Takagi-Sugeno Model Identification Toolbox

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Example of a NOE TS model for the Narendra function

V1.0

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$Id: NOE_Narendra.m | Fri Feb 26 16:25:05 2021 +0100 | Axel Dürrbaum $%

Contents

Example of the identification of a NOE MISO TS model for given multiple inputs u and single output y.

Determine the NOE TS model

$$\hat{y}_{k+1} = \sum_{i=1}^{n_v} \phi_i(z) \cdot \left(\sum_{l=0}^{l_y} A_i \cdot \hat{y}_{k-l} + \sum_{j=0}^{n_u} \sum_{l=0}^{l_u} B_{i,j} \cdot u_{k-l} + c_i \right) + e_k$$

- for given $u_j, j = 1, \ldots, n_u$ of n_u input vectors, output error e and
- input lags x_u with length l_u
- intial vector y of single output,
- output lags x_y with length l_y
- with FCM membership function

$$\mu_i(z) = \left(\sum_{j=1}^{n_v} \left(\frac{||z - v_i||}{||z - v_j||} \right) \frac{2}{\nu - 1} \right)^{-1}$$

- or Gauss membership function

$$\mu_i(z) = e^{-\frac{||z - v_i||^2}{2 \cdot \sigma_i^2}}$$

- norm

$$||z - v_j|| = (z - v_j)^T \cdot w_j \cdot (z - v_j)$$

- and fuzzy basis functions

$$\phi_i(z) = \frac{\mu_i(z)}{\sum_{j=1}^{n_v} \mu_j(z)}$$

- with the scheduling variable $z = u$ (for input space clustering) or $z = [u, y]$ (for product space clustering), and
- cluster centers $v_i, i = 1, \ldots, n_v$.

1 Structural settings

```
nc = 3;    % number of clusters = local models
nu = 1;    % number of inputs
ny = 1;    % number of outputs
nue = 1.2; % fuzziness parameter
```

Scheduling lags z = regressor lags $x = [\hat{y}_{k-1}, \hat{y}_{k-2}, u_k]^\top$

```
z_lag_u = {0};
z_lag_y = [2];
x_lag_u = {0};
x_lag_y = [2];
```

2 Identification data

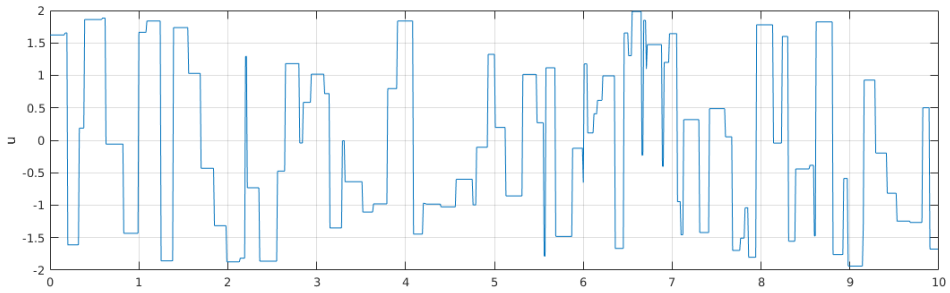
Create input u as steps with width $l = [1, \ldots, 20]$ for $N = 1000$ time steps (sampling rate is 0.01) and compute the output y from the Narendra function

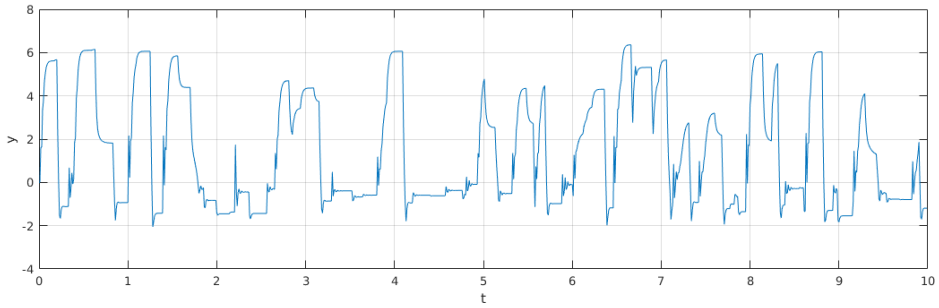
```
N = 1000;
dt = 1e-2;                              % Sampling time
t = dt * transpose( 0:size(u,1)-1 ); % time vector $t$
rng(0);
[u,y] = Narendra_fct( N );
```

Plot of the identification data

```
h=figure(1);clf

subplot(2,1,1)
plot(t,u)
grid on
ylabel('u')
subplot(2,1,2)
plot(t,y)
grid  on
ylabel('y')
xlabel('t')
```





3 Creation of TS model

```
ts = TSModel( 'OE', nc, nu, 'Name','OE Narendra', 'Comment','Narendra function');
ts.setSchedulingLags( z_lag_u, z_lag_y );
ts.setRegressorLags( x_lag_u, x_lag_y );
```

Set the identification data

```
ts.setData( u, y, 'SampleTime',dt, 'Labels', { 'u', 'y' } );
ts.setDataLimits( [-2,2 ; -5,10] );
```

4 Clustering

Clustering in product-space $z = [u, y]$ with FCM membership functions and $\nu = 1.2$ with $s = 3$ multi-start tries and ficed initialized random number generator (seed 0)

```
ts.clustering( 'FCM', 'nue', nue, 'tries',3, 'seed', 0 )
```

```
ans =
TS-Model: Type=OE
Name: 'OE Narendra'
 Type: 'TSModel'
 Date: '18-Mar-2021 13:49:52'
 Comments:
  'Narendra function'
 Structural parameters: nu = 1, ny = 1, nv = 3
 Identification data: N=1000
, ts=0.01 Initial model estimation:
 lags: u_1:0, y = 2
  Membership function type = FCM
  Clustering: FCM, nue=1.2 norm=Euclidian in input space
 Estimation of local models:
 lags:    u_1:0,   y = 2
```

Cluster centers of the inital model

```
v1 = getCluster( ts )
```

```
v1 =
   -0.7733   -0.9077
    2.0289    0.3502
    5.0925    1.0995
```

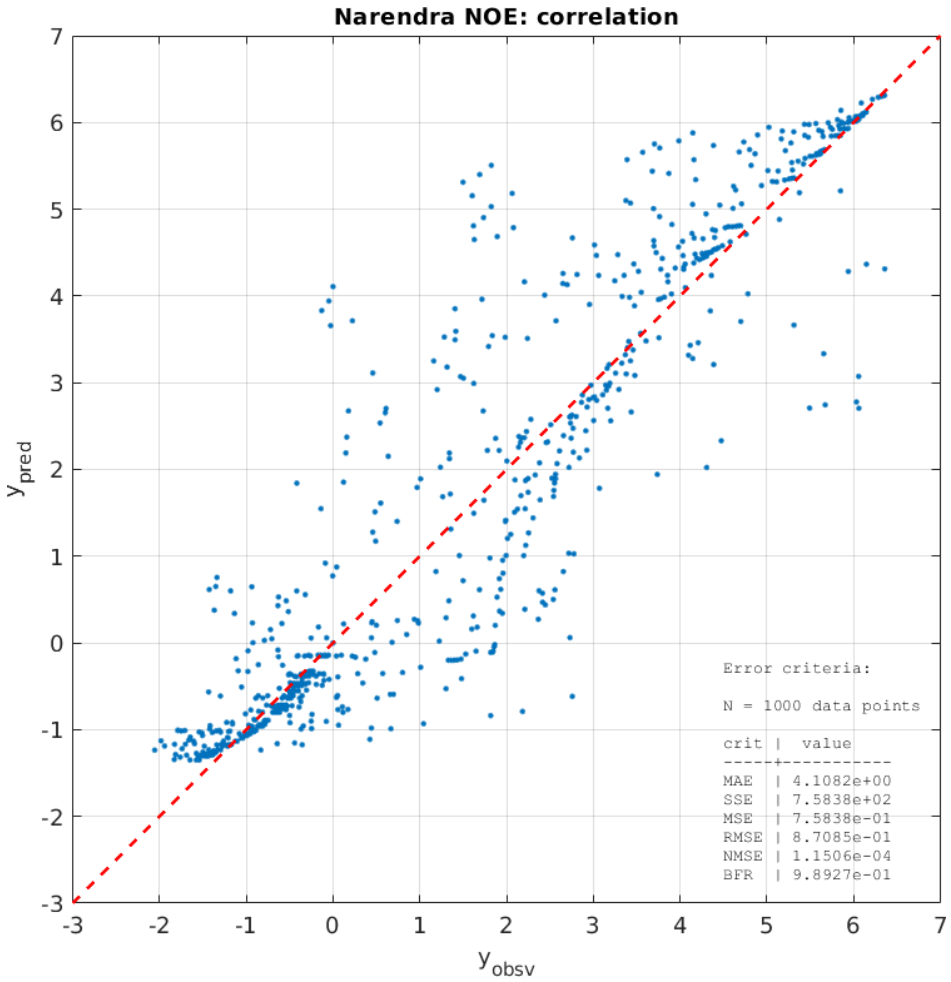
5 Initialization of local models

with global Least-Squares, FCM membership functions and $\nu = 1.2$

```
ts.initialize( 'FCM', 'nue', nue, 'method','global'  );
```

6 Predicted NOE TS model ouput

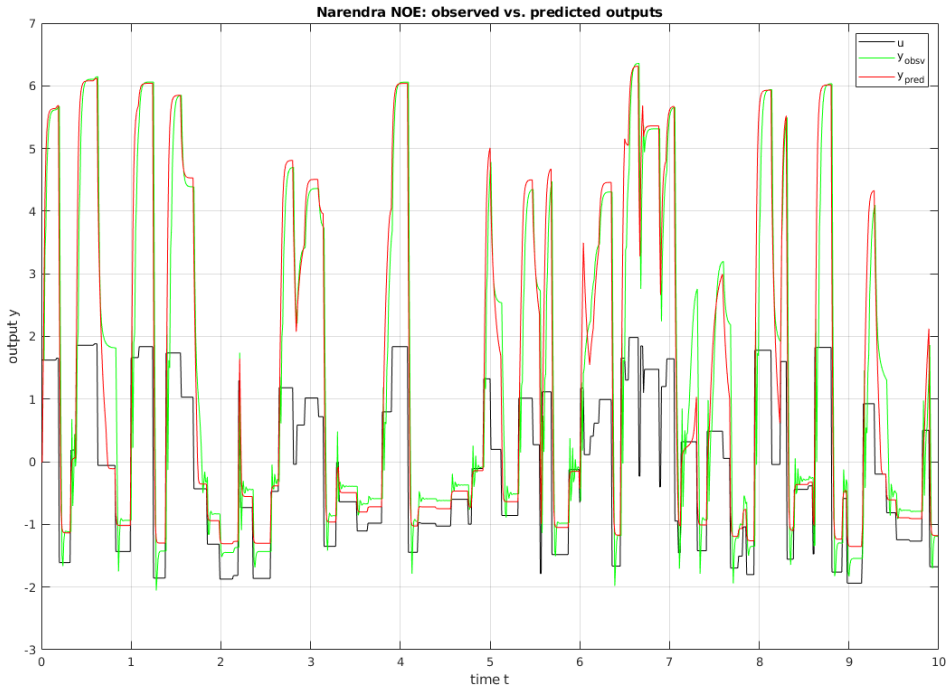
```
y_pred = ts.predict( u,y );
plotResiduals( y, y_pred, 'figure', 2, 'title', 'Narendra NOE: correlation' );
set(gcf,'WindowState', 'maximized' );
```



Plot of the observed vs. predicted outputs

```
figure(3);clf

plot(t,u,'k-',t,y,'g-',t,y_pred,'r-')
grid on
xlabel( 'time t' )
ylabel( 'output y' )
title('Narendra NOE: observed vs. predicted outputs')
legend('u','y_{obsv}','y_{pred}')
set(gcf,'WindowState', 'maximized' );
```

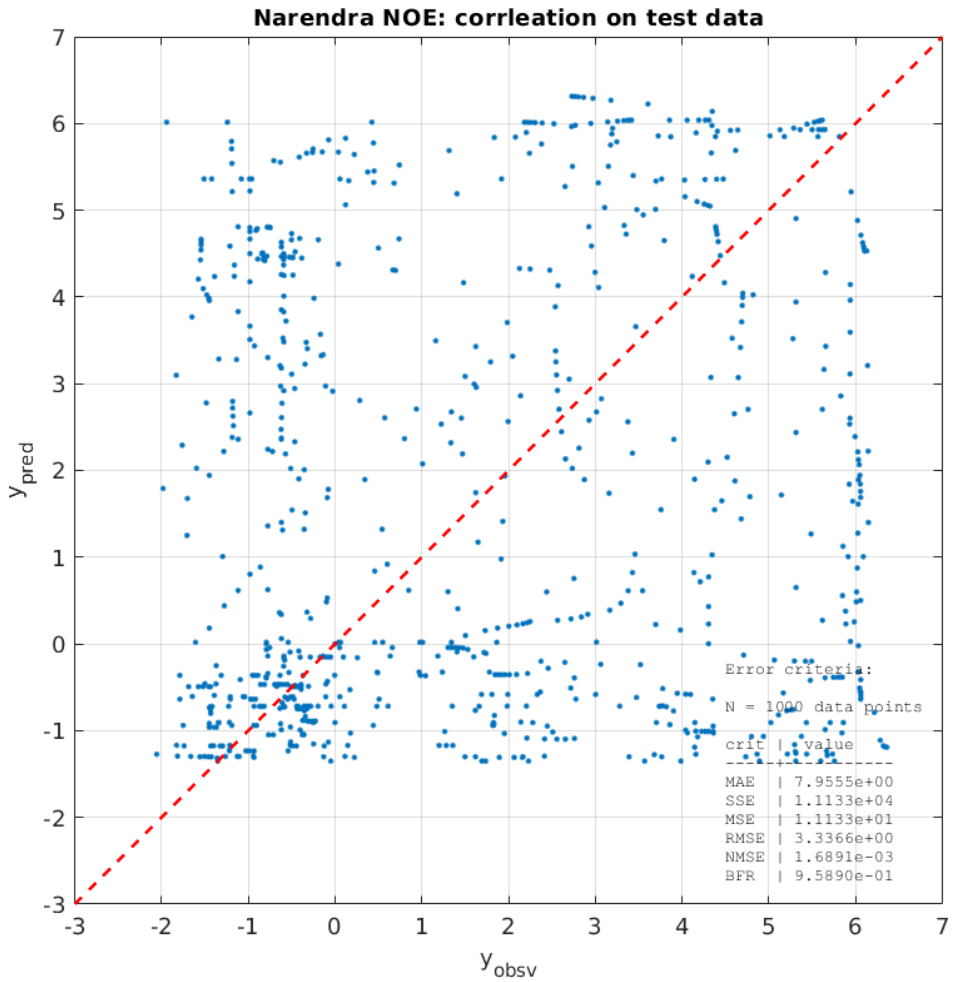


7 Prediction on validation data

```
[u_val,y_val] = Narendra_fct( N );
y_val_pred = ts.predict( u_val,y_val );
```

Plot the correlation

```
plotResiduals( y, y_val_pred, 'figure', 4, 'title', 'Narendra NOE: corrleation on test data' );
set(gcf,'WindowState', 'maximized' );
```



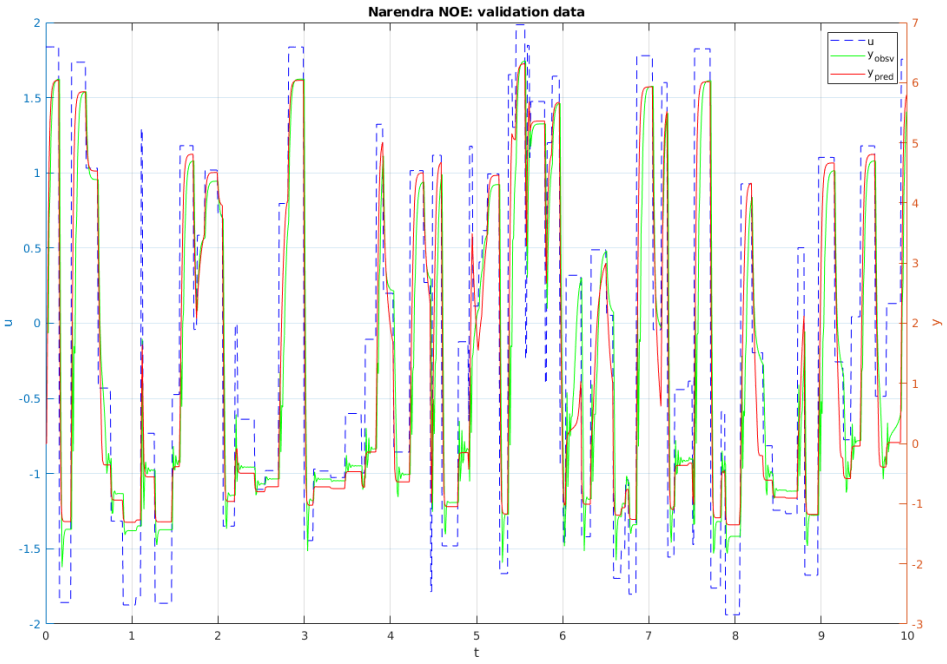
Plot of observed vs. predicted validation data

```
figure(5);clf

yyaxis left
plot(t,u_val,'b--')
ylabel( 'u' )
yyaxis right

plot(t,y_val,'g-',t,y_val_pred,'r-')
ylabel( 'y' )
xlabel( 't' )

grid on
title('Narendra NOE: validation data')
legend('u','y_{obsv}','y_{pred}')
set(gcf,'WindowState', 'maximized' );
```



8 Optimize the TS model parameters

Optimize both, the cluster centers v (MF) and the local model parameters A_i, B_i, c_i

```
optimopts = optimoptions('lsqnonlin');
optimopts.FunctionTolerance = 1e-6;
optim.OptimalityTolerance = 1e-6;
optimopts.StepTolerance = 1e-12;
optimopts.Display = 'iter-detailed';

ts.optimize( 'B', 'optimopts', optimopts )
%ts.optimize( 'B'  )
```

```
                                         Norm of      First-order
 Iteration  Func-count     f(x)            step          optimality
     0         16         758.382                          802
     1         32         758.382          1.9881          802
     2         48         526.799        0.497024          413
     3         64         361.411        0.960259          359
     4         80          277.15        0.425853          497
     5         96         271.183        0.114441         99.4
     6        112         270.479       0.0512704         42.5
     7        128         270.316       0.0405038           39
     8        144         270.222       0.0271255         22.5
     9        160         270.187        0.021413         18.5
    10        176         270.121      0.00535325         10.5
    11        192         270.114      0.00574168         3.98
    12        208         270.113      0.00411673          3.4
    13        224          270.11      0.00102918         2.73
    14        240          270.11      0.00136231         1.06
    15        256         270.109     0.000957319        0.898
    16        272         270.109      0.00023933        0.819

Optimization stopped because the relative sum of squares (r) is changing
by less than options.FunctionTolerance = 1.000000e-06.

ans =
TS-Model: Type=OE
```

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, ts=0.01 Initial model estimation:
 lags: u_1:0, y = 2
  Membership function type = FCM
  Clustering: FCM, nue=1.2 norm=Euclidian in input space
 Estimation of local models:
 lags:    u_1:0,    y = 2
  Initialization of local models: global
  Optimization of model parameters: MF&LM
```

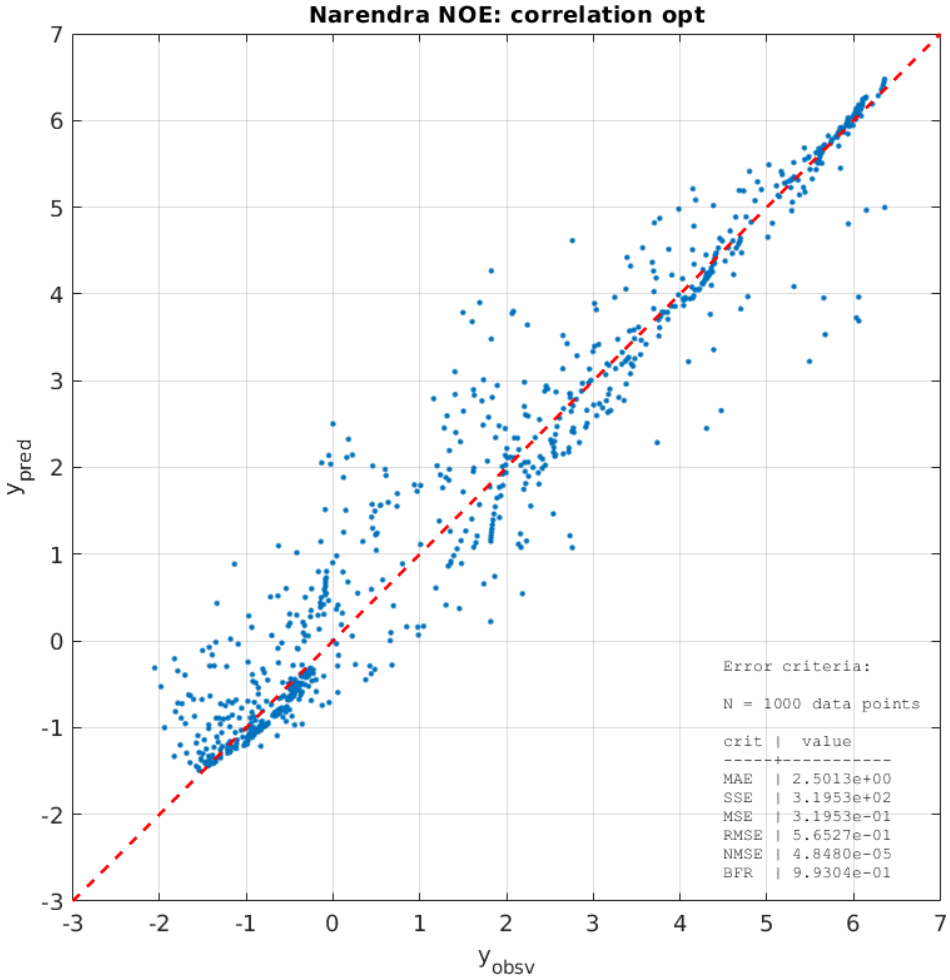
Get the cluster centers of the optimized NOE TS model

```
v2 = getCluster( ts )
```

```
v2 =
   -0.7733   -0.9077
         0    0.3502
    5.0925    1.0995
```

Plot the correlation on the validation data

```
y_pred_opt = ts.predict( u,y );
plotResiduals( y, y_pred_opt, 'figure', 6, 'title', 'Narendra NOE: correlation opt' );
set(gcf,'WindowState', 'maximized' );
```



Plot the observed vs. the predicted validation data

```
figure(7);clf
yyaxis left
plot(t,u_val)
ylabel( 'input u' )
yyaxis right
plot(t,y_val,'g-',t,y_val_pred,'r-')
grid on
ylabel( 'output y' )
xlabel( 'time t' )
title('Narendra NOE opt: predicted validation output')
legend('u','y_{obsv}','y_{pred}')
set(gcf,'WindowState', 'maximized' );
```