NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2018-2019

EE6203 – COMPUTER CONTROL SYSTEMS

November/December 2018 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 8 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed-book examination.
5. The Transform Table is included in Appendix A on pages 7 to 8.

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1. (a) Consider the following sampled-data signal:

$$x(kT) = \begin{cases} kT, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

where T is the sampling period and k is an integer. If

$$y(kT) = \sum_{h=0}^{k} x(hT),$$

determine the Z-transform of $y(kT)$ and verify the Initial Value Theorem.

(6 Marks)

(b) Suppose that $x(kT)$ is the sampled signal from $x(t)$ with sampling period T=1 second and its Z-transform is given as

$$X(z) = \frac{z}{(z-1)(z^2 - \sqrt{2}z + 1)}.$$

Find $x(kT)$.

(11 Marks)

Note: Question No. 1 continues on page 2

(c) Based on the Final Value Theorem, a student found that $x(kT)$ in Question 1(b) satisfies

$$\lim_{k\to\infty} x(kT) = \frac{1}{2} + \frac{\sqrt{2}}{2}.$$

Justify, with reasons, whether the answer from the student is correct.

(3 Marks)

2. Consider the control system shown in Figure 1.

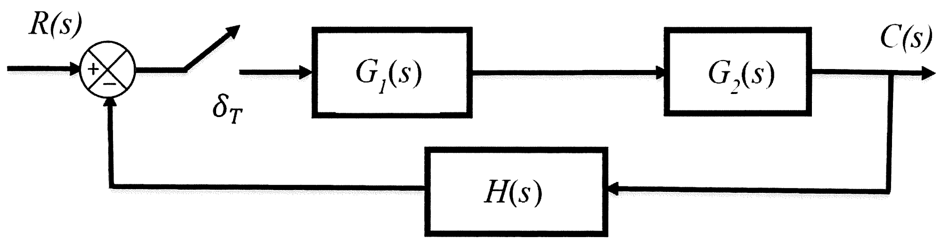

Figure 1

(a) Suppose that

$$G_1(s) = \frac{1-e^{-Ts}}{s}, \quad G_2(s) = \frac{\alpha}{s+1} \text{ where } \alpha \text{ is a constant.}$$

If $H(s) = 1$, the sampling period T is 0.1 second and the input $r(t)$ is a unit-step function, determine the Z-transform of the system output.

(11 Marks)

(b) Find the range of α in Question 2(a) to ensure the stability of the system.

(5 Marks)

(c) If the reference input $r(t) = t$ for $t \geq 0$, justify whether there exists a constant α such that $\lim_{k\to\infty} |e(kT)| \leq 0.1$, where $e(kT) = r(kT) - c(kT)$.

(4 Marks)

3. (a) The dynamics of a room temperature control system are given by the following system:

$$\begin{bmatrix} \dfrac{dx_1(t)}{dt} \\ \dfrac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} -10 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} -2 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where $x_1(t)$ and $x_2(t)$ are the states, $u(t)$ and $y(t)$ are the input and output variables, respectively. The system is sampled with a zero-order hold at a sampling period $T = 0.1$ second.

(i) Determine a discretised state-space model for the system.

(ii) Where are the poles of the discrete-time system located?

(10 Marks)

(b) A discrete-time system has a state-space representation given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \mathbf{C} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Give a matrix $\mathbf{C}$ such that the system is observable and has a single variable output $y(k)$.

(5 Marks)

(c) Consider a system which is described by the following state equation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k)$$

and with an associated performance index given by

$$J = \frac{1}{2} \sum_{k=0}^{\infty} \left(\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k) + ru^2(k) \right)$$

Note: Question No. 3 continues on page 4

The control law that minimises J is of the following form:

$$u^*(k) = -\mathbf{K}\mathbf{x}(k)$$

where the optimal control gain is given by

$$\mathbf{K} = \left(\mathbf{B}^T\mathbf{S}\mathbf{B} + r\right)^{-1} \mathbf{B}^T\mathbf{S}\,\mathbf{A}$$

and $\mathbf{S} > 0$ solves the following equation:

$$\mathbf{S} = \mathbf{A}^T\,\mathbf{S}\,\mathbf{A} + \mathbf{Q} - \mathbf{A}^T\,\mathbf{S}\mathbf{B}\left(r + \mathbf{B}^T\mathbf{S}\mathbf{B}\right)^{-1} \mathbf{B}^T\mathbf{S}\,\mathbf{A}$$

Now, consider the following system and associated performance index:

$$x(k+1) = x(k) + u(k)$$

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left(qx^2(k) + 8u^2(k)\right)$$

If the optimal control law that minimises J is

$$u^*(k) = -0.5x(k)$$

determine q. Determine also the location of the closed-loop pole.

(5 Marks)

4. (a) A discrete-time system has a state-space representation given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} e^{-T} & 0 \\ 1-e^{-T} & 1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1-e^{-T} \\ T-1+e^{-T} \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

where $\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the states, input and output variables, respectively. If the sampling period T is 1 second, design the deadbeat state-feedback controller.

(7 Marks)

Note: Question No. 4 continues on page 5

(b) A discrete-time system has a state-space representation given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.78 & 0 \\ 0.22 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.22 \\ 0.03 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

where $\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the states, input and output variables, respectively. Design a full-order prediction estimator of the following form:

$$\overline{\mathbf{x}}(k+1) = \mathbf{A}\overline{\mathbf{x}}(k) + \mathbf{B}u(k) + \mathbf{L}_o\left(y(k) - \mathbf{C}\overline{\mathbf{x}}(k)\right)$$

such that the desired estimator poles are at $z_{1,2} = 0$. Verify that the error response is indeed deadbeat with the designed estimator.

(7 Marks)

(c) A discrete-time system has a state-space representation given by

$$\mathbf{x}(k+1) = \mathbf{A}\,\mathbf{x}(k) + \mathbf{B}\,u(k)$$
$$y(k) = \mathbf{C}\,\mathbf{x}(k)$$

where $\mathbf{x}(k)$, $u(k)$ and $y(k)$ are the state vector, input and output variables, respectively. The following observer is employed to give an estimate of the state vector $\overline{\mathbf{x}}(k)$:

$$\overline{\mathbf{x}}(k) = \mathbf{A}\overline{\mathbf{x}}(k-1) + \mathbf{B}u(k-1) + \mathbf{L}_o\left\{y(k) - \mathbf{C}\left(\mathbf{A}\overline{\mathbf{x}}(k-1) + \mathbf{B}u(k-1)\right)\right\}$$

If the error vector is defined as

$$\mathbf{x}_e(k) = \mathbf{x}(k) - \overline{\mathbf{x}}(k)$$

obtain an equation for the error dynamics of the observer in the following form:

$$\mathbf{x}_e(k+1) = \mathbf{A}_e \mathbf{x}_e(k)$$

i.e. obtain an expression for $\mathbf{A}_e$ in terms of **A, B, C** and $\mathbf{L}_o$, where applicable.

(6 Marks)

5. (a) Consider the following second-order analog filter:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

where $\omega_n = 5$ rad/s and $\varsigma = 0.5$. Find its pole-zero matched strictly proper digital filter approximation with the same DC gain and a sampling period of 1 second.

(8 Marks)

(b) Consider the closed-loop system shown in Figure 2.

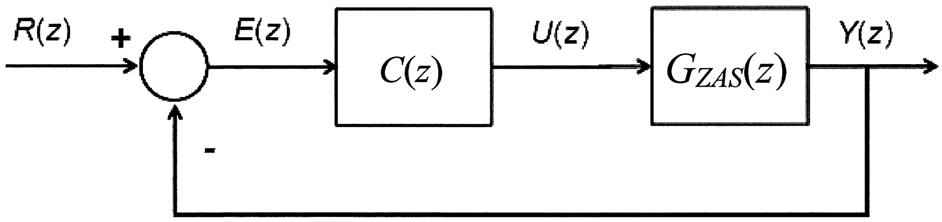

Figure 2

The desired closed-loop transfer function $G_{cl}(z)$ has the same specifications as those of the filter in Question 5(a). With a sampling period of 1 second, $G_{ZAS}(z)$ is given as

$$G_{ZAS}(z) = \frac{0.3679z + 0.2642}{(z-1)(z-0.3679)}$$

Design the controller $C(z)$.

(8 Marks)

(c) Implement the controller $C(z)$ obtained in Question 5(b) with the direct programming approach and show the relevant block diagram.

(4 Marks)

Appendix A

Properties and Table of Z-Transform

Discrete function	z Transform
$x(k+4)$	$z^4X(z)-z^4x(0)-z^3x(1)-z^2x(2)-zx(3)$
$x(k+3)$	$z^3X(z)-z^3x(0)-z^2x(1)-zx(2)$
$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
$x(k+1)$	$zX(z)-zx(0)$
$x(k)$	$X(z)$
$x(k-1)$	$z^{-1}X(z)$
$x(k-2)$	$z^{-2}X(z)$
$x(k-3)$	$z^{-3}X(z)$
$x(k-4)$	$z^{-4}X(z)$

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ $1, \quad k=0$ $0, \quad k\neq 0$	1
2.	—	—	$\delta_0(n-k)$ $1, \quad n=k$ $0, \quad n\neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$

Note: Transform Table continues on page 8.

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$	$\sin\omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos\omega t$	$\cos\omega kT$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$	$e^{-akT}\sin\omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$	$e^{-akT}\cos\omega kT$	$\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.			a^k	$\frac{1}{1-az^{-1}}$
19.			a^{k-1} $k=1,2,3,\ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.			ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.			k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.			k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.			k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.			$a^k\cos k\pi$	$\frac{1}{1+az^{-1}}$
25.			$\frac{k(k-1)}{2!}$	$\frac{z^{-2}}{(1-z^{-1})^3}$
26.		$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}$		$\frac{z^{-m+1}}{(1-z^{-1})^m}$
27.			$\frac{k(k-1)}{2!}a^{k-2}$	$\frac{z^{-2}}{(1-az^{-1})^3}$
28.	$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}a^{k-m+1}$			$\frac{z^{-m+1}}{(1-az^{-1})^m}$

$x(t)=0$, for $t<0$.

$x(kT)=x(k)=0$, for $k<0$.

Unless otherwise noted, $k=0,1,2,3,\cdots$

END OF PAPER

EE6203 COMPUTER CONTROL SYSTEMS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.