Digital Image Processing

Lecture 9: Unitary Image transforms

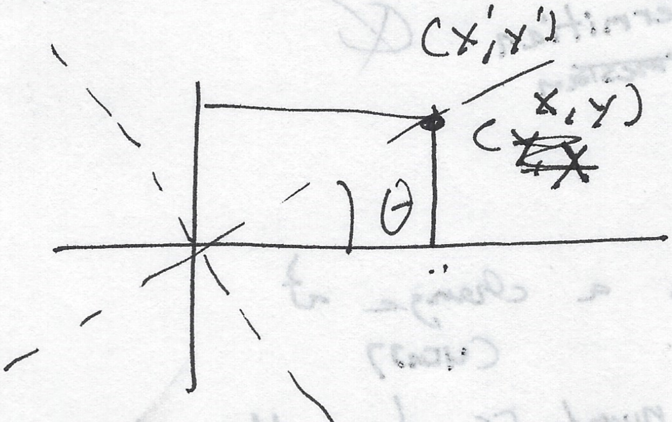
General Image transforms (other than the DFT)

↓

Think of a change of basis

In 2D, A Rotation is Like a change of basis of the Coordinate System

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$



describe by new coordinate, no info is lost in this manner

Go to higher Dimensional from 2D

— Create a unitary transform

2

UNITARY TRANSFORM

$$V[k] = \sum_{n=0}^{N-1} a[k,n]\, u[n] \qquad 0 \le k \le N-1$$

↑ OUTPUT ↑ input

funky way of writting matrix multiplications

$$\underset{N\times 1}{V} = \underset{N\times N}{A}\, u_{N\times 1}$$

what makes a transform unitary?

Unitary ← conjugate

$$A^{-1} = (A^T)^*$$

$$= A^H \leftarrow A \text{ Hermitian}$$

permestian

Idea, A represents a change of basis from the old n numbers (u[n]) to the new n numbers that represent V

Ex: $\qquad A \qquad\qquad\qquad A^T$

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$AA^T = I$ unitary Property satisfied

* columns of A are unit length and $\perp$ to Each other

DFT is like a change of Bases

Time $\rightarrow$ Freq

(What's the signal doing at every point in time?)

(How much ~~freq~~ content is there at that ~~freq~~ every freq)

IN 1D, we also have the discrete Fourier Transform Basis

$$F[k] = \sum_{n=0}^{N-1} f[n] e^{-j\frac{2\pi k n}{N}}$$

our A is

$$A(k,n) = e^{-j\frac{2\pi n k}{N}}$$

(think "Fourier Matrix")

(4) make $N \times N$ matrix out of these exp.

'A'

$F = A f_k$

output signal

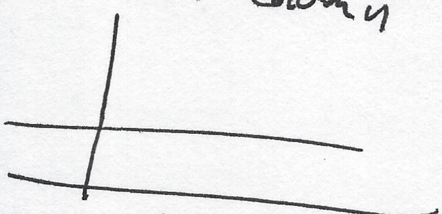
$N \times N$ Fourier matrix

complex exponential

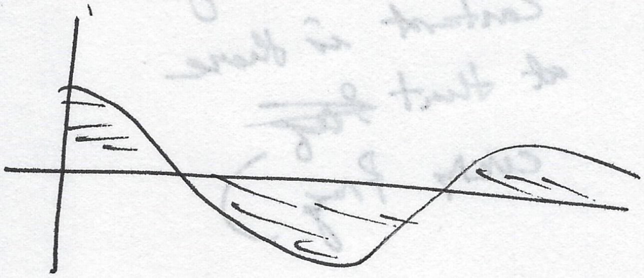
input signal

columns of A are unit length and orthogonal

1st column 'DC'



2nd column "complex cosine"



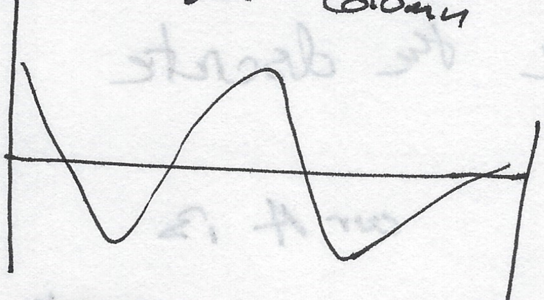
$\perp$?

dot product

half above x axis

half below

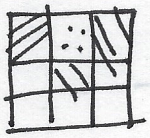
3rd column



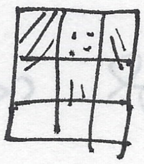
Pieces above and below the line between the two columns

Extra: In Image Processing we have

The spatial Basis

 =  $\cdot \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}$ +  $\begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}$ +

AND The DFT Basis

 $= F[0,0] \cdot \begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix} + F[1,0]$ 

$+ F[0,1]$  +

2-D DFT

$$F[u,v] = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f[x,y]\, e^{-j\left(\frac{2\pi u x}{M} + \frac{2\pi v y}{N}\right)}$$

$F[u,v]$: M×N output Image

$f[x,y]$: M×N input Image

$$= A_M f A_N$$

A_M: M×M DFT Matrix

f: M×N

A_N: N×N DFT Matrix

2D unitary Transformation

$$(M\times M)(M\times N)(N\times N) = M\times N$$

6 SPATIAL Basis

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \\ & & O \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ & & O & \end{bmatrix}, \begin{bmatrix} 0 & 0 & \\ 1 & 0 & O \\ & & \end{bmatrix}$

$I(x,y) = \underbrace{I(1,1)}_{\text{Scalars}} \underset{\uparrow \text{ 2D matrix}}{B_{11}} + I(1,2)\, B_{12} + \cdots$

How is the image made ~~out~~ up of contribution from these matrices

Fourier Basis

$\begin{bmatrix} & \\ & \end{bmatrix}$, $\begin{bmatrix} & \\ & \end{bmatrix}$

DC matrix

DC in X, single cos in Y

$\begin{bmatrix} & \\ & \end{bmatrix}$

DC in Y, single in X cosine

$I(x,y) = C_{11}(DC) + C_{12}($

8 x 8 Image 15:00

Skew basis MatLab 16:00

'spatial'

17:25 18:47

Real Part of DFT

Not a lot of High freq stuff,

if it was checkerd Board, then

Perhaps a lot of high freq.

— almost No high frequency

material and zero highest

frequency

* A lot of contribution from Low freq and DC

Image compression gets rid of the ~~Low~~ freq stuff

Not need, as we can reconstruct the Image mostly and

Sufficiently with the low freq and DC terms.

A unitary transform satisfies

$$\sum_x \sum_y (f[x,y])^2 = \sum_u \sum_v (F[x,y])^2$$

i.e. signal energy is preserved (Parseval)

→ CAN SAVE 90% of the Images energy via the DFT

world over the versions in the spatial world.

2D DFT: (Pros of the DFT)

Pros: • Energy is packed usually into Low frequency coefficients

• Convolution Property (LTI SYSTEMS)

• Fast (fft) Implementation

Cons • Transform is complex even for a real Image (real signal)

• The Basis functions span IMAGE HEIGHT/WIDTH

Say we have an Image

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & & & & & \\ 0 & & & 0 & & \\ 0 & & & & & \\ \vdots & & & & & \end{bmatrix}$$

Impulse in time domain is a constant in the frequency domain

and show basis has

$$\begin{bmatrix} 255 & & 255 \\ 255 & & \\ & & 255 \end{bmatrix}$$

28:10

Isolated in Spatial Domain

- Need a lot of frequency coefficients to describe it