Soal latihan

Edwin Wona Jaya
IF-4
10121134

1 $\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ untuk semua $n \geq 1$

Hipotesa

$\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} \dots + \frac{1}{n(n+1)} + \frac{1}{(n+1)((n+1)+1)} = \frac{(n+1)}{(n+1)+1}$

Bukti :

$n = 1$

$\frac{1}{1(2)} = \frac{1}{1+1}$

$\frac{1}{2} = \frac{1}{2}$ ✓

$n = 2$

$\frac{1}{1(2)} + \frac{1}{2(3)} = \frac{2}{2+1}$

$\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

$\frac{4}{6} = \frac{2}{3}$

$\frac{2}{3} = \frac{2}{3}$ ✓

$n = 3$

$\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} = \frac{3}{3+1}$

$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} = \frac{3}{4}$

$\frac{6+2+1}{12} = \frac{3}{4}$

$\frac{9}{12} = \frac{3}{4} \Leftrightarrow \frac{3}{4} = \frac{3}{4}$ ✓

EDWIN LUHA JAYA
10121154

No.
Date:

$n = k$

$$\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \dots + \frac{1}{k(k+1)} = \frac{k}{k+1}$$

$n = k+1$

$$\frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)((k+1)+1)} = \frac{k+1}{((k+1)+1)}$$

$$\frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

$$\frac{(k+2)k}{(k+2)(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

$$\frac{k^2+2k+1}{(k+2)(k+1)} = \frac{k+1}{k+2}$$

$$\frac{\cancel{(k+1)}(k+1)}{(k+2)\cancel{(k+1)}} = \frac{k+1}{k+2}$$

$$\frac{k+1}{k+2} = \frac{k+1}{k+2}$$

TERBUKTI

2 $1^2+3^2+5^2+\ldots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}, \forall n \geq 1$

Hipotesa

$\frac{1}{3} n(2n-1)(2n+1)$

EDWIN WONAJAYA

No.
Date:

Bukti :

$n=1$

$$1^2 = \frac{1(2(1)-1)(2(1)+1)}{3}$$

$$1 = \frac{(1)(3)}{3}$$

$$1 = \frac{3}{3}$$

$$1 = 1$$

$n=2$

$$1^2+3^2 = \frac{2(2(2)-1)(2(2)+1)}{3}$$

$$10 = \frac{(6)(5)}{3}$$

$$10 = \frac{30}{3}$$

$$10 = 10$$

$n=3$

$$1^2+3^2+5^2 = \frac{3(2(3)-1)(2(3)+1)}{3}$$

$$35 = \frac{(15)(7)}{3}$$

$$35 = \frac{105}{3}$$

$$35 = 35$$

$n=k$

$$1^2+3^2+5^2+ \ldots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$$

[illegible]

No.

Date:

$$1^2+3^2+5^2+\ldots(2k-1)^2+[2(k+1)-1]^2=\frac{1}{3}k[2k-1)(2k+1)+[2(k+1)-1]^2$$

$$1^2+3^2+5^2+\ldots(2k-1)^2+[2(k+1)-1]^2=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$$

$$1^2+3^2+5^2+\ldots(2k-1)^2+[2(k+1)-1]^2=\frac{1}{3}(k+1)(2k+1)(2k+3)$$

$$1^2+3^2+5^2+\ldots(2k-1)^2+[2(k+1)-1]^2=\frac{1}{3}(k+1)(2(k+1)-1)(2(k+1)+1)$$

$$1^2+3^2+5^2+\ldots(2k-1)^2+[2(k+1)-1]^2=\frac{1}{3}n(2n-1)(2n+1)$$

EDWIN VONNA JAYA

3 $1+a+a^2+\dots+a^n = \frac{1-a^{n+1}}{1-a}$ untuk semua $n \geq 0$ dan $a \neq 1$

Hipotesa

$$\frac{(1-a^{n+1})}{(1-a)} + a^{n+1} = \frac{(1-a^{n+2})}{(1-a)}$$

Bukti :

$$a^n = \frac{(1-a^{n+1})}{(1-a)}$$

$$a^0 = \frac{(1-a^1)}{(1-a)} \cdot 1 = 1 \checkmark$$

$$1+a+a^2+\dots+a^n = \frac{(1-a^{n+1})}{(1-a)}$$

$$1+a+a^2+\dots+a^n+a^{n+1} = \frac{(1-a^{n+1})}{(1-a)}$$

$$\frac{(1-a^{n+1})}{(1-a)} + a^{n+1} = \frac{(1-a^{n+2})}{(1-a)}$$

Edwin Lucian Jonyn

No.

Date:

$$(1-a^{n+1}) + \frac{(1-a)\,a^{n+1}}{(1-a)} = \frac{(1-a^{n+2})}{(1-a)}$$

$$\frac{1-a^{n+1}+a^{n+1}-a.a^{n+1}}{(1-a)} = \frac{(1-a^{n+2})}{(1-a)}$$

$$\frac{1-a.a^{n+1}}{(1-a)} = \frac{(1-a^{n+2})}{(1-a)}$$

$$\frac{(1-a^{n+2})}{(1-a)} = \frac{(1-a^{n+2})}{(1-a)}$$

No.

Date:

EDWIN LUNA JAYA

4) n^4-4n^2 habis dibagi 3. $\forall$ bil. bulat ≥ 2

Hipo

habis dibagi 3

bukti

$n=2$

$2^4-4(2)^2=0$

$0:3=0$

$n+1$

$= n^4-4n^2=(n+1)^4-4(n+1)^2=n^4+4n^3+6n^2+4n+1-4(n^2+2n+1)$

$= n^4+4n^3+6n^2+4n+1-4n^2-8n-4$

$= (n^4-4n^2)+4n^3+6n^2-4n-3$

$= (n^4-4n^2)+6n^2+4n^3-4n-3$

$= (n^4-4n^2)+6n^2+4n(n^2-1)-3$

$= (n^4-4n^2)+6n^2+4n(n-1)(n+1)-3$

$= (n^4-4n^2)+6n^2+4(n-1)n(n+1)-3$

$= (n^4-4n^2)+4n^3+6n^2-4n-3$

$= (n^4-4n^2)+6n^2+4n^3-4n-3$

$= (n^4-4n^2)+6n^2+4n(n^2-1)-3$

$= (n^4-4n^2)+6n^2+4n(n-1)(n+1)-3$

$= (n^4-4n^2)+6n^2+4(n-1)n(n+1)-3$

(n^4-4n^2) : Terbukti dari basis 3

$6n^2$: bilangan kelipatan 6 habis dibagi 3

$4(n-1)n(n+1)$: Perkalian 3 buah bilangan bulat berurutan $(n-1)$, n dan $(n+1)$ pasti kelipatan 3. misal $1\times2\times3$ atau $4\times5\times6$

-3 : Sudah jelas kelipatan 3

EDWIN LIONA JAYA

C Jumlah pangkat 3 dari 3 buah bil. bul positif berurutan selalu habis dibagi 9

Hipo

$k^2 + k + a$ = habis dibagi 9

bukti:

$(n-1)^3 + n^3 + (n+1)^3 = n^3 - 3n^2 + 3n - 1 + n^3 + n^3 + 3n^2 + 3n + 1$

$= 3n^3 + 6n$

Jika

$n = 1 \quad 3 + 6 = 9$ ✓

$n = k \quad 3k^3 + 6k$ ✓

$n = k + 1$

$3(k+1)^3 + 6(k+1)$

$k^2 + k + a$ = habis dibagi 9

bukti:

$(n-1)^3 + n^3 + (n+1)^3 = n^3 - 3n^2 + 3n - 1 + n^3 + n^3 + 3n^2 + 3n + 1$

$= 3n^3 + 6n$

Jika

$n = 1$ $3 + 6 = 9$ ✓

$n = k$ $3k^3 + 6k$ ✓

$n = k + 1$

$3(k+1)^3 + 6(k+1)$

$3(k^3 + 3k^2 + 3k + 1) + 6k + 6$

$3k^3 + 9k^2 + 9k + 3 + 6k + 6$

$3k^3 + 6k + 9k^2 + 9k + 9$

$3k^3 + 6$ habis dibagi 9 ✓

$9k^2 + 9k + 9$ habis dibagi 9 ✓

$3(k^3+3k^2+3k+1)+6k+6$

$3k^3+9k^2+9k+3+6k+6$

$3k^3+6k+9k^2+9k+9$

$3k^3+6$ habis dibagi 9

$9k^2+9k+9$ habis dibagi 9

6 Buktikan bahwa surat pos yang menggunakan perangko 24 sen atau lebih dapat hanya menggunakan perangko 5 atau 7 sen

Hipo :

Membeli perangko dengan $n+1$.

Erwin Cloma Jaya

No.

Date:

bukti :

misal : $p(n) = n \geq 24$ dapat menggunakan $5a + 7b$

$p(n+1)$ = biaya pos sebesar $n+1$ dan dapat menggunakan $5a+7b$

$n = 5a + 7b$, ≤ 2 buah perangko 5 sen dan 2 buah perangko 7 sen dengan 23 buah perangko 5 sen sehingga 5 buah perangko 5 sen dapat dibayar $n+1$