I need to Learn How to recite mystery persuasion (1)
~~off the cuff~~

# Chapter 5 : Signal waveforms Notes

a signal is normally thought of as an electric current $i(t)$ or voltage $v(t)$. The time variation of the signal is called a waveform.

→ A waveform is an equation or graph that defines the signal as a function of time

Waveforms that are constant for all time are called DC signals
DC: Direct current

$$\left.\begin{aligned} v(t) &= V_0 \\ i(t) &= I_0 \end{aligned}\right\} \quad \text{for } -\infty < t < \infty \qquad (5-1)$$

This approximates signals from devices Like Batteries
time variation is expressed lowercase letters $v(t), i(t), w(t)$
time variation is implicit when written $v, i, w$

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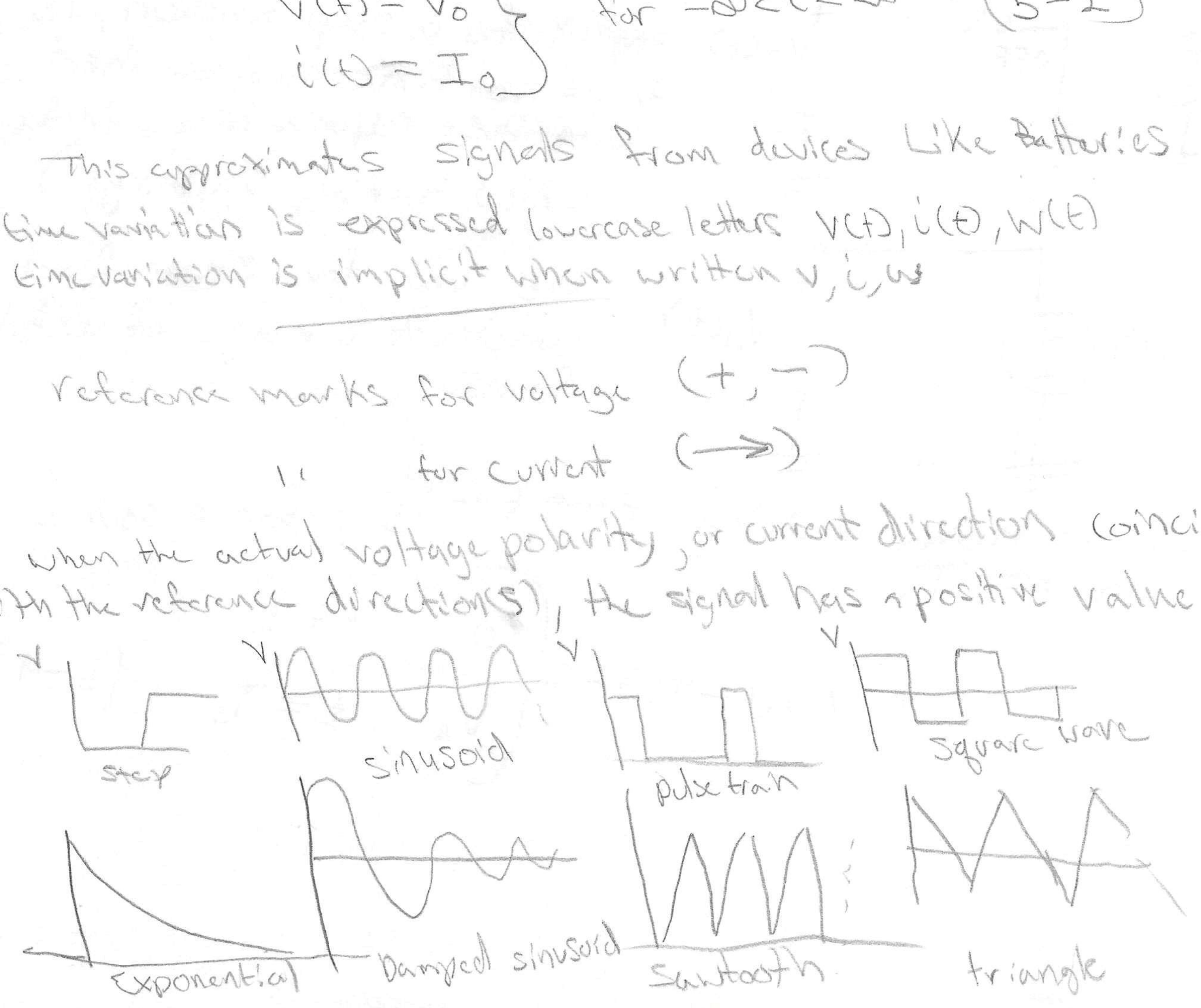
reference marks for voltage $(+, -)$

" for current $(\rightarrow)$

when the actual voltage polarity, or current direction coincides with the reference direction(s), the signal has a positive value



# 5-2 The Step Waveform p241

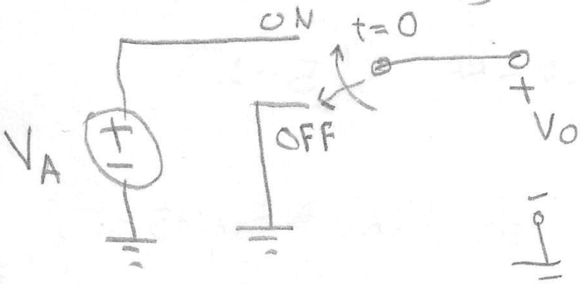
from Oliver Heaviside

rewrote Maxwell's equation form commonly used telegrapher's equations, changed telecommunications, mathematics, and science. Very Heavy personality unfortunately he was not a trained mathematician

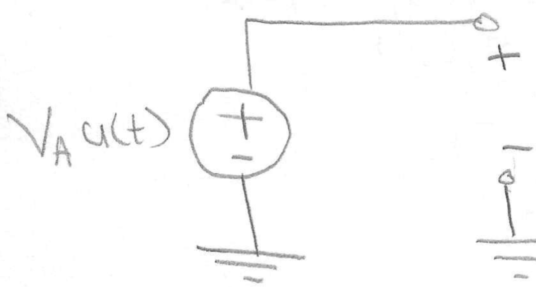
We have the Heaviside function, we'll call it the <u>unit step function</u> as Electrical Engineers do

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \qquad (5\text{-}2)$$

discontinuity at $t=0$



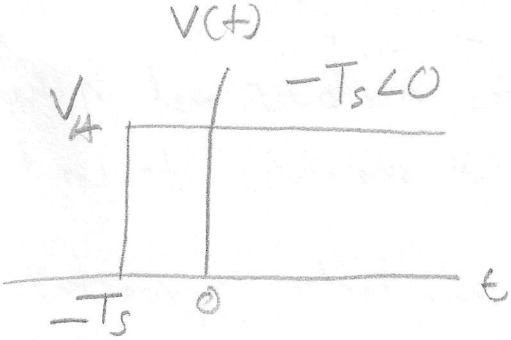

Practically, we can generate a good approx. to a step function, The switch transition time must be short compared with other response times in the circuit. (Also turning on things)

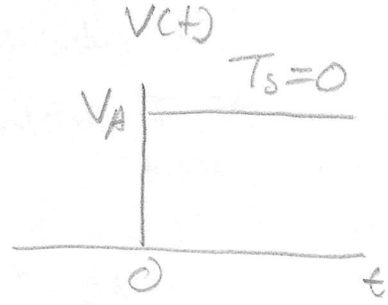


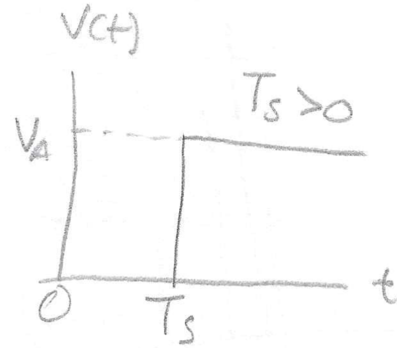

We use the step function to construct a range of useful wave forms.

We can make a delay, $t \to t - T_S$, for a source turning on at $T_S$

$$V_A u(t-T_S) = \begin{cases} 0 & t \leq T_S \\ V_A & t \geq T_S \end{cases} \qquad (5\text{-}4)$$



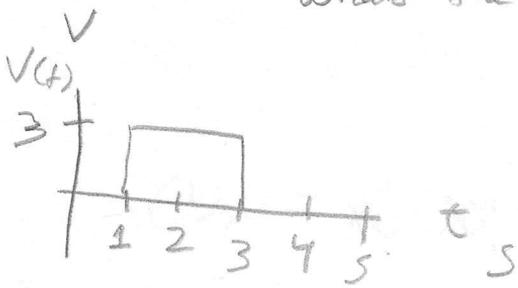




$v(t) = V_A u(t + T_s)$ $\quad$ $v(t) = V_A u(t)$ $\quad$ $v(t) = V_A u(t - T_s)$

Ex) $v(t) = 5u(t+6)$ V, signal shifts 5 volts at $t = -6$s
$i(t) = 2u(t-1)$ mA, signal shifts 2mA at $t = 1$s

Ex 5-1 what's the signal expression? p242




$v(t) = 3u(t-1) - 3u(t-3)$ V

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## The Impulse function p242

<u>from 5-1</u> we have $v(t) = V_A[u(t - T_1) - u(t - T_2)]$ V

We are turning on at $T_1$ and turning off at $T_2$,
we can call these <u>gating functions</u> because they are used in conjunction w/ electronic switches to enable or inhibit passage of another signal

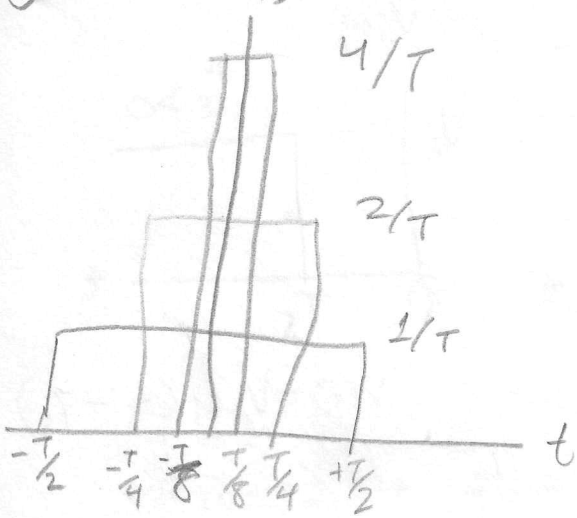
→ A unit-area pulse centered on $t=0$ is written in terms of step functions as

$$v(t) = \frac{1}{T}\left[u\left(t + \frac{T}{2}\right) - u\left(t - \frac{T}{2}\right)\right] \text{ V}$$

within the range $-\frac{T}{2} \le t \le \frac{T}{2}$ the value is $\frac{1}{T}$

→ The area under the pulse is 1 because its scale factor is inversely proportional to its duration

④




The pulse becomes narrower and higher as T decreases but maintains its unit area

$\lim_{T \to 0}$, The scale factor approaches infinity but the area remains 1

The function obtained with this condition is the unit Impulse (Dirac delta)

$\delta(t)$


idealized model of a large-amplitude, short duration pulse

$\delta(t) = 0$ for $t \neq 0$ $\qquad \int_{-\infty}^{t} \delta(x)\,dx = u(t)$

$\rightarrow \delta(t) = \frac{d}{dt} u(t) \qquad \delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{all other } t \end{cases}$ $\quad s$

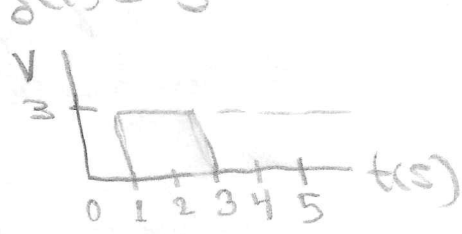
We need more math to justify this, but we aren't prepared to do so now, → Signals & Systems, Oppenheim/Willsky

→ Think of the Impulse as the derivative of the step function

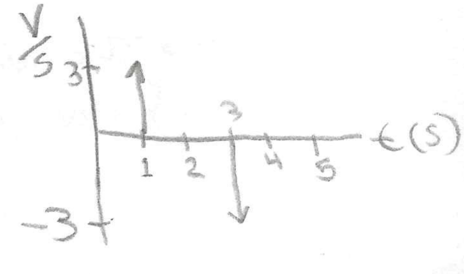
$\rightarrow$ so $v(t) = K\delta(t)$, so $K = V\text{-}s$ as $\delta(t) = s^{-1}$
volts seconds

Ex 5-2

$v(t) = 3u(t-1) - 3u(t-3)$ V




$\frac{d}{dt}v(t) = 3[\delta(t-1) - \delta(t-3)]$ V/s



# The Ramp Function p244

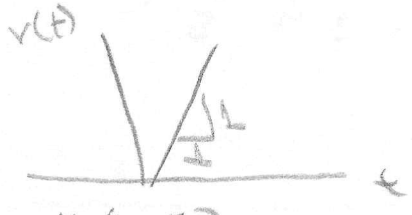
<u>unit ramp</u>

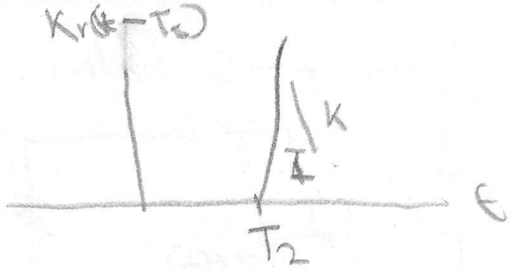
units seconds

$$r(t) = \int_{-\infty}^{t} u(x)\,dx = t\,u(t) \quad s \qquad (5\text{-}8)$$

ramp of strength K, $\rightarrow v(t) = K\,r(t)$

$K = V/s$ This is the slope of ramp




## Singularity Functions

Used to generate other waveforms and as test inputs to linear systems to characterise their responses.

unit impuls, unit step, unit ramp

$$u(t) = \int_{-\infty}^{t} \delta(x)\,dx \qquad (5\text{-}9)$$

$$r(t) = \int_{-\infty}^{t} u(x)\,dx$$

$$\delta(t) = \frac{d}{dt}u(t) \qquad 5\text{-}10$$

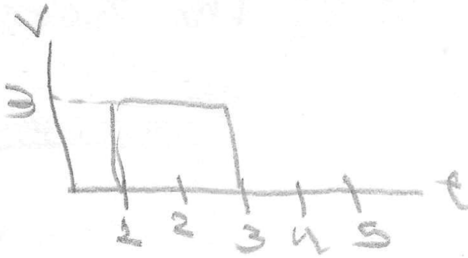
$$u(t) = \frac{d}{dt}r(t)$$

for circuit analysis, $u(t)$ dimensionless

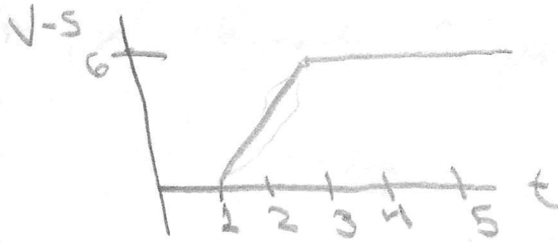
$\delta(t) = s^{-1}$ $\quad r(t) =$ seconds

(6) Example 5-3

$v(t) = 3u(t-1) - 3u(t-3)$ V




$\rightarrow \int_{-\infty}^{t} v(x)\,dx = 3r(t-1) - 3r(t-3)$ V-s




$0, \; t < 1$

$3(t-1), \; 1 \le t \le 3$

[as $3(t-1)u(t-1)$]

$6, \; t \ge 3$

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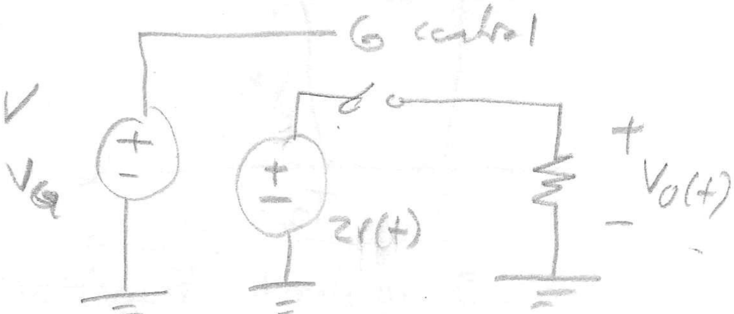
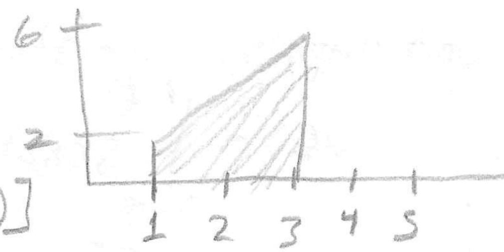
p245, Some derivation

$V_G(t) = 3u(t-1) - 3u(t-3)V$




So $V_o(t) = \begin{cases} 0 & t < 1 \\ 2t & 1 \le t < 3 \\ 0 & 3 \ge t \end{cases}$




We can write,

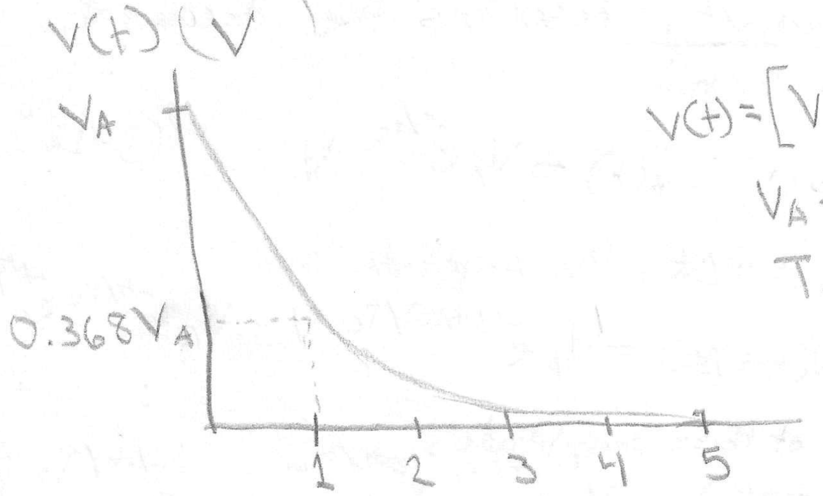
$V_o(t) = 2t[u(t-1) - u(t-3)]$

and further

$$\begin{aligned} V_o(t) &= 2tu(t-1) - 2tu(t-3) \\ &= 2(t-1+1)u(t-1) - 2(t-3+3)u(t-3) \\ &= 2[tu(t-1) - u(t-1) + u(t-1)] - 2(t-3+3)u(t-3) \\ &= 2\underbrace{(t-1)u(t-1)}_{r(t-1)} + 2u(t-1) - 2\underbrace{(t-3)u(t-3)}_{r(t-3)} - 6u(t-3) \\ &= 2r(t-1) + 2u(t-1) - 2r(t-3) - 6u(t-3) \quad \checkmark \end{aligned}$$

# 5-3 The Exponential Waveform

5-14




$$v(t) = \left[V_A e^{-t/T_C}\right] u(t)$$

$V_A$ = Volts , Amplitude

$T_C$ = seconds , Time constant

An exponent decays to about 36.8 % of its initial amplitude $[v(0) = V_A]$ in one <u>time constant</u>

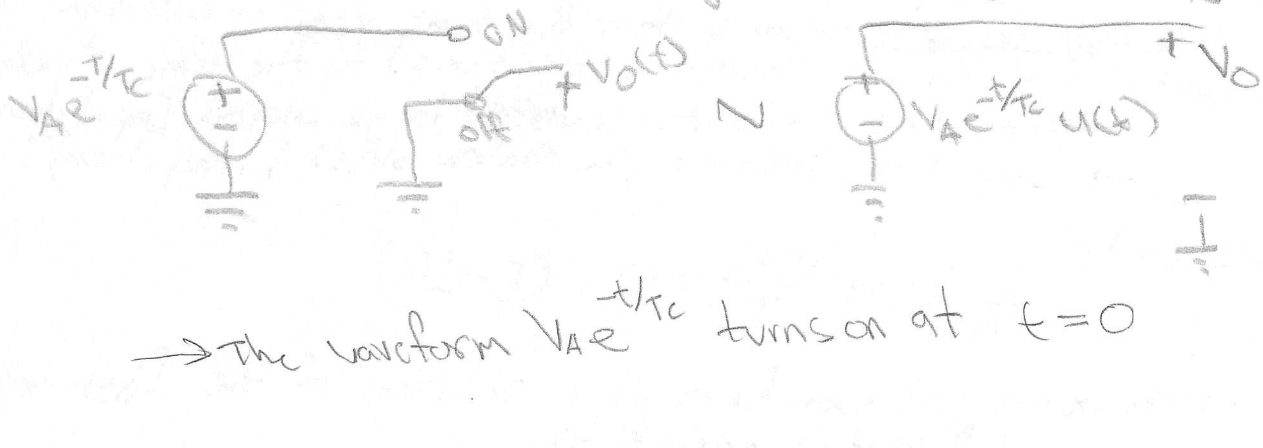
→ $t = T_C$, $v(T_C) = V_A e^{-1} \sim 0.368\, V_A$

→ At $5T_C$, $v(5T_C) = V_A e^{-5} \sim 0.00674\, V_A$

The time constant determines the rate at which the waveform Decays to zero. An exponential signal decays to less than <u>1%</u> of its initial amplitude in a span of <u>5</u> time constant

The <u>duration</u> of a waveform to be the interval of time ouside of which the waveform is everywhere less than a stated value

→ duration of an exponential waveform is $5T_C$




→ The waveform $V_A e^{t/T_C}$ turns on at $t = 0$

# Properties of Exponential Waveforms

decrement property: describes the decay rate of an exponential signal

for $t > 0$, $v(t) = V_A e^{-t/T_C}$ V (5-12)

at time $t+\Delta t$, the Amplitude is

$$v(t+\Delta t) = V_A e^{-(t+\Delta t)/T_C} \text{ V} = V_A e^{-t/T_C} e^{-\Delta t/T_C} \text{ V} \quad (5\text{-}13)$$

The ratio of these Amplitudes

$$\frac{v(t+\Delta t)}{v(t)} = \frac{V_A e^{-t/T_C} e^{-\Delta t/T_C}}{V_A e^{-t/T_C}} = e^{-\Delta t/T_C} \quad (5\text{-}14)$$

→ The decrement ratio is independent of amplitude and time in any fixed time period $\Delta t$, the fractional decrease depends on the time constant

→ The decrement property states that the same percentage decay occurs in equal time intervals

---

The slope of the exponential waveform for $t > 0$

$$\frac{d}{dt} v(t) = -\frac{V_A}{T_C} e^{-t/T_C} = -\frac{v(t)}{T_C} \quad (5\text{-}15)$$

The Slope Property states that the time rate of change of the exponential waveform is inversely proportional to the time constant

→ Small time constants leads to large slopes [rapid decay]

→ Large time constants give shallow slopes [long decay]

Let's write $\frac{d}{dt} v(t) + \frac{v(t)}{T_C} = 0$ (5-16)

→ The exponential waveform is a solution of the first-order linear differential equation

Time-shifted Exponential waveform

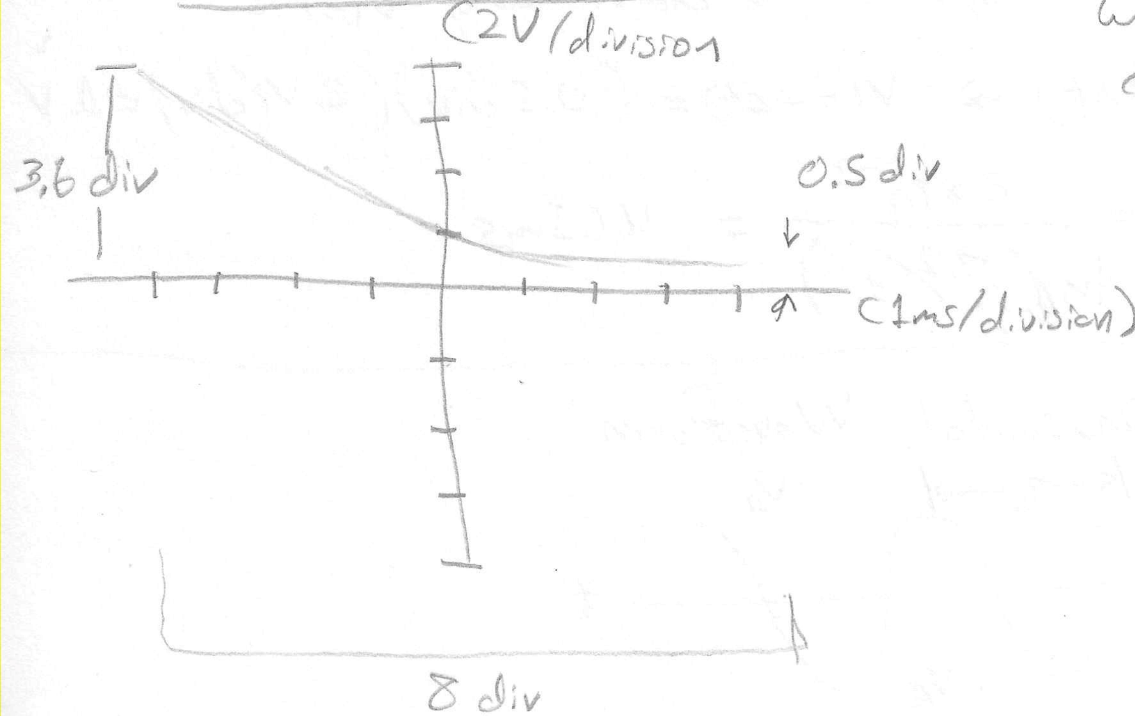
time-shifted by $T_S$ (delay if $T_S > 0$)

$$v(t-T_S) = \left[V_A e^{-(t-T_S)/T_C}\right] u(t-T_S) \text{ V} \quad (5\text{-}17)$$

if $v(t)$ = "

Then the shift is described in the original signal

Example 5-6




What's the time constant?

For $t > T_s$, the general expression for an exponential

$$V(t) = V_A e^{-(t-T_s)/T_C} \text{ V}$$

The waveform is shifted and we don't know how much, we don't know $t=0$ of the waveforms we looked at earlier

Further, we can't find $V_A$ or $T_S$.

→ We can find the time constant from the decrement property

$$\rightarrow \frac{V(t+\Delta t)}{V(t)} = e^{-\Delta t/T_C} \rightarrow \frac{-\Delta t}{T_C} = \ln\left[\frac{V(t+\Delta t)}{V(t)}\right]$$

$$\rightarrow \frac{\Delta t}{T_C} = \ln\left[\frac{V(t)}{V(t+\Delta t)}\right]$$

(10)

$$T_c = \frac{\Delta t}{\ln\left[\frac{V(t)}{V(t+\Delta t)}\right]}$$

Left Edge of Oscilloscope $\rightarrow V(t) = (3.6\text{ div})(2\text{ V/div}) = 7.2\text{ V}$

Define $\Delta t$ to be the full width of the display

$$\Delta t = (8\text{ div})(1\text{ms/div}) = 8\text{ms}$$

So therefore, at the edge at the end of $\Delta t$, $V(t)$ is

$$V(t+\Delta t) \rightarrow V(t+\Delta t) = (0.5\text{ div})(2\text{ V/div}) = 1\text{ V}$$

So $$T_c = \frac{8 \times 10^{-3}}{\ln(7.2/1)} = 4.05\text{ms}$$

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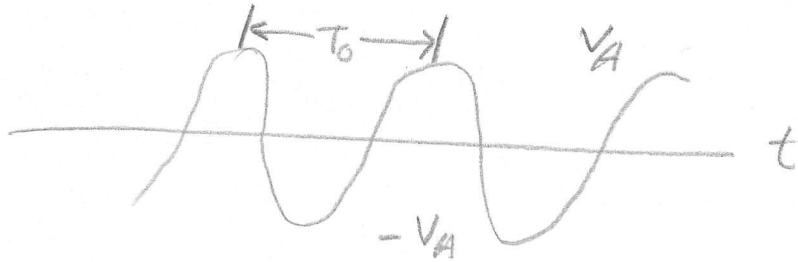
5-4 The Sinusoidal Waveform




real signals arent infinite like sinusoids, it looks unrealistic to use a sinusoid to model signals, but they make good approx. Even for signals w/ finite durations.

Amplitude $V_A$, period $T_0$
(max value) (time of 1 cycle)

if we choose the sinusoid to be 0 at $t=0$ write

$$V(t) = V_A \sin(2\pi t/T_0)\ \text{V} \quad (5\text{-}18a)$$

if max value ($V_A$) at $t=0$,

$$V(t) = V_A \cos(2\pi t/T_0)\ \text{V} \quad (5\text{-}18b)$$

for a general time shift

$$v(t) = V_A \cos[2\pi(t - T_S)/T_0] \quad V \qquad (5\text{-}19)$$

right shift, $T_S > 0$
Left shift, $T_S < 0$ from expression

Time shifting causes the positive peak nearest the origin to occur at $t = T_S$

$$v(t) = V_A \cos\left(\frac{2\pi t}{T_0}\right) \ V$$

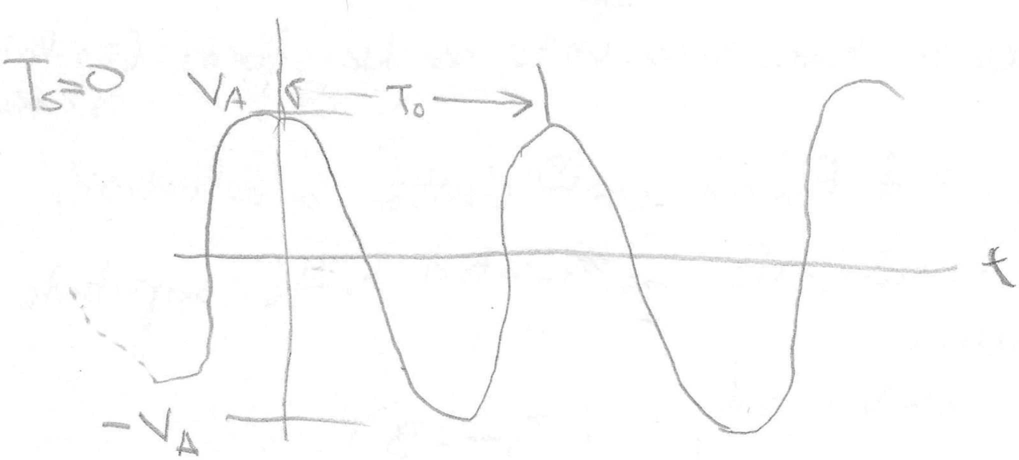



fig 5-22

The time-shifting parameter can also be represented by an angle

$$v(t) = V_A \cos[2\pi t/T_0 + \phi] \ V \qquad (5\text{-}20)$$

$\phi$ – phase angle (circular interpretation of the cosine function)

The period is divided into $2\pi$ radians
Phase angle is in between $t=0$ and the nearest positive peak

$$\phi = -2\pi \frac{T_S}{T_0} \qquad (5\text{-}21)$$

Changing phase angle $\phi$ shifts the waveform left or right, revealing different phases of the oscillating waveform

$\phi$ radians

(12)

recall $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$$v(t) = V_A \cos[2\pi t/T_0 + \phi] \quad V$$
$$= [V_A \cos(\phi)]\cos(2\pi t/T_0) + [-V_A \sin(\phi)]\sin(2\pi t/T_0) \; V$$

we can write

$$v(t) = a\cos(2\pi t/T_0) + b\sin(2\pi t/T_0) \; V \quad (5\text{-}22)$$

$a, b$ have same units as waveform [so Volts] this time

$a, b$ are called Fourier coefficients, by definition

The Fourier coefficients are related to the amplitude and phase parameters

$$a = V_A \cos\phi$$
$$b = -V_A \sin\phi \quad (5\text{-}23)$$

$$V_A = \sqrt{a^2 + b^2}, \quad \phi = \tan^{-1}\frac{-b}{a} \quad (\text{arctan})$$

Cyclic frequency $f_0$ is number of periods per unit time

$T_0$ is number of seconds per cycle

so cycles per second, $f_0 = \dfrac{1}{T_0}$ (5-26)

$f_0$ Hz (Hertz)

angular frequency, $\omega_0$ in radians per second is related to the cyclic frequency by the relationship

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad (5\text{-}27)$$

($2\pi$ radians per cycle)

Radian frequency is more convenient when describing the characteristics of circuits driven by sinusoidal inputs

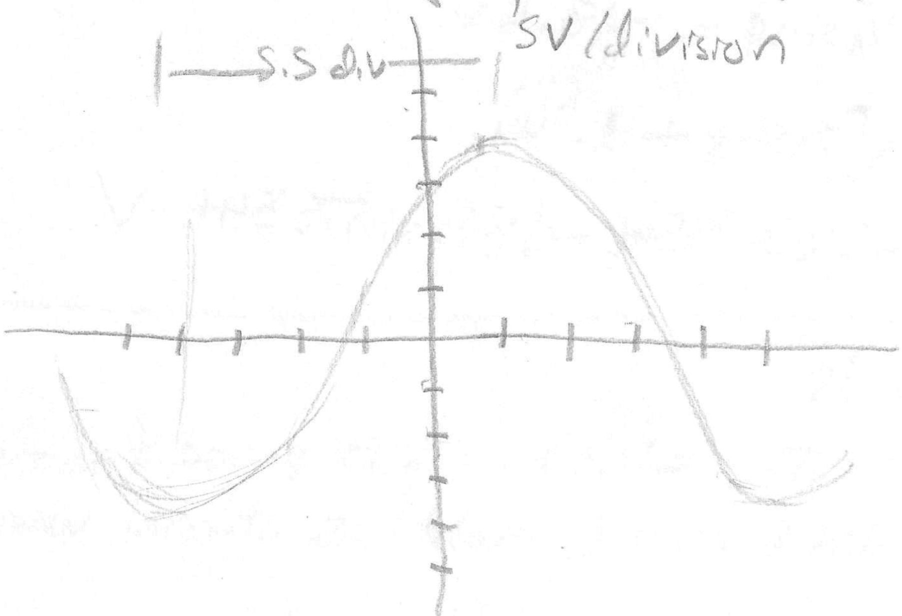
$$v(t) = V_A \cos\left[\frac{2\pi(t-T_S)}{T_0}\right] = V_A \cos\left(\frac{2\pi t}{T_0} + \phi\right) = a\cos\left(\frac{2\pi t}{T_0}\right) + b\sin\left(\frac{2\pi t}{T_0}\right) V$$

$$= V_A \cos\left[2\pi f_0(t-T_S)\right] = V_A \cos(2\pi f_0 t + \phi) = a\cos(2\pi f_0 t) + b\sin(2\pi f_0 t) V$$

$$= V_A \cos\left[\omega_0(t-T_S)\right] = V_A \cos(\omega_0 t + \phi) = a\cos(\omega_0 t) + b\sin(\omega_0 t) \; V$$

1.) Amplitude: either $V_A$ or the Fourier coefficients a and b
2.) Time shift: either $T_s$ or the phase angle $\phi$
3.) Time/frequency: either $T_0, f_0$, or $\omega_0$

what's the expression




Amplitude: $V_A = (4\,div)(5\,V/div) = 20V$

$T_0 = (8\,div)(0.1\,ms/div) = 0.8ms$, $f_0 = \frac{1}{T_0} = 1.25\,kHz$, $\omega_0 = 2\pi f_0 = 7854\,\frac{rad}{s}$

To determine $T_s$, we need to define a time origin

Say $t=0$ at left edge (arbitrarily chosen)

5.5 divisions to the right is the positive peak

→ more than ½ a cycle (4 division)

→ The positive peak closest to $t=0$ is not shown

(14) The positive peak shown is at $t = T_S + T_0$ since it is one cycle after $t = T_S$

$$T_S + T_0 = (5.5\,\text{div})(0.1\,\text{ms/div}) = 0.55\,\text{ms}$$

So $T_S = 0.55 - T_0 = \underline{-0.25\,\text{ms}}$

negative because the nearest peak is to the Left of $t=0$

So then, $\phi = -\frac{2\pi T_S}{T_0} = 1.96\,\text{rad}$ $(112.5°)$

So $a = V_A \cos\phi = -7.65\,\text{V}$

$b = -V_A \sin\phi = -18.5\,\text{V}$

$$v(t) = 20\cos[7854t + 1.96]$$
$$= -7.65\cos 7854t - 18.5\sin 7854t\ \text{V}$$

---

Properties of Sinusoids

Periodic: $v(t+T_0) = v(t)$, $T_0$ is period of waveform

must be valid for all $t$, model has eternal waveform

$$v(t+T_0) = V_A\cos[2\pi(t+T_0)/T_0 + \phi]$$
$$= V_A\cos[2\pi t + 2\pi + \phi]$$
$$= V_A\cos[2\pi t + \phi]$$
$$= v(t)$$

additive property of sinusoids states that summing two or more sinusoids w/ the same frequency yields a sinusoid w/ a different amplitude and phase parameters but the same frequency.

$$v_1(t) = a_1\cos(2\pi f_0 t) + b_1\sin(2\pi f_0 t) \text{ V}$$
$$v_2(t) = a_2\cos(2\pi f_0 t) + b_2\sin(2\pi f_0 t) \text{ V}$$

$$v_3(t) = v_1(t) + v_2(t)$$
$$= (a_1 + a_2)\cos(2\pi f_0 t) + (b_1 + b_2)\sin(2\pi f_0 t) \text{ V}$$

The fourier coefficients of the sum is the sum of the Fourier coefficients
(Must be Fourier coefficient form)

---

$$\frac{d}{dt} V_A\cos\omega t = \underline{-\omega V_A\sin\omega t} = \omega V_A\cos(\omega t + \pi/2)$$

Another sinusoid w/ same $f_0$

$$\int V_A\cos(\omega t)\,dt = \frac{V_A}{\omega}\sin(\omega t) = \frac{V_A}{\omega}\cos(\omega t - \pi/2)$$

Amplitude changes, phase change, $f_0$ remains
This is the key property of sinusoids (They preserve frequency)

5-8 p257

$$v_1(t) = 17\cos(2000t - 30^\circ) \text{ V}$$
$$v_2(t) = 12\cos(2000t + 30^\circ) \text{ V}$$

$$\pi = 180$$
$$2\pi = 360$$

$$\omega_0 = 2000 \frac{\text{rad}}{\text{s}}, \quad f_0 = \frac{\omega_0}{2\pi} = 318.3 \text{ Hz}$$

$$\frac{30}{360} = \frac{x}{2\pi}$$

$$T_0 = \frac{1}{f_0} = 3.14 \text{ ms} \qquad 30^\circ \rightarrow 2\pi\frac{30}{360} = 0.52 \text{ radians}$$

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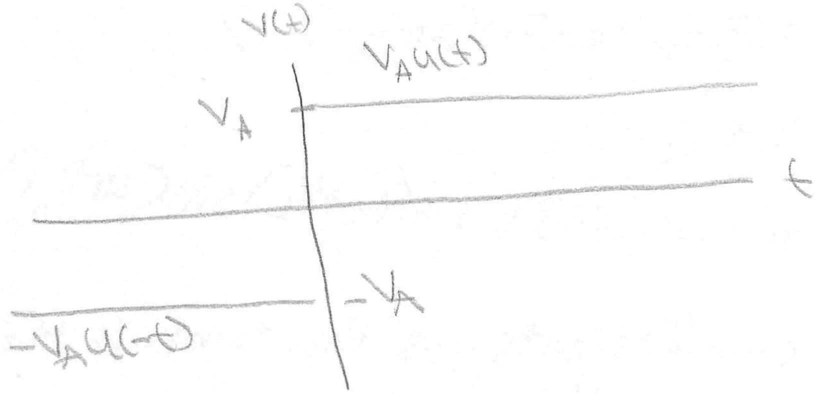
# 5-5 Composite Waveform

composite signals

Example 5-9

$$v(t) = V_A u(t) - V_A u(-t) \text{ V}$$




Signum waveform

(Signum function)

jump discontinuity at $t=0$

$(2V_A)$

$$u(-t) = \begin{cases} 1 & -t > 0, \quad t < 0 \\ 0 & -t \leq 0, \quad t \geq 0 \end{cases}$$

Exercise 5-13

$$v(t) = [V_A u(t) - V_A u(-t)][\delta(t+1) + \delta(t) + \delta(t-1)] \text{ V}$$

$$= -V_A \delta(t+1) + V_A \delta(t-1) \text{ V}$$

Note, at $t=0$ exactly, the signum function is apparently zero

$$\rightarrow H(x) = \begin{cases} 1 & t > 0 \\ 0 & t \leq 0 \end{cases}$$

This is different from the definition in the text.

for $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$, which is what the text uses

at $t=0$, both step functions are <u>on</u> so they cancel out

Example 5-10

Characterise the composite waveform generated from subtracting an exponential from a step function withe the same amplitude

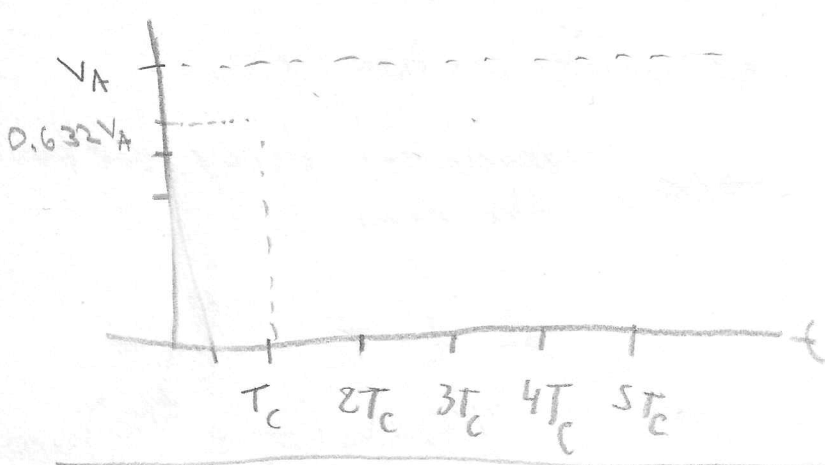
$$V(t) = V_A u(t) - [V_A e^{-t/T_C}] u(t) \ V$$

$$= V_A [1 - e^{-t/T_C}] u(t) \ V$$

for $t<0$, $v(t)=0$

at $t=0$, waveform is still zero because the step and $e$ cancel out

we have exponential rise




$v(t)$ is less than 1% of final value $V_A$ when $t=5T_c$

$V(T_c) = V_A(1-e^{-1}) = 0.632 V_A$

rises to 63% of final value after one time constant

Sometimes called "charging exponential"

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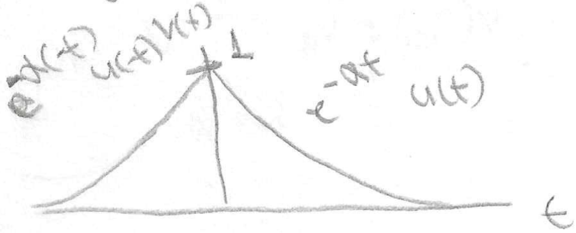
Exercise 5-15 p260




double sided exponential

write an expression

$V(t) = e^{-\alpha|t|}$ V

---

Example 5-11 characterize the composite waveform obtained by multiplying the ramp $r(t)/T_c$ multiplied by an exponential

So Lets write! $V(t) = \frac{r(t)}{T_c} [V_A e^{-t/T_c}] u(t)$ V

$$= t/T_c [V_A e^{-t/T_c}] u(t) \ V$$

(18) for $t<0$, $V(t)=0$

$t=0$, $V(t)=0$

for $t>0$, the ramp wants to increase linearly w/ time but the exponential wants to go to zero

→ The composite waveform is the product of the two, → so which dominates

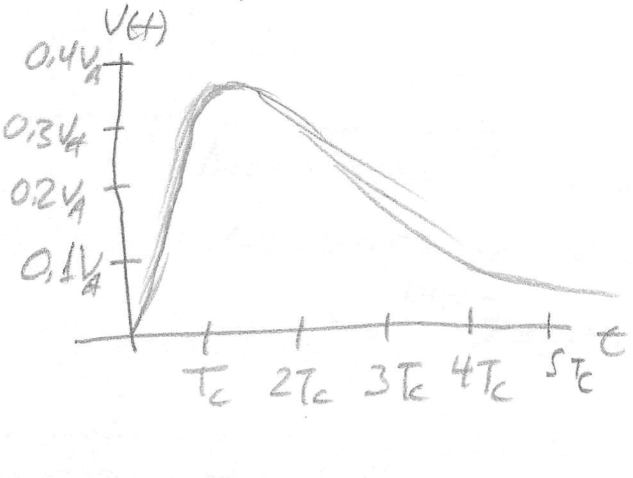
→ as $t \to \infty$, we have a case where $\infty * 0$, which is indeterminate

→ Use L'Hôpital's rule $\lim\limits_{x \to c} \frac{f(x)}{g(x)} = \lim\limits_{x \to c} \frac{f'(x)}{g'(x)}$

so clearly the exponent will dominate

So $V(t) \to 0$ as $t \to \infty$, exponential decay overpowers the ramp

damped ramp




at $t = T_c$

$$V(t) = V_A \left(\frac{t}{T_c} e^{-t/T_c}\right) u(t) \text{ V}$$

$$V(T_c) = V_A e^{-1} \text{ V}$$

$$= 0.368 V_A$$

Example 5-12

characterise the composite waveform obtained by multiplying $\sin \omega_0 t$ by an exponential

So we write

$$v(t) = \sin \omega_0 t \left[V_A e^{-t/T_C}\right] u(t) \text{ V}$$

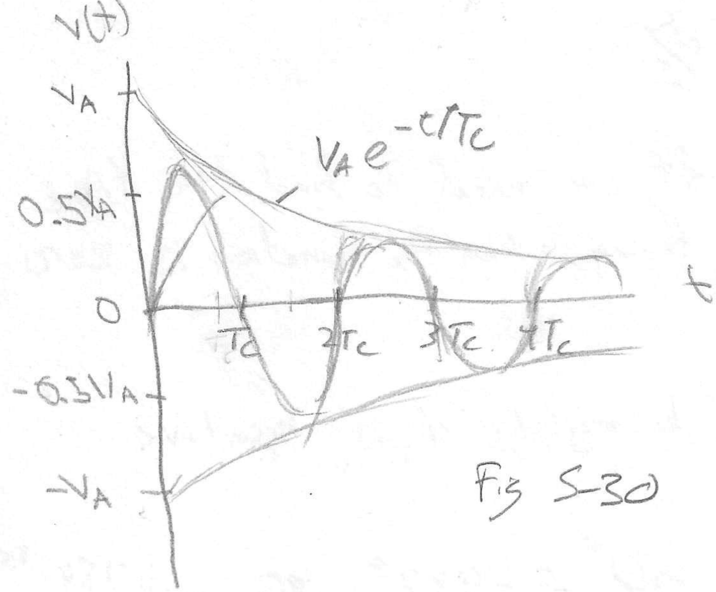



Fig 5-30

$T_0 = 2T_C$

The waveform passes thru 0 cuz $\sin(n\pi) = 0$

The waveform is not periodic after all because of the decay

the oscillations become negligibly small for $t > 5T_C$

Waveform is called a <u>damped sine</u>

---

Example 5-13 p262

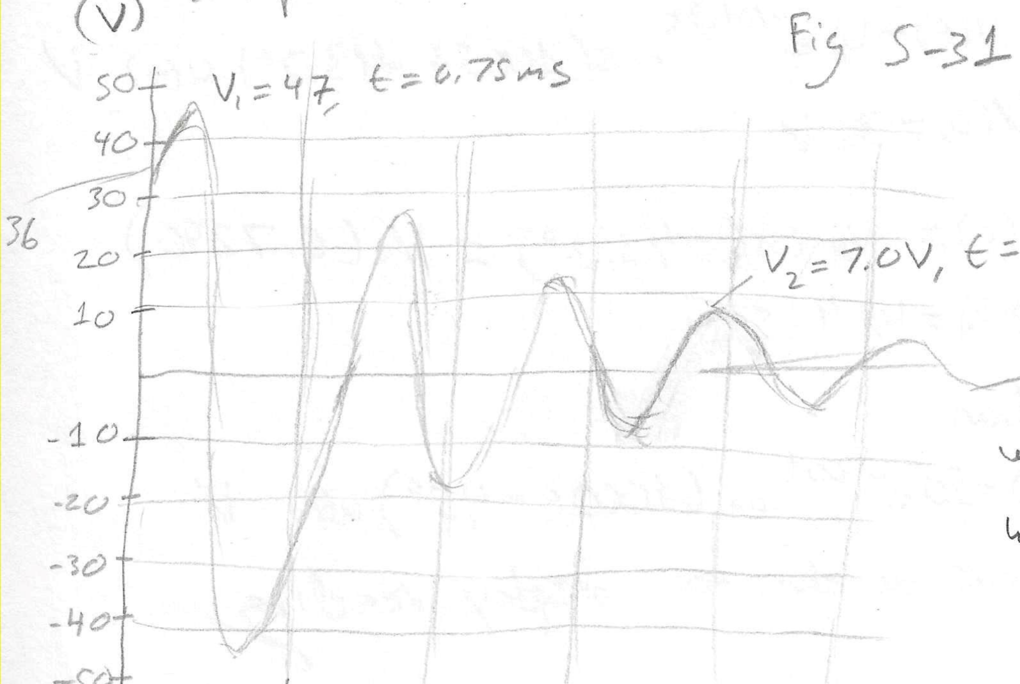



Fig 5-31

what's the approx expression

we need $V_A$, $T_C$, $\omega_0$

estimate coordinates for 1st peak. Select another peak to get $T_C$, choose a later peak to get more accurate result — 4th peak

from the decrement property

$$T_C = \frac{\Delta t}{\ln\left(\frac{v(t)}{v(t+\Delta t)}\right)} = \frac{.0195 - .00075}{\ln(47/7.0)} = \frac{0.01875}{1.90} = 9.87\text{ms}$$

(20)

to find $T_0$, make sure to use 1 cycle so $\frac{\Delta t}{3} = 6.25\,ms$

$$T_0 = \frac{\Delta t}{3} = 6.25\,ms$$

So $\omega_0 = \frac{2\pi}{T_0} = 1005$ rad/s

The waveform has a phase shift we need to find the time shift $T_s$ by measuring the time from when the function is zero to the first peak of the cosine

→ since it shifted to right, $\phi$ is negative

$$\phi = \frac{-.00075}{.00625} 360^\circ = -43.2^\circ \text{ or } -0.754 \text{ rad}$$

So we can write $v(t) = V_A e^{-101.3t} \cos(1005t - 43.2^\circ)\, u(t)$ V

We know $v(0) = 36$ V

So $v(0) = V_A \cos(-43.2^\circ) = V_A(0.7290)$

$\rightarrow V_A = 49.4$

The actual function

$$v(t) = 50e^{-100t} \cos(1000t - 45^\circ)\, u(t) \text{ V}$$

errors are due to display reading

Ex 5-14

characterize the composite waveform obtained as the difference of two exponentials with the same amplitude

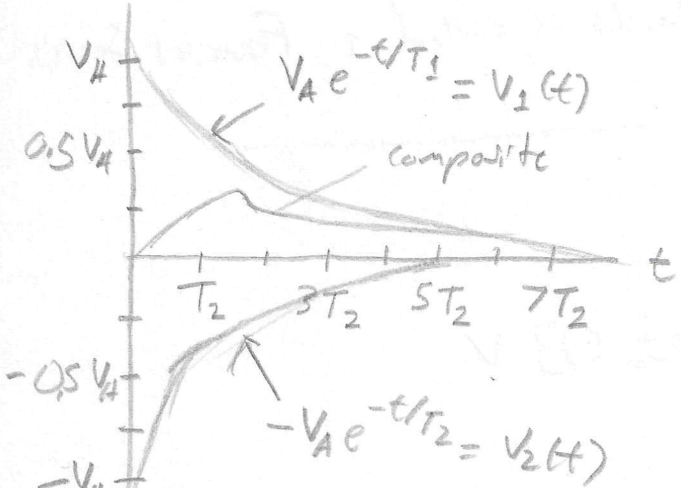
we write,

$$v(t) = \left[V_A e^{-t/T_1}\right] u(t) - \left[V_A e^{-t/T_2}\right] u(t)$$

Larger $T_C$ means the exponent decays more slowly

for $T_1 > T_2$

plot for $T_1 = 2T_2$




for $t >> T_1$ waveform goes to 0

for $5T_1 > t > 5T_2$, the 2nd exp. decays to zero, and $v(t)$ goes to the 1st exponential

for $t << T_1$, the second exponential determines the early time variation of the waveform

This waveform is called <u>double exponential</u>

---

Example 5-15 characterize

$$v(t) = 5 - \frac{10}{\pi}\sin(2\pi 500t) - \frac{10}{2\pi}\sin(2\pi 1000t) - \frac{10}{3\pi}\sin(2\pi 1500t) \quad V$$

we have a DC component and 3 sinusoids at different frequencies,

1st sinusoid component is ~~the~~ <u>fundamental</u> because it has the lowest $f_0$

$\rightarrow f_0 = 500\ Hz$ is called the <u>fundamental frequency</u>

The other sinusoidal terms are said to be harmonics because their frequencies are integer multiples of $f_0$

$\rightarrow$ The second sinusoidal term is called the <u>second harmonic</u>

$\rightarrow 2f_0 = 1000\ Hz$

(22)

third harmonic $3f_0 = 1500\ Hz$

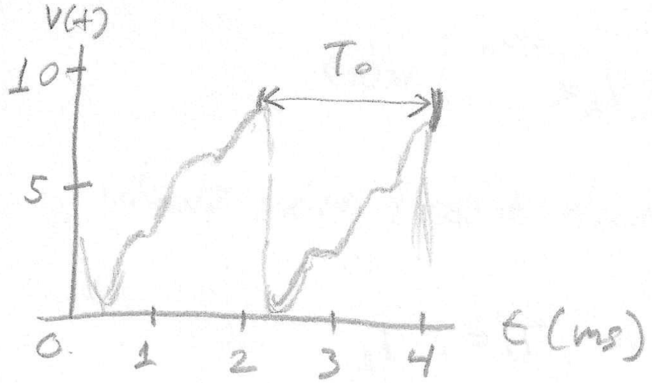



Fig 5-34

The waveform is periodic with a period equal to the fundamental component, $T_0 = \frac{1}{f_0} = 2ms$

→ The decomposition of a periodic waveform into a sum of harmonic sinusoids is called a Fourier Series

---

5-20 characterize

$$V(t) = \sum_{n=1}^{\infty} [b_n \sin(2\pi n f_0 t)]\ V$$

where $b_n = \frac{4V_A}{\pi n}$, $V_A = 10V$ $f_0 = 1000\ Hz$, $n = 1, 3, 5, 7, 9, 11 \ldots$

plotting the first 6 n terms, we get something that Looks Like a square wave

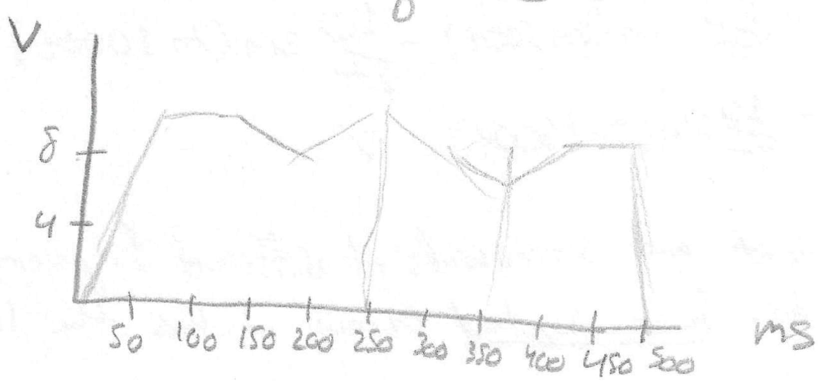



Fig 5-35

# 5-6 Waveform Partial Descriptors

an equation or graph defines a waveform for all time. The value of a waveform $v(t)$ or $i(t)$ at time $t$ is called the instantaneous value of the waveform.

Often we use Partial descriptors that categorizes important features of a waveform but do not give a complete description

1) Temporal features
2) Amplitude features

## Temporal Descriptors

- identify waveform attributes relative to the time axis
  waveforms that repeat themselves at fixed time intervals
  → Periodic $\quad v(t+T_0)=v(t)$, where $T_0$ is the smallest value that meets that condition

a signal being periodic is a partial descriptor so is $T_0$, Triangle wave, Square wave

step, exponential, damped sine are aperiodic

Causal: waveforms that are identically zero prior to some specified time.

→ A signal $v(t)$ is casual if there exist a value of $T$ such that $v(t)=0$ for all $t<T$, otherwise it's non-causal

Typically a causal signal is zero for $t<0$ (usually assumed)

ex) step, exponential ($e^{-t}u(t)$), and damped sine

noncausal, sine, cosine

To note, if the input driving signal $x(t)$ is causal, output $y(t)$ must also be causal

a physical realized circuit cannot anticipate and respond to an input before its applied

Causality is a temporal partial descriptor

---

Amplitude Descriptors

- positive scalars that describe signal strength

Generally, a waveform varies between two extremes values denoted as $V_{max}$ and $V_{min}$

The Peak-to-peak value ($V_{pp}$) describes the total excursion of $v(t)$

$$V_{pp} = V_{max} - V_{min}$$

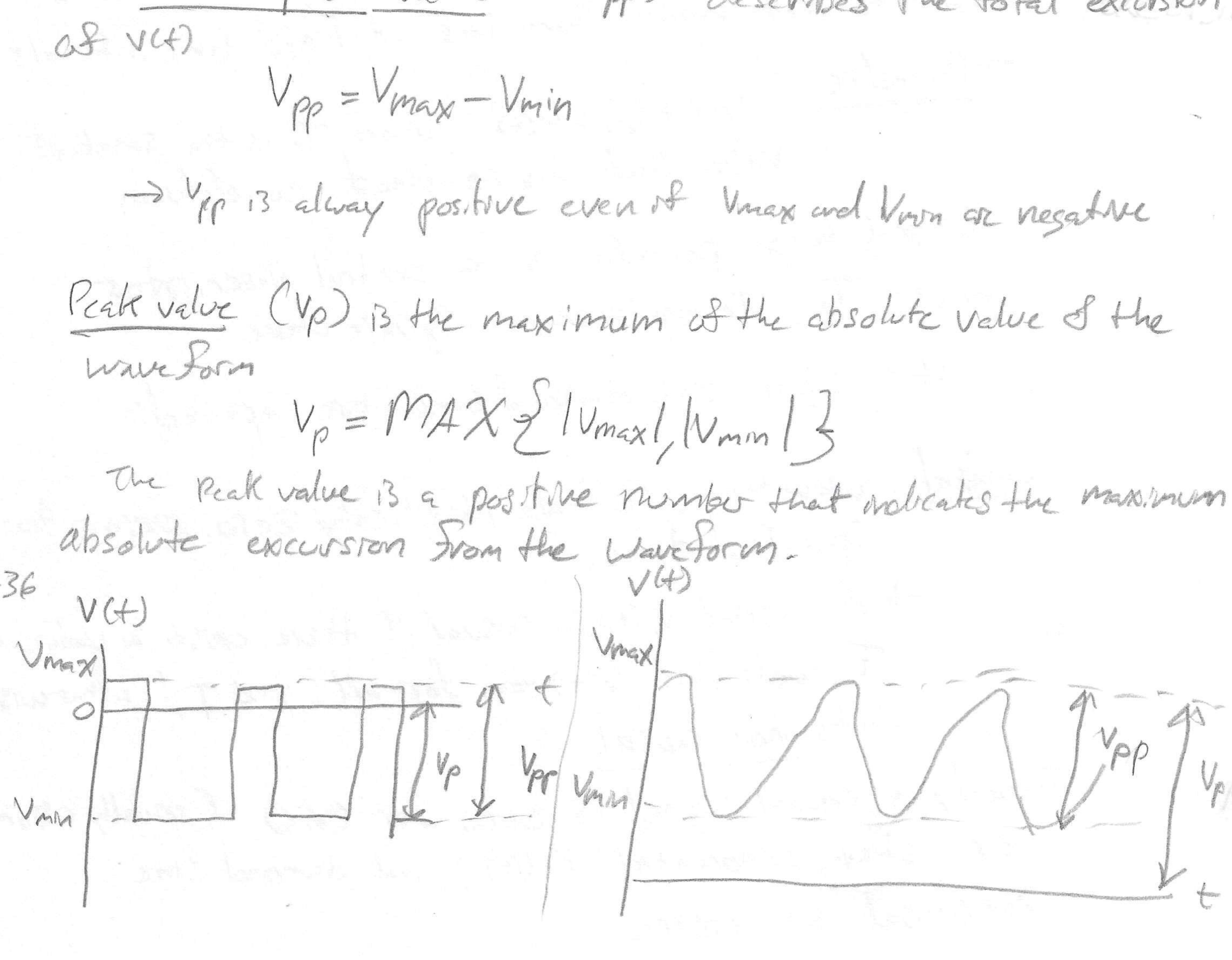
→ $V_{pp}$ is alway positive even if $V_{max}$ and $V_{min}$ are negative

Peak value ($V_p$) is the maximum of the absolute value of the waveform

$$V_p = MAX\{|V_{max}|, |V_{min}|\}$$

The Peak value is a positive number that indicates the maximum absolute excursion from the waveform.

Fig 5-36



The average value smooths things out to reveal the underlying waveform baseline.

Average value is the area under the waveform over some period of time T, divided by that time period

average value ($V_{avg}$) over the time interval T

$$V_{avg} = \frac{1}{T}\int_{t}^{t+T} v(x)\,dx \qquad (5\text{-}30)$$

for periodic signals, $T_0$ is used as the averaging interval T

Periodic waveforms, the integral can be estimated graphically

→ The net area under the waveform is the area above the time axis minus the area below the time axis

The signals in 5-36 obviously have non zero $V_{avg}$

1st waveform has negative area

The Average Value indicates whether the waveform contains a constant, non-varying component.

→ The Average Value is also called the dc component

→ because dc signals are constant for all t

→ AC components have zero value and are periodic

for $v(t) = 5 - \frac{10}{\pi}\sin(2\pi 500t) - \frac{10}{2\pi}\sin(2\pi 1000t) - \frac{10}{3\pi}\sin(2\pi 1500t)$ V

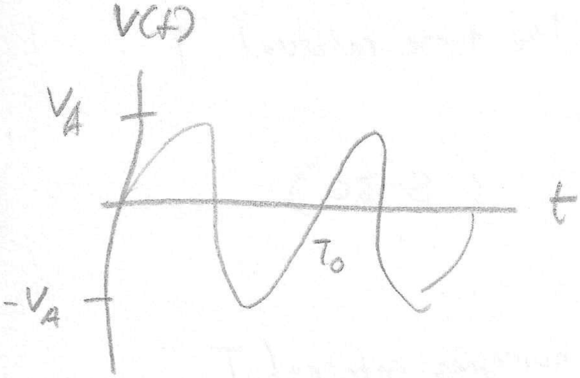
$V_{avg} = 0$ for sinusoids

over $T_0$, the sinusoids positive areas are cancelled by the negative areas

(26) Example 5-16

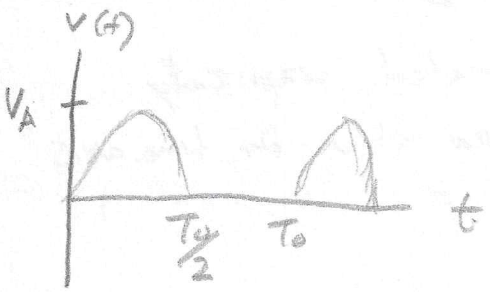
Periodic input to a half-wave rectifier

$V_p$? $V_{pp}$? $V_{avg}$?




$V_{pp} = 2V_A \quad V_{avg} = 0$

$V_p = V_A$




$V_{pp} = V_p = V_A \qquad v(t) = 0$ for $\frac{T_0}{2}$ to $T_0$

$$V_{avg} = \frac{1}{T_0}\int_0^{T_0/2} V_A \sin\left(\frac{2\pi t}{T_0}\right)dt$$

$$= -\frac{V_A}{2\pi}\cos\left(\frac{2\pi t}{T_0}\right)\Big|_0^{T_0/2} = -\frac{V_A}{2\pi}\left[\cos\left(\frac{2\pi T_0}{2T_0}\right) - \cos(0)\right]$$

$$= -\frac{V_A}{2\pi}[-1-1] = \frac{V_A}{\pi}$$

---

## Root-Mean-Square Value

the <u>root-mean-square</u> ($V_{rms}$)

is a measure of the average power carried by the signal

The instantaneous power delivered to a resistor R by a voltage v(t) is

$$p(t) = \frac{1}{R}[v(t)]^2 \text{ W} \quad (5\text{-}31)$$

The average power delivered to the resistor in time span $T$ is defined as

$$P_{avg} = \frac{1}{T}\int_{t}^{t+T} p(t)\,dt \qquad (5\text{-}32)$$

So then

$$P_{avg} = \frac{1}{R}\underbrace{\left[\frac{1}{T}\int_{t}^{t+T} [v(x)]^2\,dx\right]} \qquad (5\text{-}33)$$

the average value of the square of the waveform

$$\frac{1}{T}\int_{t}^{t+T} [v(x)]^2\,dx. \quad V^2 \quad \text{volts square}$$

The square root of this defines the amplitude partial descriptor $V_{rms}$

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t}^{t+T} [v(x)]^2\,dx} \quad V \qquad (5\text{-}34)$$

$V_{rms}$ is called root-mean-square (rms) because it is obtained by taking the square root of the average (mean) of the square of the waveform.

→ For periodic signals, the average interval is one cycle

We can express the average power delivered to a resistor in terms of $V_{rms}$ as

$$P_{avg} = \frac{1}{R} V_{rms}^2 \qquad (5\text{-}35)$$

28

This has the same form as the power delivered by a dc signal.

→ the rms value was once called <u>effective value</u>
no longer common

1.) if waveform doubled, rms doubled, $P_{avg}$ is quadrupled
→ commercial electrical power systems use transmission voltages in the range of several hundred kilovolts (rms) to efficiently transmit power over long distances.

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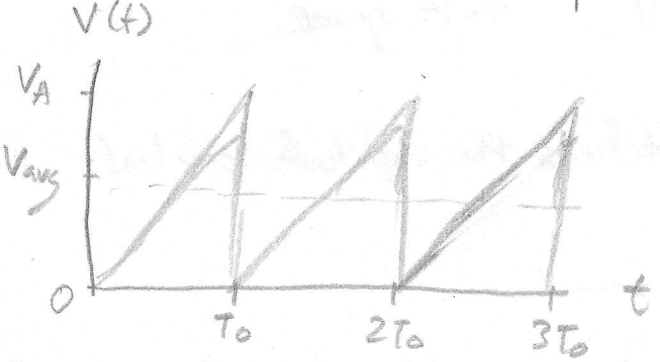
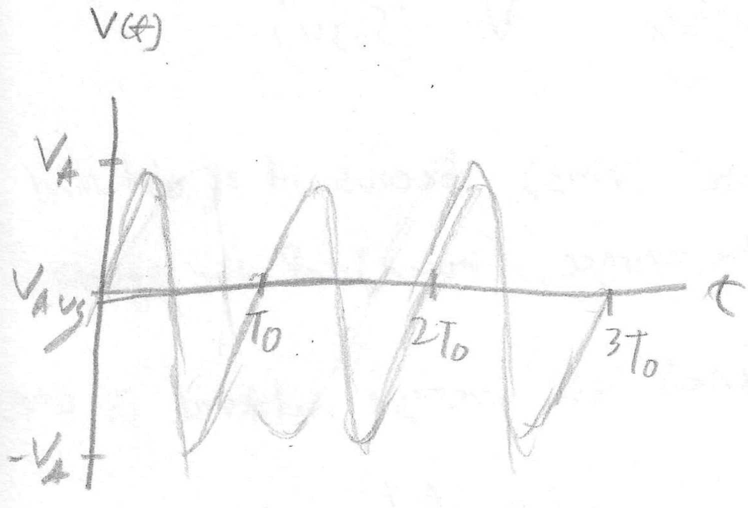
Example 5-17 p269




Find $V_{avg}$ $V_{rms}$ for both

$V_{avg}$ of sinusoid is 0
Sawtooth has nonzero $V_{avg}$

$$V_{avg} = \frac{1}{T}\int_t^{t+T} v(x)\,dx$$

Think area of waveform

$$= \frac{1}{T_0}\left(\frac{V_A T_0}{2}\right) = \frac{V_A}{2}$$

$V_{rms}$ of sinusoid

$$V_{rms} = \sqrt{\frac{1}{T_0}\int_0^{T_0} V_A^2 \sin^2\left(\frac{2\pi x}{T_0}\right) dx}$$

$$= \sqrt{\frac{V_A^2}{T_0}\left[\frac{t}{2} - \frac{\sin(4\pi t/T_0)}{8\pi/T_0}\right]_0^{T_0}}$$

$$= \frac{V_A}{\sqrt{2}}\ V$$

Saw tooth rms

$$V_{rms} = \sqrt{\frac{1}{T_0}\int_0^{T_0} \left(V_A t / T_0\right)^2 dt}$$

$$= \sqrt{\frac{V_A^2}{T_0^3}\left[\frac{t^3}{3}\right]_0^{T_0}} = \frac{V_A}{\sqrt{3}} \quad V$$

→ digital Applications p 271