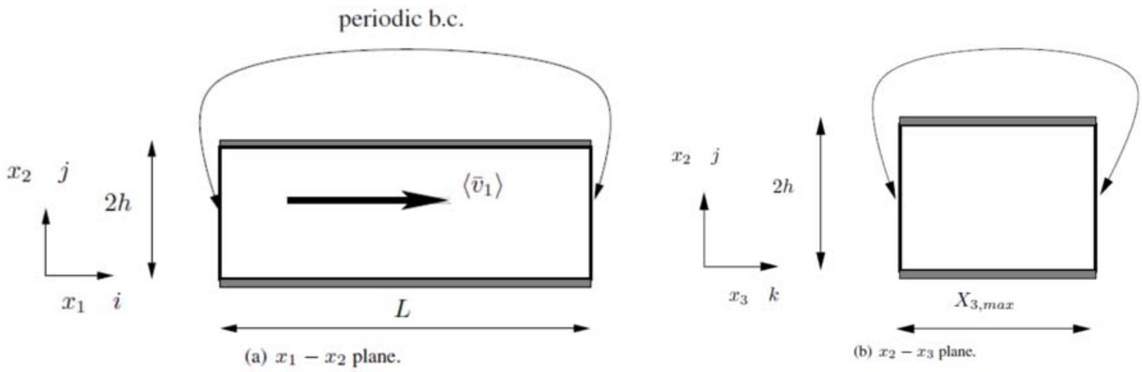
Homework #2 (Chapters 2-5)

The non-dimensionalized Navier-Stokes equations (M. 1 of Appendix M of reference [2]) have been solved in a fully developed channel flow using Direct Numerical Simulation (DNS).



The DNS data of 3 instances of the solution is given in "HW#2-Materials.zip" (see also part M6). A sample of processing of the data is performed in file "read_files.m" where the data of V_1 velocity is read and the profile of $\langle V_1 \rangle$ versus y is plotted. Note that the variables are non-dimensionalized, i.e. (superscript "*" indicates dimensional variables)

$$x_i = \frac{x_i^*}{h}, V_i = \frac{V_i^*}{u_\tau} = V_i^+, p = \frac{p^*}{\rho u_\tau^2}, Re_\tau = \frac{u_\tau h}{\nu} = 500, x_i^+ = \frac{u_\tau x_i^*}{\nu} = Re_\tau x_i$$

1. By integrating $\langle V_1 \rangle$ across the channel, compute the bulk velocity, $\langle V_1 \rangle_{\text{bulk}}$. What is the value of Reynolds number based on the bulk velocity, $Re = \langle V_1 \rangle_{\text{bulk}} h/\nu$?
2. Compute the 6 components of the (normalized) Reynolds stress tensor, $\langle v_i v_j \rangle$, (see Part M.6 Resolved stresses) and plot them across the channel versus $y^+ = x_2^+$. Note that you should use averaging in (x,z,t) directions. Compare your profiles with the ones in Figs. 7.14, 7.15, and 7.17 of reference [1] (for $Re = 13750$).
3. Note that in the fully developed channel flow, the only mean velocity gradient is $\partial \langle V_1 \rangle / x_2$. Knowing this, simplify the relations of the production rate, P, dissipation rate, ε, dissipation of the mean, $\bar{\varepsilon}$, and pseudo-dissipation, $\tilde{\varepsilon}$, and compute and plot their profiles across the channel (for a hint, see Part M.7 Resolved production and pressure-strain).
4. Plot the profiles of P/ε and $\text{S}k/\varepsilon$, where $S = \partial \langle V_1 \rangle / x_2$, across the channel and compare your results with Fig. 7.16 of reference [1] (for $Re = 13750$).