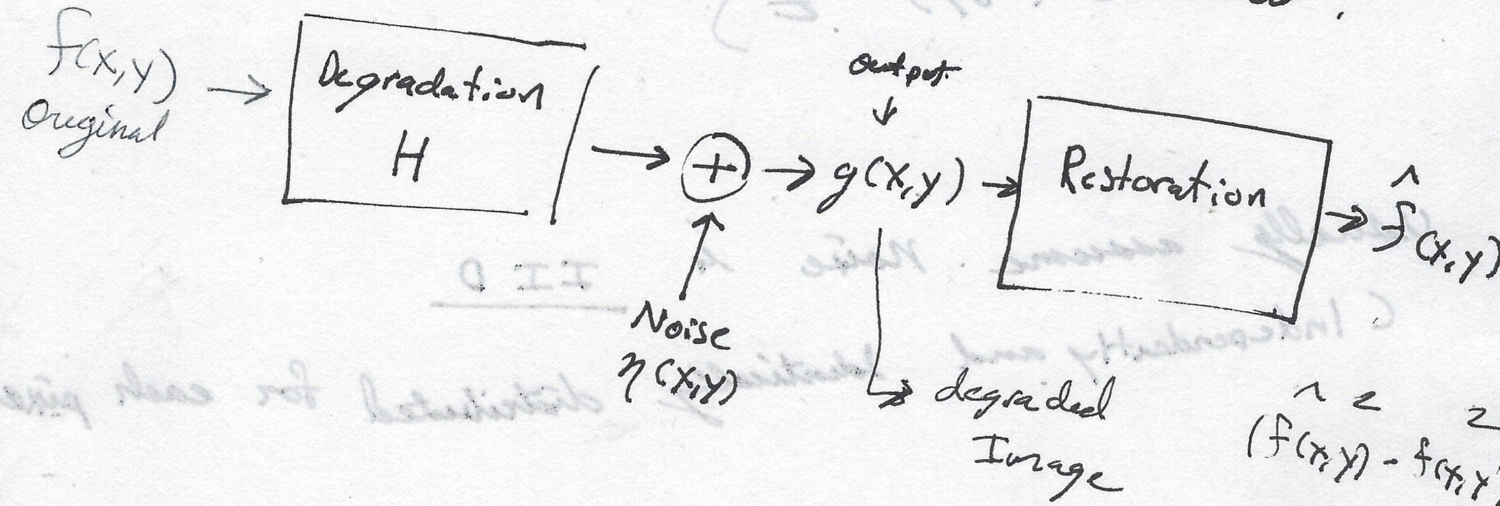
Digital Image Processing

Lecture 17

(In class)

Image reconstruction

Unlike earlier lectures where we "cleaned up" an image visually – is there an "optimal" method?



$(\hat{f}(x,y) - f(x,y))^2$

small as possible (Least squared sense

Spatial Domain:

$$g(x,y) = f(x,y) * h(x,y) + \eta(x,y)$$

Frequency Domain:

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

Can we design a restoration filter so that $\hat{f}(x,y)$ is as close as possible to $f(x,y)$

(Assuming we know something about h, η.)

Objective vs Subjective

②

With No degradation case: $H = I$ (IDENTITY)

Then we only have noise.

Noise is typically characterized by a PDF (PMF)

$$P(\eta(x,y) = z)$$

Usually assume noise is IID
(Independently and Identically distributed for each pixel)

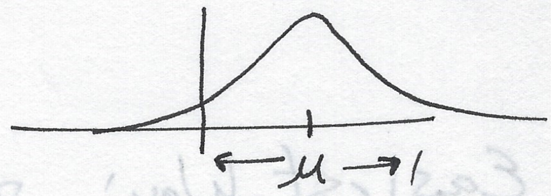
We have ~~note~~ noise sources:

- Non-Ideal Sensor Element (crappy camera)
- Environmental conditions (light level or temperature) → Noise may occur
- Corruption During Transmission

Common Noise PDFs:

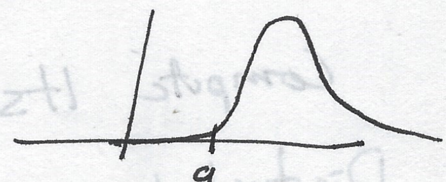
- Gaussian
(Sensor, Thermal)

$$P(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(z-\mu)^2}{2\sigma^2}}$$



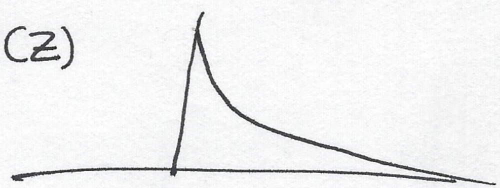
- Rayleigh
(Range Imaging)

$$\begin{cases} \frac{2}{b}(z-a)e^{\frac{-(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$$



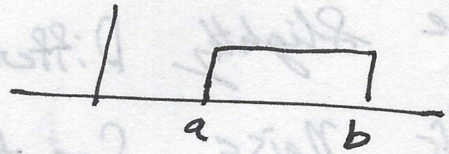
Exponential
(Laser Imaging)

$$ae^{-az}u(z)$$

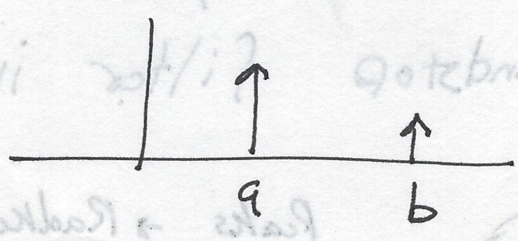


- Uniform

$$\frac{1}{b-a} \quad z \in [a, b]$$



- Salt and Pepper
(faulty component)



$$P(z) = \begin{cases} P_w & z = w \\ P_b & z = b \\ 0 & \end{cases}$$

Pg 318 - 319

4

How to know what kind of Noise is in our Image?

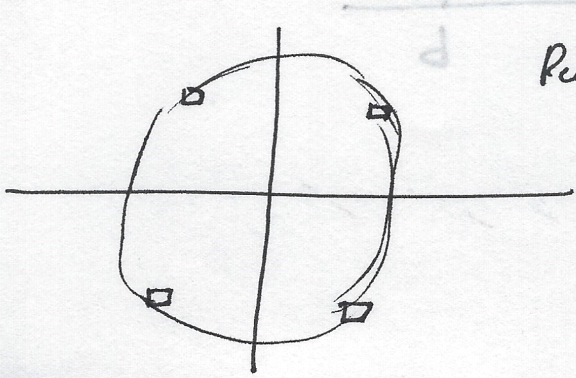
Easiest Way: Find a Region of the Image that should be constant, (flat w/ constant intensity) Look at pdf

Compute Its Histogram, fit a Noise Distribution.

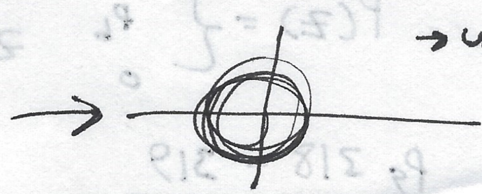
(Note Not IID):

One Slightly Different kind of Noise:

Periodic Noise. For this, we could use a Notch/Bandstop filter in the frequency domain



Peaks → Radke used a Ring to Notch it out



→ we knocked out quite a bit of Noise

Spatial Periodic Noise -13:48

fft of image

2 pixels → White and out of place

→ too much contribution.

Notchclick → do a Notch filter

→ we preserved high and low frequency

noise est -21:08

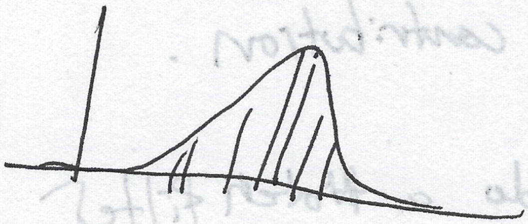
click back on image

→ look at histogram

took at μ and σ

→ fit gaussian to histogram to see how good it is [mu sig] = NoBe est (im)

estimate mean and variance from Sample distribution



$$\hat{\mu} = \sum_{z_i \in S} z_i p_i(z_i)$$

$$\hat{\sigma}^2 = \sum_{z \in S} (z_i - \mu)^2 p(z_i)$$

How to restore the Image in the presense of noise?

Problem: $g(x,y) = f(x,y) + \eta(x,y)$

we can't just subtract this. (it's unknown)

$G(u,v) = F(u,v) + N(u,v)$

May be able to estimate and subtract if noise is periodic

In IID (Independently and Identically distributed for each pixel) noise case, Not much to do other than ~~Spatial filtering~~.

→ Not periodic so No Notch filter

(6) For Gaussian Noise

- Arithmetic Mean filter : $\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$ i.e $\frac{\text{ones}(n)}{n^2}$

- Geometric Mean : $\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$

Achieves good smoothing, but loses less image detail.

Median for Impulse Noise - for Salt & Pepper Noise

- Median filter

$\hat{f}(x,y) = \underset{(s,t) \in S_{xy}}{\text{median}} \, g(s,t)$

- Alpha - Trimmed Mean filter
  - Remove $\frac{\alpha}{2}$ Lowest, $\frac{\alpha}{2}$ highest values in S_{xy}, Then average the rest

Instead of using Regular filters

Better Method is Adaptive Filter

Changes Depending on noise characteristics in a local window

Suppose we know

- $g(x, y)$ corrupted/Noisy Image
- σ_{η}^2 Noise variance over the whole Image
- $\hat{\mu}_L$ local mean around (x, y)
- $\hat{\sigma}_L^2$ local variance around (x, y)

Say our reconstructed Image is

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_{\eta}^2}{\hat{\sigma}_L^2}\left(g(x,y) - \hat{\mu}_L\right)$$

- If $\sigma_{\eta}^2 = 0$ (No Noise), $\hat{f}(x,y) = g(x,y)$
- If $\hat{\sigma}_L^2 >> \sigma_{\eta}^2$, $\hat{f}(x,y)$ is close to $g(x,y)$

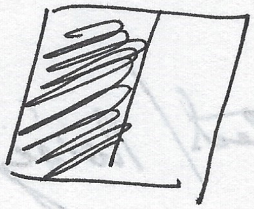
the Local variance is different? So then, do we are we evaluating only parts of the Image?

8 why is $\hat{\sigma}_L^2 >> \sigma_\eta^2$ good?

A high local variance means an Edge

→ Preserve edges.

High Low Intensities

→ 

→ don't want to blur this out

Keep as is

if $\hat{\sigma}_L^2 \simeq \sigma_\eta^2 \rightarrow \hat{f}(x,y) = \hat{\mu}_L$

Average intensities in "Normal" Regions → Nothing unusual

→ Better than using averaging filter for the whole image.

drawback: we need an estimate of $\cancel{\sigma^2}$ σ_η^2 for this to work

but we have this from

(e.g. from previous method)

$Sig^2 = 13.0932$

- if local varian around 13, keep things the way they are

- if greater (much greater) take the average

doc wiener2

J = wiener2(I, [m n], noise)

↑ variance right?

→ we mitigated the noise and we kept the sharp white to dark boundary.

Try

old → [9 9]

wiener2(I, [3 3], ~~9~~ noise) 13

→ better → smaller block size

→ Not much detail lost but we are better