Takagi-Sugeno Model Identification Toolbox

May 21, 2021

Example of a NARX TS model for a Narendra function

V 1.0

Axel Dürrbaum (axel.duerrbaum@mrt.uni-kassel.de)

Department of Measurement and Control (MRT)

Institute for System Analytics and Control (ISAC)

University of Kassel, Germany (http://www.uni-kassel.de/go/mrt)

$Id: NARX_Narendra.m | Fri Feb 26 16:25:05 2021 +0100 | Axel Dürrbaum $

Contents

Example of the identification of a NARX MISO TS model for given multiple inputs u and single output y.

Determine the NARX TS model

$$\hat{y}_{k+1} = \sum_{i=1}^{n_v} \phi_i(z) \cdot \left(\sum_{l=1}^{l_y} A_i \cdot y_{k-l} + \sum_{j=1}^{n_u} \sum_{l=0}^{l_u} B_{i,j} \cdot u_{k-l} + c_i \right)$$

- for given $u_j, j = 1, \ldots, n_u$ of n_u input vectors and
- input lags x_u with length l_u
- vector y of single output,
- output lags x_y with length l_y

- with FCM membership function

$$\mu_i(z) = \left(\sum_{j=1}^{n_v} \left(\frac{||z - v_i||}{||z - v_j||} \right)^{\frac{2}{\nu - 1}} \right)^{-1}$$

- or Gauss membership function

$$\mu_i(z) = e^{-\frac{||z - v_i||^2}{2 \cdot \sigma_i^2}}$$

- norm

$$||z - v_j|| = (z - v_j)^T \cdot w_j \cdot (z - v_j)$$

- and fuzzy basis functions

$$\phi_i(z) = \frac{\mu_i(z)}{\sum_{j=1}^{n_v} \mu_j(z)}$$

- with the scheduling variable $z = u$ (for input space clustering) or $z = [u, y]$ (for product space clustering), and
- cluster centers $v_i, i = 1, \ldots, n_v$.

1 Algorithm

1. Choose number of local models: $n_v = 3$ and fuzzy parameter $\nu = 1.2$ or $\sigma_i =$???
2. Select the TS model with minimal MSE of s multi-start tries with clustering and global LS-estimation.
3. Optimize the TS model parameters (v_i, B_i, c_i) for each try or the best found model.

2 Structural parmeters

```
nv = 3;    % number of local models
nu = 1;    % number of inputs
ny = 1;    % number of outputs
nue = 1.2; % Fuzziness parameter
```

Scheduling lags z_{lag} = regressor lags $x_{lag} = [y_{k-1}, y_{k-2}, u_k]^\top$

```
z_lag_u = {0};
z_lag_y = [1,2];
x_lag_u = {0};
x_lag_y = [1,2];
```

3 Identification data

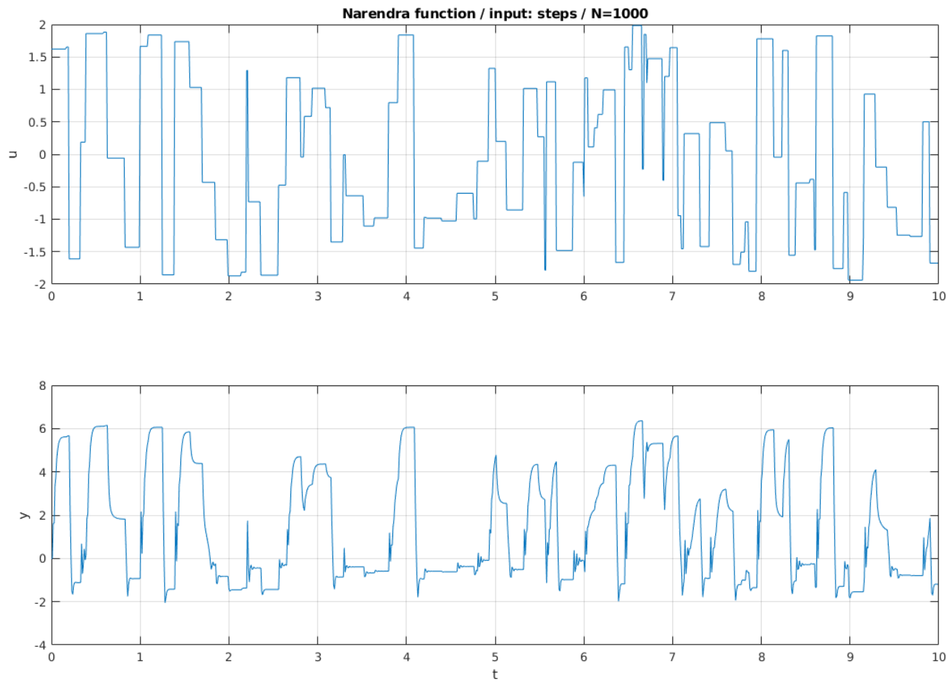
Create input u as steps with amplitude 2 and random width $l = [1, \ldots, 20]$ for $N = 1000$ time steps (sampling rate is 0.01s) and compute the output y from the Narendra function

```
N = 1000;
dt = 1e-2;                      % Sampling time
t = dt * transpose( 0:N-1 );   % time vector: $t$
rng(0);                         % Initalize random number generator
[ u, y ] = Narendra_fct( N );
```

Plot of the identification data

```
figure(1);clf

subplot(2,1,1)
plot(t,u)
```

```
grid on
ylabel('u')
title(sprintf('Narendra function / input: steps / N=%d',N))

subplot(2,1,2)
plot(t,y)
grid on
ylabel('y')
xlabel('t')

set(gcf,'WindowState', 'maximized' );
```



4 Create the NARX TS model

```
addpath( '../TSModel' );  % Path to TSModel class
ts = TSModel( 'ARX', nv, nu, 'Name','NARX Narendra', ...
    'Comment','Narendra function');
```

Set the identification data u, y

```
ts.setData( u, y, 'SampleTime',dt, 'Labels', { 'u', 'y' } );
```

Set the data limits: u=[-2,2], y=[-5,10]

```
ts.setDataLimits( [-2,2 ; -5,10] );
```

Set the scheduling and regressor lags

```
ts.setSchedulingLags( z_lag_u, z_lag_y );
ts.setRegressorLags( x_lag_u, x_lag_y );
```

5 Clustering

Clustering in product-space $z = [u, y]$ with FCM membership functions

```
ts.clustering( 'FCM', 'nue',nue, 'tries',1, 'seed',0 );
```

Estimated cluster centers: colums y_{k-1}, y_{k-1}, u_k

```
v1 = getCluster( ts )
```

```
v1 =
    5.0968     5.0864     1.1329
    2.1117     2.0720     0.3699
   -0.7434    -0.7214    -0.9016
```

6 Initialize the local models

Estimate the local model parameters with global Least Squares

```
ts.initialize( 'FCM', 'nue', nue, 'method','global'  );
```

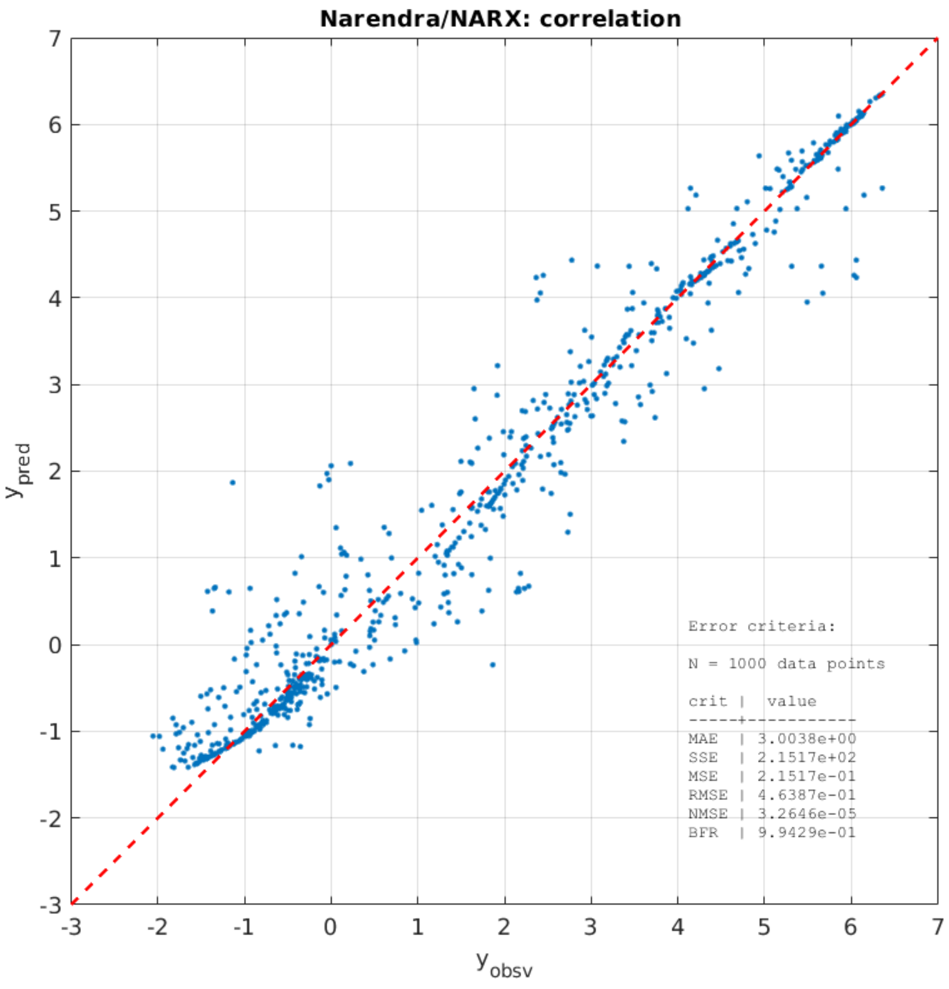
7 Predict the TS model output

for given data u: y_{pred}

```
y_pred = ts.predict( u, y );
```

Plot the correlation

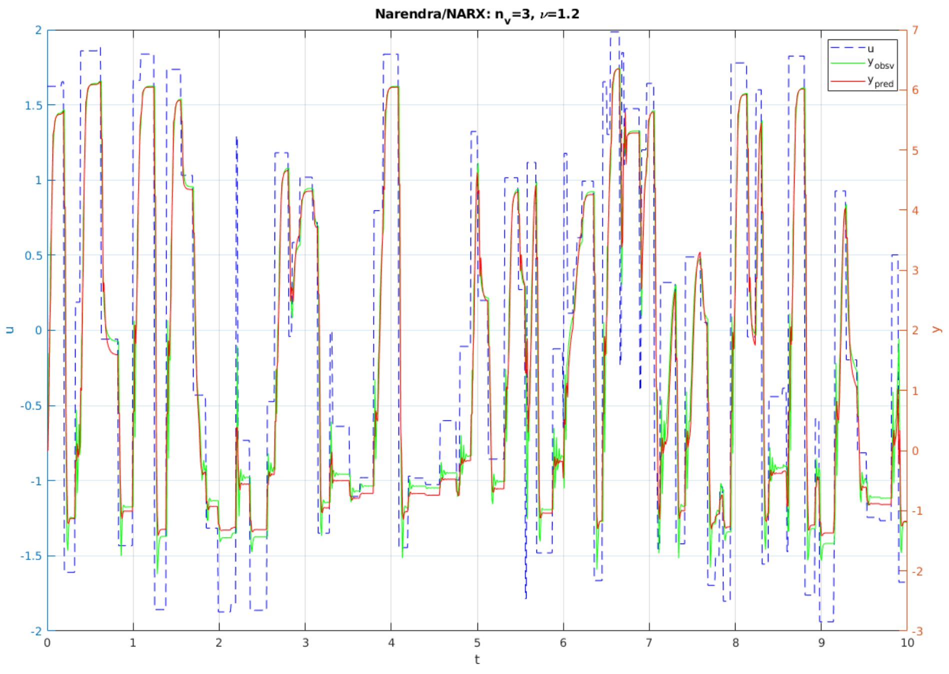
```
plotResiduals( y, y_pred, 'figure', 2, ...
    'title', 'Narendra/NARX: correlation ' );
set(gcf,'WindowState', 'maximized' );
```



```
Error criteria:

N = 1000 data points

crit |  value
-----+-----------
MAE  | 3.0038e+00
SSE  | 2.1517e+02
MSE  | 2.1517e-01
RMSE | 4.6387e-01
NMSE | 3.2646e-05
BFR  | 9.9429e-01
```

Plot of identification data y and predicted data y_{pred}

```
figure(3);clf

yyaxis left
plot(t,u,'b--')
ylabel('u')

yyaxis right
plot(t,y,'g-',t,y_pred,'r-')

grid on
ylabel('y')
xlabel('t')
title( sprintf('Narendra/NARX: n_v=%d, \\nu=%g',ts.nv,ts.nue))
legend('u','y_{obsv}','y_{pred}')
set(gcf,'WindowState', 'maximized' );
```



8 Validate with new data

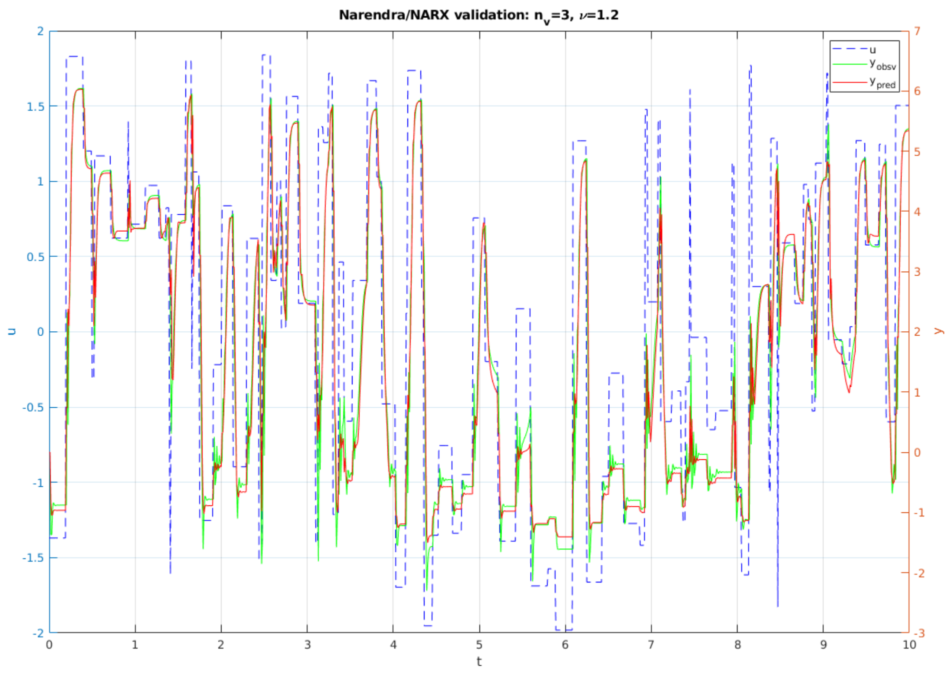
Test with new step inputs u_{val}, y_{val}

```
[u_val,y_val] = Narendra_fct( N );
y_pred_val = ts.predict( u_val,y_val );
```

Plot of observed and predicted outputs

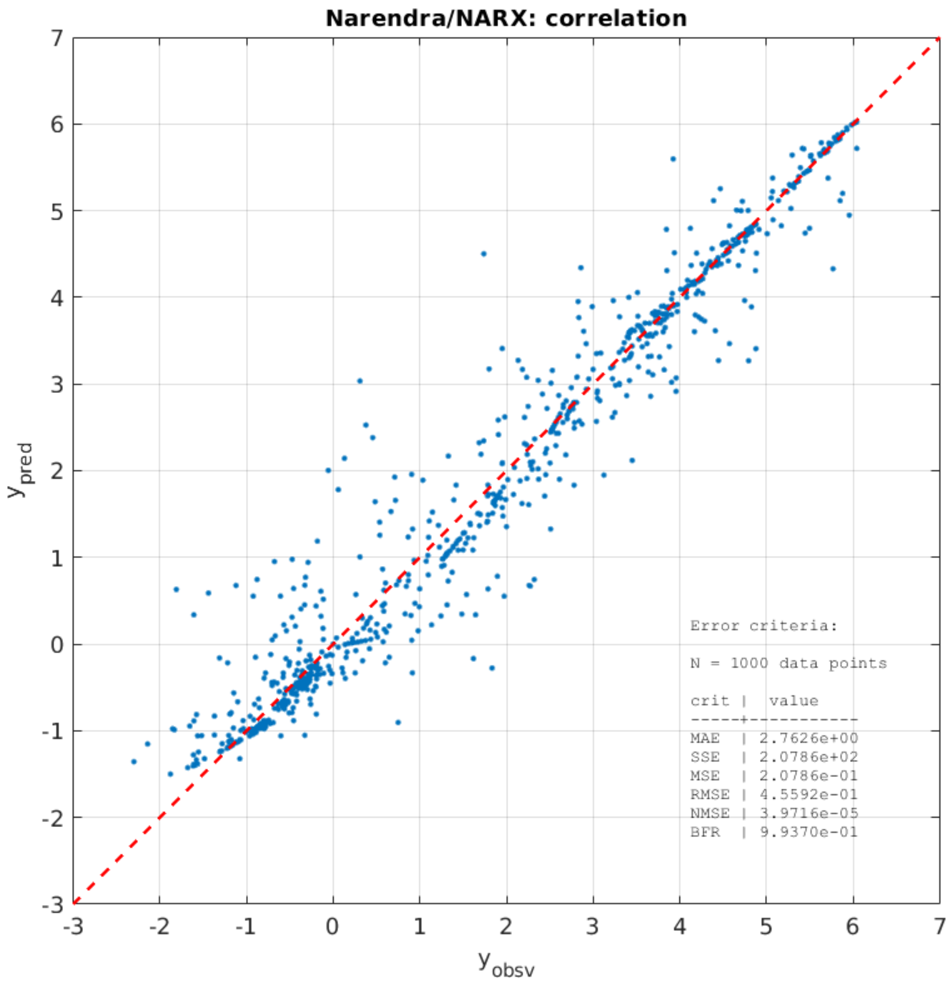
```
figure(5),clf

yyaxis left
plot(t,u_val,'b--')
ylabel('u')

yyaxis right
plot(t,y_val,'g-',t,y_pred_val,'r-')

grid on
ylabel('y')
xlabel('t')
title( sprintf('Narendra/NARX validation: n_v=%d, \\nu=%g',ts.nv,ts.nue))
```

```
legend('u','y_{obsv}','y_{pred}')
set(gcf,'WindowState', 'maximized' );
```



Plot the correlation

```
plotResiduals( y_val, y_pred_val, 'figure', 4, 'title', 'Narendra/NARX: correlation' );
set(gcf,'WindowState', 'maximized' );
```



```
Error criteria:

N = 1000 data points

crit |  value
-----+-----------
MAE  | 2.7626e+00
SSE  | 2.0786e+02
MSE  | 2.0786e-01
RMSE | 4.5592e-01
NMSE | 3.9716e-05
BFR  | 9.9370e-01
```

9 Optimize the TS model parameters

Optimize both, clusters centers v (MF) and local model parameters A_i, B_i, c_i

```
ts.optimize( 'B' )
```

```
                                        Norm of      First-order
 Iteration  Func-count     f(x)          step          optimality
     0          22        204.749                           87.1
     1          44        191.512         1.83408            172
     2          66        163.567        0.603534           19.2
     3          88        163.567         1.01328           19.2
     4         110         161.41        0.253319           14.3
     5         132        159.489        0.506638           28.2
     6         154        156.458         0.91203           29.5
     7         176        155.028        0.452131           61.6
     8         198        155.028        0.775852           61.6
     9         220        152.899        0.193963           37.7
    10         242        151.819        0.387926           9.55
    11         264        150.229        0.775852           67.5
    12         286        150.229        0.619305           67.5
    13         308        147.767        0.154826           45.9
    14         330        146.075        0.309653            109
    15         352         145.29        0.155379           6.71
    16         374        145.251        0.409002           51.1
    17         396        144.944         0.10225           3.53
    18         418        144.789        0.173526           22.1
    19         440        144.779         0.18164           63.2
    20         462         144.64       0.0454101           3.41

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to
its initial value is less than the value of the function tolerance.

ans =
TS-Model: Type=ARX
Name: 'NARX Narendra'
 Type: 'TSModel'
 Date: '21-May-2021 11:37:23'
 Comments:
  'Narendra function'
 Structural parameters: nu = 1, ny = 1, nv = 3
 Identification data: N=1000
, ts=0.01 Initial model estimation:
 lags: u_1:0, y = [1 2]
  Membership function type = FCM
  Clustering: FCM, nue=1.2 norm=Euclidean in input space
 Estimation of local models:
 lags:    u_1:0,   y = [1 2]
  Initialization of local models: global
  Optimization of model parameters: MF&LM
```

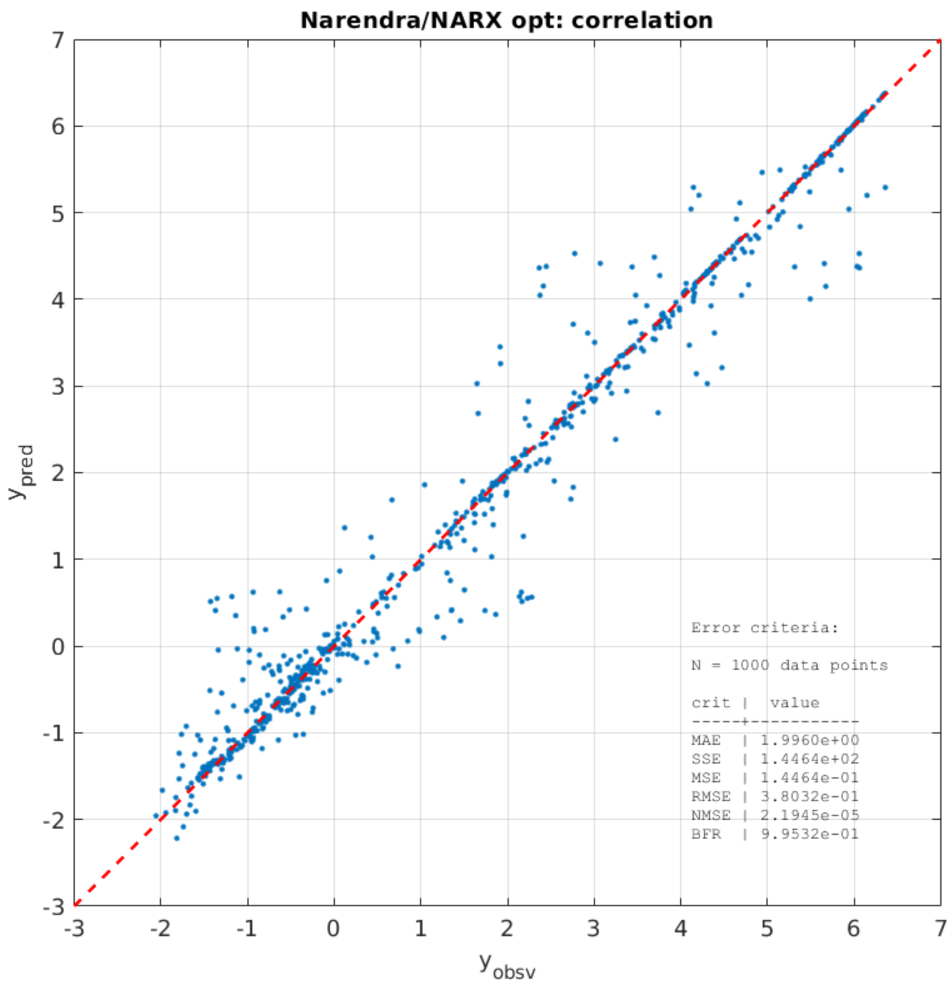
Cluster centers of the optimized TS model

```
v2 = getCluster( ts )
```

```
v2 =
   -1.1413    3.3203   -0.2958
    0.0847   -0.0034    1.7030
    0.2234   -0.7193    1.5269
```

Plot the correlation

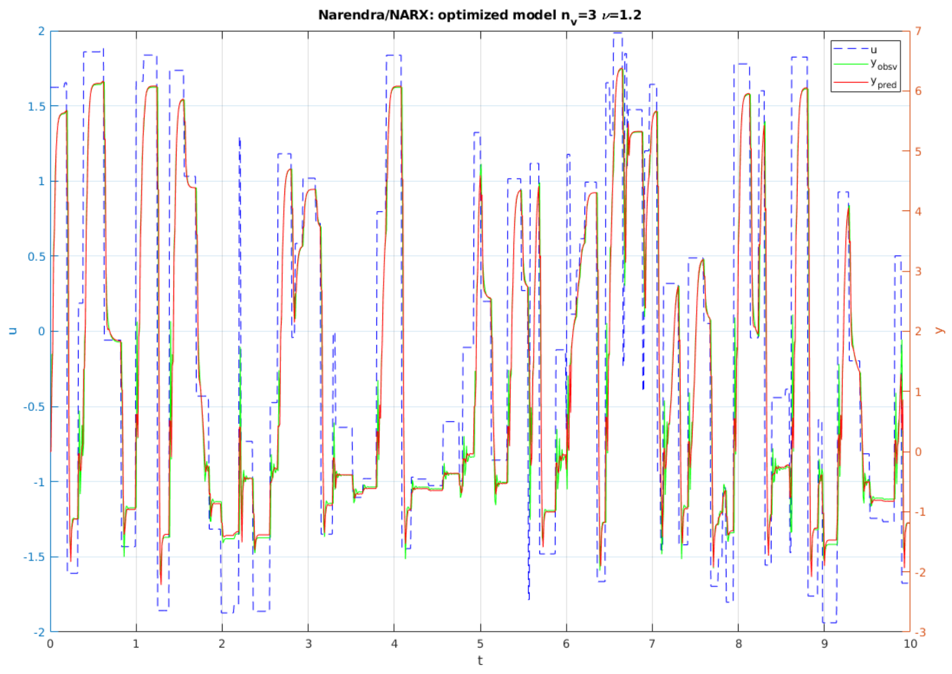
```
y_pred_opt = ts.predict( u,y );
plotResiduals( y, y_pred_opt, 'figure', 5, ...
    'title', 'Narendra/NARX opt: correlation' );
set(gcf,'WindowState', 'maximized' );
```



```
Error criteria:

N = 1000 data points

crit |  value
-----+------------
MAE  | 1.9960e+00
SSE  | 1.4464e+02
MSE  | 1.4464e-01
RMSE | 3.8032e-01
NMSE | 2.1945e-05
BFR  | 9.9532e-01
```

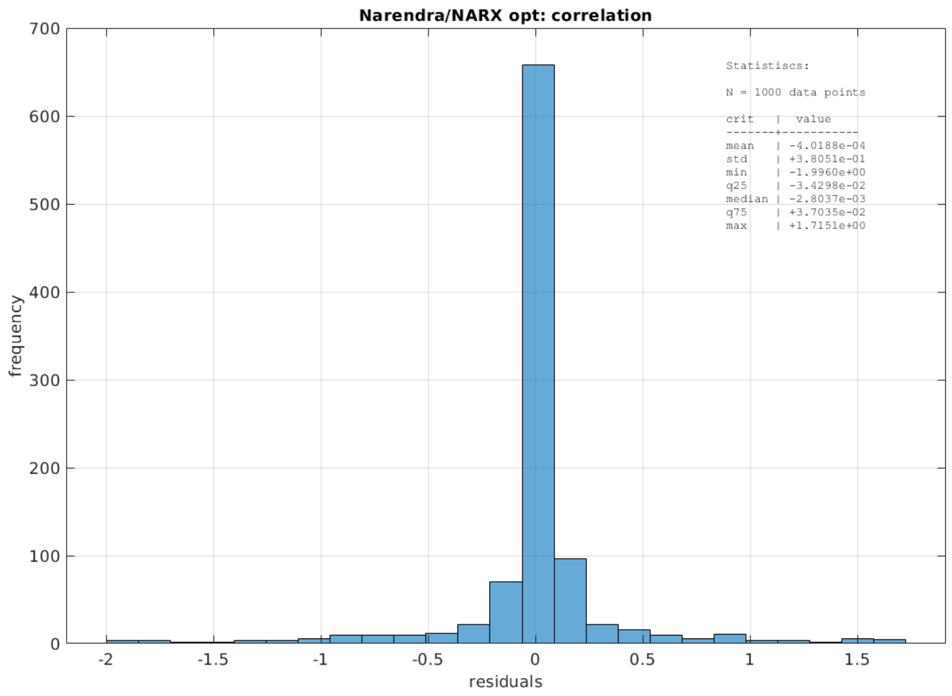
Plot of observed and predicted outputs for optimized TS model

```
figure(6);clf

yyaxis left
plot(t,u,'b--')
grid on
ylabel('u')
title( sprintf('Narendra/NARX: optimized model n_v=%d \\nu=%g',ts.nv,ts.nue))

yyaxis right
plot(t,y,'g-',t,y_pred_opt,'r-')
grid on
ylabel('y')
xlabel('t')
legend('u','y_{obsv}','y_{pred}')
set(gcf,'WindowState', 'maximized' );
```



Plot the residual histogram

```
plotResidualHist( y, y_pred_opt, 'figure', 7, 'title', 'Narendra/NARX opt: correlation', 'nbins
set(gcf,'WindowState', 'maximized' );
```



```
Statistiscs:

N = 1000 data points

crit   |  value
-------+-----------
mean   | -4.0188e-04
std    | +3.8051e-01
min    | -1.9960e+00
q25    | -3.4298e-02
median | -2.8037e-03
q75    | +3.7035e-02
max    | +1.7151e+00
```