Line of Best Fit for Circles

Michael Reinhart
Student Number: 20001556
CISC 271

ABSTRACT

PURPOSE: The purpose of this lab is to find a least-squares line of best fit to approximate the size and location of a circle given data points.

METHODS: The methods I used were to find the center, radius and RMS error of the approximation. I used the center and radius to get a circle prediction based on the data given and then used this circle to compare it to the actual circle data given.

RESULTS:

The results were plotted circles over an image that represents the graphic location of the images. The other result given was the RMS error of each individual circle based on predicted values and the values given

CONCLUSIONS:

I have concluded that using linear algebra to find a line of best fit works quite well on non-linear shapes such as circles. This can be done using QR decomposition and back substitution to find the center and radius of each individual circle.

INTRODUCTION

I was given data points that would represent different shapes. Some where very nice circles and others were rectangles and shapes that should not be fit well with circles. The goal of the lab was to make the data into a matrix and use it to generate circles that best represent the actual shape of each data set.

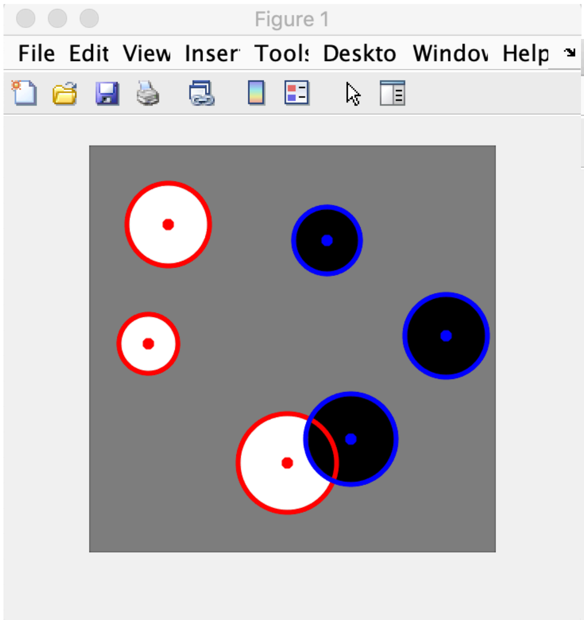
METHODS

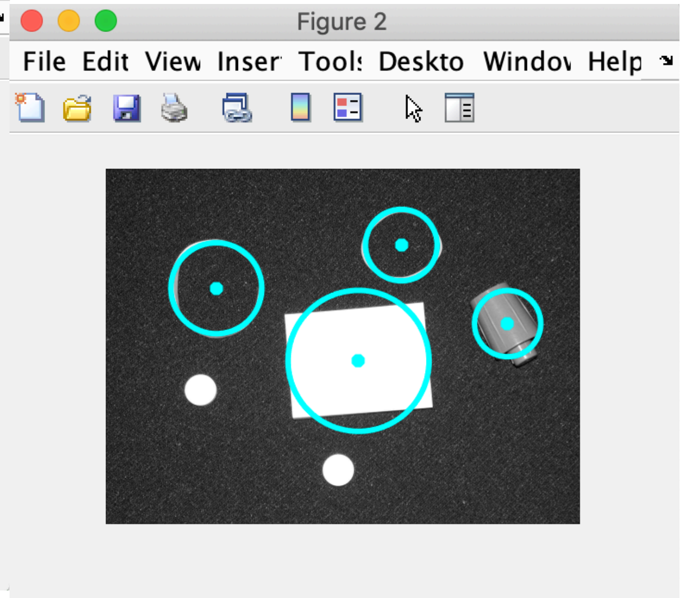
The first method was to get A in the form that Ax=b. I did this by multiplying each vector by -2 and then adding in a column of 1 so that the dimension is greater. This allows for better calculations later on for using QR decomposition, QR decomposition breaks the A matrix into a Q and and R the R is very important in finding the coefficients because it is a lower triangular matrix which allows for back substitution.

The second method I used was to find b in the equation Ax = b. I did this through taking the data given and multiplying it by -1. Then for each element in the rows of the data given I took the dot product of that vector with itself and added it into b as a row entry. B is the vector of answers in the equation Ax = b. This gives the data for what the individual indices in the matrix x should represent.

The last method within the function was to find the coefficients. This was done by using back substitution of the QR decomposition of A. I used two for loops the first generates what the transpose of Q * B is in the location that is related to the proper R matrix value. Then for each row index within R I added up each corresponding value such that each coefficient value represents the entire row.

RESULTS





The two pictures shown are circles that have been plotted over the images given. The images show what the actual data points should look like graphically. The colored circles show the generated circles made from the data sets representing the circular objects. As shown the circles generated are quite accurate to the data.

The rms error for the data plot in object 1 is: 0.1216
The rms error for the data plot in object 2 is: 0.07881
The rms error for the data plot in object 3 is: 0.11192
The rms error for the data plot in object 4 is: 0.078547
The rms error for the data plot in object 5 is: 0.1216
The rms error for the data plot in object 6 is: 0.11999
The rms error for the data plot in object 7 is: 3.8099
The rms error for the data plot in object 8 is: 3.3738
The rms error for the data plot in object 9 is: 11.0829
The rms error for the data plot in object 10 is: 7.602

The root mean squared error's of each object are shown above. I am very happy with these results as they are very low for the more accurate ellipsoids. I also feel that this is accurate as the 9th and 10th object RMS is quite high. This makes sense due to the two objects being a square and a cap to a highlighter. These objects should not fit well into circles so the error for them should also be quite high compared to the more circular objects.

DISCUSSION

I am very happy with my results and have learned that it is possible and quite accurate to use lines of best fit for linear equations to make strong approximations of shapes based on data sets given.

REFRENCES

[1] IEEE: MathWorks. Linked from
https://www.mathworks.com/matlabcentral

[2] IEEE: Jim Lamberss
chrome-extension://oemmndcbldboiebfnladdacbdfmadadm/https://www.math.usm.edu/lambers/mat610/sum10/lecture4.pdf