Digital Signal Processing ECSE 4530

1st we have

Signals that come from Sensors

Sensors as in Thermometer, Voltmeters, microphones, and cameras.

2nd we have

Systems: process signals to produce other signals

for example, look at a car, there is a gas pedal, when you put your foot on the pedal, the car drives with velocity that spins the wheels.

GAS Pedal → $\boxed{\text{CAR}}$ → Velocity

The car is the system

Another example is the digital Data on a CD

CD → $\boxed{\text{STereo}}$ → Sound

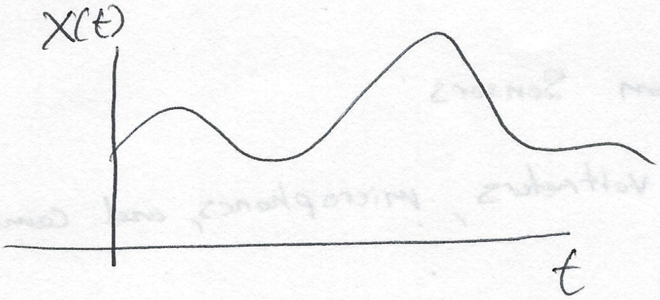
This is a digital signal

This is a continuous signal

In the early days (earlier): So to speak, Power engineering drove this signals and System stuff. The model of sending information back and forth applies to other fields.

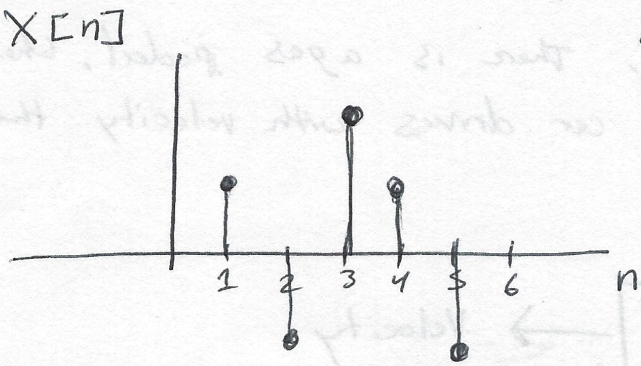
Continuous time Vs discrete time

for <u>continuous example</u>



we can ask whats the value at time $t=0.01489$ and there is a distinct value.

Discrete example



X[n] is only sampled at certain fixed discrete intervals

→ nothing occurs between the Gaps of these stem Lines

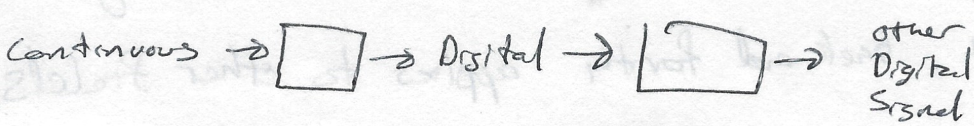
→ no such thing as X[1.5] for a digital signal

The question is; How do I go from a continuous signal to a discrete signal without losing information?

— There is a Sampling theorem for that

→ In certain conditions, you can sample a continuous time signal so that the samples are good enough to reconstruct the original continuous time signal, In a sense, you haven't Lost information.

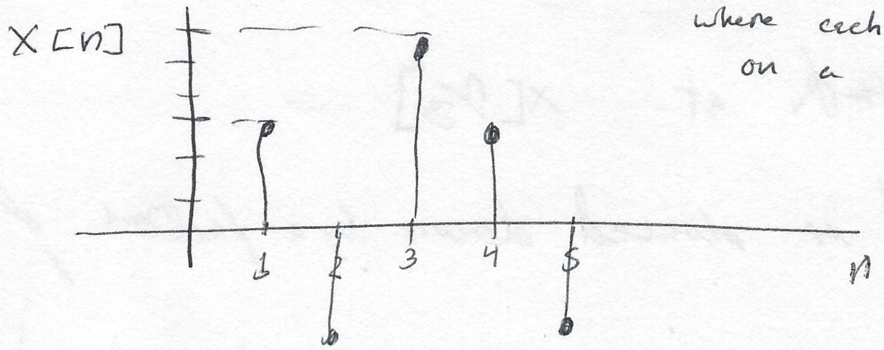
→ The focus is Discrete as computers these days are digital.



One thing to note about Discrete time is that there may not be a continuous resolution on the Y axis.

Discrete time

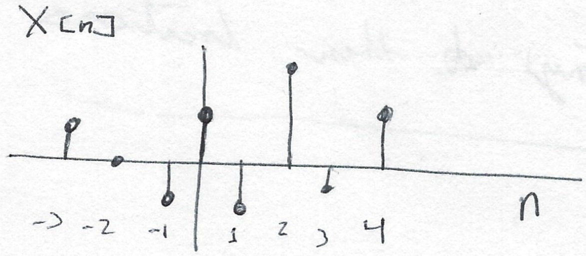
There are discrete Locations where each "stick" must fall on a "y" Location.

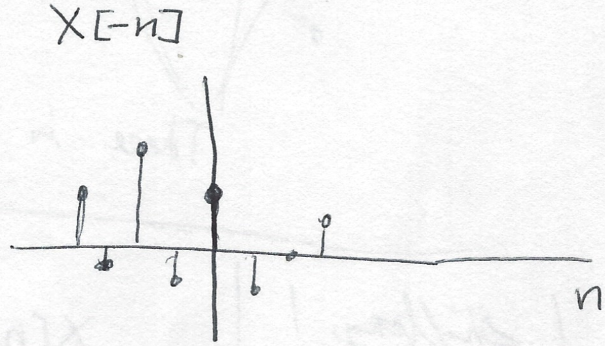


→ known as Quantization

Processing Signals

Flipping



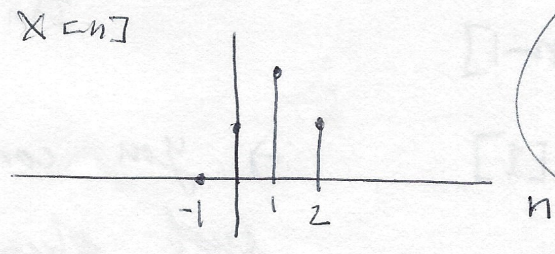


SCALING 7:00

X[2n] Think! Playing the signal twice as fast

→ thats ~~A~~ parrallel → the duration of ~~the~~ signal should decrease by $\underline{2}$

what that does to a digital signal



You Lose information

(4)

8:00

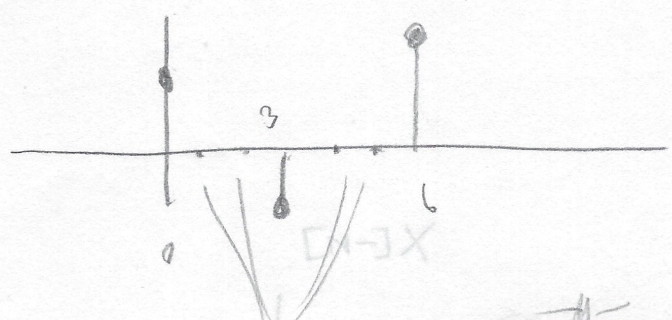
→ for a continuous time signal, information is not lost.

get ready

Next, look at $x[n/3]$

→ Signal is slowed down by a factor of $\underline{3}$

$x[n/3]$



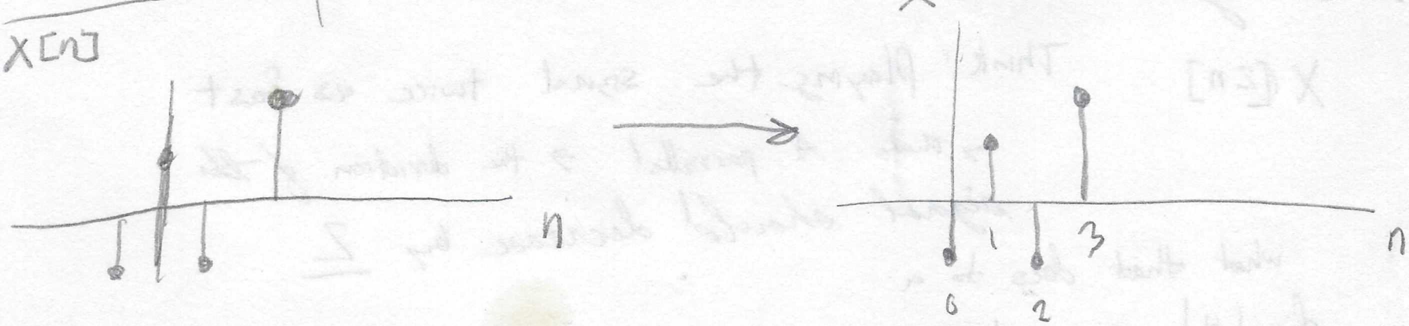
What use to be 1 is now 3

what used to be 2 is now 6

There is nothing at these locations

Shifting! $x[n-n_0]$

$X[n]$ $X[n-1]$

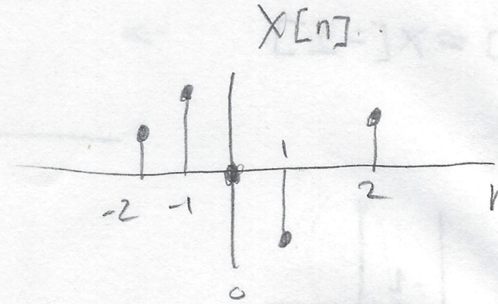


Think delay by 1

$y[n] = x[n-1]$

$y[2] = x[1]$ → you can see this by just pluging in some numbers more complicated

$x[-2n+3]$



order:

shift, flip, scale ← do this

I did shift, scale, flip

$z[n] = x[n+3]$

$w[n] = z[-n]$

$y[n] = w[2n]$

want ~~this~~ result → $y[n] = w[2n] = z[-2n]$

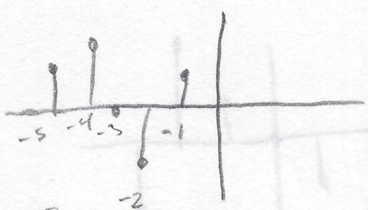
$= x[-2n+3]$??

(my guess)

→ (Seems like it. Think: only affects 'n')

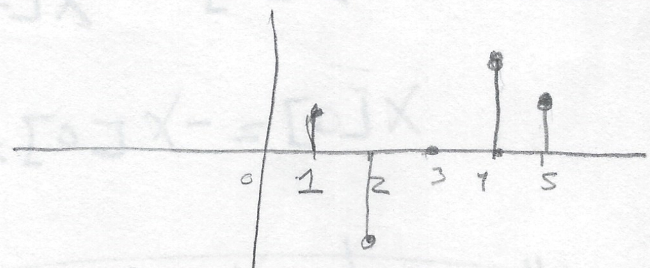
So

$z[n] = x[n+3]$



(Shift)

$w[n] = z[-n] = x[-n+3]$



(flip)

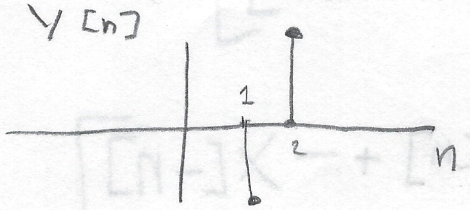
now

$y[n] = w[2n] = z[-2n]$

→ ~~y[n]~~ scales w, w flips z, z shifts x

So $y[n]$ scales, flips, and shifts x

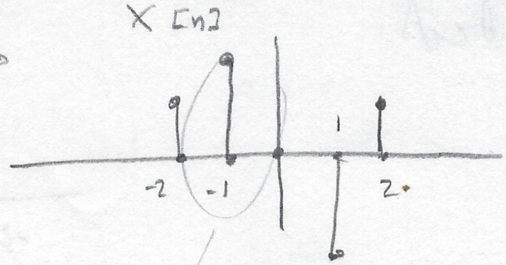
So $y[n]$



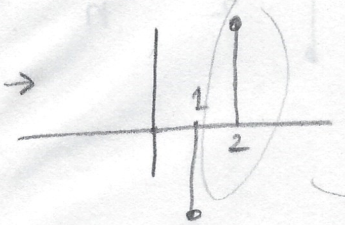
$y[n] = x[-2n+3]$

⑥ check

$Y[2] = X[-1]$ →



→

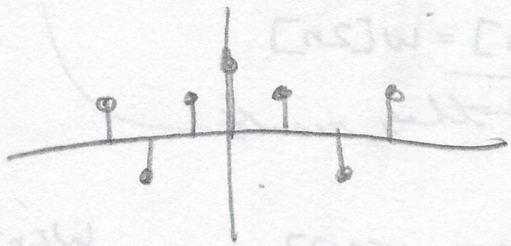


They are the Same so it checks

And $y[1] = X[1]$

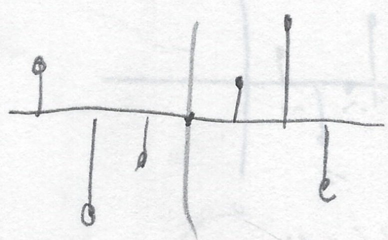
Property of Signals

Even → $X[n] = X[-n]$



ODD $X[n] = -X[-n]$

$X[0] = -X[0] = 0$



all signals have an even part and odd part

$$Ev\{x[n]\} = \tfrac{1}{2}\left[x[n] + x[-n]\right]$$

$$Od\{X[n]\} = \tfrac{1}{2}\left[x[n] + - X[-n]\right]$$

mat Lab script

```
x = rand(1,9) - 0.5

>> stem(x)  ← digital signal plot

>> stem([-4:4],x)   {same plot but centered at zero}

>> negx = fliplr(x);
>> figure(2)
>> stem([-4:4], negx)
% even and odd parts
>> evx = (x + negx)/2;
>> odx = (x - negx)/2;
>> figure(3)
>> stem([-4:4], evx);
>> figure(4)
>> stem([-4:4], odx);

q = evx + odx;

figure(5)
stem([-4:4], q);
```

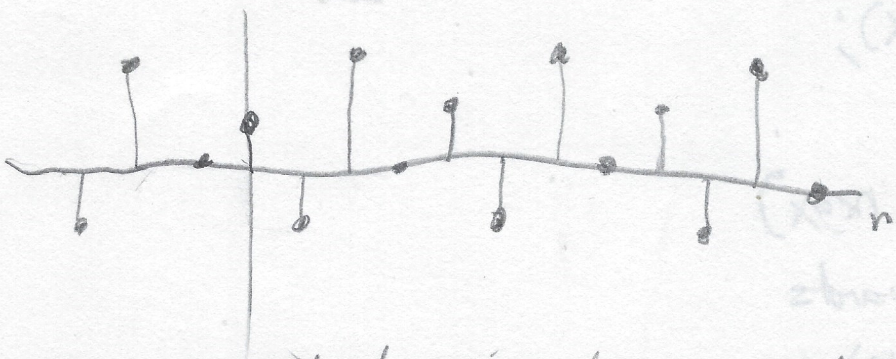
} figure(1) and figure(5) are the same

How does one prove this?

(8)

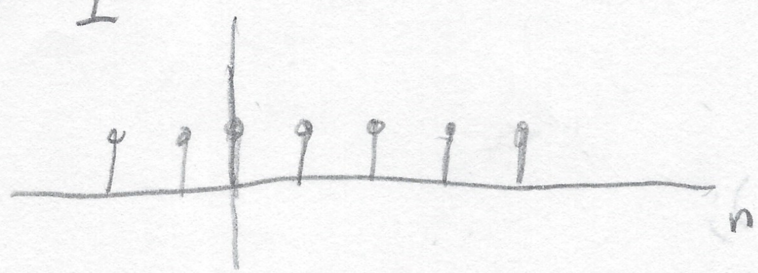
periodicity ; The signal repeats itself after a certain number of integer steps

$$x[n] = x[n+N]$$



Periodicity, 4 (repeats itself after every four units)

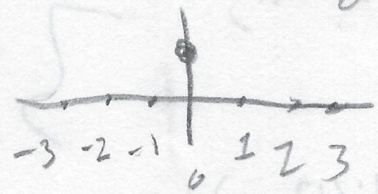
a constant is also periodic with a periodicity of 1



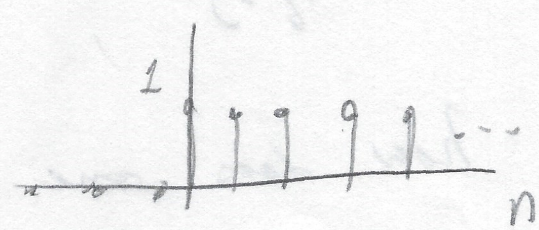
SPECIAL Signals

the delta function

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & \text{o.w.} \end{cases}$$



$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



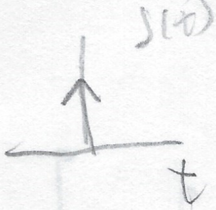
in continuous time; the delta was the derivative of the Heaviside the relationship of $\delta[n]$ and $u[n]$ in discrete are just sums and differences

→ the $u[n]$ is the sum of a bunch of $\delta[n]$'s

$$\rightarrow \quad u[n] = \delta[n] + \delta[n-1] + \delta[n-2] \cdots$$

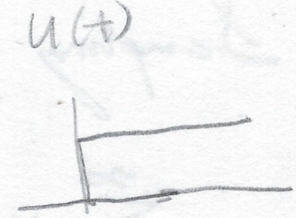
$$= \sum_{k=0}^{\infty} \delta[n-k]$$

continuous

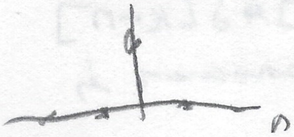


$$\rightarrow \int_{\tau=-\infty}^{t} \delta(\tau)\, d\tau = u(t)$$

(Think: Let t be τ for the integral)



discrete



$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

(add, don't integrate)

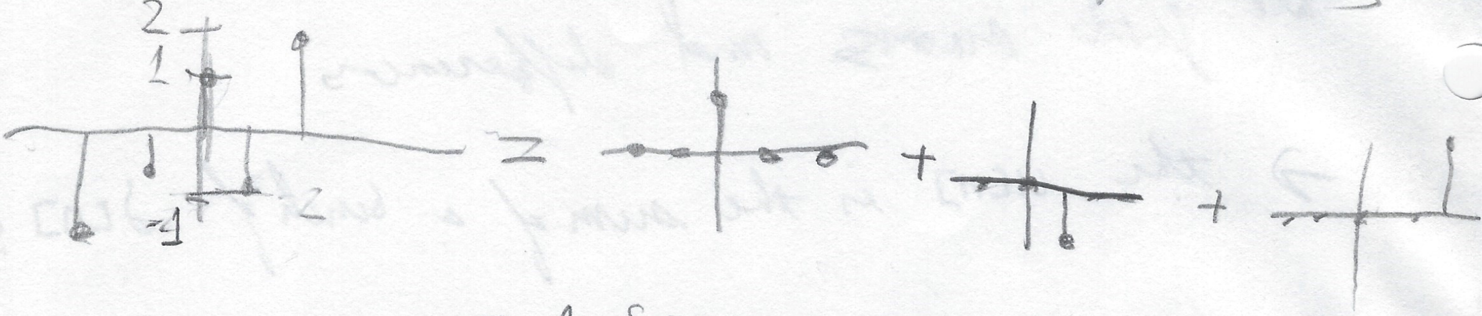
<u>running sum</u>

note

$$\delta[n] = u[n] - u[n-1]$$

$$\delta(t) = \frac{d}{dt} u(t)$$

(10) Making a signal out of delta functions

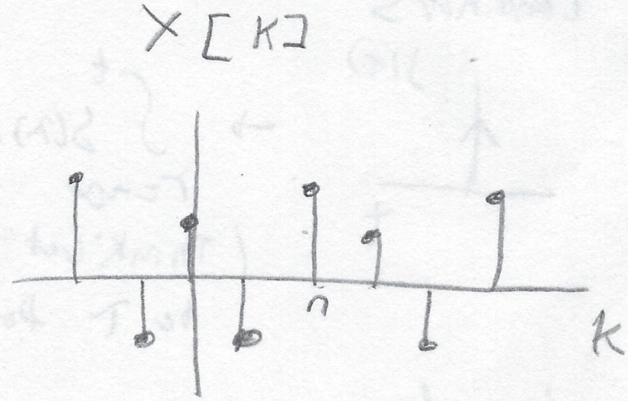


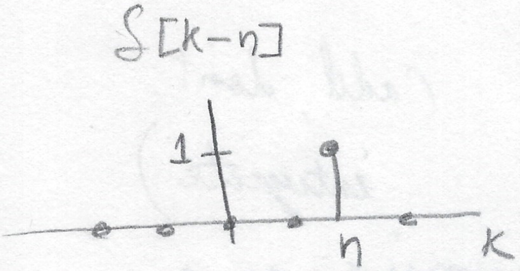
$$= 1 \cdot \delta[n] + -1 \cdot \delta[n-1] + 2 \cdot \delta[n-2]$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$ each piece of $x[n]$ ~~add together~~

Sampling property:

$$\sum_{k=-\infty}^{\infty} x[k]\,\delta[k-n]$$





So when $x[k] \cdot \delta[k-n]$ the non-zero answer is $x[n]$

38.13

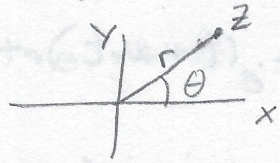
→ use a delta function to pick of a certain value of the signal complex numbers.

→ cartesian coordinants / rectangular

→ polar form (coordinates)

Eulers formula

recall



recall

$\cos\theta = \frac{1}{2}[e^{i\theta} + e^{-i\theta})$

$\sin\theta = \frac{1}{2i}[e^{i\theta} - e^{-i\theta})$

sines and cosines

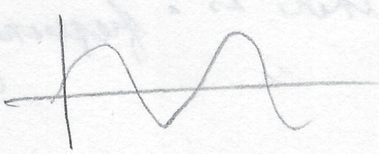
Sinusoids

$\omega_0 = 2\pi f_0$ $\qquad T_0 = 1/f_0$

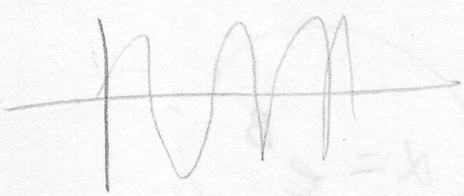
$x(t) = A\sin(\omega_0 t + \theta)$

phase shift

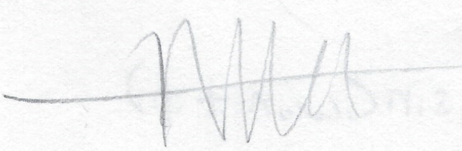
→ shifted in proportion to θ (Like delay)



sin(t)



sin(2t)



sin(3t)

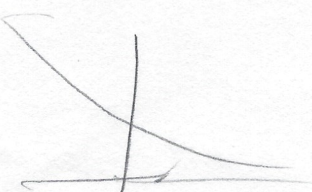
Low freq

Higher freq

Exponentials

$x(t) = Ce^{at}$

Compound interest



friction

(12)

say $c, a \in \mathbb{C}$

$x(t) = Ce^{at} \Longrightarrow C = |c|e^{i\theta}$

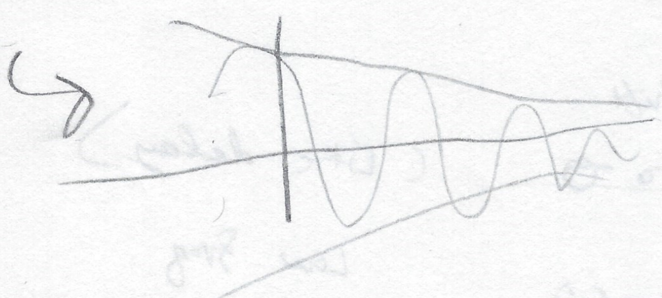
$a = |a|e^{i\phi} = r + i\omega_0$

$x(t) = |c|e^{i\theta}e^{(r+i\omega_0)t}$

$= |c|e^{i\theta + rt + i\omega_0 t} = |c|e^{rt}e^{i(\theta+\omega_0 t)rt}$

$= |c|e^{rt}\left[\cos(\theta+\omega_0 t) + i\sin(\theta+\omega_0 t)\right]$

if $\underline{r<0}$



periodic within the boundaries

→ not actually periodic

→ there is a frequency

→ In a similar way

$x[n] = C\alpha^n = \underline{Ce^{\beta n}}$ $\alpha = e^{B}$

$= |c||\alpha|^n(\cos(\omega_0 n + \phi) + i\sin(\omega_0 n + \phi))$

note $C \in \mathbb{C}$, $B \in \mathbb{C}$

Discrete time sinusoids are different

$e^{j\omega_0 n}$ what is $e^{j(\omega_0+2\pi)n}$

$= e^{j\omega_0 n + j2n\pi}$ n is an integer

$= e^{j2\pi n} e^{j\omega_0 n}$ (with $e^{j2\pi n} = 1$)

$= e^{j\omega_0 n}$

→ In discrete time world there are only a certain number of frequencies

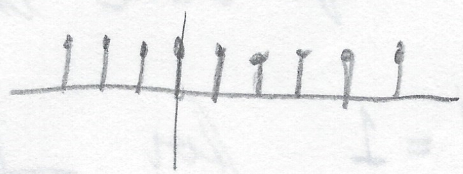
2π wide range of frequencies

all because of $\underline{n}$

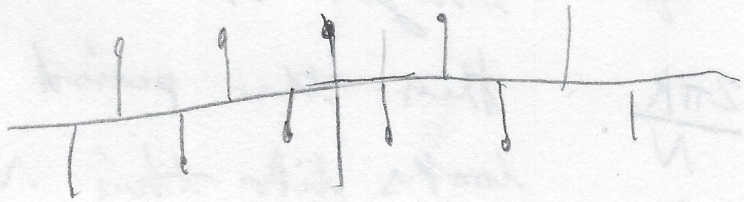
→ the lowest frequency is a constant

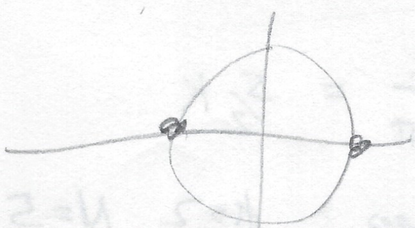
$\omega = 0$

$e^{j\omega} = 0$



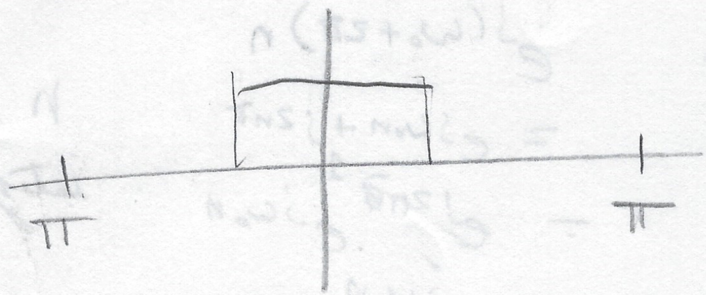
the highest frequency $\omega = \pi$

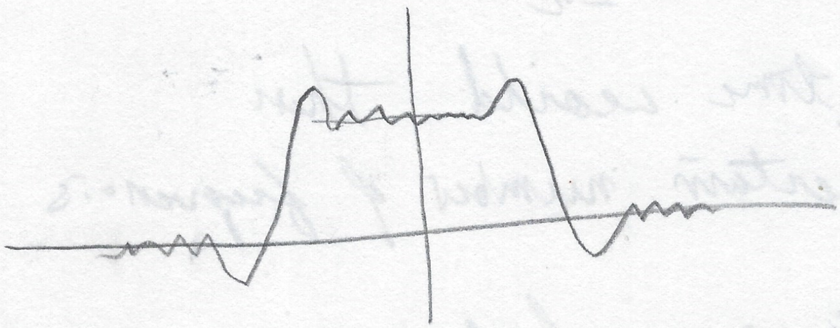




$$e^{\pi n} = \begin{cases} 1 & n = \text{even} \\ -1 & n = \text{odd} \end{cases}$$

(4) Discrete time low-pass filter



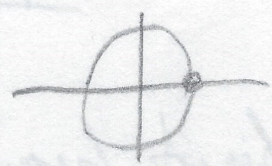


→ gotta know when a signal is periodic ~~in the~~ discrete world

when is $e^{i\omega_0 n}$ periodic?

$e^{i\omega_0(n+N)} = e^{i\omega_0(n)}$ for some integer N

$e^{i\omega_0 n} e^{i\omega_0 N} = 1$ for



$\omega_0 N = 2\pi k$ for integer k.

So $\omega_0 = \frac{2\pi k}{N}$ then the period looks like this $N = \frac{2\pi k}{\omega_0}$

$x[n] = \cos(\frac{4}{5}\pi n)$ $N = \frac{2\pi k}{(4/5)\pi} = \frac{5}{2}k$

N must be an integer so $k=2$, $N=5$

$x[n] = \cos(7n)$ ← Not periodic (Looks periodic but not