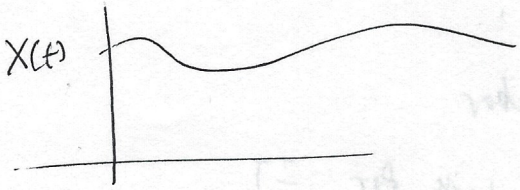
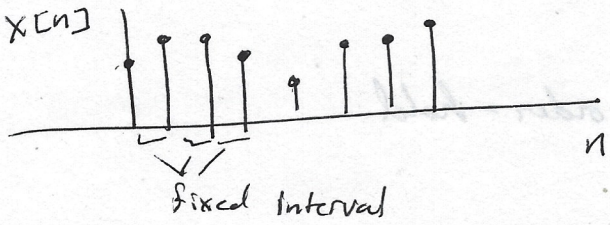
Lecture 13

(Incomplete → Notes on the back)

The Sampling Theorem



Analog signal



Discrete-time signal

Period Sampling

$$x[n] = x_c(nT) \qquad n \in \mathbb{Z}$$

$x[n]$ → discrete, $x_c(nT)$ → continuous

T = Sampling period

$\omega_s = \frac{2\pi}{T}$ = Sampling frequency (Radian)

$\frac{1}{T}$ = Sampling frequency Hz

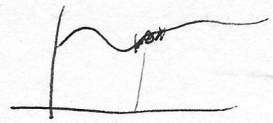
$$x_c(t) \longrightarrow \boxed{A/D} \longrightarrow x[n]$$

T

Non Ideal Effects

1) may not ideally sample

$x_c(t)$



... the average value of a short interval

$x_c(t) * h(t)$

$h(t)$ ← Impulse response of ~~sampler~~

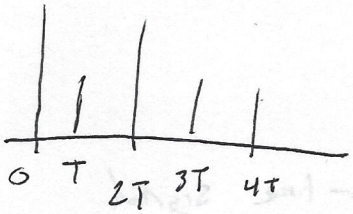
2) May have noise

$$x[n] = y(nT) + z[n]$$

noise

② Reconstruction: How to re-create/estimate $x_c(t)$ given $x[n]$

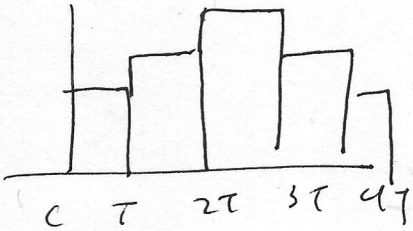
Simple Estimates

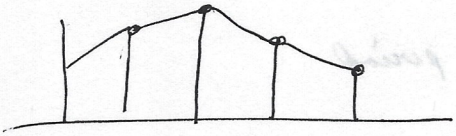


nearest neighbor

not causal Look for $\pm T$

zero order-hold

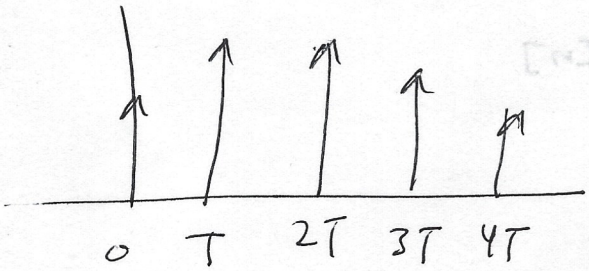




first-order hold

linear interpolation Not causal either

Think of impulses



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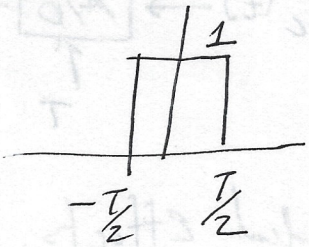
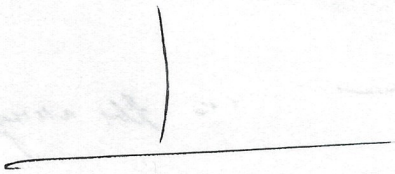
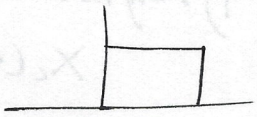


reconstruction impulse response

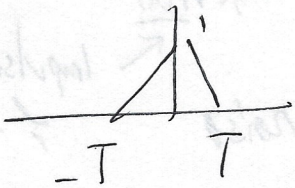
nearest neighbor





zero order

$*$



1st order hold

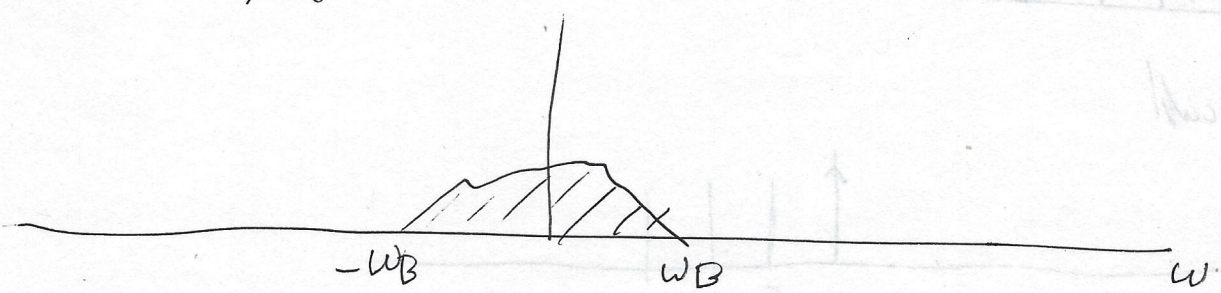
how fast should we sample?

A signal is band limited if there is some frequency ω_B

s.t.

$$X_c(\omega) = 0 \quad \text{for } |\omega| > \omega_B$$

spectral of x_c

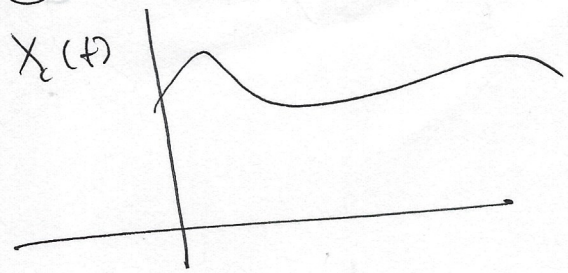


Sampling theorem — Shannon 1949, Nyquist 1928

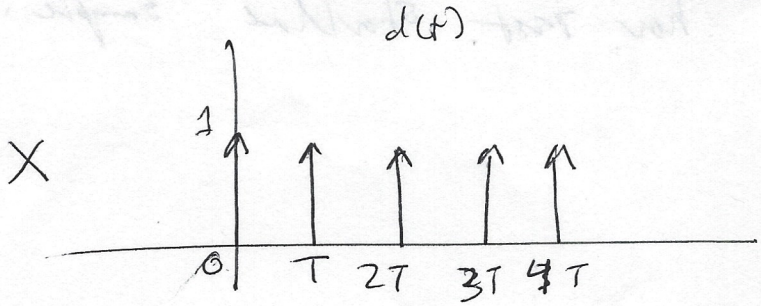
A band limited signal with max frequency ω_B can be perfectly recovered from evenly-spaced samples if the frequency ω_s satisfies $\omega_s > 2\omega_B$

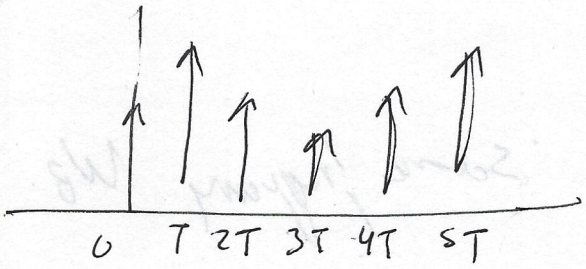
Nyquist rate

(4)



$\times$

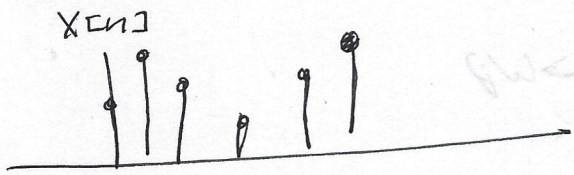




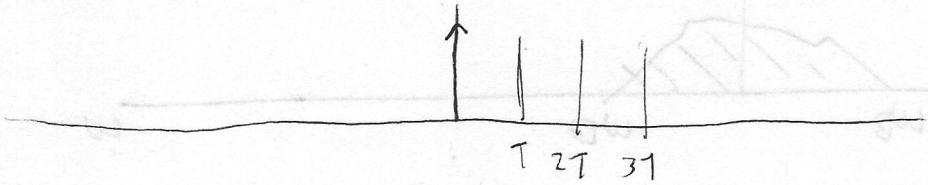
$$x_s(t) = x_c(t) \times d(t)$$

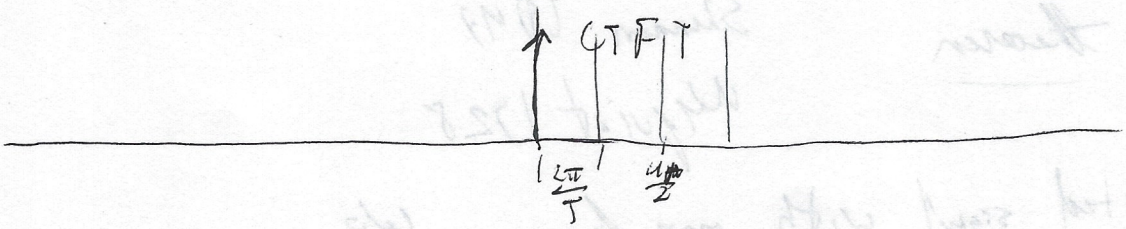
CTFT

$$X_s(j\omega) = \frac{1}{2\pi} X_c(j\omega) * D(j\omega)$$



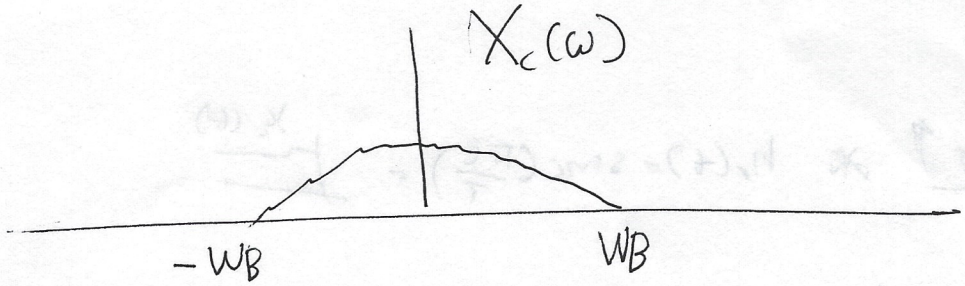
Recall

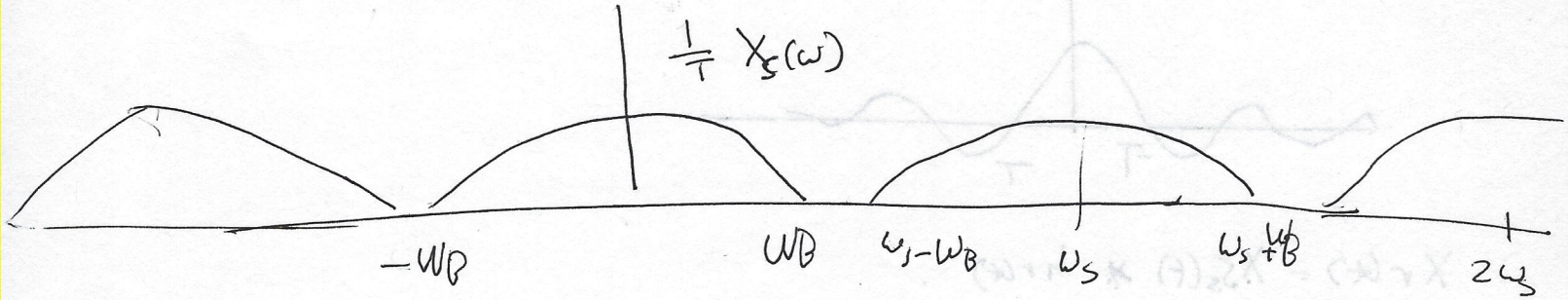




$$X_s(\omega) = \frac{1}{2\pi} X_c(\omega) * \left[\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)\right]$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(\omega - k\omega_s)$$

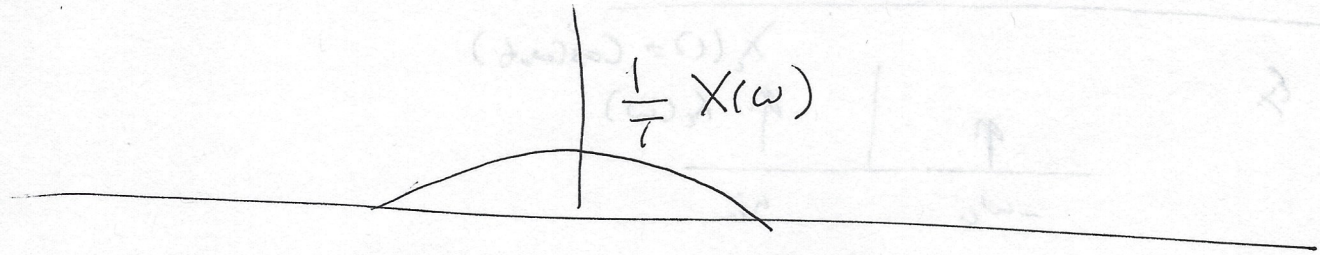




(Section 6.1)

DTFT

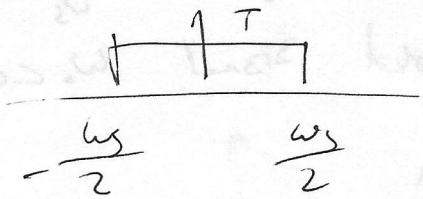
$$X(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c\left(\frac{\omega - 2\pi k}{T}\right) = X_s(\omega) = X(\omega T)$$



$$W_B \frac{2\pi}{\omega_s} = W_B T$$

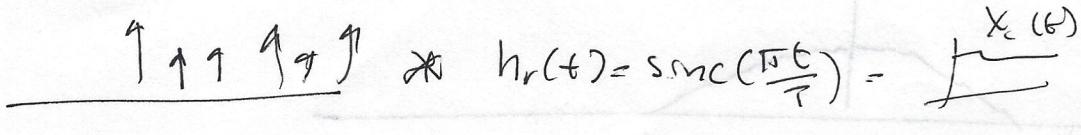
Good use $\quad \omega_s - W_B > W_B \quad \omega_s > 2W_B$

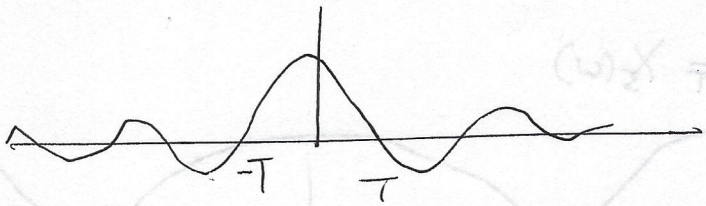
<u>reconstruction filter</u>



(6)

Time domain

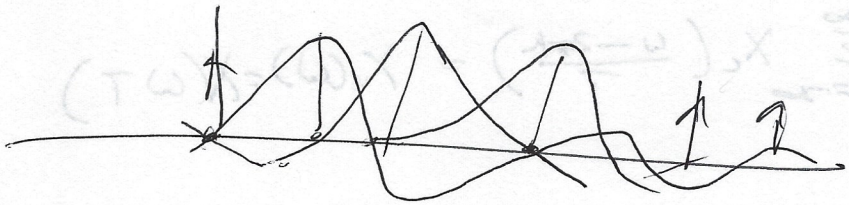




$$X_r(t) = X_{S_s}(t) * h_r(t)$$

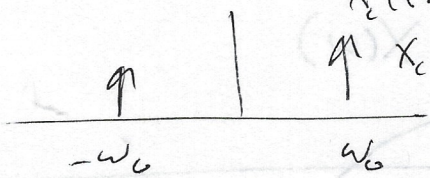
$$= \sum_{n=-\infty}^{\infty} x[n]\,\delta(t - nT) * \left(\mathrm{sinc}\,\frac{\pi t}{T}\right)$$

$$= \sum_{n=-\infty}^{\infty} x[n]\,\mathrm{sinc}\left(\frac{\pi}{T}(t - nt)\right)$$

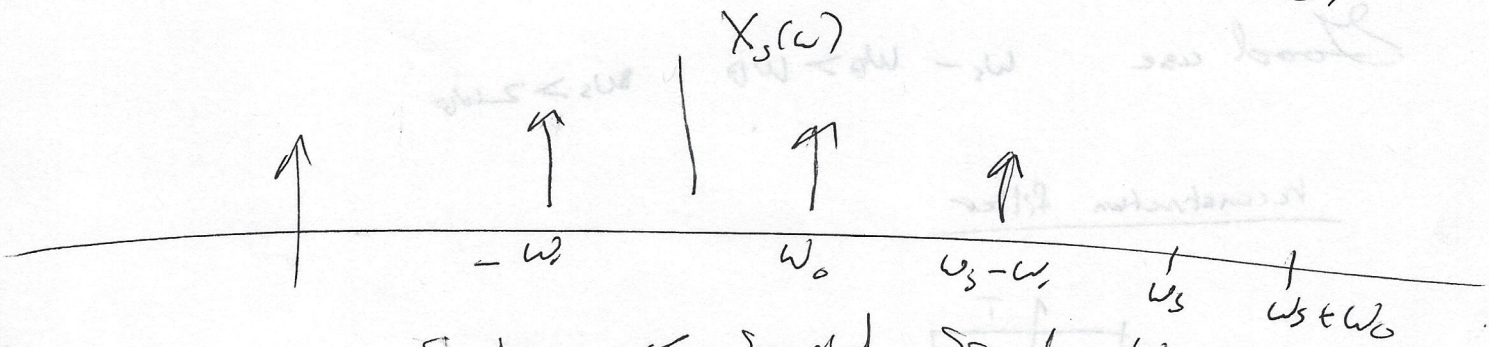


Ex

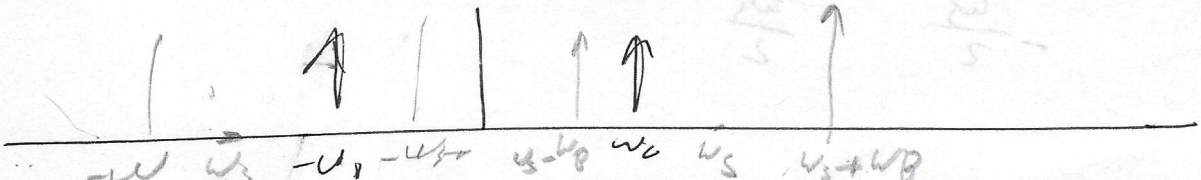
$x_c(t) = \cos(\omega_0 t)$



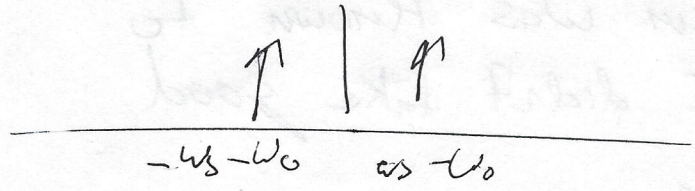
Spectrum of sampled signal $(\omega_s > 2\omega_B)$



Spectrum of sampled signal $\omega_0 < \omega_s < 2\omega_0$

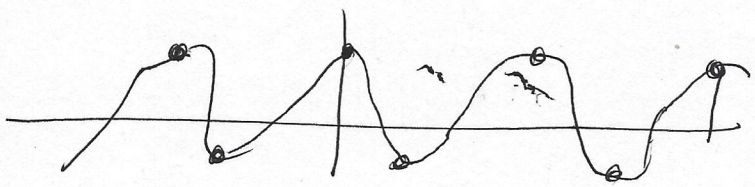


reconstruction filter cut off



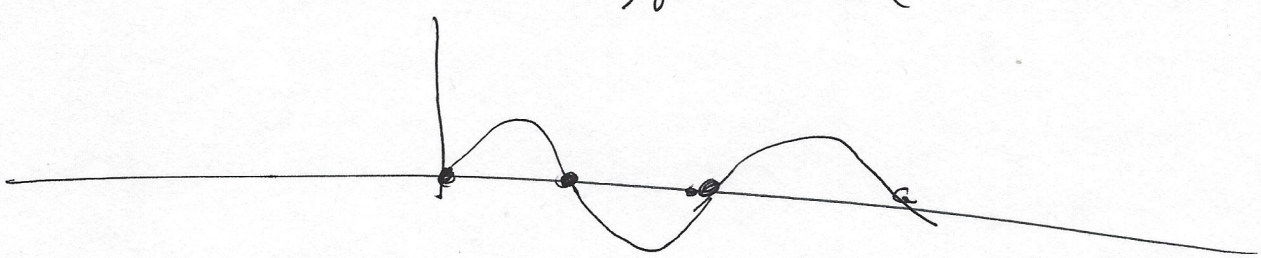
reconstructed signal

Can we sample right at the Nyquist rate?



$T = \frac{2\pi}{\omega_s}$

need to sample at a rate higher than the Nyquist rate



If $\omega_s < 2\omega_B$

$x(t) = \cos(\omega_0 t + \phi)$

$x_r \simeq \cos((\omega_s - \omega_0)t - \phi)$ phase reversal

going backward

The Sampling theorem was known to me at the time, So I didn't take good Notes for the Lecture