Lect 16 DSP

(1)

filter design

Read 5.4.1 – 5.4.2 ~~10.1 – 10.2~~

$x[n] \rightarrow \boxed{H} \rightarrow y[n]$

Design H

$$y[n] = x[n] * h[n]$$

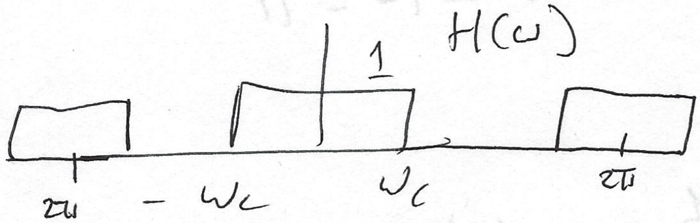
$$Y(\omega) = X(\omega)H(\omega)$$

$$|Y(\omega)| = |X(\omega)||H(\omega)|$$

$$\angle Y(\omega) = \angle X(\omega) + \angle H(\omega)$$

Ideal Low pass filter

$H(\omega)$



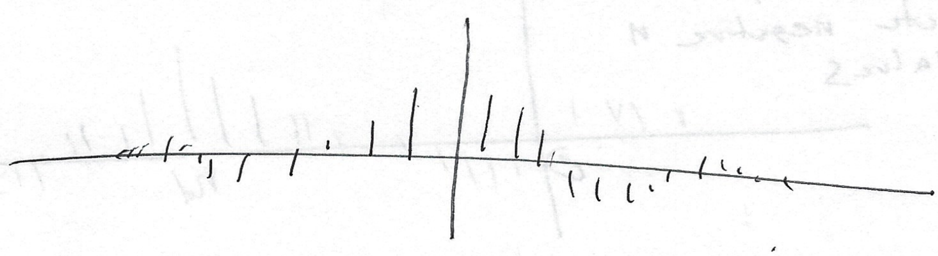
Lowpass filter

$$H_{lp}(\omega) = \begin{cases} 1 & \omega < |\omega_c| \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

No phase shift

There fore $\quad h_{lp}[n] = \dfrac{\sin \omega_c n}{\pi n} \quad -\infty \le n \le \infty$

$-\infty$ - Length



Not causal

(2) Ideal Relay $\qquad h_d[n] = \delta[n-n_d]$

Output = Input shifted in time

→ do not consider distortion.

16

$$H_d(\omega) = e^{-j\omega n_d}$$

$$|H_d(\omega)| = 1 \qquad \angle H_d(\omega) = -\omega n_d$$

↓ phase shift is a linear function of ω

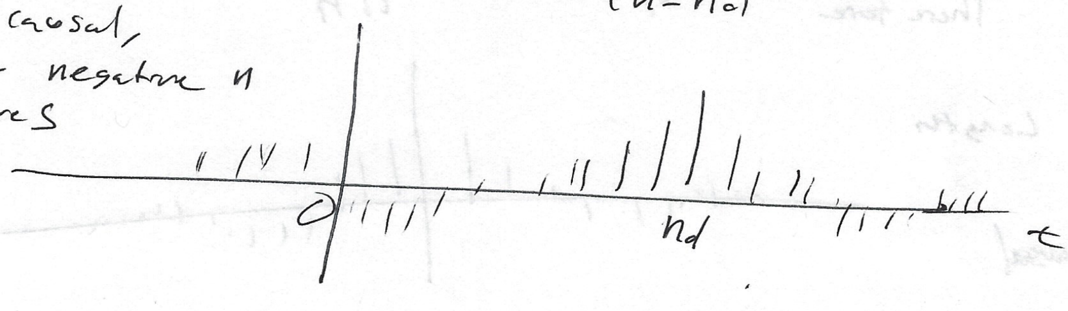
We desire filters with linear phase.

$$H_{lp}(\omega) = \begin{cases} e^{-j\omega n_d} & |\omega| < \omega_c \\ 0 & \omega_c \le |\omega| \le \pi \end{cases}$$

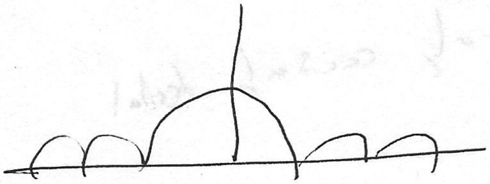
$$|H_{lp}(\omega)| = \begin{cases} 1 \\ 0 \end{cases} \qquad \angle H_{lp}(\omega) = -\omega n_d$$

$$h_{lp}[n] = \frac{\sin \omega_c (n-n_d)}{\pi (n-n_d)}$$

To make causal, delete negative n values

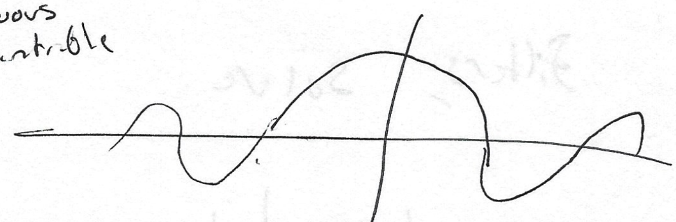


$|H(\omega)|$

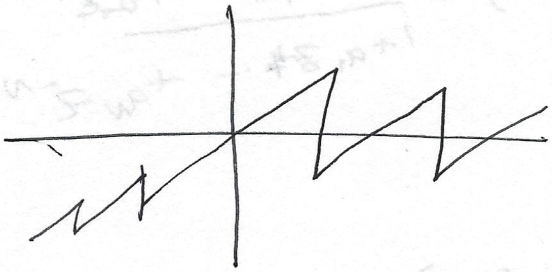


$A(\omega)$ amplitude

continuous differentiable

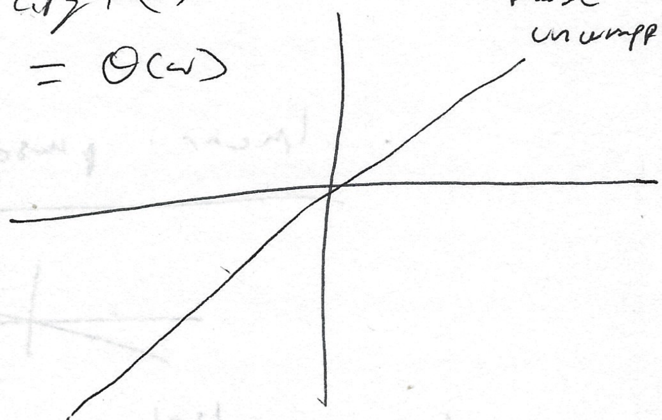


$\angle H(\omega)$



$\arg H(\omega)$
$= \theta(\omega)$

"phase unwrapping"



!?

$H = |H(\omega)| e^{-j\omega}$ $\qquad$ $H = A(\omega)\, e^{j\theta(\omega)}$

We want real, causal, digital filters of the form

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-1} + \cdots + b_m z^{-m}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}$$

finite ~~response~~ impulse response (FIR) filter $\quad a \in \mathbb{R}$, $b \in \mathbb{R}$

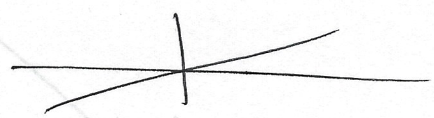
if $\{a_i\} = 0$

Infinite impulse Response IIR filter o.w.

(4) In general, to design real, causal digital filters, solve

$$\min ||E(z)|| = \left|\left| H_{des}(z) - \frac{b_0 + b_1 z^{-1} + \dots + b_n z^{-n}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}} \right|\right|$$

Linear phase filters



$\Theta(\omega)$

$$H(\omega) = \sum_{n=0}^{N-1} h[n]\, e^{-j\omega n}$$

Say N is odd

$$M = \frac{N-1}{2}$$

$$= e^{-j\omega M} \sum_{n=0}^{N-1} h[n]\, e^{-j\omega n}\, e^{j\omega M}$$

$$= e^{-j\omega M} \sum_{n=0}^{N-1} h[n]\, e^{j\omega (M-n)}$$

$$= e^{-j\omega M} \Big(h[0]\, e^{j\omega M} + h[1]\, e^{j\omega (M-1)} + \dots + h[N-2]\, e^{j\omega (M-N+2)} + \underbrace{h[N-1]\, e^{j\omega (M-N+1)}}_{e^{-j\omega M}} \Big)$$

$$= e^{-j\omega M}\Big((h[0] + h[N-1]) \cos(\omega M) + j\,(h[0] - h[N-1]) \sin(\omega M)$$

$$+ (h[1] + h[N-2]) \cos(\omega(M-1)) + j\,(h[1] - h[N-2]) * \sin(\omega(M-1))$$

$$j\,(h[n] - h[\quad])$$

If $h[n] = h[N-n-1]$, all the j-sin terms drop away

$$H(\omega) = e^{-j\omega M}\Big[(h[0] + h[N-1]) \cos(\omega M) + (h[1] + h[N-2]) \cos(\omega(M-1)) + \cdots \Big]$$

desired form

$$H(\omega) = \underbrace{A(\omega)}_{\text{real Amplitude}} e^{\overbrace{j(k_1 + k_2\omega)}^{\text{linear phase}}}$$

$$A(\omega) = \sum_{n=0}^{M-1} 2\,h[n] \cos(\omega(M-n)) + h[M]$$

if N is odd

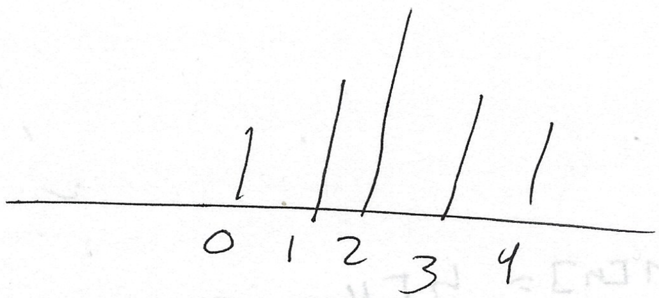
$$\underline{A(\omega)\, e^{j\omega M}}$$

(6)

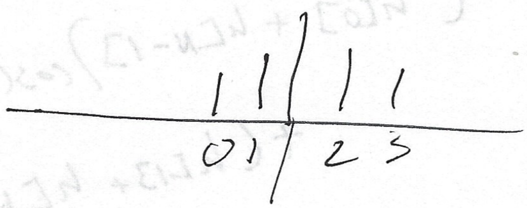
$h[n] = h[N-n-1]$

$h[n]$ is symmetric around its middle point

$N = 5 \quad M = \frac{5-1}{2} = 2$



Similarly if N is even



$h[n] = h[N-n]$

Another way

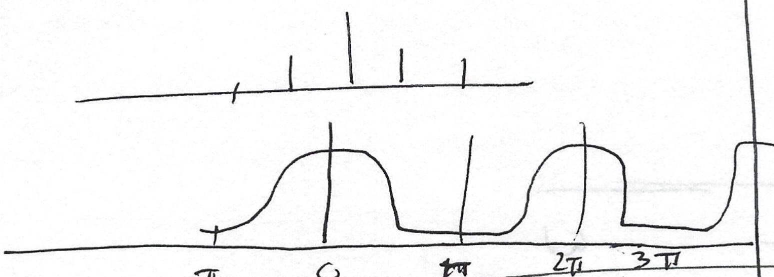
$h[n] = -h[N-1-n]$ N odd

$h[n] = -h[N-n]$ N even

$H(\omega) = A(\omega) e^{j(\omega M + \frac{\pi}{2})}$

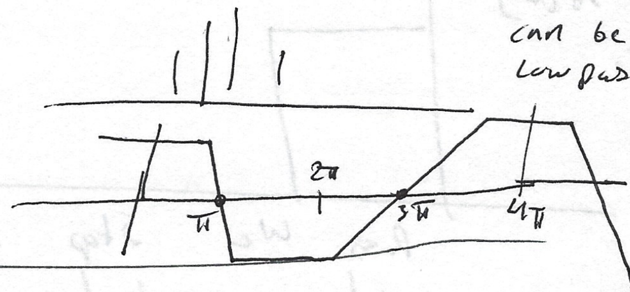
Linear phase

Type I N odd $h[n] = h[N-n-1]$



Type II N even

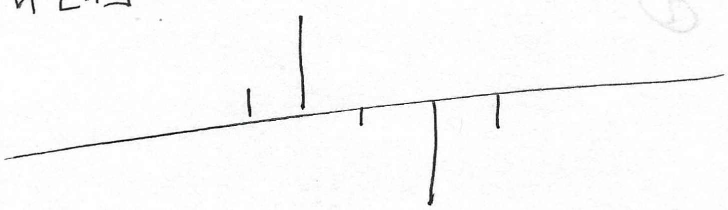
$h[n] = h[N-n-1]$



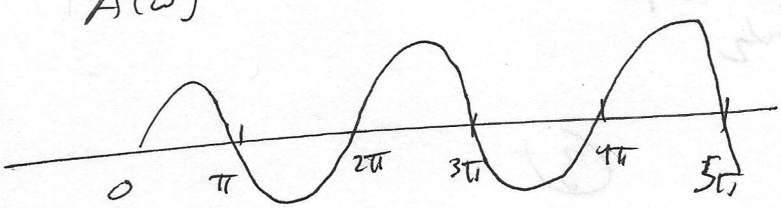
can be low pass

Type III N odd $h[n] = -h[N-n-1]$

h[n]



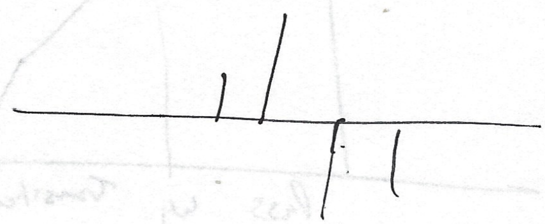
A(ω)



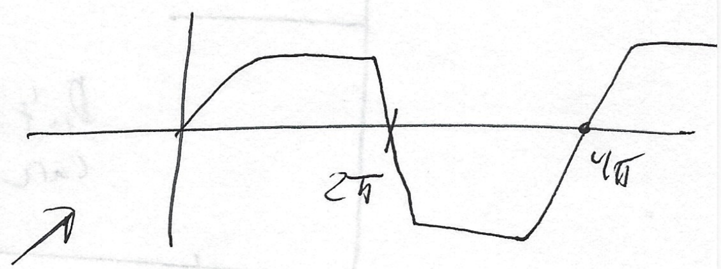
Type IV

N even $h[n] = -h[N-n-1]$

h[n]

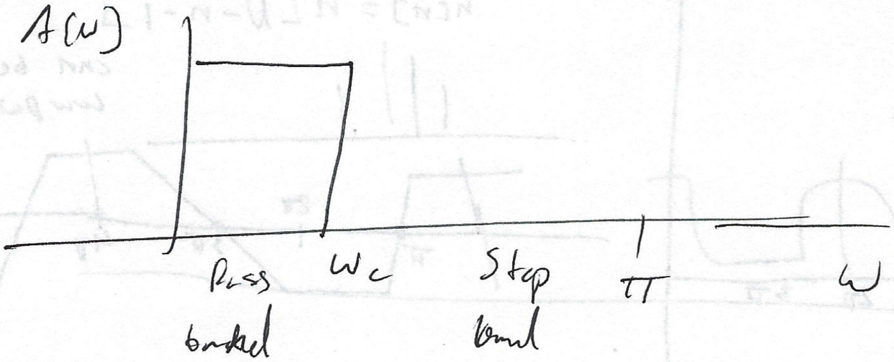


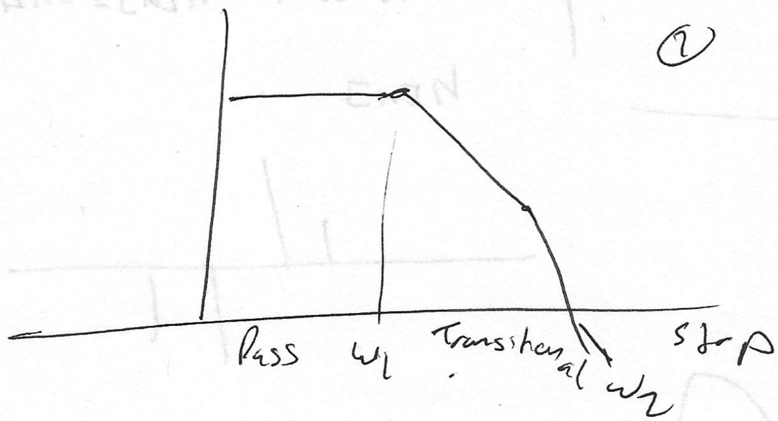
A(ω)

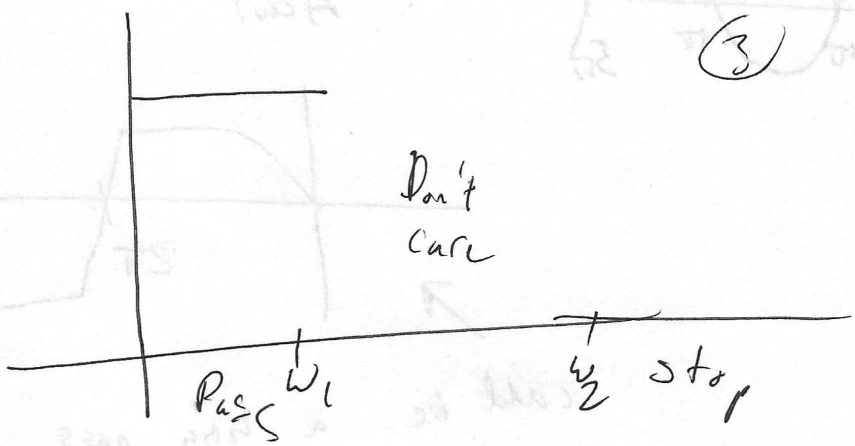


could be a high pass

8 We have a desired amplitude response

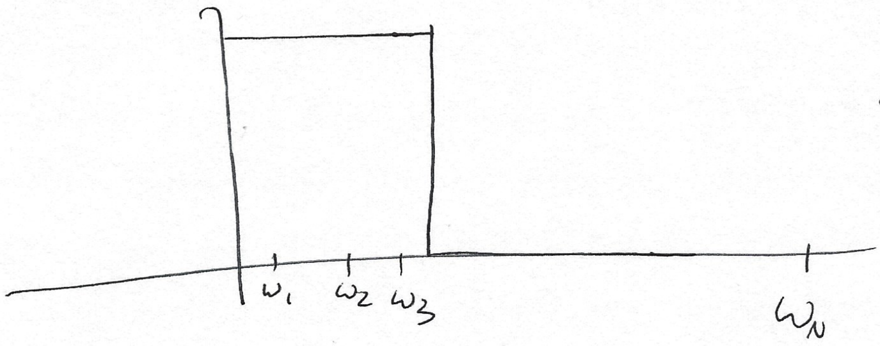






Approximation Criteria

$A_d(\omega)$



Sample N frequencies

choose Type I filter

$$A(\omega) = \sum_{n=0}^{\frac{N-1}{2}} 2h[n] \cdot \cos(\omega(M-n)) + h[M]$$

N linear equations

$$A(\omega_1) = \sum_{n=0}^{\frac{N-1}{2}} 2h[n]\cos(\omega_1(M-n)) + h[M]$$

$$A(\omega_2) = \text{\_\_\_\_\_\_\_\_} \omega_2 \text{\_\_\_\_\_\_\_\_}$$

$$A(\omega_N) = \sum_{n=0}^{\frac{N-1}{2}} 2h[n]\cos(\omega_1(M-n)) + h[M]$$

Solve for $h[n]$

Project proposal + HW 6