DSP Lect 3

Impulse response

Impulse response

$$\delta[n] \to \boxed{H} \to h[n]$$

LTI

$$x[n] \to \boxed{H} \to y[n]$$

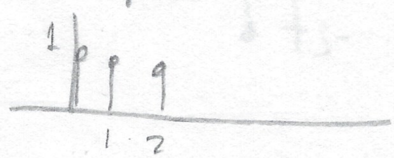
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

Consider this LTI System

$$y[n] = x[n] - 2x[n-1] + 3x[n-2]$$

what is the response to

$x[n] =$



?

1) ~~Direct~~

$y[-1] = 0$

$y[0] = 1$

$y[2] = 2$

$y[1] = 1 - 2(1) = -1$

$y[3] = -2 + 3 = 1$

$y[4] = 3$

Instead, write $x[n] = [\underline{\underline{1}}\ 1\ 1]$

(double underline means $n = 0$)

→ what is at $n = 0$? $\underline{1}$!

So, the output for $y[n] = [\underline{\underline{1}}\ -1\ 2\ 1\ 3]$

② Using convolution Sum

$$Y[n] = \sum_{k=-\infty}^{\infty} X[k]\, h[n-k]$$

What is the Impulse response?

$\delta[n] = [\underline{\underline{1}}]$

$h[0] = 1$
$h[1] = 0-2 = -2$
$h[2] = 0-0+3 = 3$
$h[3] = 0$

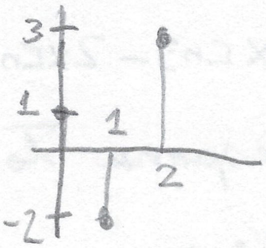
So for the impulse response;

recall

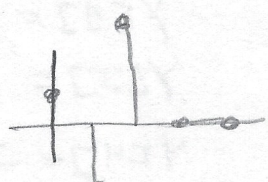
$\delta[n] \rightarrow \boxed{H} \rightarrow h[n]$

impulse response

$\rightarrow$ $[\underline{\underline{1}}\ -2\ 3]$



$y[n] = x[0]h[n] + x[1]h[n-1] + x[2]h[n-2]$



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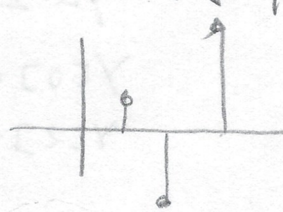

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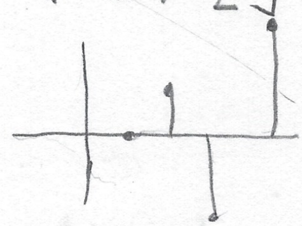

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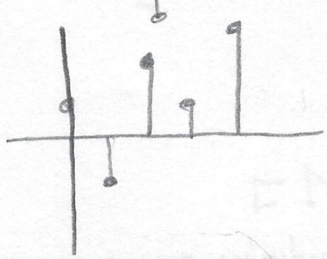


$$\begin{array}{r} [\underline{\underline{1}}\ -2\ 3] \\ +\ [\underline{\underline{0}}\ 1\ -2\ 3] \\ +\ [\underline{\underline{0}}\ 0\ 1\ -2\ 3] \\ \hline [\underline{\underline{1}}\ -1\ 2\ 1\ 3] \end{array}$$

Easier than writing the sticks

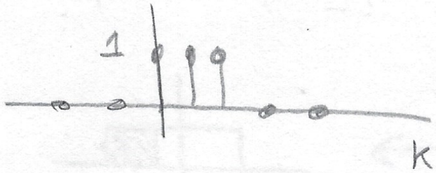
3) Flip and SLIDE one signal 12:49 (3)

$\sum_{k=-\infty}^{\infty} x[k]h[n-k]$

Well, what is $h[n-k]$?

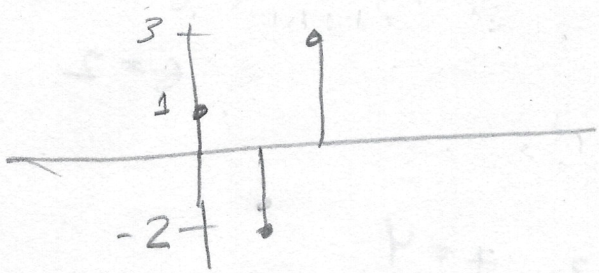
K is like my dependent variable, n is const.

X[k]

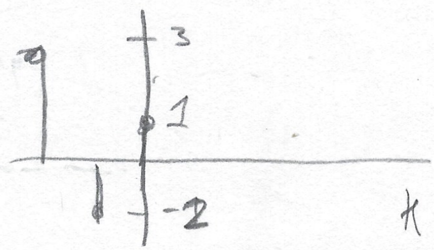


and Σ

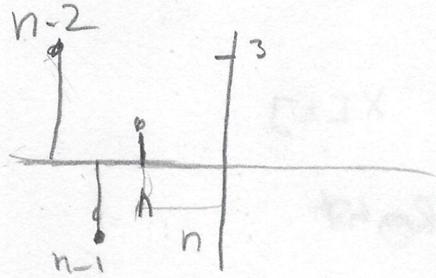
h[k]



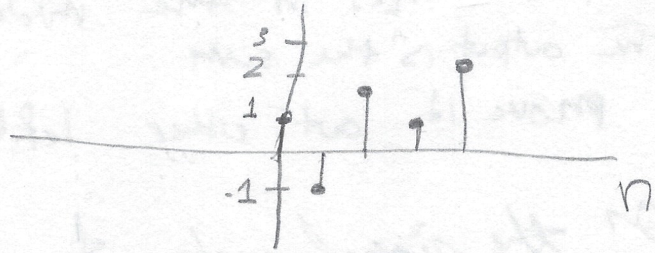
h[-k]



So h[n-k]

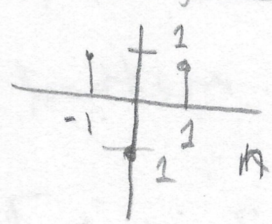


So the plotted Σ

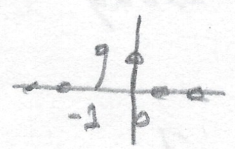


flip it and jam it through X[k]

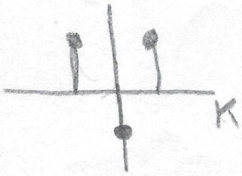
X[n] =



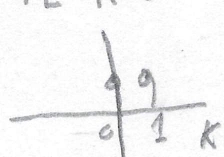
h[n]



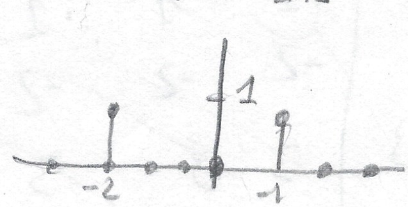
X[k]



h[-k]



x[n] * h[n]

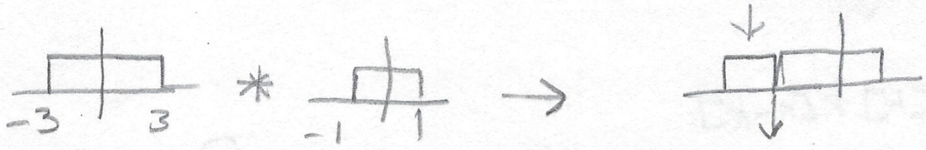


n = -1 at h[n] 1st one!

so $n+1=-1$

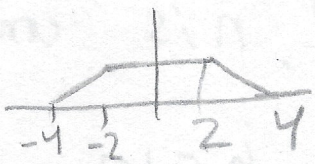
(4) recall the picture method

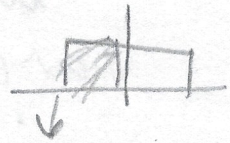
flipped



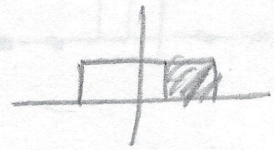
$t+1$ $\quad$ $t-1$ $\qquad$ $t+1=-3$

$t=-4$





$\rightarrow$

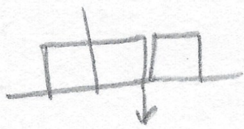


$t-1=-3$ $\quad$ $t=-2$

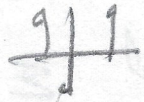
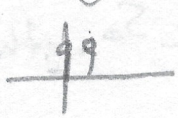
$t-1=$

$t+1=3$

$t=2$



$t-1=3$ $\quad$ $t=4$

for  and 

put h[k] in the middle of x[k]

→ The output is the sum
move it out either left and right

or, the signal meets at -2

4) convolution array (Wozny) method

h \ x	1	1	1
1	1	1	1
-2	-2	-2	-2
3	3	3	3

$\rightarrow [\,\underline{1}\ \ -1\ \ 2\ \ 1\ \ 3\,]$

zero element of output

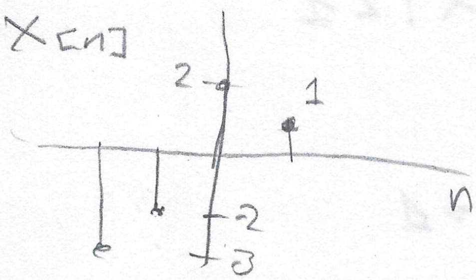
$h \backslash x$	1	$\underline{-1}$	1
1	1	-1	1
$\underline{1}$	1	$\underline{-1}$	1

$\rightarrow [1\ 0\ \underline{0}\ 1]$

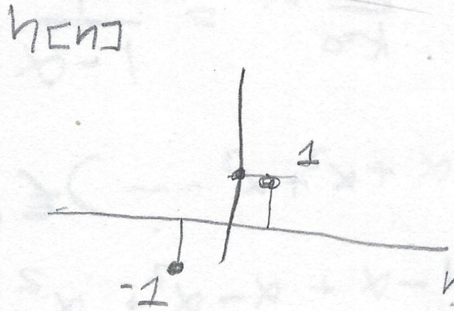
correct

Ex 2

$x[n]$



$h[n]$



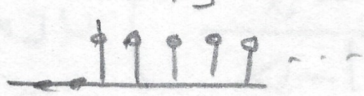
$h \backslash x$	-3	-2	$\underline{2}$	1
-1	+3	2	-2	-1
$\underline{1}$	-3	-2	2	1
1	-3	-2	2	1

$= [+3\ -1\ -7\ \underline{-1}\ 3\ 1]$

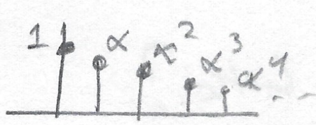
But we may deal with infinitely long inputs

Ex $\quad x[n] = \alpha^n u[n] \qquad \alpha \in (0,1)$

$h[n] = u[n]$

$h[n] =$ 

$\rightarrow$ doesn't matter which signal we flip

$x[n] =$ 

$\rightarrow$ choose $h[n]$

flip $h[n]$ then ram it into $x[n]$

So we get $y[0] = 1$, $y[1] = 1+\alpha$, $y[2] = 1+\alpha+\alpha^2$

(6) So $y[n] = \begin{cases} 0 & n<0 \\ \sum_{k=0}^{n} \alpha^k & n \geq 0 \end{cases}$

or write $= \left(\sum_{k=0}^{n} \alpha^k \right) u[n]$

→ recall $\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha} \quad |\alpha| < 1$

$(1 + \alpha + \alpha^2 + \alpha^3 \cdots)(1-\alpha) = 1$

$= 1 - \alpha + \alpha - \alpha^2 + \alpha^2 - \alpha^3 + \alpha^3 - \alpha^4 \cdots$

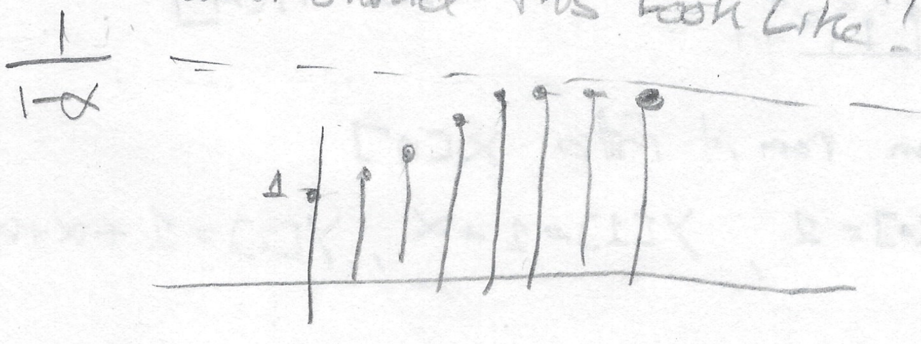
$= 1 + \alpha + \alpha^2 + \alpha^3 + \cdots$
$\quad - \alpha - \alpha^2 - \alpha^3 - \alpha^4 \cdots \quad = 1$

$\sum_{k=0}^{n} \alpha^k = \sum_{k=0}^{\infty} \alpha^k - \sum_{k=n+1}^{\infty} \alpha^k = \sum_{k=0}^{\infty} \alpha^k - \alpha^{n+1} \left(\sum_{k=0}^{\infty} \alpha^k \right)$

$= \frac{1}{1-\alpha} - \frac{\alpha^{n+1}}{1-\alpha} = \frac{1-\alpha^{n+1}}{1-\alpha}$

So finally $y[n] = \left(\frac{1-\alpha^{n+1}}{1-\alpha} \right) u[n]$

what should this look like?



So in certain situations you have to use the convolution sum

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \alpha^k u[k]\, u[n-k]$$

$$= \sum_{k=0}^{\infty} \alpha^k u[n-k]$$

$$= \sum_{k=0}^{n} \alpha^k \quad \text{if } \underline{n<0} \text{ then force it}$$

$\rightarrow$ add $u[n]$

$$\longrightarrow \left(\sum_{k=0}^{n} \alpha^k\right) u[n]$$

Properties of LTI Systems (How convolution behaves)

1) LTI system determined by Impulse response

<u>NOT</u> true for non LTI systems

Like $y[n] = n x[n]$ <u>not time invariant</u>

$\rightarrow$ use $\delta[n]$ <u>test</u> (for impulse response)

$\delta[n] \rightarrow 0$ but not for other signals

If we know the response to some special input, other signals in terms of that special input
$\rightarrow$ that also characterizes the system too

8 2) Communative Property

Special note!
we could put the step function into a system → find the step response → as long as we know how to take the arbitrary input and decompose it in terms of step functions, then we could characterise things in the step response
→ in a similar way with the impulse response

Another note
what does ~~the~~ impulse response mean?
→ want to know the impulse response of a concert Hall? fire a shot in it,
→ Loud shot to hear the reverberations, that's the impulse response
→ This may be difficult (to produce) so an easier way to analyse a systems response may just be to ~~turn it on~~ like a step response.
Step response may be easier to produce
→ then find impulse response from that
→ or do pulse's

2) Commutative

$$x[n] * h[n] = h[n] * x[n]$$

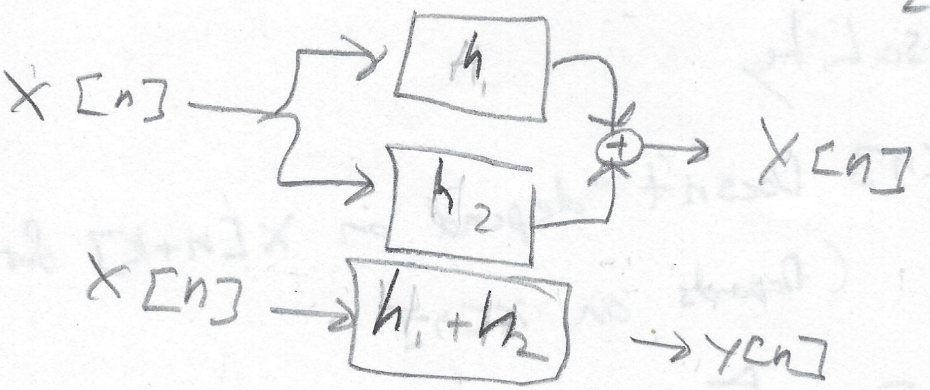
$$\sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \qquad m = n-k \quad k = n-m$$

$$\rightarrow \sum_{m=-\infty}^{\infty} x[n-m]\, h[m]$$

3) Distributive property

$x[n] * (h_1[n] + h_2[n])$

$= x[n] * h_1[n] + x[n] * h_2[n]$



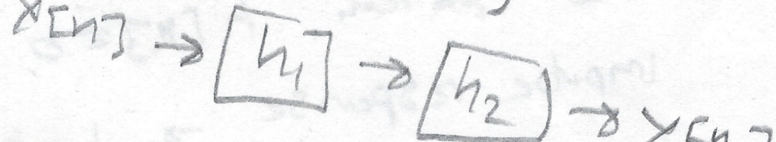
→ Like putting in parallel

4) Associative

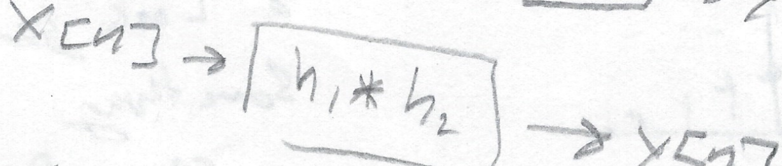
$x[n] * (h_1[n] * h_2[n])$

$(x[n] * h_1[n]) * h_2[n])$

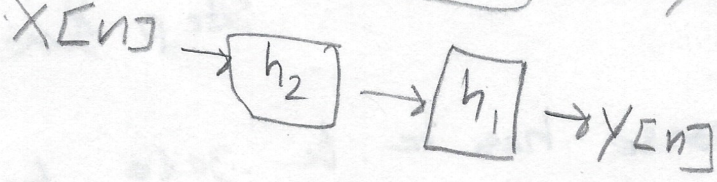
Like



equivalently



order can be interchanged



→ can put systems in a blob and in any order

→ Not true for non-Linear Systems

$x \rightarrow \boxed{2} \rightarrow \boxed{x^2} \rightarrow (2x)^2 = 4x^2$

$x \rightarrow \boxed{x^2} \rightarrow \boxed{2} \rightarrow 2x^2$ see?

5) Causality

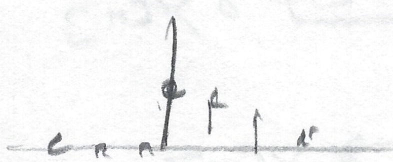
$Y[n]$ Doesn't depend on $X[n+k]$ for $k>0$
(Depends on $X[n]$)

$$Y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$ if k, $\geq -|k|$ then we are looking at a future X

— So for a causal system, $h[k]$ must be zero for future values of X

For causal system $h[k] = 0$ for $k < 0$

Impulse response



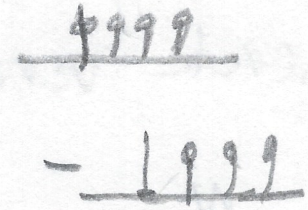
→ Look generally like something times the step function.

→ Impulse response has to be zero before the impulse occurs

shouldn't do anything before zero

b) STEP Response.

$\delta[n] = u[n] - u[n-1]$



$h[n] = s[n] - s[n-1]$

↳ step response

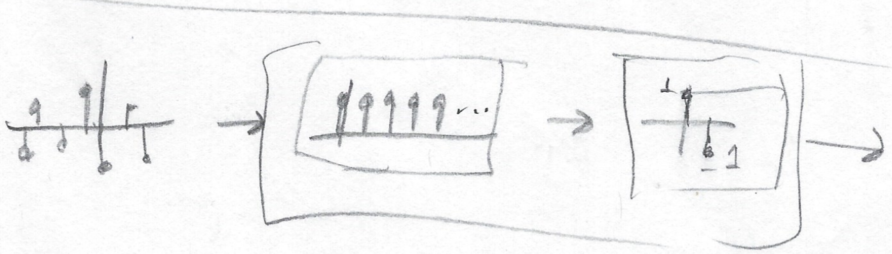
How to get impulse response $H(u[n])$

58:58

Since $u[n] = \sum_{k=-\infty}^{n} \delta[k]$

$s[n] = \sum_{k=-\infty}^{n} h[k]$ *

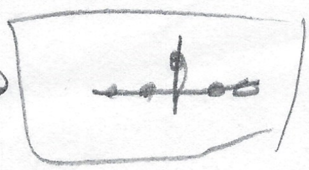
Step response also characterizes the LTI system.



(use associative property)

Don't do a series of convolution,

→ convolute both systems →



The Identity system

(12) many electrical and mechanical systems are described by Dif. eq's

Discrete versions are Difference equations

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = y_h[n] + y_p[n]$$

homogeneous eq — Particular solution

$$y[n] = \sum b_k x[n-k]$$ (finite Impulse response System)