Interpretation of Coefficients - Summary

* when there are 2 or more predictors, interpretation of a single coefficient is <u>always</u> <u>conditional</u> on the other(s). $\longrightarrow$ "all else held equal"

levels

(1) $y = \alpha + \beta x$

"An increase in x of one results in a change in y of β."

(2) $y = \alpha + \beta_1 x_1 + \beta_2 x_2$

"Holding x_2 constant, an increase in x_1 of one results in a change in y of β_1."

"all else held equal"

natural log

(1) $\ln(y) = \alpha + \beta x$

"An increase in x of one results in a e^{β} % change in y."

(2) $y = \alpha + \beta \ln(x)$

"An increase in x of 1% results in a change in y of $(1.01)^{\beta}$"

(3) $\ln(y) = \alpha + \beta \ln(x)$

"A 1% increase in x leads to a $(1-\beta)$% change in y

Interpretation of Coefficients - No Transformation

$y = mx + b$

bivariate

$y = 5x + 3$

when $x = 0, \; y = 3$ $\quad > +5$

$x = 1, \; y = 8$ $\quad > +5$

$x = 2, \; y = 13$

- ☆ for each increase in x of one, y increases by five
- ☆ an increase in x of one results in an increase in y of five

<u>the slope is the elasticity</u>

$y = \alpha + \beta_0 x_0 + \beta_1 x_1$

multivariate

$y = 3 + 5x_0 + 4x_1$

when $x_0 = 0$ & $x_1 = 0$, $y = 3$ $\quad > +5$

$x_0 = 1$ & $x_1 = 0$, $y = 8$

☆ y increases by 5 for each increase in x_0 of 1, <u>holding x_1 constant</u>

x_0:

☆ all else held equal/constant, an increase in x_0 of one leads to an increase in y of five

x_1:

☆ all else held equal/constant, an increase in x_1 of one leads to an increase in y of four

Interpretation of Coefficients (math) - No Transformation

$\Delta X \rightarrow ?\ \Delta Y$

$X_b - X_a \rightarrow ?\ Y_b - Y_a$

$\alpha + \beta X_b - (\alpha + \beta X_a) \rightarrow ?\ Y_b - Y_a$

$\downarrow$

$\alpha + \beta X_b - \alpha - \beta X_a = Y_b - Y_a$

$\downarrow$

$\beta X_b - \beta X_a = Y_b - Y_a$

$\beta (X_b - X_a) = Y_b - Y_a$

$$\beta = \frac{Y_b - Y_a}{X_b - X_a}$$ $\leftarrow$ change in Y for change in X is equal to β

Interpretation of Log-Transformed Variables

Case 1: dependent variable transformed

$$\ln(y) = \alpha + \beta x$$

$$\ln(y_2) - \ln(y_1) = \alpha + \beta x_2 - (\alpha + \beta x_1)$$

property of logs $\rightarrow$ $\ln\left(\frac{y_2}{y_1}\right) = \beta(x_2 - x_1)$

$$e^{\ln\left(\frac{y_2}{y_1}\right)} = e^{\beta(x_2 - x_1)}$$

$\alpha = 4$
$\beta = 1.10$ $\rightarrow$ $e^{1.10(1)}$ $\longrightarrow$ 3.00 $\longrightarrow$ $\ln\left(\frac{3.00}{1.00}\right) = 1.1(1)$

$e^{1.10(1)}$ $\downarrow$ change in x of one (<u>not</u> percent) $\longrightarrow$ leads to an increase in y of <u>200%</u>

$\ln\left(\frac{3.00}{1.00}\right)$ $\uparrow$ 200% change

Case 2: independent variable transformed

$$y = \alpha + \beta \ln(x)$$

$$y_2 - y_1 = \alpha + \beta \ln(x_2) - (\alpha + \beta \ln(x_1))$$

$$y_2 - y_1 = \beta(\ln(x_2) - \ln(x_1))$$

$$y_2 - y_1 = \beta\left(\ln\left(\frac{x_2}{x_1}\right)\right)$$

$$y_2 - y_1 = e^{\beta(\ln \frac{x_2}{x_1})}$$

$$= \left(e^{\ln\left(\frac{x_2}{x_1}\right)}\right)^{\beta}$$

$$= \left(\frac{x_2}{x_1}\right)^{\beta}$$

eg, $\alpha = 4$, $\beta = 50$ $\longrightarrow$ $1.01^{\beta} = 1.01^{50} = 1.64$

1% increase in x $\longrightarrow$ leads to an increase in y of 1.64 (not %)

Case 3: both dependent & independent transformed

$$\ln(y) = \alpha + \beta \ln(x)$$

$$\ln(y_2) - \ln(y_1) = \alpha + \beta \ln(x_2) - (\alpha + \beta \ln(x_1))$$

$$\ln\left(\frac{y_2}{y_1}\right) = \beta \ln\left(\frac{x_2}{x_1}\right)$$

$$e^{\ln\left(\frac{y_2}{y_1}\right)} = e^{\beta \ln\left(\frac{x_2}{x_1}\right)}$$

$$\frac{y_2}{y_1} = \left(e^{\ln\left(\frac{x_2}{x_1}\right)}\right)^{\beta} \Leftarrow \text{property of } \underline{\text{logs}}$$

$$\frac{y_2}{y_1} = \left(\frac{x_2}{x_1}\right)^{\beta}$$

eg, $\alpha = 4$, $\beta = 0.9$ $\rightarrow (1.01)^{0.9} \rightarrow 1.008996$

(1.01): 1% increase in X

1.008996: 0.8996% increase in y

approximation method: "0.9%"