NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2017-2018

EE6203 – COMPUTER CONTROL SYSTEMS

November/December 2017 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains FIVE (5) questions and comprises EIGHT (8) pages.
2. Answer all FIVE (5) questions.
3. This is a closed-book examination.
4. All questions carry equal marks.
5. The Transform Table is included in Appendix A on pages 7 to 8.

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1. (a) The Laplace Transform *X(s)* of a signal *x(t)* is given as

$$X(s) = \frac{1}{s^2 + 5s + 4}.$$

Assume that *x(kT)* is the sampled sequence of *x(t)* with a sampling period T of 0.5 second. Determine the Z-transform of *x(kT)*.

(5 Marks)

(b) Solve the following difference equation:

$$x(k+2) + (\alpha + 0.5)x(k+1) + 0.5\alpha\, x(k) = \delta(k), \text{ and } x(k) = 0 \text{ for } k < 0,$$

where α is real number and $\delta(k)$ is a unit impulse function.

(11 Marks)

(c) Discuss the convergence of *x(k)* when $k \to \infty$ for different values of α in part 1(b).

(4 Marks)

2. Consider the control system shown in Figure 1.

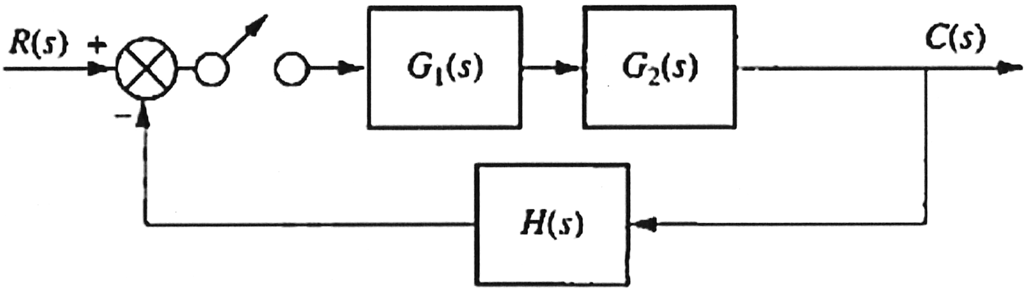

Figure 1

(a) Determine whether the pulse transfer function of the system exists.

(6 Marks)

(b) Suppose that $G_1(s)=\frac{1-e^{-Ts}}{s}$, $G_2(s)=\frac{\beta}{s(s+1)}$, $H(s)=1$, the sampling period T is 0.1 second and the input is a unit-impulse function. Determine the Z-transform of the output signal.

(9 Marks)

(c) Find the range of β in part 2(b) to ensure the stability of the system. Also, determine the output $c(kT)$ as $k \to \infty$ for $\beta = 1$ and 1000, respectively.

(5 Marks)

3. (a) An oscillatory system can be described by the following equation

$$\begin{bmatrix} \dfrac{dx_1(t)}{dt} \\ \dfrac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Determine the response $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ when the initial state is $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(6 Marks)

(b) A continuous-time system has a state-space representation given by

$$\begin{bmatrix} \dfrac{dx_1(t)}{dt} \\ \dfrac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where $x_1(t)$ and $x_2(t)$ are the states, $u(t)$ and $y(t)$ are the input and output variables, respectively. Suppose that the system is sampled with a zero-order hold at a sampling period $T = 1$ second, obtain a discretised state-space model for the system.

Is the discretised system stable?

(7 Marks)

Note: Question No. 3 continues on page 4

(c) Consider a system which is described by a state-space representation

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\,u(k)$$

The performance index for the system is given by

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left(\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k) + u^2(k)\right)$$

and the control law that minimises J is of the following form

$$u^*(k) = -\mathbf{K}\mathbf{x}(k)$$

where the optimal control law is given by

$$\mathbf{K} = \left(\mathbf{B}^T\mathbf{S}\mathbf{B} + 1\right)^{-1}\mathbf{B}^T\mathbf{S}\,\mathbf{A}$$

and $\mathbf{S} > 0$ solves the following

$$\mathbf{S} = \mathbf{A}^T\,\mathbf{S}\,\mathbf{A} + \mathbf{Q} - \mathbf{A}^T\,\mathbf{S}\mathbf{B}\left(1 + \mathbf{B}^T\mathbf{S}\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{S}\,\mathbf{A}$$

Determine the optimal control law for the following system and given performance index:

$$x(k+1) = 2x(k) + 3u(k)$$

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left(x^2(k) + u^2(k)\right)$$

(7 Marks)

4. A discrete-time system has a state-space representation given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \mathbf{A} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \mathbf{B}\, u(k)$$

$$y(k) = \mathbf{C} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the states, input and output variables, respectively.

(a) Determine the following control law

$$u(k) = -\mathbf{Kx}(k)$$

such that the closed-loop poles are at $z_{1,2} = 0.25 \pm j0.6614$.

(7 Marks)

(b) Design a full-order prediction estimator of the following form

$$\bar{\mathbf{x}}(k+1) = \mathbf{A}\bar{\mathbf{x}}(k) + \mathbf{B}u(k) + \mathbf{L}_o(y(k) - \mathbf{C}\bar{\mathbf{x}}(k))$$

such that the desired estimator poles are at $z_{1,2} = 0.1 \pm j0.1$.

(7 Marks)

(c) The true state $\mathbf{x}(k)$ in the control law in part 4(a) is replaced with the estimated state $\bar{\mathbf{x}}(k)$ from the estimator in part 4(b). Obtain the combined controller-estimator transfer function $\dfrac{U(z)}{Y(z)}$ in terms of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{L}_o$ and $\mathbf{K}$, where applicable.

(6 Marks)

5. (a) A digital controller is designed for a particular industrial process based on certain specifications. Its pulse transfer function is given as follows:

$$G_c(z) = \frac{U(z)}{E(z)} = \frac{(z+0.7)(z+0.8)}{(z-1)(z^2-\sqrt{3}z+1)}$$

Draw a block diagram representation if it is desired to implement the controller using the standard programming approach.

(10 Marks)

(b) Implement the controller $G_c(z)$ in part 5(a) with the series programming approach and show the relevant block diagrams.

(8 Marks)

(c) Comment on the two implementation approaches in parts 5(a) and 5(b).

(2 Marks)

Appendix A

Transform Table

Discrete function	z Transform
$x(k+4)$	$z^4X(z)-z^4x(0)-z^3x(1)-z^2x(2)-zx(3)$
$x(k+3)$	$z^3X(z)-z^3x(0)-z^2x(1)-zx(2)$
$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
$x(k+1)$	$zX(z)-zx(0)$
$x(k)$	$X(z)$
$x(k-1)$	$z^{-1}X(z)$
$x(k-2)$	$z^{-2}X(z)$
$x(k-3)$	$z^{-3}X(z)$
$x(k-4)$	$z^{-4}X(z)$

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ $1, \quad k=0$ $0, \quad k\neq 0$	1
2.	—	—	$\delta_0(n-k)$ $1, \quad n=k$ $0, \quad n\neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$

Note: Transform Table continues on page 8

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2\, e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$	$\sin\omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos\omega t$	$\cos\omega kT$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin\omega t$	$e^{-akT}\sin\omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$	$e^{-akT}\cos\omega kT$	$\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.			a^k	$\frac{1}{1-az^{-1}}$
19.			a^{k-1} $k=1,2,3,\ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.			ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.			k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.			k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.			k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.			$a^k\cos k\pi$	$\frac{1}{1+az^{-1}}$
25.			$\frac{k(k-1)}{2!}$	$\frac{z^{-2}}{(1-z^{-1})^3}$
26.		$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}$		$\frac{z^{-m+1}}{(1-z^{-1})^m}$
27.			$\frac{k(k-1)}{2!}a^{k-2}$	$\frac{z^{-2}}{(1-az^{-1})^3}$
28.	$\frac{k(k-1)\cdots(k-m+2)}{(m-1)!}a^{k-m+1}$			$\frac{z^{-m+1}}{(1-az^{-1})^m}$

$x(t)=0$, for $t<0$.

$x(kT)=x(k)=0$, for $k<0$.

Unless otherwise noted, $k=0,1,2,3,\cdots$

END OF PAPER

EE6203 COMPUTER CONTROL SYSTEMS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.