MRB MATH LIBRARY

DOCUMENTATION

This library is intended for use in lower-cost, slower microcontrollers not dedicated for mathematical calculations (e.g. ATmega/Arduino, ARM Cortex M0 and M4, ESP, Raspberry Pi pico) to speed up floating point operations. It can also be used in DSP to reduce cycle time, wherever cycle time is very critical, but the accuracy of calculations is not so critical. The MRB_MATH library will also significantly speed up the execution time of math calculations on microcontrollers without FPU or without hardware support for the default math.h library. An additional function not included in the math.h library is the function for calculating the RMS of a signal. This function is available in three variants: normal, fast and rapid.

In summary, this library allows faster calculations at the expense of lower precision and higher memory usage.

The documentation describes the functions contained in the MRB_MATH library and compares them with their counterparts from the math.h library. Execution times and precision are compared. The results are summarized in graphs or tables. The possibility to parameterize each function is also described - the MRB_MATH library makes it possible to increase the precision of a function at the cost of additional cycles or FLASH/RAM memory usage.

Functions included in the library

• ***sin_f***(*float x*)	*sinus from argument x*
• ***cos_f***(*float x*)	*cosinus from argument x*
• ***fast_invsqrt***(*float x*)	*fast inverse root square from argument x*
• ***fast_sqrt***(*float x*)	*fast root square from argument x*
• ***RMS***(*float x*)	*root mean square function (with normal sqrt function)*
• ***fast_RMS***(*float x*)	*fast root mean square function (with fast sqrt function)*
• ***rapid_RMS***(*float x*)	*rapid root mean square fun. (approach without sqrt)*

Execution speed overview

Table 1 Comparison of function execution time in terms of cycles on STM32 microcontroller with Cortex M-7 core

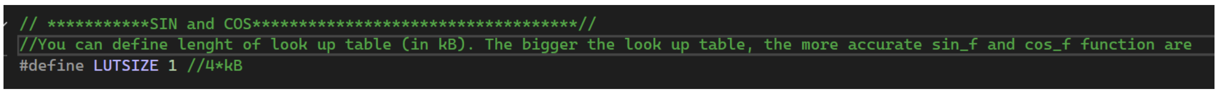
Function / Library	**<math.h>**	**"MRB_MATH_LIB.h"**
sinus	762 cycles	90 cycles
cosinus	762 cycles	92 cycles
sqrt	674 cycles	28 cycles
RMS	-	694 cycles
fast RMS	-	46 cycles
rapid RMS	-	13 cycles
FFT	-	

Library created by Maciej Brzycki ------ February 2024

Trigonometric functions

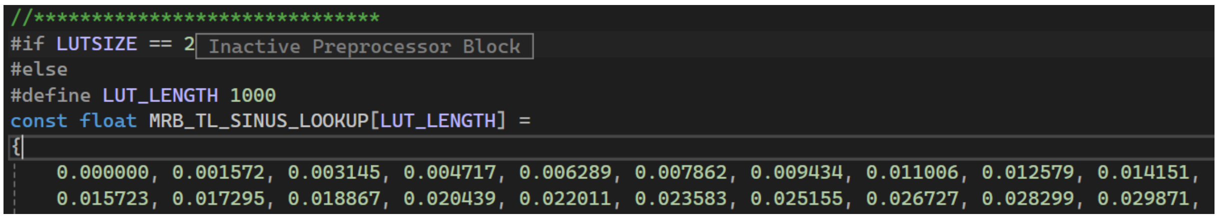
The sine and cosine functions in the MRB_MATH library are based on the look up table, which will be stored in the microcontroller's flash memory when the program is uploaded. While the program is running, it permanently occupies space in RAM memory. User can define the size of memory usage with *LUTSIZE* parameter. When equal to 1 - look up table will take about 4kB of memory (1000 float values), when 2 – 8kB and when 4 – 16kB.

Important note: the range of the function's arguments is not as large as that of the **sin** function from the math.h library. The argument of the **sin_f** function must be from the range -2π to 4π.



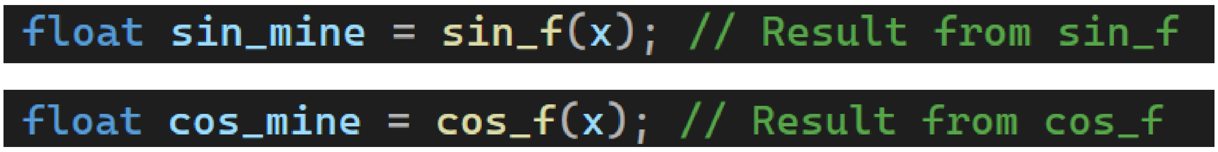
```
// ***********SIN and COS***********************************//
//You can define lenght of look up table (in kB). The bigger the look up table, the more accurate sin_f and cos_f function are
#define LUTSIZE 1 //4*kB
```

Figure 1 LUT size choice



```
//*******************************
#if LUTSIZE == 2 Inactive Preprocessor Block
#else
#define LUT_LENGTH 1000
const float MRB_TL_SINUS_LOOKUP[LUT_LENGTH] =
{
    0.000000, 0.001572, 0.003145, 0.004717, 0.006289, 0.007862, 0.009434, 0.011006, 0.012579, 0.014151,
    0.015723, 0.017295, 0.018867, 0.020439, 0.022011, 0.023583, 0.025155, 0.026727, 0.028299, 0.029871,
```

Figure 2 LUT view - beginning of the declaration



```
float sin_mine = sin_f(x); // Result from sin_f
```

```
float cos_mine = cos_f(x); // Result from cos_f
```

Figure 3 Use of trigonometric functions

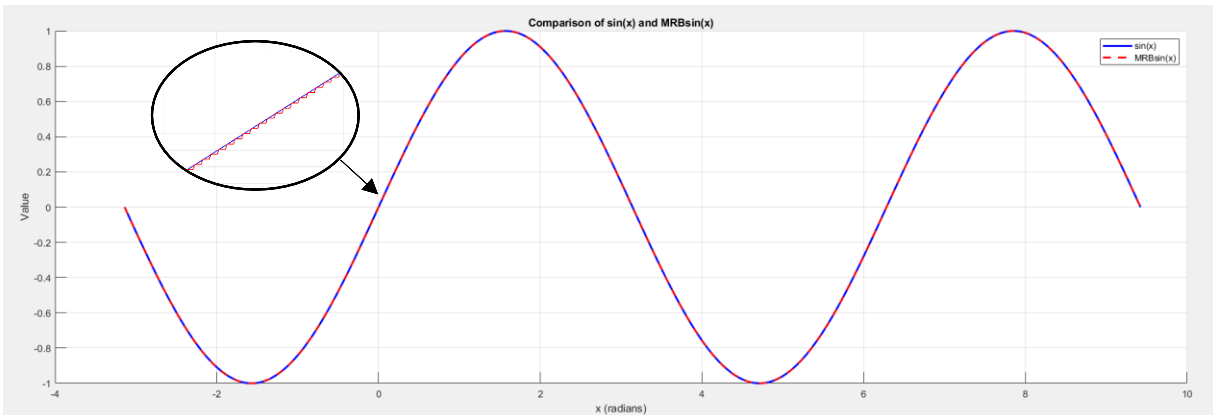


Figure 4 math.h sin(x) function and MRB_MATH_LIB.h sin_f(x) comparison (LUTSIZE = 1)

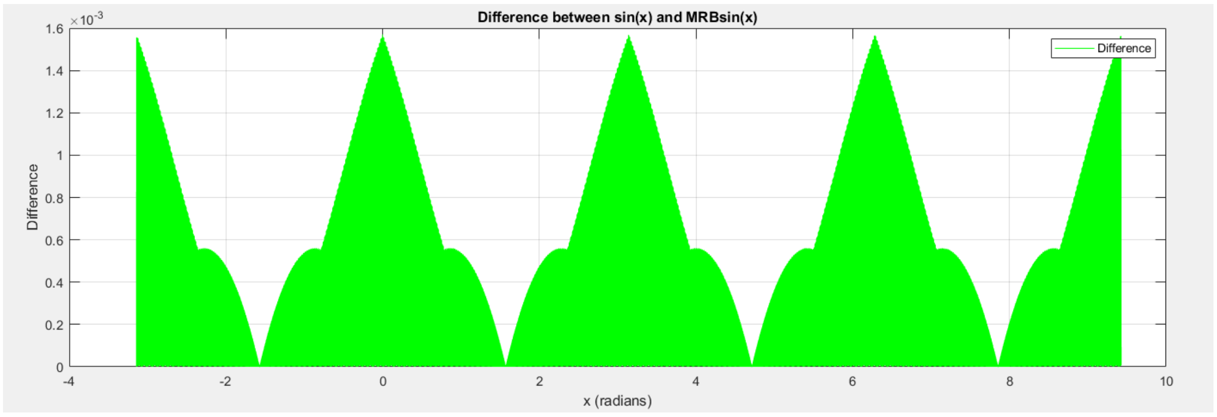


Figure 5 Absolute error between sin and sin_f functions (LUTSIZE = 1)

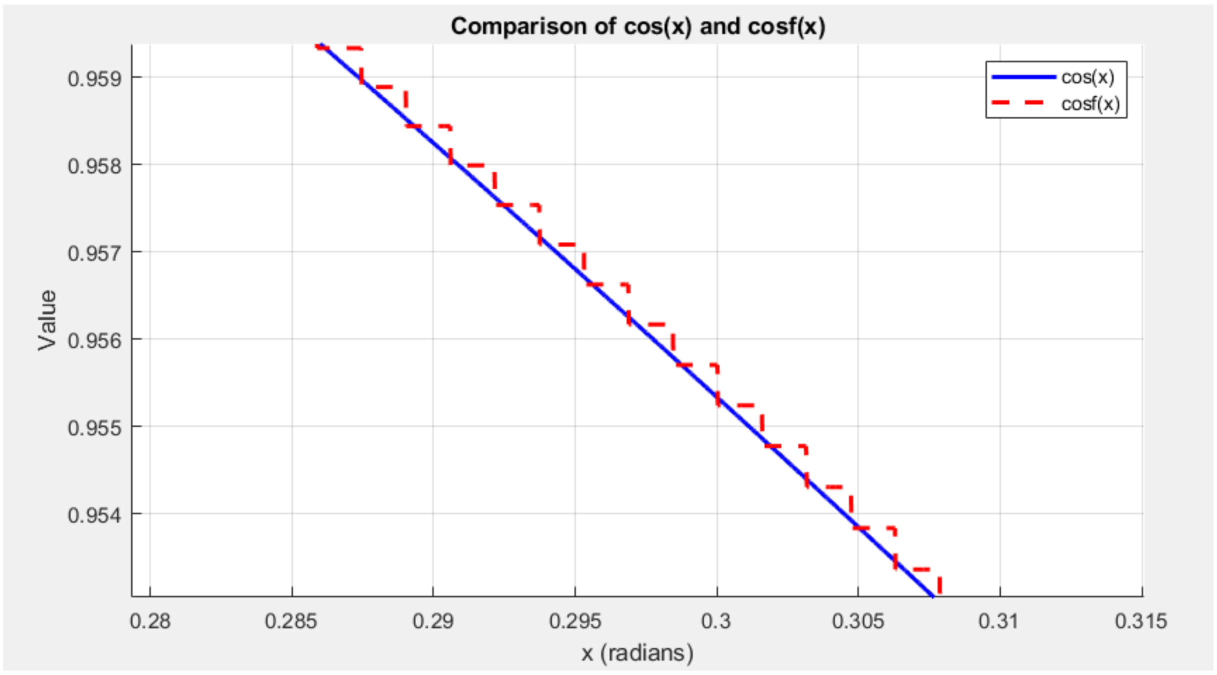


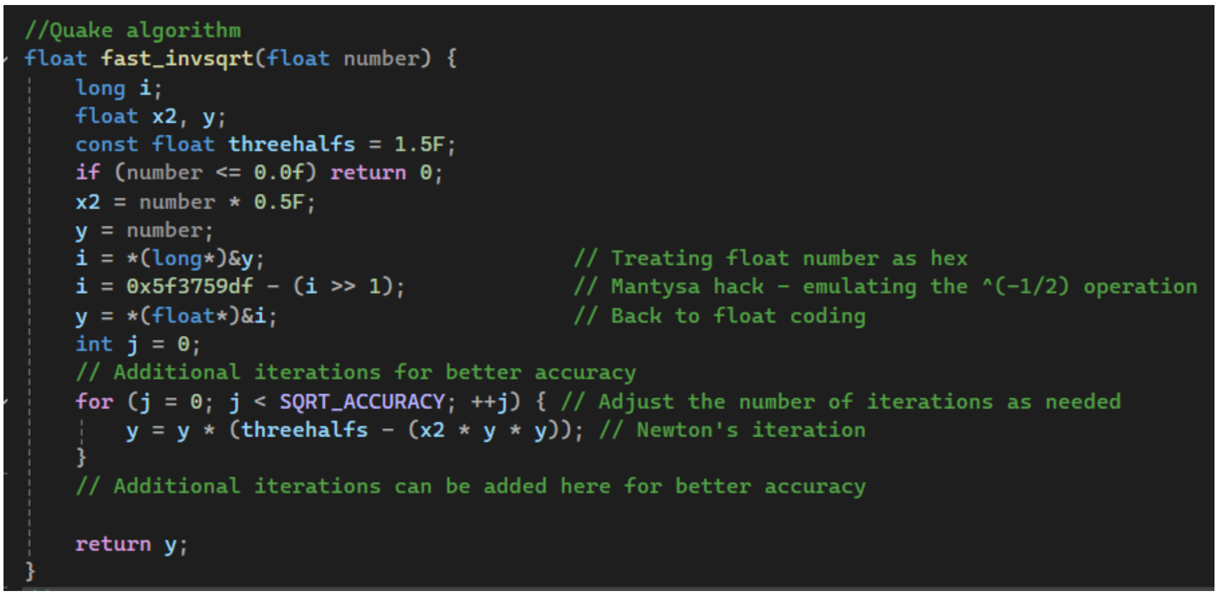
Figure 6 cos and cos_f comparison - zoom in

Table 2 Mean absolute difference between sin and sin_f (or cos and cos_f) with 1e-5 angle step.

LUTSIZE(n*4kB)	**1n**	**2n**	**4n**
Mean Absolute error	0.000326	0.000163	0.000082

Square root function

The fast square root function is based on inverse square root Quake's algorithm. The Quake algorithm avoids directly calculating the square root. Instead, it uses a fast approximation, combined with Newton's method for refinement. The key trick is in using bit-level manipulation of the floating-point number to produce a rough initial guess.



```
//Quake algorithm
float fast_invsqrt(float number) {
    long i;
    float x2, y;
    const float threehalfs = 1.5F;
    if (number <= 0.0f) return 0;
    x2 = number * 0.5F;
    y = number;
    i = *(long*)&y;                           // Treating float number as hex
    i = 0x5f3759df - (i >> 1);                // Mantysa hack - emulating the ^(-1/2) operation
    y = *(float*)&i;                          // Back to float coding
    int j = 0;
    // Additional iterations for better accuracy
    for (j = 0; j < SQRT_ACCURACY; ++j) { // Adjust the number of iterations as needed
        y = y * (threehalfs - (x2 * y * y)); // Newton's iteration
    }
    // Additional iterations can be added here for better accuracy

    return y;
}
```

Figure 7 Quake algorithm

Step-by-step Quake's algorithm description:

Approximation using a "magic number":

- The number *x* is first reinterpreted as an integer (using **(int*)&x*), which allows bit manipulation – e.g. float number *1.0f* would be reinterpreted as *1065353216*.
- This integer is then modified using a "magic constant" **0x5f3759df**, which was empirically determined. The expression *i = 0x5f3759df - (i >> 1)* performs the bit manipulation to get an initial approximation of $\frac{1}{\sqrt{x}}$.
- This step essentially tricks the floating-point representation into giving an initial estimate that's pretty close to the correct result.

Convert the bits back to a float:

- After the bit manipulation, the result is cast back into a floating-point value (using **(float*)&i*), so now *i* is a rough approximation of $\frac{1}{\sqrt{x}}$.

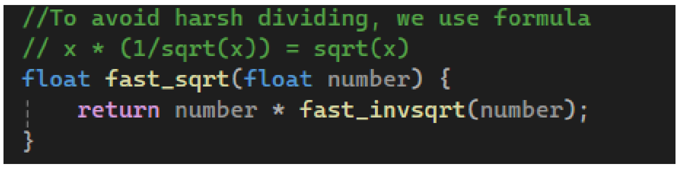
Refinement using Newton's method:

The result is then refined with one iteration of Newton's method to improve the approximation. This step reduces the error in the approximation. The formula used is:

$$x = x \cdot (1{,}5 - 0{,}5 \cdot x^2 \cdot y)$$

After that, result of Quake's algorithm is multiplied by input value, so division is avoided again (which increases execution speed a lot). Instead $sqrt = 1\ /\ (invsqrt)$ function does:

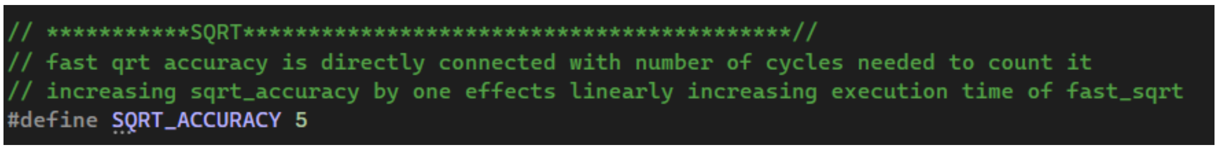
$$sqrt = x \cdot (invsqrt)$$



```
//To avoid harsh dividing, we use formula
// x * (1/sqrt(x)) = sqrt(x)
float fast_sqrt(float number) {
    return number * fast_invsqrt(number);
}
```

Figure 8 Inverse-Inverse square root

User can adjust the function by changing *SQRT_ACCURACY* parameter. Increasing SQRT_ACCURACY also increases linearly execution speed of the function. The relative error (relative to **sqrt** function from the math.h library) and the execution time of the function for a given parameter value are summarized below. The error was presented in relative form due to the very large range of values tested by the **fast_sqrt** function.



```
// ***********SQRT*******************************************//
// fast qrt accuracy is directly connected with number of cycles needed to count it
// increasing sqrt_accuracy by one effects linearly increasing execution time of fast_sqrt
#define SQRT_ACCURACY 5
```

Figure 9 Adjusting fast quare root accuracy - in default equal to 5

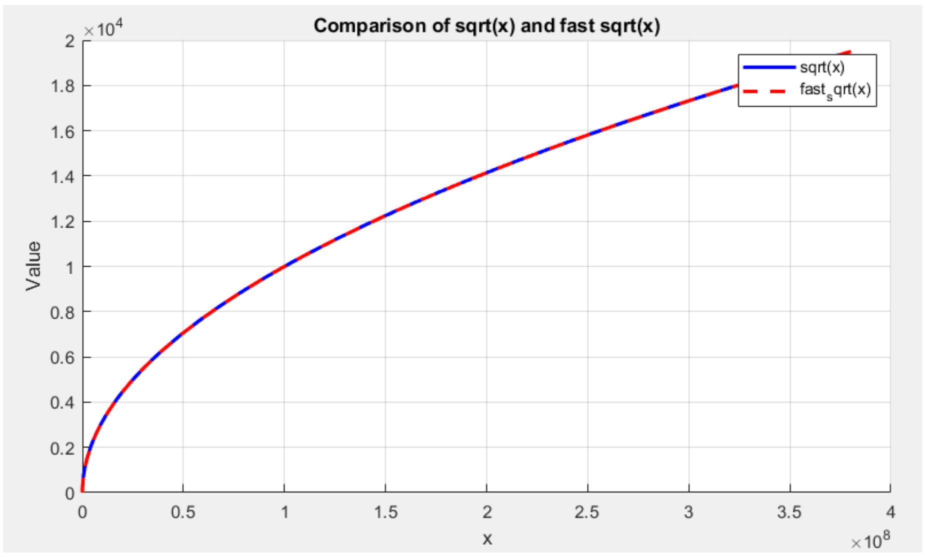


Figure 10 sqrt from math.h and fast_sqrt comparison

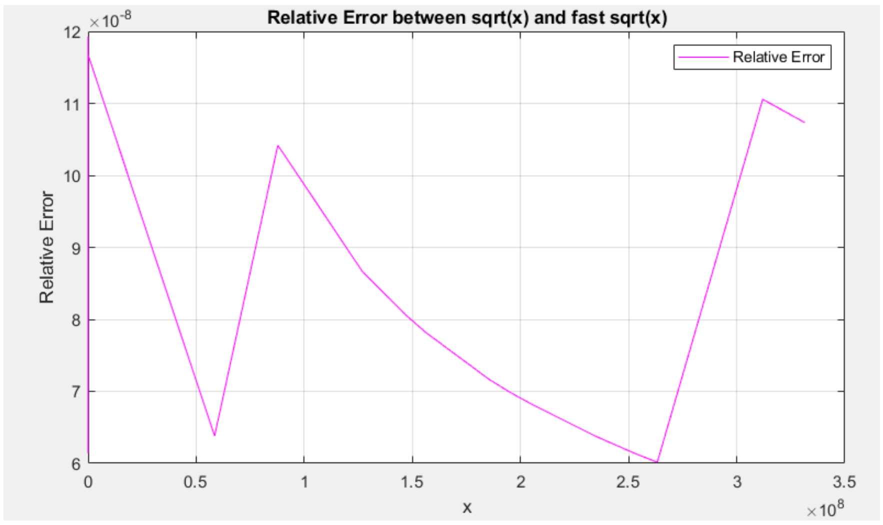


Figure 11 Relative error [%] of fast_sqrt function in comparison to sqrt function from math.h library

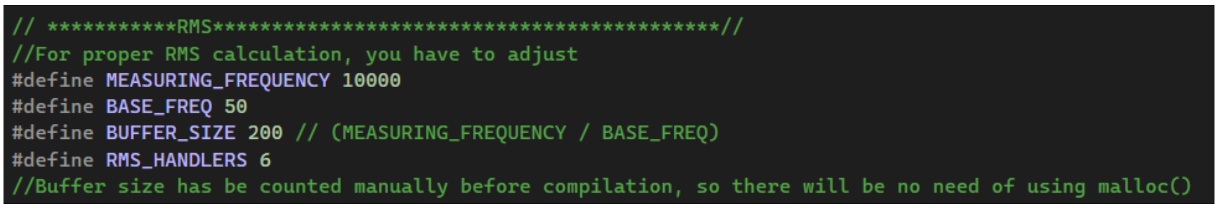
Table 3 Mean relative error of fast_sqrt function with SQRT_ACCURACY parameter change

SQRT_ACCURACY	**Mean relative error**	**Function execution speed**
2	1,6e-4 [%]	17 cycles
3	3,5e-6 [%]	21 cycles
5	2,3e-6 [%]	26 cycles
10	2,2e-6[%]	44 cycles
20	2,1e-6[%]	90 cycles

Root mean square function

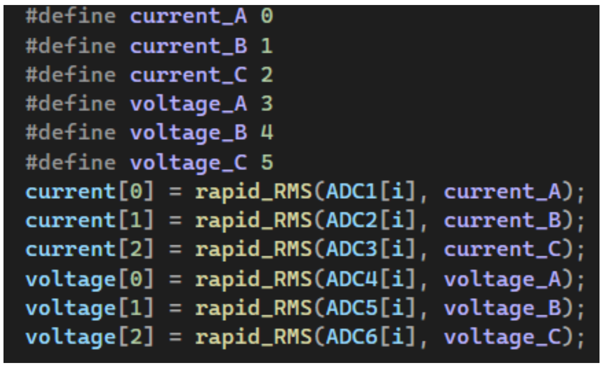
As is this one of the most commonly needed tools in digital signal processing, the root mean square function (RMS) has also been added to MRB_MATH library. Three variants of the RMS function are included in this library: normal, fast and rapid. The normal variant is based on the sqrt function included in the math.h library. The fast variant is based on the fast_sqrt function contained in the MATH_MRB library. The rapid variant uses mathematical relationships assuming a perfectly sinusoidal waveform, so it does not use division or root operations. The accuracy of the various variants and their execution times for different signals is summarized below.

The user must set the parameters of the measured RMS signal in the preprocessor directives before using any of this functions (this must be done in the preprocessor, in order to avoid using the malloc() function to create a sample buffer). Important information is signal measuring frequency, base frequency for which RMS should be calculated and buffer size (should be counted as *measuring frequency divided by base frequency*). *RMS_HANDLERS* corresponds to number of buffer arrays that are declared for this scope. Each signal should have different buffer. That means, e.g. for three phase measurement, we could have three current measurements and three voltage measurement, so *RMS_HANDLERS* can be equal to 6.



```
// ***********RMS********************************************//
//For proper RMS calculation, you have to adjust
#define MEASURING_FREQUENCY 10000
#define BASE_FREQ 50
#define BUFFER_SIZE 200 // (MEASURING_FREQUENCY / BASE_FREQ)
#define RMS_HANDLERS 6
//Buffer size has be counted manually before compilation, so there will be no need of using malloc()
```

Figure 12 Setting parameters of RMS functions



```
#define current_A 0
#define current_B 1
#define current_C 2
#define voltage_A 3
#define voltage_B 4
#define voltage_C 5
current[0] = rapid_RMS(ADC1[i], current_A);
current[1] = rapid_RMS(ADC2[i], current_B);
current[2] = rapid_RMS(ADC3[i], current_C);
voltage[0] = rapid_RMS(ADC4[i], voltage_A);
voltage[1] = rapid_RMS(ADC5[i], voltage_B);
voltage[2] = rapid_RMS(ADC6[i], voltage_C);
```

Figure 13 Approach for use of RMS function with multiple occurrences

Table 4 Execution speed comparison for different RMS functions

Execution speed	
RMS	694 cycles
fast_RMS	46 cycles
rapid_RMS	13 cycles

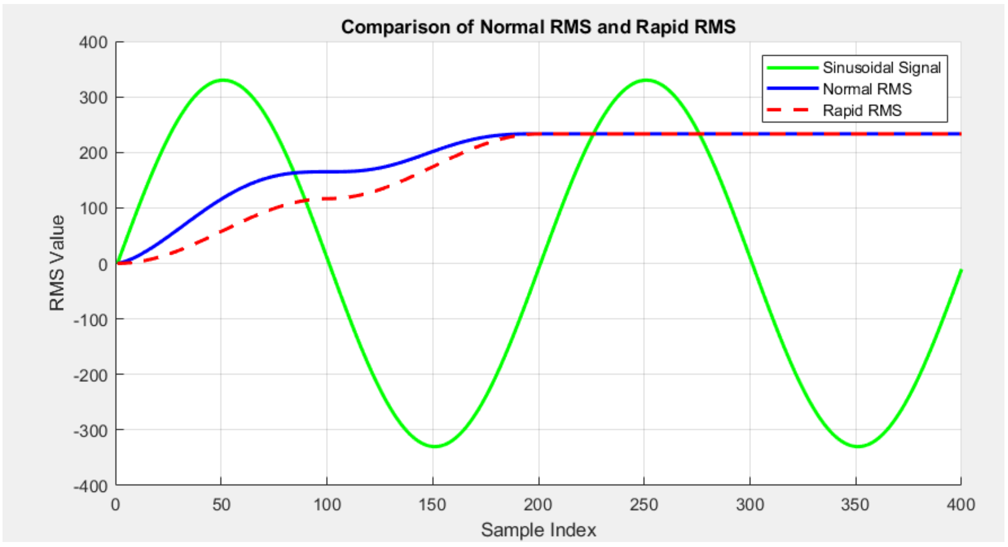


Figure 14 Comparison of transient state of normal RMS and rapid RMS for ideal sinusoidal signal

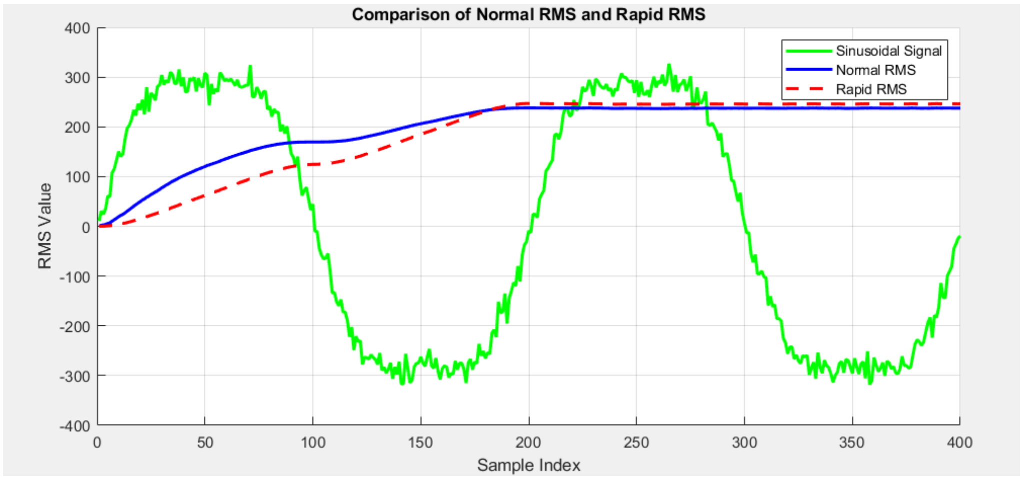


Figure 15 Comparison for noisy signal with third harmonic injected

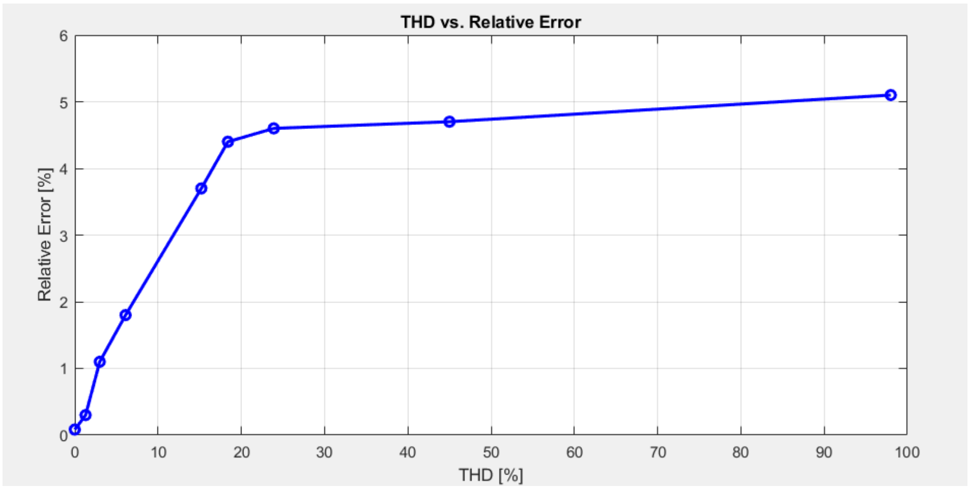


Figure 16 Relative error of rapid_RMS in comparison to normal RMS

Fast Fourier transform function

The last tool in the library is a fast Fourier transform function designed for online computing on a microcontroller.