Homework #5 (Chapter 7 – Part II)

1. From the definition of $\hat{R}_{ij}(\vec{k})$, show that

$$\hat{R}_{ij}(\vec{k}) \geq 0, \quad \text{for } i = j$$
$$\hat{R}_{ij}(\vec{k}) = \hat{R}_{ji}(-\vec{k}) = \hat{R}^{*}_{ji}(\vec{k})$$

   And from the incompressibility show that

$$k_i \hat{R}_{ij}(\vec{k}) = \hat{R}_{ij}(\vec{k}) k_j = 0$$

2. Show that $\hat{E}(\vec{k})$ is real, non-negative, with

$$\hat{E}(-\vec{k}) = \hat{E}(\vec{k})$$

3. First prove the following relation

$$\langle \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_l} \rangle = \iiint_{-\infty}^{+\infty} k_k k_l \phi_{ij}(\vec{k}) d\,\vec{k}$$

   by substituting u_i and u_j from Eq. (7.72) and using Eq. (7.86). Then, verify Eq. (7.83).