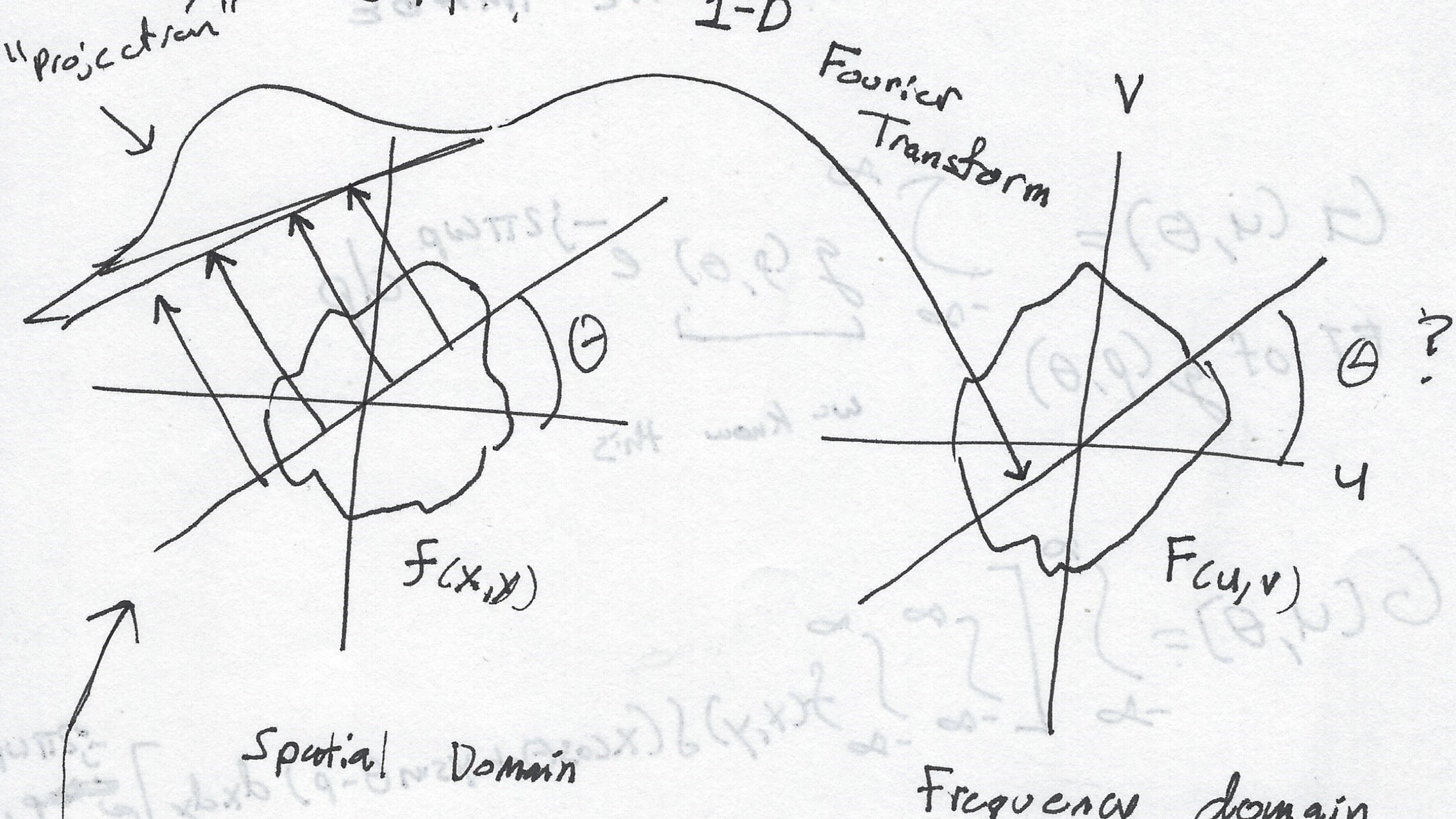
(6)

$$= F(u,v)\Big|_{\substack{u=\omega\cos\theta \\ v=\omega\sin\theta}} = F(\omega\cos\theta, \omega\sin\theta)$$

Key result!

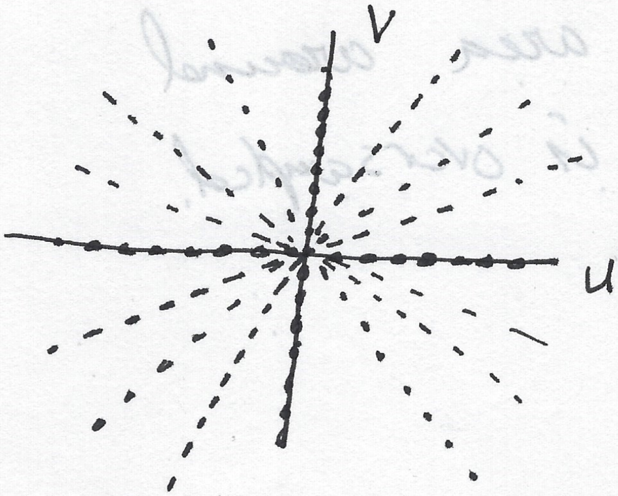


Integral in that direction, θ, is the slice of the 2D FT of f(x,y)

Projection related to the 2D FT of the image

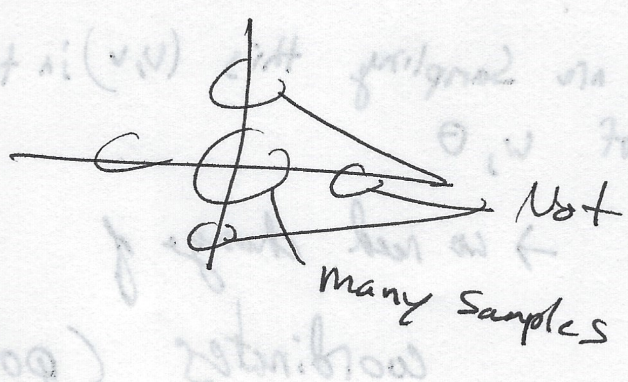
1-D FT of Projection corresponds to the slice through the 2D FT AT the Same Angle!

In Practice taking the FT of ~~the~~ Each projection at a fixed angle Gives us Samples of the 2D FT That we could use To Reconstruct.



IF we had Enough Samples, we could estimate $F(u,v)$ And invert to get original $f(x,y)$

Crux: now we see why back projection is not so good.

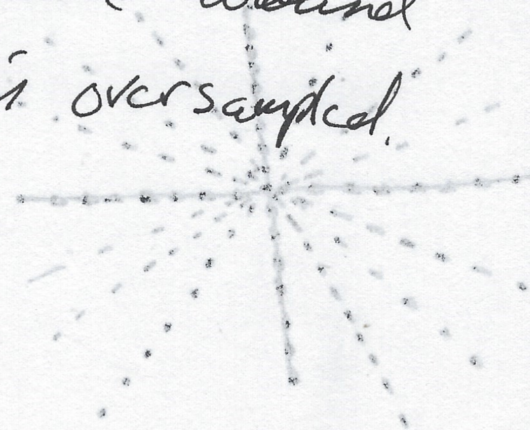


(12)

down weight the middle so the other samples have equal contribution

---

We can see ~~the~~ that the area around (0,0) in the freq domain is oversampled.



We should Down weight samples in this Area So contributions are all even.

Filtered Back projection

We want

$$f(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\, e^{j2\pi(ux+vy)}\, du\,dv$$

We are sampling this (u,v) in terms of w, θ

→ we need change of coordinates (polar)

$u = w\cos\theta$

$v = w\sin\theta$

$du\,dv = w\,dw\,d\theta$

[$\approx r\,dr\,d\theta$ from polar]

$$f(x,y) = \int_0^{2\pi}\int_{-\infty}^{\infty} F(w\cos\theta, w\sin\theta)\, e^{j2\pi(w\cos\theta x + w\sin\theta y)}\, w\,dw\,d\theta$$

by fourier slice thm

$$= \int_0^{2\pi}\int_{-\infty}^{\infty} G(\omega,\theta)\, e^{j2\pi\omega(x\cos\theta + y\sin\theta)}\, \omega\,d\omega\,d\theta$$

Note $G(\omega, \theta+\pi) = G(-\omega, \theta)$

$$= \int_0^{\pi}\int_{-\infty}^{\infty} |\omega|\, G(\omega,\theta)\, e^{j2\pi\omega(x\cos\theta + y\sin\theta)}\, d\omega\,d\theta$$

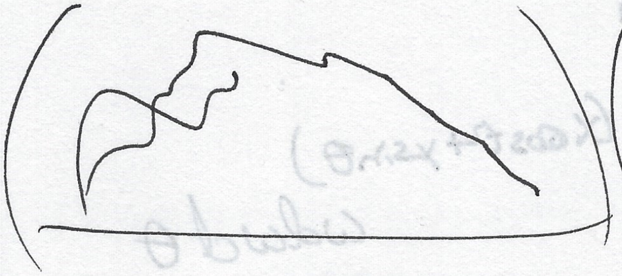
$$= \int_0^{\pi}\left[\int_{-\infty}^{\infty} |\omega|\, G(\omega,\theta)\, e^{j2\pi\omega\rho}\, d\omega \Big|_{\rho = x\cos\theta + y\sin\theta}\right] d\theta$$

FT at spacial coordinates ?

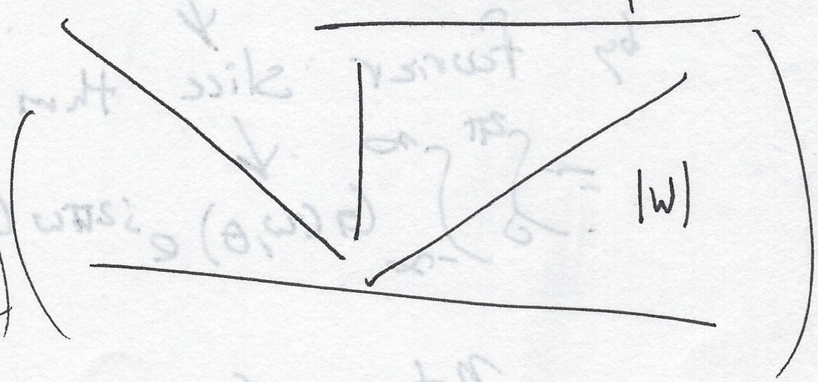
14

$$\int_0^{\pi} \left[\begin{array}{c} \text{inverse FT of } G(\omega,\theta) \\ \text{Multiplied by a filter function } |\omega| \end{array} \right] d\theta$$

With out $|\omega|$, we have inverse FT

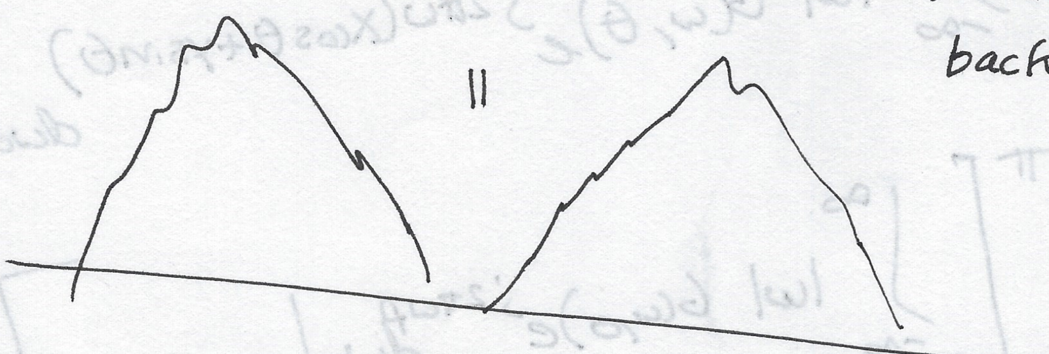


$G(\omega,\theta)$



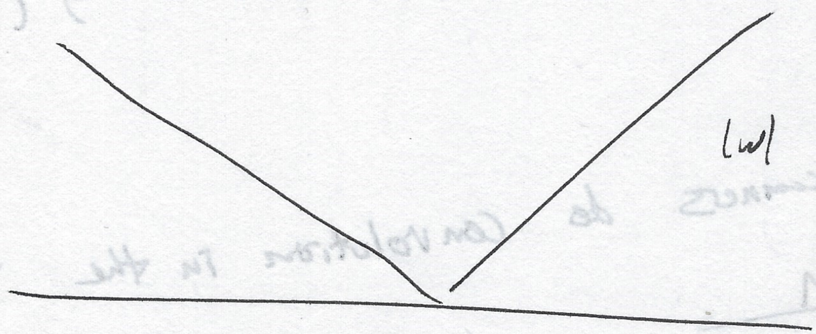
$|\omega|$

Filtered back projections



$|\omega| G(\omega,\theta)$

To Recap: We can still add up Backprojections; Just filter each one by $|\omega|$ in the freq Domain before Summing, To make sure we get the right answer. → Look at Another Recap (Pg 16)

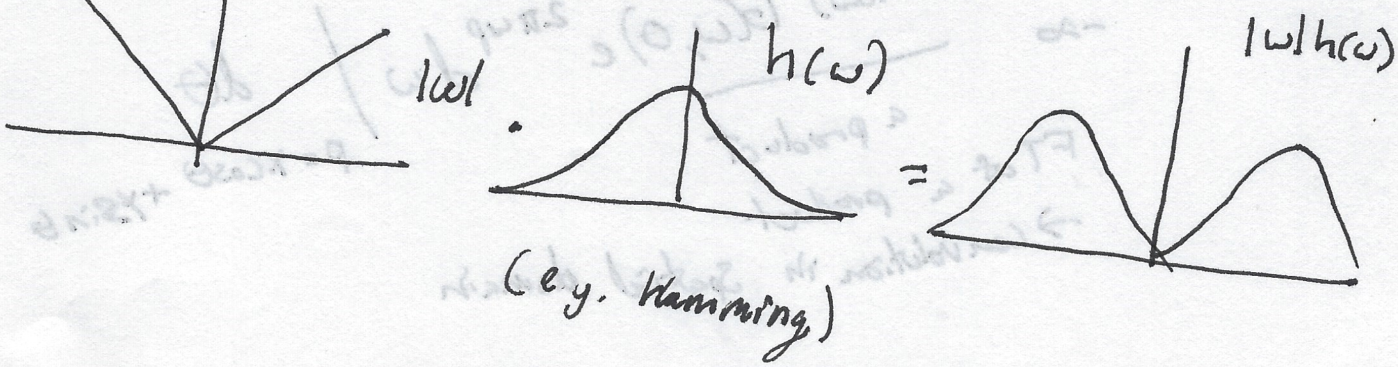


This has Infinite energy

In Practice, since $|\omega|$ is not Integrable, We can't take the Inverse FT.

In Practice, we window $|\omega|$ so it becomes 0 outside of a Given interval,

e.g.



⑱ Another Recap

1) Compute 1-D FT of Each Projection
2) Multiply by $|\omega|\, h(\omega)$
3) Take 1-D Inverse FT
4) Sum all the result to Get $\hat{f}(x,y)$

→ CAT Scanners do convolution in the Time domain

Final comment

$$f(x,y) = \int_0^{\pi} \int_{-\infty}^{\infty} |\omega|\, \underbrace{h(\omega)\, G(\omega,\theta)}_{\text{a product}}\, e^{2\pi \omega p}\, d\omega \Big/\, d\theta$$

$p = x\cos\theta + y\sin\theta$

FT of a product
→ Convolution in Spatial domain

$$f(x,y) = \int_0^{\pi} g(\rho,\theta) * s(\rho)\, d\theta$$

↳ Inv. FT of $|\omega| h(\omega)$

machines may be quicker to do convolution

then sum them up.