NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

EE6203 – COMPUTER CONTROL SYSTEMS

November/December 2015 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 8 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. The Transform Table is included in Appendix A on pages 6 to 8.

---

1. (a) Consider a first order system that has the following transfer function:

$$G(s) = \frac{C(s)}{E(s)} = \frac{s+1}{s+3} .$$

Given a sampling period T of 1 second, discretise the system using the forward difference method.

If the input signal $e(kT)$ to the system is a unit-step function, determine the output response sequence $c(kT)$ for k = 0, 1, 2, 3.

(10 Marks)

(b) Given the following digital controller

$$D(z) = \frac{z - 0.5}{(z-1)(z+0.3)}$$

Show its implementation using the series programming approach.

(7 Marks)

(c) Give your comment on part 1(a) if the backward difference approach is used.

(3 Marks)

2. (a) Given the following position control system for a robotic arm

$$G(s) = \frac{2}{s(s+6)^2}$$

find its discretized transfer function $G_{ZAS}(z)$ if a zero-order-hold is used. Assume that the desired sampling period is 0.2 s.

(10 Marks)

(b) For a finite settling time of k samples, we have the following deadbeat controller:

$$C(z) = \frac{1}{G_{ZAS}(z)}\left[\frac{z^{-k}}{1-z^{-k}}\right]$$

Determine the deadbeat controller for the position control system in part 2(a) for $k = 2$.

(6 Marks)

(c) Comment on how the time response of the controller in part 2(b) can be improved.

(4 Marks)

3. (a) A continuous-time system has a state-space representation given by

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where $x_1(t)$ and $x_2(t)$ are the states, $u(t)$ and $y(t)$ are the input and output variables, respectively. Suppose that the system is sampled with a zero-order hold.

Obtain a discretised state-space model for the system in terms of the sampling period T.

(9 Marks)

(b) Determine the values of the sampling period T that make the discretised system obtained in part 3(a) unobservable.

(4 Marks)

Note:Question No. 3 continues on Page 3

(c) A discrete-time system has a state-space representation given by

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k)$$
$$y(k) = \mathbf{C}\mathbf{x}(k) + du(k)$$

where $\mathbf{x}(k)$, $u(k)$ and $y(k)$ are the states, input and output variables, respectively. If

$$u(k) = r(k) - y(k)$$

where $r(k)$ is the reference input, obtain an overall closed-loop state-space representation of the following form:

$$\mathbf{x}(k+1) = \mathbf{A}_{cl}\mathbf{x}(k) + \mathbf{B}_{cl}r(k)$$
$$y(k) = \mathbf{C}_{cl}\mathbf{x}(k) + \mathbf{D}_{cl}r(k)$$

State the assumptions that you have made, if any.

(7 Marks)

4. (a) A discrete-time system has a state-space representation given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \mathbf{A}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \mathbf{B}u(k)$$
$$y(k) = \mathbf{C}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.125 \\ 0.25 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the states, input and output variables, respectively. A controller of the following form

$$u(k) = -\mathbf{K}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + k_r r(k)$$

is to be implemented, where $r(k)$ is the reference input.

Note: Question No. 4 continues on Page 4

(i) Design k_r and $\mathbf{K}$ such that the closed-loop poles are at $z_{1,2} = 0.5 \pm j0.5$ and the final value of $y(k)$ is unity for a unit step input $r(k)$.

(9 Marks)

(ii) If $k_r = 0$ and $\mathbf{K} = \begin{bmatrix} 4 & 6 \end{bmatrix}$, discuss the type of response that the closed-loop system will exhibit when subjected to a non-zero $\mathbf{x}(0)$.

(4 Marks)

(b) A process is described by the following state-space representation

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}u(k)$$

where $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The performance index for the system is given by

$$J = \frac{1}{2}\mathbf{x}^T(N)\mathbf{S}(N)\mathbf{x}(N) + \frac{1}{2}\sum_{k=0}^{N-1}\left(\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k) + ru^2(k)\right)$$

and the design equations for $k = N-1, \ldots, 1, 0$ are

$$\mathbf{K}(k) = \left(\mathbf{B}^T\mathbf{S}(k+1)\mathbf{B} + r\right)^{-1}\mathbf{B}^T\mathbf{S}(k+1)\mathbf{A}$$

$$u^*(k) = -\mathbf{K}(k)\mathbf{x}(k)$$

$$\mathbf{S}(k) = \left(\mathbf{A} - \mathbf{B}\mathbf{K}(k)\right)^T\mathbf{S}(k+1)\left(\mathbf{A} - \mathbf{B}\mathbf{K}(k)\right) + r\mathbf{K}(k)^T\mathbf{K}(k) + \mathbf{Q}$$

If $\mathbf{Q} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $\mathbf{S}(N) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $r = 2$ and $N = 2$, find the optimal controls $u^*(0)$ and $u^*(1)$ such that J is minimised. Let the initial state $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(7 Marks)

5. (a) A conveyor belt system uses a DC motor for direct drive for its position control and has the following loop transfer function:

$$H(s) = \frac{50}{s(1+0.01s)(1+0.003s)}$$

What is the system phase margin at $\omega_c = 150$ rad/s if a ZOH is used and the sampling period is 2 ms?

(10 Marks)

(b) Given a digital controller that has the following transfer function

$$G(z) = \frac{Y(z)}{X(z)} = \frac{z}{z+0.9}$$

If the input is a unit impulse at $k = 0$, what is the output of this controller for the time steps $k = 0, 1, 2, \ldots, 6$ assuming that there is a rounding error of one decimal significant digit. Repeat for the case if it is a truncation error of one decimal significant digit.

(8 Marks)

(c) Comment on the results that you obtained in part 5(b).

(2 Marks)