STA 260 OH 1

The following practice problems are designed to assist with review material from STA256. Some of the solutions will be discussed during office hours on Tuesday from 11:00 AM to 12:00 PM in MN 3220. These problems were prepared, either partially or entirely, by a previous STA260 instructor, Jerry Brunner.

1. Let the continuous random variable X have density $f_{_X}(x) = 2x\,e^{-x^2}\,I(x>0)$.
   (a) Write the cumulative distribution function $F_{_X}(x)$ using indicator functions. Show your work.
   (b) Calculate $P(X > \frac{1}{2})$.
2. Let $X \sim N(\mu, \sigma^2)$. Show $Z = \frac{X-\mu}{\sigma} \sim N(0,1)$.
3. Let $X_1, \ldots X_n$ be independent and identically distributed $N(\mu, \sigma^2)$ random variables. Find the distribution of $Y = a + \sum_{i=1}^n b_i X_i$. Show your work.
4. Let $Z \sim N(0,1)$. Show $Z^2 \sim \chi^2(1)$.
5. Let $Y_1, \ldots Y_n$ be independent $\chi^2(\nu_i)$ random variables. Show $Y = \sum_{i=1}^n Y_i \sim \chi^2\left(\sum_{i=1}^n \nu_i\right)$.
6. Let the random variable X have distribution function $F_{_X}(x) = 1$ for all real x. Is this possible? Answer Yes or No and briefly explain.
7. Let the continuous random variable X have density $f_{_X}(x)$. What's wrong with this?
$$F_{_X}(x) = \int_{-\infty}^{\infty} f_{_X}(t)\,dt$$
8. What's wrong with this? $F_{_{X|Y}}(x|y) = \frac{F_{X,Y}(x,y)}{F_Y(y)}$. To see it more easily, let X and Y be discrete.
9. Let X be a continuous random variable. Either prove that the following proposition is true in general, or show that it is not by giving a simple counter-example: $E\left(\frac{1}{X}\right) = \frac{1}{E(X)}$.
10. What's wrong with this? $Var(X) = E\left((X-\mu)^2\right) = \left(E(X-\mu)\right)^2 = \left(E(X) - E(\mu)\right)^2 = (\mu-\mu)^2 = 0$.
