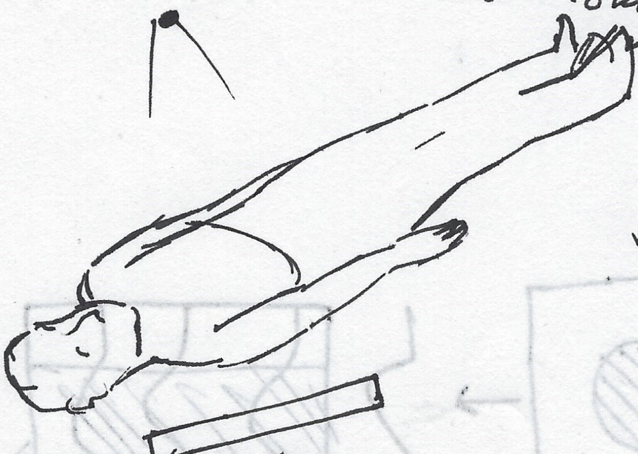
Digital Image Processing (In Class)

Lecture 18

Image Reconstruction from projection

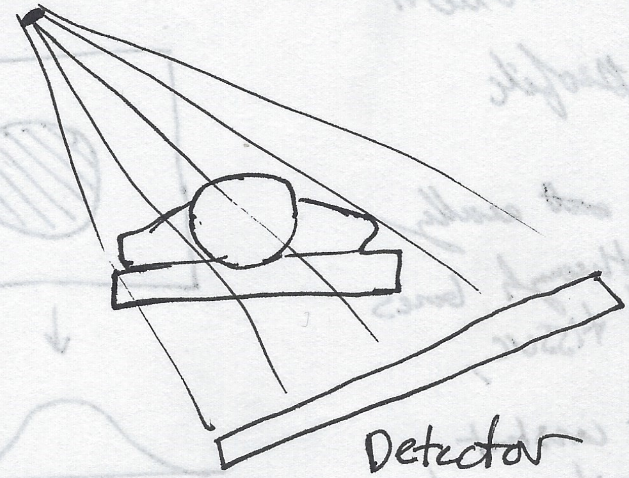
Mathematical Basis for a CAT/CT Scan

(Computed axial tomography)



Detector

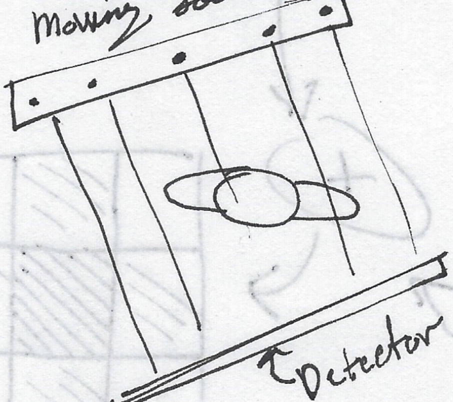
Radiation Source



Detector

Different kinds of scans

Moving Source

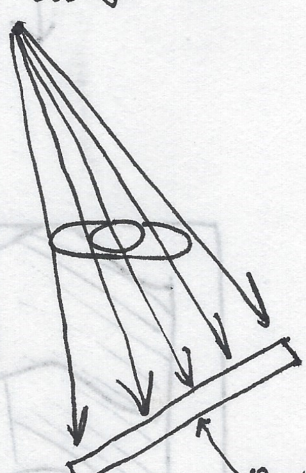


Detector

Parallel Beam

(Generation 1)

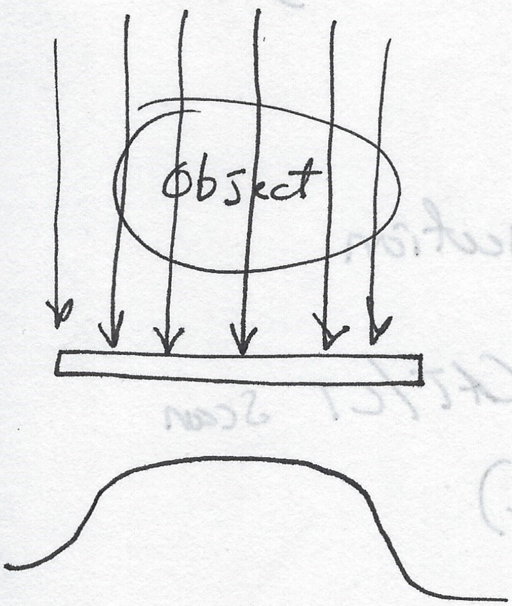
Source



Detector

Fan Beam

② Set up

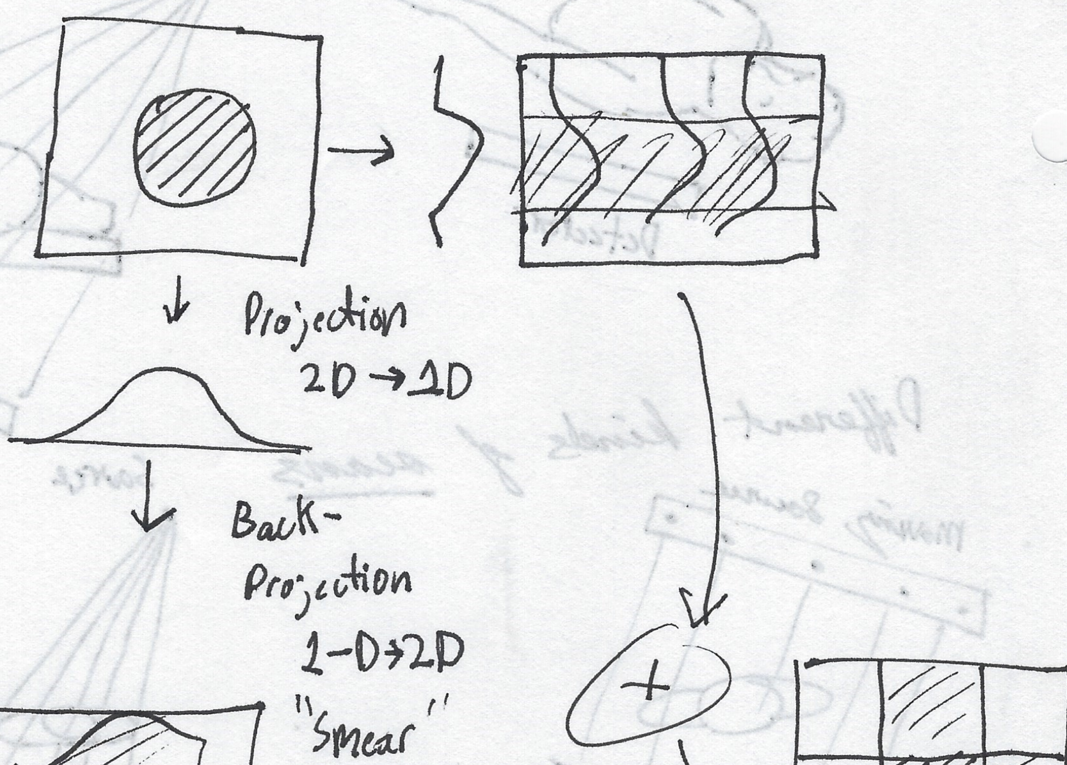


Absorbtion profile

(They ~~not~~ actually go through bones and tissue,

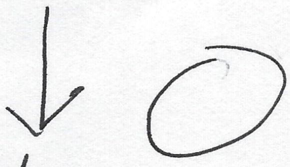
→ Not constant Density)

Suppose we have one absorption profile. How would we estimate the 2D image it came from?



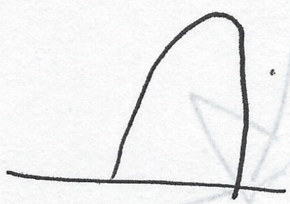
Matlab Example

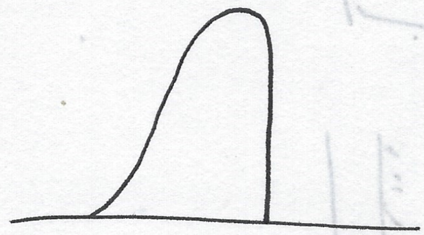
Integrate in this angle



integrate down

70°



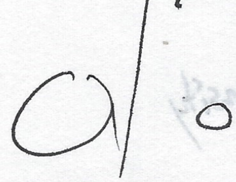


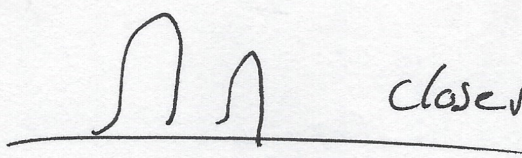
Same?

10°

radon (c2, 0)

radon (c2, 20)

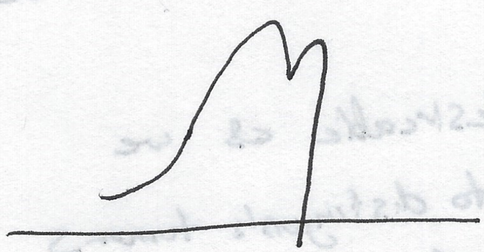


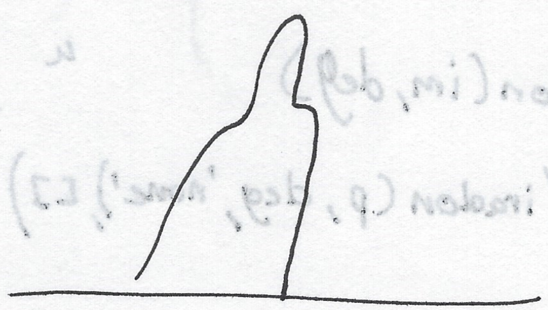


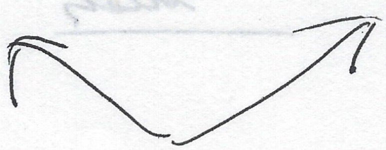
closer

radon(c2, 65)

Radon (c2, 90)



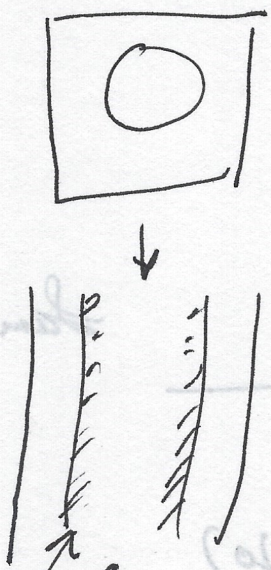




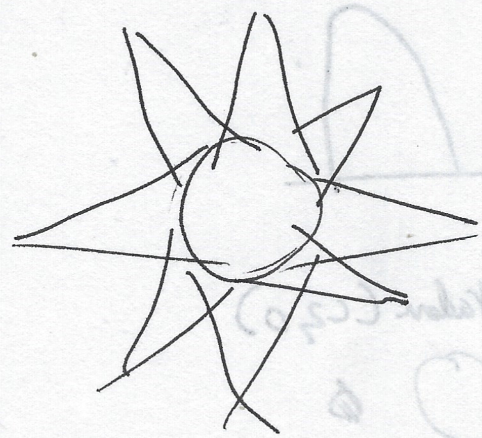
These are smears?

```
p= radon(c,0);
imshow(iradon(p,0,'none'),[])
```

→ 12:30 pm.



deg=[0:30:179]



deg=[0:2:178]

Note

```
deg= 0:90:179;
p= radon(im,deg)
imshow(iradon(p,deg,'none'),[])
```

We have the circle but we have a haze

↳ undesireable as we need to distinguish tumors or such

More Summed Back projections

→ higher accuracy reconstruction

But there will always be artifacts

e.g. "Halo Effect" around

sharp-Edged Objects

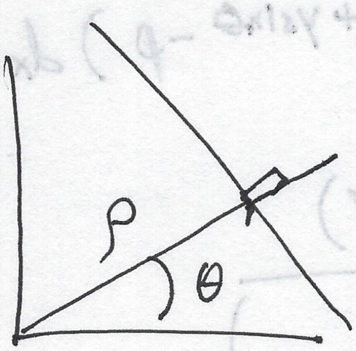
Can't tolerate this in Medical Applications

if higher contrast of original object → we may not even recognize it.

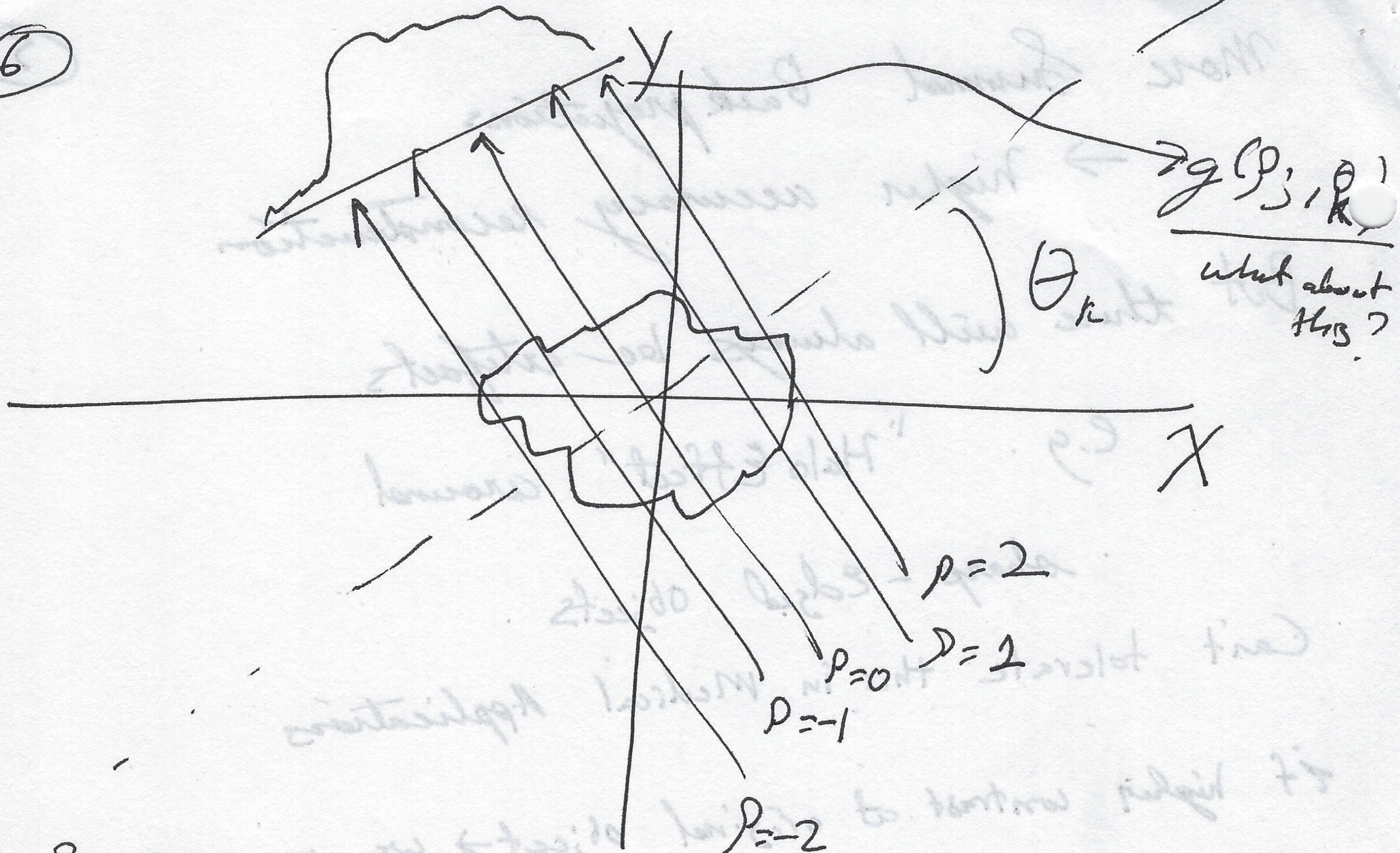
Mathematics of Reconstruction

We describe projection lines in the same way as for the Hough transform

but $x\cos\theta + y\sin\theta = \rho$ Not $y = mx + b$



(θ, ρ) corresponds to a line in (x, y) plane



for a fixed (θ_k, ρ_j)

we are integrating

the image along a line to get a value $g(\rho_j, \theta_k)$

Value $g(\rho_j, \theta_k)$

$$g(\rho, \theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy$$

(Actually a line integral)

This is called $\boxed{\text{Radon Transform}}$

We can display $g(p, \theta)$ Like an Image;

This is called the Sinogram.

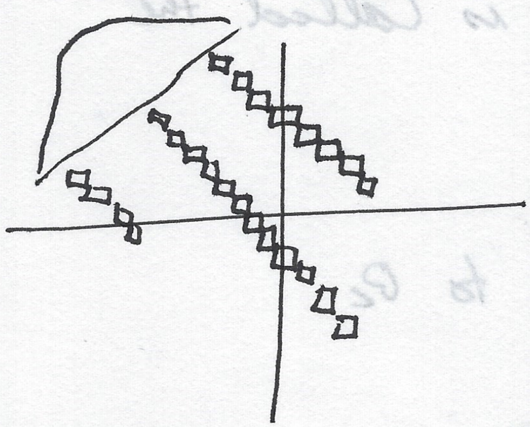
$\downarrow$

23:53

try it out.

Say we have $g(p, \theta_k)$ for a fixed

Angle θ_k. What is the back projection?



Copy $g(p_j, \theta_k)$ Back along the appropriate Pixels.

$$f_{\theta_k}(x, y) = g(p, \theta_k)$$
$$= g(x \cos\theta_k + y \sin\theta_k, \theta_k)$$

8

Adding up back-projections at all angles

$$f(x,y) = \int_0^{\pi} f_\theta(x,y)\, d\theta$$

$$\approx \sum_{0}^{\pi} f_\theta(x,y)$$

This sum of Back projections is called the <u>Laminogram</u>

As seen ~~before~~ Earlier, these tend to be unacceptably Blurry, How to improve this?

Fourier-slice Theorem (Projection-Slice Thm)

Relates the 1-D FT of a Projection
To the 2-D FT of The IMAGE

$$G(u,\theta) = \int_{-\infty}^{\infty} \underbrace{g(\rho,\theta)}_{\text{we know this}} e^{-j2\pi\omega\rho}\, d\rho$$

FT of $g(\rho,\theta)$

$$G(u,\theta) = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - \rho)\,dx\,dy\right] e^{-j2\pi\omega\rho}\,d\rho$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\left[\int_{-\infty}^{\infty} e^{-j2\pi\omega\rho}\,\delta(x\cos\theta + y\sin\theta - \rho)\,d\rho\right]dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi\omega(x\cos\theta + y\sin\theta)}\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\,dx\,dy \Big|_{\substack{u=\omega\cos\theta \\ v=\omega\sin\theta}}$$