Takagi-Sugeno Model Identification Toolbox

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Static LiP model for the 3-dimensional Friedman test function.

V1.0

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1 Identification data

Use the 3-dimensional Friedman function:

nu = 3;

$$y = 10 \cdot \sin(\pi \cdot u_1 \cdot u_2) + 20 \cdot (u_3 - 0.5)^2$$

Choose the fuzziness parameter $\nu = 1.2$

```
nue = 1.2;
```

Choose the input matrix u as random data with N data-points: $u_{1,2} \in [0,1]$

```
N = 500;
u = rand( N, nu );
```

Compute the output vector y:

```
y = Friedman_fct( u, nu );
```

2 Structural parameters

```
Number of clusters n_v = number of local models
```

```
nv = 5;
Membership function Type: FCM
MSF = 'FCM';
```

Estimation of the static LiP TS model

```
addpath( '../TSModel' ); % Path to TSModel class
ts = TSModel( 'Static', nv, nu, 'comment', 'Friedman 3D');
Set the identification data: u, y
ts.setData( u, y );
Clustering:
   • FCM: fuzziness parameter \nu = 1.2 with Euclidean norm (default)
   ullet clustering in product-space
   • Multi-Start: 5 tries
ts.clustering( MSF, 'nue', nue, 'productspace', true, 'tries', 5 );
Initialisation of local models: global least squares estimation
```

```
ts.initialize( MSF, 'nue', nue, 'method', 'global');
```

Optimization of both: membership and local model parameters

ts.optimize('Both');

Iteration Func-count f(x) step optimality 0 36 938.137 1.62e+03 1 72 782.094 10 803 2 108 623.957 20 616 3 144 623.957 23.7711 616 4 180 514.589 5.94278 267 5 216 401.823 11.8856 227 6 252 303.388 10.4834 389 7 288 256.621 15.789 212 8 324 221.423 4.20916 105 9 360 201.17 6.32526 25.6 10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.07311 12.2 15 576 166.587 0.699664				Norm of	First-order
1 72 782.094 10 803 2 108 623.957 20 616 3 144 623.957 23.7711 616 4 180 514.589 5.94278 267 5 216 401.823 11.8856 227 6 252 303.388 10.4834 389 7 288 256.621 15.789 212 8 324 221.423 4.20916 105 9 360 201.17 6.32526 25.6 10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95	Iteration	Func-count	f(x)	step	optimality
2 108 623.957 20 616 3 144 623.957 23.7711 616 4 180 514.589 5.94278 267 5 216 401.823 11.8856 227 6 252 303.388 10.4834 389 7 288 256.621 15.789 212 8 324 221.423 4.20916 105 9 360 201.17 6.32526 25.6 10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.0	0	36	938.137		1.62e+03
3 144 623.957 23.7711 616 4 180 514.589 5.94278 267 5 216 401.823 11.8856 227 6 252 303.388 10.4834 389 7 288 256.621 15.789 212 8 324 221.423 4.20916 105 9 360 201.17 6.32526 25.6 10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759	1	72	782.094	10	803
4 180 514.589 5.94278 267 5 216 401.823 11.8856 227 6 252 303.388 10.4834 389 7 288 256.621 15.789 212 8 324 221.423 4.20916 105 9 360 201.17 6.32526 25.6 10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999	2	108	623.957	20	616
5 216 401.823 11.8856 227 6 252 303.388 10.4834 389 7 288 256.621 15.789 212 8 324 221.423 4.20916 105 9 360 201.17 6.32526 25.6 10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	3	144	623.957	23.7711	616
6 252 303.388 10.4834 389 7 288 256.621 15.789 212 8 324 221.423 4.20916 105 9 360 201.17 6.32526 25.6 10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	4	180	514.589	5.94278	267
7 288 256.621 15.789 212 8 324 221.423 4.20916 105 9 360 201.17 6.32526 25.6 10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	5	216	401.823	11.8856	227
8 324 221.423 4.20916 105 9 360 201.17 6.32526 25.6 10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	6	252	303.388	10.4834	389
9 360 201.17 6.32526 25.6 10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	7	288	256.621	15.789	212
10 396 185.071 3.37852 32 11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	8	324	221.423	4.20916	105
11 432 177.916 1.64542 30.3 12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	9	360	201.17	6.32526	25.6
12 468 172.451 1.31403 21 13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	10	396	185.071	3.37852	32
13 504 169.526 1.1683 12.5 14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	11	432	177.916	1.64542	30.3
14 540 167.852 1.07311 12.2 15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	12	468	172.451	1.31403	21
15 576 166.587 0.699664 10.6 16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	13	504	169.526	1.1683	12.5
16 612 166.119 0.230535 9.73 17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	14	540	167.852	1.07311	12.2
17 648 165.619 0.248118 8.95 18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	15	576	166.587	0.699664	10.6
18 684 165.067 0.283859 8.09 19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	16	612	166.119	0.230535	9.73
19 720 164.502 0.31759 7.41 20 756 163.983 0.342999 6.46	17	648	165.619	0.248118	8.95
20 756 163.983 0.342999 6.46	18	684	165.067	0.283859	8.09
	19	720	164.502	0.31759	7.41
21 792 163.564 0.350972 5.3	20	756	163.983	0.342999	6.46
	21	792	163.564	0.350972	5.3

22	828	163.267	0.334196	4.22
23	864	163.079	0.294595	3.34
24	900	162.969	0.243338	2.65

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to its initial value is less than the value of the function tolerance.

Show the resultiung TS model parameters:

disp(ts)

```
TS-Model: Type=Static
Name: 'undefined'
Type: 'TSModel'
Date: '21-May-2021 11:37:39'
Comments:
 'Friedman 3D'
Structural parameters: nu = 3, ny = 1, nv = 5
Identification data: N=500
Initial model estimation:
 Clustering: FCM, nue=1.2 norm=Euclidean in product space
Estimation of local models:
 Initialization of local models: global
 Optimization of model parameters: MF&LM
Plot the cluster centers: v
v = getCluster( ts )
ts.plotCluster( v, 'figure',1);
```

√ =

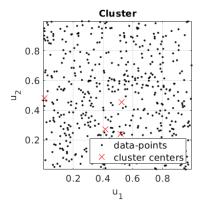
 0.5188
 0.2393
 0.5837

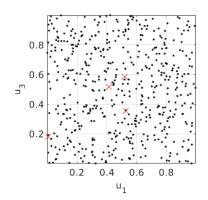
 0.0076
 0.4809
 0.1834

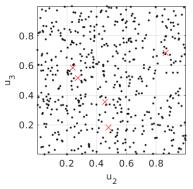
 0.5253
 0.4544
 0.3524

 0.4137
 0.2714
 0.5162

 0.9984
 0.8658
 0.6838





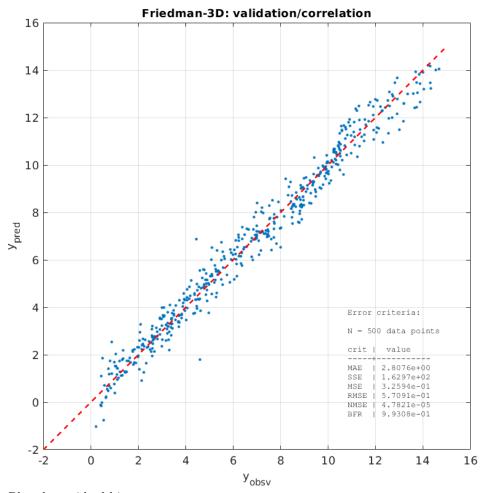


Predict the TS model output: y_{pred}

y_pred = ts.predict(u, y);

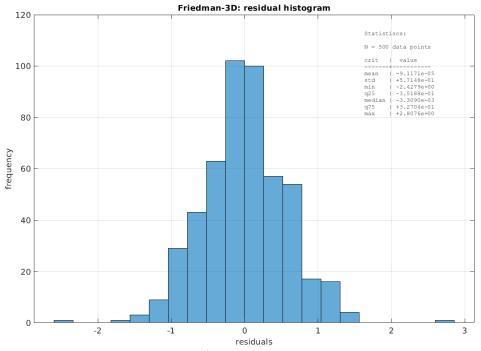
Plot the correlation

plotResiduals(y, y_pred, 'figure', 2, 'title', 'Friedman-3D: validation/correlation'); set(gcf,'WindowState','maximized');



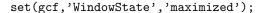
Plot the residual histogram:

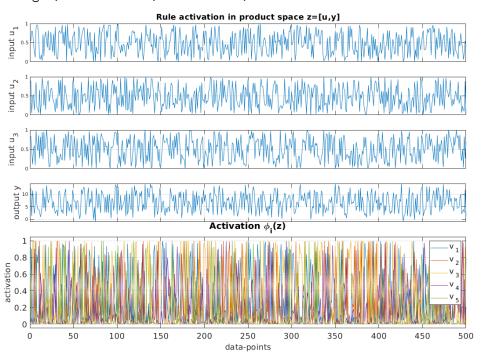
```
plotResidualHist( y, y_pred, 'figure', 3, 'nbins', 21, ...
   'title', 'Friedman-3D: residual histogram');
set(gcf,'WindowState','maximized');
```



Plot the rule activation and input/output data:

```
plotRuleActivation( u,y_pred, ts, 'figure', 4 );
```





4 Validation of the TS model

```
Choose another N random inputs [u_1, u_2, u_3]
```

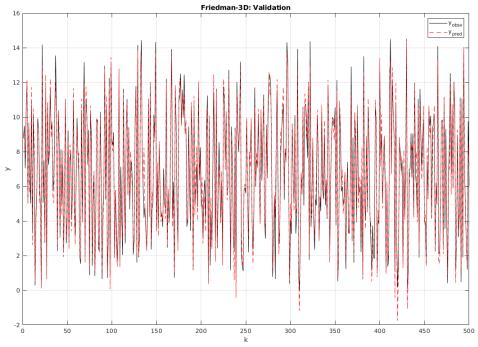
```
u_val = rand( N, nu );
y_obsv = Friedman_fct( u_val, nu );

Compute the output vector: ypred

y_val_pred = ts.predict( u_val );

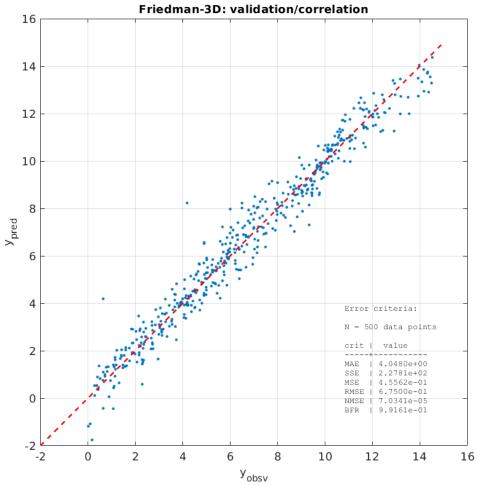
Plot the outputs

figure(5);clf
plot( 1:N, y_obsv, 'k-',1:N, y_val_pred, 'r--')
grid on
xlabel('k')
ylabel('y')
title( 'Friedman-3D: Validation')
legend( 'y_{obsv}', 'y_{pred}')
set(gcf, 'WindowState', 'maximized');
```



Plot the correlation

plotResiduals(y_obsv, y_val_pred, 'figure', 6, ...
 'title', 'Friedman-3D: validation/correlation');
set(gcf,'WindowState','maximized');



Plot the correlation histogram

plotResidualHist(y_obsv, y_val_pred, 'figure', 7, 'nbins', 31, ...

'title', 'Friedman-3D: validation/correlation histogram');
set(gcf,'WindowState','maximized');

