MRB MATH LIBRARY

DOCUMENTATION

This library is intended for use in lower-cost, slower microcontrollers not dedicated for mathematical calculations (e.g. ATmega/Arduino, ARM Cortex M0 and M4, ESP, Raspberry Pi pico) to speed up floating point operations. It can also be used in DSP to reduce cycle time, wherever cycle time is very critical, but the accuracy of calculations is not so critical. The MRB\_MATH library will also significantly speed up the execution time of math calculations on microcontrollers without FPU or without hardware support for the default math.h library. An additional function not included in the math.h library is the function for calculating the RMS of a signal. This function is available in three variants: normal, fast and rapid.

In summary, this library allows faster calculations at the expense of lower precision and higher memory usage.

The documentation describes the functions contained in the MRB\_MATH library and compares them with their counterparts from the math.h library. Execution times and precision are compared. The results are summarized in graphs or tables. The possibility to parameterize each function is also described - the MRB\_MATH library makes it possible to increase the precision of a function at the cost of additional cycles or FLASH/RAM memory usage.

Functions included in the library

* **sin\_f**(float x) sinus from argument x
* **cos\_f**(float x) cosinus from argument x
* **fast\_invsqrt**(float x) fast inverse root square from argument x
* **fast\_sqrt**(float x) fast root square from argument x
* **RMS**(float x) root mean square function (with normal sqrt function)
* **fast\_RMS**(float x) fast root mean square function (with fast sqrt function)
* **rapid\_RMS**(float x) rapid root mean square function (approach without sqrt)

Execution speed overview

Table 1 Comparison of function execution time in terms of cycles on STM32 microcontroller with Cortex M-7 core

|  |  |  |
| --- | --- | --- |
| **Function / Library** | **<math.h>** | **“MRB\_MATH\_LIB.h”** |
| sinus | 762 cycles | 90 cycles |
| cosinus | 762 cycles | 92 cycles |
| sqrt | 674 cycles | 28 cycles |

Library created by Maciej Brzycki ------ February 2024

# Trigonometric functions

The sine and cosine functions in the MRB\_MATH library are based on the look up table, which will be stored in the microcontroller's flash memory when the program is uploaded. While the program is running, it permanently occupies space in RAM memory. User can define the size of memory usage with *LUTSIZE* parameter. When equal to 1 - look up table will take about 4kB of memory (1000 float values), when 2 – 8kB and when 4 – 16kB.

**Important note**: the range of the function's arguments is not as large as that of the **sin** function from the math.h library. The argument of the **sin\_f** function must be from the range -2π to 4π.

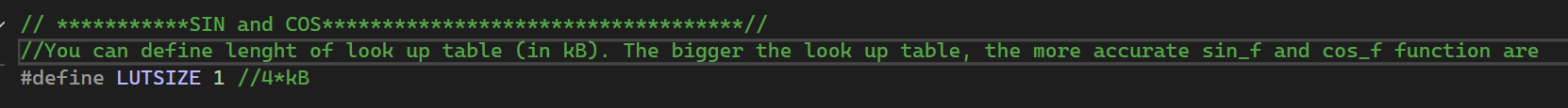


Figure 1 LUT size choice

A screenshot of a computer

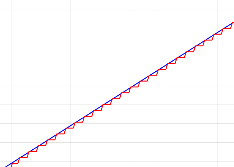
Description automatically generated

Figure 2 LUT view - beginning of the declaration





Figure 3 Use of trigonometric functions

A graph of a function

Description automatically generated

Figure 4 math.h sin(x) function and MRB\_MATH\_LIB.h sin\_f(x) comparison (LUTSIZE = 1)

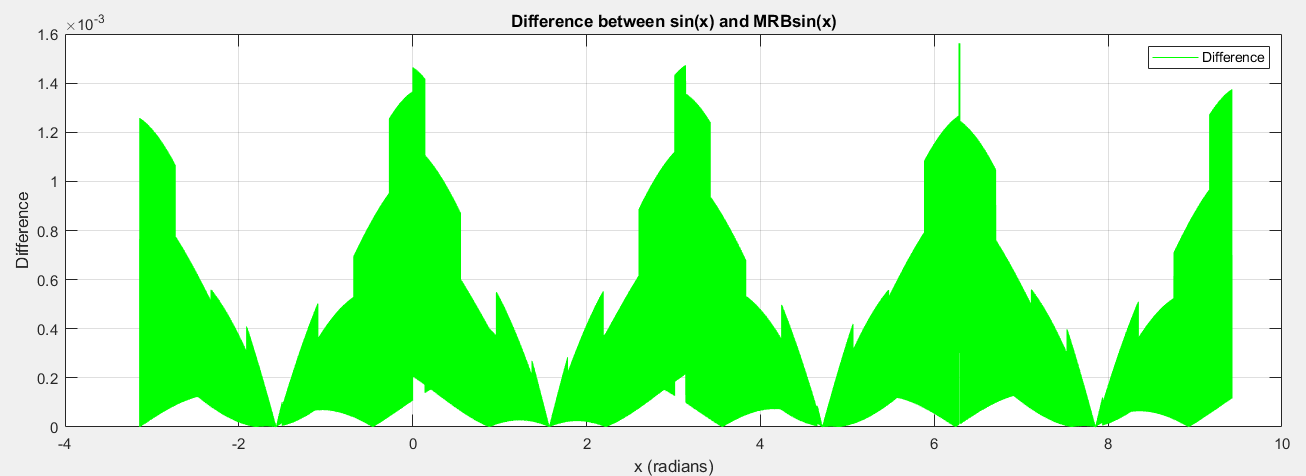


Figure 5 Absolute error between sin and sin\_f functions (LUTSIZE = 1)

Table 2 Mean absolute difference between sin and sin\_f

|  |  |  |  |
| --- | --- | --- | --- |
| **LUTSIZE(n\*4kB)** | **1n** | **2n** | **4n** |
| **Mean Absolute error** | 0.000326 | 0.000163 |  |

# Square root function

The fast square root function is based on inverse square root Quake’s algorithm. The Quake algorithm avoids directly calculating the square root. Instead, it uses a fast approximation, combined with Newton’s method for refinement. The key trick is in using bit-level manipulation of the floating-point number to produce a rough initial guess.

**Step-by-step Quake’s algorithm description:**

**Approximation using a "magic number":**

* The number *x* is first reinterpreted as an integer (using *\*(int\*)&x*), which allows bit manipulation – e.g. float number *1.0f* would be reinterpreted as *1065353216*.
* This integer is then modified using a "magic constant" **0x5f3759df**, which was empirically determined. The expression *i = 0x5f3759df - (i >> 1)* performs the bit manipulation to get an initial approximation of .
* This step essentially tricks the floating-point representation into giving an initial estimate that’s pretty close to the correct result.

**Convert the bits back to a float:**

* After the bit manipulation, the result is cast back into a floating-point value (using *\*(float\*)&i*), so now *i* is a rough approximation of ​.

**Refinement using Newton's method:**

* The result is then refined with one iteration of Newton’s method to improve the approximation. The formula used is:

This step reduces the error in the approximation.

After that, result of Quake’s algorithm is multiplied by input value, so division is avoided again (which increases execution speed a lot). Instead function does:

User can adjust the function by changing *SQRT\_ACCURACY* parameter. Increasing SQRT\_ACCURACY also increases linearly execution speed of the function. The relative error (relative to **sqrt** function from the math.h library) and the execution time of the function for a given parameter value are summarized below. The error was presented in relative form due to the very large range of values tested by the **fast\_sqrt** function.

|  |  |  |
| --- | --- | --- |
| **SQRT\_ACCURACY** | **Mean relative error** | **Function execution speed** |
| 2 | % | Xx cycles |
| 5 |  |  |
| 10 |  |  |
| 20 |  |  |

# Root mean square function

As is this one of the most commonly needed tools in digital signal processing, the root mean square function (RMS) has also been added to MRB\_MATH library. Three variants of the RMS function are included in this library: normal, fast and rapid. The normal variant is based on the sqrt function included in the math.h library. The fast variant is based on the fast\_sqrt function contained in the MATH\_MRB library. The rapid variant uses mathematical relationships assuming a perfectly sinusoidal waveform, so it does not use division or root operations. The accuracy of the various variants and their execution times for different signals is summarized below.

The user must set the parameters of the measured RMS signal in the preprocessor directives before using any of this functions (this must be done in the preprocessor, in order to avoid using the malloc() function to create a sample buffer).

|  |  |
| --- | --- |
| **Execution speed** | |
| **RMS** |  |
| **fast\_RMS** |  |
| **rapid\_RMS** |  |

# Fast Fourier transform function

The last tool in the library is a fast Fourier transform function designed for online computing on a microcontroller.