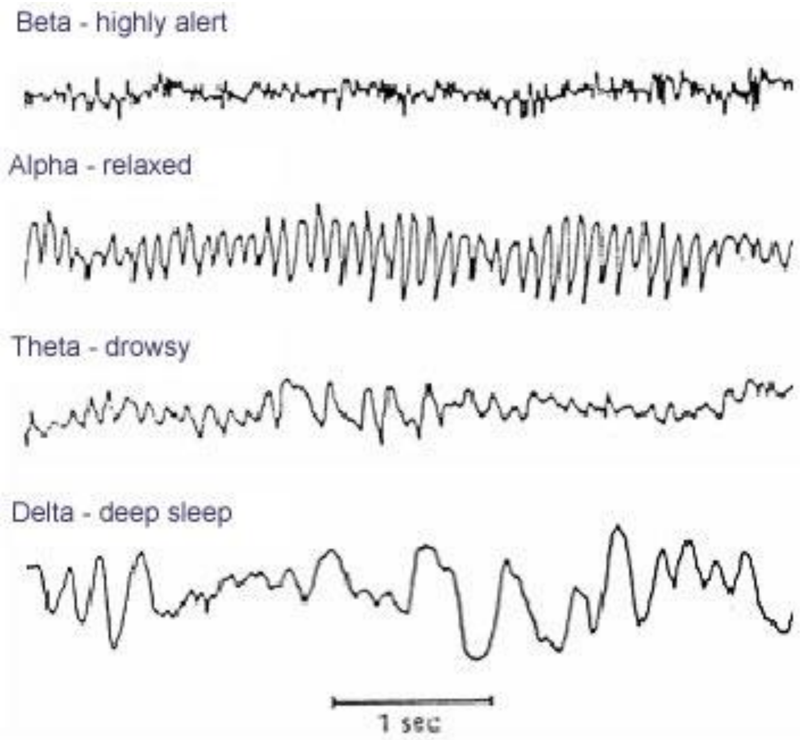
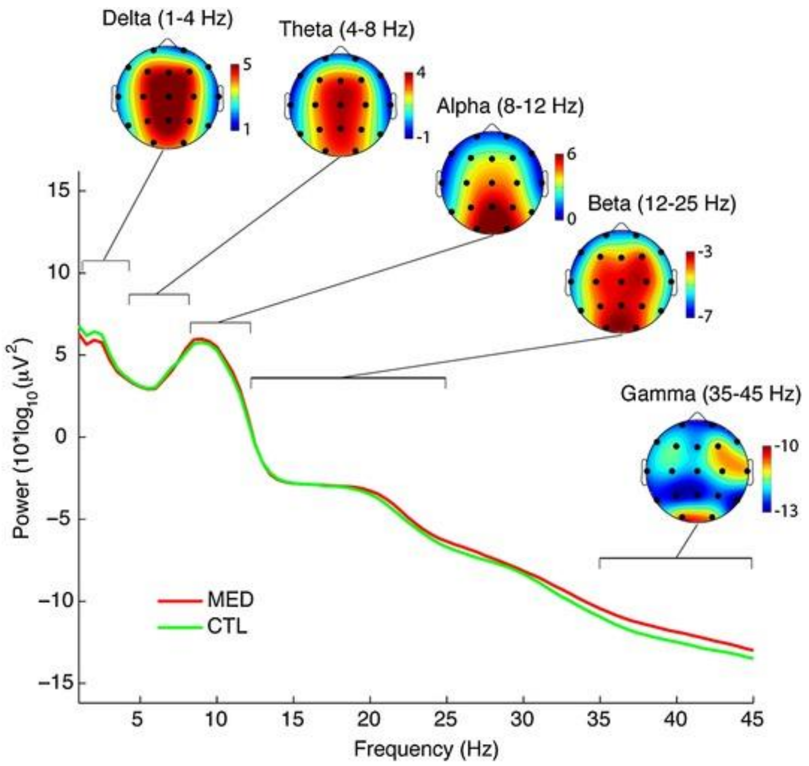
“Brain Rhythms” and “Oscillations”

Time course and topography may differ among different frequency bands

(and may depend on task, environment, subject group etc.)

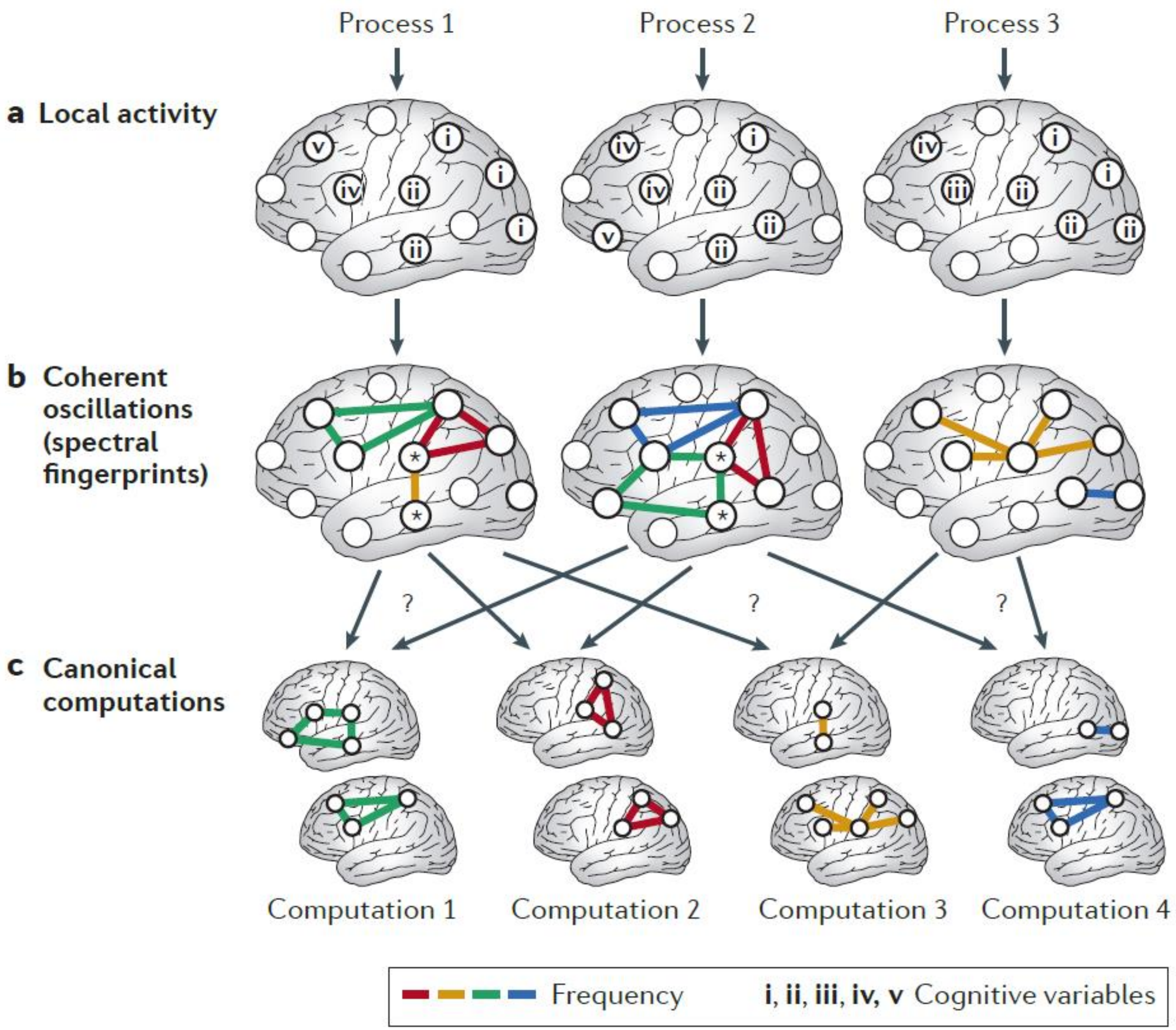
=> Different frequency “bands” may reflect different processes/computations, systems/networks, etc.





Cahn et al., Cogn Proc 2010,
http://link.springer.com/article/10.1007%2Fs10339-009-0352-1/

“Brain Rhythms” and “Oscillations”



Siegel et al., Nat Nsc 2012, https://pubmed.ncbi.nlm.nih.gov/22233726/

Polar Representation Of Periodic Signals

Euler's Formula

"Complex" numbers can capture the two axes of the coordinate system for the circle around which the vector rotates periodically – this is rather abstract but helps the notation enormously.

$$e^{-i\theta} = \boldsymbol{cos}(\theta) + \boldsymbol{i} * \boldsymbol{sin}(\theta) \qquad i=\sqrt{-1}$$

Therefore:

$$cos(\theta) = real(e^{-i\theta})$$

$$sin(\theta) = imag(e^{-i\theta})$$

An oscillation at a particular frequency can be described in a "polar representation":

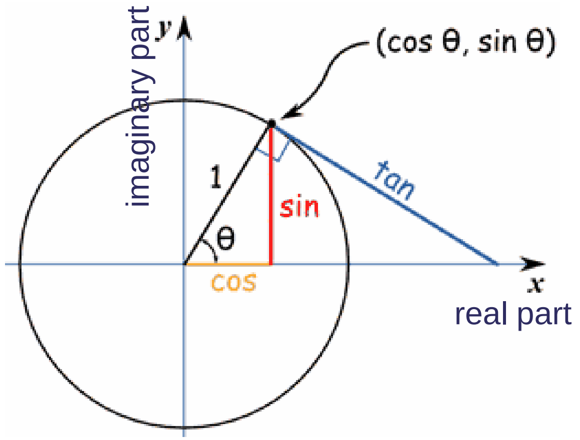
$$a * e^{-i2\pi ft}$$

a: amplitude

2π: circumference of unit circle

f: frequency

t: time



The Polar Representation Of Periodic Signals

Convenient To Compare Periodic Signals

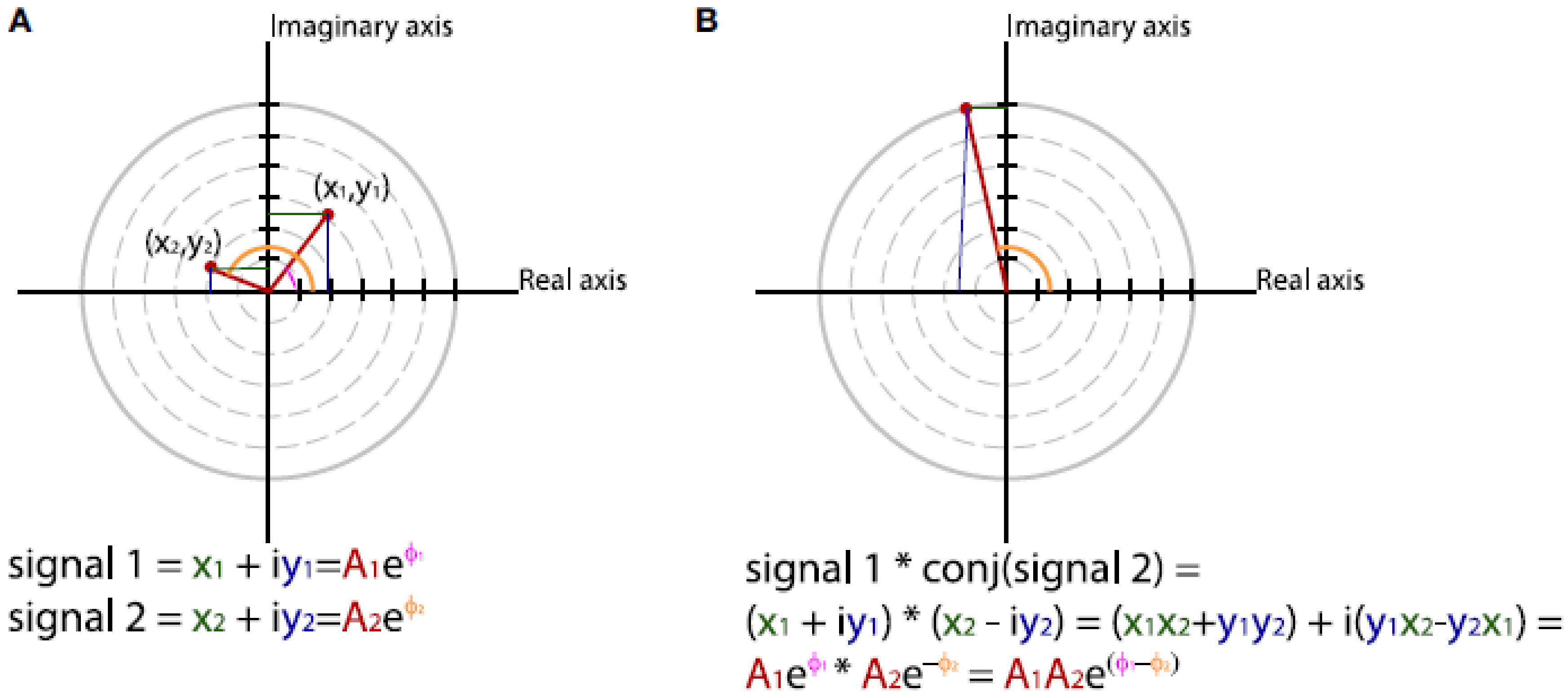


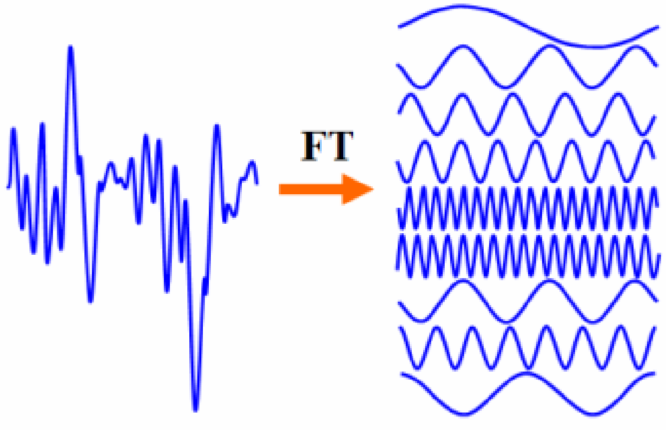
FIGURE 2 | Using polar coordinates and complex numbers to represent signals in the frequency domain. (A) The phase and amplitude of two signals. **(B)** The cross-spectrum between signal 1 and 2, which corresponds to multiplying the amplitudes of the two signals and subtracting their phases.

Bastos & Schoeffelen, Front Syst Nsc 2016
https://www.frontiersin.org/articles/10.3389/fnsys.2015.00175/full

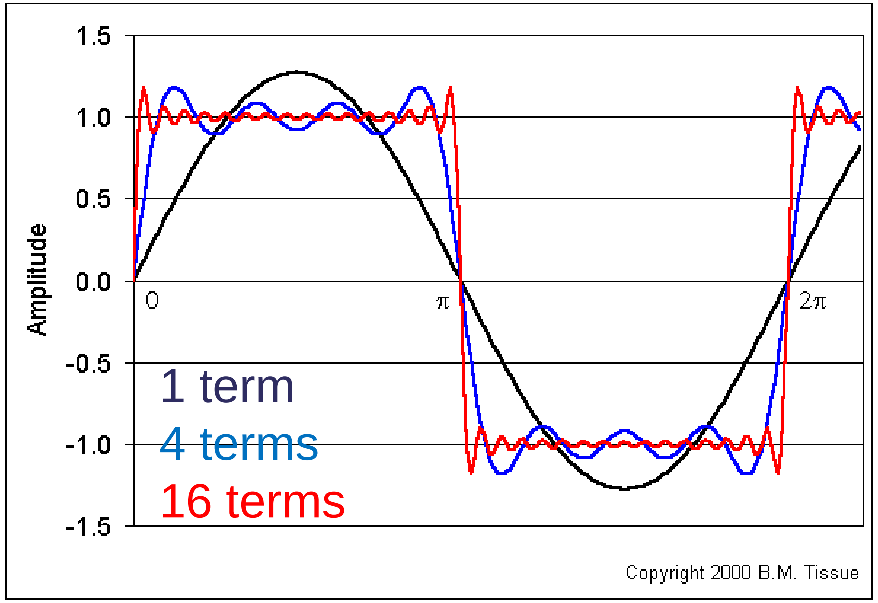
The Fourier Decomposition

The Fourier (De-)Composition

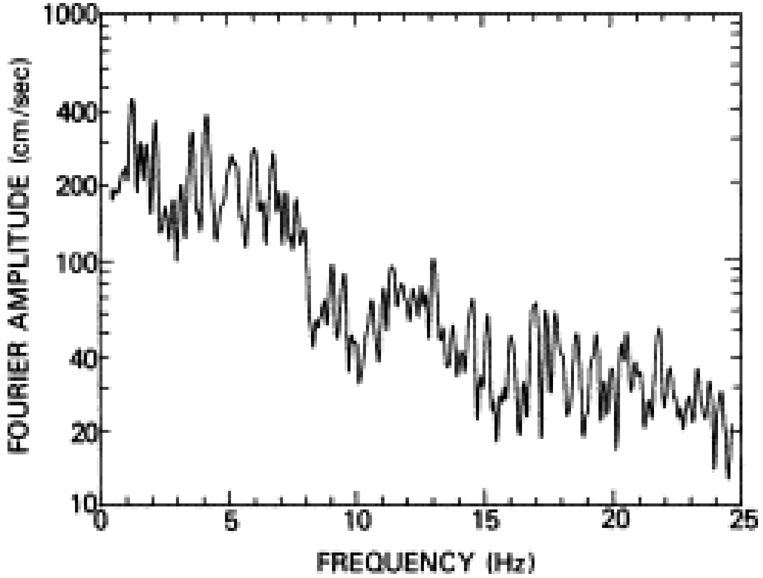
Decomposing signals
into sine/cosine terms



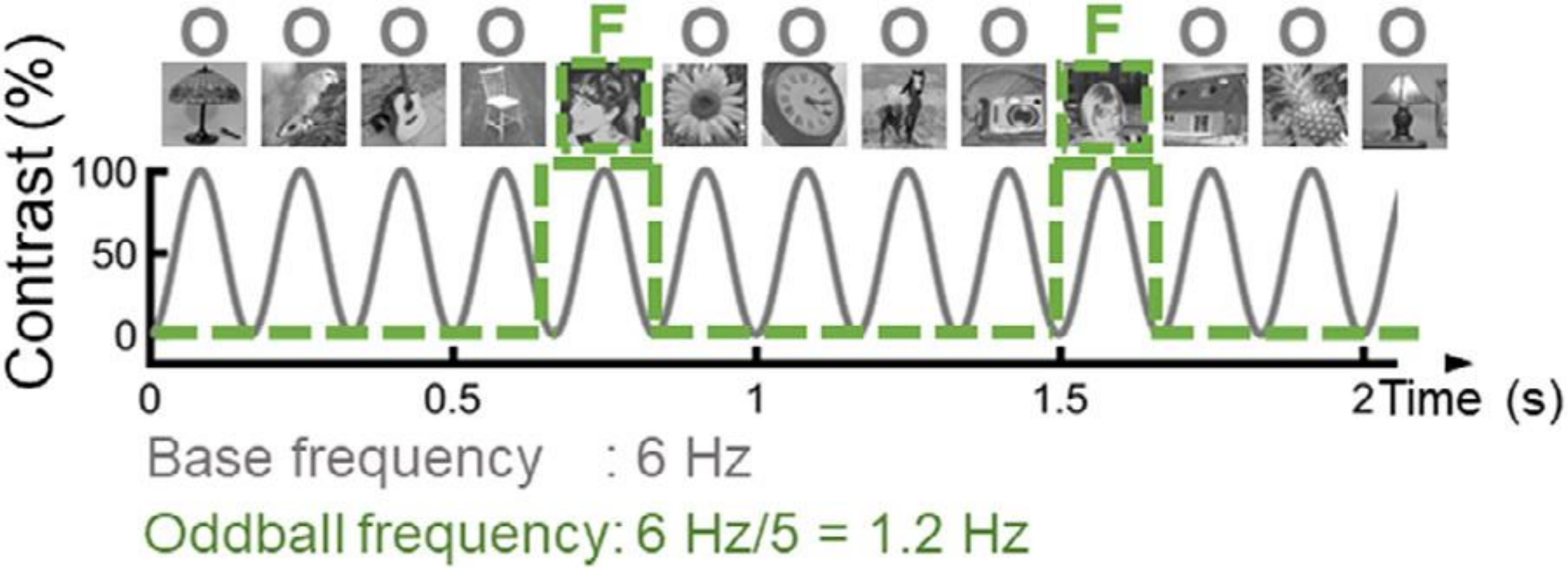
Approximating a step function
with Fourier terms



Frequency Spectrum

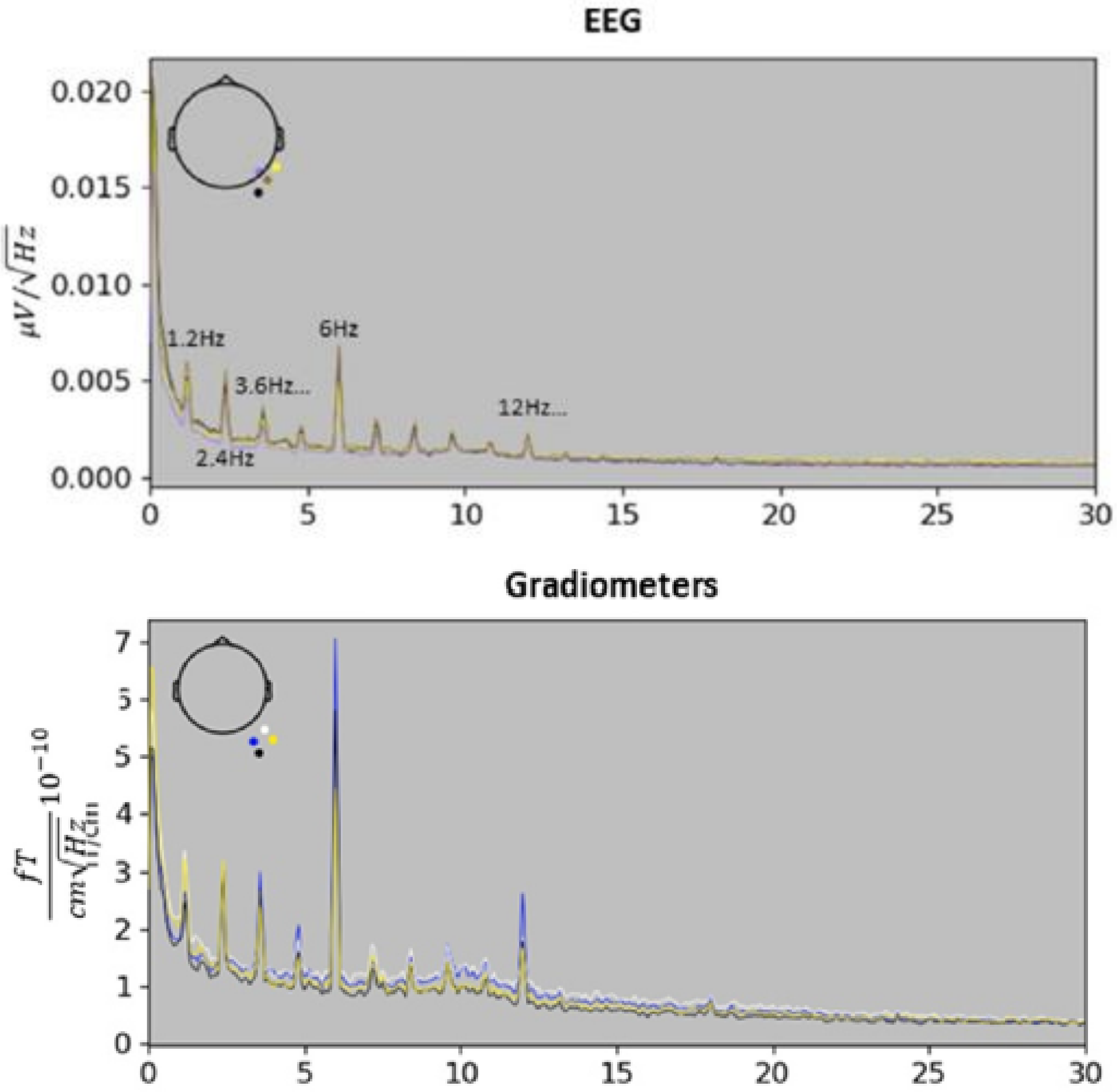


Example: Fast Periodic Visual Stimulation (FPVS)



Rossion et al., J Vis 2015, https://pubmed.ncbi.nlm.nih.gov/25597037/
Hauk et al., NI 2021, https://www.sciencedirect.com/science/article/pii/S1053811921007345

Fast Periodic Visual Stimulation (FPVS)



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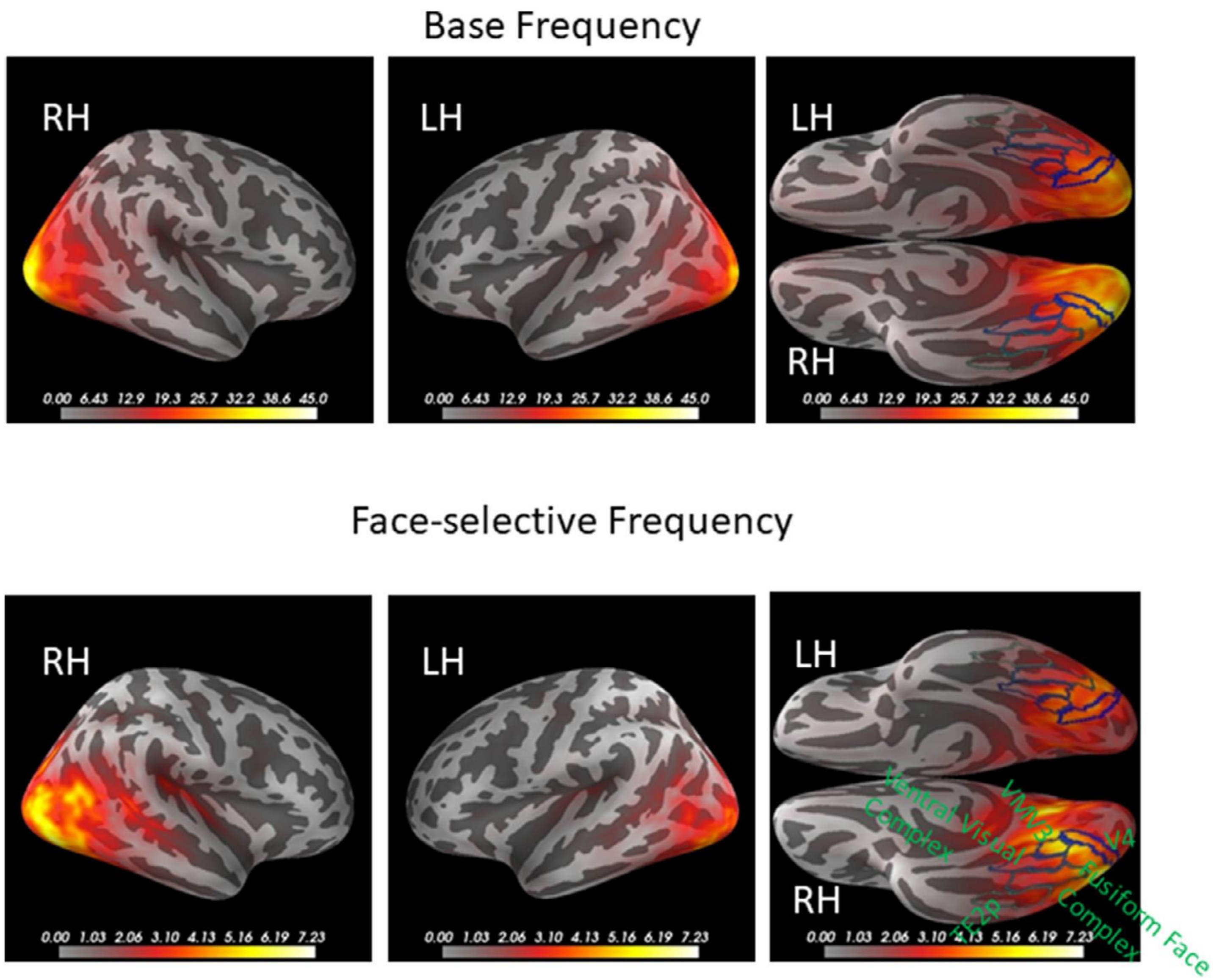
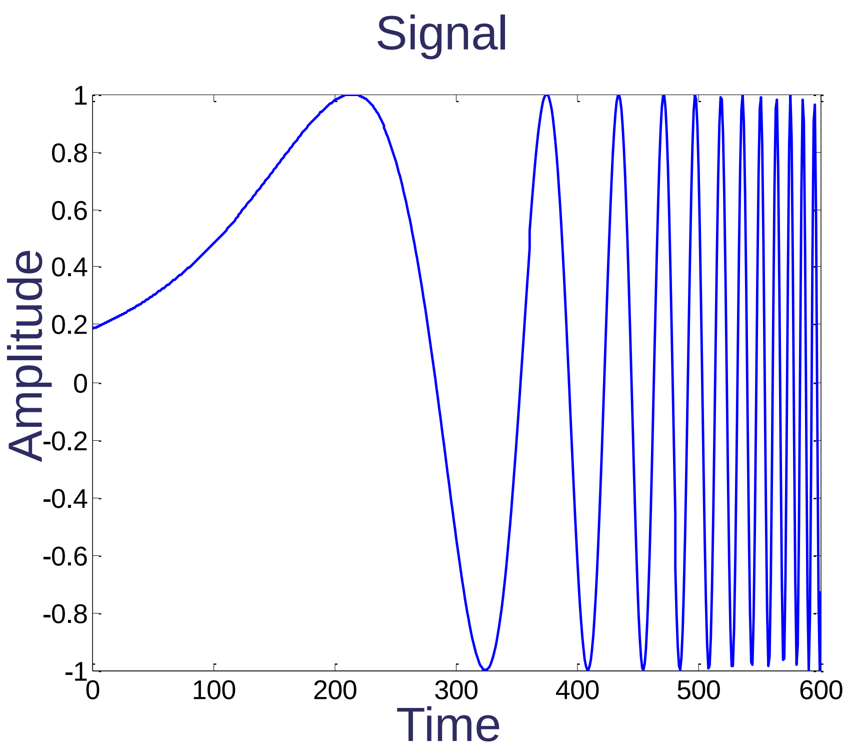


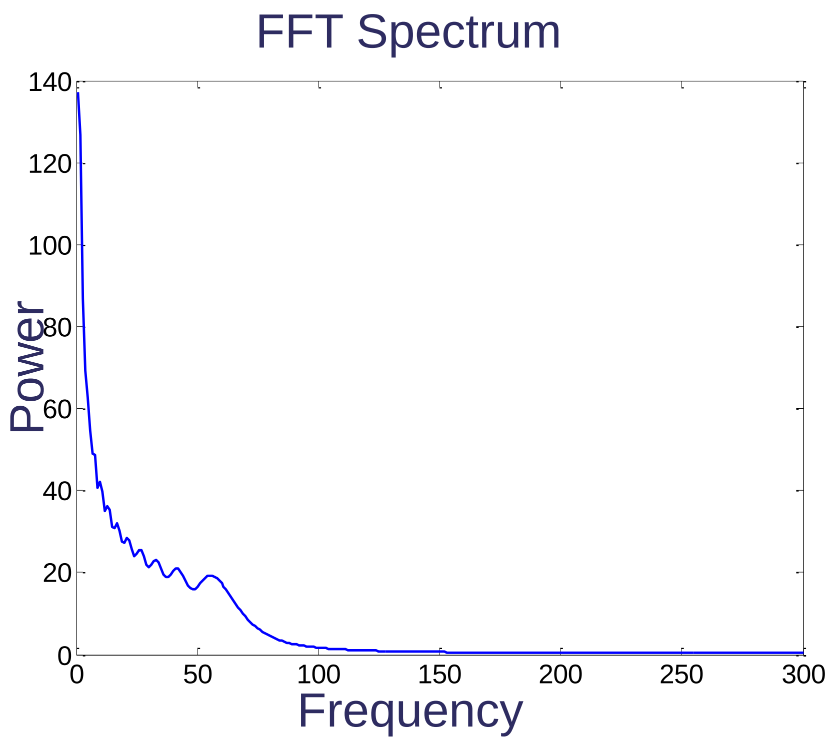
Hauk et al., NI 2021, https://www.sciencedirect.com/science/article/pii/S1053811921007345

Time-Frequency Analysis

Motivation for Time-Frequency Analysis

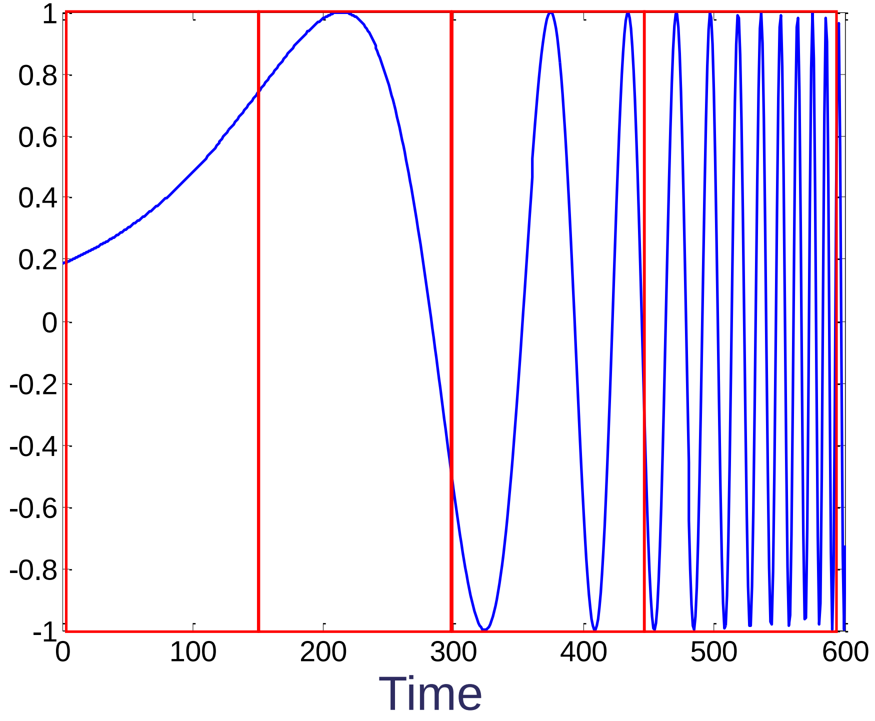
Fourier Transform assumes sines and cosines with constant amplitudes across the whole time series (“stationarity”).
What are we going to do with signals where activity in frequency bands changes over time, e.g. with a signal like this?





Motivation for Time-Frequency Analysis

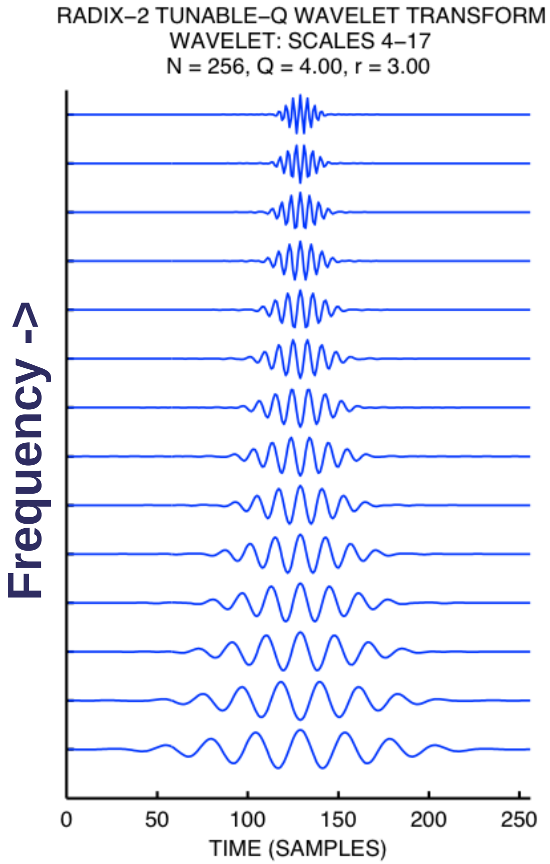
You could run separate FFTs for different (sliding) time windows:



But different window sizes are more or less optimal for different frequencies.
Run different FFTs with different window sizes for different frequency ranges? Ouff.

<u>Time</u>-Frequency Analysis: Wavelets (“little waves”)

Wavelets provide an optimal trade-off between frequency and time resolution.



Wavelets are getting “broader” with decreasing frequency

=>

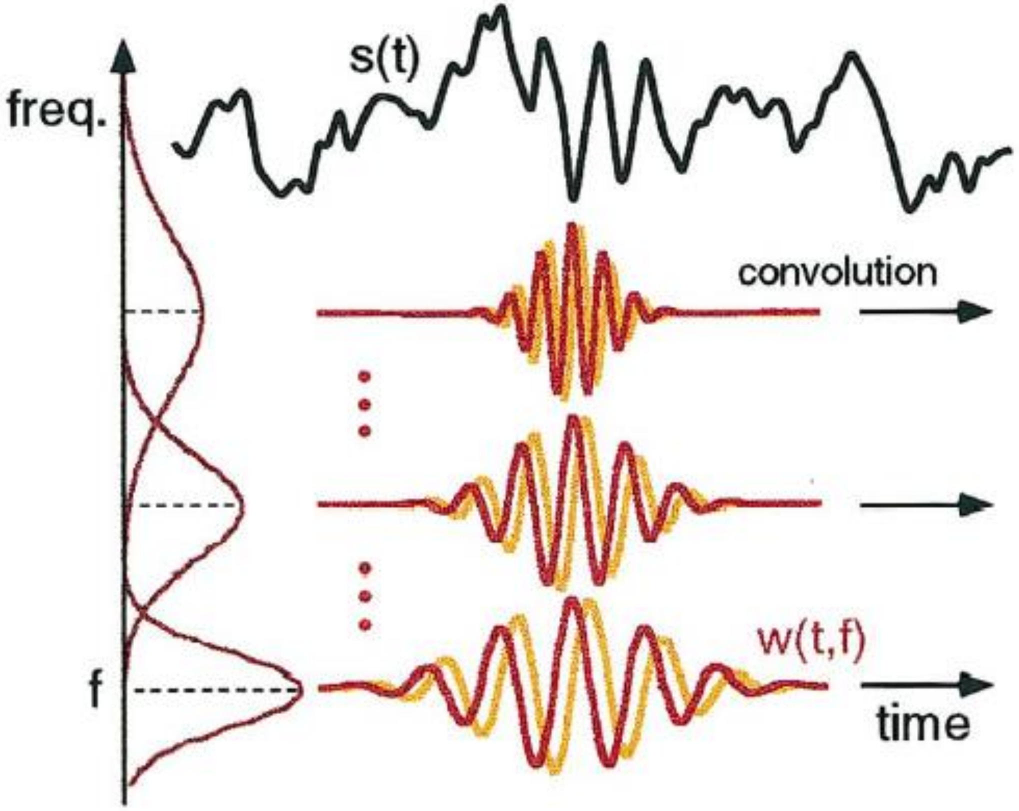
Time resolution decreases as frequency decreases

Wavelets are convolved with the data to give instantaneous amplitude and phase estimates for different frequency ranges.

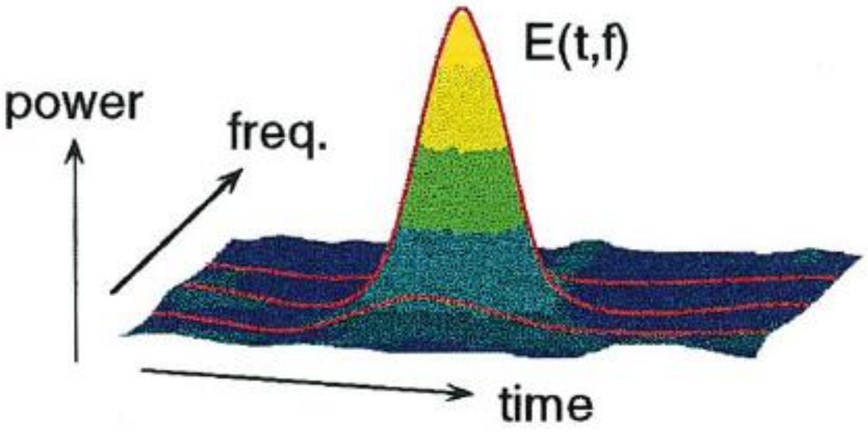
Time-Frequency Analysis: Wavelets

Wavelet Transform

Trade-off between time and frequency resolution



Time-Frequency Power



Tallon-Baudry & Bertrand, TICS 1999
https://pubmed.ncbi.nlm.nih.gov/10322469/

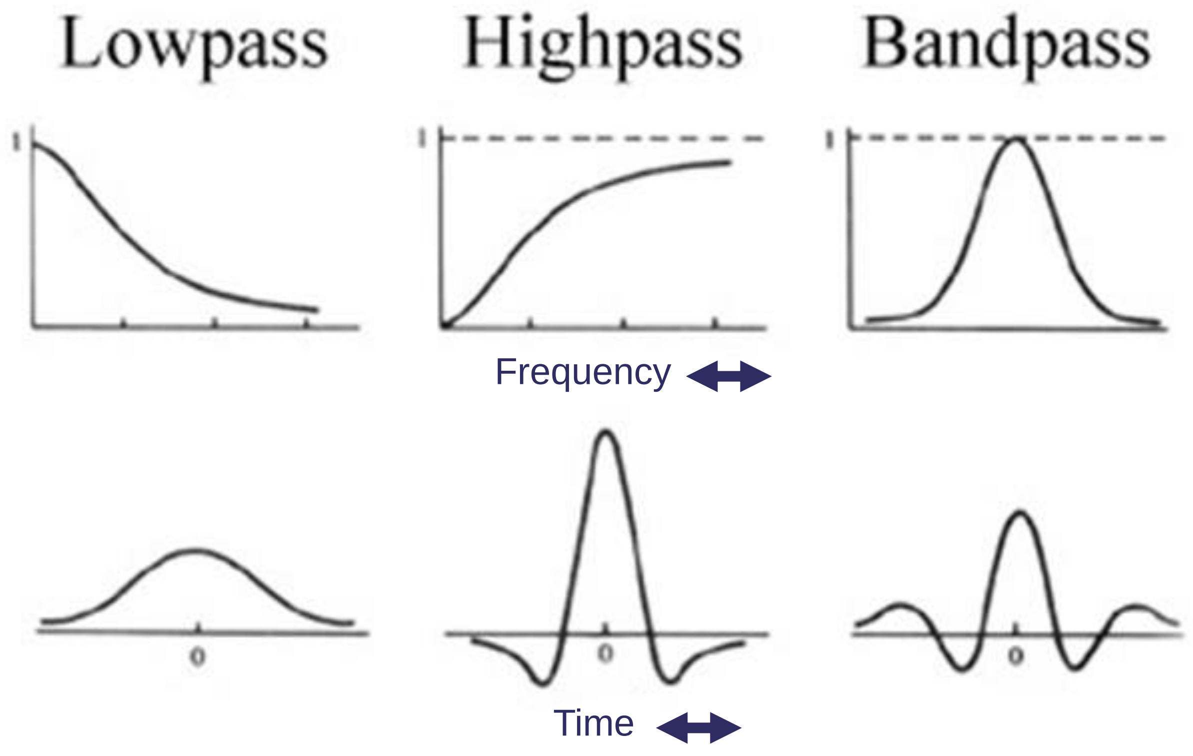
Basic Principals of Frequency Filtering

Time-domain and frequency-domain filtering are two sides of the same coin:

One type of frequency-domain filtering corresponds to one type of time-domain filtering.

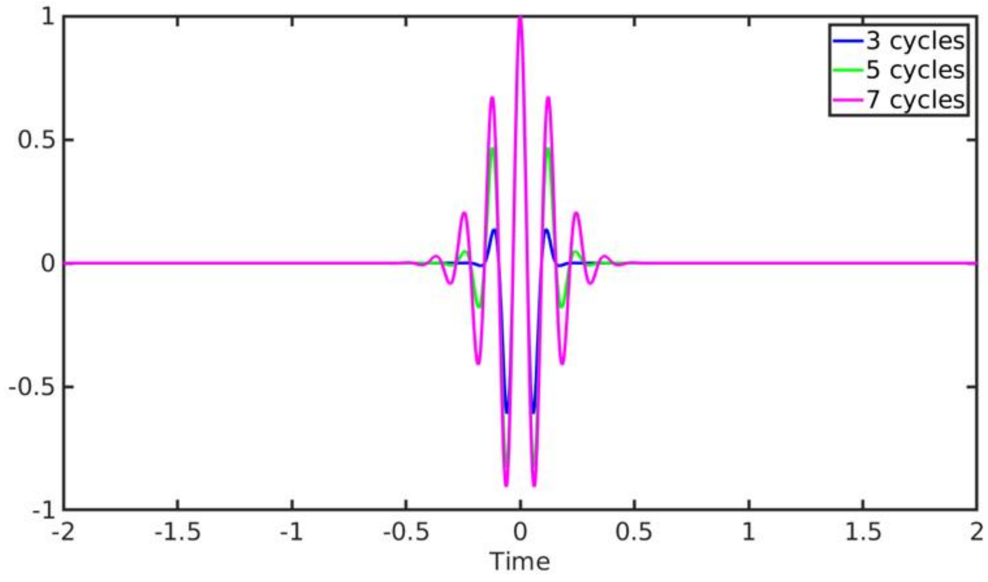
Frequency domain

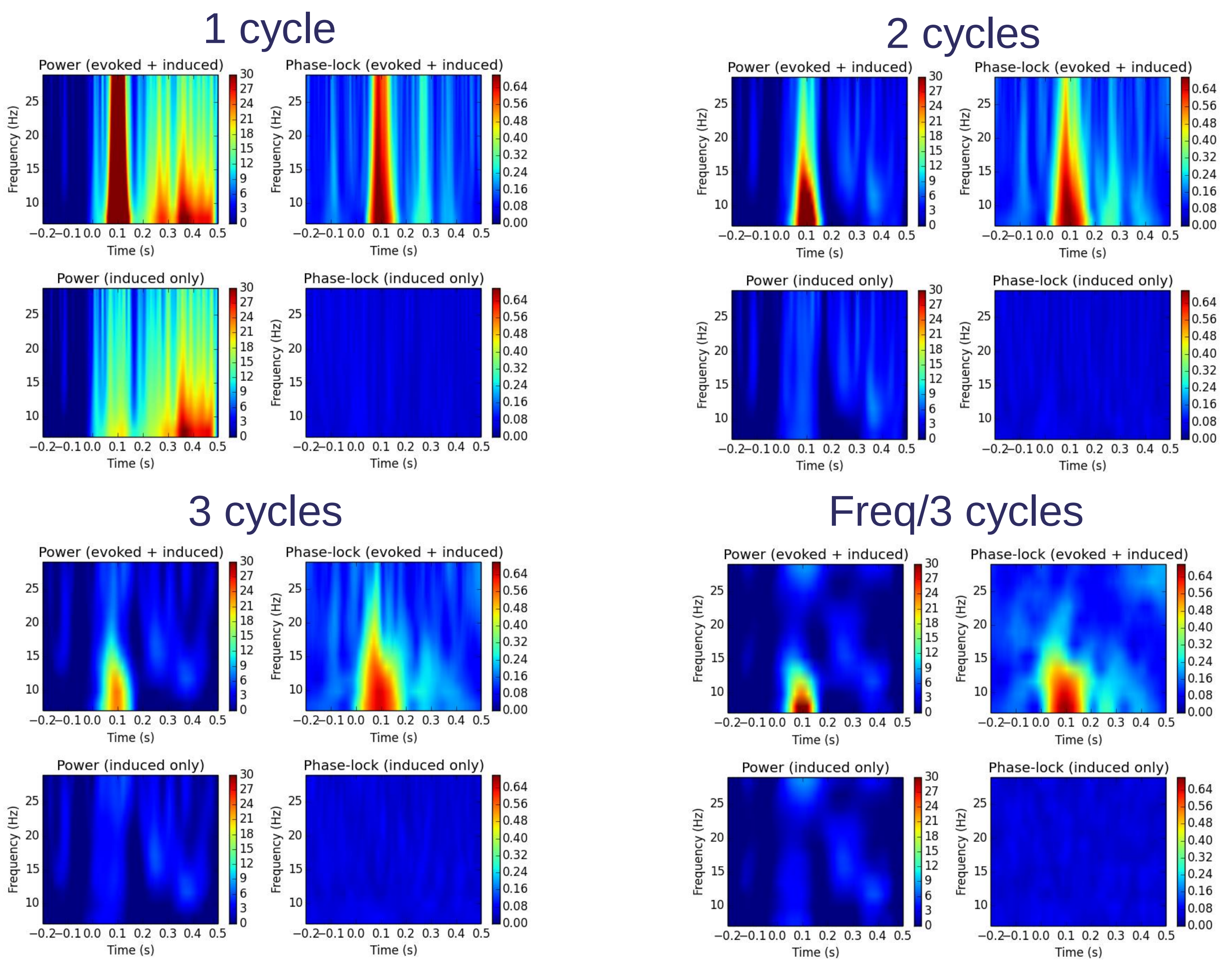
Time domain



https://uk.mathworks.com/matlabcentral/fileexchange/51155-time-domain-filtering-vs-frequency-domain-filtering-in-images

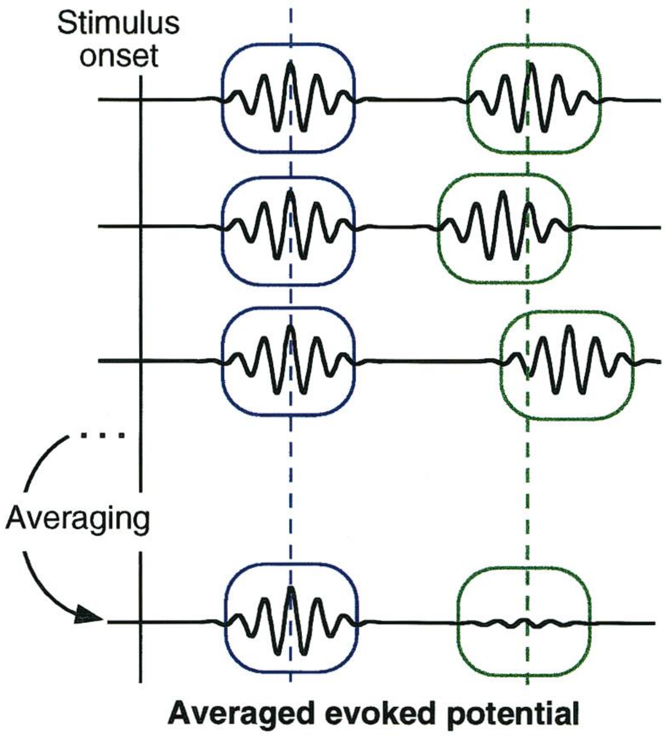
Effect of Number of Cycles

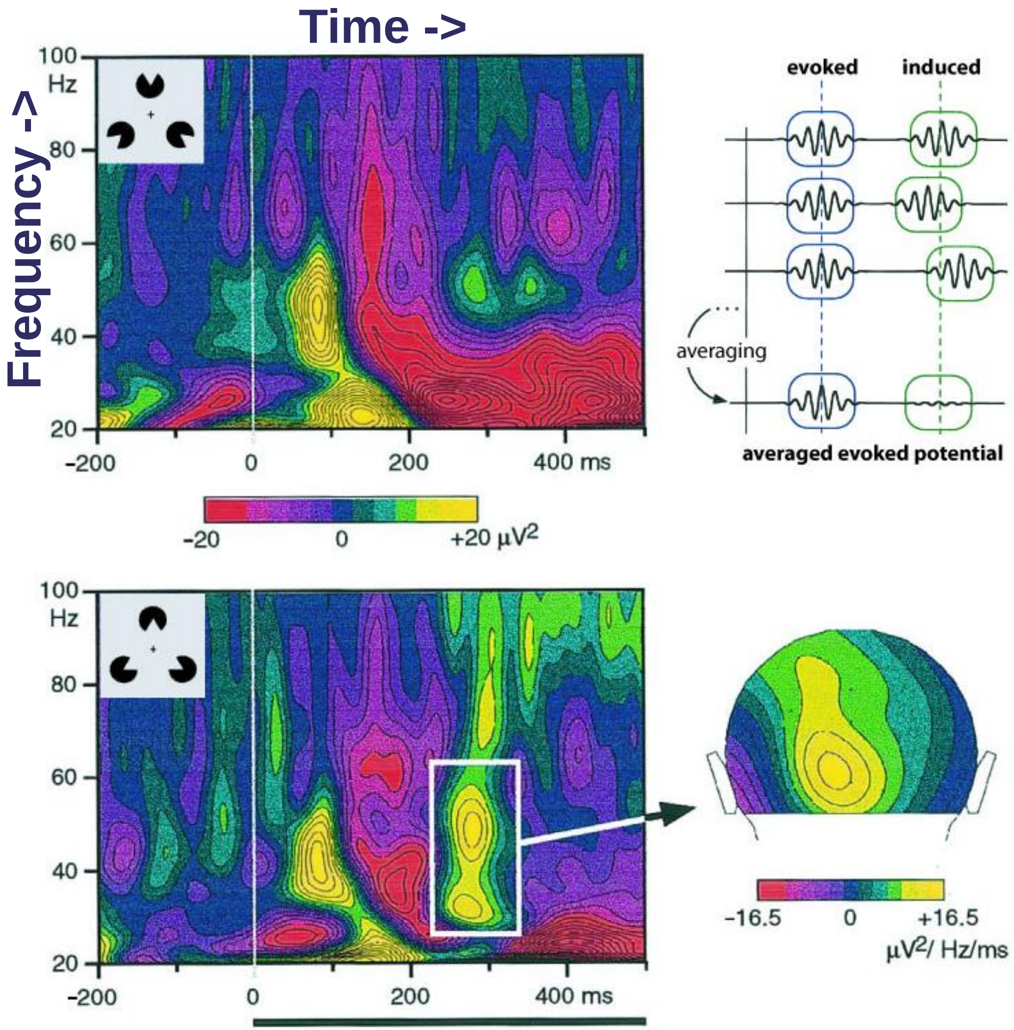




Rule of thumb: For low frequencies (<~10Hz), n=2 or 3; for higher frequencies n=f/3.

Evoked and Induced Rhythmic Activity



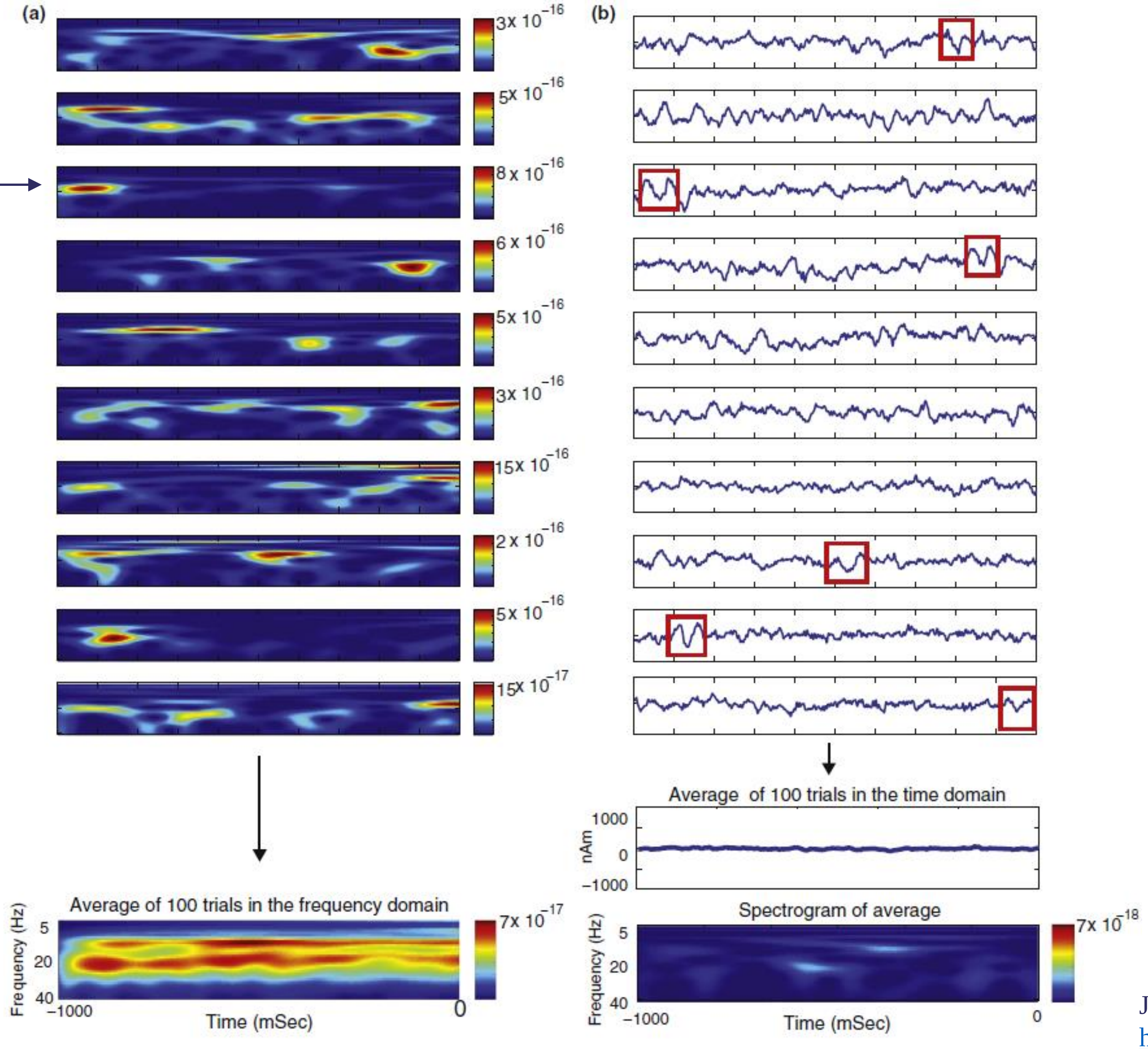


Tallon-Baudry & Bertrand, TICS 1999
https://pubmed.ncbi.nlm.nih.gov/10322469/

When brain rhythms aren't "rhythmic" – the example of beta "oscillations"

"beta bursts"

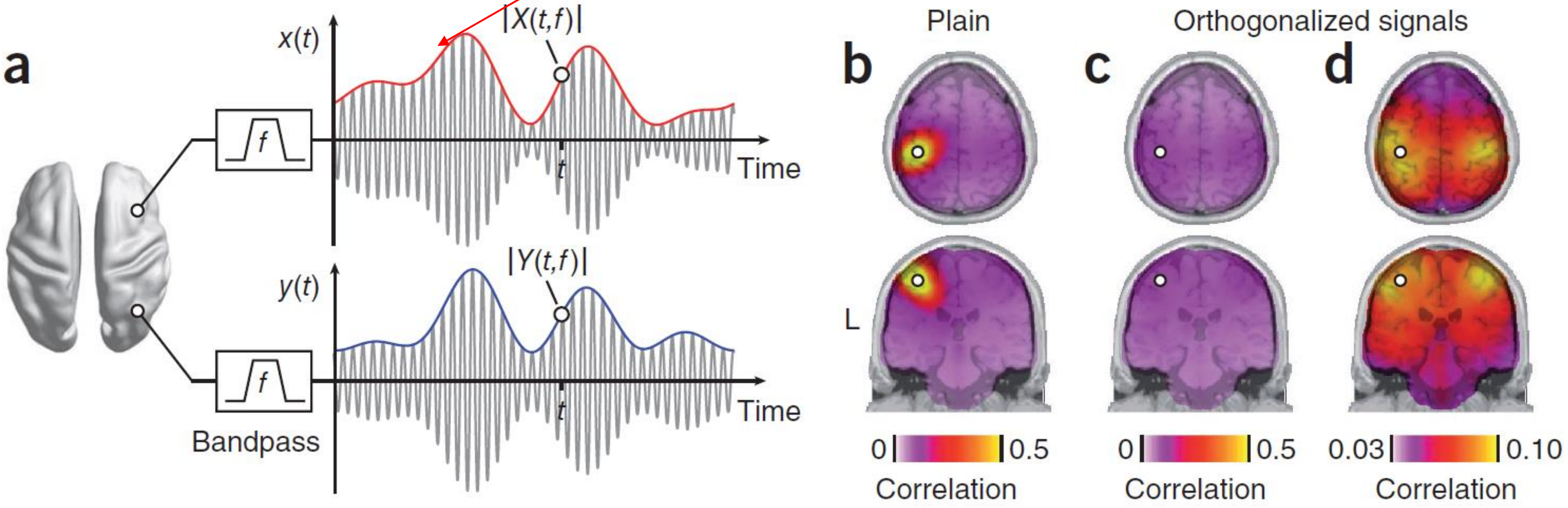
rather than "oscillations"



Jones et al., Curr Op Neurobiol 2016
https://pubmed.ncbi.nlm.nih.gov/27400290/

Alternative to wavelets: Hilbert Transform

(“Hilbert”) Envelope for a frequency band



Hipp et al., Nat Nsc 2012, https://www.nature.com/articles/nn.3101

Thank you