Interpretation of Coefficients - Summary

* when there are 2 or more predictors, interpretation of a single coefficient is <u>always</u> <u>conditional</u> on the other(s). ⟶ "all else held equal"

levels

(1) $y = \alpha + \beta x$

"An increase in x of one results in a change in y of β."

(2) $y = \alpha + \beta_1 x_1 + \beta_2 x_2 \ldots$

"Holding x_2 constant, an increase in x_1 of one results in a change in y of β_1."

"all else held equal"

natural log

(1) $\ln(y) = \alpha + \beta x$

"An increase in x of one results in a e^{β}% change in y."

(2) $y = \alpha + \beta \ln(x)$

"An increase in x of 1% results in a change in y of $(1.01)^{\beta}$"

(3) $\ln(y) = \alpha + \beta \ln(x)$

"A 1% increase in x leads to a $(\beta - 1)$% change in y

Interpretation of Coefficients – No Transformation

bivariate

$y = mx + b$

$y = 5x + 3$

when $x = 0,\ y = 3$ $\Big\rangle +5$
$x = 1,\ y = 8$ $\Big\rangle +5$
$x = 2,\ y = 13$

* for each increase in x of one, y increases by five

* an increase in x of one results in an increase in y of five

the slope is the elasticity

multivariate

$y = \alpha + \beta_0 x_0 + \beta_1 x_1$

$y = 3 + 5x_0 + 4x_1$

when $x_0 = 0$ & $x_1 = 0$, $y = 3$ $\Big\rangle +5$
$x_0 = 1$ & $x_1 = 0$, $y = 8$

* y increases by 5 for each increase in x_0 of 1, holding x_1 constant

x_0:
* all else held equal/constant, an increase in x_0 of one leads to an increase in y of five

x_1:
* all else held equal/constant, an increase in x_1 of one leads to an increase in y of four

Interpretation of Coefficients (math) – No Transformation

$\Delta X \rightarrow ?\ \Delta Y$

$X_b - X_a \rightarrow ?\ Y_b - Y_a$

$\alpha + \beta X_b - (\alpha + \beta X_a) \rightarrow ?\ Y_b - Y_a$

$\downarrow$

$\alpha + \beta X_b - \alpha - \beta X_a = Y_b - Y_a$

$\downarrow$

$\beta X_b - \beta X_a = Y_b - Y_a$

$\beta (X_b - X_a) = Y_b - Y_a$

$$\beta = \frac{Y_b - Y_a}{X_b - X_a}$$ $\leftarrow$ change in Y for change in X is equal to β

Interpretation of Log-Transformed Variables

Case 1: dependent variable transformed

$$\ln(y) = \alpha + \beta x$$

$$\ln(y_2) - \ln(y_1) = \alpha + \beta x_2 - (\alpha + \beta x_1)$$

property of logs $\rightarrow$ $$\ln\left(\frac{y_2}{y_1}\right) = \beta(x_2 - x_1)$$

$$e^{\ln\left(\frac{y_2}{y_1}\right)} = e^{\beta(x_2 - x_1)}$$

$\alpha = .4$
$\beta = 1.10$ $\rightarrow$ $e^{1.10(1)}$ $\longrightarrow$ 3.00 $\rightarrow$ $\ln\left(\frac{3.00}{1.00}\right) = 1.1(1)$

$e^{1.10(1)}$ $\downarrow$ change in x of one (not percent) $\longrightarrow$ leads to an increase in y of 200%

3.00 $\downarrow$ leads to an increase in y of 200%

$\ln\left(\frac{3.00}{1.00}\right)$ $\uparrow$ 200% change

Case 2: independent variable transformed

$$y = \alpha + \beta \ln(x)$$

$$y_2 - y_1 = \alpha + \beta \ln(x_2) - (\alpha + \beta \ln(x_1))$$

$$y_2 - y_1 = \beta(\ln(x_2) - \ln(x_1))$$

$$y_2 - y_1 = \beta\left(\ln\left(\frac{x_2}{x_1}\right)\right)$$

$$y_2 - y_1 = e^{\beta(\ln \frac{x_2}{x_1})}$$

$$= \left(e^{\ln\left(\frac{x_2}{x_1}\right)}\right)^{\beta}$$

$$= \left(\frac{x_2}{x_1}\right)^{\beta}$$

eg, $\alpha = 4$ $\longrightarrow$ $1.01^{\beta} = 1.01^{50} = 1.64$

$\beta = 50$

1% increase in x $\longrightarrow$ leads to an increase in y of 1.64 (not %)

Case 3: both dependent & independent transformed

$$\ln(y) = \alpha + \beta \ln(x)$$

$$\ln(y_2) - \ln(y_1) = \alpha + \beta \ln(x_2) - (\alpha + \beta \ln(x_1))$$

$$\ln\left(\frac{y_2}{y_1}\right) = \beta \ln\left(\frac{x_2}{x_1}\right)$$

$$e^{\ln\left(\frac{y_2}{y_1}\right)} = e^{\beta \ln\left(\frac{x_2}{x_1}\right)}$$

$$\frac{y_2}{y_1} = \left(e^{\ln\left(\frac{x_2}{x_1}\right)}\right)^{\beta} \Leftarrow \text{property of logs}$$

$$\frac{y_2}{y_1} = \left(\frac{x_2}{x_1}\right)^{\beta}$$

eg, $\alpha = 4$ $\rightarrow (1.01)^{0.9} \rightarrow 1.008996$
$\beta = 0.9$

1% increase in X

0.8996% increase in y

approximation method: "0.9%"