NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2016-2017

EE6203 – COMPUTER CONTROL SYSTEMS

November/December 2016 | Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains FIVE (5) questions and comprises NINE (9) pages.
2. Answer all FIVE (5) questions.
3. This is a closed-book examination.
4. All questions carry equal marks.
5. The Transform Table is included in Appendix A on pages 7 to 9.

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1. (a) Consider the following second order system:

$$G(s) = \frac{C(s)}{E(s)} = \frac{s}{(s+2)(s+4)}$$

Assume that a sampling period T of 2 seconds is used. Discretize the system using the bilinear transformation method.

If a unit-step input signal $e(kT)$ is applied to the system at $k = 0$, determine the response $c(kT)$ for $k = 0, 1$ and 2.

(10 Marks)

(b) The signal $g(t) = 2\pi + 5\cos 2\omega t$ is sampled with a zero-order hold at a sampling rate of m samples per second. Determine the theoretical worst case error.

For a sampling period of $T = 1$ second, determine the actual maximum error with $m = 1$ to 5.

(8 Marks)

(c) Comment on the results you obtained in part 1(b).

(2 Marks)

2. (a) A process has the following transfer function:

$$G(s) = \frac{2}{s(s+7)}$$

Find its discretized transfer function $G_{ZAS}(z)$ if a zero-order hold is used. Assume that the sampling period is 0.2 second.

(6 Marks)

(b) It is desired to design a digital controller $C(z)$ using the ripple-free deadbeat control approach for a unit-step reference input $R(z)$. Assume that the transfer function of the system is

$$G_{ZAS}(z) = \frac{(z+0.6)}{(z-1)(z-0.01)}$$

and the control variable can be expressed in terms of the closed-loop transfer function $G_{cl}(z)$ by

$$U(z) = G_{cl}(z)\frac{R(z)}{G_{ZAS}(z)}$$

Moreover, the system is of type 1 and the control variable becomes zero after two sampling steps, i.e. $U(z) = a_0 + a_1 z^{-1}$. Show with details how you design the controller.

(12 Marks)

(c) Comment on the controller obtained in part 2(b).

(2 Marks)

3. (a) A certain tracking system is described by the following equations:

$$\frac{d^2\theta_o(t)}{dt^2} = 2.5\,w(t)$$

$$w(t) = 10\,\theta_i(t) - v(t)$$

$$v(t) = 4k_d\frac{d\theta_o(t)}{dt} + 10\,\theta_o(t)$$

where $w(t)$ and $v(t)$ are some process variables. Let the state variables be $x_1(t) = \theta_o(t)$ and $x_2(t) = \dfrac{d\theta_o(t)}{dt}$. Also, let the input variable $u(t) = \theta_i(t)$ and the output variable $y(t) = \theta_o(t)$. Obtain a state-space model of the continuous-time system in terms of the process variable k_d.

(5 Marks)

(b) A continuous-time system has a state-space representation given by

$$\begin{bmatrix} \dfrac{dx_1(t)}{dt} \\ \dfrac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

where $x_1(t)$ and $x_2(t)$ are the states, $u(t)$ and $y(t)$ are the input and output variables, respectively. Suppose that the system is sampled with a zero-order hold at a sampling period $T = 1$ second, obtain a discretised state-space model for the system.

(8 Marks)

(c) A process is described by the following state-space representation:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\,u(k)$$

where $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The performance index for the system is given by

$$J = \frac{1}{2}\sum_{k=0}^{\infty}\left(\mathbf{x}^T(k)\mathbf{Q}\mathbf{x}(k) + u^2(k)\right), \quad \mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note: Question No. 3 continues on page 4

The control law that minimises J is of the following form:

$$u^*(k) = -\mathbf{K}\mathbf{x}(k)$$

where

$$\mathbf{K} = \left(\mathbf{B}^T\mathbf{S}\mathbf{B}+1\right)^{-1}\mathbf{B}^T\mathbf{S}\,\mathbf{A}$$

and $\mathbf{S} > 0$ satisfies the following:

$$\mathbf{S} = \mathbf{A}^T\,\mathbf{S}\,\mathbf{A} + \mathbf{Q} - \mathbf{A}^T\,\mathbf{S}\mathbf{B}\left(1+\mathbf{B}^T\mathbf{S}\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{S}\,\mathbf{A}$$

Determine the optimal control law $\mathbf{K}$ and comment on the stability of the closed-loop system.

(7 Marks)

4. A discrete-time system has a state-space representation given by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \mathbf{A}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \mathbf{B}\,u(k)$$

$$y(k) = \mathbf{C}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -0.5 & 1.5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.8 \\ 1.2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$, $u(k)$ and $y(k)$ are the states, input and output variables, respectively.

(a) A control law of the following form

$$u(k) = -\,\mathbf{K}\mathbf{x}(k) + k\,r(k), \quad \mathbf{K} = \begin{bmatrix} 0.6775 & -0.635 \end{bmatrix}$$

is to be implemented, where $r(k)$ is the reference input.

(i) Determine the value of k such that the output will track a unit-step reference input signal $r(k)$ with zero steady-state error.

(ii) What is damping ratio ζ and natural undamped frequency ω_n associated with the equivalent s-plane closed-loop poles if the sampling period $T = 0.2$ second?

(10 Marks)

Note: Question No. 4 continues on page 5

(b) Design a full-order prediction estimator of the following form

$$\bar{\mathbf{x}}(k+1) = \mathbf{A}\bar{\mathbf{x}}(k) + \mathbf{B}u(k) + \mathbf{L}_o(y(k) - \mathbf{C}\bar{\mathbf{x}}(k))$$

that will give deadbeat error response.

(5 Marks)

(c) The true state $\mathbf{x}(k)$ in the control law in part 4(a) is replaced with the estimated state $\bar{\mathbf{x}}(k)$ from the estimator in part 4(b). Obtain a state-space model of the following form

$$\bar{\mathbf{x}}(k+1) = \mathbf{A}_c\bar{\mathbf{x}}(k) + \mathbf{B}_c\begin{bmatrix} r(k) \\ y(k) \end{bmatrix}$$

$$u(k) = \mathbf{C}_c\bar{\mathbf{x}}(k) + \mathbf{D}_c\begin{bmatrix} r(k) \\ y(k) \end{bmatrix}$$

i.e., obtain expressions for $\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c$ and $\mathbf{D}_c$ in terms of $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{L}_o, \mathbf{K}$ and k, where applicable.

(5 Marks)

5. (a) For a particular process system, a digital controller has been designed to meet the system requirements and it can be described by the following transfer function:

$$C(z) = \frac{U(z)}{E(z)} = \frac{(z+0.1)(z-0.5)}{(z-1)(z-0.89)(z-0.2)}$$

Show a block diagram representation if it is desired to implement the controller using the direct programming approach.

(10 Marks)

(b) Assume that the process loop transfer function has a phase margin of 8^0. A phase-lead controller of the following form

$$G_c(s) = 50\frac{s+\omega_1}{s+\omega_2}$$

is to be designed to achieve a phase margin of 40^0 at 150 rad/s.

Note: Question No. 5 continues on page 6

(i) Determine ω_1 and ω_2 of the controller.

(ii) Find the corresponding digital controller using the backward difference approach with a sampling time of 1 second.

(Hint: For optimal results in phase-lead controller design, $\omega_c = \sqrt{\omega_1 \omega_2}$ where $\frac{\omega_2}{\omega_1} = \frac{(1+\sin\phi)}{(1-\sin\phi)}$ and ϕ is the required lead angle.)

(8 Marks)

(c) If a bilinear transformation approach is used instead in part 5(b), comment on the resulting digital controller that you would have obtained as compared to the one obtained in part 5(b) as an approximation of the original continuous-time controller.

(2 Marks)

Appendix A

TRANSFORM TABLE

Laplace Transform $F(s)$	Time Function $f(t),\ t>0$	z-Transform $F(z)$	Modified z-Transform $F(z,m)$
1	$\delta(t)$	1	0
e^{-kTs}	$\delta(t-kT)$	z^{-k}	z^{-k-1+m}
$\frac{1}{s}$	$u_s(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1}+\frac{T}{(z-1)^2}$
$\frac{2}{s^3}$	t^2	$\frac{T^2z(z+1)}{(z-1)^3}$	$T^2\frac{m^2z^2+(2m-2m^2+1)z+(m-1)^2}{(z-1)^3}$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{z}{z-e^{-aT}}\right]$	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left(\frac{e^{-amT}}{z-e^{-aT}}\right)$
$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$	$\frac{e^{-amT}}{z-e^{-aT}}$
$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$	$\frac{Te^{-amT}\left[e^{-aT}+m(z-e^{-aT})\right]}{(z-e^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	t^ke^{-at}	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{z}{z-e^{-aT}}\right]$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{e^{-amT}}{z-e^{-aT}}\right]$
$\frac{a}{s(s+a)}$	$1-e^{-at}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$	$\frac{(1-e^{-amT})z+(e^{-amT}-e^{-aT})}{(z-1)(z-e^{-aT})}$

Laplace Transform $F(s)$	Time Function $f(t),\ t > 0$	z-Transform $F(z)$	Modified z-Transform $F(z, m)$
$\frac{1}{(s+a)(s+b)}$	$\frac{1}{(b-a)}(e^{-at}-e^{-bt})$	$\frac{1}{(b-a)}\left[\frac{z}{z-e^{-aT}}-\frac{z}{z-e^{-bT}}\right]$	$\frac{1}{(b-a)}\left[\frac{e^{-amT}}{z-e^{-aT}}-\frac{e^{-bmT}}{z-e^{-bT}}\right]$
$\frac{a}{s^2(s+a)}$	$t-\frac{1}{a}(1-e^{-at})$	$\frac{Tz}{(z-1)^2}-\frac{(1-e^{-aT})z}{a(z-1)(z-e^{-aT})}$	$\frac{T}{(z-1)^2}+\frac{amT-1}{a(z-1)}+\frac{e^{-amT}}{a(z-e^{-aT})}$
$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$	$\frac{Te^{-amT}\left[e^{-aT}+m(z-e^{-aT})\right]}{(z-e^{-aT})^2}$
$\frac{a}{s^3(s+a)}$	$\frac{1}{2}\left(t^2-\frac{2}{a}t+\frac{2}{a^2}-\frac{2}{a^2}e^{-at}\right)$	$\frac{T^2z}{(z-1)^3}+\frac{(aT-2)Tz}{2a(z-1)^2}$ $+\frac{z}{a^2(z-1)}-\frac{z}{a^2(z-e^{-aT})}$	$\frac{T^2}{(z-1)^3}+\frac{T^2(m+\frac{1}{2})a-T}{a(z-1)^2}$ $+\frac{(amT)^2/2-amT+1}{a^2(z-1)}-\frac{e^{-amT}}{a^2(z-e^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$u_s(t)-(1+at)e^{-at}$	$\frac{z}{z-1}-\frac{z}{z-e^{-aT}}-\frac{aTe^{-aT}z}{(z-e^{-aT})^2}$	$\frac{1}{z-1}-\left[\frac{1+amT}{z-e^{-aT}}+\frac{aTe^{-aT}}{(z-e^{-aT})^2}\right]e^{-amT}$
$\frac{a^2}{s^2(s+a)^2}$	$t-\frac{2}{a}+\left(t+\frac{2}{a}\right)e^{-at}$	$\frac{1}{a}\left[\frac{(aT+2)z-2z^2}{(z-1)^2}+\frac{2z}{z-e^{-aT}}\right.$ $\left.+\frac{aTe^{-aT}z}{(z-e^{-aT})^2}\right]$	$\frac{1}{a}\left\{\frac{aT}{(z-1)^2}+\frac{amT-2}{z-1}+\left[\frac{aTe^{-aT}}{(z-e^{-aT})^2}\right.\right.$ $\left.\left.-\frac{amT-2}{z-e^{-aT}}\right]e^{-amT}\right\}$
$\frac{\omega}{s^2+\omega^2}$	$\sin\omega t$	$\frac{z\sin\omega T}{z^2-2z\cos\omega T+1}$	$\frac{\sin m\omega T+\sin(1-m)\omega T}{z^2-2z\cos\omega T+1}$

Laplace Transform $F(s)$	Time Function $f(t)$, $t > 0$	z-Transform $F(z)$	Modified z-Transform $F(z, m)$
$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\frac{z(z-\cos\omega T)}{z^2-2z\cos\omega T+1}$	$\frac{\cos m\omega T-\cos(1-m)\omega T}{z^2-2z\cos\omega T+1}$
$\frac{\omega}{s^2-\omega^2}$	$\sinh \omega t$	$\frac{z\sinh\omega T}{z^2-2z\cosh\omega T+1}$	$\frac{\sinh m\omega T+\sinh(1-m)\omega T}{z^2-2z\cosh\omega T+1}$
$\frac{s}{s^2-\omega^2}$	$\cosh \omega t$	$\frac{z(z-\cosh\omega T)}{z^2-2z\cosh\omega T+1}$	$\frac{\cosh m\omega Tz-\cosh(1-m)\omega T}{z^2-2z\cosh\omega T+1}$
$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$\frac{ze^{-aT}\sin\omega T}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$	$\frac{e^{-amT}\left[z\sin m\omega T+e^{-aT}\sin(1-m)\omega T\right]}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$
$\frac{a^2+\omega^2}{s\left[(s+a)^2+\omega^2\right]}$	$1-e^{-at}\sec\phi\cos(\omega t+\phi)$ $\phi=\tan^{-1}(-a/\omega)$	$\frac{z}{z-1}-\frac{z^2-ze^{-aT}\sec\phi\cos(\omega T-\phi)}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$	$\frac{1}{z-1}-\frac{e^{-maT}\sec\phi(Az-B)}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$ $A=\cos(m\omega T+\phi)$ $B=e^{-aT}\cos[(1-m)\omega T-\phi]$
$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos\omega t$	$\frac{z^2-ze^{-aT}\cos\omega T}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$	$\frac{e^{-maT}\left[z\cos m\omega T+e^{-aT}\sin(1-m)\omega T\right]}{z^2-2ze^{-aT}\cos\omega T+e^{-2aT}}$

Suppose $X(z)z^{k-1}$ contains a multiple pole z_j of order q, then the residue k is given by

$$k=\frac{1}{(q-1)!}\lim_{z\to z_j}\frac{d^{q-1}}{dz^{q-1}}\left[(z-z_j)^q X(z)z^{k-1}\right]$$

End of Paper

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1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.