(10) Denote the matrix as R

and Note $R^{-1} = R^T$

---

Any combination of scale, shift, Rotate

is called a similarity Transformation

- Preserves Parallel Lines

IF $\alpha, \beta = \pm 1$, Isometric transformation

(Rigid motion) (No scaling)

- Preserves shapes, Angles

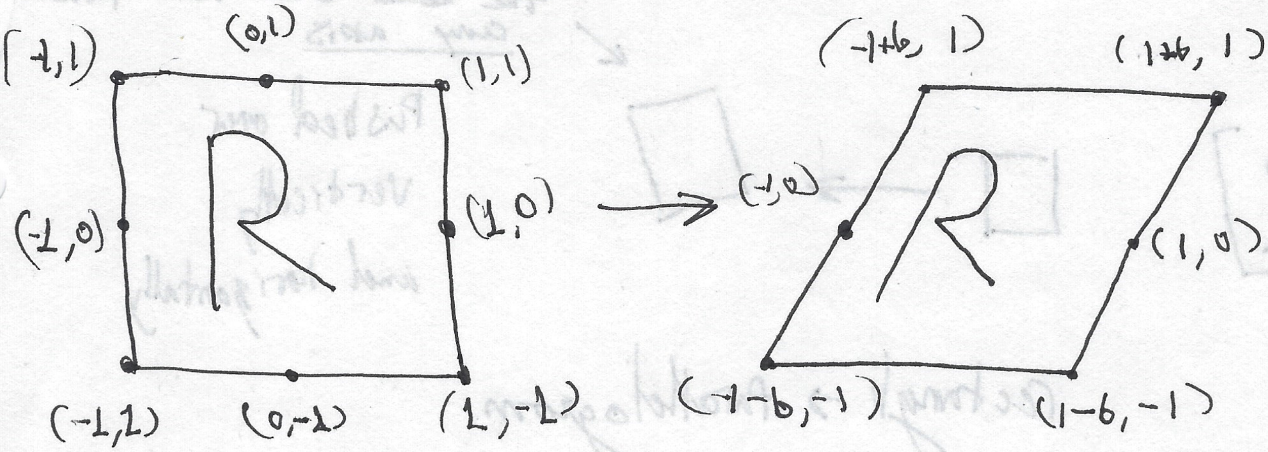
(for reflection, rotation, and translation)

We can also "Bend" The Image

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{vmatrix} x \\ y \end{vmatrix} + \begin{vmatrix} t_1 \\ t_2 \end{vmatrix}$$

What if

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{vmatrix} x \\ y \end{vmatrix} = \begin{bmatrix} x + by \\ y \end{bmatrix}$$




$$\begin{bmatrix}x'\\y'\end{bmatrix}=\begin{bmatrix}x+by\\y\end{bmatrix}$$

$\boxed{\text{SHEAR}}$

Y values will not change

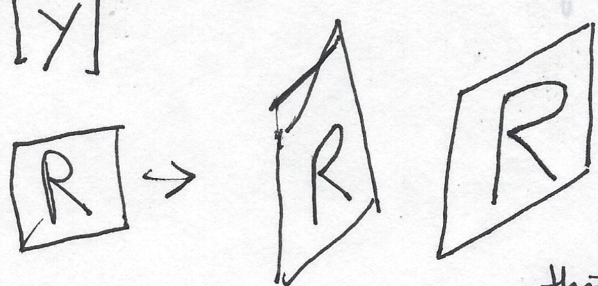
$(0,0) \rightarrow (0,0)$
$(1,0) \rightarrow (1,0)$
$(-1,0) \rightarrow (-1,0)$
$(-1,-1) \rightarrow (-1-b,-1)$

$(0,1) \rightarrow (b,1)$
$(1,1) \rightarrow (1+b,1)$
$(-1,1) \rightarrow (-1+b,1)$
$(0,-1) \rightarrow (0,-1)$
$(1,-1) \rightarrow (-b+1,-1)$

If $b<0$, the shear would be to the left

See that

$$\begin{vmatrix}x'\\y'\end{vmatrix}=\begin{bmatrix}1 & 0\\c & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$



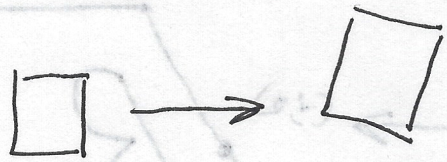

Vertical Shear

~~the sides are not~~

11

The sides are not parallel w/ any axis

$$\begin{bmatrix} 1 & b \\ c & 1 \end{bmatrix}$$



Pushed over vertically and horizontally

rectangle → parallelogram

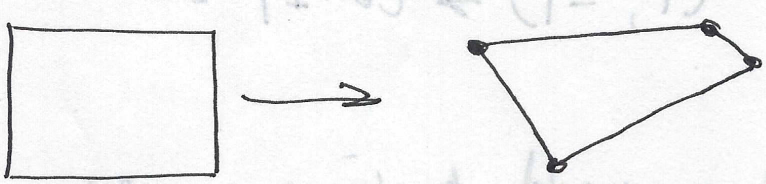
---

A Transformation of the form

$$\begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{vmatrix} x \\ y \end{vmatrix} + \begin{vmatrix} e \\ f \end{vmatrix}$$

is called an affine transformation

turn a rectangle to a weird quadrilateral



Projective Transformation

Rectangle → Quadrilateral

matlab examples 35:22

# Projective Transformation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \dfrac{a_{11}x + a_{12}y + b_1}{c_1 x + c_2 y + 1} \\[2ex] \dfrac{a_{21}x + a_{22}y + b_2}{c_1 x + c_2 y + 1} \end{bmatrix}$$

$$\begin{bmatrix} \tilde{x}' \\ \tilde{y}' \\ \tilde{z}' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ \tilde{c}_1 & \tilde{c}_2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = \frac{\tilde{x}'}{\tilde{z}'}$$

$$y' = \frac{\tilde{y}'}{\tilde{z}'}$$

These are the variables that skew the image

→ A transformation matrix in Matlab

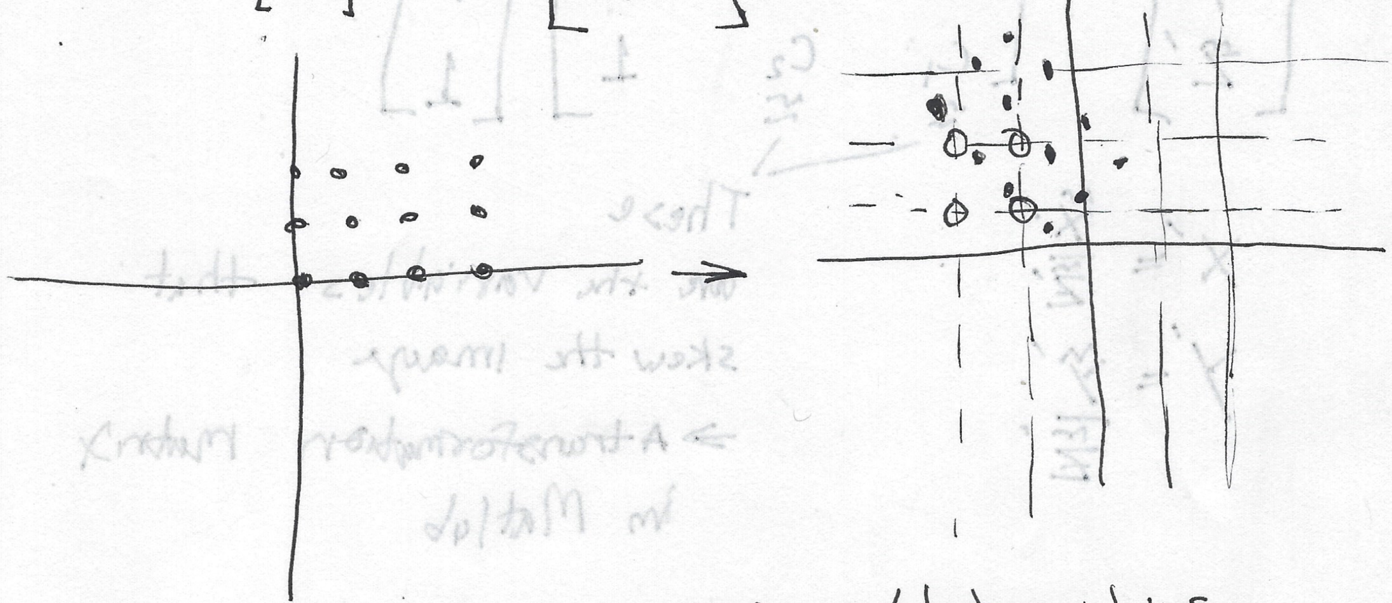
ex at 45:57 ~~Dualign~~ Dualalign

(4) How to create the OUTPUT IMAGE?

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1.2 & 1 \\ 1 & 0.8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3.2 \\ -1.6 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 5.4 \\ 0.2 \end{bmatrix}$$

Now we have non-integer points



How to get Image colors/Intensities on the new Grid? (like the circles)

Take the closest black dot? Could get significant errors

IT's More conventional To use Backwards Mapping

If this was over forward mapping

$$\begin{bmatrix}x'\\y'\end{bmatrix} = \underset{2\times 2}{A}\begin{bmatrix}x\\y\end{bmatrix} + \underset{2\times 1}{b}$$

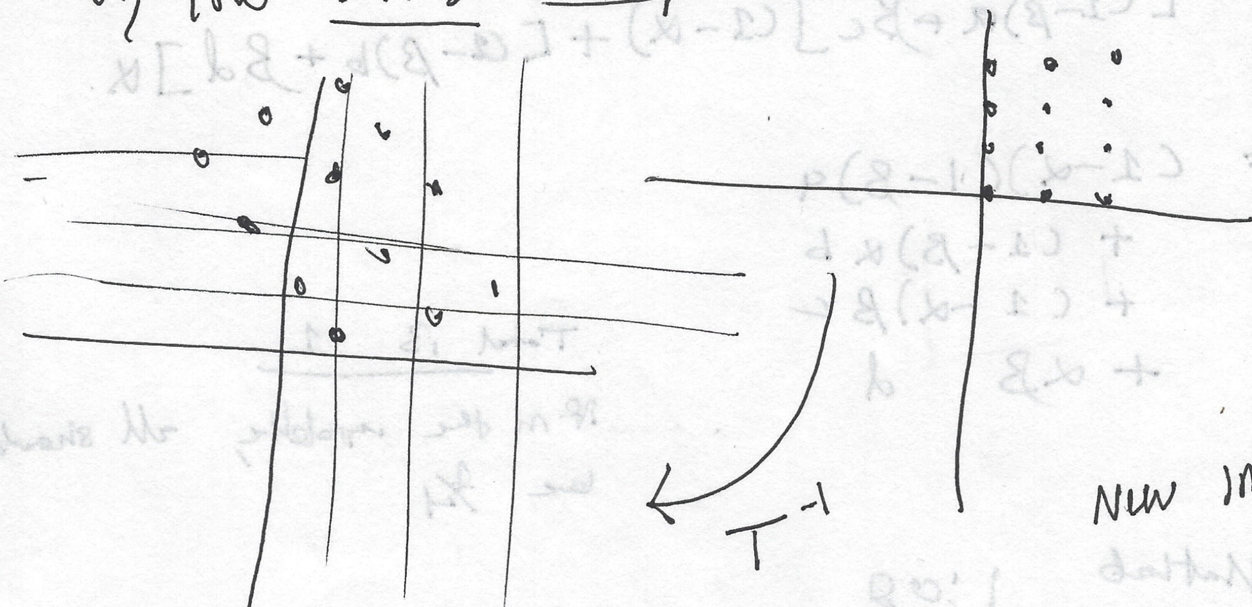
undo this process?

$$\begin{bmatrix}x\\y\end{bmatrix} = A^{-1}\left(\begin{bmatrix}x'\\y'\end{bmatrix} - b\right)$$

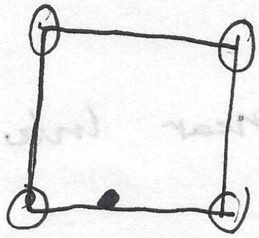
$$= A^{-1}\begin{bmatrix}x'\\y'\end{bmatrix} - A^{-1}b$$

Inverse Transformation

Now for the new Image, we have points on that Image, and we know where those points come from by the Inverse Transformation

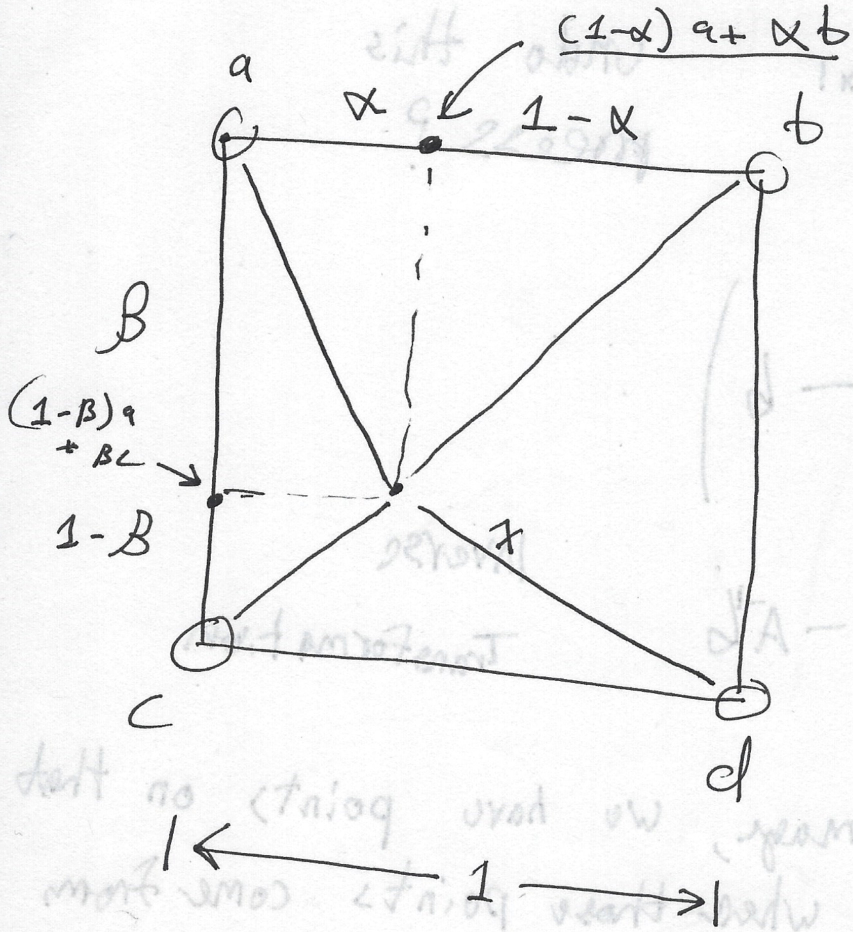



New image



MatLab 1:04 ish

# (16) Bilinear Interpolation 1:05:12




- if the dot lands on a, take a, if in between a and b, take some of a and less of b. [combo of a and b]
- if in the middle, take values from a, b, c, and d

$$x = [(1-\beta)a + \beta c](1-\alpha) + [(1-\beta)b + \beta d]\alpha$$

$$\begin{aligned} x = \; & (1-\alpha)(1-\beta)a \\ & + (1-\beta)\alpha b \\ & + (1-\alpha)\beta c \\ & + \alpha\beta \; d \end{aligned}$$

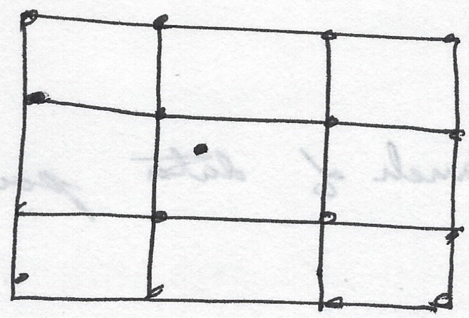
Total β 1

If in the middle, all should be 1/4

Matlab 1:08

└→ doesn't do bilinear interpolation by default

# Bicubic Interpolation



→ uses more points and looks smoother.

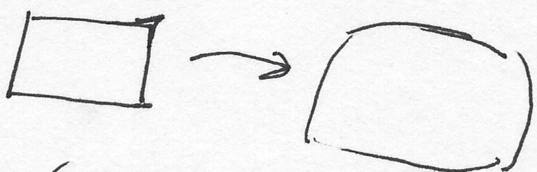
we use these 16 neighbors to find the intensity.

→ Not necessary in practice, an option

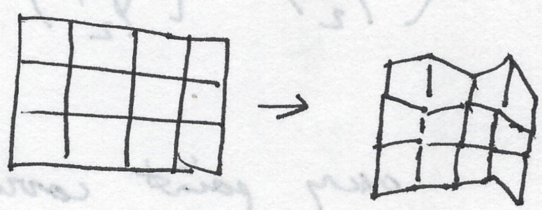
→ other Geometric Transformations exist

Image mosaic

( bow the edges 'out'



( Lens (on) Distortion )



Warp every rectangle on its own to make a weird local deformation of the image

→ bilinear ~~transformation~~ interpolation inside these deformed quadrangles

Geometric transformation could be a complex function (non linear perhaps)

$$X' = f(x, y)$$
$$Y' = g(x, y)$$

(18)

HW, workout a bilinear Interpolation

→ fit a transformation to a bunch of data points

→ ex. of points going to other (New) points

$$\begin{pmatrix}X_1\\Y_1\end{pmatrix} \to \begin{pmatrix}X_1'\\Y_1'\end{pmatrix} \qquad \begin{bmatrix}x'\\y'\end{bmatrix} = \begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}X\\Y\end{bmatrix} + \begin{bmatrix}e\\f\end{bmatrix}$$

$$\begin{pmatrix}X_2\\Y_2\end{pmatrix} \to \begin{pmatrix}x_2'\\y_2'\end{pmatrix} \cdots$$

every point corresponds yields $\underline{2}$ eqns in the $\underline{6}$ unknowns

So we need $\underline{3}$ point correspondences to get an affine transformation

$6 \times 6$

$$\begin{bmatrix} \quad \end{bmatrix}\begin{bmatrix}a\\b\\c\\d\\e\\f\end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

if we have more than 3, use an approximate way.