DSP Lecture 2

Linear, time-invariant systems

Systems process signals to create other signals.

$p(t) \rightarrow$ CAR $\rightarrow v(t)$ Velocity

pressure on gas

$o[n]$ Price of Barrel of oil on day n $\rightarrow$ Economy $\rightarrow g[n]$ Price of a gallon of gas Day n

$x[n] \rightarrow \boxed{H} \rightarrow y[n]$

$x[n] \rightarrow y[n] \quad y[n] = H(x[n])$

Same Idea

The entries of $y[n]$ as function of the entries of $x[n]$.

Ex $y[n] = \frac{1}{5} \sum_{k=-2}^{2} x[n-k]$

$= \frac{1}{5} [x[n+2] + x[n-1] + x[n] + x[n-1] + x[n-2]]$

$\rightarrow$ moving avg filter.

② matLab

```
>>x= rand(1,300);
>> close all
>> plot(x)  % nosy signal
>> h=[1 1 1 1 1]/5
>> y= filter (h,1,x);
>> figure
>> plot(y)
```

← nominally ~~smoother~~ signal compared ~~to the~~ original

(Taking away the spikyness)

```
>> g= filter(h,1,y);
```

~~>> figure~~

```
>> plot(g)
```

← now things are getting smoother

>> y = filter → Take my buddies around me and average them out.

Systems described by Differential equations

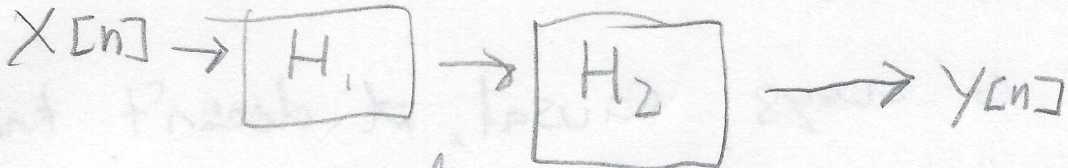
$$y'(t) + ay(t) = bx(t)$$

difference equation in discrete

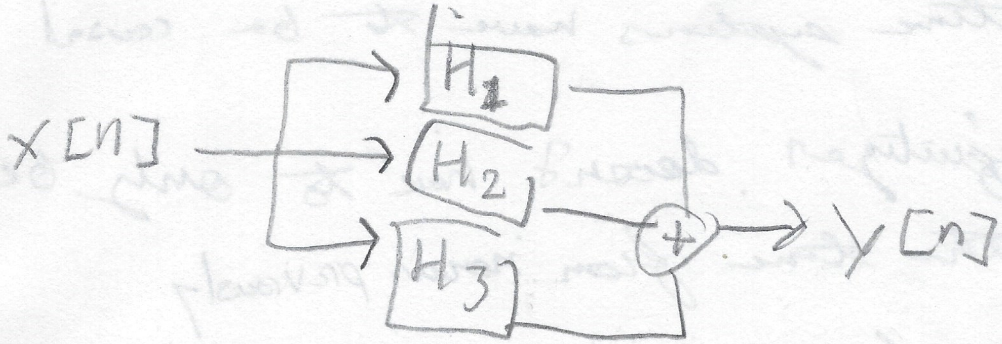
$$y[n] + ay[n-1] = bx[n]$$

(arise from elect. and mechanical problems)

Connected systems

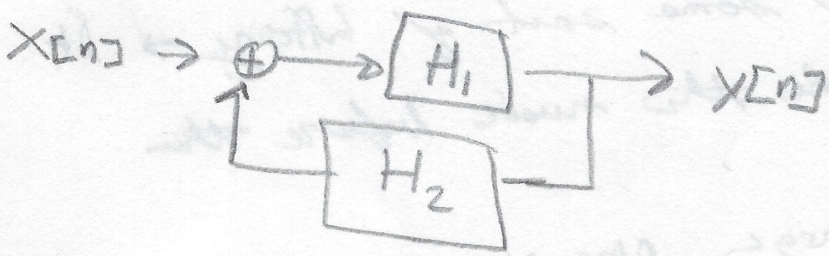


Serial, cascade



(parallel

Feed back system



Cruise control on car
→ input: gas pedal
→ output: feed it back into input

10:24

System Properties

1) causality , A system is causal if the output at time n only depends on the input to time n.

$$Y[n] = x[n] - 2x[n-1] \leftarrow \text{causal}$$

$$y[n] = X[n+3] \leftarrow \text{Not causal}$$ (something exists before $n=0$)

does some future time appear in the equation for the system?

4

Is every real world system causal?

→ Car is always causal, it doesn't know when you are going to steer or whatever.

→ real-time systems have to be causal

→ ~~Graph~~ equitizer doesn't have to only be processing the time from now. previously

→ read the signal into memory and process it into the future → there is some sort of buffering → It makes sure to provide the music before the listener hears it.

Not causal: Image processing

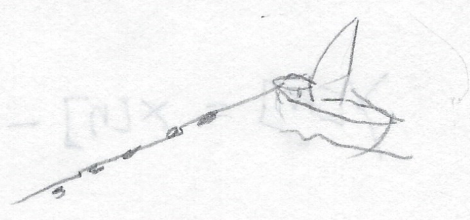
→ 1 dimensional signal

units of signal

time now (only looking at this point and previous times

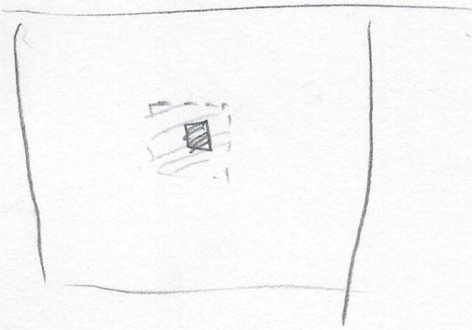
→ this is causal

→ Say that those signal samples came from a boat stringing along a bunch of sensors



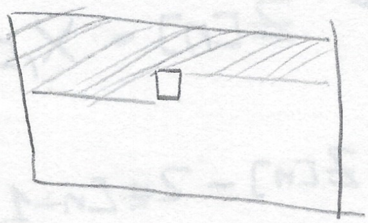
there a natural order of the sensors (sequence), but at l the data is retrieved at the same time (can be processed at once)

Think about an Image



Take the pixel of interest then average it with its Nieghbours

→ if causal, then we could only focus on the Pixels (neighbours) before ~~getting to the~~ pixels of interest.



nothing from the future can be seen, but n can be if construed in such a way

2) next property, Linearity

sum of two signals

$x_1 \to y_1$, $x_2 \to y_2$

$x_1[n] + x_2[n] \to \boxed{H} \to y_1[n] + y_2[n]$

→ System is Linear → additivity

$a\,x_1[n] \to \boxed{H} \to a\,y_1[n]$

Homogeneity

system acting proportionally on input

So then

$a x_1[n] + b x_2[n] \to \boxed{H} \to a y_1[n] + b y_2[n]$

must be true

(6) not all systems are Linear

→ find a Linear system that's close to the system we have.

Ex $y[n] = x[n] - 2x[n-1]$

$x_1[n] \longrightarrow x_1[n] - 2x_1[n-1] = y_1[n]$

$x_2[n] \longrightarrow x_2[n] - 2x_2[n-1] = y_2[n]$

what ~~is the~~ responce ~~of~~ $z[n] = x_1[n] + x_2[n]$?

$z[n] \rightarrow \boxed{H} \rightarrow z[n] - 2z[n-1]$

$= \left(x_1[n] + x_2[n]\right) - 2\left(x_1[n-1] + x_2[n-1]\right)$

$= \left(x_1[n] - 2x_1[n-1]\right) + \left(x_2[n] - 2x_2[n-1]\right)$

$= y_1[n] + y_2[n]$ Verified additivity

$z[n] = ax_1[n]$ $z[n] \rightarrow \boxed{H} \rightarrow az[n]$

$z[n] - 2z[n-1]$

$\hookrightarrow ax_1[n] - 2ax_1[n-1] = a(\;) = ay_1$ $a[x_1[n] - 2x_1[n-1]])$

Homogenaity confirmed.

All at once

$$z[n] = a x_1[n] + b x_2[n]$$

$$z[n] \rightarrow \boxed{H} \rightarrow z[n] - 2z[n-1]$$

$$= a x_1[n] + b x_2[n] - 2(a x_1[n-1] + b x_2[n-1])$$

$$= = a(x_1[n] - 2(x_1[n-1])$$ things only affect the 'n'

$$+ b(x_2[n] + 2x_2[n-1])$$

$$= a y_1[n] + b y_2[n]$$ Linear!

$$y[n] = 3x[n] + 5$$ Linear or not Linear?

$$x_1[n] \rightarrow 3x_1[n] + 5$$

$$x_2[n] \rightarrow 3x_2[n] + 5$$

→ check additivity

$$z[n] = x_1[n] + x_2[n]$$

$$z[n] \rightarrow \boxed{H} \rightarrow 3z[n] + 5 = 3(x_1[n] + x_2[n]) + 5$$

$$\neq (3x_1[n] + 5) + (3x_2[n] + 5)$$

→ not Linear

If something's Linearity is in question, provide a counter example. If one property fails, then ~~the~~ whole thing is proven non-linear.

(6) counter Ex

$x_1[n] = 1$, $x_2[n] = 2$ if linear, 2 is twice 1

$\rightarrow$ $y_2 = 2y_1[n]$

$y_1[n] = 8$, $y_2[n] = 11$ $\leftarrow$ for linearity $\rightarrow$ Not the case

$\rightarrow$ use constants or delta functions

Linear system must be Homogeneous

$a x_1[n] \rightarrow a y_1[n]$ for any constant

as in $0 \rightarrow 0$

Q: must $\underline{a}$ be real? , $a \in \mathbb{C}$? YUP!

Time invariance

System behaves the same way regardless of when input is applied

$x[n] \rightarrow \boxed{} \rightarrow y[n]$

$x[n-n_0] \rightarrow \boxed{} \rightarrow y[n-n_0]$

There are non-time invariant things

$\rightarrow$ Like the speed it takes to download something at 3pm vs 12am

$y[n] = x[n] - 2x[n-1]$ Time invariant?

$z[n] = x[n-n_0]$

$z[n] \rightarrow \boxed{} \rightarrow z[n] - 2z[n-1]$

$= x[n-n_0] - 2x[n-1-n_0]$

$= x[n-n_0]$?

To see if this checks, look at $\underline{y[n]}$

$\rightarrow$ plug in $x[\underline{n-n_0}]$

So ~~then~~ $y[n-n_0] = x[n-n_0] - 2x[n-n_0-1]$

So Time invariance holds

System is linear.

Consider $y[n] = x[n^2]$

$z[n] = x[n-n_0]$ (That only n is affected)

$z[n] \rightarrow \boxed{} \rightarrow z[n^2] = x[n^2-n_0]$

~~Is~~ is it equivalent ~~to~~ $y[n-n_0]$?

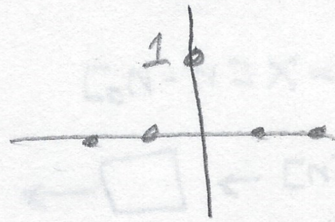
$y[n-n_0] = x[(n-n_0)^2] \neq z[n^2]$

NOT time invariant

For counter examples, try δs

(10) $Y[n] = X[n^2]$

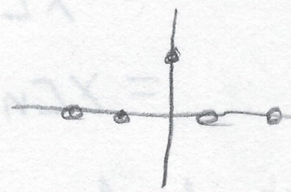
response to $x[n] = \delta[n]$



$Y[0] = X[0]$

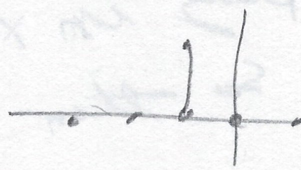
$Y[1] = Y[-1] = X[1]$

$Y[2] = Y[-2] = X[2]$ $\rightarrow$

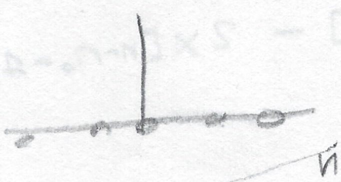


(Think: plotting the response)

Response ~~to~~ $X[n] = \delta[n+1]$



$Y[n]$



(Think: n is always positive due ~~to~~ n^2)

This demonstration shows not time-invariant

Linear, Time-invariant Systems (LTI)

→ Assume systems of Interest are all LTI

Real-World systems are often modeld as LTI

(Good approx)

a) often a good approx

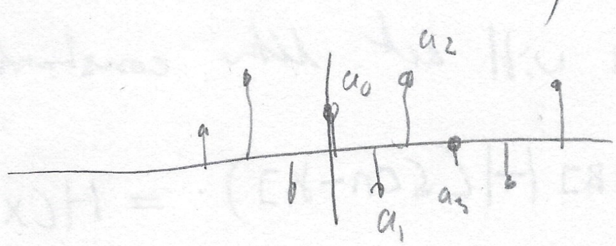
b) Analysis is easy/powerful

Key concept/ Superposition

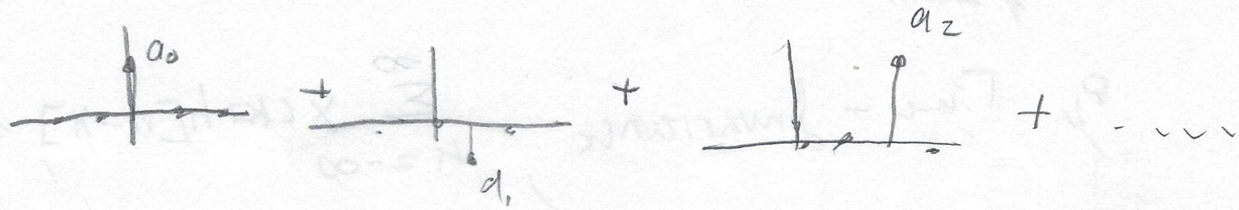
$$a_0 x[n] + a_1 x[n-1] + a_2 x[x-2] + \dots \to \boxed{LTI}$$

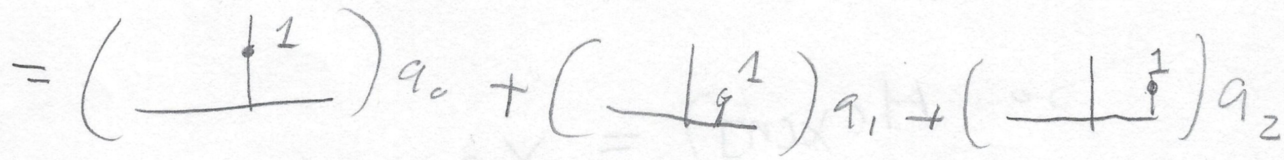
$$\Rightarrow a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] + \dots$$

consider an arbitrary signal



This looks like





$$= a_0(\delta[n]) + a_1(\delta[n-1]) + a_2(\delta[n-2]) + \dots$$

So we can say

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

"sifting" property of δ function

"non-zero at $k=n$"

what happens when we put this signal through the system

LTI ~~Sys~~ System

Impulse response

$$\delta[n] \rightarrow \boxed{H} \rightarrow h[n]$$

What is the response to $x[n]$?

$$H(x[n]) = H\left(\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]\right)$$

$x[k]$ will act like constants

By linearity, $\sum_{k=-\infty}^{\infty} x[k]\, H(\delta[n-k]) = H(x[n])$

(Take constants out of sum)

By Time-Invariance, $\sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = H(x[n])$

So $H(x[n]) = x[n] * h[n] = y[n]$

Input, Impulse response, output

Impulse response fully characterizes the system