Defining a Figure of Merit

1 Introduction

The primitive merit functions that can be calculated over a region of space include field energy (including absorption), transmission and mode-match:

Measurements

$T\ (tranmission)$
$E\ (field\ energy, absorption)$
$M\ (mode\ match)$

The allowable dimensions over which the figure of merit can depend are separated into three categories:

Dimensions

$freq\ (frequency)$
$mon\ (monitor)$
$user\ (user-defined)$

In an optical cloaking problem, for example, one might want to optimize (minimize in this case) the scattered transmission through a box of four monitors (*monitor*) over a range of frequencies (*frequency*) and under a variety of manufacturing perturbations (*user-defined*). There will be a different transmission value across each of the dimensions, but ultimately one has to take some combination of these values to arrive at a single figure of merit defining the design problem. There is a variety of combinations one could imagine taking: weighted addition, subtraction, multiplication, division, or even the minimum value across a dimension (i.e. optimizing the worst-performing frequency across some frequency band).

Operators

$\Sigma\ (sum)$
$\Pi\ (product)$
$\min\ (minimum)$

In abstract form, the figure of merit could be written as:

Figure of Merit

$F = \Theta_3^j \Theta_2^k \Theta_1^l\ w_i \cdot f_i^{e_i}$
$\Theta = \{\Sigma, \Pi, min\}$
$(j, k, l) = \{frequency, monitor, user\ defined)$
$f = \{transmission, field\ energy, mode\ match\}$
$w = dimension\ dependent\ weights$
$e = dimensions\ dependent\ exponents$

where j,k,l are the three dimensions in some order, and i is a linear index for (j,k,l). Θ_1, Θ_2, and Θ_3 are the operators contracting the many values of the objective function down to a single value. f_i is the type of figure of merit (i.e. transmission, field energy, etc.), and w_i and e_i are a weight and exponent, respectively. Because of the inclusion of w_i and e_i, subtraction (addition with a negative weight) and division (multiplication with a negative exponent) do not need to be explicitly included as potential operators.

The ordering of the operators is important. Taking the sum over frequencies then minimizing between two monitors, for example, is very different from minimizing between the monitors and then performing the sum over frequency. The values of j, k, and l specify the ordering. Θ_1 operates first on the l^{th} dimension, where the first dimension varies across monitors, the second dimension is frequency, and the third dimension corresponds to user-defined variations. Then Θ_2 operates, then Θ_3.

2 Restrictions

There are a few restrictions we have placed on the definition of the figure of merit:

- All three operators must be defined. For a simple problem where only one operator is needed (i.e. Θ_1 = min over frequency), Θ_2 and Θ_3 can be defined as dummy summation operators and the second and third dimensions will each be of length 1.
- Of the three operators defined, ONLY ONE may be a min operator, and IF IT IS USED, IT MUST BE Θ_3 (the operator that is evaluated last). Θ_1 and Θ_2 cannot be min functions. This is because of the way we solve the minmax (actually maxmin) optimization problem through transformation to a linear programming problem given all shape derivatives. In practice, this is not a severe limitation.

3 Examples

3.1 Waveguide Splitter

In a waveguide splitter, one generally wants to branch a single waveguide into multiple waveguides, and efficiently direct different wavelengths to the different branches. Imagine you want to maximize the minimum transmission through branch 1 at frequency ω_1, branch 2 at frequency ω_2, etc. for N branches. We will assume there is no user-defined variation included in the merit function. In this case, we would define the merit function in the following way:

$f = T\ (transmission)$
$\Theta_3 = \min$
$\Theta_2 = \Sigma$ (dummy operator)
$\Theta_1 = \Sigma$
$j = \{monitor\}$
$k = \{user\ defined\}$

$l = \{frequency\}$
$w = [1,0,0; 0,1,0; 0,0,1]$
$e\ = [1,1,1; 1,1,1; 1,1,1]$

Defined in this way, the figure of merit is

$$F\ =\ \min_{mon} \sum_{user} \sum_{freq} w_i \cdot T_i$$

$$=\ \min_{mon} \{\ T_1(\omega_1) +\ T_2(\omega_2) + T_3(\omega_3)\}$$