Other Unitary Transforms Exist That maintain The DFT's Advantages But Have their own.

Transforms are all in the form

$$\underset{M \times N}{\text{OUTPUT IMAGE}} = \underset{M \times M}{A} \underset{M \times N}{(\text{input Image})} \underset{N \times N}{A^T}$$

Discrete Cosine Transform

~~$C(k,n) = \begin{cases} \frac{1}{\sqrt{N}} \\ \sqrt{\frac{2}{N}} \cos\left(\pi(2n+1\right. \end{cases}$~~

1st row is constant all other rows have a real valued cosine

$$C(k,n) = \begin{cases} \frac{1}{\sqrt{N}} & k=0, \quad 0 \leq n \leq N-1 \\ \sqrt{\frac{2}{N}} \cos\left(\frac{\pi(2n+1)k}{2N}\right) & \text{otherwise } k = 1, \dots N-1 \end{cases}$$

$$\underset{M \times N}{F} = C_M f C_N$$

f ← M×N input

C_M ← M×M DCT Matrix

C_N ← N×N DCT MATRIX

- C is real

real input real transform

Note $C^{-1} = C^T$ (Unitary transform)

(10) Unitary ~~prop~~ property indicates that

There is equal energy in the (spatial) Image Domain and

The <u>DCT</u> domain

- Excellent Energy Compaction for Natural Images

- Fast Transforms (We don't have to multiply element by element)

<u>32:40</u>

Basis looks like DFT, Higher <u>frequency</u> from left to right (low freq)

low to high freq top to bottom

→ bottom corner has the checkered board base.

→ we have real numbers

For the Impulse-like Image, we have higher freq contribution (mostly) but we only have real <u>numbers</u> (same)

DCT is basis for a lot of Image compression algorithms

DCT is the Critical Part of the JPEG Algorithm

Input Image → split into 8×8 blocks

- Take DCT of each block
- Quantize DCT coefficients

use more fidelity (more bits) for lower DCT frequencies → keep DCT contributions where signal (image) is likely to have multiples of those basis functions

(Example: if the high frequency checkerboard basis has little contribution, just Round to zero)

Save some space!

- Code up Quantized coefficients

→ Inside jpeg image algo

→ we can see the information turn back into the image.

(12)

Similarly we have the Discrete Sine transform

(There's an ∞ amount of Image transforms)

$$S(k,n) = \sqrt{\frac{2}{N+1}} \sin\left(\frac{\pi(k+1)(n+1)}{N+1}\right)$$

$$0 \le k,n \le N-1$$

— Square matrix of sines

Properties

- $S^{-1} = S^{T}$, S real and unitary)

  also $S = S^{T}$ (symmetric)

→ A disadvantage with this and the DCT are all the trigonometric functions in the operations

---

11)

HADAMARD Transform

Basis Functions only contain ± 1

To implement, add and subtract things

$$\left[\begin{array}{cc|cc|cccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
\hline
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1
\end{array}\right] = H_3 \quad 2^3 = 8$$

Not the same

bottom

right Not same

$$H_n = \begin{bmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{bmatrix} \frac{1}{\sqrt{2}}$$

To make unitary

- ADD/Subtracts → Fast
- $H = H^T = H^{-1}$
- Good compaction for real Images

Hadamard Basis 41:50

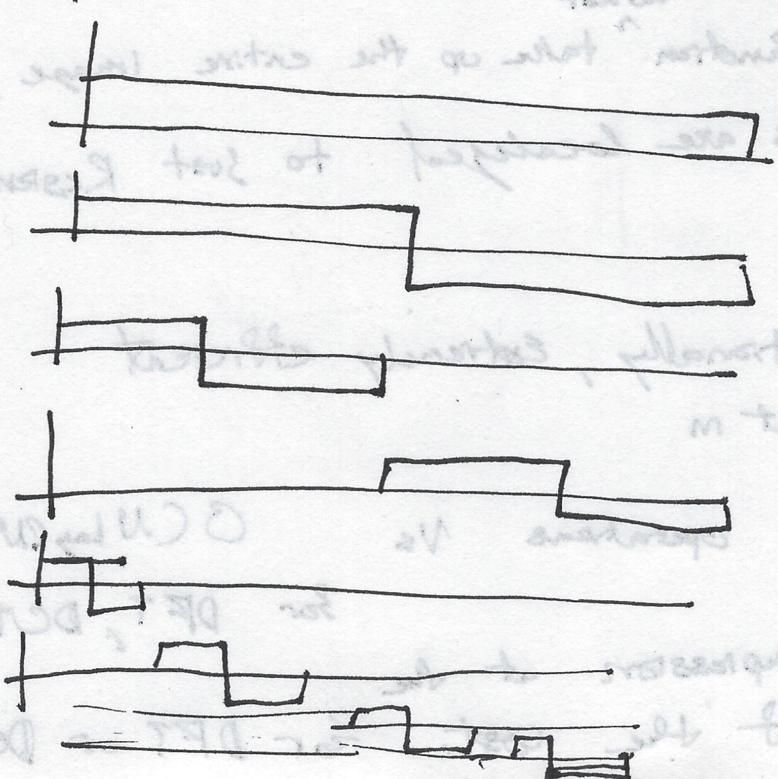
Black and white checkard board

Some large contributions, some Not so large

HAAR TRANSformation

$$R_3 = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & \sqrt{2} & -\sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2 \end{bmatrix}$$

The Rows Look Like



(14) Easy to see these basis functions are perpendicular

Think, orthogonal integral from communication systems

The 2nd Row is squished and moved around

* Simplest Wavelet Transform (Important in modern image processing)
→ good project idea

Wavelet Transforms

Similar 'Good' properties to the Fourier transform

- Unitary
- Can represent both smooth and discontinuous images efficiently
- Local basis functions

47:13 Look at basis of the Haar transformation, The basis function do not take up the entire image, Think; The oscillations are localized to just regions of the image

- Computationally, extremely efficient
  can do wavelet in

$O(N)$ operations Vs $O(N\log(N))$ for DFT, DCT

- Great compression at the fraction of the cost for DFT or DCT

Wavelets early 90s

Jpeg is DCT

Not all standards have moved on to the wavelets

(.jp2 , .j2c)

Jpeg 2000 Image files 20% better Compression

Hard drive space is cheap. you would want to have space even if say you have a chip with limited space, why not do it with wavelets ?

→ Good project idea to look into more deeply

Wavelet transform combines Time Spatial AND Frequency Domain

F

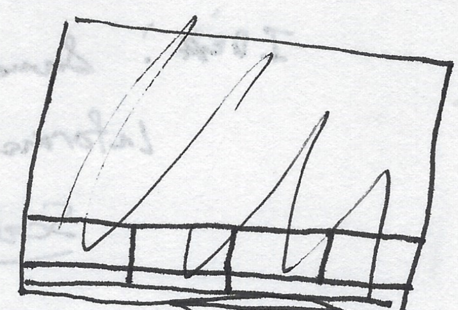
T

Spatial Basis

F

T

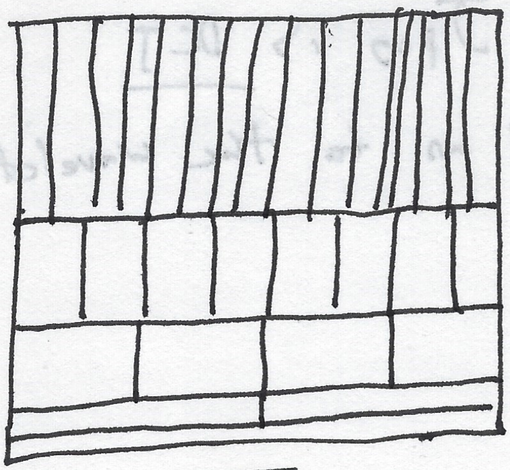
Fourier Basis



N-fine pieces of the Image in No Notion of Frequency (No oscillation Noticed)

No idea at whats Going on ith the time space

F



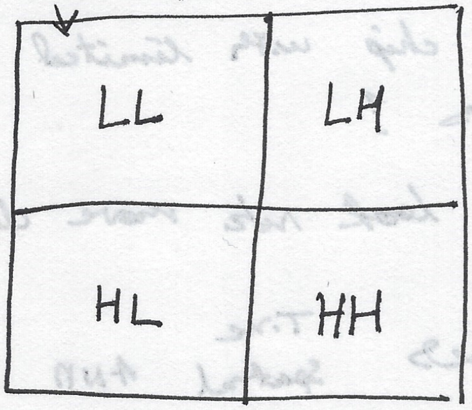
T

Trade off between Time and frequency

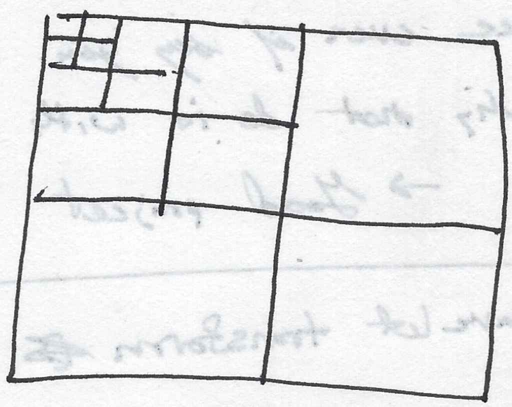
⇒ Investigate further

Wavelet Basis

Low Pass Filtered Image



Tiny blurry Image



Edges at Smallest level of detail

Rest are edge Maps at different Levels of Detail

IDEA: Same ~~#~~ of Bits of Information Rearranged at Different Scales

Wavelet Advantages

- Great compression
- computationally fast
- Local Basis functions
- CAN Represent discontinuous edges
- CAN Represent Smoothing functions