

The price impact of order book events

Rama Cont, Arseniy Kukanov and Sasha Stoikov

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Abstract

We study the price impact of order book events - limit orders, market orders and cancellations - using the NYSE TAQ data for 50 U.S. stocks. We show that, over short time intervals, price changes are mainly driven by the *order flow imbalance*, defined as the imbalance between supply and demand at the best bid and ask prices. Our study reveals a linear relation between order flow imbalance and price changes, with a slope inversely proportional to the market depth. These results are shown to be robust to intraday seasonality effects, and stable across time scales and across stocks. This linear price impact model, together with a scaling argument, implies the empirically observed “square-root” relation between the magnitude of price moves and trading volume. However, the latter relation is found to be noisy and less robust than the one based on order flow imbalance. We discuss a potential application of order flow imbalance as a measure of adverse selection in limit order executions, and demonstrate how it can be used to analyze intraday volatility dynamics.

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1 Introduction

The availability of high-frequency records of trades and quotes has stimulated an extensive empirical and theoretical literature on the relation between order flow, liquidity and price movements in order-driven markets. A particularly important issue for applications is the impact of orders on prices: the optimal liquidation of a large block of shares, given a fixed time horizon, crucially involves assumptions on price impact (see Bertsimas and Lo [7], Almgren and Chriss [2], Obizhaeva and Wang [40]). Understanding price impact is also important from a theoretical perspective, since it is a fundamental mechanism of price formation.

Various aspects of price impact have been studied in the literature but there is little agreement on how to model it [8], and the only consensus seems to be the intuitive notion that imbalance between supply and demand moves prices. Theoretical studies draw a distinction between instantaneous price impact of orders and its decay through time, and show that the form of instantaneous impact has important implications. Huberman and Stanzl [27] show that there are arbitrage opportunities if the instantaneous effect of trades on prices is non-linear and permanent. Gatheral [19] extends this analysis by showing that if the instantaneous price impact function is non-linear, impact needs to decay in a particular way to exclude arbitrage and if it is linear, it needs to decay exponentially. Bouchaud et al. [9] associated the decay of price impact of trades with limit orders, arguing that there is a “delicate interplay between two opposite

tendencies: strongly correlated market orders that lead to super-diffusion (or persistence), and mean reverting limit orders that lead to sub-diffusion (or anti-persistence)”. This insight implies that looking solely at trades, without including the effect of limit orders amounts to ignoring an important part of the price formation mechanism.

However, most of the empirical literature on price impact has primarily focused on trades. One approach is to study the impact of “parent orders” gradually executed over time using proprietary data (see Engle et. al [14], Almgren et. al [3]). Alternatively, empirical studies on public data [16, 18, 20, 30, 31, 48, 42, 43] have analyzed the relation between the direction and sizes of trades and price changes and typically conclude that the instantaneous price impact of trades is an increasing, nonlinear function of their size. This focus on trades leaves out the information in quotes, which provide a more detailed picture of price formation [15], and raises a natural question: is volume of trades truly the best explanatory variable for price movements in markets where many quote events can happen between two trades?

In our view, a price impact model that encompasses limit orders, market orders and cancellations, and relates their impact to the concurrent market liquidity would provide a more detailed description of price formation. Obtaining such model is also desirable from the practical point of view because modern order execution algorithms increasingly use limit orders and incorporate market state variables in their decisions. There is also ample empirical evidence that limit orders play an important role in determining price dynamics. Arriving limit orders significantly reduce the impact of trades [49] and the concave shape of the price impact function changes depending on the contemporaneous limit order arrivals [46]. The outstanding limit orders (also known as market depth) significantly affect the impact of an individual trade ([32]), low depth is associated with large price changes [50, 17], and depth influences the relation between trade sizes and returns [24]. The emphasis in the aforementioned studies remains, however, on trades and there are few empirical studies that focus on limit orders from the outset. Notable exceptions are Engle & Lunde [15], Hautsch and Huang [25] who perform an impulse-response analysis of limit and market orders, Hopman [26] who analyzes the impact of different order categories over 30 minute intervals and Bouchaud et al. [12] who examine the impact of market orders, limit orders and cancellations at the level of individual events.

1.1 Summary

We conduct an empirical investigation of the instantaneous impact of order book events – market orders, limit orders and cancellations – on equity prices. Although previous studies give a relatively complex description of their impact, we show that their instantaneous effect on prices may be modeled parsimoniously through a *single* variable, the *order flow imbalance* (OFI). This variable represents the net order flow at the best bid and ask and tracks changes in the size of the bid and ask queues by

- increasing every time the bid size increases, the ask size decreases or the bid/ask prices increase,
- decreases every time the bid size decreases, the ask size increases or the bid/ask prices decrease.

Interestingly, this variable treats a market sell and a cancel buy of the same size as equivalent, since they have the same effect on the size of the best bid queue. This aggregate variable explains mid-price changes over short time scales in a linear fashion, for a large sample of stocks, with an average R^2 of 65%. In contrast, order flows deeper in the order book do not substantially contribute to price changes. Our model based on *OFI* relates prices, trades, limit orders and cancellations in a simple way: it is *linear*, requires the estimation of a single *price impact coefficient* and it is robust across stocks and across timescales.

Most of variability in the instantaneous price impact, both across time and across stocks is explained by variations in market depth. In fact, we establish an exact inverse relation between the two variables. The coefficient of proportionality in that relation depends dramatically on the depth definition, showing that arbitrary measures of market depth are biased proxies for price impact and may lead to misleading conclusions on market liquidity.

The price impact coefficient exhibits substantial intraday variability coinciding with known intraday patterns observed in spreads, market depth and price volatility [1, 5, 35, 39]. We explain the diurnal effects in price volatility using the volatility of order flow imbalance and market depth, as opposed to unobservable parameters previously invoked in the literature, such as information asymmetry [38] or informativeness of trades [21]. The strong link between price volatility and standard deviation of *OFI* suggests that our price impact coefficient is a better estimate of Kyle's λ (a useful metric of liquidity [4, 33]) than traditional estimates based on trades data. We also show that intraday price volatility is mainly driven by *OFI* and not by trading volume. The positive correlation between price volatility and volume, widely confirmed by empirical studies [29], can be a statistical artifact due to aggregation of data over time, and we establish how such spurious relation can arise in our model.

The *OFI* variable exhibits positive autocorrelation over short time scales, which can be exploited to improve the quality of order executions. In particular, we show that a limit order fill is more likely to be followed with a price change in the same direction as the order flow imbalance before that fill. For example, a limit sell order is more likely to be adversely selected when order flow imbalance is positive. Monitoring *OFI* can therefore help reduce adverse selection in limit order fills.

1.2 Outline

The article is structured as follows. In Section 2, we specify a parsimonious model that links stock price changes, order flow imbalance and market depth and motivate it by a stylized example of the order book. Section 3 describes our data and presents estimation results for our model. Section 4 discusses potential applications of our results: in 4.1 we use order flow imbalance as a measure of adverse selection in limit order executions, in 4.2 we demonstrate how diurnal effects in depth and order flow imbalance generate intraday patterns in price impact and price volatility, and in 4.3 we show how a spurious relation between volume and the magnitude of price moves emerges as a statistical artifact from our simple model. Section 5 presents our conclusions.

2 Price impact model

2.1 Stylized order book

To motivate our approach we first consider a stylized example of the order book where the instantaneous effect of order book events can be explicitly computed.

Consider an order book in which the number of shares (depth) at each price level beyond the best bid and ask is equal to D . Order arrivals and cancelations occur only at the best bid and ask. Moreover, when bid (or ask) size reaches D , the next passive order arrives one tick above (or below) the best quote, initializing a new best level. Consider a time interval $[t_{k-1}, t_k]$ and denote by L_k^b, C_k^b respectively the total size of buy orders that arrived to and canceled from current best bid during that time interval. Also denote by M_k^b the total size of marketable buy orders that arrived to current best ask, and by P_k^b the bid price at time t_k . The quantities L_k^s, C_k^s, M_k^s for sell orders are defined analogously and P_k^s is the ask price.

In this simple order book model there exists a linear relation between order flows $L_k^{b,s}, C_k^{b,s}, M_k^{b,s}$ and price changes $\Delta P_k^{b,s} = (P_k^{b,s} - P_{k-1}^{b,s})$ (also illustrated on Figures 1-3):

$$\Delta P_k^b = \delta \left\lceil \frac{L_k^b - C_k^b - M_k^s}{D} \right\rceil \quad (1)$$

$$\Delta P_k^s = -\delta \left\lceil \frac{L_k^s - C_k^s - M_k^b}{D} \right\rceil, \quad (2)$$

where δ is the tick size¹. These relations are remarkably simple - they involve no parameters, the impact of all order book events is *additive* and depends only on their net imbalance. Although all of the subsequent analysis can be carried out separately for bid and ask prices, for simplicity we consider mid-price changes normalized by tick size $P_k = \frac{P_k^b + P_k^s}{2\delta}$:

$$\Delta P_k = \frac{OFI_k}{2D} + \epsilon_k, \quad (3)$$

$$OFI_k = L_k^b - C_k^b - M_k^s - L_k^s + C_k^s + M_k^b \quad (4)$$

where OFI_k is the order flow imbalance (or net order flow) and ϵ is the truncation error. We can also rewrite (3) as:

$$\Delta P_k = \frac{TI_k}{2D} + \eta_k, \quad (5)$$

$$TI_k = M_k^b - M_k^s \quad (6)$$

where TI_k is the trade imbalance and $\eta_k = \frac{L_k^b - C_k^b - L_k^s + C_k^s}{2D} + \epsilon_k$. When limit order activity dominates, i.e. absolute values of terms $|L_k^{b,s}|, |C_k^{b,s}|$ are much larger than $|M_k^{b,s}|$, the correlation of price changes with TI_k is weaker than with OFI_k , because limit order submissions and cancelations manifest as noise in (5).

¹This is easily proven by induction over the number of price changes in $[t_{k-1}, t_k]$. The statement is clearly true when there are no price changes or a single price change of $\pm\delta$. Since any price change of $\pm k\delta$ consists of jumps of size 1, we simply need to sum the order flow imbalances across these jumps on the right side of the equation.

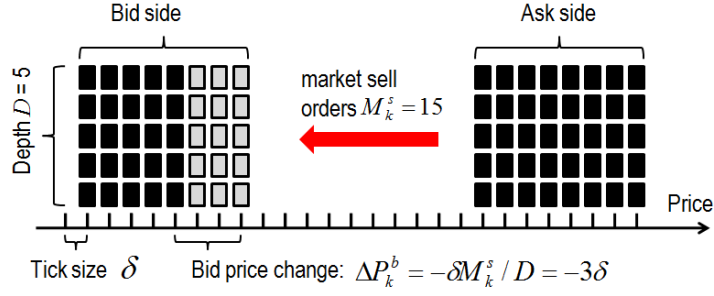


Figure 1: Market sell orders remove M^s shares from the bid (gray squares represent net change in the order book).

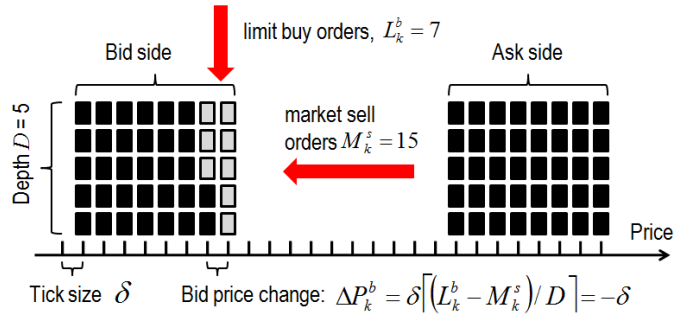


Figure 2: Market sell orders remove M^s shares from the bid, while limit buy orders add L^b shares to the bid.

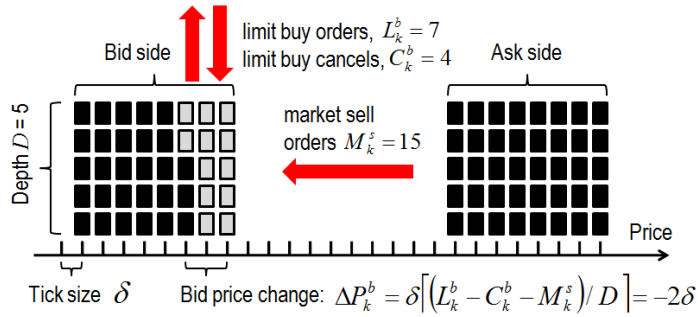


Figure 3: Market sell orders and limit buy cancels remove $M^s + C^b$ shares from the bid, while limit buy orders add L^b shares to the bid.

2.2 Model specification

Actual order books have complex dynamics: arrivals and cancelations occur at all price levels, the depth distribution across levels has non-trivial features [43, 45, 52], and hidden orders together with data-reporting issues create additional errors [6, 22]. Motivated by the stylized order book example we assume a noisy relation between price changes and OFI , which holds locally for short intervals of time $[t_{k-1,i}, t_{k,i}] \subset [T_{i-1}, T_i]$, where $[T_{i-1}, T_i]$ are longer intervals.

$$\Delta P_{k,i} = \beta_i OFI_{k,i} + \epsilon_{k,i}, \quad (7)$$

In this model β_i is a *price impact coefficient* for an i -th time interval and $\epsilon_{k,i}$ is a noise term summarizing influences of other factors (e.g. deeper levels of the order book). We allow β_i and the distribution of $\epsilon_{k,i}$ to change with index i , because of well-known intraday seasonality effects. Our discussion from the previous section allows us to interpret $\frac{1}{2\beta_i}$ as an *implied* order book depth. The stylized order book model suggests that price impact coefficient is inversely related to market depth, and we consider the following model:

$$\beta_i = \frac{c}{D_i^\lambda} + \nu_i, \quad (8)$$

where c, λ are constants and ν_i is a noise term. The stylized order book model corresponds to $c = \frac{1}{2}, \lambda = 1$. We also consider a relation between price changes and trades:

$$\Delta P_{k,i} = \beta_i^T TI_{k,i} + \eta_{k,i}, \quad (9)$$

but expect it to be much noisier than (7).

The specification (7-8) may be regarded as a model of instantaneous price impact of order book events, arriving within time interval $[t_{k-1}, t_k]$. An order submitted or canceled at time $\tau \in [t_{k-1}, t_k]$ contributes a signed quantity e_τ to supply/demand. In any given time interval, these contributions are likely to be unbalanced, leading to an order flow imbalance OFI_k , which affects supply/demand and leads to a corresponding price adjustment. If an individual order goes in the same direction as the majority of orders ($\text{sgn}(e_\tau) = \text{sgn}(OFI_k)$), it reinforces the concurrent order flow imbalance and can affect the price. If the order goes against the concurrent order flow imbalance ($\text{sgn}(e_\tau) = -\text{sgn}(OFI_k)$), it is compensated by other orders and has an instantaneous impact of zero. In our model all events (including trades) have a linear price impact, on average equal to β_i during the i -th interval. Their realized impact however depends on the concurrent orders.

The idea that the concurrent limit order activity can make a difference in terms of trades' impact was demonstrated in [46], where authors show that the shape of the price impact function essentially depends on the contemporaneous limit order activity. Our approach can also be related to the model proposed in [12], where order book events have a linear impact on prices, which depends on their signs and types². The major difference of our model lies in the aggregation across time and events. As shown in [12], time series of individual order book events have complicated auto- and cross-correlation structures, which typically vanish after 10 seconds. In our data the autocorrelations at a timescale of 10 seconds are small and quickly vanish as well (ACF plots for a representative stock are shown on Figure 4). Finally, the model used in [24] for explaining the price impact of trades is similar to (9). Although the focus there is on trades, authors allow the price impact coefficient to depend on contemporaneous liquidity factors and change through time.

²Note that in our case all order book events have the same average impact, equal to β_i , regardless of their type. As shown in [12], average impacts of different event types are empirically very similar, allowing to reasonably approximate them with a single number.

At the same time, the linear relation (7) is different from many earlier models that consider only the effect of transactions [18, 20, 31, 48, 42, 43]. Instead of modeling price impact of trades as a (nonlinear) function of trade size, we show that the instantaneous price impact of a series of events (including trades) is a linear function of their size after these events are aggregated into a single imbalance variable. We will show that, first, the effect of trades on prices is adequately captured by the order flow imbalance and, second, that if one leaves out all events except trades, the relation 7 leads to an apparent concave relation between the magnitude of price changes and trading volume.

The next section provides an overview of the estimation results for our model.

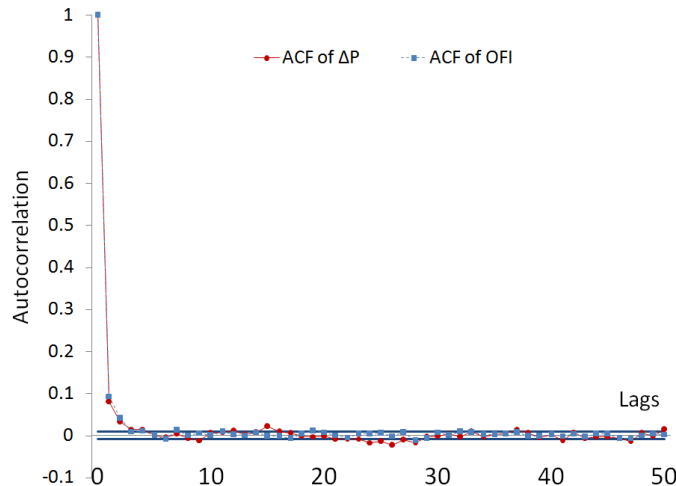


Figure 4: ACF of the mid-price changes $\Delta P_{k,i}$, the order flow imbalance $OFI_{k,i}$ and the 5% significance bounds for the Schlumberger stock (SLB).

3 Estimation and results

3.1 Data

Our main data set consists of one calendar month (April, 2010) of trades and quotes data for 50 stocks. The stocks were selected by a random number generator from S&P 500 constituents, which were obtained from Compustat. The data for individual stocks was obtained from the TAQ consolidated quotes and TAQ consolidated trades databases³.

Consolidated quotes contain best bid/ask price changes and round-lot changes in best bid/ask sizes. Quote data entries consist of a stock ticker, a timestamp (rounded to the nearest second), bid price and size, ask price and size and various flags including exchange flag. Consolidated trade entries consist of timestamps, prices, sizes and various flags. These two data sets are often referred to as Level 1 data, as opposed to Level 2 data, which includes quote updates deeper in the book, or information on individual orders.

At the same time TAQ data has important limitations - the timestamps are rounded to the nearest second, and it may omit odd-lot trades and quotes. To perform several detailed robustness checks we also use an auxiliary data set consisting of NASDAQ ITCH 4.0 messages for the same calendar month (April, 2010) for one representative stock from our main data

³The TAQ data were obtained through Wharton Research Data Services (WRDS).

set (Schlumberger). This data is accessible through LOBSTER website⁴ which also provides NASDAQ order book history for the selected stock. We used LOBSTER data for the top five order book levels without any additional pre-processing.

The TAQ data was used to compute the National Best Bid and Offer sizes and prices (NBBO) at each quote update. We find that the ratio between the number of NBBO quote updates and the number of trades is roughly 40 to 1 in our data. Many empirical studies have focused exclusively on trades rather than quotes, but the sheer difference in sizes of these data sets suggests that more information may be conveyed by quotes than by trades. Using the exchange flag, we also considered one exchange at a time and obtained similar empirical results.

3.2 Variables

Every observation of the bid and the ask consists of the bid price P^b , the the bid queue size q^b (in number of shares), the ask price P^s and size q^s . We enumerate them by n and compute differences between consecutive observations $(P_n^b, q_n^b, P_n^s, q_n^s)$ as follows:

$$e_n = q_n^b \mathbb{1}_{\{P_n^b \geq P_{n-1}^b\}} - q_{n-1}^b \mathbb{1}_{\{P_n^b \leq P_{n-1}^b\}} - q_n^s \mathbb{1}_{\{P_n^s \leq P_{n-1}^s\}} + q_{n-1}^s \mathbb{1}_{\{P_n^s \geq P_{n-1}^s\}} \quad (10)$$

The variables e_n are signed contributions of order book events to supply/demand. When a passive buy order arrives, q^b increases but P^b remains the same, leading to $e_n = q_n^b - q_{n-1}^b$ which is the size of that order. If q^b decreases, we have $e_n = q_n^b - q_{n-1}^b$, representing the size of a marketable sell order or buy order cancelation. If P^b changes, then $e_n = q_n^b$ or $e_n = -q_{n-1}^b$, representing respectively the size of a price-improving order or the last order in the queue that that was removed. Symmetric computations are done for the ask side.

We use two uniform time grids $\{T_0, \dots, T_I\}$ and $\{t_{0,0}, \dots, t_{I,K}\}$ with time steps $T_i - T_{i-1} = 30$ minutes and $t_{k,i} - t_{k-1,i} = \Delta t = 10$ seconds⁵. Within each long time interval $[T_{i-1}, T_i]$ we compute 180 price changes and order flow imbalances indexed by k :

$$\Delta P_{k,i} = \frac{P_{N(t_{k,i})}^b + P_{N(t_{k,i})}^s}{2\delta} - \frac{P_{N(t_{k-1,i})}^b + P_{N(t_{k-1,i})}^s}{2\delta} \quad (11)$$

$$OFI_{k,i} = \sum_{n=N(t_{k-1,i})+1}^{N(t_{k,i})} e_n, \quad (12)$$

where $N(t_{k-1,i}) + 1$ and $N(t_{k,i})$ are the index of the first and the last order book event in the interval $[t_{k-1,i}, t_{k,i}]$. The tick size δ is equal to 1 cent in our data. Note that in our empirical study OFI is computed from fluctuations in best bid/ask prices and their sizes according to (12), because data on individual orders is not available in our main dataset. If that data is available, OFI can be computed according to (4). We believe that a computation based on (4) can lead to better empirical results because aggressive order terms M^b, M^s will capture information on hidden orders and unreported odd-lot sized orders within the spread, to the extent that aggressive orders interact with hidden orders. Since TAQ data reports only round-lot sized quote changes, we note that units of OFI are round lots (100 shares), and assume in (12) that both sides of the market are equally affected by missing quote updates⁶.

We define trade imbalance during a time interval $[t_{k-1,i}, t_{k,i}]$ as the difference between volumes of buyer- and seller-initiated trades during that interval, and also define trading volume within that time interval:

⁴<http://lobster.wiwi.hu-berlin.de/Lobster/about/About.WhatIsLOBSTER.jsp>

⁵results for other timescales are reported in the appendix

⁶As we demonstrate in the appendix, neither missing odd-lot sized observations nor potential mis-sequencing of quote updates across different exchanges during NBBO computation change our qualitative findings.

$$TI_{k,i} = \sum_{n=N(t_{k-1,i})+1}^{N(t_{k,i})} b_n - s_n \quad VOL_{k,i} = \sum_{n=N(t_{k-1,i})+1}^{N(t_{k,i})} b_n + s_n \quad (13)$$

where b_n, s_n are sizes of buyer- and seller-initiated trades (in round lots) that occurred at the n -th quote (equal to zero if no trade occurred at that quote). In contrast with TI , the OFI measure computed using (12) does not hinge on trade classification, which is known to be problematic for TAQ data (see appendix for more details on matching trades with quotes and trade classification). Whereas previous studies [11, 20, 24, 31, 42, 48] focused on trade imbalance⁷, the order flow imbalance is a more general measure. It encompasses effects of all order book events, including trades.

For each interval $[T_{i-1}, T_i]$ we also estimate depth by averaging the bid/ask queue sizes right before or right after a price change, consistently with the definition of depth in the stylized order book model:

$$D_i = \frac{1}{2} \left[\frac{\sum_{n=N(T_{i-1})+1}^{N(T_i)} \left(q_n^b \mathbb{1}_{\{P_n^b < P_{n-1}^b\}} + q_{n-1}^b \mathbb{1}_{\{P_n^b > P_{n-1}^b\}} \right)}{\sum_{n=N(T_{i-1})+1}^{N(T_i)} \mathbb{1}_{\{P_n^b \neq P_{n-1}^b\}}} + \frac{\sum_{n=N(T_{i-1})+1}^{N(T_i)} \left(q_n^s \mathbb{1}_{\{P_n^s > P_{n-1}^s\}} + q_{n-1}^s \mathbb{1}_{\{P_n^s < P_{n-1}^s\}} \right)}{\sum_{n=N(T_{i-1})+1}^{N(T_i)} \mathbb{1}_{\{P_n^s \neq P_{n-1}^s\}}} \right]$$

⁷Hopman [26] computes the supply/demand imbalance based on limit orders and trades, but not cancelations.

3.3 Empirical findings

This section reports detailed results for a representative stock, Schlumberger (SLB) and some average results across stocks. Detailed results for other stocks in our sample are presented in the appendix. During the sample period the average price of Schlumberger stock was 67.94 dollars and the average daily volume was 947.6 million shares. The daily average number of NBBO quote updates is about 440 thousands, and the average daily number of trades is around 10 thousands. The average spread is one cent, its 95-th percentile is 2 cents and the average best NBBO quote size is 39 round lots (3900 shares).

The model (7) is estimated by an ordinary least squares regression:

$$\Delta P_{k,i} = \hat{\alpha}_i + \hat{\beta}_i OFI_{k,i} + \hat{\epsilon}_{k,i}, \quad (14)$$

with separate half-hour subsamples indexed by i . Figure 5 presents a scatter plot of $\Delta P_{k,i}$ against $OFI_{k,i}$ for one of such subsamples.

In general we find that $\hat{\beta}_i$ is statistically significant⁸ in 98% of samples, and $\hat{\alpha}_i$ is significant in 10% of samples, which is close to the Type-I error rate. The average t-statistics for $\hat{\alpha}_i, \hat{\beta}_i$ are respectively -0.21 and 16.27 for SLB (cross-sectional averages are -0.02 and 12.08). To check for higher order/nonlinear dependence we estimate an augmented regression:

$$\Delta P_{k,i} = \hat{\alpha}_i^Q + \hat{\gamma}_i OFI_{k,i} + \hat{\gamma}_i^Q OFI_{k,i} |OFI_{k,i}| + \hat{\epsilon}_{k,i}^Q, \quad (15)$$

The coefficients $\hat{\gamma}_i^Q$ have an average t-statistic of -0.32 across stocks and are statistically significant only in 17% of our samples. We reject the hypothesis of quadratic (convex or concave) instantaneous price impact, and take this as strong evidence for a linear price impact model (7), because other kinds of non-linear dependence would likely be picked up by this quadratic term.

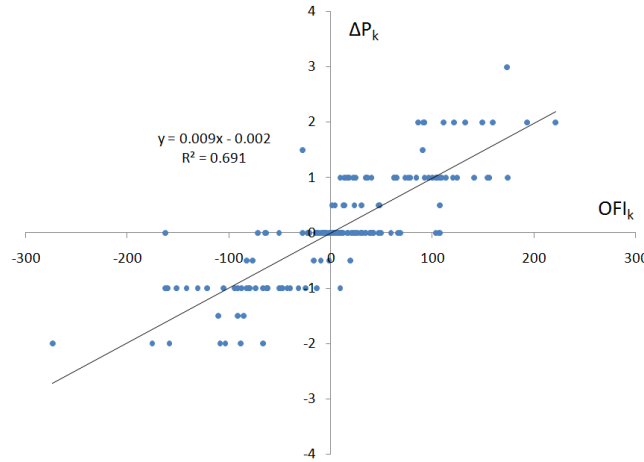


Figure 5: Scatter plot of $\Delta P_{k,i}$ against $OFI_{k,i}$ for the Schlumberger stock (SLB), 04/01/2010 11:30-12:00pm.

The goodness of fit is surprising for high-frequency data, with an R^2 of 76% for SLB and 65% on average across stocks⁹, suggesting that a one-parameter linear model (7) performs well

⁸Given a relatively large number of observations we use the z-test with a 95% significance level. Since regression residuals demonstrate heteroscedasticity and autocorrelation, Newey-West standard errors are used to compute t-statistics.

⁹We note that OFI includes the contributions e_n of price-changing order book events, leading to a possible

regardless of stock-specific features, such as average spread, depth or price level. The definition of R^2 as a percentage of explained variance has an interesting consequence in our case. Since OFI is constructed from order book events taking place only at the best bid/ask, our results show that activity at the top of the order book is the most important factor driving price changes. In the appendix we confirm this by showing that order flow imbalances from deeper order book levels only marginally contribute to short-term price dynamics. Even though large price movements sometimes occur at this timescale, they mostly correspond to large readings of OFI . Figure 6 confirms this by demonstrating a relatively low level of excess kurtosis in regression residuals.

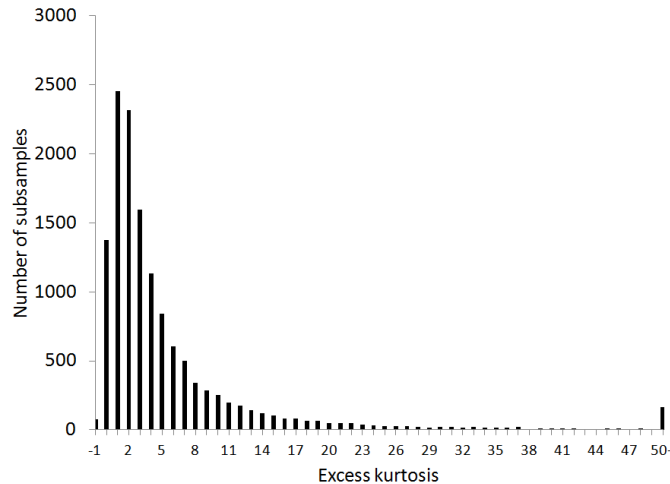


Figure 6: Distribution of excess kurtosis in the residuals $\hat{\epsilon}_{k,i}$ across stocks and time.

When the amount of passive order submissions and cancelations is much larger than the amount of trades, the stylized order book model predicts that trade imbalance TI explains price changes significantly worse than OFI . To empirically confirm this we estimate following regressions using the same half-hour subsamples ¹⁰:

$$\Delta P_{k,i} = \hat{\alpha}_i^T + \hat{\beta}_i^T TI_{k,i} + \hat{\eta}_{k,i} \quad (16a)$$

$$\Delta P_{k,i} = \hat{\alpha}_i^D + \hat{\theta}_i^O OFI_k + \hat{\theta}_i^T TI_{k,i} + \hat{\epsilon}_{k,i}^D \quad (16b)$$

When either OFI or TI variable is taken individually, that variable has a statistically significant correlation with price changes. The average t-statistics of slope coefficients in simple regressions (14, 16a) are, correspondingly 16.27 and 5.31 for SLB (cross-sectional averages are 12.08 and 5.08). The average R^2 for the two regressions are 65% and 32%, respectively, confirming the prediction that relation between price changes and trade imbalance is more noisy. When the two variables are used in a multiple regression (16b), the dependence of price changes on trade imbalance becomes much weaker. The average t-statistic of TI coefficient drops to 1.56 for SLB

endogeneity in the regression (14). This problem is inherent to all price impact modeling, because the explanatory variables (events or trades) sometimes mechanically lead to price changes. To test that the high R^2 in our regressions is not due to this endogeneity, we estimated (14) on a subsample of stocks, excluding the price-changing events from OFI . With this change the R^2 declined, but remained high, in the 35%-60% region.

¹⁰These regressions contain only linear terms, because we found no evidence of non-linear price impacts in our data (for neither OFI nor TI).

(1.51 across stocks) and it remains statistically significant in only 47% of SLB samples (43% of all stock samples). The dependence on *OFI* remains strong with an average t-statistic 13.91 for SLB (9.53 across stocks), and the coefficient is statistically significant in almost all samples. We conclude that *OFI* explains price movements better than trade imbalance, and *OFI* is a more general measure of supply/demand imbalance because it adequately includes the effect of trade imbalance.

Finally, we use time series of D_i and $\hat{\beta}_i$ for each stock to estimate the relation (8) with the following two regressions:

$$\log \hat{\beta}_i = \alpha_{L,i} - \hat{\lambda} \log D_i + \hat{\epsilon}_{L,i} \quad (17)$$

$$\hat{\beta}_i = \alpha_{M,i} + \frac{\hat{c}}{D_i^{\hat{\lambda}}} + \hat{\epsilon}_{M,i} \quad (18)$$

Both regressions are estimated using ordinary least squares¹¹. For SLB we find $\hat{c} = 0.56$, $\hat{\lambda} = 1.08$ and an R^2 of (17) is 92%. The results for all stocks are shown in Table 4. We observe that depth significantly correlates with price impact coefficients for the vast majority of stocks, confirming our intuition that $\frac{1}{2\beta_i}$ is the implied order book depth. Interestingly, estimates \hat{c} , $\hat{\lambda}$ across stocks are very close to values predicted by the stylized order book model. With the t-statistics¹² in Table 4 the null hypotheses $\{c = 0.5\}$ and $\{\lambda = 1\}$ cannot be rejected for most stocks based on conventional significance levels. The restricted model with $\lambda = 1$ also demonstrates a good quality of fit, making this a good approximation. Figure 7 illustrates these results with a log-log scatter plot for D_i and $\hat{\beta}_i$. Some stocks (namely APOL, AZO and CME) have poor fits in regression (17), mainly due to outliers in the dependent variable. After removing these outliers and re-estimating the regression, the estimates \hat{c} , $\hat{\lambda}$ for these stocks fell in line with estimates for other stocks.

To assess the stability of these findings, we re-estimated (17,18) with observations pooled across days but not across intraday time intervals, resulting in 13 estimates \hat{c}_i , $\hat{\lambda}_i$ for each stock. Although these estimates demonstrate some diurnal variability, they are relatively stable and most of variability in price impact coefficients is explained by variations in depth (e.g. see Figure 9).

We repeated the analysis with different depth variables, taking D_i to be equal to arithmetic or geometric average of queue sizes over the i -th time interval. Overall, the results were the same, except for the level of \hat{c} estimates, which were about 40% lower across stocks for the arithmetic average depth, and even lower for the geometric average. The systematic difference in these coefficients implies that taking an arbitrary measure of depth (such as arithmetic average of queue sizes) as a proxy of price impact may lead to significant biases, i.e. one would dramatically under- or over-estimate price impact in a given stock. Instead of looking at arbitrary depth measures, we suggest computing price impact coefficients β_i and/or implied depth $\frac{1}{2\beta_i}$ to precisely characterize price sensitivity to order flow.

¹¹We note that an estimate $\hat{\lambda}_i$ is used in regression (18). This “plug-in” approach leads to potential errors in explanatory variable, and standard errors for \hat{c} may be underestimated. However, the good quality of fit in regression (17) with an average R^2 of 76% indicates that $\hat{\lambda}_i$ are estimated with good precision. We believe that errors in variable $\frac{1}{D_i^{\hat{\lambda}}}$ are small and do not affect our results.

¹²Since the residuals of these regressions appear to be autocorrelated, the t-statistics are computed with Newey-West standard errors.

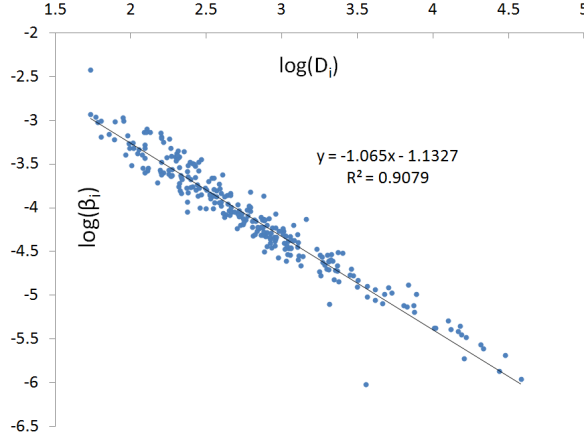


Figure 7: Log-log scatter plot of the price impact coefficient estimate $\hat{\beta}_i$ against average market depth D_i for the Schlumberger stock (SLB).

4 Applications

4.1 Monitoring adverse selection

Time intervals that are involved in modern high-frequency trading applications are usually so short that price changes are relatively infrequent events. Therefore price changes provide a very coarse and limited description of market dynamics. However, *OFI* tracks best bid and ask queues and fluctuates on a much faster timescale than prices. It incorporates information about build-ups and depletions of order queues and it can be used to interpolate market dynamics between price changes (see Figure 8 for example). Our results confirm that such interpolation is in fact valid because *OFI* closely approximates price changes over short time intervals (e.g. results for 50 millisecond time intervals are shown in the appendix). To study one possible application of *OFI* for high-frequency trading we turn to our auxiliary dataset, because it contains accurate timestamps up to a millisecond.

Given the strong link between *OFI* and price changes, and the positive autocorrelation of *OFI* over short time intervals (see Figure 4), we propose to use it as a measure of adverse selection in the order flow. For example, when a limit order is filled, and its execution was preceded by positive *OFI*, a positive price change is more likely to happen after the limit order execution. This is because the pre-execution positive *OFI* is likely to persist in the future, and can lead to a post-execution positive price change. For a limit sell order a positive post-execution price change implies that the order was executed at a loss, i.e. adversely selected.

To test our hypothesis, we consider all limit order executions in our auxiliary dataset. For each execution we compute the pre-execution order flow imbalance OFI_k^{pre} and the post-execution mid price change ΔP_k^{post} . The pre-execution order flow imbalance is computed from best bid and ask quote updates with timestamps in $[t_k - 200, t_k - 1]$ milliseconds, where t_k is the time of the k -th limit order execution. Similarly the post-execution price change is defined as the difference in mid-quote prices between $t_k + 200$ milliseconds and t_k ¹³. Then we consider 30-minute subsamples of data indexed by i , and estimate the following regression:

$$\Delta P_{k,i}^{post} = \alpha_i^p + \beta_i^p OFI_{k,i}^{pre} + \epsilon_{k,i}^p, \quad (19)$$

¹³If there are multiple quotes with timestamp $t_k + 200$ or t_k , we take the last one.

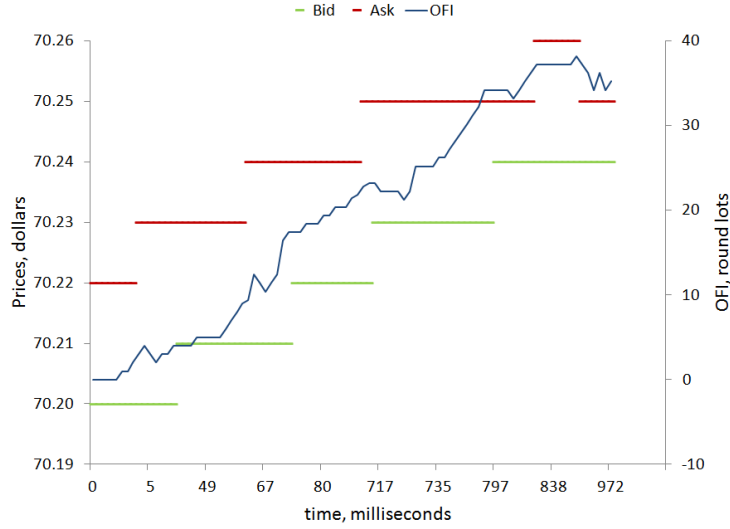


Figure 8: Price dynamics and cumulative *OFI* on NASDAQ for a 1-second time interval starting at 11:16:39.515 on 04/28/2010, Schlumberger stock (SLB).

The average R^2 of these regressions across a month is 2.93%, the average t-statistic¹⁴ of β_i^p is 2.68 and this coefficient is significant at a 5% level in 63% of subsamples. The average β_i^p is 0.0105. We conclude that pre-execution *OFI* are positively correlated with post-execution price changes.

We also estimated regression (19) with 50- and 100-millisecond time intervals for pre- and post-execution variables, and obtained similar results, with stronger correlations for smaller time intervals¹⁵. When we split $OFI_{k,i}^{pre}$ into multiple order flow imbalance variables over non-overlapping subintervals of $[t_k - 200, t_k - 1]$, we find that only the variable closest to t_k - the execution time - is statistically significant and positively correlated with post-execution price change. These results suggest that limit order traders need to actively monitor order flows and react to emerging order flow imbalances as quickly as possible to avoid being adversely selected.

4.2 Intraday volatility dynamics

The link between price impact and market depth established here has important implications for intraday volatility. Market depth is known to follow a predictable diurnal pattern ([1], [35]), and equation (8) implies that instantaneous price impact must also have a *predictable* intraday pattern. To demonstrate it, we averaged $\hat{\beta}_i$ for each stock and each intraday half-hour interval across days, resulting in diurnal effects for that stock, normalized these effects by a grand average $\hat{\beta}_i$ for that stock and averaged normalized diurnal effects across stocks. The same procedure was repeated for depths D_i . We also re-estimated (17,18) with observations pooled across days but not across intraday time intervals, resulting in 13 estimates $\hat{\lambda}_i, \hat{c}_i$ for each stock. The overall average diurnal effects for these quantities are shown on Figure 9.

We found that between 9:30 and 10am the depth is two times lower than on average, indicating that the market is relatively shallow. In a shallow market, incoming orders can easily affect mid-prices and price impact coefficients between 9:30 and 10am are in fact two times

¹⁴Here we also use Newey-West standard errors because residuals demonstrate significant autocorrelation.

¹⁵For instance, with 50-millisecond time intervals the average t-statistic of β_i^p is 3.41 and this coefficient is significant in 75% of samples. The average R^2 becomes 3.32%

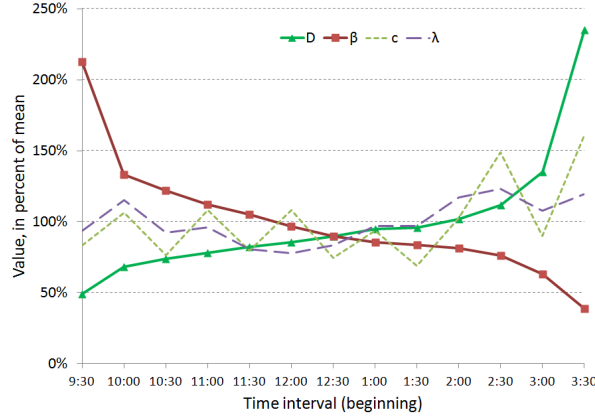


Figure 9: Diurnal effects in the price impact coefficient $\hat{\beta}_i$, the average depth D_i and the parameters $\hat{c}_i, \hat{\lambda}_i$. Most of the intraday variation in price impact coefficients comes from variations in depth, while parameters $\hat{c}_i, \hat{\lambda}_i$ are relatively more stable.

higher than on average. Moreover, price impact coefficients between 9:30 and 10am are five times higher than between 3:30 and 4pm.

The intraday pattern in price impact can be used to explain intraday patterns in price volatility, observed by many studies ([1], [5], [21], [38]). Similarly to the price impact coefficient and the market depth, we computed the intraday patterns in variances of $\Delta P_{k,i}$ and $OFI_{k,i}$, using our half-hour subsamples. Taking the variance on both sides in equation (7), we obtain a link between $\text{var}[\Delta P_{k,i}]$, $\text{var}[OFI_{k,i}]$ and β_i :

$$\text{var}[\Delta P_{k,i}] = \beta_i^2 \text{var}[OFI_{k,i}] + \text{var}[\epsilon_{k,i}] \quad (20)$$

The average variance patterns are plotted on Figure 10. Notice that price volatility has a sharp peak near the market open, while volatility of OFI peaks near the market close. The latter peak is offset by low price impact, which gradually declined throughout the day. For the i -th half-hour interval, equation (20) implies that $\text{var}[\Delta P_{k,i}] \approx \hat{\beta}_i^2 \text{var}[OFI_{k,i}]$ because $\text{var}[\epsilon_{k,i}]$ is relatively small, which is also demonstrated¹⁶ on Figure 10. Since the R^2 in regression (14) is high, the ratio $\frac{\text{var}[\epsilon_{k,i}]}{\text{var}[OFI_{k,i}]}$ is small, and we can rewrite (20) as $\beta_i \approx \frac{\sigma_{P,i}}{\sigma_{O,i}}$, where $\sigma_{P,i} = \sqrt{\text{var}[\Delta P_{k,i}]}$ and $\sigma_{O,i} = \sqrt{\text{var}[OFI_{k,i}]}$. This bears strong resemblance to the definition of Kyle's λ (see [33]) - a metric that is used in the asset pricing literature to gauge liquidity risk (see [4] and references therein). This metric is traditionally estimated as a slope β_i^L in regression (16a), but our analysis suggests that β_i is a better estimate. Although one could also write $\beta_i^L \approx \frac{\sigma_{P,i}}{\sigma_{T,i}}$, where $\sigma_{T,i} = \sqrt{\text{var}[TI_{k,i}]}$, this would be a poorer approximation because $\frac{\text{var}[\eta_{k,i}]}{\text{var}[TI_{k,i}]} > \frac{\text{var}[\epsilon_{k,i}]}{\text{var}[OFI_{k,i}]}$ as shown by R^2 values in Table 3.

The intraday pattern in price variance was explained in an earlier study [38] using a structural model. The authors argued that price volatility is higher in the morning because of a higher inflow of public and private information. In another study [21] the morning peak of price volatility is explained mostly by higher intensity of public information. Both studies agree that the impact of trades is larger in the morning. Our model contributes to this discussion by explaining the peak of price volatility using tangible quantities, rather than unobservable information variables. Our findings also suggest that price impact and information asymmetry

¹⁶ $\hat{\beta}_i^2 \text{var}[OFI_{k,i}]$ was computed from the average patterns of $\hat{\beta}_i$ and $\text{var}[OFI_{k,i}]$

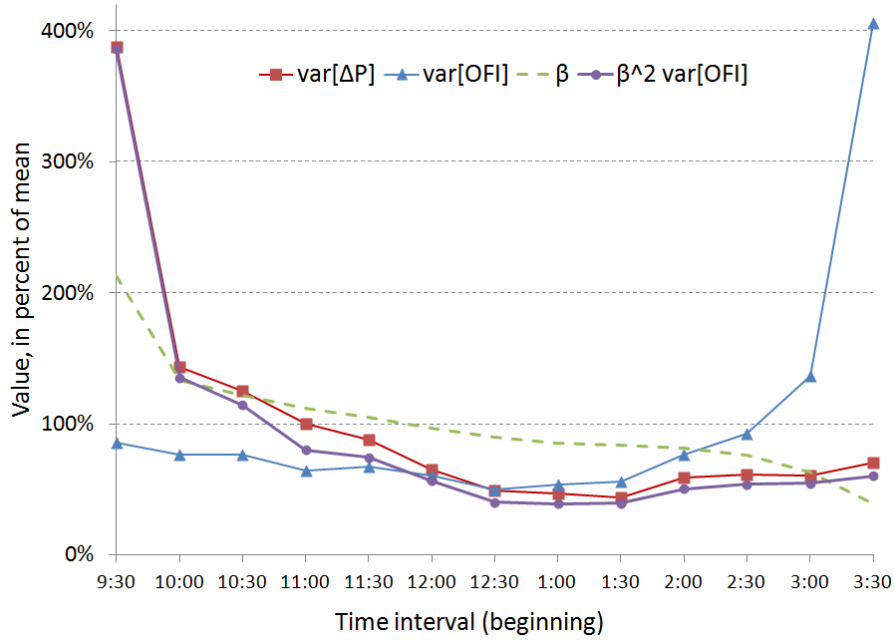


Figure 10: Diurnal variability in variances $\text{var}[\Delta P_{k,i}]$, $\text{var}[OFI_{k,i}]$, the price impact coefficient $\hat{\beta}_i$ and the expression $\beta_i^2 \text{var}[OFI_k]_i$.

may be, in fact, two sides of the same coin. If there is more private information in the morning than in the evening and if limit order traders are aware of this information asymmetry, their participation will likely diminish in the morning, leading to lower depth near market open. At the same time, low depth implies a higher price impact in our model, making the information advantages harder to realize at the market open.

4.3 Volume and volatility

The positive correlation between magnitudes of price changes and trading volume is empirically confirmed by many authors (see [29] for a review). Recently, trading volume became an important metric for order execution algorithms - these algorithms often attempt to match a certain percentage of the total traded volume to reduce the price impact. However, it remains unclear whether trading volume truly determines the magnitude of price moves and whether it is a good metric for price impact. Casting doubt on this assertion, it was found in [28] that the relation between *daily* volatility and *daily* volume is essentially due to the number of trades and not the volume per se (also see [10] for a following discussion).

We provide further evidence that volume is not a driver of price volatility, now on *intraday* timescales. First, we prove that even when prices are purely driven by *OFI* and not by volume, a concave relation between magnitude of price changes and transaction volume emerges as an artifact due to aggregation of data across time. Second, we confirm that such relation exists in the data, but it becomes statistically insignificant after accounting for magnitude of *OFI*.

Comparing the definitions of *VOL* and *OFI* we note that both quantities are sums of random variables. As the aggregation time window $[t_{k-1}, t_k]$ becomes progressively larger, the behavior of these sums (under certain assumptions) will be governed by the Law of Large Numbers and the Central Limit Theorem. We consider a general time interval $[0, T]$ and denote by $N(T)$ the number of order book events during that time interval. We also denote by $OFI(T)$

and $VOL(T)$, respectively, the order flow imbalance and the traded volume during $[0, T]$. The following proposition shows a link between $OFI(T)$ and $VOL(T)$ as T grows.

Proposition 1 *Assume that*

1. $\frac{N(T)}{T} \rightarrow \Lambda$, as $T \rightarrow \infty$, where Λ is the average arrival rate of order book events.
2. $\{e_i\}_{i \geq 1}$ form a covariance-stationary sequence and have a linear-process representation $e_i = \sum_{j=0}^{\infty} a_j Y_{i-j}$, where Y_i is a two-sided sequence of i.i.d random variables with $E[Y_i] = 0$ and $E[Y_i^2] = 1$, and a_j is a sequence of constants with $\sum_{j=0}^{\infty} a_j^2 = \sigma^2 < \infty$. Moreover, $cov(e_1, e_{1+n}) \sim cn^{2(H-1)}$ as $n \rightarrow \infty$, where $0 < H < 1$ is a constant that governs the decay of the autocorrelation function.
3. $\{w_i\}_{i \geq 1}$, $w_i = b_i + s_i$ are random variables with a finite mean $\mu\pi$, where π is the proportion of order book events that correspond to trades and μ is the mean trade size. $E|w_i|^p < \infty$ for some $p > 1$ and $\sum_{N \geq 1} \frac{1}{N} (E|\frac{1}{N} \sum_{i \leq N} w_i|^q)^{r/q} < \infty$ for some r, q such that $0 < r \leq q \leq \infty$ and $r/q \leq 1 - 1/p$.

$$\text{Then } \frac{(\mu\pi)^H}{\sigma} \frac{OFI(T)}{VOL^H(T)} \xrightarrow{T \rightarrow \infty} \xi \sim N(0, 1)$$

where \Rightarrow denotes convergence in distribution.

The proof of this Proposition is given in the appendix. If the time interval $[0, T]$ includes a large enough number of order book events, Proposition 1 implies that

$$OFI(T) \sim \xi \frac{\sigma}{(\mu\pi)^H} VOL^H(T) \simeq N\left(0, \frac{\sigma^2}{(\mu\pi)^{2H}} VOL^{2H}(T)\right) \quad (21)$$

If the time intervals $[t_{k-1,i}, t_{k,i}]$ are large enough to support this approximation then substituting (21) in (7) yields

$$\Delta P_{k,i} \sim N\left(0, \frac{\sigma^2 \beta_i^2}{(\mu\pi)^{2H}} VOL_{k,i}^{2H} + \sigma_i\right)$$

where $\sigma_i = var[\epsilon_{k,i}]$. Note that even if $\sigma_i = 0$, i.e. even if volume cannot affect price volatility through the residual variance, Proposition 1 predicts a spurious relation between price volatility and volume.

Interestingly, the recent theory of market microstructure invariants (see [34]) also predicts a relation between the volatility of order flow imbalance and trading volume. In their analysis, order flow imbalance is defined differently based on unobservable “bets”, however it is natural to assume positive correlation between OFI and the imbalance of “bets”, since the latter reach exchanges in form of actual orders.

We can recast this statement in a testable form for the magnitudes (absolute values) of price changes. Assuming $\epsilon_{k,i} \approx 0$, the scaling argument in Proposition 1 together with our linear price impact model imply that

$$|OFI_{k,i}| \approx \frac{\sigma}{(\mu\pi)^H} VOL_{k,i}^H |\xi_{k,i}| \quad (22)$$

$$|\Delta P_{k,i}| \approx \frac{\beta_i \sigma}{(\mu\pi)^H} VOL_{k,i}^H |\xi_{k,i}| \quad (23)$$

We denote by $\theta_i = \frac{\beta_i \sigma}{(\mu\pi)^H}$ and take logarithms in (23) to obtain

$$\log |\Delta P_{k,i}| = \log \hat{\theta}_i + \hat{H}_i \log VOL_{k,i} + \log |\hat{\xi}_{k,i}| \quad (24)$$

Based on Proposition 1, we expect this relation to be indirect (i.e. come through $|OFI_{k,i}|$) and noisy. To confirm this empirically, we estimate three regressions¹⁷:

$$|\Delta P_{k,i}| = \hat{\alpha}_i^O + \hat{\beta}_i^O |OFI_{k,i}| + \hat{\epsilon}_{k,i}^O \quad (25a)$$

$$|\Delta P_{k,i}| = \hat{\alpha}_i^V + \hat{\beta}_i^V VOL_{k,i}^{\hat{H}_i} + \hat{\epsilon}_{k,i}^V \quad (25b)$$

$$|\Delta P_{k,i}| = \hat{\alpha}_i^W + \hat{\phi}_i^O |OFI_{k,i}| + \hat{\phi}_i^V VOL_{k,i}^{\hat{H}_i} + \hat{\epsilon}_{k,i}^W \quad (25c)$$

These regressions are estimated for every half-hour subsample with the exponents \hat{H}_i pre-estimated by (24). The averages of \hat{H}_i and their standard deviation for each stock are presented on the left panel in Table 5. The exponent varies considerably across stocks and time, but is generally below 1/2 in our data. The average results of regressions (25a-25c) for each stock are presented on the middle and right panels. We observe that $|OFI_{k,i}|$ explains the magnitude of price moves better than $VOL_{k,i}^{\hat{H}_i}$. Although both variables appear to be statistically significant when taken individually, the t-statistics for $VOL_{k,i}$ drop to marginally significant levels in the multiple regression. Thus, the dependence between absolute values of price moves and traded volume seems to come mostly from correlation between $VOL_{k,i}$ and $|OFI_{k,i}|$. Interestingly, the number of trades variable (suggested in [28]) is also statistically significant on a stand-alone basis, but becomes insignificant when added to (25c) as a third variable. Given the recent proliferation of order splitting, the size of most orders is equal to one lot, so $VOL_{k,i}$ is almost the same as the number of trades variable.

5 Conclusion

We have introduced *order flow imbalance*, a variable that cumulates the sizes of order book events, treating the contributions of market, limit and cancel orders equally, and provided empirical and theoretical evidence for a linear relation between high-frequency price changes and order flow imbalance for individual stocks. We have shown that this linear model is robust across stocks and timescales, and the price impact coefficient is inversely proportional to market depth. These relations suggest that prices respond to changes in the supply and demand for shares at the best quotes, and that the impact coefficient fluctuates with the amount of liquidity provision, or depth, in the market. Moreover, we have demonstrated that order flow imbalance is a more general metric of supply/demand dynamics than trade imbalance, and it can be used to analyze intraday changes in volatility, and monitor possible adverse selection in limit order executions. Trades seem to carry little to no information about price changes after the simultaneous order flow imbalance is taken into account. If trades do not help to explain price changes after controlling for the order flow imbalance, it is highly possible that the relation between the magnitude of price changes, or price volatility and traded volume simply captures the noisy scaling relation between these variables.

¹⁷Here we estimate linear regressions rather than log-linear ones to directly test whether the effect of VOL is consumed by $|OFI|$ variable

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A Appendix: TAQ data processing

We considered only quotes with timestamps $\in [9:30 \text{ am}, 4:00 \text{ pm}]$, positive bid/ask prices and sizes and quote mode $\notin \{4, 7, 9, 11, 13, 14, 15, 19, 20, 27, 28\}$. Similarly, trades were considered only if they had timestamps $\in [9:30 \text{ am}, 4:00 \text{ pm}]$, positive price and size, correction indicator ≤ 2 and condition $\notin \{ "O", "Z", "B", "T", "L", "G", "W", "J", "K" \}$.

From the filtered quotes data we construct the National Best Bid and Offer (NBBO) quotes. This is done by scanning through the filtered quotes data, while maintaining a matrix with the best quotes for every exchange. When a new entry is read, we check the exchange flag of that entry and update the corresponding row in the exchange matrix. Using this matrix, the NBBO prices are computed at each entry as the highest bid and the lowest ask across all exchanges. The NBBO sizes are simply the sums of all sizes at the NBBO bid and ask across all exchanges. For more details on TAQ dataset we refer the reader to [22], which discusses some particularities of that data, such as possible mis-sequencing of data across exchanges and lack of odd-lot sized orders. With our auxiliary dataset we checked that neither of these issues significantly affects our results.

After the NBBO quotes are computed, we applied a simple quote test to the NBBO quotes and the filtered trades data. This test matches trades with NBBO quotes and computes the direction of matched trades. A trade is matched with a quote, if:

1. Trade is not inside the spread, i.e.
 - (a) Trade price \geq NBBO ask: in this case the trade is considered to be a buy trade.
 - (b) Trade price \leq NBBO bid: in this case the trade is considered to be a sell trade.
2. Trade date = quote date.
3. Trade timestamp $\in [\text{quote timestamp}, \text{quote timestamp} + 1 \text{ second}]$.
4. If the above conditions allow to match a trade with several quotes, it is matched with the earliest quote.

This matching algorithm cannot identify the direction of trades occurring within the bid-ask spread. By comparing the number of matched trades with the overall number of trades in our sample, we found that 59-95% of trades depending on the stock cannot be matched. Although these percentages appear to be extremely large, the volume percentage of unmatched trades is only 10-39% depending on the stock with an average of 17% across stocks, and we believe that omitting these trades does not affect our results. There are other routines to estimate trade direction, including the tick test and the Lee-Ready rule [36]. Although the latter is used quite frequently, there seems to be no compelling evidence of superiority of either of these heuristics [41, 47]. To test the robustness of our findings to the choice of a trade direction test, we compared our results on a subsample of stocks, applying alternatively the tick test or our quote test and results were virtually the same.

Finally, we removed observations with high bid-ask spreads to filter out “stub quotes” and data errors. To apply this filter coherently across stocks, we computed the 95-th percentile of bid-ask spread distribution for each stock and removed 5% of that stock’s quotes with spreads above that percentile. For the representative stock in our sample (SLB), the removed observations fall mostly on the first minutes after market opening: 15.8% of them occur between 9:30 am and 9:35 am, and 42.1% of them occur between 9:30 am and 10:00 am. The average bid-ask spread of the removed quotes is 3.44 cents with a standard deviation 11.98 cents, the average queue size of these quotes is 11.78 round lots with a standard deviation 12.89 lots. The average time interval between two removed quotes is 1.03 seconds with a standard deviation 41.64 seconds. All of the results and tables in this paper are generated using the filtered data.

B Appendix: Robustness checks

B.1 Cross-sectional evidence

Table 1. Descriptive statistics

Name	Ticker	Price	Daily volume, round lots	Number of best quote updates	Number of trades	Average Spread, cents	Maximum spread, cents	Best quote size, round lots
Advanced Micro Devices	AMD	9.61	20872996	417204	6687	1	1	1035
Apollo Group	APOL	62.92	1949337	172942	4095	2	5	15
American Express	AXP	45.21	8678723	559701	7748	1	24	79
Autozone	AZO	179.03	243197	43682	1081	9	35	7
Bank of America	BAC	18.43	164550168	1529395	15008	1	1	3208
Becton Dickinson	BDX	78.07	1130362	61029	2968	2	5	15
Bank of New York Mellon	BK	31.77	6310701	285619	5518	1	1	122
Boston Scientific	BSX	7.13	25746787	309441	6768	1	1	2965
Peabody Energy corp	BTU	47.14	5210642	298616	7267	1	3	29
Caterpillar	CAT	67.20	6664891	392499	8224	1	2	38
Chubb	CB	52.22	1951618	149010	3601	1	2	43
Carnival	CCL	40.16	4275911	215427	5503	1	2	53
Cincinnati Financial	CINF	29.41	688914	51373	1528	1	2	42
CME Group	CME	322.83	418955	38504	1412	31	103	5
Coach	COH	41.91	3126469	176795	4458	1	2	41
ConocoPhillips	COP	56.09	9644544	426614	8621	1	2	84
Coventry Health Care	CVH	24.16	1157022	79305	2213	1	2	38
Denbury Resources	DNR	17.88	5737740	263173	4643	1	1	186
Devon Energy	DVN	66.98	3260982	177006	5805	2	4	18
Equifax	EFX	35.34	799505	62957	1945	1	3	39
Eaton	ETN	78.53	1757136	67989	3580	2	6	13
Fiserv	FISV	52.56	1038311	58304	2208	1	3	20
Hasbro	HAS	39.48	1322037	86040	2672	1	2	34
HCP	HCP	32.63	2872521	213045	4357	1	2	48
Starwood Hotels	HOT	50.59	3164807	150252	5106	2	4	22
Kohl's	KSS	56.88	3064821	128196	4936	1	3	27
L-3 Communications	LLL	94.64	670937	72818	2141	2	6	9
Lockheed Martin	LMT	84.14	1416072	88254	3333	2	5	15
Macy's	M	23.40	8324639	491756	6469	1	1	176
Marriott	MAR	34.45	5014098	238190	5499	1	2	65
McAfee	MFE	40.04	2469324	109073	3561	1	2	40
McGraw-Hill	MHP	34.90	1954576	102389	3261	1	2	42
Medco Health Solutions	MHS	63.22	2798098	109382	4680	1	3	25
Merck	MRK	36.03	13930842	448748	7997	1	1	231
Marathon Oil	MRO	32.33	5035354	341408	5522	1	1	143
MeadWestvaco	MWV	26.96	1035547	92825	2312	1	3	37
Newmont Mining	NEM	53.43	5673718	435295	7717	1	2	38
Omnicom	OMC	41.17	3357585	150800	4359	1	2	65
MetroPCS Communications	PCS	7.53	4424560	107967	2901	1	1	523
Pultegroup	PHM	11.80	6834683	262420	4604	1	1	319
PerkinElmer	PKI	23.98	1268774	78114	2127	1	2	72
Ryder System	R	44.01	631889	47422	2085	2	5	11
Reynolds American	RAI	54.44	773387	56236	2076	1	4	22
Schlumberger	SLB	67.94	9476060	440839	10286	1	2	39
Teco Energy	TE	16.52	1070815	70318	1807	1	1	148
Time Warner Cable	TWC	53.21	1770234	88286	3554	2	3	22
Whirlpool	WHR	97.73	1424264	134152	3348	4	9	10
Windstream	WIN	11.03	2508830	104887	2937	1	1	798
Watson Pharmaceuticals	WPI	42.51	895967	63094	2024	1	3	29
XTO Energy	XTO	48.13	7219436	612804	5040	1	7	225
Grand mean		51.75	7512376	223232	4552	2	6	227

Table 1 presents the average mid-price, daily transaction volume, daily number of best quote updates, daily number of trades, spread and the depth at the best bid and ask for 50 randomly chosen U.S. stocks. One round lot is equal to 100 shares. All values are calculated from the filtered data, that consists of 21 trading day during April, 2010.

Table 2. Relation between price changes and order flow imbalance.

Ticker	Average results							Hypothesis testing		
	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}_i$	$t(\hat{\beta}_i)$	$\hat{\gamma}_i^Q$	$t(\hat{\gamma}_i^Q)$	R^2	$\{\alpha_i \neq 0\}$	$\{\beta_i \neq 0\}$	$\{\gamma_i^Q \neq 0\}$
AMD	-0.0032	-0.24	0.0008	11.10	1.4E-07	0.93	64%	0%	100%	36%
APOL	0.0038	0.13	0.0555	10.74	-2.2E-04	-2.42	63%	17%	96%	6%
AXP	0.0019	0.11	0.0082	14.12	-3.8E-06	-1.37	69%	16%	100%	8%
AZO	0.0101	0.34	0.1619	7.02	-9.3E-04	-1.40	47%	25%	99%	6%
BAC	-0.0018	-0.13	0.0002	19.08	1.9E-09	-0.08	79%	3%	100%	14%
BDX	-0.0008	-0.07	0.0536	10.77	-1.1E-04	-0.74	63%	12%	100%	12%
BK	-0.0078	-0.26	0.0069	15.56	-4.0E-06	-0.89	74%	6%	100%	8%
BSX	0.0000	0.01	0.0003	7.55	7.8E-08	3.51	58%	3%	88%	51%
BTU	0.0048	0.15	0.0242	14.75	-3.5E-05	-2.05	72%	16%	100%	3%
CAT	0.0147	0.30	0.0194	14.80	-1.9E-05	-1.72	71%	19%	100%	5%
CB	-0.0086	-0.05	0.0191	12.61	-3.5E-07	-0.04	64%	10%	100%	18%
CCL	-0.0067	-0.24	0.0140	14.16	-1.2E-05	-1.03	70%	7%	100%	11%
CINF	-0.0030	-0.02	0.0260	11.66	-7.0E-06	0.38	70%	4%	99%	30%
CME	0.0506	0.06	0.6262	5.46	-7.2E-03	-1.66	35%	18%	96%	7%
COH	-0.0221	-0.54	0.0179	13.13	-1.7E-05	-1.18	69%	5%	100%	7%
COP	-0.0008	0.10	0.0084	12.79	-5.8E-06	-1.86	68%	13%	100%	5%
CVH	-0.0034	-0.07	0.0217	11.74	7.6E-06	0.37	65%	7%	99%	20%
DNR	-0.0008	-0.07	0.0045	13.78	-1.3E-07	0.19	69%	5%	99%	22%
DVN	0.0112	0.20	0.0370	12.11	-1.0E-04	-2.72	65%	19%	100%	2%
EFX	-0.0032	-0.06	0.0222	9.47	6.4E-05	0.87	56%	6%	99%	32%
ETN	-0.0076	0.10	0.0712	11.01	-2.3E-04	-1.81	65%	17%	100%	4%
FISV	-0.0002	0.10	0.0397	11.09	-2.3E-05	-0.28	63%	10%	100%	16%
HAS	-0.0031	-0.01	0.0222	12.36	4.7E-06	0.28	67%	6%	100%	23%
HCP	-0.0078	-0.21	0.0150	13.82	-1.4E-05	-0.63	67%	5%	100%	10%
HOT	-0.0012	0.05	0.0345	12.94	-7.2E-05	-2.06	68%	14%	100%	4%
KSS	-0.0030	-0.05	0.0317	14.10	-5.4E-05	-1.38	71%	13%	100%	5%
LLL	0.0160	0.42	0.1000	12.34	-3.8E-04	-1.56	67%	22%	98%	7%
LMT	0.0006	0.00	0.0520	14.14	-1.2E-04	-1.49	72%	17%	100%	4%
M	-0.0010	0.07	0.0043	16.61	8.8E-08	0.15	75%	6%	100%	19%
MAR	-0.0039	0.02	0.0121	15.10	-4.1E-06	-0.43	71%	10%	100%	10%
MFE	0.0087	0.22	0.0205	13.19	-3.8E-05	-0.63	68%	11%	100%	11%
MHP	-0.0073	-0.18	0.0211	12.41	5.8E-06	0.18	68%	5%	99%	24%
MHS	-0.0055	-0.20	0.0334	11.97	-8.3E-05	-1.64	66%	12%	100%	4%
MRK	-0.0065	-0.26	0.0032	13.26	-5.4E-07	-0.61	69%	4%	100%	14%
MRO	0.0018	0.12	0.0058	14.16	-3.6E-07	0.32	69%	8%	100%	23%
MWV	-0.0011	0.02	0.0205	12.55	-1.7E-05	-0.31	68%	9%	100%	17%
NEM	-0.0102	-0.26	0.0170	13.90	-1.9E-05	-2.15	71%	12%	100%	5%
OMC	-0.0099	-0.36	0.0144	12.40	-4.5E-06	-0.19	65%	4%	100%	20%
PCS	-0.0006	-0.05	0.0015	6.52	1.8E-06	3.79	53%	2%	86%	51%
PHM	0.0006	0.02	0.0027	11.27	8.4E-07	1.20	66%	3%	99%	36%
PKI	-0.0004	-0.05	0.0102	7.96	4.1E-05	2.15	53%	3%	96%	51%
R	0.0006	0.03	0.0667	10.90	3.7E-05	-0.21	63%	14%	100%	16%
RAI	-0.0070	-0.10	0.0396	11.39	2.6E-05	-0.03	66%	9%	100%	19%
SLB	-0.0077	-0.21	0.0198	16.27	-1.8E-05	-1.67	76%	10%	100%	2%
TE	0.0011	0.05	0.0049	7.76	1.4E-05	3.27	54%	4%	91%	55%
TWC	-0.0130	-0.15	0.0384	12.24	-5.6E-05	-0.73	64%	12%	99%	9%
WHR	0.0628	0.73	0.1278	11.10	-3.3E-04	-1.44	65%	25%	100%	7%
WIN	-0.0004	-0.04	0.0009	4.32	1.5E-06	3.98	44%	1%	72%	43%
WPI	-0.0090	-0.27	0.0270	11.46	2.9E-05	0.26	66%	5%	99%	23%
XTO	-0.0088	-0.25	0.0029	13.26	2.7E-07	0.48	65%	3%	100%	28%
Average	0.0002	-0.02	0.0398	12.08	-2.0E-04	-0.32	65%	10%	98%	17%

Table 2 presents a cross-section of results (averaged across time) for regressions:

$$\begin{aligned}\Delta P_{k,i} &= \hat{\alpha}_i + \hat{\beta}_i OFI_{k,i} + \hat{\epsilon}_{k,i}, \\ \Delta P_{k,i} &= \hat{\alpha}_i^Q + \hat{\beta}_i^Q OFI_{k,i} + \hat{\gamma}_i^Q OFI_{k,i}|OFI_{k,i}| + \hat{\epsilon}_{k,i}^Q,\end{aligned}$$

where $\Delta P_{k,i}$ are the 10-second mid-price changes in ticks and $OFI_{k,i}$ are the contemporaneous order flow imbalances. These regressions were estimated using 273 half-hour subsamples (indexed by i) for each stock and their outputs were averaged across subsamples. Each subsample typically contains about 180 observations (indexed by k). The t-statistics were computed using Newey-West standard errors. For brevity, we report the R^2 , the average $\hat{\alpha}_i$ and the average $\hat{\beta}_i$ only for the first regression (with a single $OFI_{k,i}$ term). There is almost no difference between averages of estimates $\hat{\beta}_i$ and $\hat{\beta}_i^Q$ and the R^2 in two regressions. The last three columns report the percentage of samples where the coefficient(s) passed the z-test at the 5% significance level.

Table 3. Comparison of order flow imbalance and trade imbalance.

Ticker	Order flow imbalance				Trade imbalance				Both covariates					
	R^2	$t(\beta_i)$	$\{\beta_i \neq 0\}$	F	R^2	$t(\beta_i^T)$	$\{\beta_i^T \neq 0\}$	F	R^2	$t(\theta_i^O)$	$t(\theta_i^T)$	$\{\theta_i^O \neq 0\}$	$\{\theta_i^T \neq 0\}$	F
AMD	64%	11.10	100%	382	39%	5.06	95%	140	67%	7.64	1.59	99%	45%	214
APOL	63%	10.74	96%	396	30%	5.04	95%	83	66%	8.95	1.58	96%	44%	211
AXP	69%	14.12	100%	449	34%	5.55	92%	101	71%	11.31	1.90	100%	55%	241
AZO	47%	7.02	99%	179	30%	4.88	96%	87	54%	5.78	2.87	98%	81%	118
BAC	79%	19.08	100%	774	45%	7.03	98%	157	80%	13.55	0.80	99%	25%	397
BDX	63%	10.77	100%	362	28%	4.85	92%	79	65%	8.90	1.53	100%	46%	195
BK	74%	15.56	100%	610	36%	5.36	93%	117	75%	11.90	0.80	100%	26%	313
BSX	58%	7.55	88%	338	31%	3.60	71%	106	62%	5.74	0.88	82%	24%	189
BTU	72%	14.75	100%	527	35%	6.03	97%	103	74%	11.96	1.63	100%	44%	277
CAT	71%	14.80	100%	498	33%	5.75	94%	94	72%	12.14	1.55	100%	46%	262
CB	64%	12.61	100%	378	33%	5.47	95%	102	66%	9.41	1.57	99%	44%	202
CCL	70%	14.16	100%	478	32%	5.31	94%	93	71%	11.44	1.17	100%	37%	247
CINF	70%	11.66	99%	552	39%	5.35	96%	141	72%	8.28	1.28	98%	40%	297
CME	35%	5.46	96%	112	24%	4.31	88%	63	44%	4.73	2.78	96%	71%	78
COH	69%	13.13	100%	457	29%	4.75	93%	80	70%	11.06	1.12	100%	31%	238
COP	68%	12.79	100%	450	35%	5.69	92%	107	70%	10.25	1.76	100%	49%	240
CVH	65%	11.74	99%	418	35%	5.05	93%	114	67%	8.43	1.35	97%	37%	222
DNR	69%	13.78	99%	471	32%	4.89	92%	101	70%	10.43	1.27	99%	37%	246
DVN	65%	12.11	100%	414	33%	5.57	95%	96	68%	9.61	2.12	98%	60%	226
EFX	56%	9.47	99%	289	31%	4.75	89%	101	60%	7.13	2.26	98%	55%	167
ETN	65%	11.01	100%	389	25%	4.43	86%	69	67%	9.85	1.47	99%	43%	209
FISV	63%	11.09	100%	380	28%	4.82	93%	79	65%	9.08	1.25	100%	38%	201
HAS	67%	12.36	100%	427	32%	5.15	95%	97	68%	9.67	1.17	100%	34%	223
HCP	67%	13.82	100%	417	31%	5.07	90%	91	68%	10.92	1.33	100%	42%	217
HOT	68%	12.94	100%	438	27%	4.75	88%	74	70%	11.00	1.48	100%	40%	231
KSS	71%	14.10	100%	525	31%	5.16	93%	91	72%	11.86	1.14	100%	37%	274
LLL	67%	12.34	98%	485	36%	6.00	95%	117	70%	9.68	2.14	98%	57%	270
LMT	72%	14.14	100%	516	35%	5.80	96%	105	73%	11.35	1.83	100%	51%	277
M	75%	16.61	100%	640	35%	5.10	93%	108	76%	12.80	1.13	100%	38%	330
MAR	71%	15.10	100%	498	34%	5.54	95%	105	72%	11.41	1.18	100%	36%	258
MFE	68%	13.19	100%	463	31%	4.82	88%	93	69%	10.27	0.89	100%	30%	239
MHP	68%	12.41	99%	489	31%	5.09	93%	96	70%	9.94	1.04	99%	33%	257
MHS	66%	11.97	100%	414	28%	4.81	89%	80	68%	10.03	1.50	99%	40%	218
MRK	69%	13.26	100%	451	31%	4.99	92%	93	70%	10.41	1.02	100%	29%	235
MRO	69%	14.16	100%	465	35%	5.38	96%	104	70%	10.67	1.12	100%	35%	241
MWV	68%	12.55	100%	452	34%	5.30	96%	102	69%	9.66	1.01	100%	33%	237
NEM	71%	13.90	100%	490	34%	5.77	92%	100	72%	11.38	1.90	100%	54%	260
OMC	65%	12.40	100%	411	30%	4.90	93%	88	67%	9.85	1.22	100%	39%	216
PCS	53%	6.52	86%	297	35%	4.08	74%	169	58%	4.47	1.43	81%	35%	195
PHM	66%	11.27	99%	416	35%	4.76	93%	115	68%	8.40	1.22	98%	38%	224
PKI	53%	7.96	96%	263	28%	3.98	82%	89	57%	6.16	1.70	93%	47%	148
R	63%	10.90	100%	352	27%	4.80	96%	71	65%	9.02	1.58	100%	44%	188
RAI	66%	11.39	100%	422	36%	5.60	98%	111	68%	8.64	1.42	100%	43%	224
SLB	76%	16.27	100%	644	32%	5.31	89%	94	77%	13.91	1.56	100%	47%	336
TE	54%	7.76	91%	301	37%	4.65	82%	175	60%	5.27	1.96	86%	45%	200
TWC	64%	12.24	99%	377	31%	5.21	86%	93	66%	9.67	1.70	99%	45%	201
WHR	65%	11.10	100%	394	29%	5.03	95%	85	67%	9.27	1.86	100%	52%	217
WIN	44%	4.32	72%	243	41%	4.74	75%	249	58%	2.60	2.55	58%	47%	206
WPI	66%	11.46	99%	437	32%	4.80	93%	100	68%	8.95	1.35	99%	46%	232
XTO	65%	13.26	100%	399	21%	3.78	78%	54	66%	11.72	1.42	100%	40%	209
Average	65%	12.08	98%	429	32%	5.08	91%	103	67%	9.53	1.51	97%	43%	231

Table 3 presents the average results of regressions:

$$\Delta P_{k,i} = \hat{\alpha}_i + \hat{\beta}_i OFI_{k,i} + \hat{\epsilon}_{k,i},$$

$$\Delta P_{k,i} = \hat{\alpha}_i^T + \hat{\beta}_i^T TI_{k,i} + \hat{\eta}_{k,i},$$

$$\Delta P_{k,i} = \hat{\alpha}_i^D + \hat{\theta}_i^O OFI_{k,i} + \hat{\theta}_i^T TI_{k,i} + \hat{\epsilon}_{k,i}^D,$$

where $\Delta P_{k,i}$ are the 10-second mid-price changes, $OFI_{k,i}$ are the contemporaneous order flow imbalances and $TI_{k,i}$ are the contemporaneous trade imbalances. These regressions were estimated using 273 half-hour subsamples (indexed by i) for each stock and their outputs were averaged across subsamples. Each subsample typically contains about 180 observations (indexed by k). The t-statistics were computed using Newey-West standard errors. For each of three regressions, Table 3 reports the average R^2 , the average t-statistic of the coefficient(s), the percentage of samples where the coefficient(s) passed the z-test at the 5% significance level and the F-statistic of the regression.

Table 4. Relation between the price impact coefficient and market depth.

Ticker	Parameter estimates						Fit measures		
	\hat{c}	$\hat{\lambda}$	$t(\hat{c} = 0)$	$t(\hat{c} = 0.5)$	$t(\hat{\lambda} = 0)$	$t(\hat{\lambda} = 1)$	R^2	$corr[\hat{\beta}, \hat{\beta}]^2$	$corr[\hat{\beta}, \hat{\beta}^*]^2$
AMD	0.53	1.04	31.06	2.0	22.5	1.0	78%	86%	86%
APOL	0.30	0.38	4.59	-3.2	1.1	-1.8	3%	34%	35%
AXP	0.45	1.01	26.27	-3.2	45.8	0.6	90%	87%	87%
AZO	0.45	0.70	5.47	-0.7	5.2	-2.2	14%	17%	16%
BAC	0.87	1.10	31.80	13.6	19.1	1.7	80%	89%	89%
BDX	0.48	1.03	23.91	-1.2	22.2	0.6	74%	71%	71%
BK	0.47	1.04	28.28	-1.9	68.5	2.5	94%	94%	94%
BSX	0.51	1.02	15.23	0.3	24.1	0.4	73%	81%	81%
BTU	0.58	1.10	45.36	6.2	53.8	5.0	93%	90%	90%
CAT	0.48	1.01	35.12	-1.4	20.7	0.2	91%	91%	90%
CB	0.53	1.09	32.41	2.1	63.6	5.5	93%	91%	91%
CCL	0.45	1.04	35.26	-3.6	41.9	1.5	89%	86%	86%
CINF	0.43	1.03	25.48	-4.2	52.5	1.7	93%	90%	90%
CME	1.21	0.35	2.10	1.2	1.4	-2.7	1%	2%	2%
COH	0.61	1.11	15.35	2.9	44.7	4.3	81%	83%	82%
COP	0.32	0.94	13.77	-8.1	22.6	-1.6	82%	79%	79%
CVH	0.54	1.13	26.92	2.2	37.9	4.2	88%	90%	89%
DNR	0.55	1.10	40.77	3.6	44.9	3.9	92%	90%	90%
DVN	0.34	0.91	16.15	-7.8	19.3	-2.0	48%	61%	61%
EFX	0.43	1.05	19.58	-3.0	27.1	1.2	84%	80%	80%
ETN	0.64	1.11	13.55	2.9	20.8	2.1	65%	63%	63%
FISV	0.47	1.04	25.33	-1.7	34.1	1.3	85%	80%	80%
HAS	0.52	1.08	27.80	1.3	49.8	3.8	90%	86%	85%
HCP	0.37	1.00	33.13	-11.3	64.7	0.0	95%	94%	94%
HOT	0.61	1.13	28.19	5.2	37.7	4.3	87%	87%	87%
KSS	0.59	1.09	28.99	4.3	41.7	3.4	90%	85%	85%
LLL	0.57	1.02	15.30	1.9	14.5	0.3	53%	65%	65%
LMT	0.72	1.17	7.93	2.4	15.6	2.3	69%	63%	63%
M	0.52	1.06	24.92	1.0	52.1	3.0	94%	92%	92%
MAR	0.50	1.06	22.26	0.0	52.7	3.1	92%	89%	89%
MFE	0.47	1.06	22.12	-1.3	45.3	2.7	92%	89%	89%
MHP	0.45	1.02	20.58	-2.1	38.1	0.6	83%	78%	78%
MHS	0.71	1.16	19.88	5.9	39.5	5.3	88%	87%	86%
MRK	0.31	0.94	21.38	-12.8	36.3	-2.3	87%	84%	84%
MRO	0.55	1.09	28.87	2.4	51.9	4.2	94%	94%	94%
MWV	0.54	1.13	28.16	2.2	39.3	4.6	90%	87%	87%
NEM	0.51	1.07	31.09	0.4	39.3	2.6	89%	88%	88%
OMC	0.52	1.04	36.61	1.1	19.5	0.7	86%	90%	90%
PCS	0.43	1.06	22.79	-3.4	18.6	1.1	53%	83%	83%
PHM	0.62	1.10	39.56	7.7	36.6	3.3	87%	92%	92%
PKI	0.49	1.14	29.01	-0.5	34.7	4.4	80%	87%	86%
R	0.50	1.05	17.43	-0.1	15.8	0.7	58%	59%	59%
RAI	0.51	1.07	26.19	0.4	47.3	3.1	88%	79%	79%
SLB	0.56	1.08	23.39	2.5	47.6	3.6	92%	94%	93%
TE	0.35	1.10	12.12	-5.1	25.1	2.2	70%	85%	86%
TWC	0.55	1.07	22.29	1.9	18.9	1.2	73%	85%	84%
WHR	1.09	1.25	12.66	6.9	13.4	2.7	51%	54%	53%
WIN	17.21	1.80	13.95	13.5	12.2	5.4	35%	72%	74%
WPI	0.39	0.99	19.57	-5.6	32.6	-0.4	79%	77%	77%
XTO	0.97	1.19	27.70	13.46	35.64	5.77	88%	91%	90%
Grand mean	0.88	1.05	23.55	0.59	33.40	1.99	76%	79%	79%

Table 4 presents the results of regressions:

$$\log \hat{\beta}_i = \alpha_{L,i} - \hat{\lambda} \log D_i + \hat{\epsilon}_{L,i},$$

$$\hat{\beta}_i = \alpha_{M,i} + \frac{\hat{c}}{D_i^{\hat{\lambda}}} + \hat{\epsilon}_{M,i},$$

where $\hat{\beta}_i$ is the price impact coefficient for the i -th half-hour subsample and D_i is the average market depth for that subsample. These regressions were estimated for each of the 50 stocks, using 273 estimates of $\hat{\beta}_i$ for that stock, obtained from (14). The second regression uses estimates $\hat{\lambda}$ obtained from the first regression. The t-statistics were computed using Newey-West standard errors. The last three columns provide three alternative fit measures - the R^2 of the linear regression (17), the squared correlation between $\hat{\beta}_i$ and fitted values $\hat{\beta}_i = \frac{\hat{c}}{D_i^{\hat{\lambda}}}$ and the squared correlation between $\hat{\beta}_i$ and $\hat{\beta}_i^* = \frac{\hat{c}}{D_i}$.

Table 5. Comparison of traded volume and order flow imbalance.

Ticker	Avg \hat{H}	Stddev \hat{H}	Order flow imbalance				Traded volume				Both covariates					
			R^2	$t(\beta_i^O)$	$\beta_i^O \neq 0$	F	R^2	$t(\beta_i^V)$	$\beta_i^V \neq 0$	F	R^2	$t(\phi_i^O)$	$t(\phi_i^V)$	$\phi_i^O \neq 0$	$\phi_i^V \neq 0$	F
AMD	0.06	0.08	63%	11.7	100%	356	14%	4.6	87%	34	63%	10.8	1.2	99%	38%	182
APOL	0.24	0.08	53%	9.1	97%	258	25%	6.9	100%	63	57%	7.6	3.3	94%	86%	144
AXP	0.16	0.08	55%	11.3	100%	249	20%	6.8	100%	48	57%	9.7	2.9	100%	82%	133
AZO	0.43	0.22	39%	6.3	98%	131	32%	5.8	100%	93	50%	5.0	3.9	97%	98%	98
BAC	0.09	0.08	73%	17.6	100%	560	24%	6.0	89%	61	74%	15.3	1.3	97%	40%	285
BDX	0.26	0.10	55%	9.4	100%	261	27%	6.5	100%	71	58%	7.6	3.1	99%	85%	147
BK	0.11	0.07	68%	14.1	100%	437	19%	6.7	97%	46	68%	12.6	2.0	100%	58%	225
BSX	-0.17	2.41	68%	10.3	100%	486	14%	3.4	97%	33	69%	10.1	0.0	99%	13%	246
BTU	0.24	0.07	58%	11.4	100%	283	23%	7.1	99%	57	60%	9.7	2.6	100%	81%	151
CAT	0.22	0.07	56%	11.0	100%	250	19%	6.3	99%	44	57%	9.7	2.3	100%	68%	131
CB	0.19	0.09	56%	11.1	100%	261	23%	6.5	99%	58	58%	9.1	2.8	100%	76%	141
CCL	0.14	0.07	60%	12.2	100%	309	19%	6.7	99%	45	62%	10.8	2.5	100%	77%	162
CINF	0.13	0.12	67%	12.0	100%	505	30%	6.2	98%	85	69%	10.3	2.1	100%	58%	268
CME	0.49	0.24	28%	4.8	98%	78	30%	5.3	100%	83	42%	3.9	4.1	94%	99%	71
COH	0.19	0.07	60%	11.3	100%	299	22%	6.5	99%	52	61%	9.8	2.4	100%	73%	157
COP	0.16	0.07	56%	10.5	100%	277	20%	6.1	97%	49	58%	9.2	2.5	100%	74%	145
CVH	0.18	0.10	62%	11.4	100%	352	27%	6.1	100%	72	64%	9.2	2.4	100%	73%	189
DNR	0.08	0.07	64%	13.4	100%	376	17%	6.4	95%	38	65%	12.0	1.9	99%	57%	193
DVN	0.26	0.07	52%	9.6	97%	236	24%	6.9	100%	59	55%	8.0	3.2	96%	85%	131
EFX	0.20	0.11	52%	9.1	100%	241	26%	5.6	99%	69	56%	7.3	2.8	99%	77%	137
ETN	0.26	0.10	55%	9.1	99%	252	27%	6.6	99%	70	58%	7.6	3.1	98%	85%	142
FISV	0.19	0.11	57%	10.1	100%	284	25%	6.0	100%	65	59%	8.3	2.4	100%	70%	153
HAS	0.20	0.09	61%	11.3	100%	328	26%	6.3	100%	67	63%	9.5	2.5	100%	76%	175
HCP	0.14	0.07	57%	11.8	100%	268	21%	7.1	99%	50	59%	10.0	2.8	100%	80%	143
HOT	0.23	0.08	57%	10.5	99%	263	24%	7.2	100%	60	60%	9.0	3.2	99%	88%	145
KSS	0.24	0.08	60%	11.6	100%	318	25%	6.8	99%	61	62%	9.8	2.6	99%	78%	169
LLL	0.33	0.12	58%	10.3	97%	323	34%	7.2	100%	101	63%	7.9	3.4	96%	92%	188
LMT	0.28	0.09	61%	11.6	100%	327	31%	7.6	100%	85	64%	9.3	3.1	100%	85%	182
M	0.11	0.07	69%	15.2	100%	463	20%	6.3	100%	46	69%	13.5	2.0	100%	63%	238
MAR	0.15	0.07	61%	13.3	100%	324	21%	7.0	99%	50	62%	11.5	2.5	100%	74%	170
MFE	0.16	0.09	60%	11.7	100%	318	24%	7.0	98%	62	62%	9.7	2.6	100%	73%	170
MHP	0.20	0.10	62%	11.6	100%	377	25%	6.1	100%	62	64%	9.7	2.0	100%	56%	199
MHS	0.23	0.08	56%	10.0	100%	258	24%	6.7	100%	58	58%	8.4	2.9	100%	80%	139
MRK	0.10	0.07	62%	12.1	100%	330	17%	5.5	99%	40	63%	10.8	1.9	100%	60%	170
MRO	0.09	0.06	61%	12.7	100%	333	16%	6.4	97%	36	63%	11.5	2.0	100%	56%	172
MWV	0.18	0.10	62%	11.3	100%	330	28%	6.9	100%	75	64%	9.2	2.6	100%	79%	180
NEM	0.20	0.07	56%	10.6	100%	253	20%	6.3	99%	47	58%	9.3	2.6	100%	79%	135
OMC	0.15	0.09	57%	11.0	100%	286	20%	6.4	98%	48	59%	9.4	2.5	100%	75%	151
PCS	0.11	0.18	62%	8.9	100%	411	18%	3.8	98%	54	63%	8.4	0.8	100%	28%	214
PHM	0.07	0.08	64%	11.5	100%	384	15%	5.4	91%	34	65%	10.7	1.2	100%	41%	195
PKI	0.11	0.11	55%	9.0	99%	266	20%	4.8	98%	47	57%	7.8	1.9	98%	55%	141
R	0.27	0.11	56%	9.8	99%	259	28%	6.3	100%	74	59%	7.9	3.1	99%	87%	147
RAI	0.25	0.10	61%	10.6	100%	334	28%	5.9	99%	73	63%	8.8	2.6	100%	75%	182
SLB	0.24	0.07	62%	12.5	99%	330	19%	5.8	97%	46	63%	11.2	1.9	99%	56%	171
TE	0.09	1.69	60%	9.6	100%	371	18%	4.5	84%	48	61%	8.7	1.3	99%	43%	196
TWC	0.25	0.10	55%	10.5	100%	253	27%	6.8	100%	73	58%	8.4	3.1	99%	83%	142
WHR	0.34	0.11	56%	9.2	99%	272	29%	6.6	100%	78	59%	7.5	3.2	98%	88%	156
WIN	0.06	0.26	48%	5.5	86%	340	10%	2.9	50%	34	49%	5.3	0.6	85%	31%	179
WPI	0.22	0.10	61%	11.0	100%	361	28%	5.9	100%	75	64%	9.0	2.4	99%	71%	196
XTO	0.08	0.06	53%	11.3	100%	238	15%	6.6	100%	32	55%	10.0	2.8	100%	82%	125
Average	0.18	0.18	58%	10.9	99%	313	23%	6.1	97%	58	61%	9.3	2.4	99%	70%	168

Table 5 presents the average results of regressions:

$$\begin{aligned}
|\Delta P_{k,i}| &= \hat{\alpha}_i^O + \hat{\beta}_i^O |OFI_{k,i}| + \hat{\epsilon}_{k,i}^O, \\
|\Delta P_{k,i}| &= \hat{\alpha}_i^V + \hat{\beta}_i^V VOL_{k,i}^{\hat{H}_i} + \hat{\epsilon}_{k,i}^V, \\
|\Delta P_{k,i}| &= \hat{\alpha}_i^W + \hat{\phi}_i^O |OFI_{k,i}| + \hat{\phi}_i^V VOL_{k,i}^{\hat{H}_i} + \hat{\epsilon}_{k,i}^W,
\end{aligned}$$

where $\Delta P_{k,i}$ are the 10-second mid-price changes, $OFI_{k,i}$ are the contemporaneous order flow imbalances and $VOL_{k,i}$ are the contemporaneous trade volumes. The exponents \hat{H}_i were estimated in each subsample beforehand using a logarithmic regression: $\log |\Delta P_{k,i}| = \log \hat{\theta}_i + \hat{H}_i \log VOL_{k,i} + \log |\hat{\xi}_{k,i}|$. These regressions were estimated using 273 half-hour subsamples (indexed by i) for each stock and their outputs were averaged across subsamples. Each subsample typically contains about 180 observations (indexed by k). The t-statistics were computed using Newey-West standard errors. For each of three regressions, Table 5 reports the average R^2 , the average t-statistic of the coefficient(s), the percentage of samples where the coefficient(s) passed the z-sest at the 5% significance level and the F-statistic of the regression.

B.2 Transaction prices

To reconcile our results with earlier studies that operate in transaction time, we repeated regressions (14,16a,16b) with differences between transaction prices $\Delta_L P_k^t = P_k^t - P_{k-L}^t$ for L trades, instead of differences in mid-prices ΔP_k . We picked at random five stocks from our sample (BDX, CB, MHS, PHM and PKI), and computed $\Delta_L P_k^t$ for $L = 2, 5, 10$ trades (we avoided using $L = 1$ because of possible issues with trade and quote matching). Using the same inter-trade time intervals we computed concurrent OFI and TI variables. To ensure that there is an ample amount of data for each regression, we pooled data across days for each stock and each intraday time interval, resulting in 13 samples for each stock over a month of data. The results averaged across time and stocks are presented in Table 6 and closely mirror our results for mid prices. The variable OFI_k explains price changes better than TI_k on stand-alone basis. Moreover, the effect of trades on prices seems to be captured by the order flow imbalance, i.e. the variable TI_k loses its statistical significance¹⁸. when used together with OFI_k in the regression. The increase in R^2 from adding TI_k as an extra regressor is almost nill (0.65%, 0.18%, 0.24% for $L = 1, 2, 5$ respectively).

Interestingly, we found that the relation between trade price changes and OFI_k (or TI_k) is sometimes concave. We estimated regressions (14) and (16a) for trade price changes $\Delta_L P_k^t$ with additional quadratic variable $OFI_k|OFI_k|$ and found that average t-statistics of its coefficient are, respectively -3.02, -4.10 and -3.85 for $L = 1, 2, 5$ trades. The quadratic term is significant at 5% level in 60%, 74% and 85% of samples for respective values of L , and we did not observe any pattern in these t-statistics, neither across stock nor across time. In the trade imbalance regression the coefficient near quadratic variable $TI_k|TI_k|$ is also significant with average t-statistics -3.64, -5.53, -5.48 for respective lag values and it is significant in even a larger fraction of samples.

From these results it appears that price impact is concave when prices are sampled at trade times, but it is linear when they are sampled at regular time intervals. This effect may be a consequence of sampling data at special times (i.e. trade times), which may introduce systematic down biases into the dependent variable. For example, if traders submit large orders when they expect their impact to be minimal, that would lead to a concave (sublinear) price impact. Supporting the idea of a sampling bias, we found that when mid-price changes are sampled at trade times, the price impact of OFI_k is again concave in a substantial fraction of our samples. We also regressed changes in last trade prices sampled regularly at a 1-minute frequency on OFI_k , and observed concave price impact once again. This may again be attributed to a dependent variable bias - since trades are relatively infrequent, for many time intervals the trade prices are going to be stale and trade price changes are equal to zero, while mid price changes are not.

Table 6. Comparison of order flow imbalance and trade imbalance for transaction prices.

Lag	Order flow imbalance				Trade imbalance				Both covariates					
	R^2	$t(\hat{\beta}_i)$	$\{\beta_i \neq 0\}$	F	R^2	$t(\hat{\beta}_i^T)$	$\{\beta_i^T \neq 0\}$	F	R^2	$t(\hat{\theta}_i^O)$	$t(\hat{\theta}_i^T)$	$\{\theta_i^O \neq 0\}$	$\{\theta_i^T \neq 0\}$	F
$L = 2$ trades	14%	15.03	100%	464	1%	2.97	69%	26	15%	14.19	-2.90	100%	71%	245
$L = 5$ trades	38%	16.68	98%	753	8%	4.79	88%	113	39%	15.13	-0.14	98%	14%	379
$L = 10$ trades	51%	14.85	98%	655	13%	4.85	88%	100	51%	13.21	0.70	98%	11%	329

¹⁸Here we also use Newey-West standard errors because regression residuals have statistically significant autocorrelation

B.3 Order flow at higher order book levels

The level of detail in our Level 2 auxiliary data set allows us to analyze contributions of order flows at different price levels to price formation and to confirm our claim that price changes are mostly driven by activity at the top of the order book (thus Level 1 data is sufficient to study the impact of limit orders on prices).

For example, consider the bid side of the order book with 10 shares at the top two levels. Absent any activity on the ask side and the second bid level, an OFI of -11 shares will lead to a bid price change of -1 tick. However, if 9 orders at the second bid level cancel before that order flow happens, the same OFI of -11 shares will lead to a price change of -2 ticks. In other words, if order activity up to second (third, fourth etc) level is important, tracking OFI only at the best prices will give a flawed picture of price dynamics. To test this assertion, we compute variables $OFI^m, m = 2, \dots, 5$ from m -th level queue fluctuations similarly to (12) and relabel $OFI^1 = OFI$. Then we fit five regressions, similar to (14), where variables $OFI^m, m = 2, \dots, 5$ are added one at a time:

$$\Delta P_{k,i} = \hat{\alpha}_i^M + \sum_{m=1}^M \hat{\beta}_i^{m,M} OFI_{k,i}^m + \hat{\epsilon}_{k,i}^M, \quad M = 1, \dots, 5 \quad (26)$$

The average results across time for a representative stock are shown on Figure 11. The average increase in explanatory power (measured by R^2) from adding OFI^2 as a regressor is 6.22%, which is quite small compared to the stand-alone R^2 of 70.83% for OFI^1 . The effect of $OFI^3 - OFI^5$ is very small, and their coefficients appear to be only marginally significant, in contrast with those of OFI^1 and OFI^2 . The cross-time average of coefficients $\hat{\beta}_i^{1,1}$ in the simple regression¹⁹ with OFI^1 is 0.0597. In the multiple regression with OFI^1 and OFI^2 the averages of their respective coefficients are 0.0673 and 0.0406. We conclude that second-level activity, as summarized by OFI^2 , has only a second-order influence on price changes, which are mainly driven by OFI^1 . The effect of $OFI^3 - OFI^5$ is almost null.

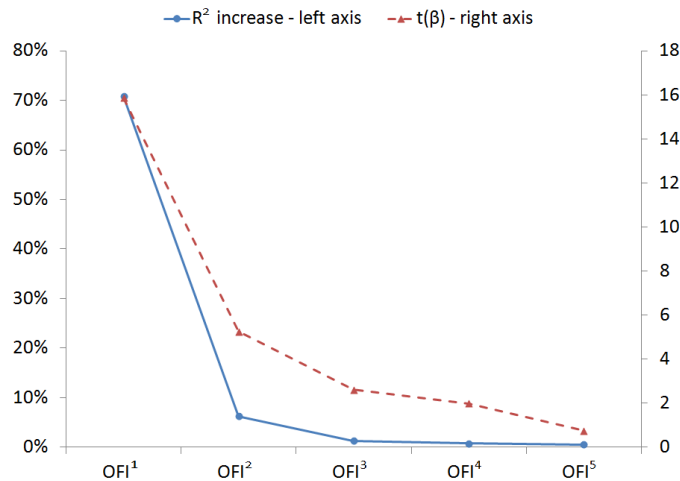


Figure 11: Cross-time average increase in R^2 from inclusion of variables $OFI^2 - OFI^5$, and cross-time average Newey-West t-statistics of their coefficient in the regression with all five variables, with NASDAQ ITCH data for the Schlumberger stock (SLB).

¹⁹This coefficient is higher than the one obtained with NBBO data, because NASDAQ best quote depth is smaller than NBBO depth

B.4 Choice of timescale

Using the auxiliary Level 2 dataset, we verify that our results are robust to potential issues in TAQ data, namely odd-lot sized orders at the best bid and offer, and mis-sequencing in quote data across exchanges during NBBO construction. We also compare our results across a wide range of timescales. The auxiliary data comes from a single exchange (NASDAQ), has information on orders of all sizes and has timestamps up to a millisecond.

We estimate the regression (14) for a variety of timescales Δt , ranging from 50 milliseconds to 5 minutes using separate intraday subsamples as before. The size of these samples was different in order to stabilize the number of observations per sample. More precisely, data for the smallest timescales (50, 100 and 500 milliseconds) was separated into 1-minute instead of 30-minute subsamples to make numerical computations feasible. Data for the largest timescales (30 seconds to 5 minutes) was pooled across days preserving separate 30-minute intraday intervals to have a large number of observations per sample. The average R^2 and Newey-West t-statistics for OFI across time for each Δt are presented on Figure 12.

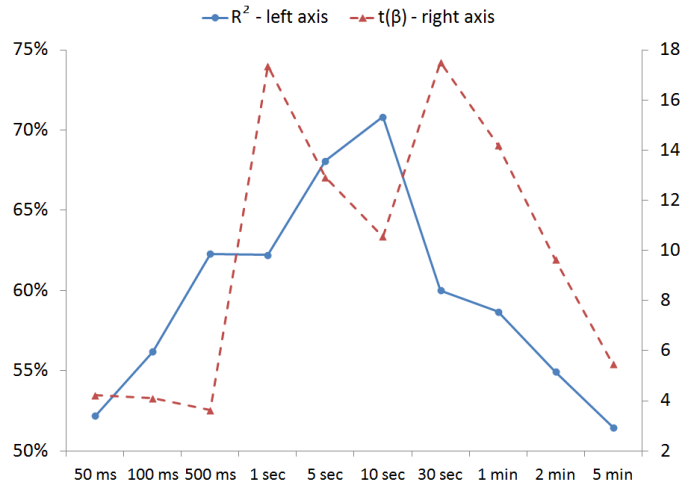


Figure 12: Average R^2 and Newey-West t-statistics for OFI coefficient across time for different Δt , with NASDAQ ITCH data for the Schlumberger stock (SLB).

The goodness of fit is stable across Δt , despite pronounced discreteness of data for very short time intervals. The OFI variable is statistically significant at a 95% level²⁰ in more than 80% of samples for Δt below one second, 100% of samples for Δt between one second and 2 minutes, and 92% of samples for Δt equal 5 minutes.

Notably there are many large price changes even when we consider Δt equal to 50 milliseconds, but they usually correspond to high values of OFI . This is consistent with findings in [23], where authors describe the sporadic character of order activity in modern markets. When a subset of traders reacts to market updates in a matter of several milliseconds, this creates short intervals of increased activity with possibly large price changes and large OFI , and many time intervals with no activity when both variables are equal to zero. From our findings it appears that the simple model (7) can capture both of these regimes.

When a quadratic term $\hat{\gamma}_i^Q OFI_{k,i} |OFI_{k,i}|$ is added to the regression, the coefficient $\hat{\gamma}_i^Q$ is significant in a handful of samples (10 out of 871) for Δt bigger or equal to one second. For Δt under one second, the quadratic term is significant in about 16% of samples, and its contribution

²⁰using Newey-West t-statistics

is marginal (about 3% increase in average R^2). We conclude that the relation between price changes and OFI is linear, irrespective of a timescale.

C Appendix: Proof of Proposition 1

Proposition 1: *Assume that*

1. $\frac{N(T)}{T} \rightarrow \Lambda$, as $T \rightarrow \infty$, where Λ is the average arrival rate of order book events.
2. $\{e_i\}_{i \geq 1}$ form a covariance-stationary sequence and have a linear-process representation $e_i = \sum_{j=0}^{\infty} a_j Y_{i-j}$, where Y_i is a two-sided sequence of i.i.d random variables with $E[Y_i] = 0$ and $E[Y_i^2] = 1$, and a_j is a sequence of constants with $\sum_{j=0}^{\infty} a_j^2 = \sigma^2 < \infty$. Moreover, $\text{cov}(e_1, e_{1+n}) \sim cn^{2(H-1)}$ as $n \rightarrow \infty$, where $0 < H < 1$ is a constant that governs the decay of the autocorrelation function.
3. $\{w_i\}_{i \geq 1}$, $w_i = b_i + s_i$ are random variables with a finite mean $\mu\pi$, where π is the proportion of order book events that correspond to trades and μ is the mean trade size. $E|w_i|^p < \infty$ for some $p > 1$ and $\sum_{N \geq 1} \frac{1}{N} (E|\frac{1}{N} \sum_{i \leq N} w_i|^q)^{r/q} < \infty$ for some r, q such that $0 < r \leq q \leq \infty$ and $r/q \leq 1 - 1/p$.

$$\text{Then } \frac{(\mu\pi)^H}{\sigma} \frac{OFI(T)}{VOL^H(T)} \xrightarrow{T \rightarrow \infty} \xi \sim N(0, 1)$$

where \Rightarrow denotes convergence in distribution.

Proof: First, we note that Assumption (1) ensures $N(T) \rightarrow \infty$ as $T \rightarrow \infty$. With this we can use Assumption (3) and apply the law of large numbers for weakly dependent variables (e.g. see Theorem 7 in [37]) to the traded volume.

$$\frac{VOL(T)}{N(T)} = \frac{\sum_{i=1}^{N(T)} w_i}{N(T)} \rightarrow \mu\pi, w.p.1, \text{ as } T \rightarrow \infty, \quad (27)$$

Second, event contributions e_i have a finite variance σ^2 and, using Assumption (2), we apply a central limit theorem for strongly dependent sequences (see Chapter 4.6 in [51]):

$$\frac{OFI(T)}{\sigma N^H(T)} \equiv \frac{\sum_{i=1}^{N(T)} e_i}{\sigma N^H(T)} \Rightarrow \xi, \text{ as } T \rightarrow \infty, \quad (28)$$

where $\xi \sim N(0, 1)$ is a standard normal random variable. Although the denominator $\sigma N^H(T)$ is random, it goes to infinity by assumption (1) and Anscombe's lemma ensures that we can use such a normalization in the central limit theorem [13, Lemma 2.5.8]. Since the function $g(x) = x^H$, $H > 0, x \geq 0$ is continuous, the convergence in (27) takes place almost-surely and the limit in (27) is deterministic, we can combine (27) and (28) in the following way:

$$\frac{(\mu\pi)^H}{\sigma} \frac{OFI(T)}{VOL^H(T)} \equiv \frac{\frac{\sum_{i=1}^{N(T)} e_i}{\sigma N^H(T)}}{\left(\frac{\sum_{i=1}^{N(T)} w_i}{\mu\pi N(T)}\right)^H} \Rightarrow \xi, \text{ as } T \rightarrow \infty \quad (29)$$

■