**Homework #6 (Chapter 7 – Part III)**

1. Show that the Fourier transform of the residual, $U_i'$, is
$$\hat{U}_i'(\vec{k}) = \mathcal{F}\{U_i'(\vec{x})\} = [1 - \hat{G}(\vec{k})]\hat{U}_i(\vec{k})$$
that the Fourier transform of the filtered residual, $\overline{U}_i'$, is
$$\hat{\overline{U}}_i'(\vec{k}) = \mathcal{F}\{\overline{U}_i'(\vec{x})\} = \hat{G}(\vec{k})[1 - \hat{G}(\vec{k})]\hat{U}_i(\vec{k})$$
and that the Fourier transform of the doubly filtered field, $\overline{\overline{U}}_i$, is
$$\hat{\overline{\overline{U}}}_i(\vec{k}) = \mathcal{F}\{\overline{\overline{U}}_i(\vec{x})\} = \hat{G}^2(\vec{k})\hat{U}_i(\vec{k})$$
Use the above results to show that
$$\overline{U}_i'(\vec{x}) = \overline{U}_i(\vec{x}) - \overline{\overline{U}}_i(\vec{x})$$

2. For the sharp spectral filter show that
$$\overline{\overline{U}}_i(\vec{x}) = \overline{U}_i(\vec{x})$$
$$\overline{U}_i'(\vec{x}) = 0$$
Generally, a filter which satisfies the above relations is called a *projection* filter.

3. For homogeneous flows and a homogeneous filter, show that
$$\phi_{ij}^R(\vec{k}) = \mathcal{F}\{R_{ij}^R(\vec{r})\} = \hat{G}^2(\vec{k})\phi_{ij}(\vec{k})$$
Then, if the filter is isotropic, from the above relation and the definition of the resolved energy spectrum, it follows that
$$E^R(k) = \hat{G}^2(k)E(k)$$

4. In the spectral solution of the 1D Burgers equation with periodic B.C., describe the steps required for determining the coefficient $C_s$ of the dynamic Smagorinsky model.