Homework #4 (Chapter 7 – Part I)

1. Consider the periodic square function (Figure 6.1, Moin) which is given by
$$f(x) = \begin{cases} 1 & ; 0 \leq x < \pi \\ -1 & ; \pi \leq x < 2\pi \end{cases}$$
   Use Matlab or "Numerical recipes" FFT subroutine to obtain the discrete Fourier transform of f on the $N = 16$ grid.

2. Using spectral method, solve the linear advection-diffusion equation
$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0.05 \frac{\partial^2 u}{\partial x^2}$$
   on $0 \leq x \leq 1$, with periodic boundary condition, and the initial condition
$$u(x, 0) = \begin{cases} 1 - 25(x - 0.2)^2 & ; 0 \leq x < 0.4 \\ 0 & ; \text{ otherwise} \end{cases}$$
   Use $N = 32$ and verify your solution with EXAMPLE 6.7 (Moin).