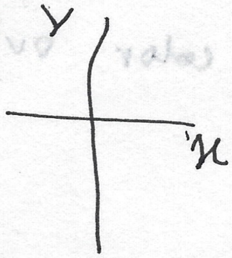
Lecture 5

# DIGITAL IMAGE Processing



## Geometric Operations

Today, we stay in euclidean coordinates




$$J(x,y) = I(T(x,y)) \quad \text{Geometric operation}$$

$J$: new Image, $I$: old Image, $T$: Transformation

Where does pixel $(x,y)$ of Image $J$ come from?

It comes from some transformed version of $(x,y)$ in the original Image $I$.

The pixel at $J(x,y)$ will take its color at some pixel at $I$ (color doesn't change) But that location in $I$ where that pixel comes from is different than $x,y$.

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Geometric vs. Point operation

Last time we said $J(x,y) = T(I(x,y))$

② $J(x,y) = T(I(x,y))$

This says change the color of ~~I(x,y)~~ (x, y) and keep it in the same place.

$J(x,y) = I(T(x,y))$ – keep the same color but get that color from some other place.

also note: coordinates of the new image

coordinates of the old Image

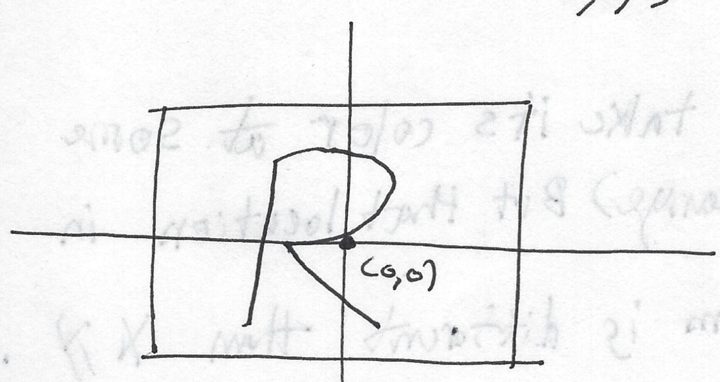
$$J(x,y) = I(T(x,y))$$

→ Scaling a signal, shifting, and flipping
we can do the same with Image

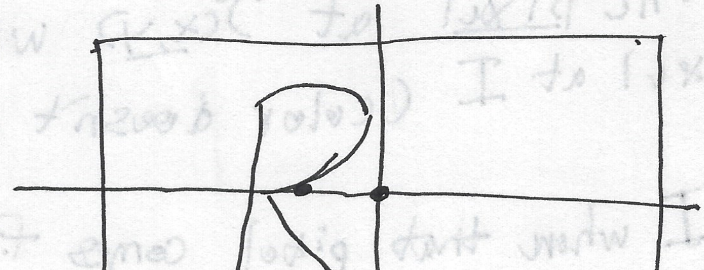
ex. 1)

$$J(x,y) = I(x+2, y)$$

assume (0,0) is in the middle of the image




$I(x,y)$



Note, $J(0,0) = I(2,0)$, so $(2,0)$ is mapped to $(0,0)$

$J(-2,0) = I(0,0)$, so $(0,0)$ is mapped to $(-2,0)$

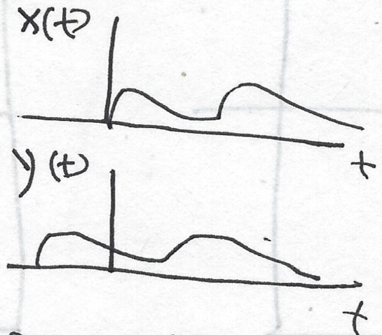
So we conclude, $J(x,y) = I(x+2,y)$

translate the image to the left by 2

Pixels

ex 2) 1D signals for a moment

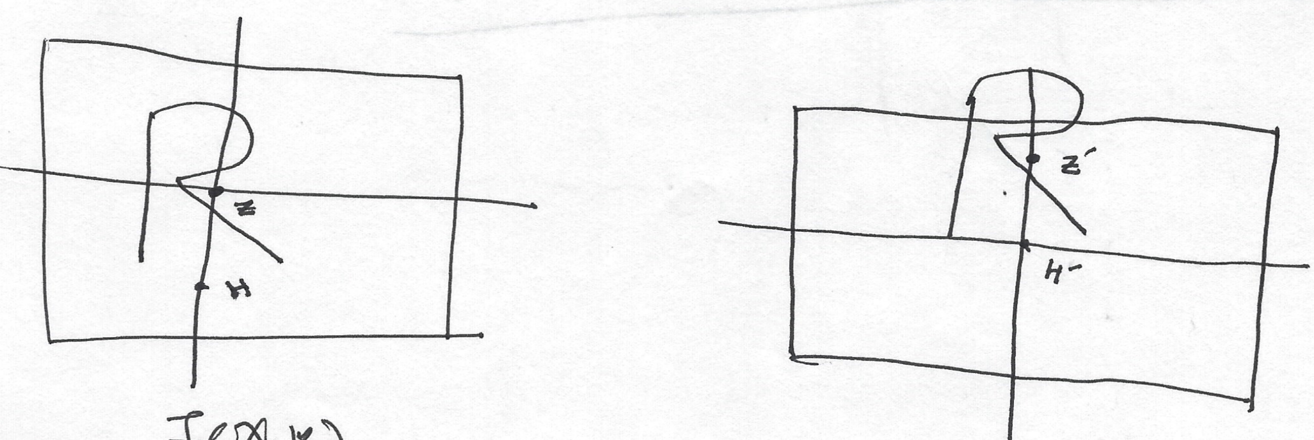
$y(t) = x(t+2)$




negative delay by 2 units

Note: plug in values to see if you are doing this correctly

ex 3) $J(x,y) = I(x, y-10)$ self note x, y belong to J for this expression




$I(x,y)$

$J(0,0) = I(0,-10)$

$J(0,10) = I(0,0)$

→ Image is shifted up by 10 pixels (+y)

(4) This is Image translation

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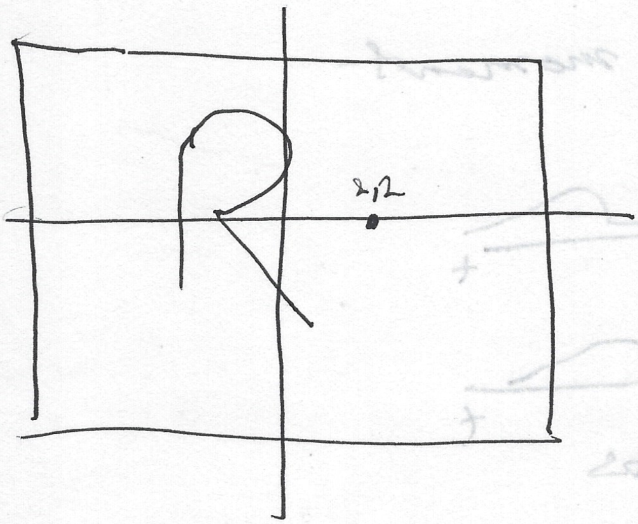
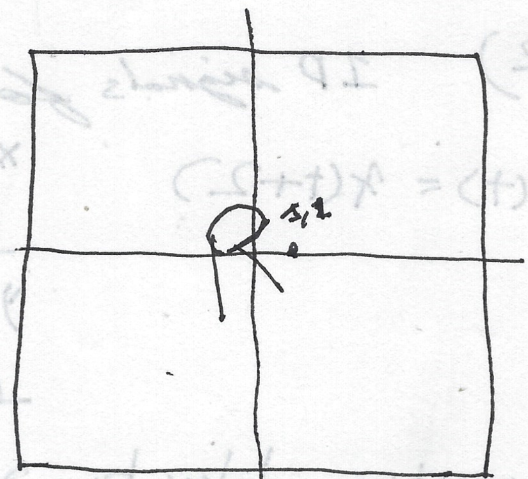
# Scaling

Ex 4) $J(x,y) = I(2x,2y)$




$I(x,y)$

$J(0,0) = I(0,0)$

$J(1,1) = I(2,2)$

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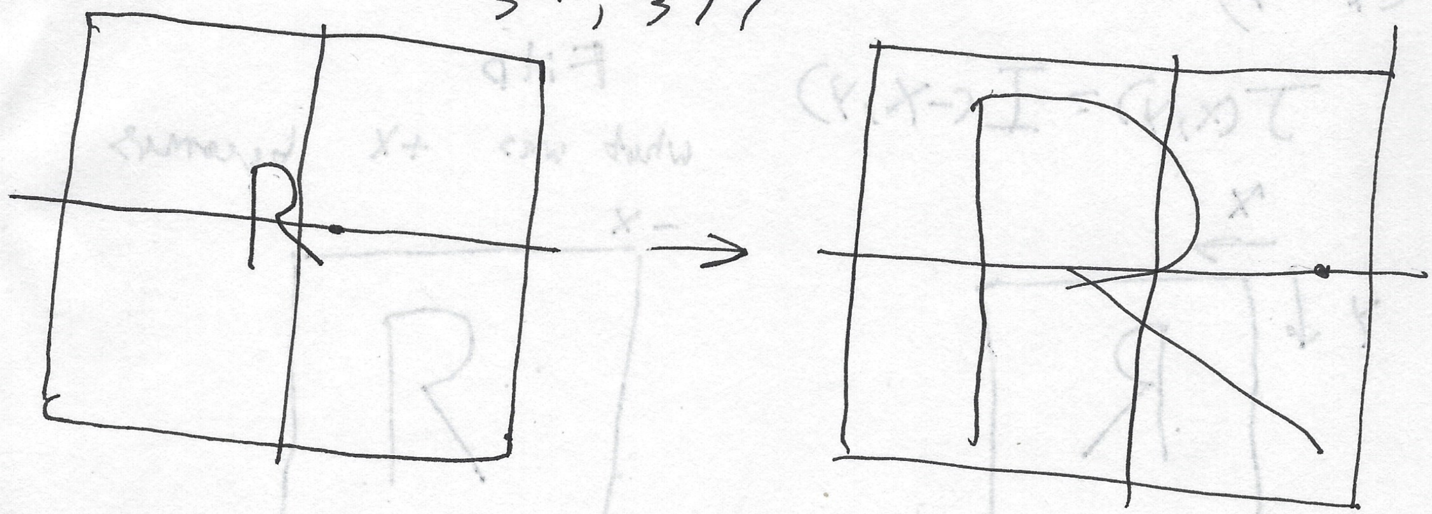
Ex 5)

$J(x,y) = I(\frac{1}{3}x, \frac{1}{3}y)$

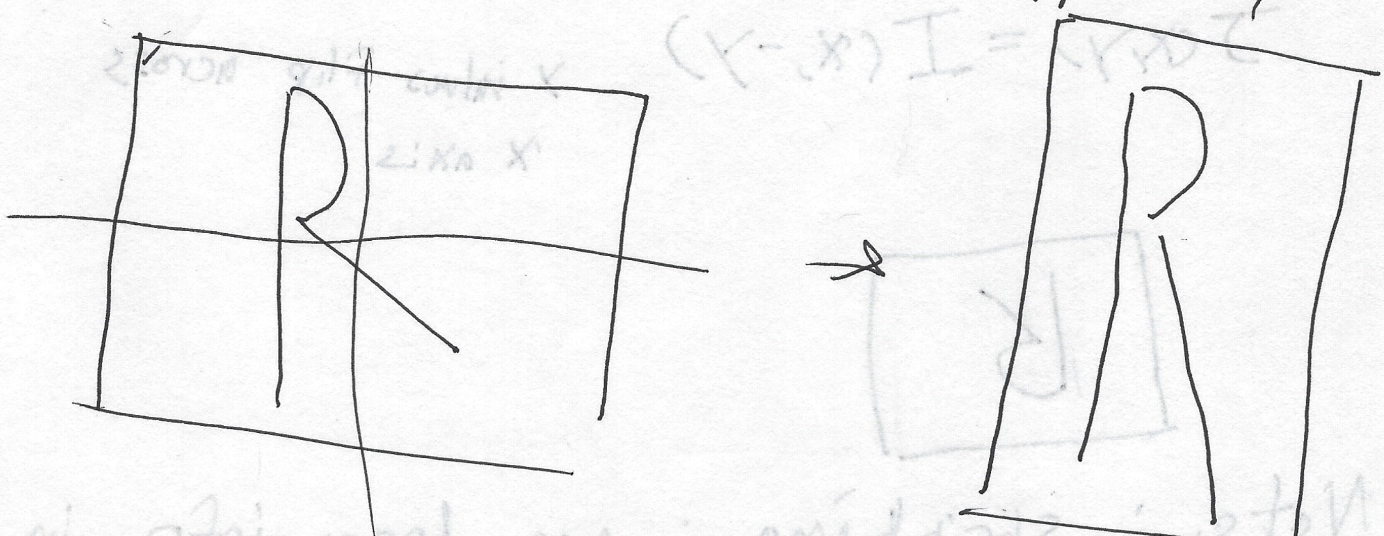


$J(3,3) = I(1,1)$

Ex 6)

$J(x,y) = I(2x, \frac{1}{2}y)$

smaller in x
bigger in y



taller in y direction,
smaller in x direction

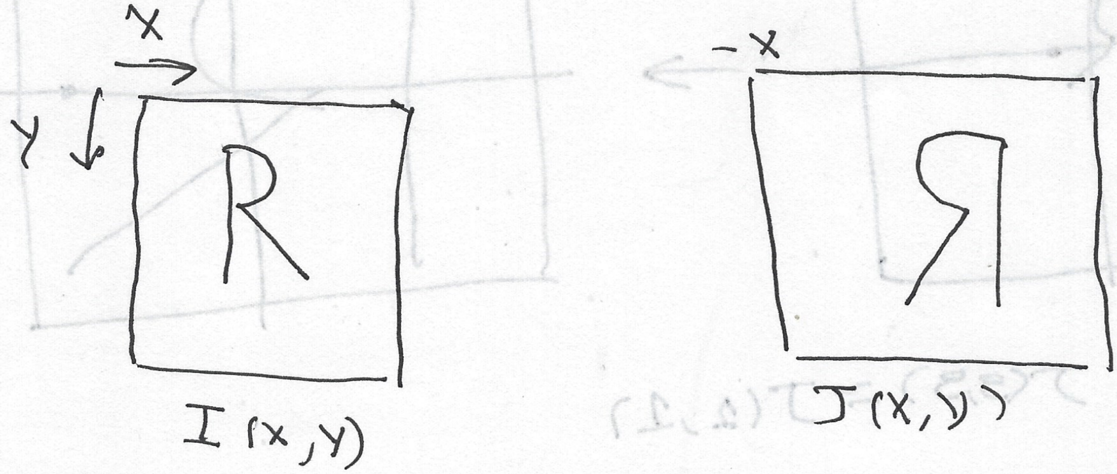
⑥ Mirror Image (flip Image)

Ex 7)

$J(x,y) = I(-x,y)$

Filp

what was +X becomes

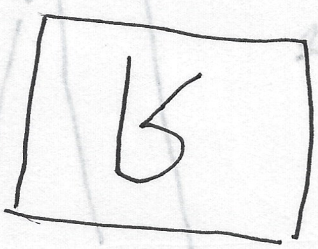



Reflect Image across y axis

Ex 8)

$J(x,y) = I(x,-y)$

y values flip across x axis



Note: Shrinking, we lose info in the Image

Expand, we need to do funky stuff

IT's common for scale + shift + flip to be combined into a 2D Linear transformation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

coordinates of transformed Image $J(x,y)$

coordinates of original Image $I(x,y)$

* Where Does $(x,y)$ in the old Image Go To? (Forward Mapping)

* The expression, $J(x,y) = I(x, y-10)$ Is like saying where did $(x,y)$ come from? (backwards Mapping)

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We may write Translation as

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{vmatrix} x \\ y \end{vmatrix} + \begin{vmatrix} t_1 \\ t_2 \end{vmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

flip

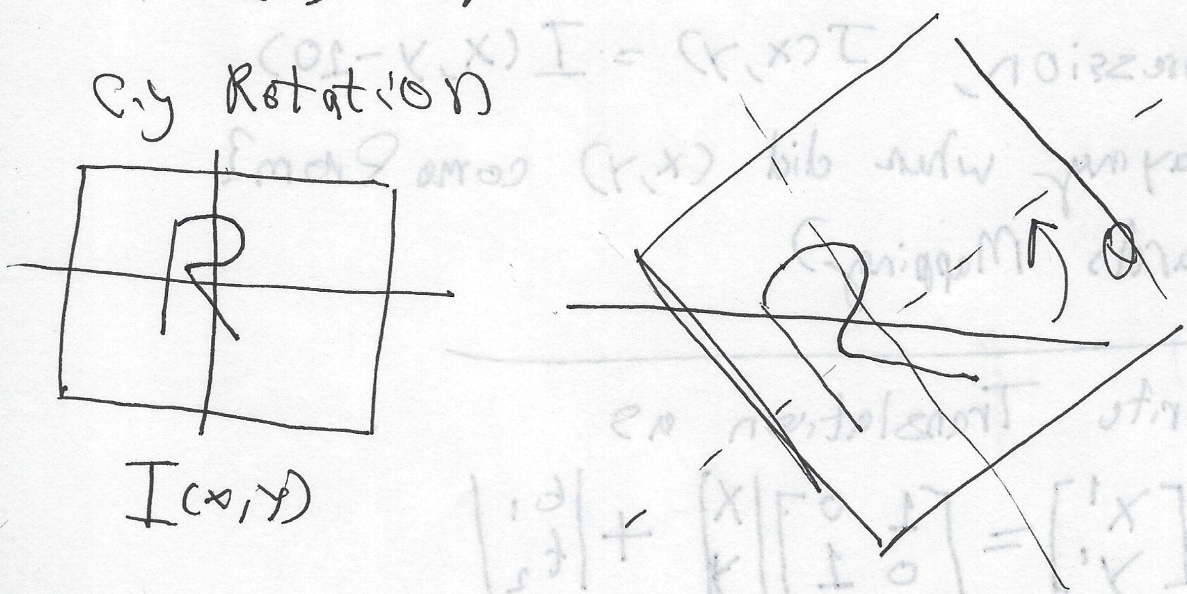
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & +1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(Special case of Scaling where $\alpha, B$ could be negative)

We can do rotation with 2D signals

→ We can do things to Images that we can't Do to 1-D Signals

e.g. Rotation





$I(x,y)$ $J(x,y)$

Rotation by $\theta°$ counter clockwise

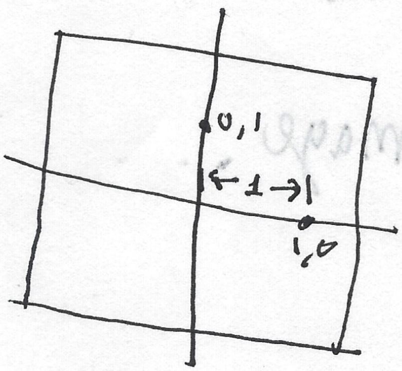
Rotation matrix?

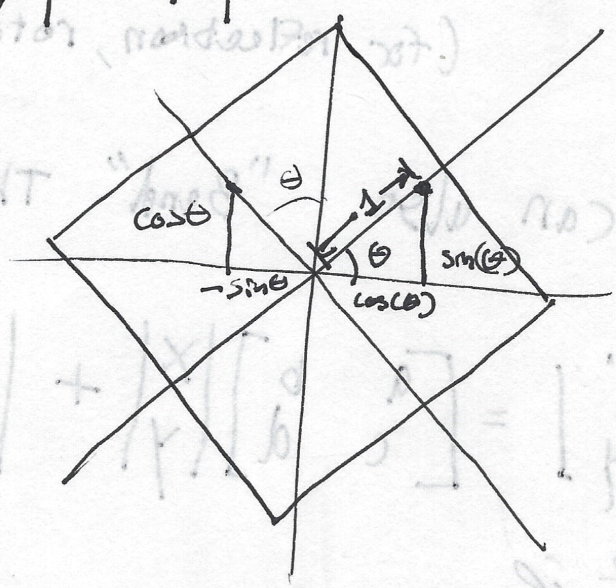
Rotate around the origin

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{vmatrix} x \\ y \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix}, \quad \begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} a \\ c \end{vmatrix}$$

$$\begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} x \\ y \end{vmatrix}, \quad \begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{vmatrix} b \\ d \end{vmatrix}$$




So we see that

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

So we have

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

<u>Note</u> flip signs on $\sin\theta$'s to rotate the other way (clockwise)