(10) We have a star, its very far away, So shouldn't we have only 1 pixel to represent it?

→ We have a smear

→ Image was blurred and noised up

→ Look

What happens when we have Blur/Degradation in addition to Noise?

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

$$G(u,v) = \underline{H(u,v)}\, F(u,v) + N(u,v)$$

Could come from moving camera or objects moving

we talked about how to estimate/model $N(u,v)$. How about estimating $H(u,v)$?

(Estimating $H(u,v)$) (11)

- Guessing; e.g. Take a piece of the degraded Image and guess what the original Image should have Looked Like

$$H(u,v) \simeq \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

corrupted Image

manual Guess

(S = ~~Guess seen~~

(we're working with regions of the Image)

- Experimentation (if you have access to the Imaging device) - Directly acquiring the Impulse response / point spread function

→ Get a camera in a dark room and have a very small very bright light Source (~~[illegible]~~ Impulse → 2D Impulse response

- Estimate / Model $H(u,v)$ (e.g. Gaussian blur)

(from simple functions e.g. Gaussian)

(12)

getpsf(in) $\underline{51:24}$

Get point spread function

We have the Degraded Image $g(x,y)$

We have the estimated Blur, $H(x,y)$

So try

Inverse filtering

We've estimated $H(u,v)$

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

Sounds good in theory but not practice

$$= F(u,v) + \frac{N(u,v)}{H(u,v)}$$

If $H(u,v) \simeq 0$ for some (u,v), or

$\frac{N(u,v)}{H(u,v)}$ is Large → Poor Result

J = deconvwnr(I, PSF, NSR)

supply Image and point spread function, assume

Noise = 0 (NSR) → we have inverse filter

```
iminv=deconvwnr(im, h);
```

from estimated filter

```
imshow(iminv)
```

→ very Bad!

why is this happening?

we see,

$$\hat{F}(u,v) = \frac{F(u,v)H(u,v) + N(u,v)}{H(u,v)}$$

$$= F(u,v) + \frac{N(u,v)}{H(u,v)}$$ ← we have noise

→ for small H, Large Noise

– mitigate?

→ Apply when the filter has content (close to low frequencies)

(14)

One Solution: ~~Only apply~~ Inverse filter at low frequencies (Small (u,v)) otherwise, keep $G(u,v)$.

$$\hat{F}(u,v) = \begin{cases} \frac{G(u,v)}{H(u,v)} & \sqrt{u^2+v^2} \le r^2 \\ G(u,v) & \text{o.w.} \end{cases}$$

Smoothing

$(r^2 < \sqrt{u^2+v^2})$

We want a smooth transition between these two (we don't want a step function)

The Right Thing to do:

Wiener FILTER

Minimum-Mean-Square error filtering

Goal: Minimize

$$\underbrace{\mathcal{E}}_{\text{expected value}}\left(\left(f(x,y) - \hat{f}(x,y) \right)^2 \right)$$

Wiener filter

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{\left(|H(u,v)|^2 + \frac{S_n(u,v)}{S_f(u,v)} \right)} \right] G(u,v)$$

$S_n(u,v) = |N(u,v)|^2$ = Power Spectrum of Noise

$S_f(u,v) = |F(u,v)|^2$ = Power Spectrum of Signal

also

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)\, S_f(u,v)}{S_f(u,v)\, |H(u,v)|^2 + S_n(u,v)} \right] G(u,v)$$

(16)

H is our psf point spread function

if we didn't have $\dfrac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_n}{S_f}}$

we would be left with the inverse filter

---

if $H(u,v) \to 0$, we don't explode

if $N(u,v) \to 0$, inverse filter

wouldn't have the bad numerical properties of the inverse filter

there are stuff for the wiener filter we must know but we don't have access to.

$S_n \rightarrow$ Estimate noise, ~~get~~ pdf
(from flat region)
or constant (white noise)

S_f is hard to estimate as we don't have access to it

tune $\frac{S_n}{S_f}$ to match up with the SNR $\rightarrow$ constant K

We may be able to estimate $S_n(u,v)$
But we don't know $S_f(u,v)$ [requires original image] Instead we usually use

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v)$$

Improves noisy and blurry images

tunable parameters
(related to estimate of SNR)

18 Wiener filter Moon Image

point spread function, h

from star

J = deconvwnr(I, PSF, NSR)

Noise to Signal Power ratio, K

imw = deconvwnr(im, h, 0.0001);

we underestimate the noise so this is close to inverse filter

try 0.001 → better but not great

try 0.01 → much better, but Not Amazing

Star is much brighter

$K = \underline{0.1}$ → Some things are better

→ Star is compact

→ Preserved edge and less noisey

needed psf and K

→ this is used in practice