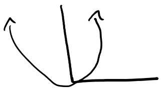
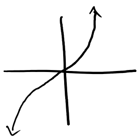
Week 1 day 1

Day 1

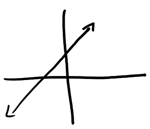
Functions (often continuous)



$y = x^2$ parabola, quadratic polynomial



$y = x^3$ cubic polynomial

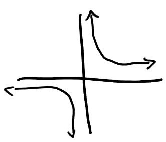


$y = mx + b$ line

m = slope $= \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$

b = y-intercept

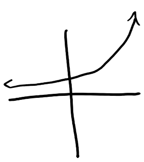
$x = 0 \rightarrow y = b$



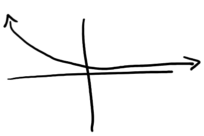
rational function "hyperbola"

$y = \frac{1}{x}$

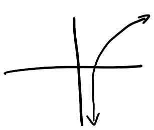
$\frac{p(x)}{q(x)}$ $\frac{\text{polynomial}}{\text{polynomial}}$



$y = 2^x$ exponential function (growth)



$y = \left(\frac{1}{2}\right)^x$ exponential function (decay)



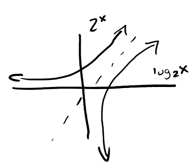
$\log_2 x$ logarithmic function

→ inverse of 2^x

ie reflected over $y = x$ line.

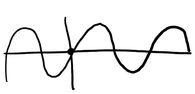
Note: $\log_2(2^x) = x$

$2^{\log_2(x)} = x$

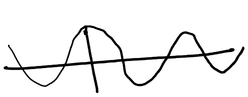


Sin x

$\sin 0 = 0$ trignometric function

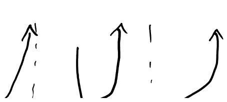


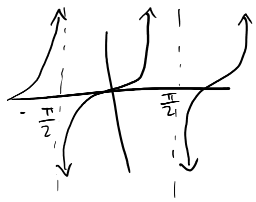
cos x



$\cos 0 = 1$

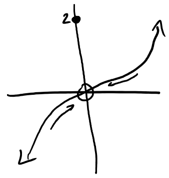
tan x





CALCULUS

1. Limits: what value is a function approaching.



f(x) =
$y = x^3$

$f(-1) = (-1)^3 = -1$
$f(1) = (1)^3 = 1$
$\underbrace{f(0) = 2}_{\text{value of function}}$

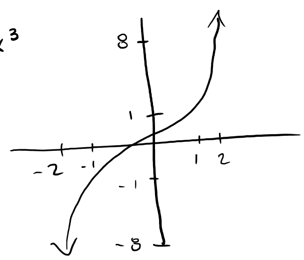
$\lim_{x \to 0} f(x) = \boxed{0}$

The limit as x approaches 0 is $\boxed{0}$.
As $x \to 0$, $y \to \boxed{0}$

This function has a discontinuity at x = 0.
This is called a removable discontinuity, because if we filled the hole, the function would be continuous.

2. Derivatives = slope of a function at a point or slope of tangent line.

$y = x^3$

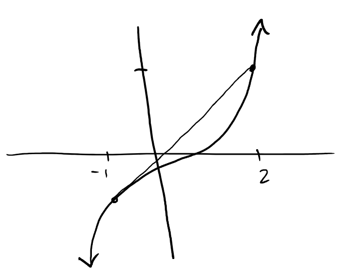


x	$y = x^3$
-2	-8
-1	-1
0	0
1	1
2	8

SLOPE OF SECANT LINE

secant = "cutting" (latin)

From x = -1
to x = 2



$m = \frac{\Delta y}{\Delta x} = \frac{9}{3} = 3$

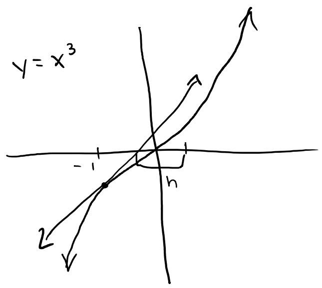
	x	y
p2	-1	-1
p1	2	8
Δ	2 -(-1)	8 -(-1)

Δ	$2-(-1)$ $=3$	$8-(-1)$ $=9$

SLOPE OF TANGENT LINE

tangent = "touching"

at $x=-1$



	x	y
P_1	-1	$(-1)^3 = -1$
P_2	$-1+h$	$(-1+h)^3$
Δ	$-1+h$ $-(-1)$ $= h$	$-1+3h-3h^2+h^3$ $-(-1)$ $= 3h-3h^2+h^3$

$$m = \frac{\Delta y}{\Delta x} = \frac{3h-3h^2+h^3}{h} = 3-3h+h^2$$

Shrink the distance between points to Ø.

ie $h=0$. (infinitessimally small)

$$m = 3-3h+h^2 \Big|_{h \to 0}$$

$$= 3-3(0)+(0)^2$$

$$= 3$$

NOTE: this is a limit

$f'(-1) = 3$

Slope of tangent line =

slope of function =

instantaneous rate of change =

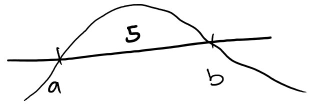
derivative of f =

$f'(x)$ =

$\frac{dy}{dx}$ =

The slope of the tangent line is the limit of the slope of secant lines as the secant lines becomes closer to the tangent line.

3. INTEGRALS / INTEGRATION

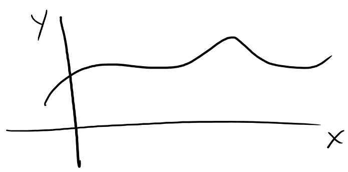


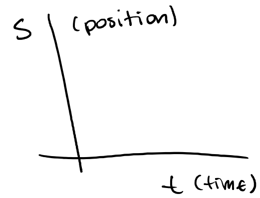
$$\int_a^b f(x)\,dx = 5$$

The integral tells you the area

The integral tells you the area under the curve.

Find the average rate of change (slope) and instantaneous rate of change





s = position of particle (m)

t = time (sec)

$s = t^2 - 2t + 4$

① Find average ROC from time = 0 to 4 sec

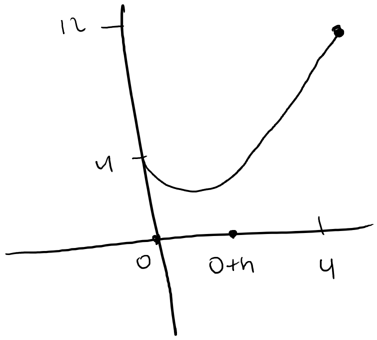
$$ROC_{ave} = \frac{\Delta s}{\Delta t} = \frac{8}{4} = 2\ \text{m/sec}$$

(= V_{ave} average velocity)

ie on average moved 2m/sec forward

	t	$s = t^2 - 2t + 4$
P1	0	$0^2 - 2(0) + 4 = 4$
P2	4	$4^2 - 2(4) + 4 = 12$
Δ	$4 - 0$ $= 4$	$12 - 4$ $= 8$
unit	sec	m

② Find instanteous ROC @ $t = \varnothing$.



$$ROC_{inst} = \frac{\Delta s}{\Delta t}\Big|_{h\to 0}$$

$$= \frac{h^2-2h}{h}\Big|_{h\to 0}$$

$$= h-2\Big|_{h\to 0}$$

$$= 0-2 = \boxed{-2}$$

	t	$s = t^2-2t+4$
	0	4
	$0+h$ $= h$	h^2-2h+4
Δ	$h-0$ $= h$	$h^2-2h+4-4$ $= h^2-2h$

$\frac{0}{0}$ limits $\quad \lim_{h\to 0} \frac{h^2-2h}{h} \quad \frac{0-0}{0} = \frac{0}{0}$

FIR "further investigation required"

$\frac{\text{polynomial}}{\text{polynomial}}$ factor and cancel,

$$\frac{h^2-2h}{h} = \frac{\cancel{h}(h-2)}{\cancel{h}} = h-2$$

Ex 2:

$s = \sqrt{t}$

① average velocity from $t=0$ to $t=4$

	t	$s=\sqrt{t}$
	0	$\sqrt{0} = 0$
	4	$\sqrt{4} = 2$
Δ	4	$2-0 = 2$

$V_{ave} = \frac{\Delta s}{\Delta t} = \frac{2}{4} = \frac{1}{2}$ m/sec

② instantaneous velocity at $t=1$

t	$s=\sqrt{t}$
1	$\sqrt{1} = 1$

multiply by conjugate

$$\frac{\Delta s}{\Delta t} = \frac{\overset{a}{\sqrt{1+h}} - \overset{b}{1}}{h} \cdot \frac{\overset{a}{\sqrt{1+h}} + \overset{b}{1}}{\sqrt{1+h}+1}$$

1	$\sqrt{1} = 1$
$1+h$	$\sqrt{1+h}$
Δ $1+h-1 = h$	$\sqrt{1+h} - 1$

$$\frac{\Delta s}{\Delta t} = \frac{\sqrt{1+h} - 1}{h} \cdot \frac{\sqrt{1+h} + 1}{\sqrt{1+h} + 1}$$

$$= \frac{\overset{a^2}{(1+h)} - \overset{b^2}{1}}{h(1+h)+1}$$

$(a-b)(a+b)$
$= a^2 - b^2$

$$= \frac{h}{h(\sqrt{1+h} + 1)} = \frac{1}{\sqrt{1+h}+1}$$

$$\left.\frac{1}{\sqrt{1+h}+1}\right|_{h \to 0}$$

$$= \frac{1}{\sqrt{1+0}+1}$$

$$= \frac{1}{1+1} = \frac{1}{2}$$

The particle is moving forward (+) at $\frac{1}{2}$ m/sec.