UNIVERSITY OF COPENHAGEN

BSc in Economics

Energy price shocks and core inflation

The energy crisis in a DDGE model

Mathias Berg (lwp713), Oliver Ding (wgl247) & Andreas Laursen (dzw863)

Supervised by Jeppe Druedahl

November 24, 2022

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Responsibilities

Jointly: Abstract, Introduction (1), Conclusion (6).

In general, the thesis is a joint collaboration between all three authors.

Mathias: 2.1, 2.4, 3.3, 4.2, 4.5, 5.3

Oliver: 2.2, 3.1, 3.4, 4.3, 5.1, 5.4

Andreas: 2.3, 3.2, 4.1, 4.4, 5.2, 5.5

Keystrokes

80,141

University of Copenhagen

Faculty of Social Sciences

Department of Economics

Øster Farimagsgade 5, Building 26

1353 Copenhagen K

The inflation is now on its highest level in decades. High inflation challenges the stability in the economy, and is therefore the most important economic challenge we face today.

Lars Rohde, Governor of the National Bank of Denmark, September 21 2022[1]

[1]Lars Rohde, Governor of *Nationalbanken*, emphasized the magnitude and importance of the high inflation on a press conference on September 21, 2022. The press conference concerned rising energy prices and inflation as a whole.

Abstract

This paper examines the current energy crisis and its effect on core inflation in Denmark. To analyze a global energy shock, we expand Jeppe Druedahl (2022)'s Baby-MAKRO, a theoretical *dynamic deterministic general equilibrium* model (DDGE) for a small open economy with a fixed exchange rate. We split the global energy shock into a domestic energy and foreign price shock and solve the model by numerical optimization in Python.

Firstly, we find only one transmission mechanism from energy prices to core inflation, which is through production. Secondly, we find that a global energy shock has an ambiguous effect on core inflation and depends on Denmark's "energy responsiveness" relative to foreign countries. However, as current data shows high core inflation, our results suggest that other factors could potentially account for the stark increase in core inflation or that our model contains a missing link from energy prices to core inflation. The latter could imply that DDGE models are insufficient in modeling price dynamics as they lack uncertainty among agents.

Keywords: Energy prices, Energy price shock, Core inflation, Energy crisis, Danish economy, Dynamic Deterministic General Equilibrium Model, Perfect foresight, Numerical optimization, Python

Code: Link to our Git repository.

Contents

1 Introduction

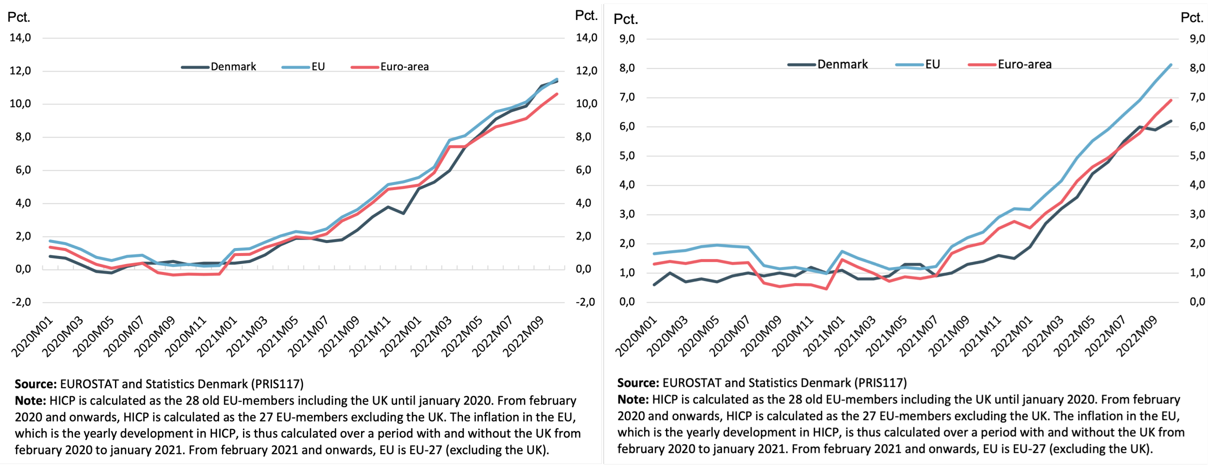
The average spot price of electricity in Denmark on the 1st of September 2021 was 832 DKK/MWh according to The Danish Energy Agency (2022); exactly one year later, it has more than quadrupled to 4,248 DKK/MWh. We see similar trends for gas, oil, coal, and renewable energy. Europe is in an energy crisis and, consequently, inflation levels have soared. Statistics Denmark (2022*a*) released a report showing that inflation, measured as the harmonized consumer price index (HICP), began to increase throughout 2021 and accelerated throughout 2022; see figure 1a. The report states that for 15 out of 27 EU countries, the primary reason for the high HICP is the elevated energy prices.

However, Statistics Denmark also states that core inflation has been increasing too; see figure 1b. This is even though core inflation, by definition, excludes energy prices (Lindgaard, 2020). Therefore, the energy prices cannot directly explain the elevated core inflation, and if they explain it at all, it must be through indirect transmission mechanisms. Alternatively, there may only be a correlation but no causal link between core inflation and energy prices. Therefore, this paper investigates the impact of the global energy shock on the Danish economy and its transmission mechanisms to core inflation to determine if the energy crisis has elevated the core inflation.

To do so, we use a *dynamic deterministic general equilibrium* model. Our model expands on Jeppe Druedahl (2022)'s original draft of Baby-MAKRO, which is a simplified version of the new macroeconomic model MAKRO used by the Danish Ministry of Finance. Baby-MAKRO describes a small open economy with a fixed exchange rate and includes overlapping generations and heterogeneous agents. The agents have perfect foresight, but the model contains imperfections such as menu costs, labor market frictions, and wage rigidities. Our expansions include elements such as nested CES functions, Nash wage bargaining, and hand-to-mouth households. Some of these expansions are inspired by DREAM, who developed MAKRO and their preliminary model documentation of MAKRO; see Ejarque and Stephensen (2021).

Figure 1: The HICP and core inflation Denmark, EU, and the Euro-area

(a) HICP (b) Core inflation



The energy crisis is not just a shock to Denmark's economy but to all of Europe. According to Eurostat (2022), more than half the EU countries experienced double-digit inflation in September 2022. From a domestic point of view, the energy crisis is not just a shock to energy prices but also to foreign prices. We define the global energy crisis as the *overall* shock and split it into its partial shocks: a *primary* shock to energy prices and a *secondary* shock to foreign prices. This separation allows us to analyze each shock's transmission mechanisms independently. If we understand the partial shocks of the global energy crisis, we can better understand its entirety. For the rest of this paper, we will call the domestic energy price shock *the primary shock* and the foreign price shock *the secondary shock*. Note that we use "energy" as an umbrella term that includes all kinds of energy sources, such as gas, oil, coal, and electricity.

When applying the shocks to our model, we find two things. Firstly, we find only one transmission mechanism from energy prices to core inflation which is through production. Secondly, we find that the energy shock has an ambiguous effect on core inflation since the primary and secondary shocks have counteracting effects on production and core inflation. The primary shock decreases core inflation as production falls due to higher input costs. In contrast, the secondary shock increases core inflation as export demand rises. Since the two effects are opposite, we cannot tell which effect dominates. It depends on the country's *energy responsiveness*, which we define as how responsive a country's producer prices are to a change in energy prices. On the one hand, if the domestic economy is *less* energy responsive relative to the foreign economy, the increase in foreign prices exceeds the increase in domestic prices, contributing positively to production and core inflation. On the other hand, if the domestic economy is *more* energy responsive relative to the foreign economy, the increase in domestic prices exceeds the increase in foreign prices, contributing negatively to production and core inflation. Initially, the energy crisis will have a negative impact on core inflation, but if the foreign economy is more affected, it could eventually lead to higher production and core inflation in the domestic economy.

Our results could suggest that the high core inflation might not be rooted in higher energy prices but in other factors. These could be the catching-up effect following the Covid-19 crisis or expansive fiscal or monetary policies. The energy prices might be a veil that hides the true reason for high core inflation. Alternatively, our model may omit a key dynamic connecting energy prices to core inflation. Such missing links could include uncertainty and imperfect information among agents. Due to perfect foresight, our model allows no expectation errors, implying that critical inflation mechanisms such as wage-price spirals and price-setting behavior of firms are hard to model.

Therefore, an idea for future research could be to expand Baby-MAKRO with the concept of *adaptive learning*. In addition, our results could indicate that there might be a missing link connecting energy prices to core inflation. Therefore it could be interesting to conduct an econometric analysis that explores if there is, in fact, a causal relationship between the two. Finally, this paper encourages a similar shock analysis with a DSGE framework to examine the implications of uncertainty and imperfect information on inflation.

2 Theory

This section covers the theory of our model, which is split into three parts. Firstly, we introduce the model type and provide an overall model description. Secondly, we present the *block structure* of the model, and finally, we briefly describe the steady state.

2.1 Model Type

Our model expands on Baby-MAKRO, a simplified version of the macroeconomic model used by the Danish Ministry of Finance, MAKRO. Baby-MAKRO is a *dynamic deterministic general equilibrium* model (DDGE). The model is *dynamic*, as all variables of economic processes and systems are functions of time. It is *deterministic* as the agents have perfect foresight, which entails perfect information for all periods, including information about future shocks and the behavior of other agents. At last, it is a *general equilibrium* model, as the supply, demand, and prices of several markets interact to achieve the overall equilibrium of the economy. An alternative to DDGE models is the *dynamic stochastic general equilibrium* models (DSGE), see DREAM (2022). Unlike DDGE models, DSGE models include stochasticity. DSGE models can easily model uncertainty, risk, and imperfect information, which is more difficult in a DDGE model with perfect foresight. However, the Danish Ministry of Finance picked a DDGE model since the absence of stochasticity allows for a higher degree of heterogeneity and complexity.

2.2 Model Description

The households are modeled as overlapping generations (OLG) and includes heterogeneity of two dimensions: 1. their age and 2. whether or not they are budget constrained. The first dimension determines whether or not the household is a part of the labor force. The second dimension determines whether the household is Ricardian or ”hand-to-mouth”. Hand-to-mouth households, abbreviated as ”HtM households”, consume all income in a given period, and only the optimizing households can smooth consumption over time. Both types of households consume a bundle of *goods* and *energy*. Excessive income is saved for either future consumption or left as bequests for future generations. The households gain utility from consuming and bequesting. This modeling of the households allows us to investigate how they reallocate consumption when introducing a shock.

The labor market is a classic matching model. The households exogenously search for jobs posted by a labor agency. The labor agency hires labor and rents it to production firms. Search friction and the cost to post job vacancies generate involuntary unemployment and unfilled positions. The labor agency, representing the firms, and the labor union, representing the workers, negotiate the wage through Nash bargaining. The final wage is a weighted average of the recently negotiated and the last period's wage, which creates rigidity and increases marginal costs for the firms in the short run, as wages are proportional to labor demand.

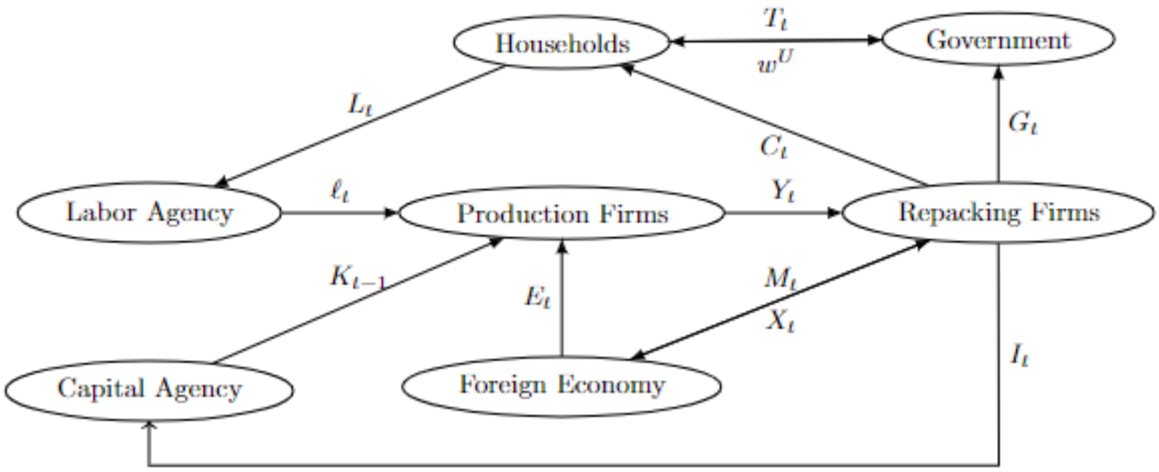
The firms are split into four parts: 1. capital agency, 2. labor agency, 3. production firms, and 4. repacking firms. The production firm uses labor, capital, and energy as inputs to produce output. Labor and capital are rented from the respective agencies while energy is imported. The repacking firm combines output from the production firms with imports to produce the final goods. These are consumed by households or the government, used as input by the capital agency to produce capital, or exported to the foreign economy.

The foreign sector is exogenous as we model a small open economy with a fixed exchange rate. This implies that import and energy prices are exogenous, and the supply of these are unlimited. The relative prices between the domestic and foreign economies determine export demand. The fixed exchange rate implies that the interest rate is determined outside the model.

The government earns revenue from income taxes paid by the households and supplies a fixed unemployment benefit. They finance deficits by accumulating debt in the short run and raising taxes in the long run.

We make several simplifying assumptions. To mention a few, we do not distinguish between nominal and real interest rates, as the interest rates are unaffected by inflation in this model. In addition, we do not model current accounts, net foreign assets, or balance of payments. These assumptions are unrealistic, but if we include them, they could create more complexity with no apparent upside for our purpose. We show the model relations in figure 2.

Figure 2: Model Diagram of Real Variables



Note: This figure is a simplified illustration of the model and only contains real variables. Technically, the tax income, T, and the unemployment benefits, w^U, are nominal, but we can deflate them into real variables.

2.3 The Block Structure

The model is divided into blocks. This structure serves as a tool when solving in Python and as an overview of the different model elements. The numerated equations within each block are contained in the code. Time is discrete with T periods, and one period corresponds to 1 year. Lowercase t denotes the current period.

Households - Search Behavior and Matching

The households search for jobs and supply of labor exogenously. Their lifetime is A, and they retire at A_R. Their current age is denoted by a. We assume homogeneity in the age-specific job-separation probability, $\delta_a^L \in (0, 1)$ though it is possible to include heterogeneity. The number of searchers is:

$$S_{a,t} = \begin{cases} 1 & \text{if } a = 0 \\ (1 - L_{a-1,t-1}) + \delta_a^L L_{a-1,t-1} & \text{if } a < A_R \\ 0 & \text{if } a \geq A_R \end{cases} \tag{2.1}$$

$$S_t = \sum_a S_{a,t} \tag{2.2}$$

The quantity of households with a job *before matching* is:

$$\underline{L}_{a,t} = \begin{cases} 0 & \text{if } a = 0 \\ (1 - \delta_a^L) L_{a-1,t-1} & \text{if } a < A_R \\ 0 & \text{if } a \geq A_R \end{cases} \tag{2.3}$$

$$\underline{L}_t = \sum_a \underline{L}_{a,t} \tag{2.4}$$

The aggregate job-separation rate is:

$$\delta_t^L = \frac{\underline{L}_t - L_{t-1}}{L_{t-1}} \tag{2.5}$$

The quantity of vacancies is v_t, and the number of matches is given by the *matching function*, $\mathcal{M}_t$. The matching function is derived from Petrongolo and Pissarides (2001) and corresponds to a rescripted CES matching function.[1]

$$\mathcal{M}_t = \frac{S_t v_t}{\left(S_t^{\frac{1}{\sigma^m}} + v_t^{\frac{1}{\sigma^m}}\right)^{\sigma^m}} \tag{2.6}$$

The job-filling rate, m_t^v, the job-finding rate, m_t^s, and the market tightness, θ_t, are thus:

$$m_t^v = \frac{\mathcal{M}_t}{v_t} \tag{2.7}$$

$$m_t^s = \frac{\mathcal{M}_t}{S_t} \tag{2.8}$$

$$\theta_t = \frac{m_t^s}{m_t^v} = \frac{v_t}{S_t} \tag{2.9}$$

The number of employed workers is:

$$L_t = \underline{L}_t + m_t^s S_t \tag{2.10}$$

And the number of unemployed workers is:

$$U_{a,t} = \begin{cases} 1 - L_{a,t} & \text{if } a < A_R \\ 0 & \text{if } a \geq A_R \end{cases} \tag{2.11}$$

$$U_t = \sum_a U_{a,t} \tag{2.12}$$

In equilibrium, the number of matches must equal the number of new hires, i.e.:

$$\mathcal{M}_t = L_t - \underline{L}_t \tag{2.13}$$

[1] σ^m is technically not the elasticity of substitution, but rather the inverse of the substitution parameter. See Appendix for an elaboration.

Labor Agency

The labor agency hires labor, L_t, at the cost w_t and rents out effective labor, ℓ_t, to production firms at the rental rate, r^ℓ_t. To hire labor, the labor agency posts vacancies, v_t, at the cost κ^L. The labor agency takes the job-filling rate as given. When solving the maximization problem, they also take the wage as given. However, it is determined by bargaining with the labor union, as described in the next section. The problem of the labor agency is thus:

$$V(L_{t-1}) = \max_{L_t} \sum_{t=0}^{\infty} \frac{1}{(1+r^{\text{firm}})^t} \left[r^\ell_t \ell_t - w_t L_t\right]$$
$$\ell_t = L_t - \kappa^L v_t$$
$$L_t = \left(1 - \delta^L_{t+1}\right) L_{t-1} + m^v_t v_t.$$

Solving the above yields the following conditions. See derivation in Appendix.

$$r^\ell_t = \frac{1}{1 - \frac{\kappa^L}{m^v_t}}(w_t - \frac{1-\delta^L_t}{1+r^{firm}} \frac{\kappa^L}{m^v_{t+1}} r^\ell_{t+1}) \tag{2.14}$$
$$\ell_t = L_t - \kappa^L v_t \tag{2.15}$$

Bargaining

The target wage, w^*_t, is determined by Nash bargaining between the labor agency and labor union and is based on the paper by Ljungqvist and Sargent (2017). J is the surplus of the labor agency arising from a filled job position, while V is the surplus arising from an unfilled position.

$$J = r^\ell - w + \beta[\delta^L V + (1-\delta^L)J]$$
$$V = -\kappa^L + \beta\left[m^v J + [1-m^v]V\right]$$

Correspondingly, we define E as the worker's value of employment and U as the value of unemployment. When employed, the worker receives the wage w_t; when unemployed, the worker receives the outside option, w^U. The outside option is a fixed ratio of the steady state wage, $U_B \in (0,1)$, and is financed by the government.

$$E = w + \beta[\delta U + (1-\delta)E]$$
$$U = w^U + \beta[\theta m^v E + [1-\theta]m^v U] \quad \text{with } w^U = U_B \cdot w_{ss}$$

We derive the target wage function based on the surpluses; see Appendix. The target wage depends on the rental rate per unit of labor, r^ℓ, labor market tightness, $\theta_t = \frac{v_t}{S_t}$, the worker outside option, w^U, and finally, the bargaining power of each party, ϕ.

$$w_t^* = \phi(r_t^\ell + \theta_t \kappa^L) + (1-\phi)w^U \tag{2.16}$$

To create rigidity, the *final wage* is a weighted average of the last period's wage and the current target wage. This is inspired by, but not identical, to Calvo (1983). The original framework proposed by Calvo also involves stochasticity.

$$w_t = \gamma^w w_{t-1} + (1-\gamma^w)w_t^* \tag{2.17}$$

The final wage creates the desired dynamics as it depends on the market tightness and thus labor demand. So, as production increases, wages and marginal costs increase as well. This decreases the scaled returns of the production firms.

Capital Agency

The capital agency accumulates capital, K_t, and rents it to the production firm at the rental rate, r_t^K. To produce capital, they buy investment goods at the price P_t^I but must account for convex adjustment costs. Hence, effective investment, ι_t, corresponds to the gross investments,

I_t, minus adjustment costs. The problem of the capital agency is thus:

$$V\left(K_{t-1}\right) = \max_{\{K_t\}} \sum_{t=0}^{\infty} \frac{1}{\left(1+r^{\text{firm}}\right)^t} \left[r_t^K K_{t-1} - P_t^I(\iota_t + \Psi(\iota_t, K_{t-1}))\right]$$

s.t.

$$I_t = \iota_t + \Psi(\iota_t, K_{t-1})$$

$$K_t = (1-\delta^K)K_{t-1} + \iota_t$$

$$\Psi(\iota_t, K_t) = \frac{\Psi_0}{2}\left(\frac{\iota_t}{K_t} - \delta\right)^2 K_t$$

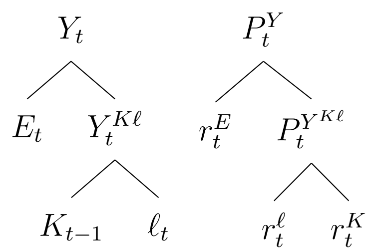
Solving yields the following condition. See derivation in Appendix:

$$P_t^I = \frac{1}{\left(1+\Psi_\iota\left(\iota_t, K_{t-1}\right)\right)} \frac{1}{1+r^{\text{firm}}} \left[r_{t+1}^K + (1-\delta^K)P_{t+1}^I(1+\Psi_\iota(\iota_{t+1}, K_t)) - P_{t+1}^I \Psi_K\left(\iota_{t+1}, K_t\right)\right] \tag{2.18}$$

Production Firms

The production firm produces output, Y_t, with a nested CES technology, see figure 3. Brockway et al. (2017) outlines the foundation for the nesting structure of CES.

Figure 3: Nest Structure in Production



The lower branches consist of capital, K_{t-1}, from the capital agency and effective labor, ℓ_t, from the labor agency, which combined creates the *core* output $Y_t^{K\ell}$. The core output is then combined with energy, E_t, to produce the *final* output, Y_t. For inputs, the production firm pays the rental price of capital, r_t^K, the rental price of labor, r_t^ℓ, and the rental price of energy r_t^E. As energy is assumed to be imported the rental price of energy is exogenous. The production firm solves one big profit maximization problem that includes the inner and the outer nest. However, it is easier to perceive it as two separate maximization problems:

$$V_t^{YK\ell}(\ell_t, K_{t-1}) = \max_{\ell_t, K_{t-1}} P_t^{YK\ell} Y^{K\ell} - r_t^K K_{t-1} - r^\ell \ell_t$$
$$\text{s.t. } Y_t^{K\ell} = \left(\left(\mu^K\right)^{\frac{1}{\sigma^{YK\ell}}} \left(K_{t-1}\right)^{\frac{\sigma^{YK\ell}-1}{\sigma^{YK\ell}}} + \left(1-\mu^K\right)^{\frac{1}{\sigma^{YK\ell}}} \left(\ell_t\right)^{\frac{\sigma^{YK\ell}-1}{\sigma^{YK\ell}}} \right)^{\frac{\sigma^{YK\ell}}{\sigma^{YK\ell}-1}}$$

and

$$V_t^{Y}(E_t, Y_t^{K\ell}) = \max_{E_t, Y_t^{K\ell}} P_t^{Y,0} Y_t - r_t^E E_t - P_t^{YK\ell} Y_t^{K\ell}$$
$$\text{s.t. } Y_t = \left(\left(\mu^E\right)^{\frac{1}{\sigma^{Y}}} \left(E_t\right)^{\frac{\sigma^{Y}-1}{\sigma^{Y}}} + \left(1-\mu^E\right)^{\frac{1}{\sigma^{Y}}} \left({}^{K\ell}_{t}\right)^{\frac{\sigma^{Y}-1}{\sigma^{Y}}} \right)^{\frac{\sigma^{Y}}{\sigma^{Y}-1}}$$

Solving the problem of the production firm, we obtain the following output, Y_t, and marginal cost, $P_t^{Y,0}$. The Appendix provides a formal description of the problem and derives the solution. Note that the price-setting behavior of the production firm is treated as a separate problem described in the next section.

$$Y_t^{K\ell} = \left(\left(\mu^K\right)^{\frac{1}{\sigma^{YK\ell}}} \left(K_{t-1}\right)^{\frac{\sigma^{YK\ell}-1}{\sigma^{YK\ell}}} + \left(1-\mu^K\right)^{\frac{1}{\sigma^{YK\ell}}} \left(\ell_t\right)^{\frac{\sigma^{YK\ell}-1}{\sigma^{YK\ell}}} \right)^{\frac{\sigma^{YK\ell}}{\sigma^{YK\ell}-1}} \tag{2.19}$$

$$Y_t = \left(\left(\mu^E\right)^{\frac{1}{\sigma^{Y}}} \left(E_t\right)^{\frac{\sigma^{Y}-1}{\sigma^{Y}}} + \left(1-\mu^E\right)^{\frac{1}{\sigma^{Y}}} \left(Y_t^{KL}\right)^{\frac{\sigma^{Y}-1}{\sigma^{Y}}} \right)^{\frac{\sigma^{Y}}{\sigma^{Y}-1}} \tag{2.20}$$

$$P_t^{YK\ell} = \left(\left(\mu^K\right) \left(r_t^K\right)^{1-\sigma^{YK\ell}} + \left(1-\mu^K\right) \left(r_t^\ell\right)^{1-\sigma^{YK\ell}} \right)^{\frac{1}{1-\sigma^{YK\ell}}} \tag{2.21}$$

$$P_t^{Y,0} = \left(\left(\mu^E\right) \left(r_t^E\right)^{1-\sigma^{Y}} + \left(1-\mu^E\right) \left(P_t^{YK\ell}\right)^{1-\sigma^{Y}} \right)^{\frac{1}{1-\sigma^{Y}}} \tag{2.22}$$

Due to constant returns to scale, there are infinitely many solutions to the above. However, the solution must satisfy the following conditions for the ratio of inputs and input prices.

$$\frac{K_{t-1}}{\ell_t} = \frac{\mu_K}{1-\mu_K}\left(\frac{r_t^\ell}{r_t^K}\right)^{\sigma^{YK\ell}} \tag{2.23}$$

$$\frac{E_t}{Y_t^{K\ell}} = \frac{\mu_E}{1-\mu_E}\left(\frac{P_t^{YK\ell}}{r_t^E}\right)^{\sigma^Y} \tag{2.24}$$

Note that $P^{YK\ell}$ is the relevant variable to describe core inflation as it is the price of output that excludes the cost of energy.

The Philips Curve

This model aims to describe short and medium-run dynamics and not just the long run. In the long run, prices are perfectly flexible. However, in the short run, we assume price rigidity. Rigidity entails that the price signals cannot adapt perfectly in the short run, implying that the economy will deviate from the optimal allocation of resources.

In this model, we assume price adjustment costs of output as presented in Rotemberg (1982). The production firm operates under monopolistic competition and faces the demand elasticity η. Thus, the firm solves the following problem. Lowercase p_t^Y denotes the price of the individual firm, and y_t denotes individual production. Uppercase P_t^Y and Y_t are the corresponding aggregate levels.

$$\max_{p_t^Y} V_t = (p_t^Y - P_t^{Y,0}) \cdot y_t - \Psi_t^P + \beta V_{t+1}$$

s.t.

$$\Psi_t^P = \frac{\iota_0}{2}\left[\frac{p_t^Y/p_{t-1}^Y}{p_{t-1}^Y/P_{t-2}^Y} - 1\right]^2 P_t^Y Y_t$$

$$y_t = \left(\frac{p_t^Y}{P_t^Y}\right)^{-\eta} Y_t$$

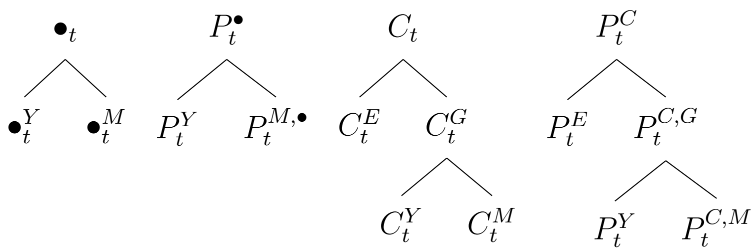
Solving and using symmetry yields the following price of output, P_t^Y. See derivations in Appendix. As the equation states a positive relationship between output and its price, it can be considered the Philips curve of our model.

$$
\begin{aligned}
P_t^Y &= \frac{\eta}{\eta-1} P_t^{Y,0} \\
&- \frac{\iota_0}{\eta-1}\left(\frac{1+\pi_t^Y}{1+\pi_{t-1}^Y}-1\right)\frac{1+\pi_t^Y}{1+\pi_{t-1}^Y} P_t^Y \\
&+ \frac{\iota_0}{\eta-1}\left(\frac{1+\pi_{t+1}^Y}{1+\pi_t^Y}-1\right)\frac{1+\pi_{t+1}^Y}{1+\pi_t^Y} P_{t+1}^Y \left(2\beta\frac{Y_{t+1}}{Y_t}\right) \qquad \text{with } \pi_t^Y = \frac{P_t^Y}{P_{t-1}^Y}-1.
\end{aligned}
\tag{2.25}
$$

Repacking Firms

The repacking firms combine output, Y_t, and imports, $\bullet_t^M$ for $\bullet \in \{C, G, I, X\}$, to produce consumption goods, C_t^G, public consumption goods, G_t, investment goods, I_t, and export goods, X_t. The consumption good, C_t^G, is combined with an energy consumption good, C_t^E, to produce to final consumption *bundle*, C_t. Hence, the repacking firm sells a bundle to the households that consists of goods and energy. It is up to the households to choose how much of the bundle to consume, given the price. The maximization problem is identical to the one for the production firm. For C_t, we use a similar nest structure as the one for the production firm. The nest structure of $\bullet_t \in \{G, I, X\}$ and C_t is illustrated in Figure 4.

Figure 4: Nest Structure of The Repacking Firm



Note that the inner nest of C_t will be substitutes, while the outer nest will be complements. The latter reflects the fact that energy generally is considered an essential good. Using the CES technology described in Appendix, we obtain the following for $\bullet_t \in \{C^G, G, I, X\}$.

$$\bullet_t^M = \mu^{M,\bullet} \left(\frac{P_t^\bullet}{P_t^{M,\bullet}} \right)^{\sigma^\bullet} \bullet_t \tag{2.26}$$

$$\bullet_t^Y = \left(1 - \mu^{\bullet,M}\right) \left(\frac{P_t^\bullet}{P_t^Y} \right)^{\sigma^\bullet} \bullet_t \tag{2.27}$$

$$P_t^\bullet = \left(\mu^{\bullet,M} \left(P_t^{M,\bullet}\right)^{1-\sigma^\bullet} + \left(1 - \mu^{\bullet,M}\right) \left(P_t^Y\right)^{1-\sigma^\bullet} \right)^{\frac{1}{1-\sigma^\bullet}} \tag{2.28}$$

$$C_t^E = \mu^E \left(\frac{P_t^C}{P_t^E} \right)^{\sigma^E} C_t \tag{2.29}$$

$$C_t^G = \left(1 - \mu^E\right) \left(\frac{P_t^C}{P_t^{CG}} \right)^{\sigma^E} C_t \tag{2.30}$$

$$P_t^C = \left(\mu^E \left(P_t^E\right)^{1-\sigma^C} + \left(1 - \mu^E\right) \left(P_t^{C,G}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma^C}} \tag{2.31}$$

Government

The government has a classic dynamic governmental budget constraint, allowing them to accumulate debt. Public consumption, G_t, and government debt, B_t^G, are exogenous. The government acquires tax revenue through income taxation on wages and unemployment benefit. The tax rate, τ_t, is defined to ensure a constant debt in steady state. However, if the debt is outside of its steady state value, we scale the tax rate with the governmental debt deviation from its steady state value to ensure a quicker convergence. Groth (2017)'s lecture notes inspired the modeling of the government.

$$\tilde{B}_t^G = (1 + r^B)B_{t-1}^G + U^B w_{SS} U_t + P_t^G G_t - \tau_{SS}(w_t L_t + U^B w_{SS} U_t) \tag{2.32}$$

$$\tilde{\tau}_t = \tau_{ss} + \varepsilon_B \frac{\tilde{B}_t^G - B_{ss}^G}{w_t L_t + U^B w_{SS} U_t} \tag{2.33}$$

$$\tau_t = \begin{cases} \tau_{ss} & \text{if } t < t_B \\ \tilde{\tau}_t & \text{if } t \geq t_B \end{cases} \tag{2.34}$$

$$B_t^G = (1 + r^B)B_{t-1}^G + U^B w_{SS} S_t + P_t^G G_t - \tau_t(w_t L_t + U^B w_{SS} U_t) \tag{2.35}$$

Foreign Economy

The foreign economy has a so-called Armington demand for domestic export goods, which depends on the relative price level.

$$X_t = \chi_t \left(\frac{P_t^Y}{P_t^F} \right)^{-\sigma^F} \tag{2.36}$$

Households - Consumption and Saving

The model has two types of households: Ricardian and Hand-to-mouth (HtM) households. The modeling of the households is inspired by Ejarque and Stephensen (2021). The Ricardian households are forward-looking and intertemporally optimizing. They choose consumption and savings according to model-consistent expectations of future prices and income, i.e., under perfect foresight. The HtM households are credit-constrained. They do not look forward, as they consume their entire income in a given year. The Ricardian households can ideally smooth consumption over time, while the presence of HtM households increases the aggregated marginal propensity to consume.

Both households earn taxed income from wages or unemployment benefits, and in addition, they get an evenly divided inheritance from older generations. Income for households is thus:

$$\text{inc}_{a,t} = (1-\tau_t)\underbrace{(w_t L_{a,t} + U^B w_{ss} U_{a,t})}_{\text{Income from labor or benefits}} + \underbrace{\frac{B_t^q}{A}}_{\text{Inheritance}} \qquad \text{and } \text{inc}_t = \sum_a \text{inc}_{a,t}$$

Aggregate consumption:

$$C_t = C_t^{OPT} + C_t^{HTM} \tag{2.37}$$

Ricardian households: The Ricardian households consume goods at the price P_t^C, and have end-of-period savings, $B_{a,t}$, from which they get a fixed nominal return given by the interest rate, r^{hh}. Excess savings in the last living period of the household, $B_{A-1,t}$, is inherited by younger generations - that is $B_{A-1,t} = B_t^q$. The optimizing household solves the following problem with υ being the share of Ricardian households.

$$\begin{aligned}
V_{t_0} &= \max_{\{C_{a,t=t_0+a}\}_{a=0}^{A-1}} \sum_{a=0}^{A-1} \beta^a \left[\frac{\left(C_{a,t}^{OPT}\right)^{1-\sigma}}{1-\sigma} + \mathbf{1}_{a=A-1}\mu^B \frac{\left(\frac{B_{a,t}}{P_t^C}\right)^{1-\sigma}}{1-\sigma} \right] \\
&\text{s.t.} \\
t &= t_0 + a \\
B_{-1,t} &= 0 \\
B_{a,t} &= (1+r^{hh})B_{a-1,t-1} + \upsilon \cdot \text{inc}_{a,t} - P_t^C C_{a,t}^{OPT}
\end{aligned}$$

Solving and using the inheritance inflows must match bequest outflows, $B_t^q = B_{A-1,t}$, we get:

$$C_{a,t}^{OPT} = \begin{cases} \left(\mu^B \left(\frac{B_t^q}{P_t^C}\right)^{-\sigma}\right)^{-\frac{1}{\sigma}} & \text{if } a = A-1 \\ \left(\beta \frac{1+r_{hh}}{1+\pi_{ss}^{hh}} \left(C_{a+1,ss}^{OPT}\right)^{-\sigma}\right)^{-\frac{1}{\sigma}} & \text{elif } t = T-1 \\ \left(\beta \frac{1+r_{hh}}{1+\pi_{t+1}^{hh}} \left(C_{a+1,t+1}^{OPT}\right)^{-\sigma}\right)^{-\frac{1}{\sigma}} & \text{else} \end{cases} \qquad \text{and } C_t^{OPT} = \sum_a C_{a,t}^{OPT} \tag{2.38}$$

$$B_t^q = B_{A-1,t} \tag{2.39}$$

See Appendix for derivations. From here, we calculate savings forwards:

$$B_{a,t} = (1 + r^{hh})B_{a-1,t-1} + \upsilon \cdot \text{inc}_{a,t} - P_t^C C_{a,t}^{OPT} \tag{2.40}$$

Hand-to-mouth households: In other models, it is customary to assume that the HtM do not have any assets, but we assume that they receive inheritance like the Ricardian households. Since they consume the entirety of their income, the HtM consumption is given by:

$$C_t^{HTM} = \frac{(1-\upsilon) \cdot \text{inc}_t}{P_t^C} \tag{2.41}$$

Goods Market Clearing

The production of the domestic output must match the output good used by the repacking firms. Import must match the sum of all imports and energy used by the repacking and production firms.

$$M_t = C_t^M + G_t^M + I_t^M + X_t^M + C_t^E + E_t \tag{2.42}$$

$$Y_t = C_t^Y + G_t^Y + I_t^Y + X_t^Y \tag{2.43}$$

2.4 Steady State

The economy is assumed to be in steady state for $t < 0$, that is, before the time horizon of our model. Steady state is the baseline equilibrium path of the model. If no shocks are applied, the economy will remain in steady state for $t = 1, ..., T$, but if we introduce a shock, we solve for a new, updated equilibrium path. Steady state exists in our model as no variable has a natural growth rate. To find a steady state, we predetermine a few variables: $\pi_{ss}^{hh} = 0$,

$m^s_{ss} = 0.5$, and $B^G_{ss} = 0$. Then, we normalize the prices:

$$P^Y_{ss} = P^F_{ss} = P^E_{ss} = P^{M,\bullet}_{ss} = \frac{\eta}{\eta - 1}, \bullet \in \{C, G, I, X\}^{2}$$

The pricing behavior of repacking firms then implies:

$$P^{\bullet}_{ss} = 1, \bullet \in \{C^G, G, I, X\}$$

We can now find steady state of all variables as shown in Appendix. The parameter ϕ is uniquely defined in steady state. This ensures a more efficient code in which we set the value of $m^s_{ss} = 0.5$, and then we can solve the rest of the model in steady state and pick the ϕ that will ensure we can solve steady state. The derivation of steady state is shown in the Appendix.

[2] Usually one normalizes prices to 1 for simplicity. However, we encountered issues when solving for steady state because the production price is a markup of marginal costs. We found that the most efficient solution was to normalize prices to the markup in steady state. It has no impact on the impulse response functions as the model depends on relative prices and not the absolute price level.

3 Method

We now proceed to our method. Firstly, we describe the model setup in Python. Secondly, we address how the model is solved through numerical optimization. Finally, we outline the calibration of the model parameters.

3.1 Solving the Model

The model is written and solved in Python. A link to our Git repository is provided on the front page. We divide the code into an *ordered series of blocks* as presented in The Block Structure in Appendix. For instance, the block *The Government* contains the model equations that determine government behavior, 2.32, 2.33, 2.34, and 2.35. Each block has input and output variables. *The Government* has the wage, w_t, as (one of its) input variables and produces the debt, B_t^G, as an output. Each block can use output variables of previous blocks and *unknown* variables (these are explained below) as input variables. Appendix illustrates the input-output structure. Generally, the block structure is trivial, as we can always update unknowns and targets. However, the order is important when we solve the model for a given set of unknowns and targets.

Some of the blocks imply *target equations* that must be satisfied in equilibrium. For instance, the block *Production Firms* has the first order condition 2.23 as a target. The targets are, in other words, the conditions that ensure optimal agent behavior and market clearing. When the targets are fulfilled, the economy is in equilibrium, so solving the model is basically ”just” solving the system of equations.

We use the *unknown* variables to solve the targets. For a solution to exist, the number of targets must equal the number of unknown variables. Our model contains 7 targets and 7 unknown variables per period t (for the total of T periods, this becomes 3500 targets and 3500 unknowns). Again, we refer to Appendix for an overview of the targets and unknowns. Once the targets are solved, the unknowns will determine the remaining variables in the model.

3.2 Solving the Targets

We solve the targets through numerical optimization. In particular, we use *Broyden's method*. As Broyden's method relies heavily on Newton's method, we will mainly describe how Newton's method works and briefly explain why using Broyden is optimal. This section aims not to describe the exact workings of Broyden's method in Python but rather to give a rough idea of how a solution is computed.

Newton's Method

To start, let us consider the one-dimensional case as it is the same underlying principle for multiple variables. Let $f(x)$ with $x \in \mathbb{R}$ be the function of interest. In our case, $f(x)$ corresponds to a target and could, for instance, be the first-order condition of optimal household behavior. The goal is to obtain x^* such that $f(x^*) \approx 0$, not by analysis but by a computational algorithm we can implement in Python.

Assuming that we know the function value and the derivative at any given point, we can compute the first-order Taylor approximation of $f(x)$ at a point x_0. These assumptions are fulfilled for all targets in our model. One could choose a higher-order approximation to find solutions quicker, but a first-order approximation suffices. Rearranging the Taylor approximation, we obtain the following:

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$
$$f(x) = 0 \iff x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

From here, we write the algorithm as follows:

0. Set $n = 0$, guess x_0 and choose $\epsilon > 0$.

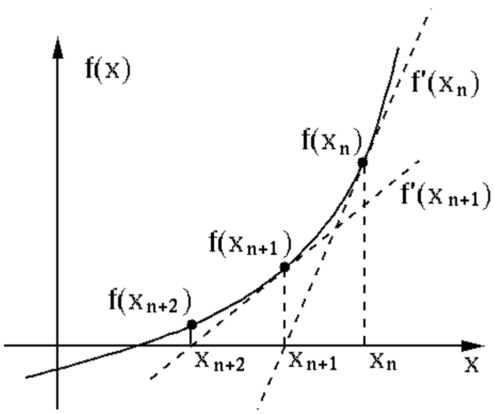
1. Calculate $f(x_n)$ and $f'(x_n)$.

2. If $|f(x_n)| < \epsilon$, then end. Otherwise, proceed to 3.

3. Let $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, then update $n = n + 1$ and go back to 1.

We choose $\epsilon = 10^{-8}$ and let the initial guess be the steady state values. The algorithm is of a simple "trial and error" nature: One computes the first order approximation of f at a guess x_n and checks if $|f(x_n)| < \epsilon$. If not, one computes the next guess x_{n+1} as the root of the previous first-order approximation, which over iterations will converge to a solution. Figure 5 illustrates the process.

Figure 5: Illustration of Newton's Method



Since we have 7 targets that must hold for all $T = 500$ periods, we have 3500 equations of which we need to find the roots. Therefore, we formally state the multi-dimensional case below.

$$\text{All unknowns:} \quad \mathbf{x} = \left(\underbrace{x_{1,1}, ..., x_{1,T}}_{\text{1st unknown}}, \underbrace{x_{2,1}, ..., x_{2,T}}_{\text{2nd unknown}}, \quad ... \quad, \underbrace{x_{7,1}, ..., x_{7,T}}_{\text{7th unknown}} \right)$$

$$\text{All targets:} \quad f(\mathbf{x}) = \left(\underbrace{f_{1,1}(\mathbf{x}), ..., f_{1,T}(\mathbf{x})}_{\text{1st target}}, \underbrace{f_{2,1}(\mathbf{x}), ..., f_{2,T}(\mathbf{x})}_{\text{2nd target}}, \quad ... \quad, \underbrace{f_{7,1}(\mathbf{x}), ..., f_{7,T}(\mathbf{x})}_{\text{7th target}} \right)$$

For instance, $x_{1,t}$ is the first unknown of period t, and $f_{1,t}(\mathbf{x})$ is the first target of period t. Similar to the one-dimensional case, the goal is to find $\mathbf{x}^*$ such that $f(\mathbf{x}^*) \approx \mathbf{0}$. Note that all targets constitute a homogeneous system of equations implying that it can be stated in the form $f(\mathbf{x}) = \mathbf{0}$. The algorithm is similar to the one stated earlier, but now computing the next guess involves the Jacobian, $\mathcal{J}$, instead of the derivative:

$$\mathbf{x}_{n+1} = \mathbf{x}_n - f(\mathbf{x}_n)\mathcal{J}_{\mathbf{x}_n}^{-1}$$

Our Jacobian becomes a 3500×3500 matrix.

$$\mathcal{J}_{\mathbf{x}_n} = \begin{bmatrix} \frac{\partial f_{1,1}(\mathbf{x}_n)}{\partial x_{1,1}} & \dots & \frac{\partial f_{1,1}(\mathbf{x}_n)}{\partial x_{7,T}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{7,T}(\mathbf{x}_n)}{\partial x_{1,1}} & \dots & \frac{\partial f_{7,T}(\mathbf{x}_n)}{\partial x_{7,T}} \end{bmatrix}$$

Computing the inverse of a matrix is very costly (especially for a 3500×3500 matrix). Therefore we use Broyden's method to update the Jacobian instead of computing a brand new one for each iteration n.

3.3 Code Overview

We divide the code into five different modules. `BabyMAKROModel.py` defines the model variables, parameters, and functions for running the model. `Blocks.py` contains the code for each block. `Steady_state.py` solves for steady-state values of each variable. `Broyden_solver.py` contains the code that solves the targets. `run_model.ipynb` is a Jupyter notebook that runs the entire model. It provides the essential information needed when solving the model, such as steady-state values and target values. Furthermore, it allows for shocks to exogenous variables, and prints the resulting impulse response functions. `para_exp.ipynb` allows one to compare impulse response functions for different parameter values and shock settings. The plots of chosen variables will be relative to steady-state levels.

Our Git repository has two branches: `main` and `extended`. The `main` branch contains the model as it is presented in The Block Structure. The `extended` branch contains the same model, but we expand the wage function to 7.1 presented in Discussion.

3.4 Calibration

We calibrate our model using external literature. We base our parameters on the parameters estimated in MAKRO; see Kronborg and Ejarque (2021). DREAM set the long-term parameters on external literature, while the short-term parameters are based on impulse response matching with SVAR econometric analyses; see DREAM (2021). See Appendix for an overview of all parameters. In some cases, there is no corresponding parameter in MAKRO. In those cases, our parameters are freely chosen, and we note that these values are somewhat arbitrary or that the empirical foundation is weak. Nonetheless, we aim for consistency among the parameters as, for instance, the weight on imports is the same for all goods C_t, G_t, I_t, and X_t. Furthermore, we set all interest rates to 0.04, which implicitly defines the household's discount factor $\frac{1}{1+r} = \frac{1}{1.04} = 0.96 \approx 0.95 = \beta$.

4 Shock Analysis

In this section, we implement shocks to the exogenous variables, plot the resulting impulse response functions, and analyze the shock dynamics. We apply two shocks: A primary shock to the domestic price of energy and a secondary shock to foreign prices. Both shocks have a negative impact on demand and supply. The shock to foreign prices reflects the fact that the energy crisis is global and therefore affects the price levels of Danish trade partners.

We will first specify how we implement the shocks. Then, we outline the overall impact of both shocks on core inflation and proceed to an in-depth analysis of each shock. Finally, we examine the implications of the shocks occurring at the *same* time. Note that each shock's in-depth analysis also examines mechanisms that do not directly affect inflation levels. It functions as a "sanity check" on the model checking that output aligns with reasonable economic results.

When the computer solves for an equilibrium path, the entire model is determined simultaneously and can run without sanity or human intuition. The intuition presented in the analysis is reserved for our understanding of the model and is irrelevant to the computer's ability to solve the model.

4.1 Shock Specification

Our shock takes the following form such that an exogenous variable, X_t, is modeled as

$$X_t = X_{ss} + \text{Shock}_t$$
$$\text{Shock}_t = \begin{cases} X_{ss} \cdot \text{Amplitude} \cdot \text{Persistence}^{t-T_{\text{start}}} & \text{if } T_{\text{start}} \leq t < T_{\text{end}} \\ 0 & \text{else} \end{cases}$$

Timing

To create reasonable impulse responses in a model with perfect foresight, we will apply *MIT shocks*. MIT shocks are unpredictable, sudden shocks occurring in the steady state where agents do not expect shocks. The paper by Boppart et al. (2018) argues that implementing MIT shocks is the best way to compute a reasonable equilibrium path for a deterministic heterogeneous-agent model with perfect foresight.

Due to perfect foresight, agents will foresee any shocks made in $t > 0$. They will know the impact of the shock and how other agents will react. The agents will use this information such that rigid variables will adapt ahead of the shock. For instance, imagine a shock in $t = 10$ that leads to higher capital. As capital is rigid, agents will prepare by letting capital grow before $t = 10$. They will accept an excessive amount of capital *before* the shock to prevent being short in capital when the shock occurs. Therefore, to create a "surprise effect", we set $T_{\text{start}} = 0$. After all, the energy price shock seen in 2022 was a surprise without warning for the agents.

Amplitude and Linearity

The shock amplitude indicates "the size" of the shock - i.e., the degree of deviation from steady state in the shock variable. The amplitude affects the shock dynamics. Why? A simple example is equation 2.18, which is the real adjustment cost in capital. This equation contains a quadratic term; therefore, the greater the shock, the greater the impact on adjustment costs. More formally, this means that our model is not linear. Suppose x and y are inputs of two shocks and $f(\bullet)$ to be the model's output. Then, the model is a linear map if it satisfies the following as stated by Edwards (1994).

1. *Homogeneity of degree 1*: $f(ax) = af(x)$
2. *Additivity*: $f(x + y) = f(x) + f(y)$

The second condition implies that it is possible to add the impact of partial shocks and get the same result as when implementing all the shocks at once. Even though our model might not be linear, we analyze each shock separately to identify the critical shock dynamics. We choose an amplitude of 1 pct. for each shock, and note that, i.e., an amplitude of 2 pct. will not just result in impulse response functions that are twice as big due to non-linearity. In other words, the impulse responses are not proportional to the amplitudes.

Persistence

The persistence of the shock indicates the degree to which the shock persists as time progresses. Note that this *exogenous* persistence setting is different from the *endogenous* persistence created by rigidity mentioned in the previous section. As the persistence setting approaches 1, the shock takes infinitely long to fade. The shock is present until $T_{\text{end}} = 20$, and we set persistence to 0.75, which is a relatively high persistence. The longer the shock is present, the longer it will take for the economy to converge. Although shocks are unlikely to last that long in reality, it does not critically affect the shock dynamics. We prefer a long and persistent shock as it is easier to illustrate the effects and transmission mechanisms of the shock.

The Shock Variables

- **The primary shock to domestic energy prices:** As we assume that households and production firms face the same energy prices, we implement a 1 pct. increase in the price of energy consumed by households, P_t^E, and in the input price of energy used in production, r_t^E. This is a supply shock since it leads to higher production costs, but also a demand shock, as increasing prices erodes real income and reduces demand.
- **The secondary shock to foreign prices:** As we assume that the foreign economy is affected by the energy price shock, we implement a 1 pct. increase in all foreign prices, which include the price of all imported goods, $P_t^{M,\bullet}$ for $\bullet \in \{C, G, I, X\}$, and the foreign price level, P_t^F. The latter enters in the export demand function 2.36.

4.2 Impact of Shocks on Inflation

Overall, the shock to energy and foreign prices results in elevated consumer prices, but the effect on core inflation is ambiguous. On the one hand, as energy prices surge, the marginal cost of production rises, which causes elevated consumer and output prices. The higher consumer and output prices result in lower activity as demand and production decrease. The lower production contributes negatively to core inflation. On the other hand, as *foreign* prices rise, imported goods are substituted for domestic output, and export increases since the domestic price competitiveness is improved. These effects increase production and, thus, core inflation.

4.3 Shock to Energy Prices

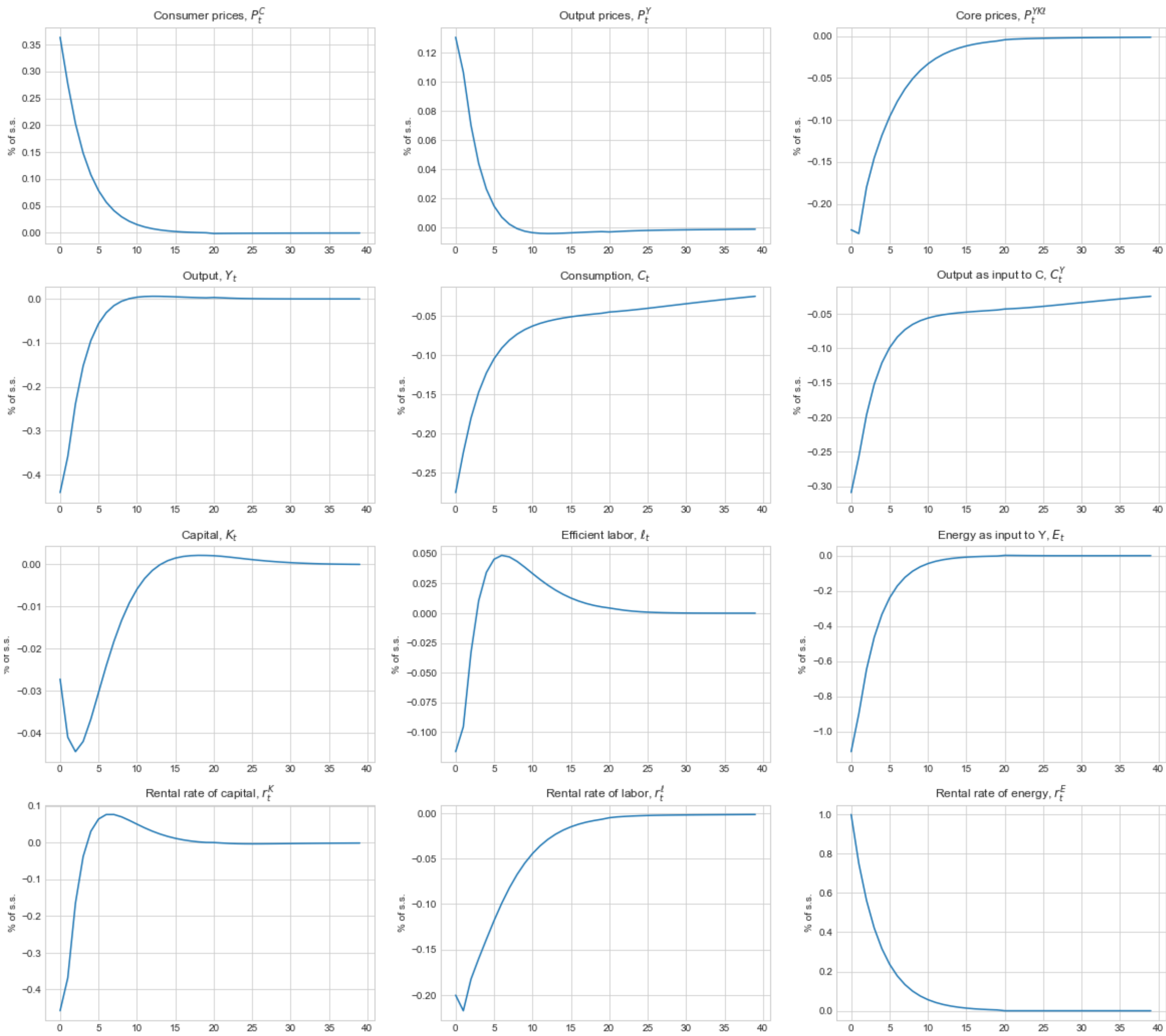
In this in-depth shock analysis, we consider how the primary shock affects 1. the production, 2. labor and capital, 3. rental rates, and 3. the households. We address production first, as production affects the rest of the model. The sections on labor & capital markets and households provide a more thorough understanding of why production was affected the way it was. Naturally, many of the causal relationships are interconnected, and this division of the analysis is superficial, but it is the most intuitive approach. The primary shock is shown on the next page in figure 6.

Production

First and foremost, as energy prices increase, input prices of production rise, which causes the marginal cost and thus output prices, P_t^Y, to rise as shown in figure 6. The more production relies on energy, i.e. the bigger the μ^E, the greater this effect will be; see figure 7. The higher producer prices will be transferred into higher "final good prices" set by repacking firms, $P_t^\bullet$ for $\bullet \in \{C, G, I, X\}$. Therefore, the demanded quantity of output shrinks and production drops leading to negative core inflation, $P_t^{YK\ell}$. However, the rise in $P_t^\bullet$ is limited as the repacking firm can substitute domestic output for import as we assume $\sigma^{M,\bullet} > 1$ for $\bullet \in \{C, G, I, X\}$. Consequently, import functions as a cushion for the repacking firm as the energy prices rise.

A decline in demand from the foreign economy reinforces the drop in demand, production, and core inflation. As domestic producer prices rise, the Danish economy becomes less competitive relative to the foreign economy resulting in a drop in export demand. This leads to a fall in production and, thus, core inflation.

Figure 6: The primary shock to r_t^E and P_t^E



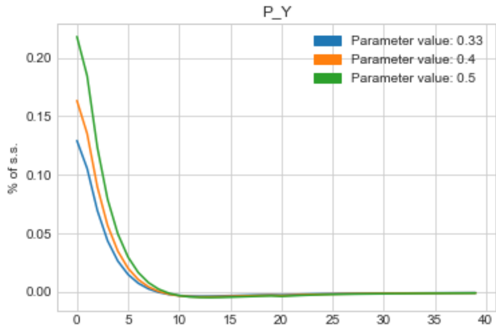
Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.

Labor and Capital

As stated above, when energy prices increase, it leads to a fall in production. The effect of the energy shock on the production inputs, labor, and capital, can mainly be explained by the following.

- **Reduced output demand:** As output demand decreases, the production firm must reduce output, which leads to a drop in *all* production inputs, i.e. labor and capital, which will fall.

Figure 7: Shock to r_t^E and P_t^E for different values of μ_E



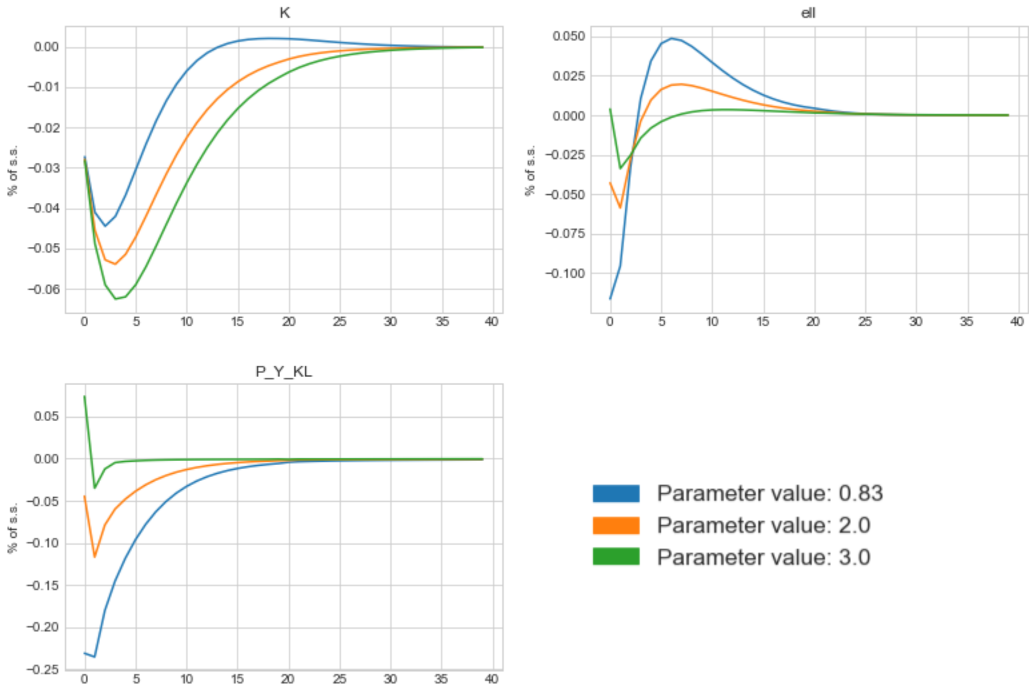
Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.

- **Rise in r_t^E:** The rise in energy prices will, in isolation (that is, irrespective of the drop in demand), reduce the use of energy as input. But since energy is assumed to be a complementary input, a drop in energy will also drag down capital and labor. This contributes to lower core production, $Y_t^{K\ell}$, and thus a lower core inflation, $P_t^{YK\ell}$. Figure 8 shows that the better substitutes energy and labor&capital are, the less it will drag down labor&capital with it. Thus, core production and core inflation will be less affected.

Although both labor and capital are negatively affected, the drop in labor exceeds the reduction in capital. In other words, labor absorbs more of the shock because of the relative rigidity between capital and labor. The capital agency faces adjustment costs, while the labor agency faces vacancy costs. In our model, we assume capital to be more rigid than labor as the adjustment cost of capital, Ψ, is a quadratic cost function, while the vacancy cost, κ^L, is a linear cost function. As a result, it takes longer for capital to adapt to shocks. Therefore, labor absorbs more of the shock. Although higher investment prices reinforce the drop in capital, P_t^I, making it more expensive to accumulate capital.

Furthermore, we observe an "over-shooting effect" for both labor and the rental rate of capital in the impulse responses in figure 6. This over-shooting effect is created by the labor agency. The more difficult it is for the labor agency to acquire labor, the greater the over-shooting effect. So, the more unfilled positions (higher σ^m), and the higher the cost of a vacancy (higher κ^L), the more the firms will hoard labor as the cost of hiring a new worker

Figure 8: Shock to r_t^E and P_t^E for different values of σ_Y



Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75. σ_Y is the substitution elasticity between energy and labor&capital.

increases. We show in Appendix that higher convexity, i.e., higher σ^m, will lead to fewer matches for each period and, thus, more unfilled positions.

Rental Rate of Inputs

As the labor and capital inputs drop, their rental rates will also decrease. We will first analyze the impact on the rental rate of labor and then the reciprocal adaptation between them.

- **The rental rate of labor:** The rental rate of labor and wages are decided simultaneously in the model. Hence, a change in wages will affect the rental rate of labor and the other way around, which can be seen in the following:

$$r_t^\ell = \frac{1}{1 - \frac{\kappa^L}{m_t^v}}(w_t - \frac{1-\delta_t^L}{1+r^{\text{firm}}}\frac{\kappa^L}{m_{t+1}^v}r_{t+1}^\ell)$$

$$w_t^* = \phi(r_t^\ell + \theta_t\kappa^L) + (1-\phi)w^U$$

An essential component of wage bargaining is the labor market tightness, $\theta_t = v_t/S_t$, which expresses demand for labor. As demand for labor falls when production falls, it results in lower wages since the labor union is in a worse bargaining position. Therefore, as wages fall, the cost for the labor agency falls. The fall in wages will be transferred into the rental rate of labor as the labor agency reduces its costs.

- **The rental rate of capital:** When production falls, the capital agency must reduce employed capital. However, the capital agency faces quadratic real adjustment costs, and therefore it is too costly to let capital adjust immediately for the capital agency. Instead, it will be more optimal for them to reduce the rental rate of capital.

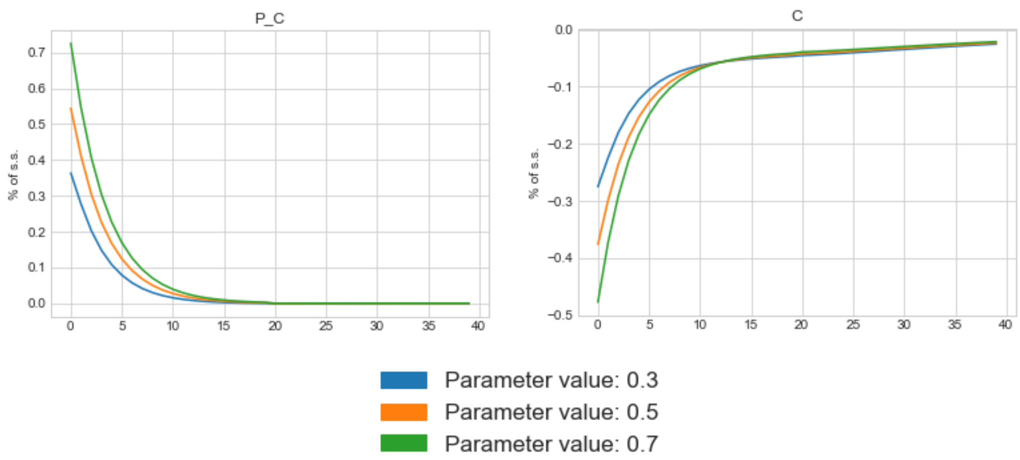
Households

Increasing energy prices result in rising consumer prices, P_t^C, which erodes real income and reduces consumption. This erosion of income is partially due to rising prices of consumption *goods*, $P_t^{C,G}$, which stems from rising input prices of energy for the production firms, and partially due to increasing prices of energy *consumption*, P_t^E. The nominal wage drop mentioned in the previous section reinforces the fall in real income and consumption. However, the government acts as a stabilizing sector as the fall in income is counteracted by fixed unemployment benefits. The unemployment benefit is financed by short-run debt, which the government repays in the long run by collecting higher taxes.

The reaction of the households to the energy price shock is especially affected by the following four parameters. These parameters recreate many of the same effects as in Kilian (2008).

- **Proportion of energy in consumption bundle, $\mu^{E,C}$:** The final consumption prices depend on the weight of energy in the consumption bundle. Thus, the greater the share of energy, the more consumer prices will rise following the energy price shock, and the drop in consumption will be even greater; see figure 9.
- **Elasticity of substitution, σ^C:** We assume consumption of energy, C_t^E, to be a complementary good with intermediate goods, C_t^G, in the consumption bundle.

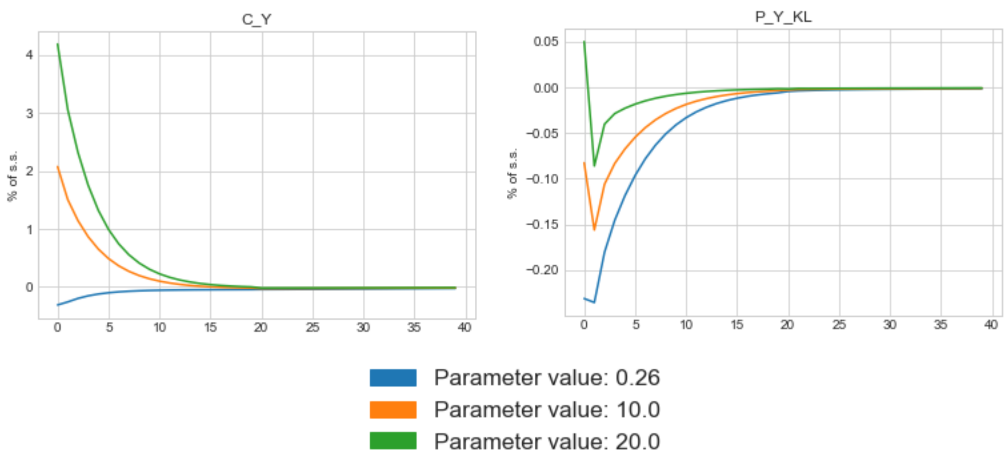
Figure 9: Shock to r_t^E and P_t^E for different values of $\mu^{E,C}$



Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.

Therefore, the income effect will dominate the substitution effect, such that a rise in energy prices will decrease consumption goods, $C_t^{C,G}$. However, if we increase the elasticity of substitution, more energy is substituted with goods affecting core inflation positively; see figure 10.

Figure 10: Shock to r_t^E and P_t^E for different values of σ^C

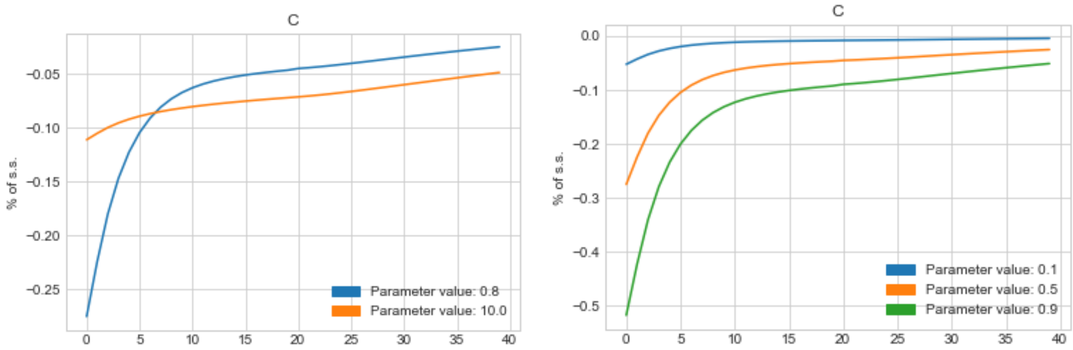


Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.

- **Proportion of HtM households,** υ**:** The HtM households are credit constrained, which means they do not have the intertemporal option to smooth consumption but are instead consuming their whole income in a given period. Therefore, the presence of HtM households implies an increased marginal propensity to consume. So, as we increase the share of HtM households $1-\upsilon$, consumption increases as shown in 11b on the next page.

Figure 11: Shock to r_t^E and P_t^E

(a) for different values of σ (b) for different values of υ



Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.

- **The intertemporal elasticity of substitution, σ:** As the energy price shock hits and prices increase, Ricardian households aim to smooth their consumption. The greater σ is, the more determined the Ricardian households are to smooth consumption. As a result, a greater σ implies that the initial effect of the energy price shock on consumption is reduced; see figure 11a.

However, since we assume $\sigma < 1$, we find that the presence of Ricardian households results in a *more volatile* consumption path. Why? If we discard the unique solutions for $A-1$ and $T-1$, then the general solution for the Ricardian household is:

$$C_t^{OPT} = \left(\left(\beta \frac{1+r^{hh}}{1+\pi_{t+1}}\right)\left(C_{t+1}^{OPT}\right)^{-\sigma}\right)^{\frac{-1}{\sigma}} = \left(\frac{1+\pi_{t+1}}{\beta(1+r^{hh})}\right)^{\frac{1}{\sigma}} C_{t+1}^{OPT}$$

When the energy price shock occurs, consumer prices initially increase and then converge back to steady state. This is technically *deflation* since $\frac{P_{t+1}^C}{P_t^C} < 1 \Leftrightarrow 1+\pi_{t+1}^{hh} < 1$. Furthermore, we previously defined $\beta \approx \frac{1}{1+r^{hh}}$, which implies $\beta(1+r^{hh}) \approx 1$. Thus $\frac{1+\pi_{t+1}}{\beta(1+r^{hh})} < 1$. Taking the limits of $\sigma \in (0,1) \vee (1,\infty)$, we get:

$$C_t \to C_{t+1} \text{ for } \sigma \to \infty$$
$$C_t \to \frac{1+\pi_{t+1}}{\beta(1+r^{hh})} C_{t+1} \text{ for } \sigma \to 1$$
$$C_t \to 0 \text{ for } \sigma \to 0^+$$

The only reason we get this result is because $\frac{1+\pi_{t+1}}{\beta(1+r^{hh})} < 1$. Therefore, as we assume $\sigma < 1$, the Ricardian households will lower current consumption to get more consumption in the future. The opposite would be true if $\frac{1+\pi_{t+1}}{\beta(1+r^{hh})} > 1$. Consequently, if we assumed a high enough σ, the Ricardian households would smooth consumption and have a less volatile consumption path than the HtM households.

4.4 Shock to Foreign Prices

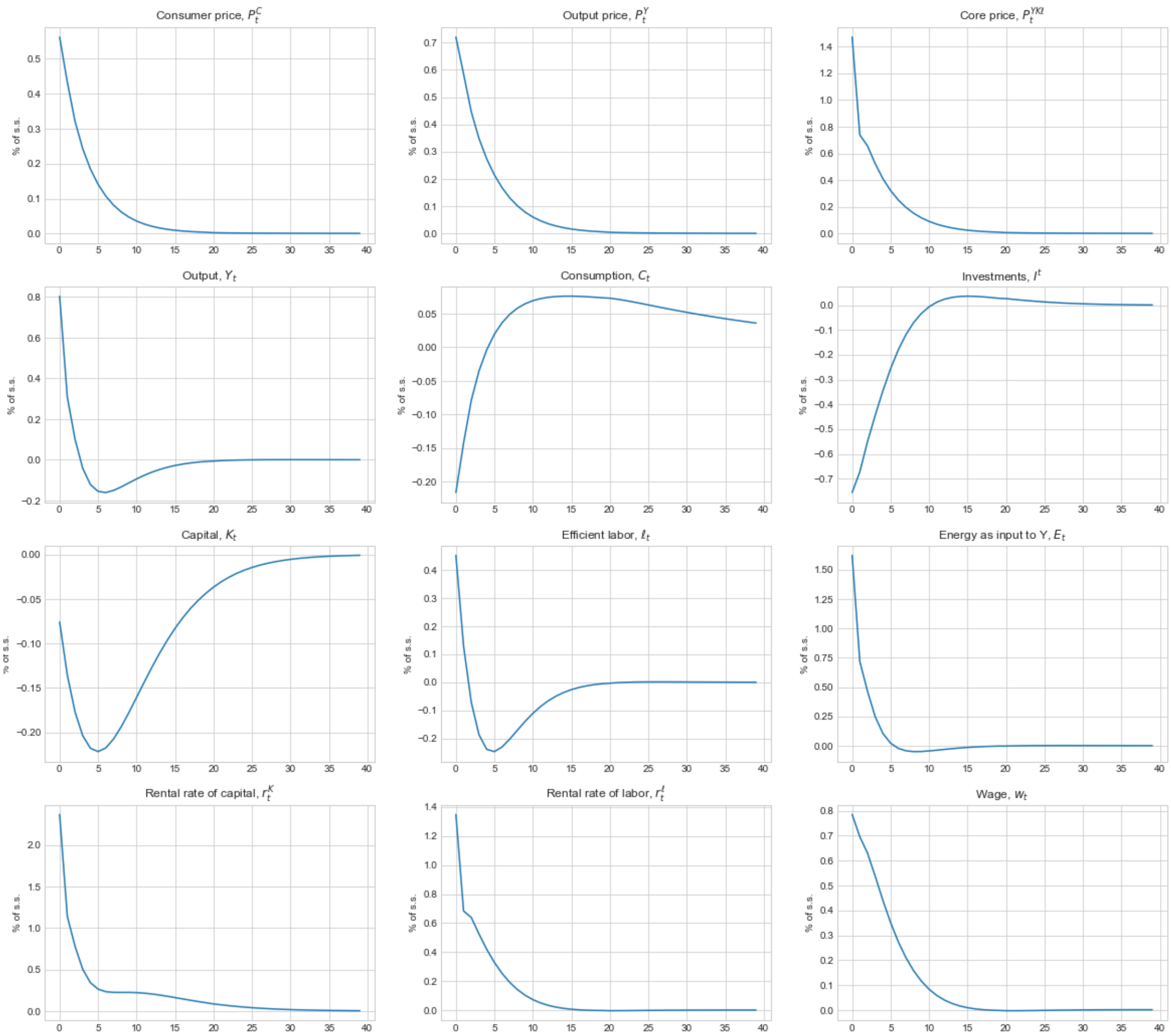
The secondary shock is a demand shock as foreign demand for domestically produced goods rise. Many of the transmission mechanisms at play in the primary shock will also be involved in the secondary shock. Therefore, we will try to avoid repetition. The secondary shock can be seen on the next page in figure 12.

Production

As illustrated in figure 12, a shock to foreign and all import prices will, like the shock to domestic energy prices, cause higher consumer prices, P_t^C, and higher producer prices, P_t^Y - but for different reasons. For the primary shock, producer prices increase as a direct result of a higher input price of energy. For the secondary shock, producer prices increase due to elevated demand. The higher production causes an increase in core inflation, $P_t^{YK\ell}$. There are two reasons for the rise in production: The first is improved price competitiveness, while the second is a substitution effect.

- **Improved price competitiveness:** As we assume a fixed exchange rate, we only have to consider the relative prices between the foreign and domestic economies. Equation

Figure 12: The secondary shock to P_t^F and $P_t^{M,\bullet}$



Note: 1 pct. shock to P_t^F and $P_t^{M,\bullet}$ for $t = 1, ..., 20$ with persistence of 0.75.

2.36 states; the higher the price elasticity of export demand, σ^F, which means the export demand function will be more convex, the more a change in relative prices will affect export. Higher export demand pressures production and implies a higher rental rate of inputs and core inflation.

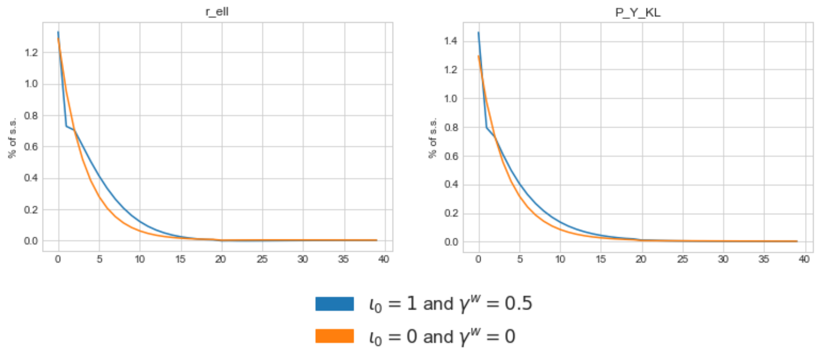
- **Substitution towards domestic output:** As we assume output and import to be substitutes, higher import prices create an incentive among the repacking firms to substitute import with domestic production. We note that this substitution could be prevented if the rise in domestic output prices exceeds the increase in import prices. However, this is not the case as output prices increase by 0.7 pct., while the import prices by assumption increase by 1 pct.

As mentioned above, the relative price between the domestic and foreign economies determines the export demand. One might ask, how is the relative price affected by the energy shock? This is a difficult question with many possible answers. However, some of it has to do with the *energy responsiveness* defined as "the change in producer prices, P_Y, given a change in energy prices, P_t^E and r_t^E". Suppose the Danish economy is *less* energy responsive than the foreign economy. In that case, foreign prices will increase more than Danish prices when the energy shock occurs, implying higher export demand in isolation. The energy responsiveness in our model depends on at least two parameters.

- **The elasticity of substitution for energy:** The more energy can be substituted with other inputs and goods, the less energy prices will affect producer prices.
- **Energy efficiency:** We define energy efficiency as Gillingham et al. (2009) to be "the services provided per unit of energy input". In our model, energy efficiency depends on the parameter μ^E. The greater this value, the more energy the production firm needs relative to labor and capital, and the more a rise in energy prices will affect the marginal cost and the price of output.

Finally, note that rigidity in wages and prices can cause the IRF to appear uneven, as shown in figure 13.

Figure 13: Shock to P_t^F and $P_t^{M,\bullet}$ for different values of ι_0 and γ^w



Note: 1 pct. shock to P_t^F and $P_t^{M,\bullet}$ for $t = 1, ..., 20$ with persistence of 0.75. The figure shows that if we remove the wage and price rigidity completely, the convergence to steady state is completely smooth.

Labor and Capital

Overall, the same dynamics are at play when the foreign prices rise as when energy prices rise, but the foreign price shock results in *higher* production and core inflation, which is the opposite of the energy shock. In addition, the allocation of the production inputs will differ from the primary shock essentially because energy prices are exogenous. So, as production increases, the rental rate of energy does not change as opposed to capital and labor. Therefore, energy is the input that increases the most. However, this effect is limited because energy complements capital & labor.

We found a drop in capital and labor for the primary shock. For the secondary shock, we find a more asymmetric development of the core inputs as capital decreases while labor increases. This can be explained by the following.

- **Rigidity in capital:** As production increases, core inputs must increase. In the absence of adjustment costs, capital and labor would increase equally. However, capital is more rigid *relative* to labor as discussed in Shock to Energy Prices, causing the shock to be absorbed mainly by labor.
- **Rise in the price of investment:** As investment goods are combinations of imports and domestic output, and because the price of both increase as the foreign shock occurs,

the price of investment becomes higher. The higher investment cost makes maintaining and expanding capital more expensive and thus causes the shock to be absorbed mainly by labor.

Due to the rigidity in capital, production firms limit their use of capital when the shock occurs while labor increases. However, as time progresses, capital is slowly adjusted, and a balance between labor and capital is restored.

Rental Rate of Inputs

The reallocation of capital and labor affects the rental rates. The drop in capital and increase in labor imply a higher ratio between labor to capital. As a result, capital is more productive relative to labor due to positive cross-derivatives:

$$\frac{\partial^2 Y^{KL}}{\partial \ell \partial K} = \left(\frac{\mu^K (1 - \mu^K)}{K\ell} \right)^{\frac{1}{\sigma^{KL}}} \frac{1}{\sigma^{KL}} (Y^{KL})^{\frac{2-\sigma^{KL}}{\sigma^{KL}}} > 0$$

The cross-derivative implies that the rental rate of capital grows more relative to the wage.

Households

We discussed these dynamics in Shock to Energy Prices. However, it is worth explaining the odd parabola shape of the consumption path illustrated in figure 12. The foreign price shock will have two opposite effects on real income. Firstly, higher prices following elevated production will, in isolation, cause a reduction in real income. However, elevated production levels will also imply greater labor demand and growing wages. Initially, the negative effect of inflation dominates, but as time progresses, consumption and real income grow above their steady state level.

4.5 Main Results

Analyzing these two shocks in the economy, we get the four following results.

The first result is the most obvious. The greater the ability to substitute energy with other goods and inputs, the less impact an energy shock has on the economy and inflation.

In particular, we found that if energy is a complementary input and good, a rise in energy prices will also cause other inputs and goods to decrease. This contributes negatively to core inflation.

The second result is not as obvious. An energy shock could potentially make a country richer in the short term, essentially because the positive demand shock from higher foreign prices could outweigh the negative supply shock from higher energy prices. However, this result depends on the country's *energy responsiveness*. The less energy responsive a country is relative to the foreign economies, the less domestic prices are affected by higher energy prices, and thus the higher export demand will be. If this effect is strong enough, it could dominate the negative effect of higher energy prices.

The third result relates to the international impact of an energy shock. Initially, the energy shock has an asymmetric effect on the international economy: Domestic price levels will be less affected in less energy-responsive economies. In comparison, domestic price levels will be affected more in more energy-responsive economies. Therefore, the more energy-responsive economies will attempt to substitute domestic output for import from less energy-responsive economies. Meanwhile, this causes "convergence to symmetry" as increased import in more energy-responsive economies contribute negatively to domestic price levels, and increased export demand in less energy-responsive economies contributes positively to domestic price levels. In this sense, the energy shock initially has an asymmetric effect on the international economy as it affects countries' price levels differently. But due to changes in relative prices, the differences in domestic price levels are minimized through international trade. However, we note that this result presumes a relaxation of the assumption that a small open economy has no impact on international price levels.

The final result is the most striking. Although the foreign price shock creates some core inflation, it seems improbable that this shock is large enough to outweigh the negative supply shock from higher domestic energy prices. Thus, the effect on core inflation from an energy shock is ambiguous. In our model, core inflation is defined by the rental rate of inputs, which are highly correlated with production levels. Hence, it is not possible to achieve core inflation without increasing production.

5 Discussion

From the analysis, we have one key finding: The energy shock's impact on core inflation is ambiguous. The primary shock affects core inflation negatively, while the secondary shock affects core inflation positively. However, since recent data presented in Introduction shows high core inflation for all of Europe, *something* must be missing. Our model could be incorrect, or perhaps something entirely different explains the high core inflation. These are the main themes of our discussion. In particular, we address the following:

1. Is the model too simple?
2. Alternative explanations for core inflation
   - Is it something else driving up core inflation?
   - Is there a missing link between core inflation and energy prices in the model?
3. Comparison of our model to other DDGE models.

5.1 Is The Model Too Simple?

As with all models that fail to predict accurately, we rush to reevaluate the underlying assumptions. However, it is essential to note that not all of our model assumptions that seem unrealistic are subjects of criticism. According to Friedman[3], the quality of a model should depend on its reliability and ability to predict accurately - not on whether it is a one-to-one description of reality. Even though some of the assumptions of Baby-MAKRO are unrealistic, e.g., perfect foresight, they are not necessarily invalid assumptions; In the very short run, perfect foresight is questionable, but in the long run, it is generally a reasonable assumption. Predictions of models with perfect foresight, like the ones presented in this paper, serve as benchmark scenarios for the perfect rational world, highlighting fundamental economic dynamics.

[3] A paper discussing Friedman's ideas: Melitz (1965).

Model Structure

Firstly, one could argue that we have created an ”ad hoc model” that shows exactly want we wanted to show in the first place. However, we attempted to pick the most critical dynamics from MAKRO to describe core inflation, and since the model is having difficulties recreating the ongoing core inflation, this criticism seems misplaced.

It is possible that our model cannot reproduce core inflation because it lacks the complexity necessary to describe the Danish economy. A possible expansion could be a domestic energy sector. This dynamic would limit the fall in production when introducing an energy shock as the energy sector would profit from higher energy prices. However, the energy market in Europe is liberal and highly interconnected. Danish energy producers sell domestically but also export to neighboring countries like the UK, Norway, and Germany. Given the size of the Danish economy and energy sector, it seems to be a fair assumption for energy to be imported and given at an exogenous price. In addition, further expansions to the model would increase the complexity, which could create noise that blurs out the main transmission mechanisms of interest. Therefore, a more complex model is not necessarily preferred.

Calibration

Secondly, one could criticize the model calibration. We calibrated the model only with external literature from MAKRO. The short-term parameters in MAKRO are based on an SVAR econometric analysis to match the model impulse response to real data, while the long-term parameters are based on external literature. The short-term parameters in our model are not calibrated in the same way. The long-term parameters, i.e. elasticities, can easily be set exogenously and are, in general, more important. They are more structural and determine how agents react to new price signals and, thus, the overall transmission mechanisms.

When shocking to exogenous variables, we assume that all parameters are fixed. According to Lucas (1976), this could be problematic. If we implement an energy shock, we cannot be sure that the agent behavior is invariant - parameter values might change with time. For instance, one could imagine a norm change during the energy crisis in which people began to reduce their consumption of energy (μ^E). This would imply that the energy shock's impact on the economy would be reduced. The Lucas critique is reasonable and could affect the transmission mechanisms, and our model disregards this aspect.

5.2 Alternative Explanations

The shock analysis revealed that our model has issues with the recreation of core inflation. We reason that it is possible that the model is correct, but that factors outside the model could drive the high core inflation in Denmark. Alternatively, the model is incorrect and omits a central dynamic that connects energy prices to core inflation.

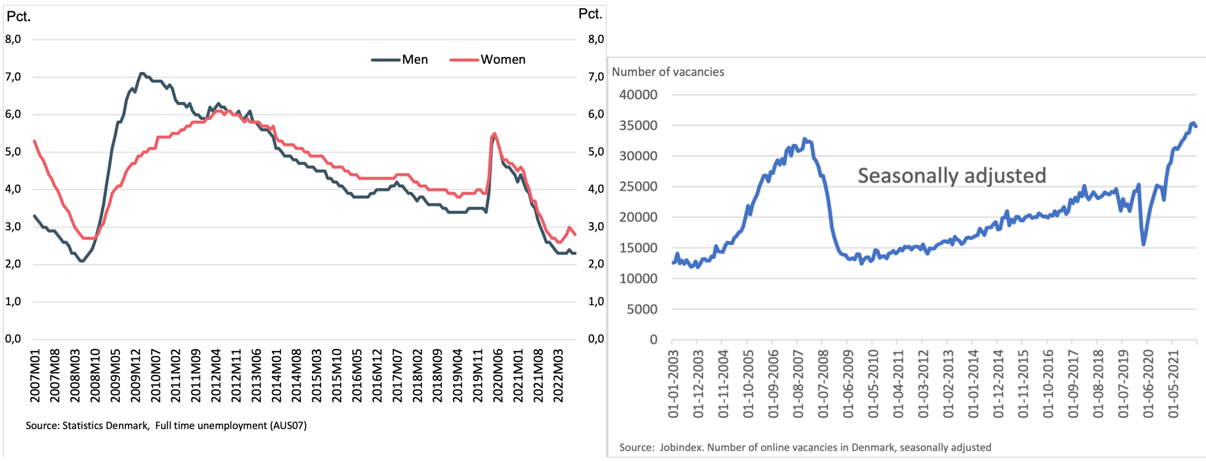
1. Denmark is less "energy responsive" than foreign countries. Thus, higher energy prices increase production and core inflation.
2. It is something different from 1. that drives production and core inflation.
3. The model omits a dynamic that connects energy prices and core inflation.

If the first point is true, the model is correct. The crucial thing is not the absolute prices but the relative prices - Denmark's energy responsiveness could be timber or water on the bonfire that increases core inflation. If the second point is true, core inflation and consumer prices will not necessarily disappear if energy prices return to normal - it is possible that, e.g., a demand shock has caused higher production and core inflation. If the third point is true, the model lacks essential mechanisms such that the potential causality between energy prices and core inflation is uncertain. We will now discuss the second and the third point in 5.3 and 5.4.

5.3 Is It Something Else?

Let's consider the possibility that something different from the energy shock is causing core inflation. The Covid-19 crisis initially resulted in an economic recession, but in 2022, the quarterly growth rates of GDP have been between 3.8 and 9.0 pct. according to Statistics Denmark (2022*b*). This sudden boost in production creates pressure on production inputs and their price, e.g., higher wages. In figure 14a, we see that the unemployment rate is nearly as low as before the financial crisis. Jobindex (a) in Denmark reported a record-high number of vacancies of 35.900 in January 2022. The previous record was set before the financial crisis in 2008 of 33.200 vacancies. However, now Jobindex (b) reports that the number of vacancies is starting to decrease. The season-corrected number of vacancies in September 2022 was 14.5 percent lower than in January 2022, as figure 14b shows.

Figure 14: The unemployment rate and number of vacancies in Denmark



(a) Unemployment rate in Denmark

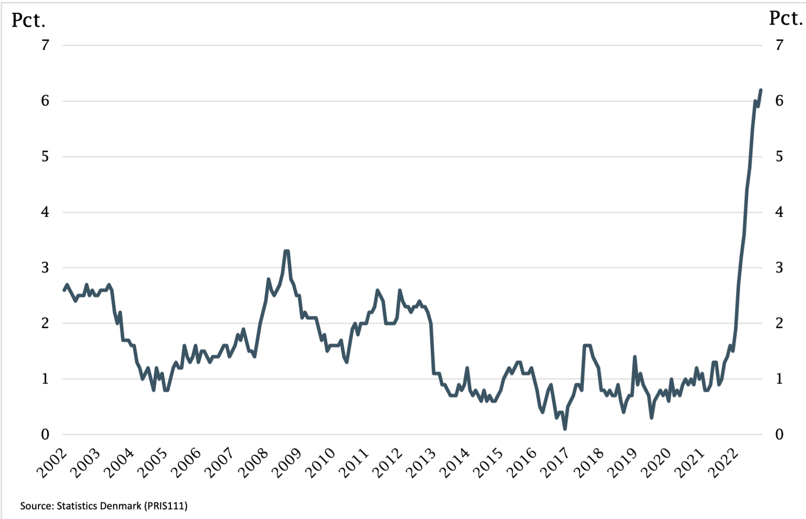
(b) Number of vacancies in Denmark

All in all, the high growth rates in 2022, a record-high number of vacancies that is stagnating, and an unemployment rate approaching the rate from the financial crisis are all factors pressuring the labor market, wages, and core inflation. It could indicate that core inflation is not a product of surging energy prices but an overheated economy. Thus, core inflation could result from the catching-up effect in the wake of Covid-19 or the expansive fiscal policies during this crisis. It could also be the expansive monetary policies by ECB in the last decade finally having an effect. There are several possibilities. On the one hand, one could argue that it is not the surging energy prices that are causing core inflation but that the energy crisis is merely a well-timed veil that hides the true reasons for the high core inflation. On the other hand, one could argue that the high energy prices have limited the growth in production, so when energy prices return to normal levels, core inflation could increase even further.

5.4 A Missing Link

It might seem obvious from the previous section; elevated production is the reason behind high core inflation. We saw the same trend around the financial crisis as shown in figure 15.

Figure 15: Core inflation in Denmark since 2002



However, core inflation levels around the financial crisis were significantly lower than core inflation levels of today, even though the number of vacancies and the unemployment rates are fairly similar. Thus, there seems to be a missing link: The labor market tightness today and before the financial crisis are approximately equivalent, yet inflation levels today are far greater. Something different from high economic activity levels seems to be at play. Although this is a very loose empirical "analysis", it could indicate that energy prices *do* impact core inflation and that such a relationship is omitted in our model.

Wage Determination

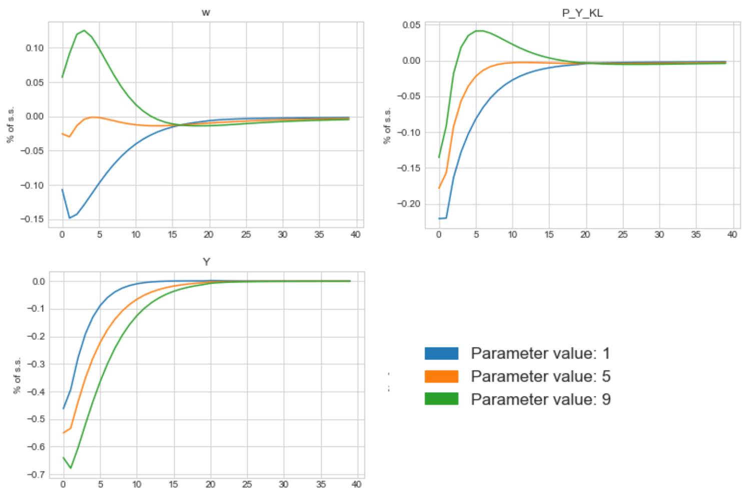
Such a relationship between core inflation and energy prices could be "ruined" by the assumption of perfect foresight. Our model allows no errors in expectation or uncertainty. Agents cannot make mistakes. In particular, perfect foresight makes it challenging to model wage-price spirals - a critical inflation mechanism. There are no incorrect expectations in the wage bargaining; the labor union and the labor agency know that the shock is temporary and can therefore plan the "perfect path" through periods that *in reality* entails a significant degree of uncertainty.

However, we can still introduce imperfections that agents will react perfectly to. If we manipulate the wage bargaining a bit, we can create core inflation without increasing production. Suppose we added a compensation feature to the original equation describing the negotiated wage, w_t^* - that is equation (2.16). With this extended wage function, the labor union would require indexation/compensation for rising energy prices.

$$\text{Compensation:} \quad w_t^* = \phi(r_t^\ell + \theta_t \kappa^L) + (1-\phi)(w^U + \xi((1 + P^E - P_{ss}^E)^2 - 1))$$
$$\text{Indexed:} \quad w_t^* = (\frac{P_t^E}{P_{ss}^E})^\zeta(\phi(r_t^\ell + \theta_t \kappa^L) + (1-\phi)w^U)$$

When running the model with a shock to the energy prices, P_t^E, with either of the extensions above, we can achieve positive levels of core inflation despite negative production levels; see figure 16.[4]

Figure 16: Shock to r_t^E and P_t^E for different values of ξ



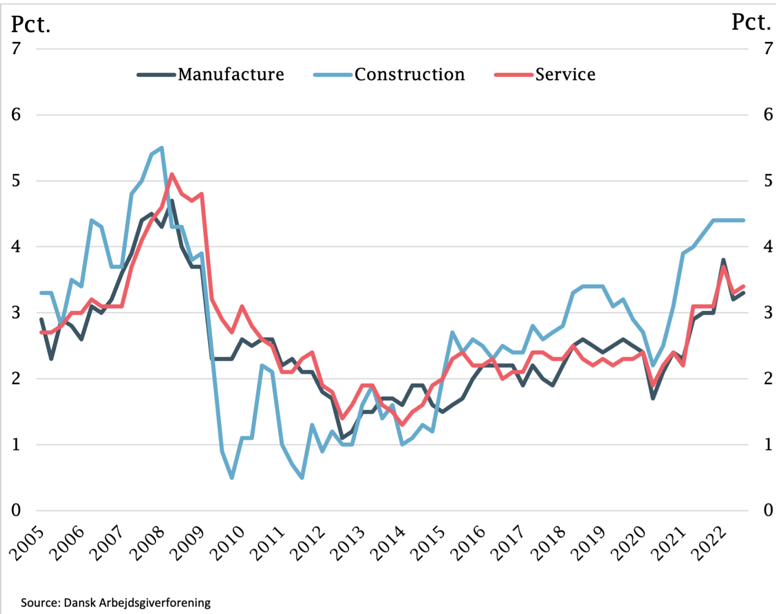
Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75 including the wage compensation. An indexation extension produces the same results.

[4]We stress that this expansion should not be considered a part of our model. The theoretical foundation is vague, but we include it to illustrate that a compensation/indexation feature in the wage bargaining process could create the desired dynamics. The expansions allow convergence and have no impact on steady state. The branch 'extended' in our Git repository contains the model with the wage extension as in 7.1.

The parameters ξ and ζ determine the magnitude of compensation or indexation. Greater values of ξ and ζ will lead to higher wages during an energy crisis. As mentioned, these two equations can both create higher core inflation without an increase in production. However, they are also specially designed to do so, emphasizing our comment earlier on "ad hoc" models. Yet, they illustrate that a compensation or indexation feature in the bargaining process can create the desired results and that models with perfect foresight can, to a certain extent, be used to *mimic* behavior under uncertainty.

Even though the wage expansions seem appealing explanations for the lack of core inflation, data does not necessarily indicate so. Looking at the growth rate in wages in Denmark, we get a different picture. The core inflation today is nearly twice as high as before the financial crisis. Though the official growth rates of wages are increasing, it is still at a different level than the wage growth around the financial crisis, see figure 17.

Figure 17: Growth rate in wages for different sectors in Denmark



Price Determination

If wages are not the answer, then what could be? One possibility could be that expected inflation affects inflation itself. Suppose that some firms cannot set prices in every period, which is a central feature of the pricing model by Calvo (1983). Consequently, these firms must set prices under their expectations of future price levels, and thus expectations of high inflation could in itself drive prices up. This is a critical characteristic of *New Keynesian Philips Curve*, and this mechanism is omitted in our model due to perfect foresight.[5] Furthermore, a study by Coibion et al. (2019) finds that in Italy, if firms expect higher inflation, that in itself will make the firms increase their prices. If we imagine firms in Denmark act according to the Calvo model and, as the study of Coibion suggests, they will likely raise their prices aggressively to avoid erosion of their relative price to other firms.

In our model, we use Rotemberg prices, which include the expectation of future price levels. However, the expectations in our model will always be correct due to perfect foresight. Firms cannot make expectation errors and do not have to fear erosion of their prices. An important difference between Rotemberg and Calvo prices is that under Rotemberg pricing firms can adjust prices every period, while those under Calvo cannot. Therefore, under Rotemberg pricing, firms will attempt to stick to minor price adjustments as their costs increase with the size of the adjustment. Thus, Leith (2016) suggests that the inflation bias problem could be greater under Calvo pricing and that the average markup is greater under Calvo.

Furthermore, it is well-established that there is a positive correlation between inflation and inflation uncertainty, but the direction of causality is not completely clear. According to the Friedman-Ball hypothesis, higher inflation uncertainty will lead to higher inflation. This hypothesis is controversial as studies tend to contradict each other on this matter. Some studies like Fountas (2001) support it, while Holland (1995) finds the reverse causality, and then other studies such as Grier and Perry (1998) find different results for different countries. The contradicting literature could indicate a mutual causality between inflation uncertainty and inflation levels. Errors in expectations and uncertainty are crucial in modeling inflation, and our DDGE model, with perfect foresight, lacks this aspect. Consequently, when the energy crisis hit Denmark, it led to inflation and inflation *uncertainty*. This could make firms overreact and set too high markups, contributing to core inflation.

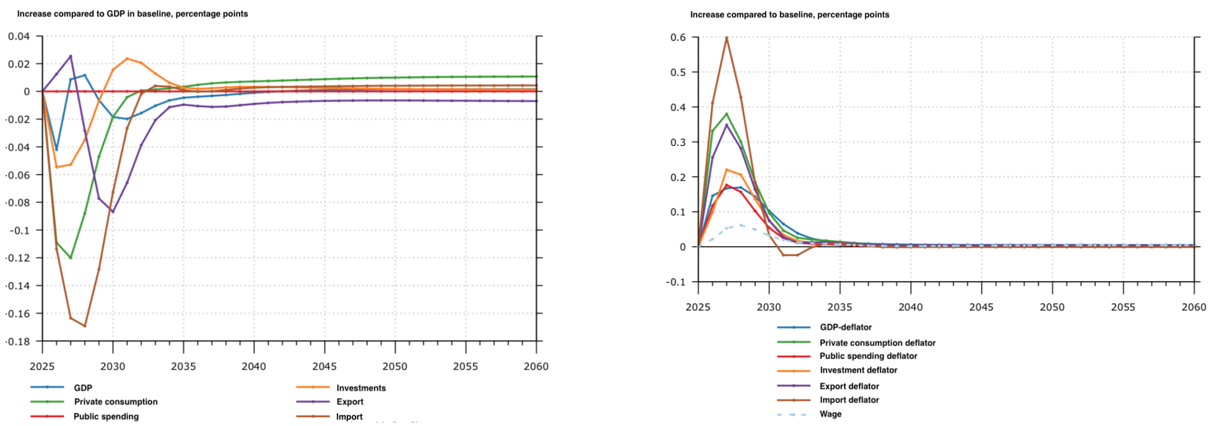
[5]For a derivation of the new Keynesian Philips Curve based on the Calvo framework, see Romer (1996).

5.5 Perspective to MAKRO

We cannot make any definitive conclusions about DDGE models in general, including MAKRO. MAKRO is much more complex than our Baby-MAKRO model. Nonetheless, we have adapted our model to imitate the inflation dynamics from MAKRO and get similar results as Høegh and Stephensen (2021) from the DREAM group, who illustrate the effect of a gas price shock on the Danish economy. Importantly, they assume that the gas price shock affects foreign prices and interest rates. In this paper, MAKRO gets:

1. Wages and employment increase marginally above the baseline[6], see figure 18.
2. GDP is shortly above the baseline but generally below the baseline.

Figure 18: MAKRO gas price shock effect on production (left) and wages (right)



A shock to oil prices in the beta MAKRO model. The right graph shows the change in production, compared to GDP in baseline. The left figure shows the change in wages compared to their baseline levels. Both changes are in percentage points.

The result relies on the assumption that Denmark is less energy-responsive relative to foreign countries. Firstly, this implies that the foreign prices increase more relative to the prices in Denmark, leading to greater export demand. Secondly, as MAKRO includes a domestic energy production sector, the Danish energy sector will increase production as they are more competitive than the energy sectors of foreign countries. This affects the wage level and other input prices positively, and consequently, a shock to gas prices in MAKRO results

[6]"Baseline" refers to the balanced growth path.

in high production, driving core inflation up. However, the resulting level of core inflation appears to be far from the level seen in Denmark for 2022.

To investigate this, we reached out to the Macro Political Center (MPC) of the Danish Ministry of Finance. As a part of their work, they collaborate with DREAM to test and improve MAKRO.[7] They stated that whether the actual MAKRO model performs better at replicating the Danish economy in 2022 is still an open question. To properly evaluate how well MAKRO captures the development in core inflation of 2022, the origins and the sizes of the shocks that the economy has been subject to must be clearly defined, which is a challenging task. Additionally, as we have already touched on, they noted that expectations are of utmost importance when dealing with inflation. A perfect-foresight model naturally lacks the aspect of inflation expectations and how they might change in response to inflation changes. Accordingly, a DSGE model, which incorporates aggregate uncertainty, might be more appropriate when modeling inflation.

Currently, DREAM is experimenting with a concept called *adaptive learning* in MAKRO. According to Evans and McGough (2020), adaptive learning is an alternative to rational expectations, where agents do not know the future but constantly make forecasts, allowing behavioral mistakes. This could potentially make the model dynamics richer and allow MAKRO to describe a broader set of scenarios.

As an end note, we emphasize that we refrain from concluding anything about MAKRO based on our model. However, Baby-MAKRO and MAKRO share many critical assumptions like perfect foresight, and this section of the discussion thus illustrates some of the main challenges of DDGE models. MAKRO's ability to forecast inflation is insignificant to its ability to analyze structural and long-term effects, which for now, is the main objective of MAKRO. However, the Danish Ministry of Finance aims to replace its current macroeconomic model, ADAM, with MAKRO for business cycle assessment in the near future, and its ability to model inflation will affect its short-term predictions.[8]

[7]We thank Max Blichfeldt Ørsnes and Niels Hvingelby for taking the time to discuss and provide us with great insights into MAKRO.

[8]See here for more information.

6 Conclusion

This paper examines the current energy crisis and its effect on core inflation in Denmark. We find that a global energy shock has an ambiguous impact on core inflation. On the one hand, the primary shock of higher energy prices will have a negative impact on core inflation. On the other hand, the secondary shock of higher foreign prices will have a positive impact on core inflation. Initially, core inflation drops below steady state, but if the effect of the secondary shock is great enough, it can more than compensate for the negative effect of the primary shock. This depends on the relative energy responsiveness between the domestic and foreign economies. The energy responsiveness of an economy depends on 1. the elasticity of substitution of energy and 2. energy efficiency. However, when comparing our results with current data, we find that our model is incapable of recreating the observed increase in core inflation. Based on this, we are left with the following possibilities:

(i) The model could be correct. Core inflation could stem from the fact that foreign countries are far more energy-responsive relative to Denmark. However, if one compares the core inflation levels across Europe, it seems unlikely. It is more likely that core inflation is caused by other economic factors than the energy shock, for instance, the Covid-19 crisis or expansive fiscal policy.

(ii) The model could be incorrect. It is possible that our model omits a critical dynamic that links energy prices to core inflation. Due to the assumption of perfect foresight, our model lacks aspects such as imperfect information, errors in expectations, and uncertainty - all crucial elements of inflation modeling. For instance, uncertainty and inflation bias are critical in wage and price-setting behavior among agents, including wage-price spirals and excessive markups set by firms to avoid erosion of relative prices.

To analyze the global energy crisis, we expand on Jeppe Druedahl's original draft of Baby-MAKRO. Baby-MAKRO is a DDGE model describing a small open economy with a fixed exchange rate. The model includes overlapping generations and perfect foresight among agents but contains imperfections such as menu costs, labor market frictions, and wage rigidities. The model is written and solved in Python by numerical optimization. To simulate the global energy crisis, we implement MIT shocks to domestic energy prices and foreign prices respectively. We analyze the transmission mechanisms of each shock independently to thoroughly understand the combined effects of both shocks.

Our results suggest that the assumption of perfect foresight is problematic when analyzing price shocks. Therefore, an idea could be to expand Baby-MAKRO with *adaptive learning*. In addition, as our results suggest that there might be a missing link between core inflation and energy prices, it encourages an econometric analysis that explores if there is, in fact, a causal relationship between the two. Finally, it encourages an equal shock analysis within a DSGE framework to examine the implications of uncertainty and imperfect information on inflation.

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7 Appendix

Table 1: Parameter Values

	Baby-MAKRO	**MAKRO**
Households		
CRRA coefficient, σ	0.8	0.8[9]
Weight on bequest, μ_B	2.5	M[10]
Households discount factor , β	0.95	M
Nominal interest rate for households, r^{hh}	0.04	V
Share of Ricardian households, υ	0.5	0.5
Life-span of households, A	80	S, max 100
Work-life-span of households, A_R	60	N/A
Model time frame, T	500	N/A
The production firm		
Internal rate of return, r^f	0.04	N/A
Depreciation of capital , δ_K	0.1	N/A
Weight on capital, μ^K	1/3	N/A
Weight on energy, μ^E	1/3	N/A
Substitution of energy versus capital and labor, σ^Y	0.83	0.83
Substitution of labor and capital, $\sigma^{Y,KL}$	0.45	0.45
Adjustment cost for capital, Ψ_0	5	5
Price elasticity of demand, η	10	N/A
Price adjustment costs, ι_0	1	1
Labor market		
Cost of vacancies, κ^L	0.025	N/A

[9]All long term parameters from MAKRO are derivde from Kronborg and Ejarque (2021) while all short term parameters are derived from DREAM (2021), unless if we state something else.

[10]"N/A" indicates that the parameter does not exist in the model or that we did not manage to find it. "V" indicates that it is a variable and not a parameter. "M" indicates the parameter is determined in steady state or within the model. "S" means it is stochastic.

Table 1: Parameter Values

	Baby-MAKRO	MAKRO
Separation probability, $\delta_L = \delta_{L,a}$	0.05	0.0976[11]
Wage persistence, γ_w	0.5	0.5
Bargaining power of firms, ϕ	M	N/A
Matching curvature, σ^M	1	1
Government		
The rate of return on government debt, r^b	0.04	V
Unemployment benefit, U^B	0.25	N/A
G share of Y, $G_{\text{share Y}}$	0.25[12]	V
Convergence of G, ϵ_B	0.2	N/A
Debt in steady state, B_G_ss	0	V
The repacking firm		
Weight on import in C, $\mu^{M,C}$	0.3	N/A
Weight on energy in C, $\mu^{E,C}$	0.3	N/A
Weight on import in G, $\mu^{M,G}$	0.3	N/A
Weight on import in I, $\mu^{M,I}$	0.3	N/A
Weight on import in X, $\mu^{M,X}$	0.3	N/A
Substitution for energy in C, σ^C	0.26	0.26
Substitution for import in C, $\sigma^{C,G}$	2.67	2.67
Substitution for import in G, σ^G	2.67	2.67
Substitution for import in I, σ^I	2.67	2.67
Substitution for import in X, σ^X	2.67	2.67
The foreign sector		
Substitution in export demand, σ^F	5.02	5.02

[11] Ejarque (2021)
[12] This values approximates the past decade's average in Denmark.

Derivations

Labor Agency Problem

$$V(L_{t-1}) = \max_{\{L_t\}} \sum_{t=0}^{\infty} \frac{1}{(1+r^{\text{firm}})^t} \left[r_t^{\ell} \ell_t - w_t L_t \right]$$

$$\ell_t = L_t - \kappa^L v_t$$

$$L_t = \left(1 - \delta_t^L\right) L_{t-1} + m_t^v v_t$$

Inserting constraints:

$$V(L_{t-1}) = \max_{\{L_t\}} \sum_{t=0}^{\infty} \frac{1}{(1+r^{\text{firm}})^t} \left[r_t^{\ell} \left(L_t - \kappa^L \frac{L_t - (1-\delta_t^L) L_{t-1}}{m_t^v} \right) - w_t L_t \right]$$

FOC:

$$\frac{\partial V(L_{t-1})}{\partial L_t} = \frac{1}{(1+r^{\text{firm}})^t} \left(r_t^{\ell} \left(1 - \frac{\kappa^L}{m_t^v} \right) - w_t \right) + \frac{1}{(1+r^{\text{firm}})^{t+1}} \left(r_{t+1}^{\ell} \frac{\kappa^L}{m_{t+1}^v} (1 - \delta_{t+1}^L) \right) = 0$$

Doing the algebra:

$$\left(r_t^{\ell} \left(1 - \frac{\kappa^L}{m_t^v} \right) - w_t \right) + \frac{1}{(1+r^{\text{firm}})} \left(r_{t+1}^{\ell} \frac{\kappa^L}{m_{t+1}^v} (1 - \delta_{t+1}^L) \right) = 0$$

$$\frac{1}{1 - \frac{\kappa^L}{m_t^v}} \left[w_t - \frac{1}{(1+r^{\text{firm}})} \left(r_{t+1}^{\ell} \frac{\kappa^L}{m_{t+1}^v} (1 - \delta_{t+1}^L) \right) \right] = r_t^{\ell}$$

Capital Agency Problem

$$V\left(K_{t-1}\right)=\max_{\{K_{t-1}\}}\sum_{t=0}^{\infty}\frac{1}{\left(1+r^{\text{firm}}\right)^{t}}\left[r_t^K K_{t-1}-P_t^I(\iota_t+\Psi(\iota_t,K_{t-1}))\right]$$

$$\text{s.t.}$$

$$I_t=\iota_t+\Psi(\iota_t,K_{t-1})$$

$$K_t=(1-\delta^K)K_{t-1}+\iota_t$$

$$\Psi(\iota_t,K_t)=\frac{\Psi_0}{2}\left(\frac{\iota_t}{K_t}-\delta\right)^2K_t$$

Inserting constraints:

$$V\left(K_{t-1}\right)=\max_{\{K_t\}}\sum_{t=0}^{\infty}\frac{1}{\left(1+r^{\text{firm}}\right)^{t}}\left[r_t^K K_{t-1}-P_t^I(K_t-(1-\delta^K)K_{t-1}+\Psi(\iota_t,K_{t-1}))\right]$$

FOC:

$$\frac{\partial V(K_{t-1})}{\partial K_t}=\frac{1}{(1+r^{\text{firm}})^t}\left(-P_t^I\left(1+\Psi_\iota(\iota_t,K_{t-1})\right)\right)$$
$$+\frac{1}{(1+r^{\text{firm}})^{t+1}}\left(r_{t+1}^K-P_{t+1}^I\left(-(1-\delta^K)+\Psi_\iota(\iota_{t+1},K_t)(1-\delta^K)+\Psi_K(\iota_{t+1},K_t)\right)\right)=0$$

From which it follows:

$$-P_t^I\left(1+\Psi_\iota(\iota_t,K_{t-1})\right)$$
$$+\frac{1}{(1+r^{\text{firm}})}\left(r_{t+1}^K-P_{t+1}^I\left(-(1-\delta^K)-\Psi_\iota(\iota_{t+1},K_t)(1-\delta^K)+\Psi_K(\iota_{t+1},K_t)\right)\right)=0\iff$$

$$\frac{1}{1+\Psi_\iota(\iota_t,K_{t-1})}\frac{1}{(1+r^{\text{firm}})}\left(r_{t+1}^K+P_{t+1}^I(1-\delta^K)\left(1+\Psi_\iota(\iota_{t+1},K_t)\right)-P_{t+1}^I\Psi_K(\iota_{t+1},K_t)\right)=P_t^I$$

Household Problem

$$V_{t_0} = \max_{\{C_{a,t=t_0+a}\}_{a=0}^{A-1}} \sum_{a=0}^{A-1} \beta^a \left[\frac{(C_{a,t})^{1-\sigma}}{1-\sigma} + \mathbf{1}_{a=A-1} \mu^B \frac{\left(\frac{B_{a,t}}{P_t^C}\right)^{1-\sigma}}{1-\sigma} \right]$$

$$\text{s.t.}$$

$$t = t_0 + a$$

$$B_{-1,t} = 0$$

$$B_{a,t} = (1 + r^{hh}) B_{a-1,t-1} + w_t L_{a,t} + \frac{B_t^q}{A} - P_t^C C_{a,t}$$

Using Lagrange we find FOC:

$$\mathcal{L}(C_{a,t}, B_{a,t}) = \sum_{a=0}^{A-1} \beta^a \Bigg[\frac{C_{a,t}^{1-\sigma}}{1-\sigma} + 1_{a=A-1} \mu^B \frac{(\frac{B_{a,t}}{P_t^C})^{1-\sigma}}{1-\sigma} + \lambda_t((1+r^{hh}) B_{a-1,t-1}$$

$$+ w_t L_{a,t} + \frac{B_t^q}{A} - P_t^C C_{a,t} - B_{a,t}) \Bigg]$$

$$\frac{d\mathcal{L}}{dC_{a,t}} = \beta^a (C^{-\sigma} - \lambda_t P_t^C) = 0 \Leftrightarrow$$

$$\lambda_t = \frac{C_{a,t}^{-\sigma}}{P_t^C}$$

And:

$$\frac{d\mathcal{L}}{dB_{a,t}} = -\beta^a \lambda_t + \beta^{a+1} \lambda_{t+1} (1 + r^{hh}) = 0 \Leftrightarrow$$

$$\lambda_t = \beta \lambda_{t+1} (1 + r^{hh})$$

We exploit the fact:

$$\lambda_{t+1} = \frac{C_{a+1,t+1}^{-\sigma}}{P_{t+1}^C}$$

We now get:

$$\begin{aligned}
\frac{C_{a,t}^{-\sigma}}{P_t^C} &= \beta \frac{C_{t+1,a+1}^{-\sigma}}{P_{t+1}^C}(1+r^{hh}) \Leftrightarrow \\
C_{a,t}^{-\sigma} &= \beta \frac{C_{a+1,t+1}^{-\sigma}}{P_{t+1}^C} P_t^C (1+r^{hh}) \Leftrightarrow \\
C_{a,t} &= \left(\beta \frac{C_{a+1,t+1}^{-\sigma}}{\frac{P_{t+1}^C}{P_t^C}} \right)^{\frac{-1}{\sigma}} = \left(\beta \frac{1+r^{hh}}{1+\pi^h h_{t+1}} C_{a+1,t+1}^{-\sigma} \right)^{\frac{-1}{\sigma}}, \text{ in which } \frac{P_{t+1}^C}{P_t^C} = 1+\pi_{t+1}^{hh}
\end{aligned}$$

We will now find the FOC for the special case $A-1$:

$$\begin{aligned}
\frac{d\mathcal{L}}{dC_{A-1}} &= \beta^{A-1}(C_{A-1}^{-\sigma} - P_t^C \lambda_t) = 0 \Leftrightarrow \\
\lambda_t &= \frac{C_{A-1}^{-\sigma}}{P_t^C} \\
\frac{d\mathcal{L}}{dB_{A-1,t}} &= \beta^{A-1}\left(\mu^B \frac{(B_{A-1,t})^{-\sigma}}{(P_t^C)^{1-\sigma}} - \lambda_t \right) = 0 \Leftrightarrow \\
\lambda_t &= \mu^B \frac{B_{A-1,t}^{-\sigma}}{P_t^{C\,1-\sigma}}
\end{aligned}$$

We can now combine these two results:

$$\begin{aligned}
\frac{C_{A-1}^{-\sigma}}{P_t^C} &= \mu^B \frac{B_{A-1,t}^{-\sigma}}{P_t^{C\,1-\sigma}} \Leftrightarrow \\
C_{A-1} &= \left(\mu^B \left(\frac{B_{A-1,t}}{P_t^C} \right)^{-\sigma} \right)^{\frac{-1}{\sigma}}
\end{aligned}$$

Therefore, if we assume that the economy has converged back to steady state for the final period, then we get the following:

$$C_{a,t} = \begin{cases} \left(\mu^B \left(\frac{B_t^q}{P_t^C} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{if } a = A-1 \\ \left(\beta \frac{1+r_{hh}}{1+\pi_{ss}^{hh}} \left(C_{a+1,ss} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{elif } t = T-1 \\ \left(\beta \frac{1+r_{hh}}{1+\pi_{t+1}^{hh}} \left(C_{a+1,t+1} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{else} \end{cases} \tag{7.1}$$

Price Adjustment Costs and Monopolistic Competition

$$\max_{p_t^Y} V_t = (p_t^Y - P_t^{Y,0}) \cdot y_t - \Psi_t^P + \beta V_{t+1}$$

s.t.

$$\Psi_t^P = \frac{\iota_0}{2}\left[\frac{p_t^Y/p_{t-1}^Y}{p_{t-1}^Y/P_{t-2}^Y} - 1\right]^2 P_t^Y Y_t$$

$$y_t = \left(\frac{p_t^Y}{P_t^Y}\right)^{-\eta} Y_t$$

FOC:

$$\frac{\partial V_t}{\partial P_t^Y} = (-\eta+1)y_t - P_t^{Y,0}\eta\left(p_t^Y\right)^{-1} y_t - \left[\frac{\partial \Psi_t^P}{p_t^Y} + \beta\frac{\partial \Psi_{t+1}^P}{p_t^Y}\right] = 0$$

$$\frac{p_t^Y}{y_t}\frac{\partial V_t}{\partial P_t^Y} = (-\eta+1)p_t^Y + P_t^{Y,0}\eta - \frac{p_t^Y}{y_t}\left[\frac{\partial \Psi_t^P}{p_t^Y} + \beta\frac{\partial \Psi_{t+1}^P}{p_t^Y}\right] = 0$$

$$\underbrace{\frac{\eta}{\eta-1}P_t^{Y,0}}_{\text{Markup in absence of price adjustment costs}} - \frac{1}{\eta-1}\frac{p_t^Y}{y_t}\left[\frac{\partial \Psi_t^P}{p_t^Y} + \beta\frac{\partial \Psi_{t+1}^P}{p_t^Y}\right] = p_t^Y$$

By inserting the derivatives below and assuming symmetry one achieves the desired result.

$$\frac{p_t^Y}{y_t}\frac{\partial \Psi_t^P}{p_t^Y} = \iota_0 \frac{p_t^Y/p_{t-1}^Y}{p_{t-1}^Y/P_{t-2}^Y}\left[\frac{p_t^Y/p_{t-1}^Y}{p_{t-1}^Y/P_{t-2}^Y} - 1\right]\frac{P_t Y_t}{y_t}$$

$$\frac{p_t^Y}{y_t}\frac{\partial \Psi_{t+1}^P}{p_t^Y} = -2\iota_0 \frac{P_{t-1}^Y \cdot p_{t+1}^Y}{p_t^Y \cdot p_t^Y}\left[\frac{P_{t-1}^Y \cdot p_{t+1}^Y}{p_t^Y \cdot p_t^Y} - 1\right]\frac{P_{t+1}Y_{t+1}}{y_t}$$

Nash Surplus Bargaining

The Nash surplus bargaining problem is inspired by Ljungqvist and Sargent (2017). The surplus of the labor agency is defined as follows (see The Block Structure for elaboration).

$$J = r^{\ell} - w + \beta[\delta V + (1-\delta)J], \tag{7.2}$$

$$V = -\kappa^{L} + \beta\left[m^{v} J + (1-m^{v})V\right] \tag{7.3}$$

If impose a zero profit condition, $V = 0$, we can rewrite 7.3:

$$J = \frac{\kappa^{L}}{\beta m^{v}} \tag{7.4}$$

A worker's surplus as employed, E, and as unemployed, U, are:

$$E = w + \beta[\delta U + (1-\delta)E], \tag{7.5}$$

$$U = w^{U} + \beta\left[\theta m^{v} E + [1-\theta m^{v}]U\right] \tag{7.6}$$

The total surplus in a matching model is defined as: $S \equiv J - V + E - U = J + E - U$, which will be split between the labor agency and the labor union according to Nash bargaining. The Nash product, $(E-U)^{\phi} J^{1-\phi}$, is maximized. We write the match surplus, S, and the Nash product, N, for $\phi \in [0,1)$:

$$S \equiv J - V + E - U = J + E - U$$

$$N = (E-U)^{\phi} J^{1-\phi} \implies$$

$$\ln(N) = \phi \ln(E-U) + (1-\phi)\ln(J)$$

We can now maximize the surplus using Lagrange:

$$
\begin{aligned}
\mathcal{L}(E-U,J) &= \phi \ln(E-U) + (1-\phi)\ln(J) - \lambda(J+E-U-S) \\
\frac{d\mathcal{L}}{d(E-U)} &= \frac{\phi}{E-U} - \lambda = 0 \\
\frac{d\mathcal{L}}{dJ} &= \frac{1-\phi}{J} - \lambda = 0 \\
\frac{d\mathcal{L}}{d\lambda} &= -(J+E-U-S) = 0
\end{aligned}
$$

From this we obtain the following:

$$
\begin{aligned}
\frac{\phi}{E-U} &= \lambda \\
\frac{1-\phi}{J} &= \lambda \\
E-U &= S-J
\end{aligned}
$$

Which yields the solutions:

$$
\begin{aligned}
\frac{\phi}{E-U} &= \frac{1-\phi}{J} \Leftrightarrow && (7.7) \\
\frac{\phi}{S-J} &= \frac{1-\phi}{J} \\
\phi J &= (1-\phi)(S-J) \\
J &= (1-\phi)S \Leftrightarrow && (7.8) \\
E-U &= S-J = S-(1-\phi)S = \phi S && (7.9)
\end{aligned}
$$

Now we have the solution to the Nash bargaining problem, and we can find an expression for the wage. We will solve equation 7.2 for J and 7.5 for E:

$$
\begin{aligned}
J &= \frac{1}{1-\beta(1-\delta)}(r^{\ell} - w + \beta\delta U) \\
E &= \frac{1}{1-\beta(1-\delta)}(w + \beta\delta U)
\end{aligned}
$$

We can insert this into 7.8 and 7.9 an get the following:

$$\frac{J}{1-\phi} = \frac{E-U}{\phi} \Leftrightarrow \tag{7.10}$$

$$\frac{r^\ell - w + \beta\delta U}{(1-\beta(1-\delta))(1-\phi)} = \frac{w + \beta\delta U - U}{(1-\beta(1-\delta))\phi} \Leftrightarrow \frac{r^\ell - w}{1-\phi} = \frac{w-(1-\beta)U}{\phi} \Leftrightarrow$$

$$\phi(r^\ell - w) = (1-\phi)(w-(1-\beta)U) \Leftrightarrow$$

$$w = \phi r^\ell + (1-\phi)(1-\beta)U \tag{7.11}$$

We solve 7.6 for $E-U$:

$$E - U = \frac{(1-\beta)U - w^U}{\beta\theta m^v}$$

And insert this and 7.4 into 7.8 and 7.9:

$$\frac{J}{1-\phi} = \frac{E-U}{\phi} \Leftrightarrow$$

$$\frac{\kappa^L}{(1-\phi)\beta m^v} = \frac{(1-\beta)U - w^U}{\phi\beta\theta m^v}$$

$$(1-\beta)U = w^U + \frac{\phi\theta\kappa^L}{1-\phi}$$

Insert this into equation 7.11:

$$w = \phi r^\ell + (1-\phi)(1-\beta)U \Leftrightarrow \tag{7.12}$$

$$w = \phi r^\ell + (1-\phi)(\frac{(1-\phi)w^U + \phi\theta\kappa^L}{1-\phi})$$

$$w = \phi r^\ell + (1-\phi)(w^U + \phi\theta\kappa^L)$$

$$w = \phi(r^\ell + \theta\kappa^L) + (1-\phi)w^U \tag{7.13}$$

And finally, we arrive at equation 7.13 as desired.

The Firm Problem and CES Technology

The CES function is an essential part of our model used by the production firm as well as the repacking firm. In generel, the problem of the firm is given by:

$$\max_{X_i,X_j} PX - P_iX_i - P_jX_j \text{ s.t. } X = (\mu_i^{\frac{1}{\sigma}} X_i^{\frac{\sigma-1}{\sigma}} + \mu_j^{\frac{1}{\sigma}} X_j^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}},$$
$$\mu_i + \mu_j = 1, 1 > \mu_i > 0, \sigma > 0$$

The first order condition for X_i is:

$$\begin{aligned}
0 &= P\mu_i^{\frac{1}{\sigma}} X_i^{\frac{-1}{\sigma}} (\mu_i^{\frac{1}{\sigma}} X_i^{\frac{\sigma-1}{\sigma}} + \mu_j^{\frac{1}{\sigma}} X_j^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} - P_i \Leftrightarrow \\
P_i &= P\mu_i^{\frac{1}{\sigma}} X_i^{\frac{-1}{\sigma}} (\mu_i^{\frac{1}{\sigma}} X_i^{\frac{\sigma-1}{\sigma}} + \mu_j^{\frac{1}{\sigma}} X_j^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} \\
P_i &= P\mu_i^{\frac{1}{\sigma}} X_i^{\frac{-1}{\sigma}} X^{\frac{1}{\sigma}} \\
X_i &= \mu_i \left(\frac{P}{P_i}\right)^{\sigma} X
\end{aligned}$$

We can replicate the above for X_j. If we combine the FOCs, then we get the following condition:

$$\frac{X_i}{X_j} = \frac{\mu_i}{\mu_j} \left(\frac{P_i}{P_j}\right)^{-\sigma}$$

The marginal cost is given by:

$$\begin{aligned}
0 &= PX - P_iX_i - P_jX_j \Leftrightarrow \\
P &= \frac{P_iX_i + P_jX_j}{X} \\
P &= \mu_i \left(\frac{P}{P_i}\right)^{\sigma} P_i + \mu_j \left(\frac{P}{P_j}\right)^{\sigma} P_j \\
P^{1-\sigma} &= \mu_i P_i^{1-\sigma} + \mu_j P_j^{1-\sigma})^{\frac{1}{1-\sigma}} \\
P &= (\mu_i P_i^{1-\sigma} + \mu_j P_j^{1-\sigma})^{\frac{1}{1-\sigma}}
\end{aligned}$$

A CES Matching Function

The following matching function is from Petrongolo and Pissarides (2001). They use U instead of S, but besides that they are the equivalent.

$$\begin{aligned}
\mathcal{M} &= (S^{-\rho} + v^{-\rho})^{-\rho\frac{-1}{\rho}} \\
&= (\frac{1}{S^\rho} + \frac{1}{v^\rho})^{\rho\frac{-1}{\rho}} \\
&= (\frac{S^\rho + v^\rho}{(Sv)^\rho})^{\frac{-1}{\rho}} \\
&= \frac{Sv}{(S^\rho + v^\rho)^{\frac{1}{\rho}}} \\
&= \frac{Sv}{(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}})^{\sigma^m}} \qquad \text{for } \rho = \frac{1}{\sigma^m}
\end{aligned}$$

The derivative of the matching function w.r.t. σ^m:

$$\mathcal{M}_t = \frac{S_t v_t}{\left(S_t^{\frac{1}{\sigma^m}} + v_t^{\frac{1}{\sigma^m}}\right)^{\sigma^m}} \implies \ln(\mathcal{M}_t) = \ln(S_t v_t) - \sigma^M \ln\left(S_t^{\frac{1}{\sigma^m}} + v_t^{\frac{1}{\sigma^m}}\right)$$

$$\implies \frac{\partial \ln(\mathcal{M}_t)}{\partial \sigma^m} = -\ln(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}}) + \frac{S^{\frac{1}{\sigma^m}}\ln(S) + v^{\frac{1}{\sigma^m}} l(v)}{\sigma^m(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}})}$$

$$= \frac{S^{\frac{1}{\sigma^m}}\ln(S) + v^{\frac{1}{\sigma^m}}\ln(v) - \ln(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}})\sigma^m(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}})}{\sigma^m(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}})}$$

$$= \frac{1}{\sigma^m(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}})}(S\frac{1}{\sigma^m}\ln(\frac{S}{(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}})^{\sigma^m}}) + v\frac{1}{\sigma^m}\ln(\frac{v}{(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}})^{\sigma^m}})) < 0$$

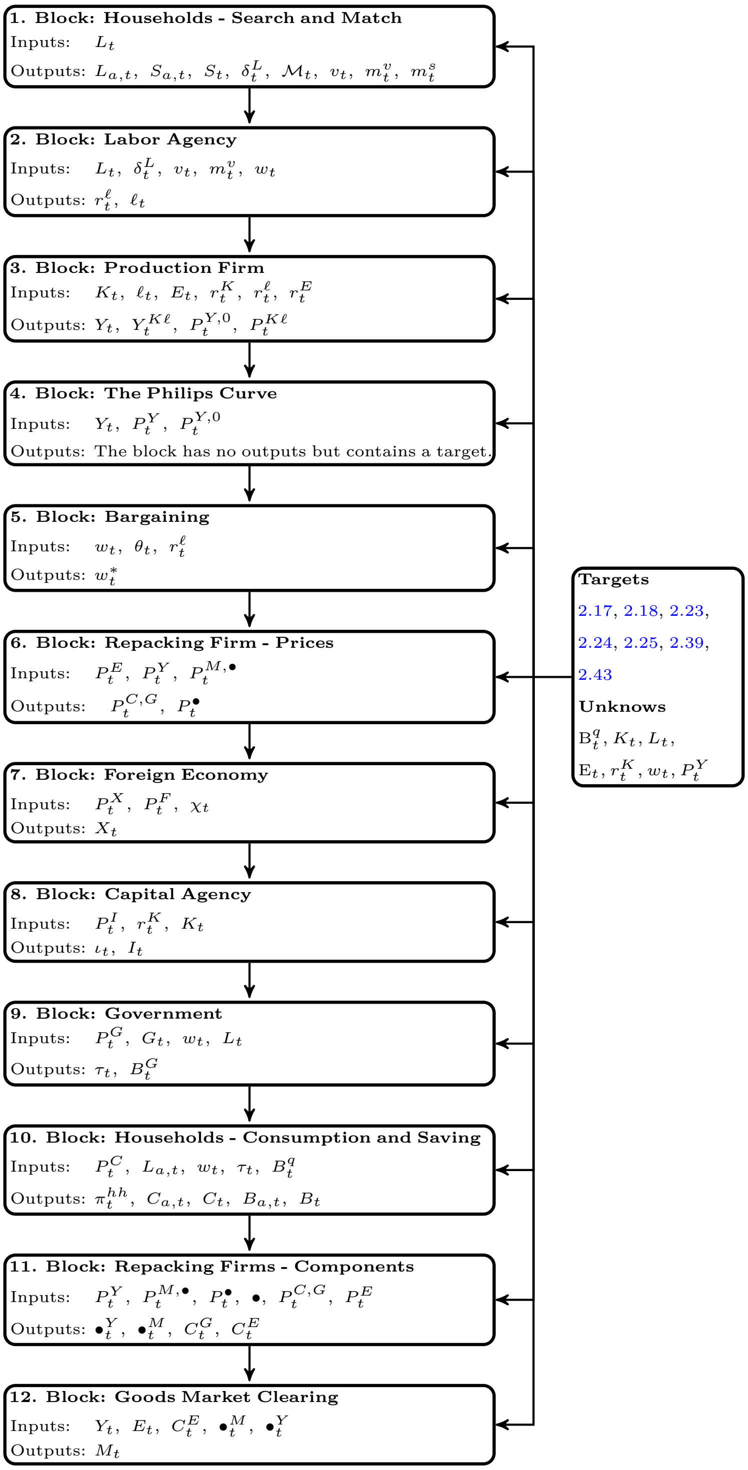
$$\text{if} \quad \frac{S}{(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}})^{\sigma^m}} < 1 \vee \frac{v}{(S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}})^{\sigma^m}} < 1$$

$$\frac{S^{\frac{1}{\sigma^m}}}{S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}}} < 1 \vee \frac{v^{\frac{1}{\sigma^m}}}{S^{\frac{1}{\sigma^m}} + v^{\frac{1}{\sigma^m}}} < 1$$

$$0 < v\frac{1}{\sigma^m} \vee 0 < S\frac{1}{\sigma^m}$$

Hence, as the last statement is always true in our model, the derivative of the matching function w.r.t. σ^m.

Input-Output Structure



Steady state

First, we fix the following variables to be able to determine steady state:

1. Job-finding probability, $m^s_{ss} = 0.5$
2. The government debt, $B^G_{ss} = 0$
3. The inflation rate, $\pi^{hh} = 0$

Then, we set normalize prices and solve the rest of steady state.

1. Price normalization:

$$P^Y_{ss} = P^F_{ss} = P^E_{ss} = P^{M,\bullet}_{ss} = \frac{\eta}{\eta - 1}, \bullet \in \{C, G, I, X\}$$

2. The pricing behavior of repacking firms then implies:

$$P^{\bullet}_{ss} = \frac{\eta}{\eta - 1}, \bullet \in \{C^G, G, I, X\}$$

3. The exogenous labor supply and search-and-matching implies:

$$S_{a,ss} = \begin{cases} 1 & \text{if } a = 0 \\ (1 - L_{a-1,ss}) + \delta_a^L L_{a-1,ss} & \text{if } a < A_R \\ 0 & \text{else} \end{cases}$$

$$\underline{L}_{a,ss} = \begin{cases} 0 & \text{if } a = 0 \\ (1 - \delta_a^L) L_{a-1,ss} & \text{if } a < A_R \\ 0 & \text{else} \end{cases}$$

$$L_{a,ss} = \underline{L}_{a,ss} + m_{ss}^s S_{a,ss}$$

$$U_{ss} = \begin{cases} 1 - L_{a,ss} & \text{if } a < A_R \\ 0 & \text{if } a \geq A_R \end{cases}$$

$$L_{ss} = \sum_a L_{a,ss}$$

$$S_{ss} = \sum_a S_{a,ss}$$

$$U_{ss} = \sum_a U_{a,ss}$$

$$\delta_{ss}^L = \frac{L_{ss} - \underline{L}}{L_{ss}}$$

$$\mathcal{M}_{ss} = \delta_{ss}^L L_{ss}$$

$$v_{ss} = \frac{\mathcal{M}_{ss}}{\left(1 + S_{ss}^{\frac{1}{\sigma_m}}\right)^{\sigma_m}}$$

$$m_{ss}^v = \frac{\mathcal{M}_{ss}}{v_{ss}}$$

4. Capital agency behavior implies:

$$r_{ss}^K = (r^{\text{firm}} + \delta^K) P_{ss}^I$$

5. The rental price of labor and energy is:

$$r^{\ell}_{ss} = \left(\frac{1 - \mu^K \left(r^K_{ss}\right)^{1-\sigma^{YK\ell}}}{1 - \mu^K} \right)^{\frac{1}{1-\sigma^{YK\ell}}}$$

$$r^{E}_{ss} = \left(\frac{1 - \mu^E (P^{YK\ell}_{ss})^{1-\sigma^{Y}}}{1 - \mu^E} \right)^{\frac{1}{1-\sigma^{Y}}}$$

6. Labor for production and wages are:

$$\ell_{ss} = L_{ss} - \kappa^L v_{ss}.$$

$$w_{ss} = r^{\ell}_{ss} \left(1 - \frac{\kappa^L}{m^v_{ss}} + \frac{1 - \delta^L_{ss}}{1 + r^{\text{firm}}} \frac{\kappa^L}{m^v_{ss}} \right)$$

7. From production firm:

$$K_{ss} = \frac{\mu_K}{1 - \mu_K} \left(\frac{r^{\ell}_{ss}}{r^K_{ss}} \right)^{\sigma^Y} \ell_{ss}$$

8. From capital accumulation equations:

$$\iota_{ss} = I_{ss} = \delta^K K_{ss}$$

9. Determine output and energy as input:

$$Y^{K\ell}_{ss} = \left(\left(\mu^K\right)^{\frac{1}{\sigma^{YK\ell}}} K_{ss}^{\frac{\sigma^{YK\ell}-1}{\sigma^{YK\ell}}} + \left(1 - \mu^K\right)^{\frac{1}{\sigma^{YK\ell}}} \ell_{ss}^{\frac{\sigma^{YK\ell}-1}{\sigma^{YK\ell}}} \right)^{\frac{\sigma^{YK\ell}}{\sigma^{YK\ell}-1}}$$

$$E_{ss} = \frac{\mu^E}{(1 - \mu^E)} \left(\frac{P^{YK\ell}_{ss}}{r^E_{ss}} \right)^{\sigma^Y} Y^{K\ell}_{ss}$$

$$Y_{ss} = \left(\left(\mu^E\right)^{\frac{1}{\sigma^Y}} E_{ss}^{\frac{\sigma^Y-1}{\sigma^Y}} + \left(1 - \mu^E\right)^{\frac{1}{\sigma^Y}} Y^{K\ell}_{ss}{}^{\frac{\sigma^Y-1}{\sigma^Y}} \right)^{\frac{\sigma^Y}{\sigma^Y-1}}$$

10. Public consumption and the tax rate:

$$G_{ss} = G_{\text{share}_Y} * Y_{ss}$$

$$\tau_{ss} = \frac{r^B B^G_{ss} + U^B w_{ss} U_{ss} + P^G_{ss} G_{ss}}{w_{ss} L_{ss} + U^B w_s U_{ss}}$$

11. We guess on B^q_{ss} and check $B_{A-1,t} = B^q_{ss}$ by:

$$C^{OPT}_{a,t} = \begin{cases} \left(\mu^B \left(\frac{B^q_{ss}}{P^C_{ss}} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{if } a = A-1 \\ \left(\beta \frac{1+r_{hh}}{1+\pi^{hh}_{ss}} \left(C^{OPT}_{a+1,t+1} \right)^{-\sigma} \right)^{-\frac{1}{\sigma}} & \text{else} \end{cases}$$

$$C^{HTM}_t = \frac{(1-\upsilon) \cdot \text{inc}_t}{P^C_t}$$

$$B_{a,t} = (1 + r^{hh}) B_{a-1,t-1} + \upsilon \cdot \text{inc}_{a,t} - P^C_t C^{OPT}_{a,ss}$$

12. Determine package components for public and private consumption as well as investments:

$$\bullet^Y_{ss} = (1 - \mu^{M,\bullet}) \left(\frac{P^\bullet_{ss}}{P^Y_{ss}} \right)^{\sigma^\bullet} \bullet_{ss}, \text{ for } \bullet \in \{C^G, G, I\}$$

$$\bullet^M_{ss} = \mu^{M,\bullet} \left(\frac{P_{ss}\bullet}{P^{M,\bullet}_{ss}} \right)^{\sigma^\bullet} \bullet_{ss}, \text{ for } \bullet \in \{C^G, G, I\}$$

$$C^E_{ss} = \mu^{E,C} \left(\frac{P^C_{ss}}{P^E_{ss}} \right)^{\sigma^C} C_{ss},$$

$$C^G_{ss} = (1 - \mu^{E,C}) \left(\frac{P^C_{ss}}{P^{C,G}_{ss}} \right)^{\sigma^C} C_{ss}$$

13. Determine χ_{ss} to get market clearing:

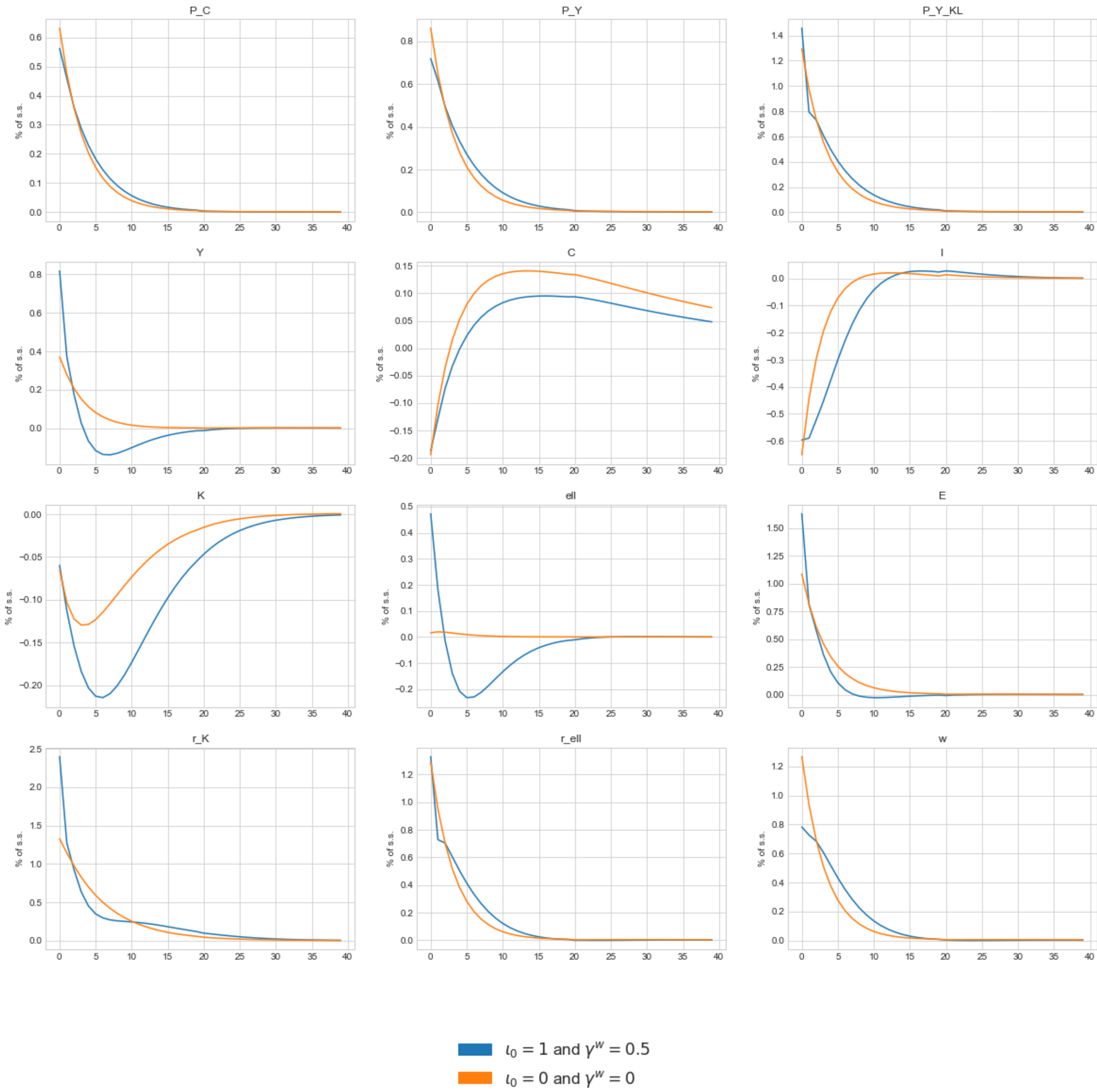
$$
\begin{aligned}
X_{ss}^{Y} &= Y_{ss} - \left(C_{ss}^{Y} + G_{ss}^{Y} + I_{ss}^{Y}\right) \\
X_{ss} &= \frac{X_{ss}^{Y}}{1 - \mu^{M,X}} \\
X_{ss}^{M} &= \mu^{M,X} \left(\frac{P_{ss}^{X}}{P_{ss}^{M,X}}\right)^{\sigma^{X}} X_{ss} \\
M_{ss} &= (C_{ss}^{M} + G_{ss}^{M} + I_{ss}^{M}) + (C_{ss}^{E} + E_{ss})
\end{aligned}
$$

14. Let φ adjust to make bargaining fit:

$$
\begin{aligned}
w_{ss}^{*} &= w_{ss} \\
\varphi &= \frac{w_{ss}^{*} - w_{ss}^{U}}{r_{ss}^{\ell} - w_{ss} + \theta_t \kappa^{L}}
\end{aligned}
$$

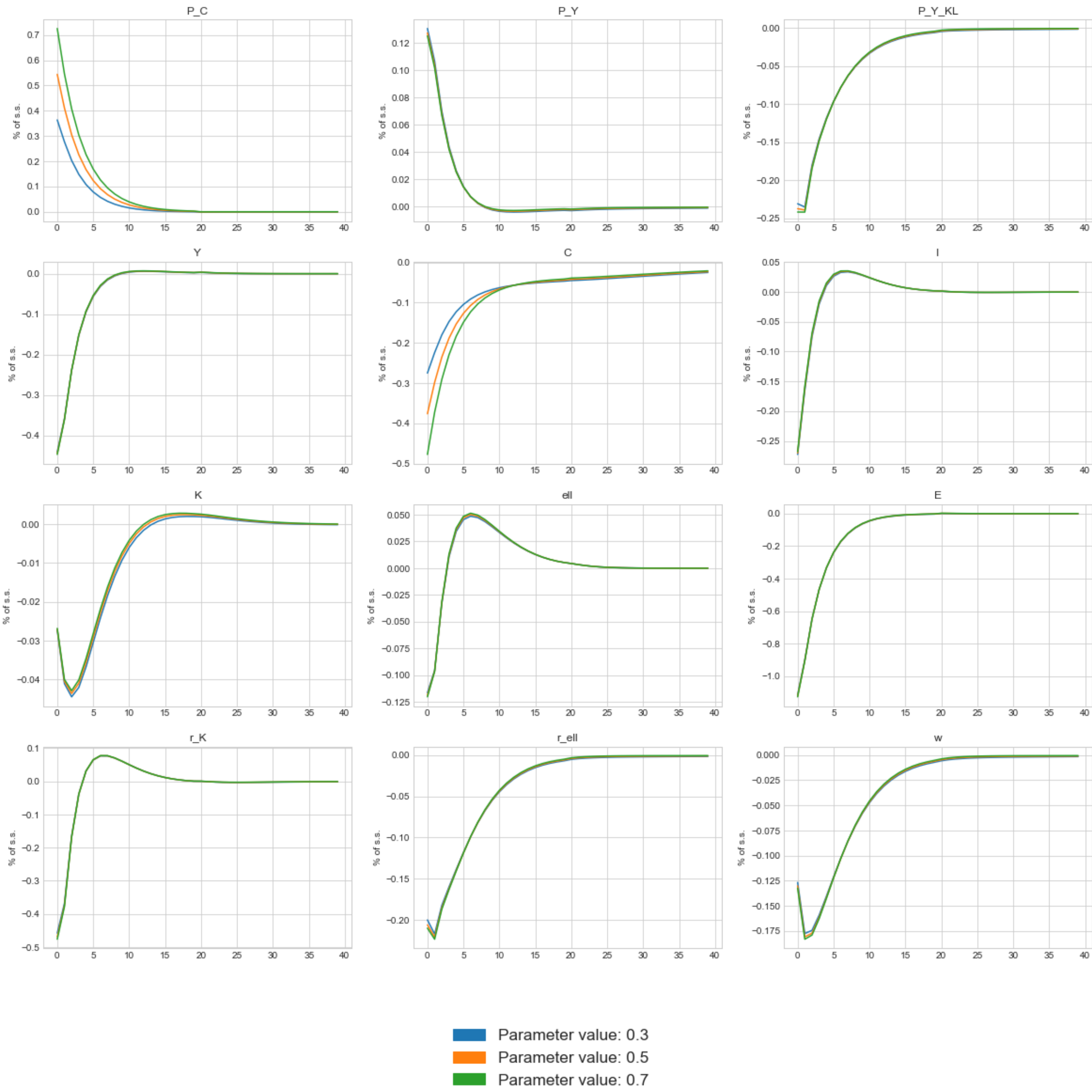
IRF's for all key macro economic variables

Figure 19: The secondary shock to P_t^F and $P_t^{M,\bullet}$ for different values of ι_0 and γ^w



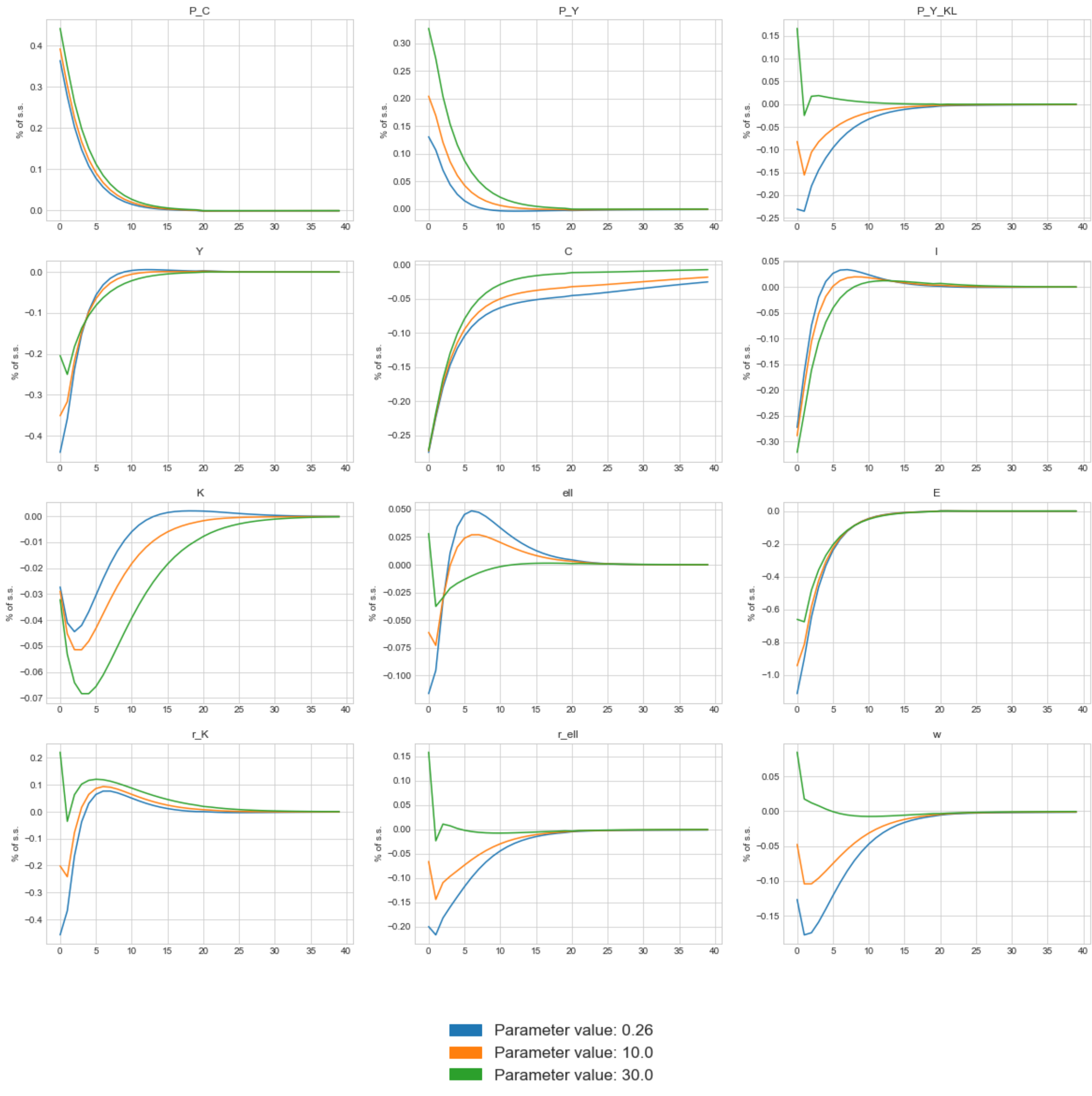
Note: 1 pct. shock to P_t^F and $P_t^{M,\bullet}$ for $t = 1, ..., 20$ with persistence of 0.75.

Figure 20: The primary shock to r_t^E and P_t^E for different values of μ^E



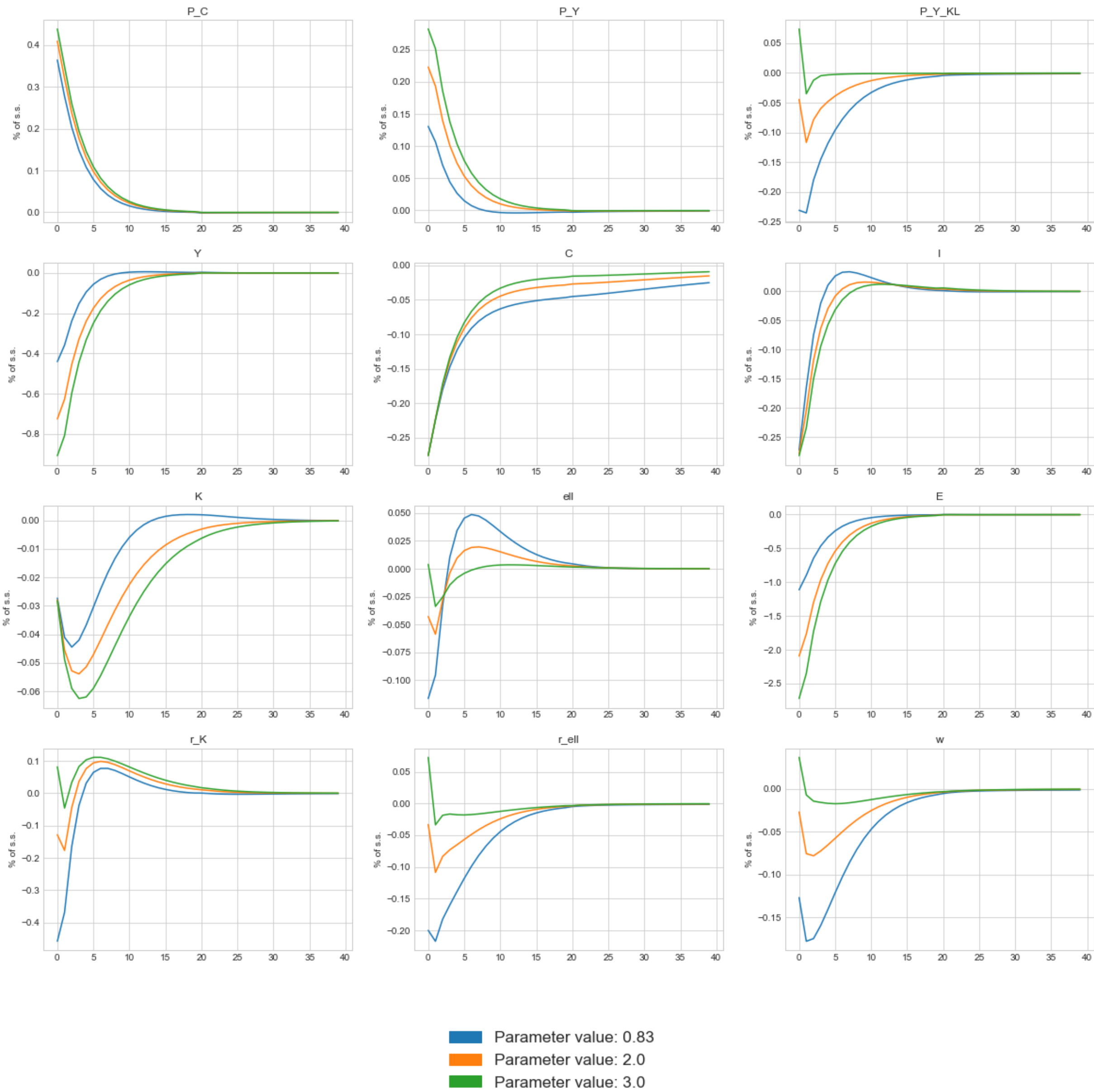
Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.

Figure 21: The primary shock to r_t^E and P_t^E for different values of σ^C



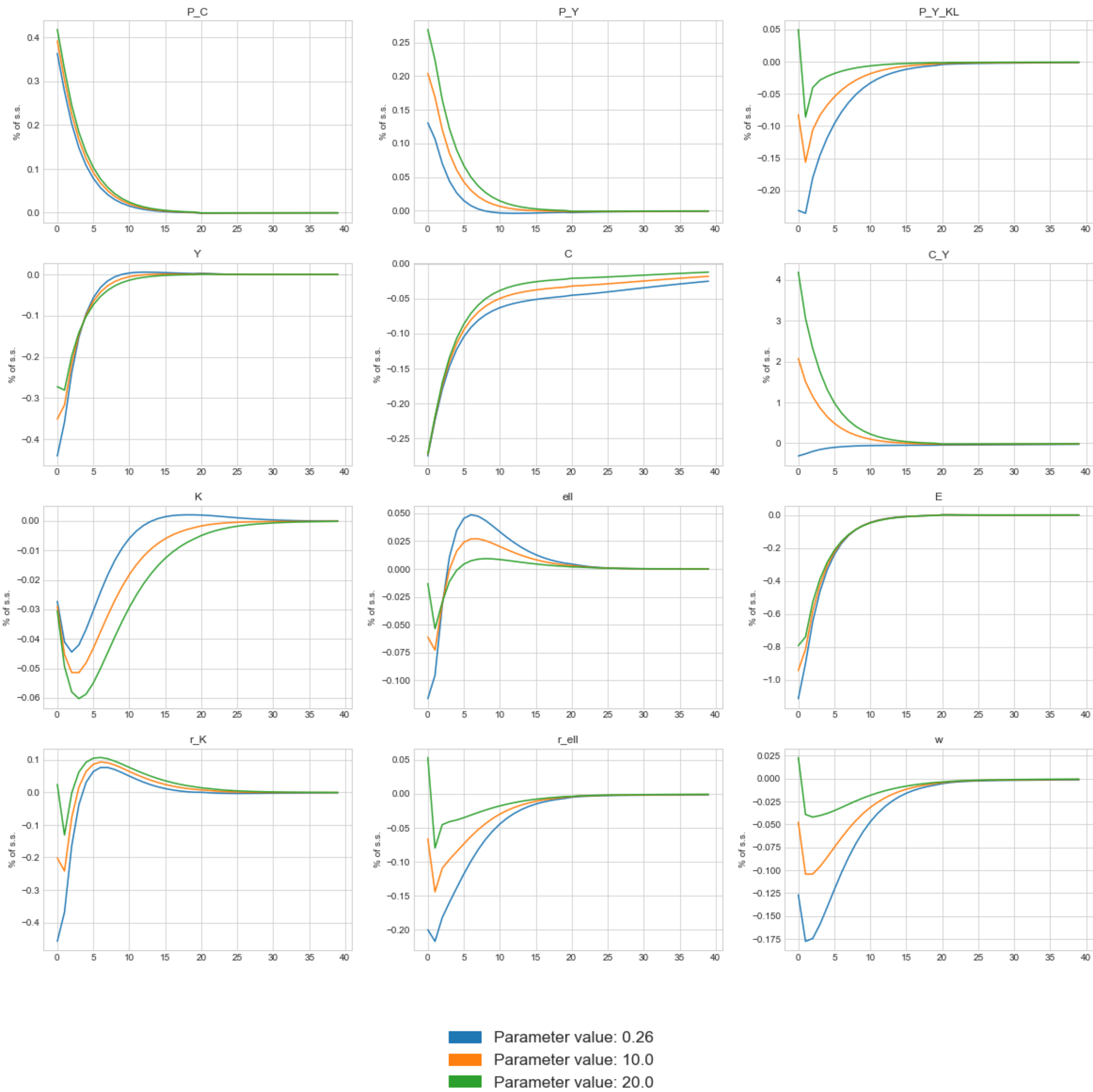
Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.

Figure 22: The primary shock to r_t^E and P_t^E for different values of σ^Y



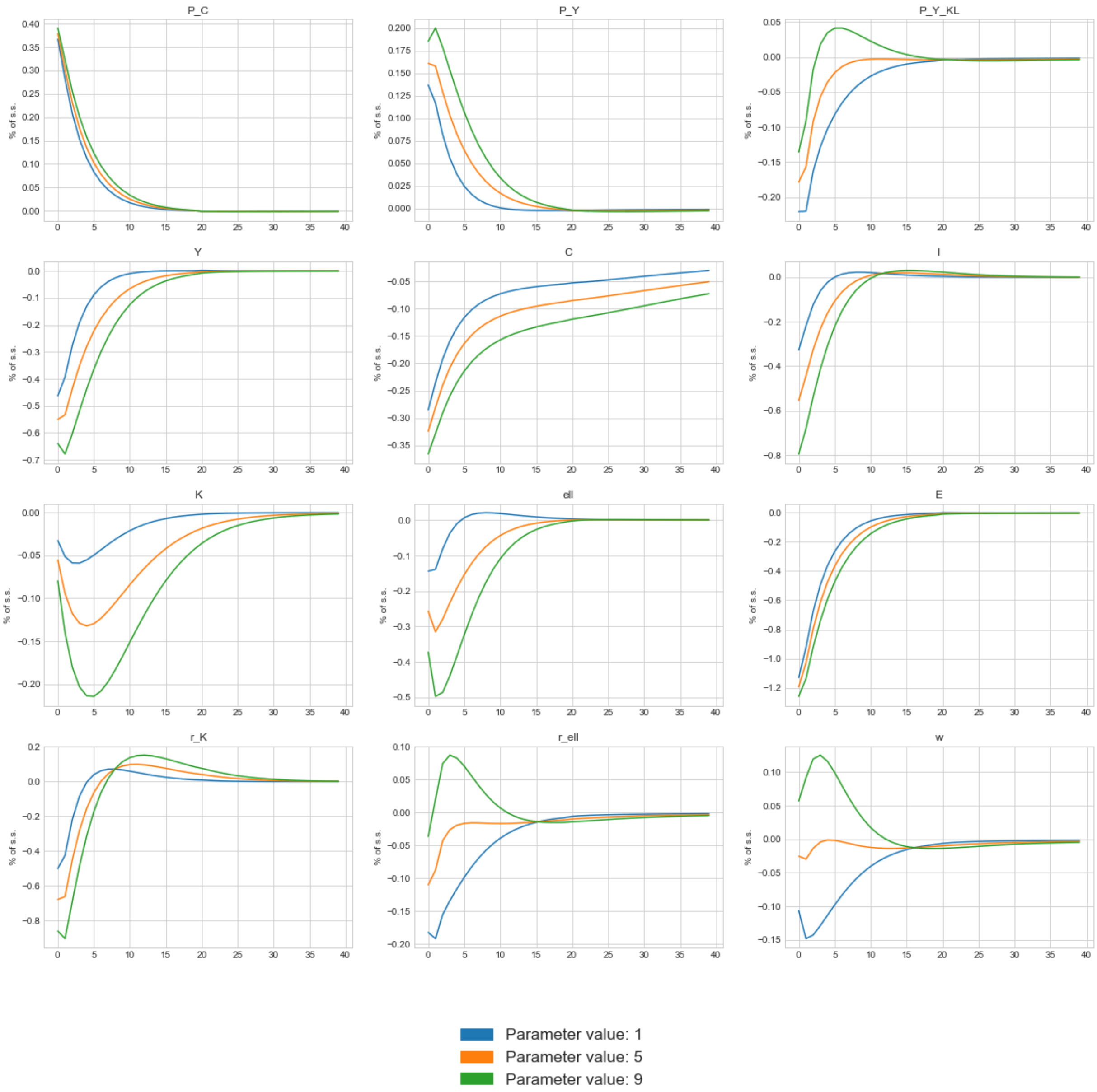
Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.

Figure 23: The primary shock to r_t^E and P_t^E for different values of σ



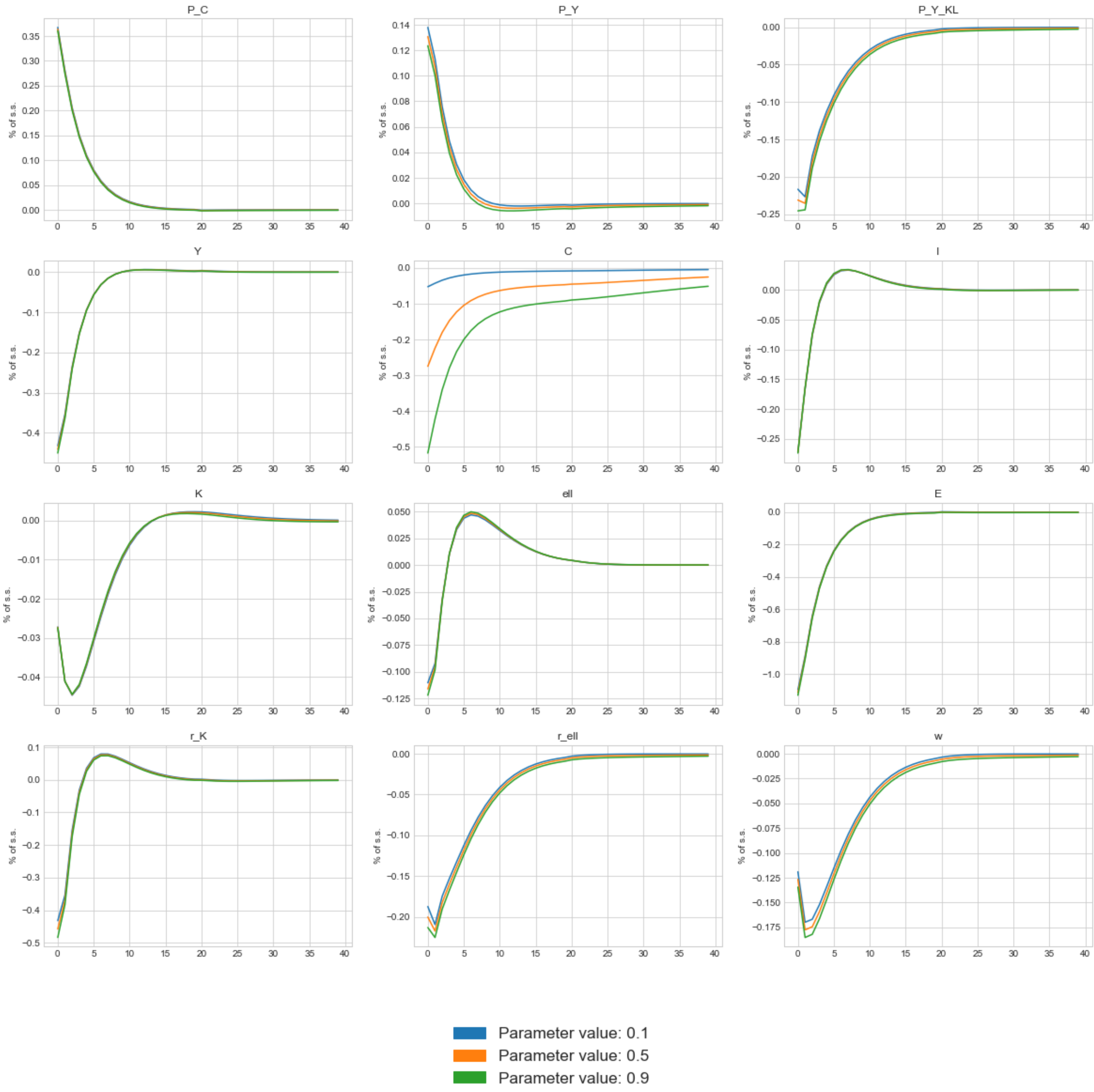
Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.

Figure 24: The primary shock to r_t^E and P_t^E for different values of ξ



Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.

Figure 25: The primary shock to r_t^E and P_t^E for different values of υ



Note: 1 pct. shock to r_t^E and P_t^E for $t = 1, ..., 20$ with persistence of 0.75.