Interpretation of Regression Coefficients for Untransformed Variables

Univariate

$y = \alpha + \beta x$

An increase in x of one results in a change in y of β.

$\boxed{\alpha = 3 \qquad \beta = 5}$

$\boxed{\Delta x = 1}$

x	y
0	3
1	8
2	13

$\boxed{\Delta y = 5 = \beta}$

Generalized Rule: $\Delta y = \beta \Delta x$

$$
\begin{aligned}
y_2 - y_1 &= (\alpha + \beta x_2) - (\alpha + \beta x_1) \\
&= \alpha + \beta x_2 - \alpha - \beta x_1 \\
&= \beta x_2 - \beta x_1 \\
&= \beta (x_2 - x_1)
\end{aligned}
$$

Bivariate & Multivariate

$y = \alpha + \beta_1 x_1 + \beta_2 x_2$

Holding x_2 constant, an increase in x_1 of one results in a change in y of β_1.

Holding x_1 constant, an increase in x_2 of one results in a change in y of β_2.

$\boxed{\alpha = 3 \quad \beta_1 = 5 \quad \beta_2 = 4}$

x_1	x_2	y
0	0	3
1	0	8
2	0	13
3	1	22
4	1	27
5	1	32

$\boxed{\Delta x = 1}$

$\boxed{\Delta y = 5 = \beta_1 \text{ when } \Delta x_2 = 0}$

$\boxed{\Delta y = 9 = \beta_1 + \beta_2 \text{ when } \Delta x_1 = 1 \text{ and } \Delta x_2 = 1}$

$\boxed{\Delta y = 5 = \beta_1 \text{ when } \Delta x_2 = 0}$

Generalized Rule*: **Holding all else constant,** $\Delta y = \beta \Delta x$

**this rule can be derived using partial derivatives*