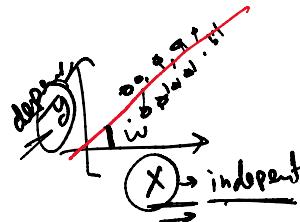


① Linear Regression



Co - Variance →

it is mathematical measurement of dependency or relationship of two variable.

X → height →
y → weight →

Regression →

predicting a continuous value at a dependent variable.

predicting a continuous value as variable on the basis of independent variables.

$$\underline{\text{Cov}(x,y)} = \frac{\sum_{i=1}^N (\underline{x} - \underline{x_i}) \cdot (\underline{y} - \underline{y_i})}{N-1}$$

$$-\infty < \underline{\text{Cov}(x,y)} < \infty$$

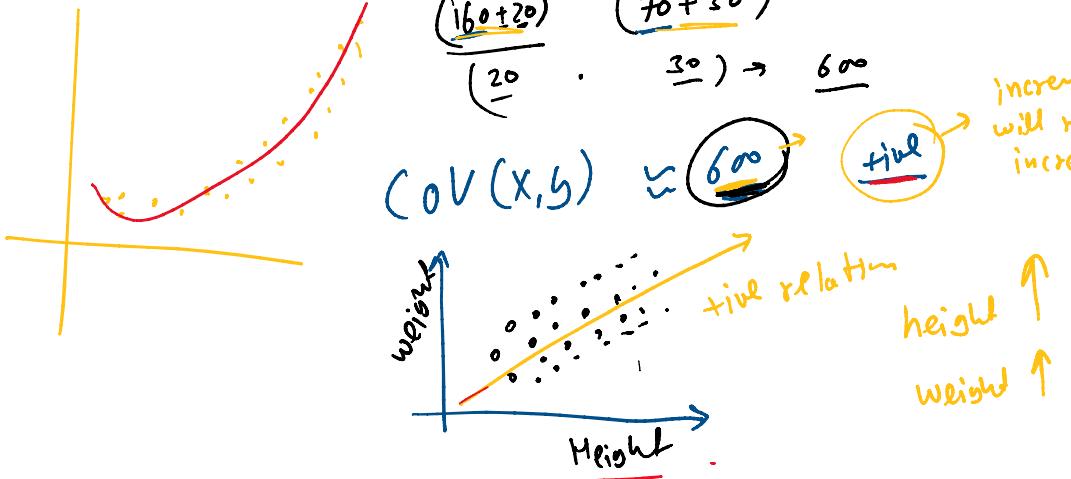
$\text{Cov}(x,y) \rightarrow \text{time}$ x and y both are dependent, and they have a positive relationship

e.g. $\underline{x} = \text{height}$, $\underline{y} = \text{weight}$

$$\frac{(160+20)}{(20)} \cdot \frac{(70+30)}{(30)} \rightarrow 600$$

$\text{Cov}(x,y) \approx 600$

increment in x will result increment in y

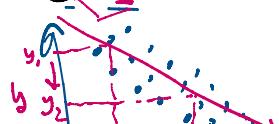


$X = \text{Age} \rightarrow$

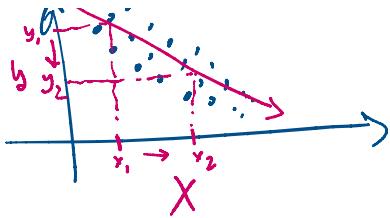
$y = \text{strength} \rightarrow$

if we increase x then y will decrease

$$\text{Cov}(x,y) = \text{-ine}$$



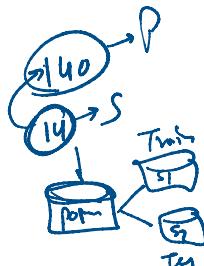
$x \uparrow y \downarrow$
 $x \downarrow y \uparrow$



$$\text{Cov}(n, y) \approx 0$$

There is no relation in $\underline{x}, \underline{y}$

$$\boxed{\text{Cov}(x, y) = \frac{\sum_{i=1}^N (\bar{x}_i - x_i)(\bar{y}_i - y_i)}{N-1}}$$



target
 $y \rightarrow \text{price}$
features
 $X \rightarrow$

- $\# \text{ rooms } x_1$
- $\text{Area } x_2$
- $\text{age } x_3$

$z^0 \approx 5$ feature importance $\rightarrow \text{population} \rightarrow N$
 $\rightarrow \text{sample} \rightarrow N-1$

① $\text{Cov}(y, x_1) \rightarrow +\text{ive}$ (15)
 ② $\text{Cov}(y, x_2) \rightarrow +\text{ive}$ (500)
 ③ $\text{Cov}(y, x_3) \rightarrow -\text{ive}$ (-75)

① $\frac{(\bar{x}_1 - x_1)(\bar{y} - y)}{N-1} \rightarrow 15 \rightarrow 10 \rightarrow 3 \rightarrow 5 - 15 \rightarrow 3$
 ② $\frac{(\bar{x}_2 - x_2)(\bar{y} - y)}{N-1} \rightarrow 500 \rightarrow 10 \rightarrow 250 + 10 \rightarrow 260$
 ③ $\frac{(\bar{x}_3 - x_3)(\bar{y} - y)}{N-1} \rightarrow 75 \rightarrow -8 \rightarrow +8$
 $8 - 50 \rightarrow 260 + 12$

\Rightarrow Pearson's Correlation \rightarrow

$$\boxed{r = \frac{\text{Cov}(x, y)}{\sigma_x \cdot \sigma_y}}$$

$$\begin{aligned}\sigma_x &\rightarrow \text{std } \underline{o+X} \\ \sigma_y &\rightarrow \text{std } \underline{oY}\end{aligned}$$

$$r = \frac{\text{cov}(x, y)}{\sigma_x \cdot \sigma_y}$$

-1 < r < 1

price (y)

Rows (x_1)
area (x_2)
age (x_3)

$r \approx 0$, no relation

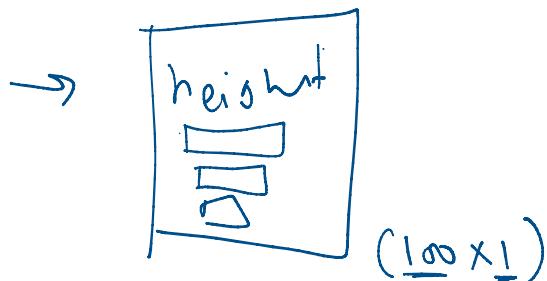
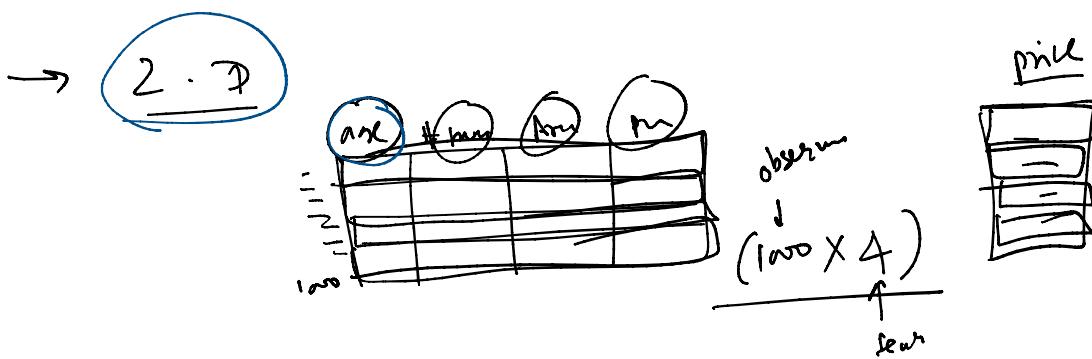
$r \approx 1$ or -1 Both variables are same

-1, 1

$r(y, x_1) = 0.6$
$r(y, x_2) = 0.8$
$r(y, x_3) = -0.7$



$$\underline{x_2} > \underline{x_3} > \underline{x_1}$$



→ Model

$y \rightarrow \underline{\text{target}}$

y is a vector containing possible outcomes for given $\sim \dots$

$y \rightarrow \underline{\text{target}}$

y is a vector
possible outcome for given
input (feature)

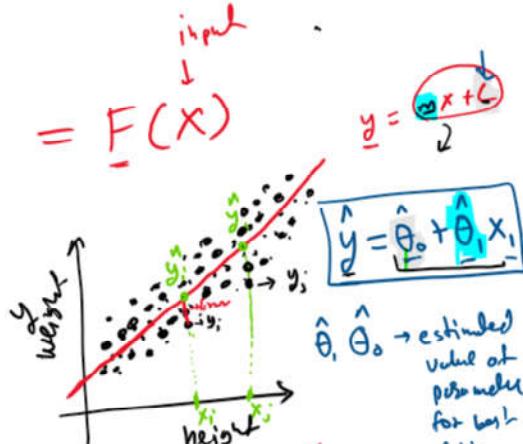
$X \rightarrow \text{features / Attribute}$

→ Linear Regression

① Hypothesis function

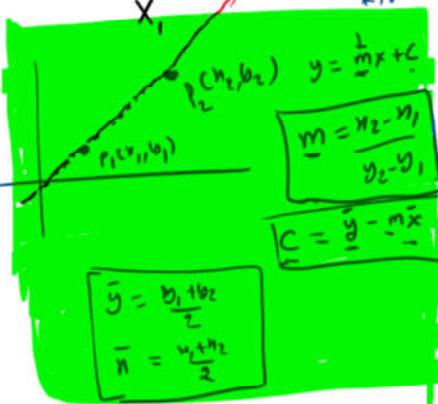
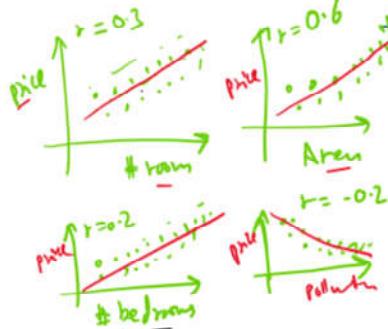
$$\hat{y} = H_{\theta}(X) = F(X)$$

$y \rightarrow \underline{\text{actual output}}$
 $\hat{y} \rightarrow \underline{\text{predicted output}}$
estimated



$$\hat{y} = H_{\theta}(X) = \hat{\theta}_0 + \hat{\theta}_1 \cdot \underline{\text{height}}$$

$y \rightarrow \underline{\text{price}}$
 $X \rightarrow \# \text{rooms} \rightarrow x_1$
 $\rightarrow \# \text{bedrooms} \rightarrow x_2$
 $\rightarrow \text{Area} \rightarrow x_3$
 $\rightarrow \text{Pollution} \rightarrow x_4$
 \vdots
 x_n



$$\hat{y} = H_{\theta}(X) = \theta_0 + \theta_1 \cdot \# \text{rooms} + \theta_2 \cdot \# \text{bedrooms} + \theta_3 \cdot \text{Area} + \theta_4 \cdot \text{Pollution}$$

$$\hat{y} = H_{\theta}(X)$$

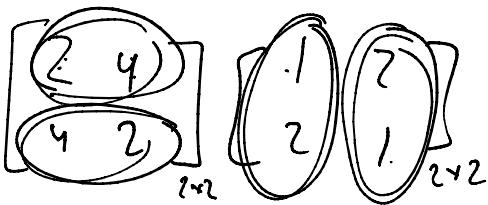
$$H_{\theta}(X) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \dots + \theta_n x_n$$

$$\theta = [\theta_0, \theta_1, \theta_2, \theta_3]_{1 \times 4}$$

$$X = [1, x_1, x_2, x_3]_{1 \times n} \quad \theta^T \cdot X$$

$$\theta^T \cdot X \rightarrow \theta_0 + \theta_1 x_1 + \dots + \theta_n x_n$$

(2 4) 1 7



$$\begin{bmatrix} 2+8 & 4+4 \\ 4+4 & 8+2 \end{bmatrix}_{2 \times 2}$$

$$X = \begin{bmatrix} 1 & x_1 & x_2 & x_3 & x_4 \end{bmatrix}_{n \times k}$$

$\theta \cdot X^T \rightarrow \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$

$$A = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}_{1 \times n}$$

$$B = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{k \times 1}$$

$$A_{m \times n} \cdot B_{n \times k} = C_{m \times k}$$

$$A_{1 \times k} \cdot B_{k \times 1} = C_{1 \times 1}$$

$$C = \underline{a \cdot x_1 + b \cdot x_2 + c \cdot x_3 + d \cdot x_4}$$

$$H_{\theta}(x) = \underline{\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4}$$

$X \rightarrow 2D$ matrix which holds all training records

$$X = \begin{array}{c|cccc} & \#rooms & \#bedrooms & Area & Polns \\ \hline x_1 & x_2 & x_3 & x_4 \\ \hline 0 & 5 & 1 & 200 & 0.2 \\ 1 & 10 & 2 & 100 & 0.1 \\ 2 & 15 & 5 & 150 & 0.5 \\ 3 & 20 & 2 & 100 & 0.3 \\ 4 & 5 & 1 & 150 & 0.4 \\ 5 & 2 & 0 & 50 & 0.5 \\ 6 & 1 & 0 & 100 & 0.6 \end{array}$$

$m \times n \rightarrow$ features
 \downarrow observations

$$y = \begin{bmatrix} 20 \\ 30 \\ 40 \\ 20 \\ 5 \\ 50 \\ 60 \end{bmatrix} \rightarrow y_0, y_1, y_2, y_3, y_4, y_5, y_6$$

$\theta_0 \rightarrow \text{Bias/Intercept}$

$\theta_1, \theta_2, \theta_3, \dots, \theta_n \rightarrow \text{Coefficients}$

$$Error = |y_i - \hat{y}_i|$$

$$\begin{aligned} \hat{y}_0 &= \hat{\theta}_0 + \hat{\theta}_1 x_{10} + \hat{\theta}_2 x_{20} + \hat{\theta}_3 x_{30} + \hat{\theta}_4 x_{40} \\ \hat{y}_1 &= \hat{\theta}_0 + \hat{\theta}_1 x_{11} + \hat{\theta}_2 x_{21} + \hat{\theta}_3 x_{31} + \hat{\theta}_4 x_{41} \\ \vdots & \\ \hat{y}_n &= \hat{\theta}_0 + \hat{\theta}_1 x_{1n} + \hat{\theta}_2 x_{2n} + \hat{\theta}_3 x_{3n} + \hat{\theta}_4 x_{4n} \end{aligned}$$

$$\theta = \begin{bmatrix} \theta_0 & \theta_1 & \theta_2 & \theta_3 & \theta_4 \end{bmatrix}_{1 \times 5}$$

$$x_i = \begin{bmatrix} 1 & x_{1i} & x_{2i} & x_{3i} & x_{4i} \end{bmatrix}_{1 \times 5}$$

$$\underline{\theta} = [\underline{\theta}_0 \ \underline{\theta}_1 \ \underline{\theta}_2 \ \underline{\theta}_3 \ \underline{\theta}_4]_{1 \times 5} \quad \underline{x}_i = [1 \ x_{i1} \ x_{i2} \ x_{i3} \ x_{i4}]_{1 \times 5}$$

$$\hat{y}_i = \underline{\theta} \cdot \underline{x}_i$$

$$\hat{y}_i = \underline{\theta} \cdot \underline{x}_i^T \rightarrow \hat{y}_i = \underline{\theta}_0 + \underline{\theta}_1 x_{i1} + \underline{\theta}_2 x_{i2} + \underline{\theta}_3 x_{i3} + \underline{\theta}_4 x_{i4}$$

prediction function

$$\hat{y} = \underline{\theta} \cdot \underline{x}$$

$$H_{\underline{\theta}}(\underline{x}) = \underline{\theta} \cdot \underline{x}$$

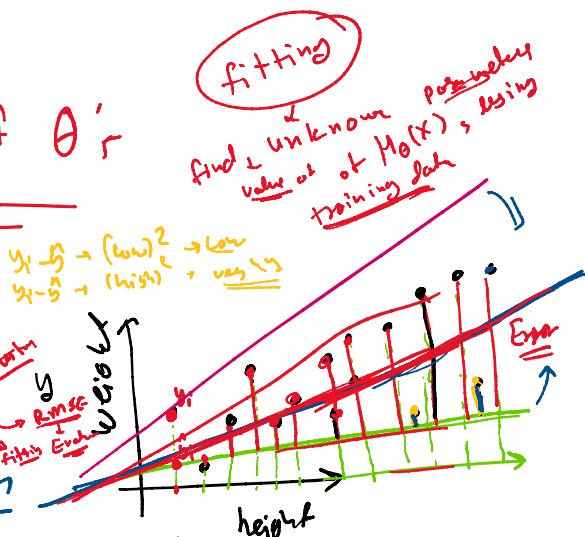
$$\hat{y} = H_{\underline{\theta}}(\underline{x})$$

→ How to estimate value of $\underline{\theta}$'s

$$\hat{y} = H_{\underline{\theta}}(\underline{x}) = \hat{\underline{\theta}}_0 + \hat{\underline{\theta}}_1 x_1$$

parameter

To estimate Best value for $\underline{\theta}$
we need to minimize Cost function



$$J_{\underline{\theta}} = \frac{1}{N} \sum_{i=1}^N (\hat{y}_i - y_i)^2$$

$$\text{MAE} = \frac{1}{N} \sum_{i=1}^N |y_i - \hat{y}_i|$$

Evaluates linear

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2$$

$$f(x)$$

$aN^2 + bN + c$

How to find
min or max

Convex

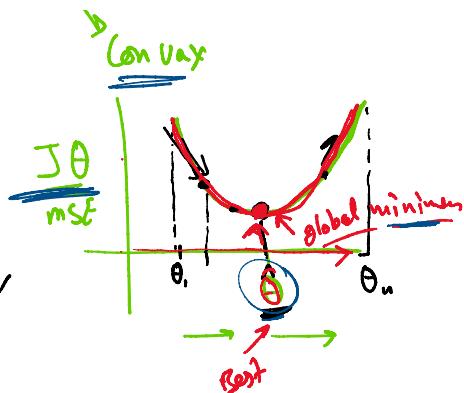
$T(X)$

How to find
min or max
of a function

$$\rightarrow f'(n) = 0, \text{ min, Max}$$

$$f(n) = 2n^2 + 3n + 4$$

$$\frac{df(n)}{x} = 2 \cdot 3n^2 + 3 \Rightarrow 6n^2 + 3$$



$$\hat{y}_i = H_{\theta}(x) = (\theta_0 + \theta_1 x_i) \quad \textcircled{1}$$

$$mse(\theta) = \sum_{i=1}^N (\hat{y}_i - y_i)^2 \quad \textcircled{2}$$

put eq. ① into eq. ②

$$mse(\theta) = \frac{\sum_{i=1}^N (\hat{\theta}_0 + \hat{\theta}_1 x_i - y_i)^2}{N}$$

$y_i \rightarrow \text{training data}$
 $N \rightarrow \text{no. of observer}$
 $\hat{y}_i \rightarrow \text{prediction}$

Training Data
 $x_i, y_i \rightarrow \text{constants}$
 $\hat{\theta}_0, \hat{\theta}_1 \rightarrow \text{variables}$

we want to find value of θ such that

$mse(\theta)$ should be minim

$$\begin{aligned} 2n + 3y &= 0 \\ 2n - 3y &= 1 \end{aligned}$$

$$\frac{\partial mse(\theta_0)}{\partial \theta_0} = 0 \quad \text{(i)}$$

To find with

$$\frac{\partial mse(\theta_1)}{\partial \theta_1} = 0 \quad \text{(ii)}$$

→ Normal Equation

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Delta_0 = \sum_{i=1}^N (\bar{y} - \bar{y}_i) \cdot (\bar{x}_i - \bar{x}_0)$$

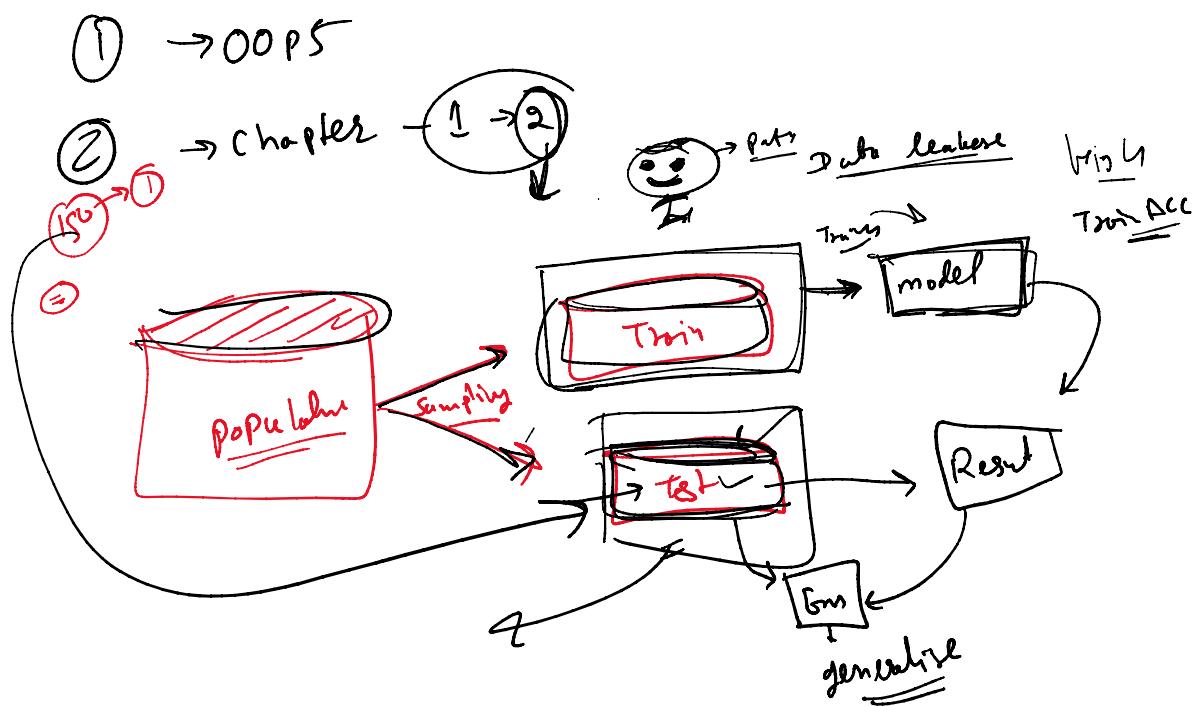
$$y = \theta_0 + \theta_1 x_1$$

$$\theta_1 = \frac{\sum_{i=1}^n (\bar{y} - \hat{y}_i) \cdot (\bar{x}_i - x_{ij})}{\sum_{i=1}^n (\bar{x}_i - x_{ij})^2}$$

$$\theta_0 = \bar{y} - \theta_1 \bar{x}$$

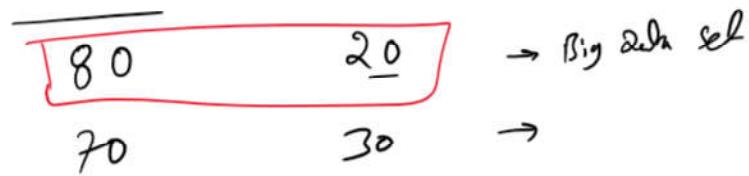
price = $\theta_0 + \theta_1 \cdot \# \text{room} + \theta_2 \cdot \text{Area} + \theta_3 \cdot \dots$

$$\hat{\theta} = (X^T X)^{-1} X^T y \rightarrow \text{normal form} \quad \begin{bmatrix} 1 & x_1 & x_2 & x_3 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

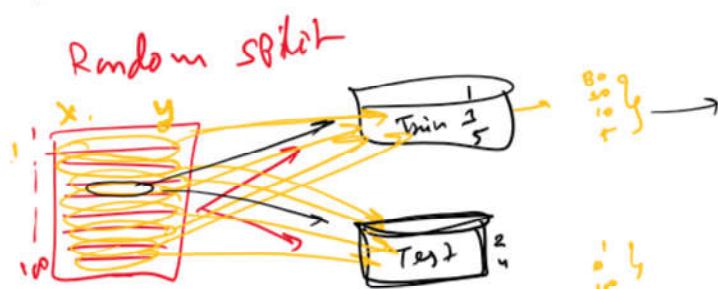
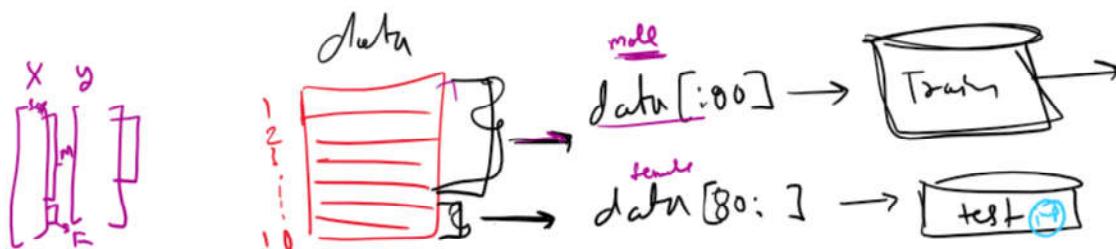
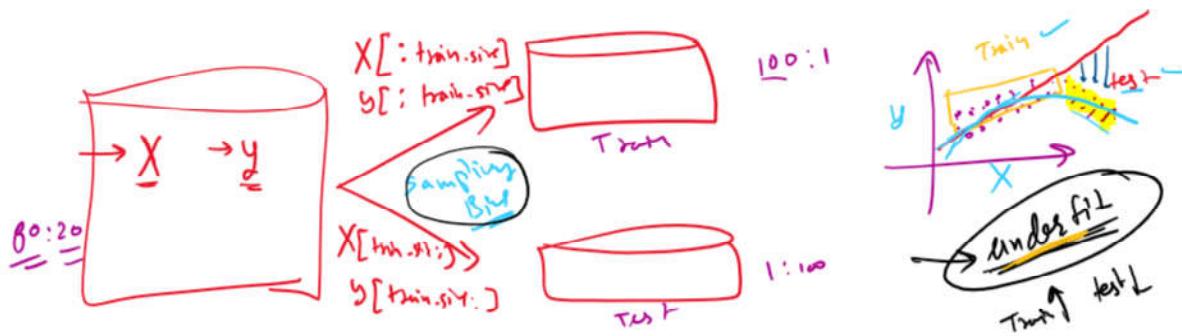


→ Sampling

$$\frac{\rightarrow \text{train size}}{80} \quad \frac{\rightarrow \text{test size}}{20} \rightarrow \text{Big data set}$$



→ Sampling Techniques



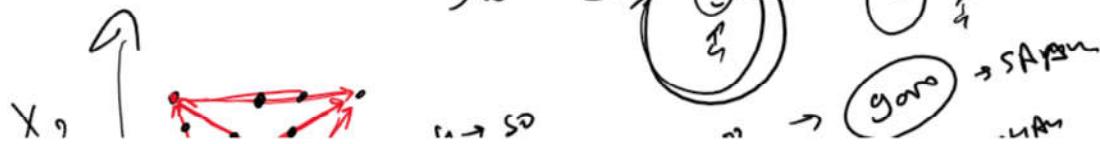
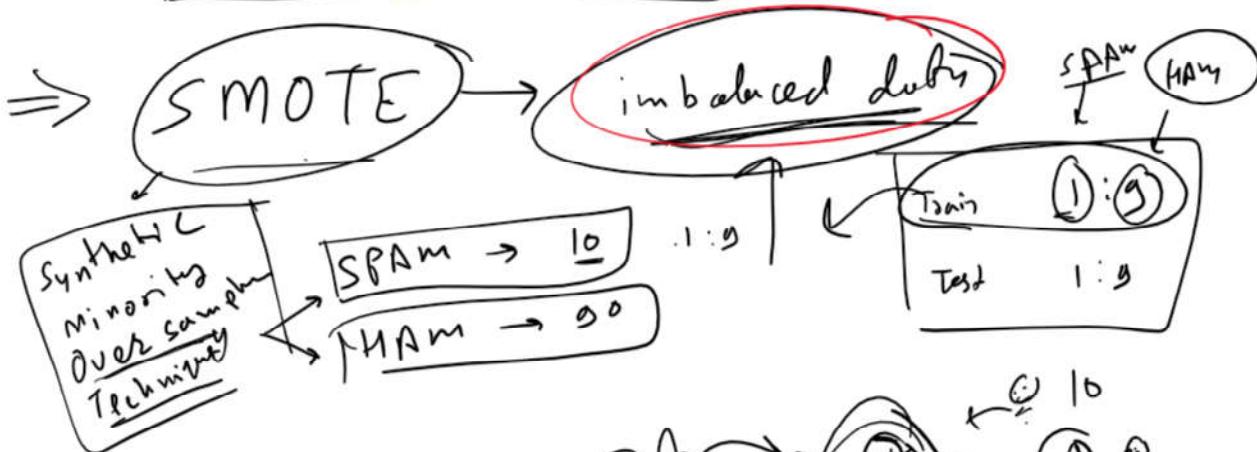
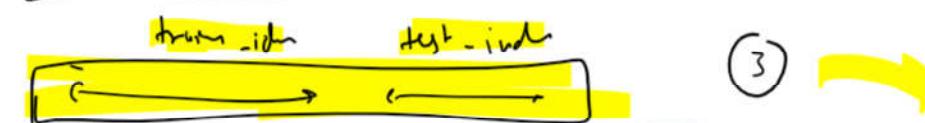
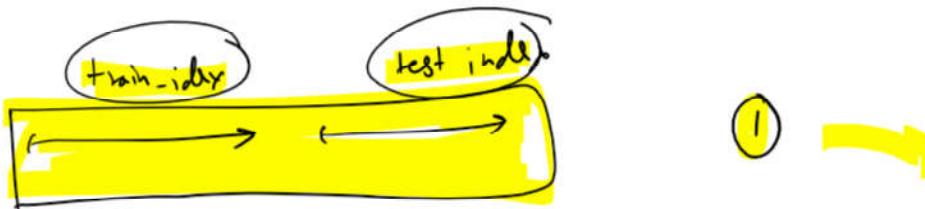
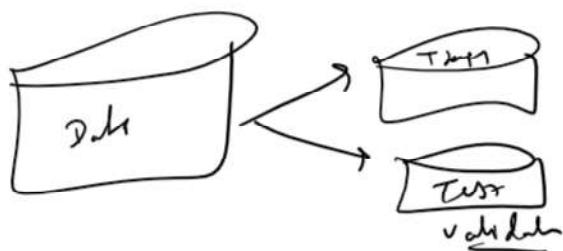
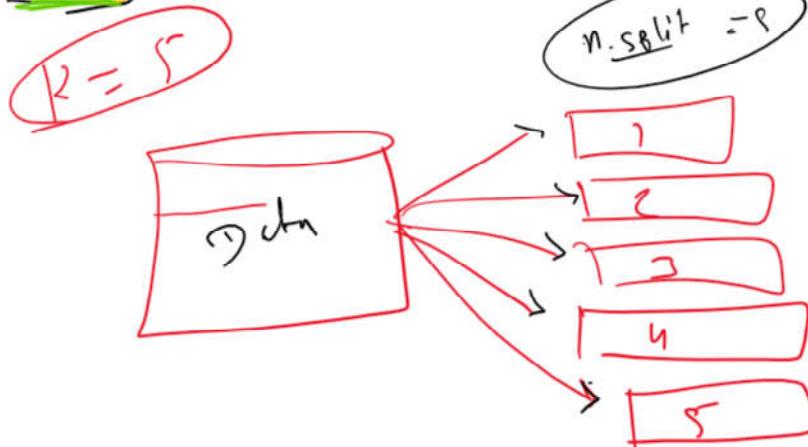
→ Stratified Sampling

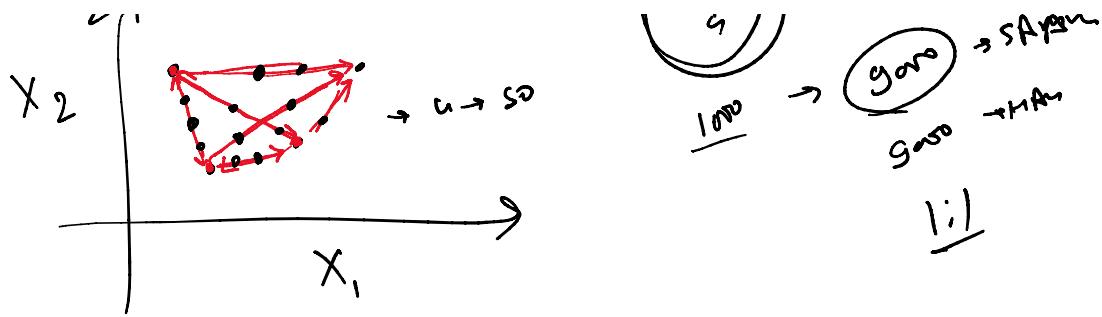
Column
↑

proportion of column will be same

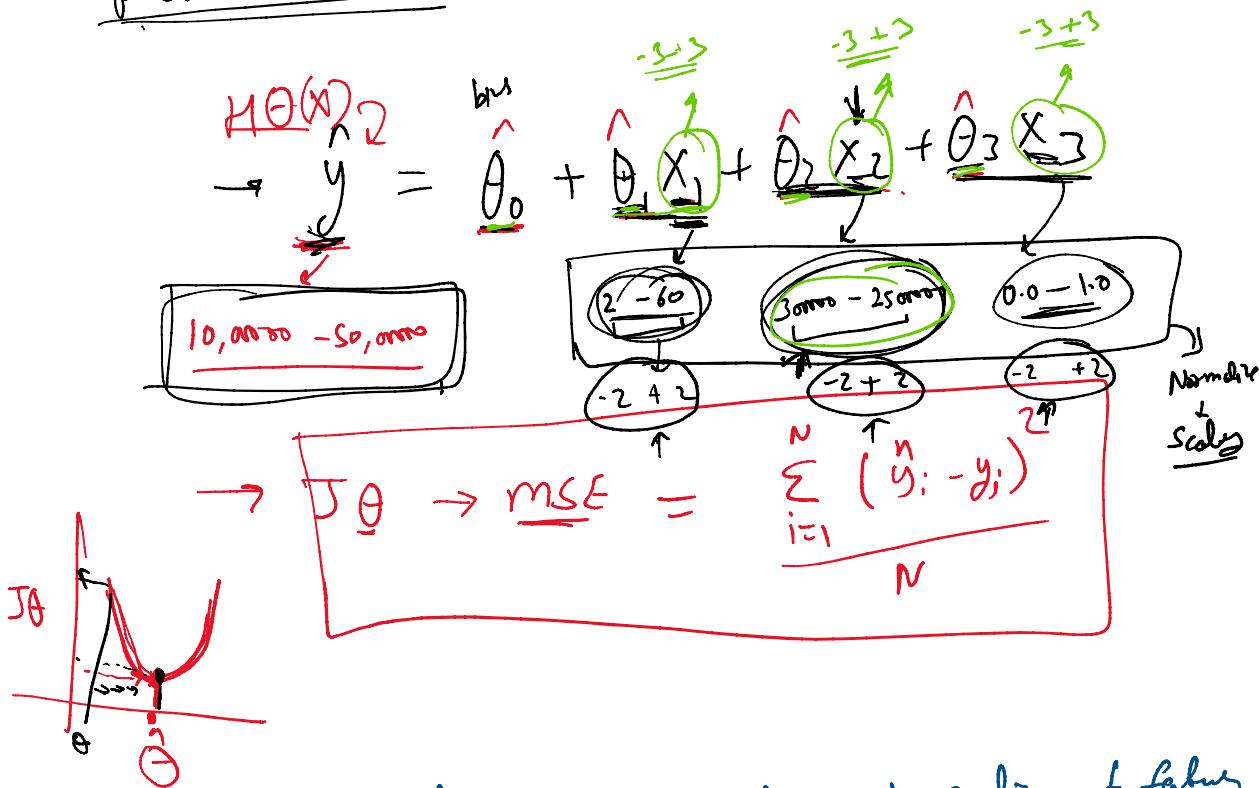
on Train & Test data sel

170-1. 30-4





Prediction function



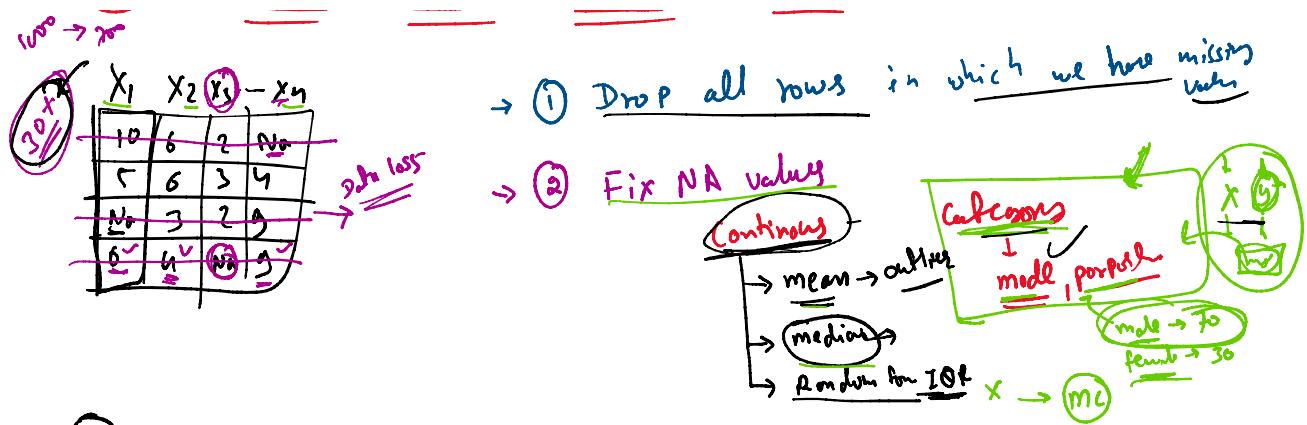
SGD is very sensitive to scaling of features

Preprocessing

(1) Scaling
Some models are sensitive to feature scaling so it need to be done before you fit your model.

(2) Deal with missing values

$x_{100} \rightarrow x_{\text{new}}$ $x_1 \ x_2 \ x_3 \dots x_n$. (i) Drop all rows in which we have missing values



③ Feature Selection

→ drop useless columns

④ Deal with text or category data

ML model or Computer always work on numerical data

Categories → ordinal → Label Encoder
Nominal → One Hot Encoding

Number
 $y = b + w_1 x_1 + w_2 x_2$

Open - proximities
NBBR API
CHI OCEN
MC
i

⑤ Feature Embedding / Feature Engineering

→ Scaling

- min-max Scaling → 0-1
- Standard Scaling / Normalization

X

1	2	3	n
2	4	6	
3	6	9	

Y

6
10

$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$

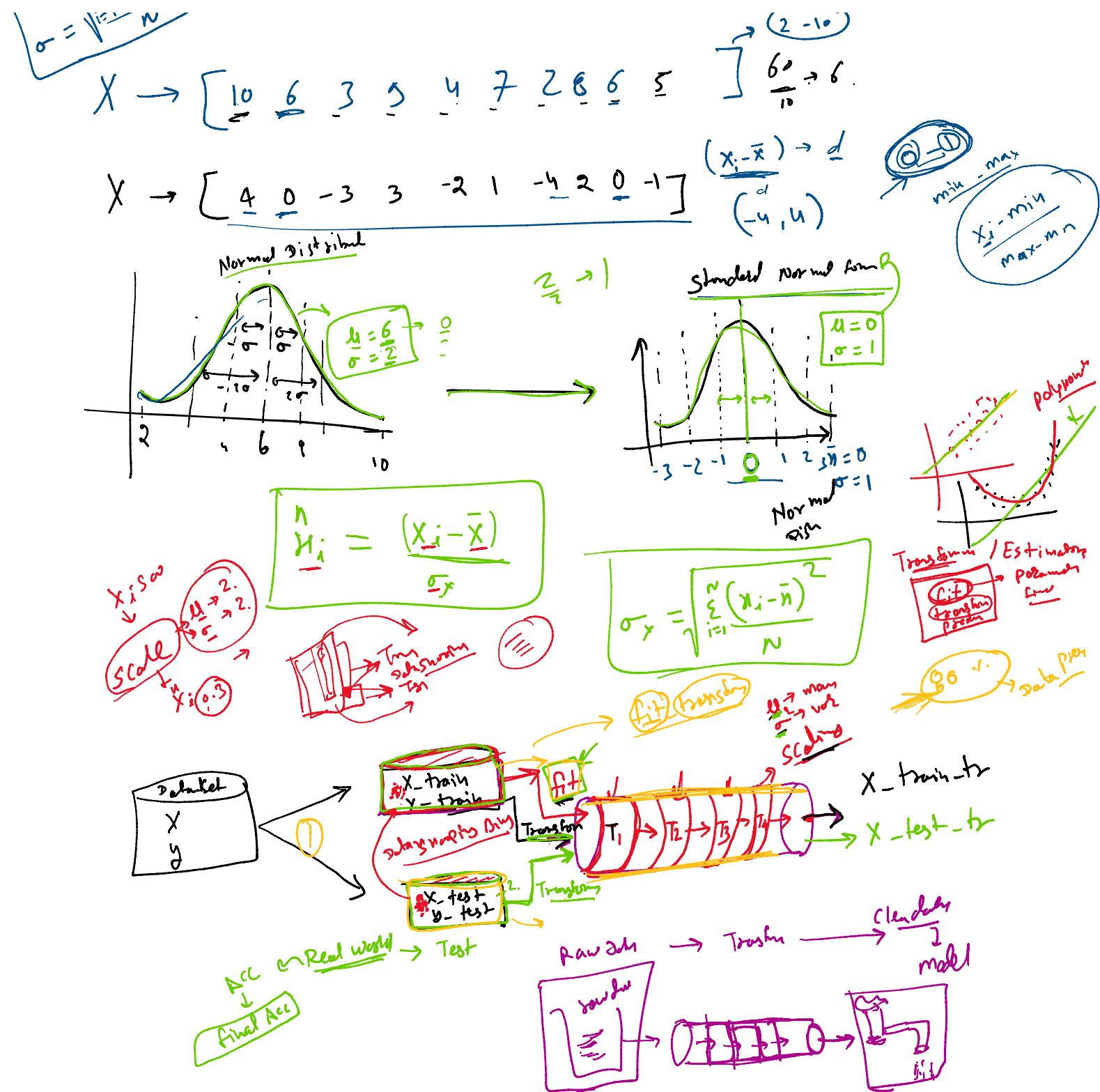
$$\hat{x}_i = \frac{(x_i - \bar{x})}{\sigma_x} \rightarrow \text{mean} = 0 \rightarrow \text{std} = 1$$

Rank

2 - 10

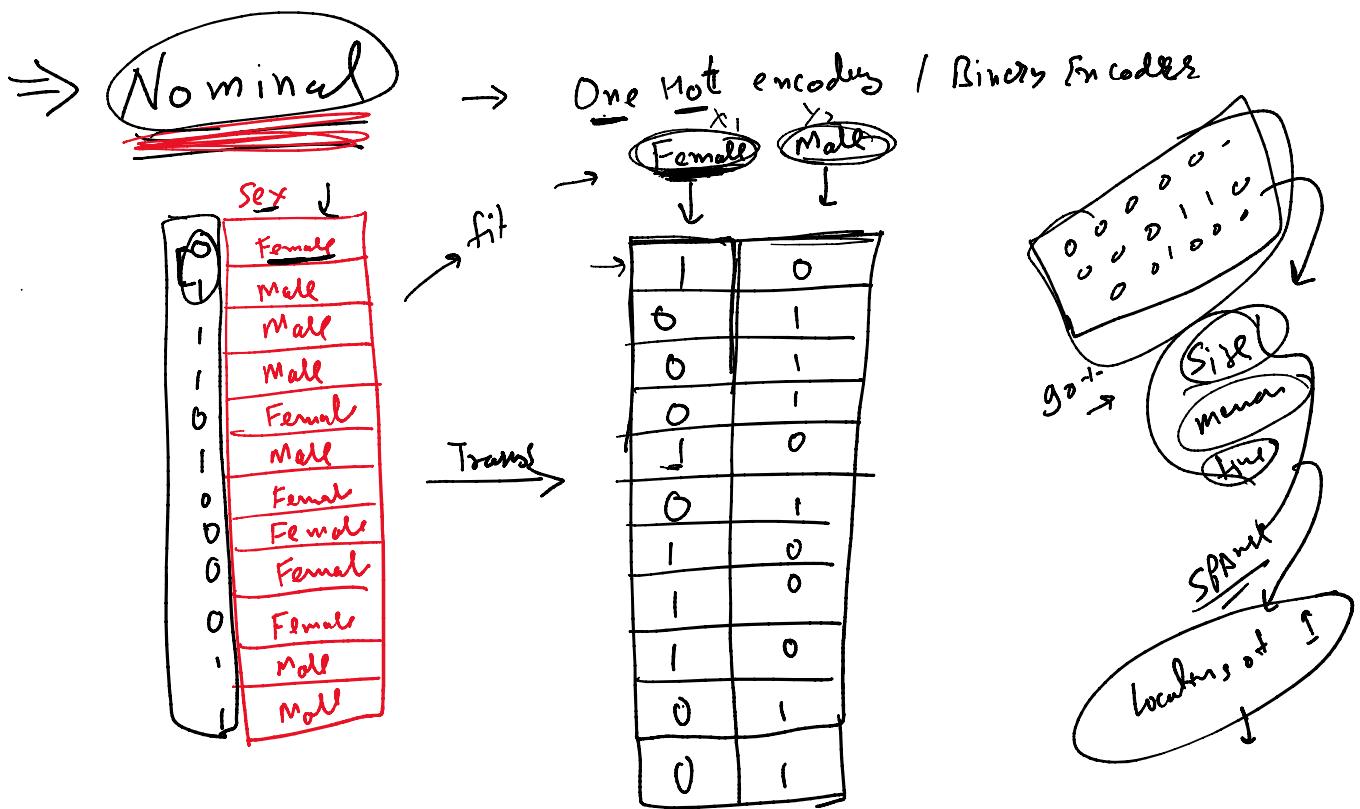
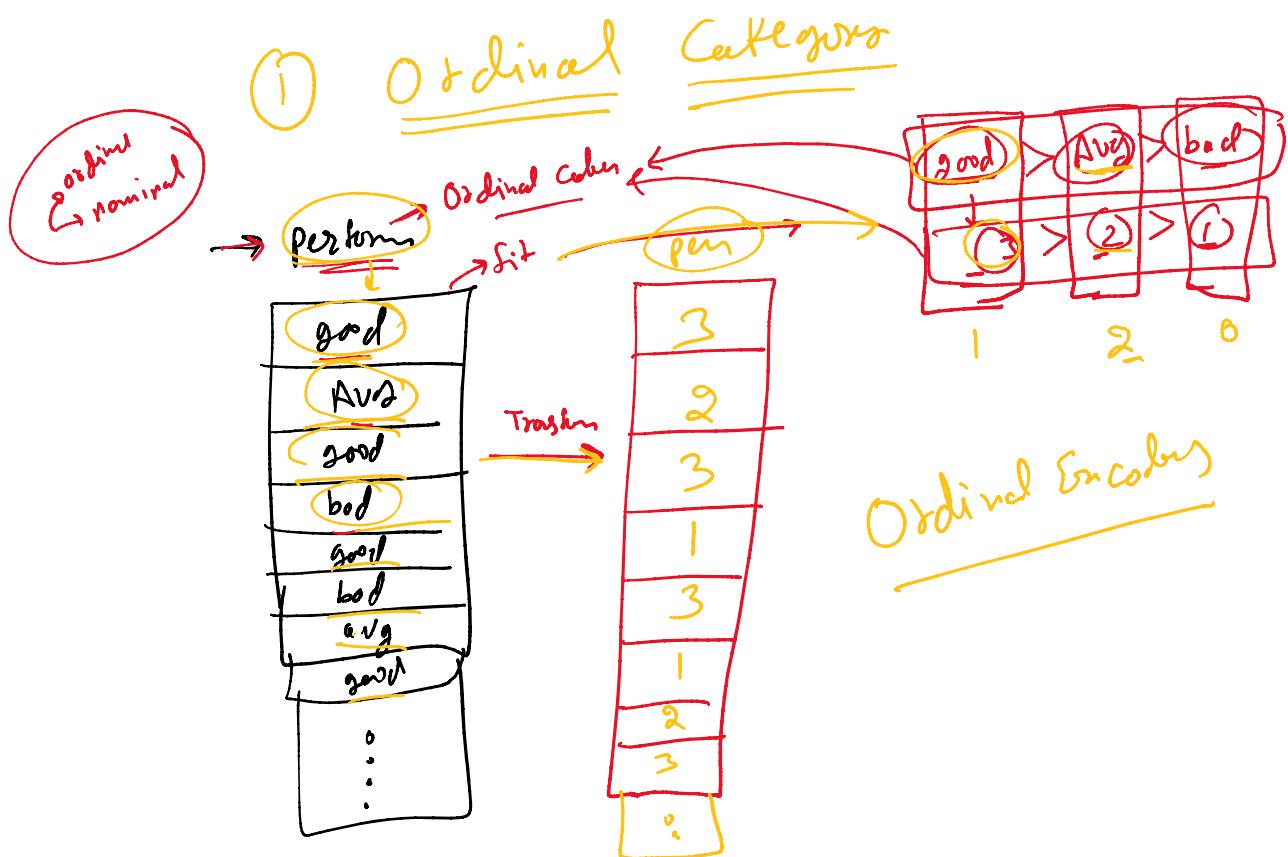
2 - 10

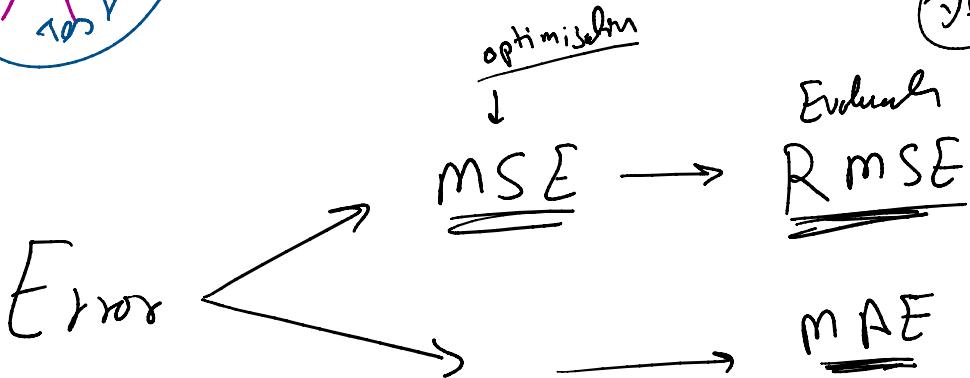
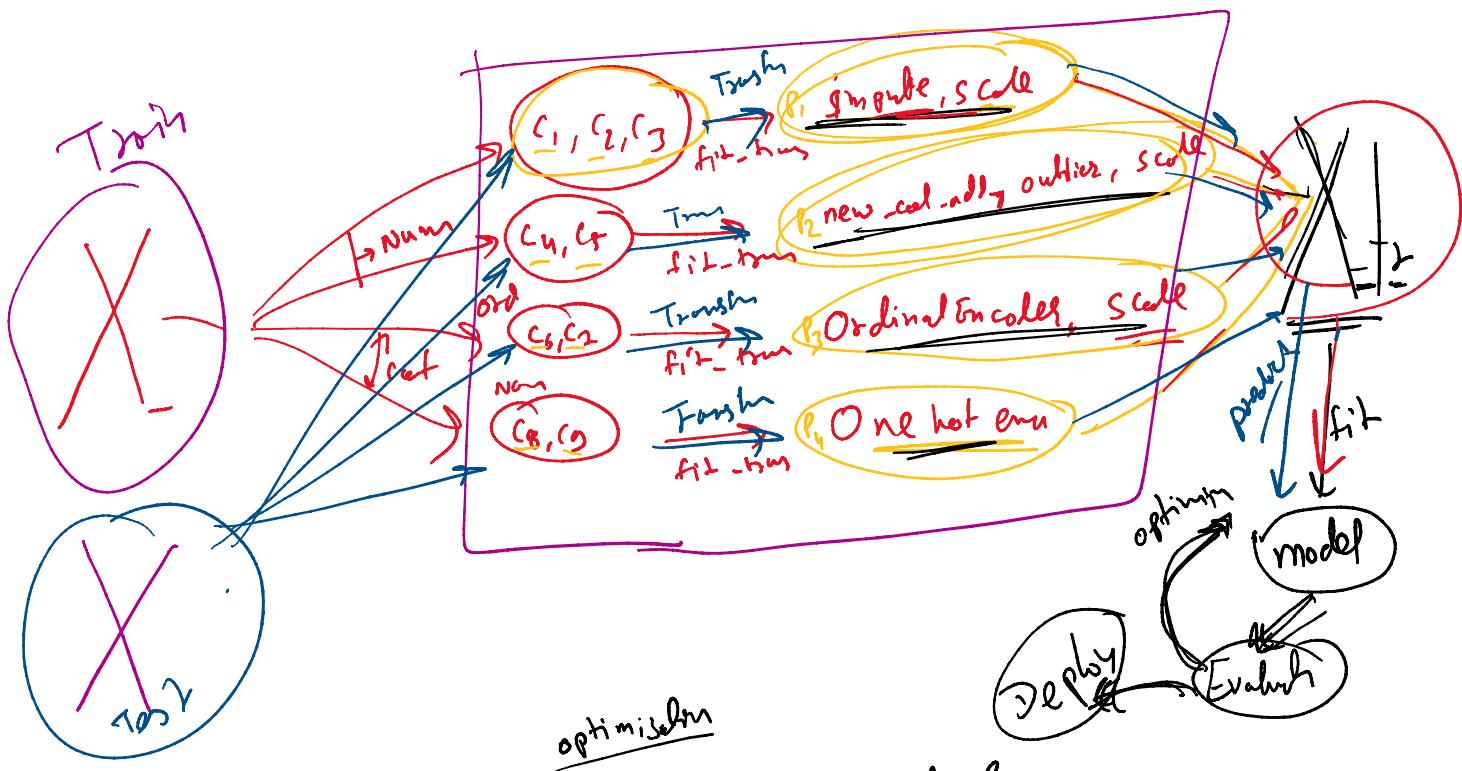
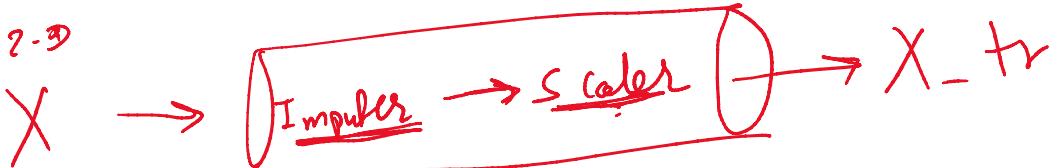
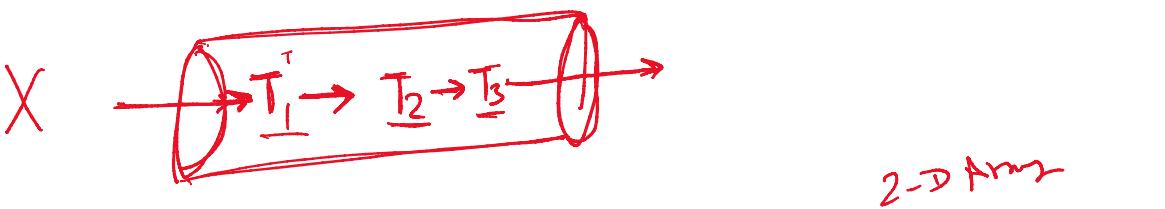
60 → 6



\rightarrow Categories \rightarrow

— (1) Ordinal Categories





Accuracy → $\underline{\underline{R^2 - Score}}$

$$r = \sqrt{y_i - \hat{y}_i}$$

$$\underline{\underline{R^2 - Score}} = 1 - \frac{RSS}{TSS}$$

Residual = $y_i - \hat{y}_i$

$R^2\text{-Score} = \dots \rightarrow$

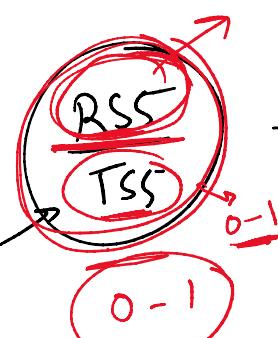
Random model

Total Sum of Squared Error = $\sum_{i=1}^N (\bar{y} - y_i)^2$ (Training values)

R^2 = Residual Sum of Squared Error = $\sum_{i=1}^N (\hat{y}_i - y_i)^2$ (Prediction error)

$$\begin{aligned} R^2\text{-Score} &\rightarrow 1 \rightarrow 100\% \\ R^2\text{-Score} &\rightarrow 0 \rightarrow 100\% \\ R^2\text{-Score} &\rightarrow 0.2 \rightarrow 70\% \end{aligned}$$

ga. error



$$\frac{0-\varnothing}{0-\varnothing} = \frac{a}{b} \rightarrow f_n$$

$$0-1 \rightarrow$$

$$0.0001 \rightarrow \frac{\text{low}}{\text{very High}}$$

$$\begin{aligned} R^2 &= \\ \frac{RSS}{TSS} &\uparrow \text{versus} \rightarrow 1000 \\ \frac{RSS}{TSS} &\rightarrow \text{low} \rightarrow 1 - 1000 \end{aligned}$$

$$-125 \times 10^4$$

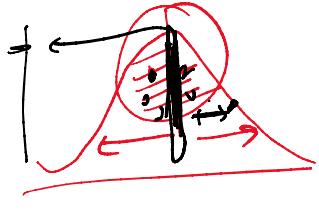
$$\frac{a}{b} > 1$$

$$b > a$$

$$\frac{RSS}{TSS} \approx 1 \rightarrow 0$$

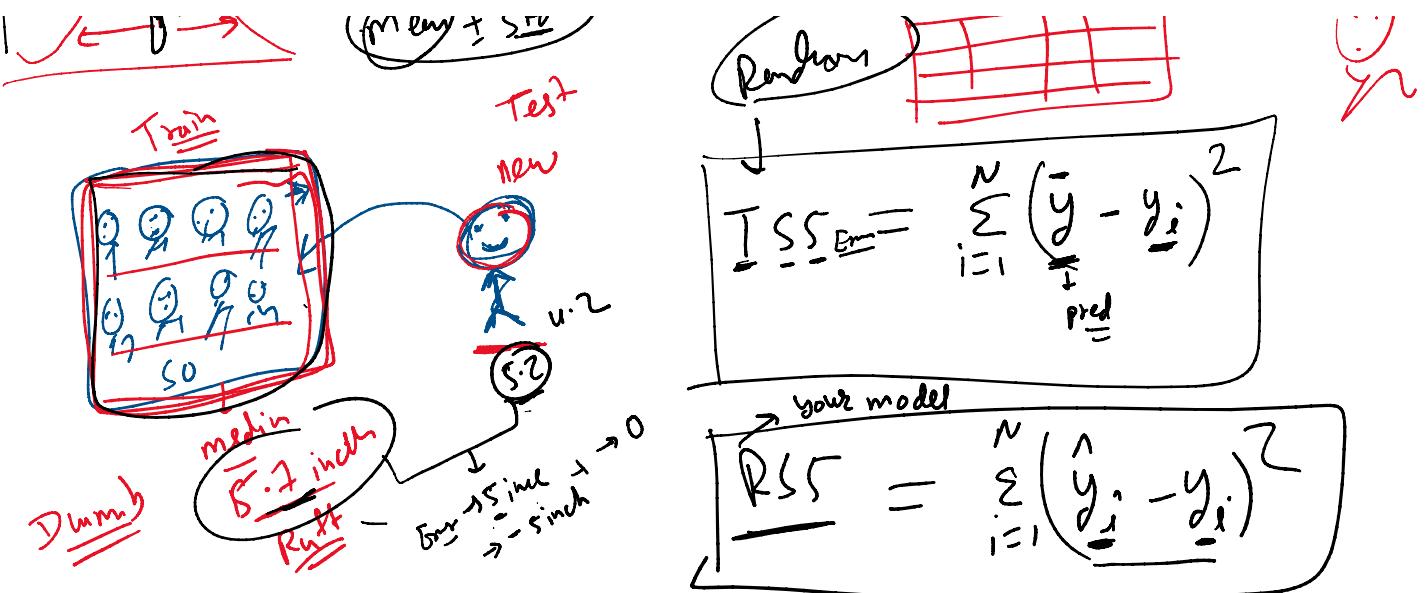
$$\frac{a}{b} < 1 \rightarrow a < b$$

$$R^2 \text{ Score} = 1 - \frac{RSS}{TSS}$$



$$\text{mean} \pm \text{std} \rightarrow 68$$





$$\frac{RSS}{TSS} \rightarrow \frac{\text{your model error}}{\text{Random model error}} = \frac{\text{low}}{\text{high}} \rightarrow 0-1$$

0-1

 $\underline{\text{Error}} + \underline{\text{Acc}} = 1$
 $AUC = (1 - \underline{\text{Error}})$
 $R^2 = 1 - \frac{RSS}{TSS}$

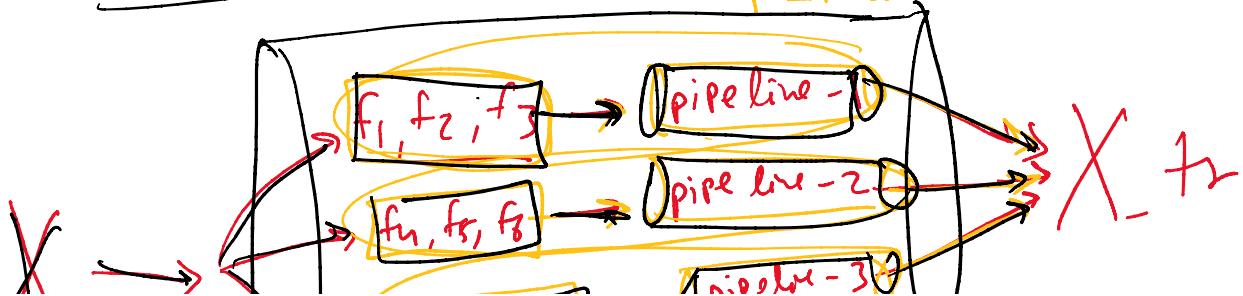
-1.79×10^9

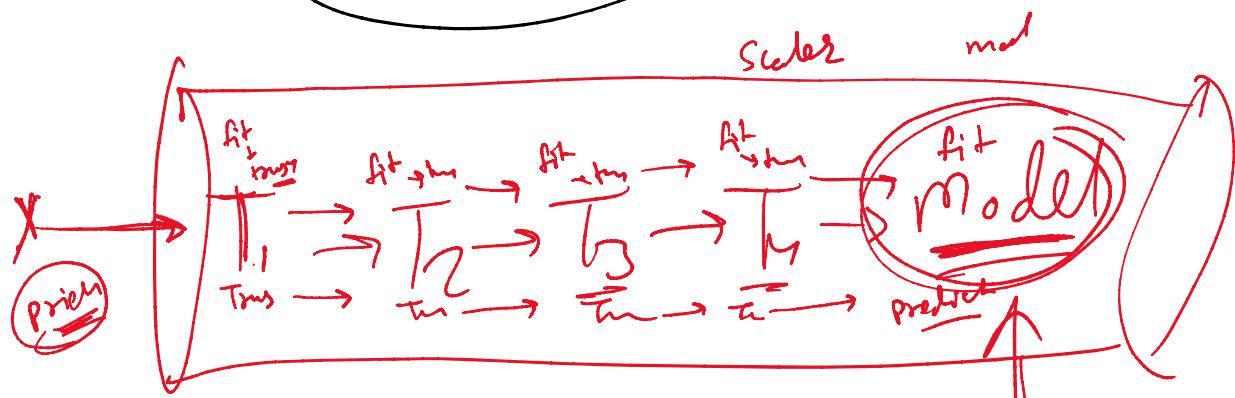
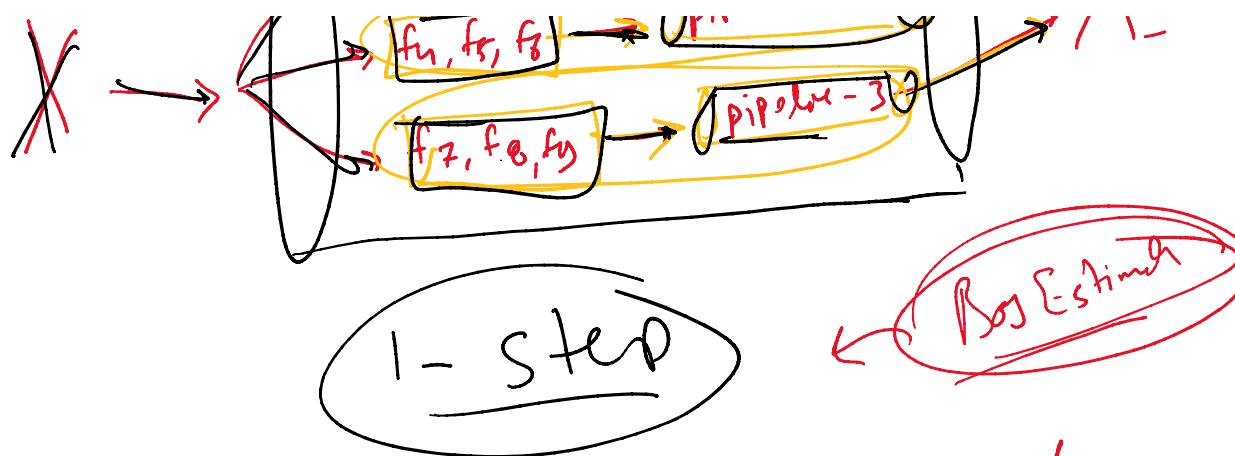
$\frac{-10^{16}}{10} \rightarrow 10^9$

-10^9

\Rightarrow Column Transformer

→ split
→ build
→ fit-trn
→ combine



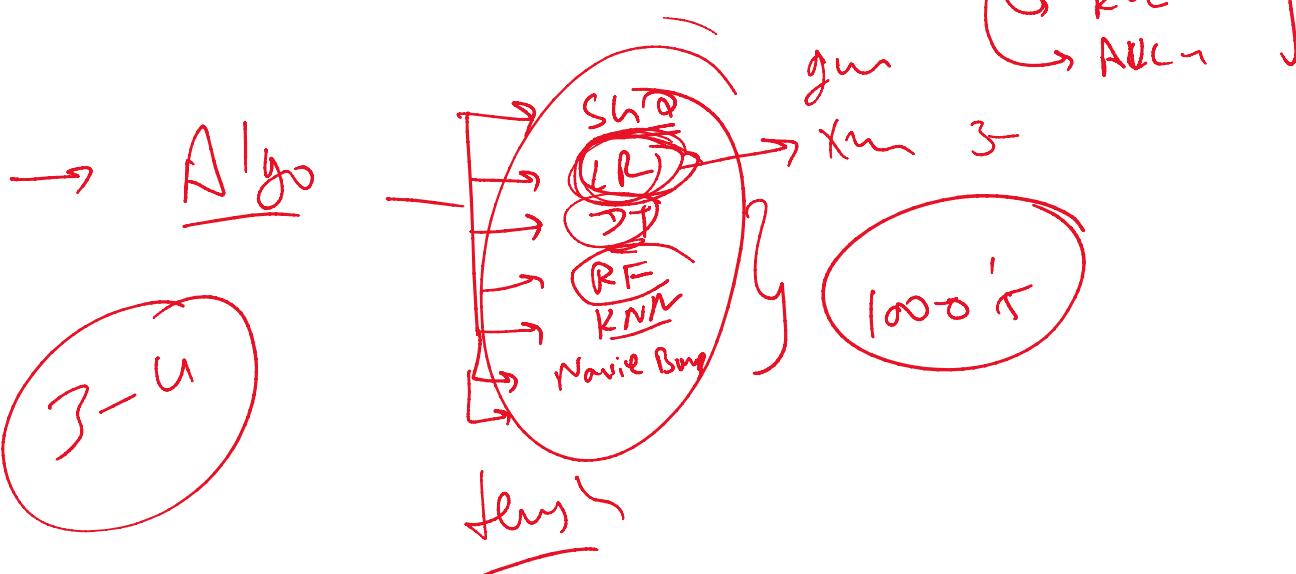


→ pipeline.fit()
 → pipeline.predict()

→ Regression
 → Classification Evalu

BO → Data Plan

metrics → precision
 recall
 F1
 ROC
 AUC



\overrightarrow{r}