International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

Diploma Programme

Mathematics SL formula booklet

For use during the course and in the examinations

First examinations 2014

Edited in 2015 (version 2)

Contents

Prior learning

Area of a parallelogram	$A = b \times h$
Area of a triangle	$A = \frac{1}{2}(b \times h)$
Area of a trapezium	$A = \frac{1}{2}(a + b)\,h$
Area of a circle	$A = \pi r^2$
Circumference of a circle	$C = 2\pi r$
Volume of a pyramid	$V = \frac{1}{3}$(area of base $\times$ vertical height)
Volume of a cuboid (rectangular prism)	$V = l \times w \times h$
Volume of a cylinder	$V = \pi r^2 h$
Area of the curved surface of a cylinder	$A = 2\pi r h$
Volume of a sphere	$V = \frac{4}{3}\pi r^3$
Volume of a cone	$V = \frac{1}{3}\pi r^2 h$
Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

Topic 1—Algebra

1.1	The *n*th term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right) = \frac{n}{2}(u_1 + u_n)$
	The *n*th term of a geometric sequence	$u_n = u_1 r^{n-1}$
	The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}, \ r \neq 1$
	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}, \ \lvert r \rvert < 1$
1.2	Exponents and logarithms	$a^x = b \iff x = \log_a b$
	Laws of logarithms	$\log_c a + \log_c b = \log_c ab$ $\log_c a - \log_c b = \log_c \frac{a}{b}$ $\log_c a^r = r \log_c a$
	Change of base	$\log_b a = \frac{\log_c a}{\log_c b}$
1.3	Binomial coefficient	$\binom{n}{r} = \frac{n!}{r!(n-r)!}$
	Binomial theorem	$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$

Topic 2—Functions and equations

2.4	Axis of symmetry of graph of a quadratic function	$f(x) = ax^2 + bx + c \quad \Rightarrow \quad$ axis of symmetry $x = -\frac{b}{2a}$
2.6	Relationships between logarithmic and exponential functions	$a^x = \mathrm{e}^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$
2.7	Solutions of a quadratic equation Discriminant	$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \ a \neq 0$ $\Delta = b^2 - 4ac$

Topic 3—Circular functions and trigonometry

3.1	Length of an arc Area of a sector	$l = \theta r$ $A = \frac{1}{2}\theta r^2$
3.2	Trigonometric identity	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
3.3	Pythagorean identity Double angle formulae	$\cos^2\theta + \sin^2\theta = 1$ $\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
3.6	Cosine rule Sine rule Area of a triangle	$c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $A = \frac{1}{2}ab\sin C$

Topic 4—Vectors

4.1	Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$
4.2	Scalar product	$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \cos\theta$ $\boldsymbol{v} \cdot \boldsymbol{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$
	Angle between two vectors	$\cos\theta = \dfrac{\boldsymbol{v} \cdot \boldsymbol{w}}{\lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert}$
4.3	Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

Topic 5—Statistics and probability

5.2	Mean of a set of data	$\overline{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$
5.5	Probability of an event A	$\mathrm{P}(A) = \dfrac{n(A)}{n(U)}$
	Complementary events	$\mathrm{P}(A) + \mathrm{P}(A') = 1$
5.6	Combined events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
	Mutually exclusive events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
	Conditional probability	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\ \mathrm{P}(B \mid A)$
	Independent events	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$
5.7	Expected value of a discrete random variable X	$\mathrm{E}(X) = \mu = \sum_x x\,\mathrm{P}(X = x)$
5.8	Binomial distribution	$X \sim \mathrm{B}(n,\ p) \quad \Rightarrow \quad \mathrm{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r},\ r = 0, 1, \ldots, n$
	Mean	$\mathrm{E}(X) = np$
	Variance	$\mathrm{Var}(X) = np(1-p)$
5.9	Standardized normal variable	$z = \dfrac{x - \mu}{\sigma}$

Topic 6—Calculus

6.1	Derivative of $f(x)$	$y = f(x) \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$
6.2	Derivative of x^n	$f(x) = x^n \quad \Rightarrow \quad f'(x) = nx^{n-1}$
	Derivative of $\sin x$	$f(x) = \sin x \quad \Rightarrow \quad f'(x) = \cos x$
	Derivative of $\cos x$	$f(x) = \cos x \quad \Rightarrow \quad f'(x) = -\sin x$
	Derivative of $\tan x$	$f(x) = \tan x \quad \Rightarrow \quad f'(x) = \frac{1}{\cos^2 x}$
	Derivative of e^x	$f(x) = \mathrm{e}^x \quad \Rightarrow \quad f'(x) = \mathrm{e}^x$
	Derivative of $\ln x$	$f(x) = \ln x \quad \Rightarrow \quad f'(x) = \frac{1}{x}$
	Chain rule	$y = g(u),\ u = f(x) \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$
	Product rule	$y = uv \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$
	Quotient rule	$y = \frac{u}{v} \quad \Rightarrow \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$
6.4	Standard integrals	$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
		$\int \frac{1}{x} \,\mathrm{d}x = \ln x + C, \ x > 0$
		$\int \sin x \,\mathrm{d}x = -\cos x + C$
		$\int \cos x \,\mathrm{d}x = \sin x + C$
		$\int \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x + C$
6.5	Area under a curve between $x = a$ and $x = b$	$A = \int_a^b y \,\mathrm{d}x$
	Volume of revolution about the x-axis from $x = a$ to $x = b$	$V = \int_a^b \pi y^2 \,\mathrm{d}x$
6.6	Total distance travelled from t_1 to t_2	distance $= \int_{t_1}^{t_2} \lvert v(t) \rvert \,\mathrm{d}t$