Approach to solving the PnP

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1 Solution Methods

Referring to the proposal, we have the reduced cost function J to be optimized in terms of the Rotation Matrix R for the n object points (P_i) in Camera Frame $\boldsymbol{\{C\}}$.

$$J = \sum_{i=1}^{n} P_i^T R^T W_i^{'} R P_i \tag{1}$$

Let us express the rotation matrix in terms of its elements as shown in (2).

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{2}$$

The cost function can now be expressed in terms of the unknown elements as shown in (3).

$$J = \sum_{i=1}^{n} P_i^T \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}^T W_i^{'} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} P_i \tag{3}$$

The cost function comprising of 9 scalar variables ($\vec{x} = \begin{bmatrix} r_{11}, r_{12}, r_{13}, r_{21}, r_{22}, r_{23}, r_{31}, r_{32}, r_{33} \end{bmatrix}^T$), with the following constraints shown below.

$$\begin{aligned} r_{11}^2 + r_{12}^2 + r_{13}^2 &= 1 \\ r_{21}^2 + r_{22}^2 + r_{23}^2 &= 1 \\ r_{31}^2 + r_{32}^2 + r_{33}^2 &= 1 \\ r_{11}r_{21} + r_{12}r_{22} + r_{13}r_{23} &= 0 \\ r_{11}r_{31} + r_{32}r_{32} + r_{13}r_{33} &= 0 \\ r_{21}r_{31} + r_{22}r_{32} + r_{23}r_{33} &= 0 \end{aligned}$$

Now, we can express the problem statement in terms of a polynomial optimization problem. The cost function is given by (4) and the constraints are given by (5).

$$J = f(\vec{x}) = \frac{1}{2}\vec{x}^T W_i \vec{x} = f(x_1, x_2, ..., x_9) \tag{4}$$

$$c_j(x_1, x_2, ..., x_9) = 0 \tag{5}$$

The order of the monomials is 2 (quadratic). This reduces to a subclass of the QCQP (Quadratically constrained Quadratic Programming) problem with equality constraints.

However, the problem at hand is simpler than a QCQP problem. The idea behind solving the global optimization is drived from [1]. The primary advantage of solving the above equation in terms of 9 variables is that the polynomial is quadratic in terms of the variables. Hence, there is only 1 potential point of minima to check for in each of the 9 variables.

If the Rotation matrix is expressed in terms of the quaternion $R = R(q)$, the equation is bi-quadratic (monomials of order 4) in terms of 3 variables (4 variables and 1 constraint) and hence there would be 27 potential points of minima/maxima to check for the global minimum.

This forms an outline of the proposed solution.

References

[1] A. Vandermeersch and B. D. Moor, "A svd approach to multivariate polynomial optimization problems," in *2015 54th IEEE Conference on Decision and Control (CDC)*, pp. 7232–7237, Dec 2015.