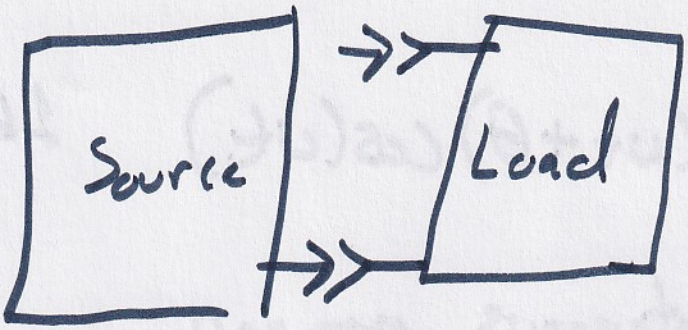
Linear Circuits

16-1



$$v(t) = V_A \cos(\omega t + \theta) \ V \qquad (16\text{-}1)$$

$$i(t) = I_A \cos(\omega t) \quad A$$

$V_A \quad I_A$ peak amplitude

$t = 0$

$$\theta = \phi_V - \phi_I$$

$$\mathbb{V} = V_A \angle \phi_V \qquad \mathbb{I} = I_A \angle \phi_I$$

$$\mathbb{V} = V_A \angle \theta \qquad \mathbb{I} = I_A \angle 0$$

$$\phi = V \times I$$

$$= V_A I_A \cos(\omega t + \theta) \cos(\omega t) \qquad 16-2$$

$\Rightarrow$ Instantaneous power

ac, dc components!

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$p(t) = V_A I_A [\cos(\omega t) \cos\theta - \sin\omega t \sin\theta] \cos\omega t$$

$$= [V_A I_A \cos(\theta)] \cos^2 \omega t$$

$$- [V_A I_A \sin\theta] \cos\omega t \sin\omega t$$

$$p(t) = \frac{V_A I_A}{2} \cos\theta + \left[\frac{V_A I_A}{2} \cos\theta\right] \cos 2\omega t$$

$$- \left[\frac{V_A I_A}{2} \sin\theta\right] \sin 2\omega t$$

Avg value of periodic waveform

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

$\cancel{\frac{\omega}{2\pi} = T}$ $\quad T = \frac{1}{f}$ $\quad \omega = 2\pi f$

$$f = \frac{1}{T} \rightarrow 2\pi f = \boxed{\frac{2\pi}{T} = \omega}$$

$$\boxed{\frac{2\pi}{\omega} = T}$$

2ω

$$\boxed{T = \frac{2\pi}{2\omega}}$$

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{2\omega}{2\pi}\int_0^T p(t)\,dt$$

$$P = \frac{V_A I_A}{2}\cos\theta \ W$$

Avg Value of Power

16-5

$$\frac{V_A I_A}{2} \sin\theta$$

Volt-Amperes Reactive

$$Q = \frac{V_A I_A}{2} \sin\theta \quad VAR$$

reactive Power & $p(t)$

$$p(t) = \underbrace{P(1+\cos 2\omega t)}_{\text{unpolar}} - \underbrace{Q \sin 2\omega t}_{\text{bipolar}}$$

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$$W = \frac{J}{s} \quad Wh \rightarrow \frac{J}{s} hr \sim J$$

$$W = \int_0^T p(t)\,dt \quad \text{Energy}$$

$$= P \int_0^T (1 + \cancel{\cos 2\omega t})\,dt \quad 0 \quad T = \frac{2\pi}{2\omega} \text{ one cycles} \quad 1 \text{ period}$$

$$- Q \int_0^T \cancel{\sin 2\omega t}\,dt \quad 0$$

$W = P \times T$ $\quad$ $P \sim$ Avg Power $\quad W$

$Q \sim$ Reactive Power $\quad$ VAR

Q provides no net energy transfer

didn't do examples

16-2 Complex Power

In power circuit analysis, it is convenient to express phasor magnitudes in rms

$$\mathbb{V} = V_{rms}\, e^{j\phi_V} \qquad \mathbb{I} = I_{rms}\, e^{j\phi_I}$$

(16-5)

$$P = \frac{V_A I_A}{2}\cos\theta \quad W$$

(16-6)

$$Q = \frac{V_A I_A}{2}\sin\theta \quad VAR$$

$$V_{rms} = \frac{V_A}{\sqrt{2}} \text{ for sinusoids}$$

$$P = \frac{V_A I_A}{2} \cos\theta = \frac{V_A}{\sqrt{2}} \frac{I_A}{\sqrt{2}} \cos\theta$$

$$= V_{rms} I_{rms} \cos\theta \qquad \theta = \phi_V - \phi_I$$

$$Q = V_{rms} I_{rms} \sin\theta$$

complex Power (S)

$$S = \mathbb{V} \mathbb{I}^*$$

$$S = \mathbb{V} \mathbb{I}^* = V_{rms} e^{j\phi_V} I_{rms} e^{-j\phi_I}$$

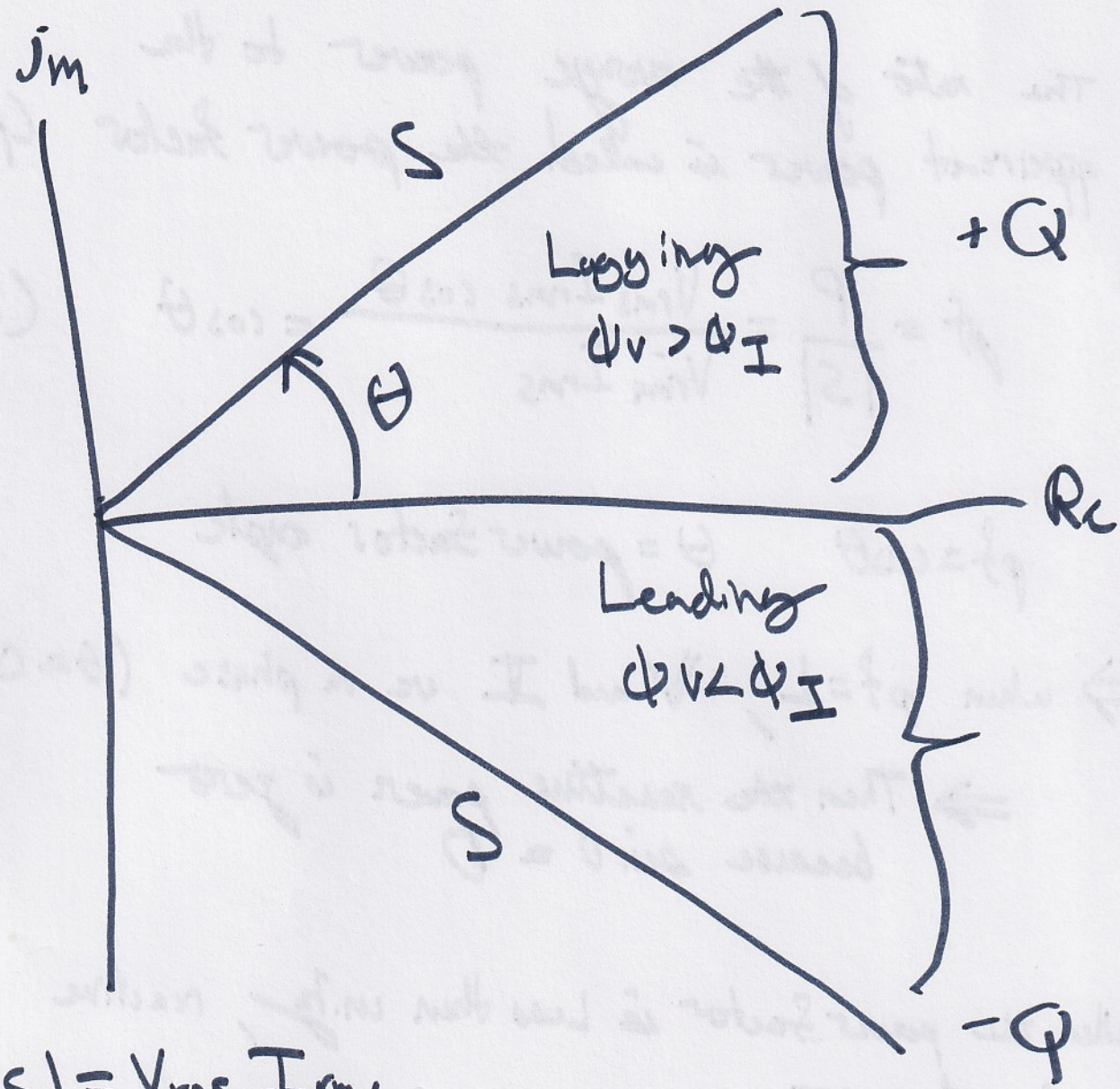
$$= \left[V_{rms} I_{rms}\right] e^{j(\underbrace{\phi_V - \phi_I}_{\theta})}$$

$$S = P + jQ \quad VA$$

avg Power

reactive power

S is a complex number, not treated as a phasor. → keeps track of the two components of power when V, I are expressed as phasors



$|S| = V_{rms} I_{rms}$

↖ Apparent Power

This triangle works to remember complex power relationships and terminology

We are confining our study to cases in which net energy is transferred from source to load.

The ratio of the average power to the apparent power is called the power factor (pf)

$$pf = \frac{P}{|S|} = \frac{V_{rms} I_{rms} \cos\theta}{V_{rms} I_{rms}} = \cos\theta \qquad (16\text{-}25)$$

$pf = \cos\theta$ $\qquad \theta$ = power factor angle

$\Rightarrow$ when $pf = 1$, $\mathbb{V}$ and $\mathbb{I}$ are in phase ($\theta = 0°$)

$\Rightarrow$ Then the reactive power is zero because $\sin 0 = 0$

When the power factor is less than unity, reactive power is not zero.

$\Rightarrow$ the sign is indicated by the modifiers Lagging or Leading

Lagging power factor (most industrial residential Loads have Lagging power factors)

$\mathbb{I}$ Lags $\mathbb{V}$

$$\phi_V - \phi_I > 0$$

$\Rightarrow$ S falls into the first quadrant

$\sin(\phi_V - \phi_I) > 0 \rightarrow$ reactive power is positive

$$Q = +|Q|$$

Leading Power Factor

$\mathbb{I}$ Leads $\mathbb{V}$

$$\phi_V - \phi_I < 0$$

$\Rightarrow$ S falls into the fourth quadrant

$\sin(\phi_V - \phi_I) < 0 \rightarrow$ reactive power is negative

$$Q = -|Q|$$

Example 16-2

Find the avg power, reactive power, and apparent power for the following voltage and current phasors. Find the power factor and state whether it is Lagging or Leading.

$$\mathbb{V} = 350\angle 45^\circ \text{ V (rms)}$$

$$\mathbb{I} = 6\angle 65^\circ \text{ A rms}$$

$$S = \mathbb{V}\,\mathbb{I}^* = (350\angle 45^\circ)(6\angle 65^\circ)$$

$$= 2100\angle -20^\circ = 2100[\cos(-20) + j\sin(-20)]$$

$$= 1973 - j718 \text{ VA}$$

$$S = P + jQ$$

complex power

Avg Power

reactive power

Avg Power = 1973 VA

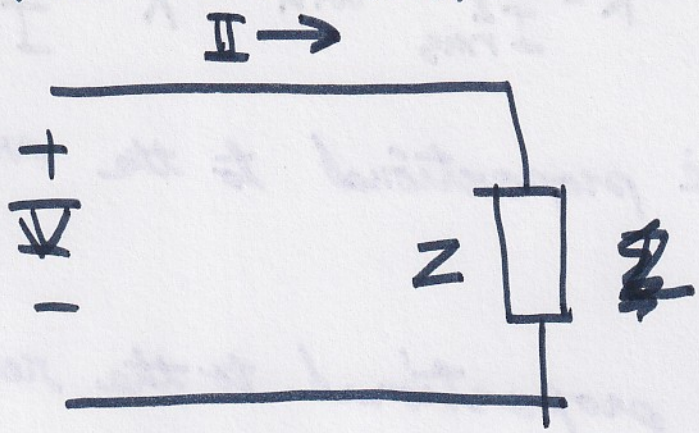
Reactive Power = $-j718$ VA

Apparent Power = $|S|$
$= 2100$

$$pf = \frac{P}{|S|} = \frac{1973}{2100} = 0.939$$

As $Q = -|Q| \rightarrow$ Leading

Complex Power and Load Impedance



$$\rightarrow \mathbb{V} = Z\,\mathbb{I}$$

~~with~~ with this constraint, $\mathbb{V} = Z\mathbb{I}$, we can write the complex power of the load as

$$S = \mathbb{V} \times \mathbb{I}^* = Z\mathbb{I} \times \mathbb{I}^* = Z\,|\mathbb{I}|^2$$

$$= (R + jX)\,I_{rms}^2$$

recall, for Power, The phasors use rms for Amplitude

R = resistance
X = reactance of the Load

As $S = P + jQ$ (16-16)

So we can see $R = \frac{P}{I_{rms}^2}$ and $X = \frac{Q}{I_{rms}^2}$

Load resistance is proportional to the average power of the load

Load reactance is proportional to the reactive power of the load

Looking at properties of Loads made up of passive resistors, capacitors, and inductors.

a) $R \geq 0$, $\rightarrow$ P (avg power) is non negative

b) For an Inductive Load ~~$X = \omega L \rightarrow$~~

$X = \omega L > 0$, Q (reactive power) is positive

c) For a capacitive Load

$X = -1/\omega C < 0 \rightarrow Q$ is negative

Inductive Load, Positive reactive power, Lagging power factor

are "equivalent statements"

Capacitive Load, Negative reactive power, Leading power factor

are "equivalent statements"

Example 16-3

At 440 V (rms) a two-terminal Load draws 3 kVA of apparent power at a Lagging power factor of 0.9.

(a) I_{rms}

(b) P

(c) Q

(d) the load impedance

Draw the power triangle for the Load.

$$|S| = V_{rms} I_{rms} \rightarrow I_{rms} = \frac{|S|}{V_{rms}} = \frac{3000}{440} = 6.82 \text{ A rms}$$

$$\frac{P}{|S|} = \cos\theta = 0.9 \qquad P = 0.9|S| = 0.9(3000) = 2.7 \text{ kW}$$

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$\arccos 0.9 = 0.451$ rads

$Q = |S| \sin 0.451 = 1.31$ kVAR

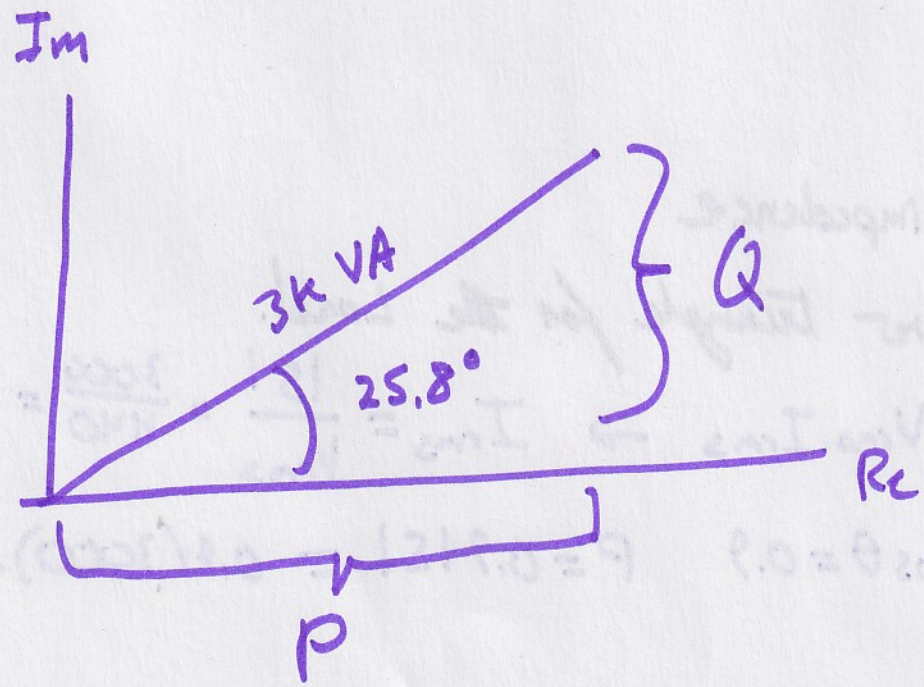
$S = \mathbb{V} \times \mathbb{I}^* = Z\mathbb{I} \times \mathbb{I}^* = Z|\mathbb{I}|^2$

$= \underline{(R + jX)\, I_{rms}^2}$

$\rightarrow R + jX = \dfrac{S}{I^2_{rms}} = \dfrac{P + jQ}{I^2_{rms}}$

$= \dfrac{2700 + j1310}{(6.82)^2} = 58.0 + j28.2\ \Omega$

0.451 rads $\times \dfrac{180}{\pi} = 25.8^\circ$, $|S| = 3$k VA



16-3 Single-Phase Circuit Analysis

single-phase circuits (analysis) deals with linear circuits in the sinusoidal steady state

~~single phase~~

Single phase: all of the ac sources have the same phase angle. (And assume they have the same frequency)

The Overall point (the objective of our analysis) is to find the complex power produced by the sources or delivered to specific loads

$\Rightarrow$ This is very similar to phasor circuit analysis but we are looking for complex power

$$S = \mathbb{V}\mathbb{I}^*, \quad S = |\mathbb{I}|^2 Z$$

Conservation of complex power

In a linear circuit the sum of the complex powers produced by all the sources is equal to the ~~sum~~ sum of the complex powers delivered to all of the passive loads.

So what does 'produced by' and 'delivered to' mean?

→ In ac power systems, we modify our practice of always using the passive sign convention...
(current directed in at the + voltage reference)

to the <u>Active convention</u> (current directed out at the + voltage reference) for all surface.

Complex power is always $S = \mathbb{V}\mathbb{I}^* = P + jQ$

→ The net result is that the average power "produced by" a source or "delivered to" a load has the same sign.

Example 16-4

A load consisting of a 2.5-kΩ resistor in parallel with a 2 μF capacitor is connected across a 440 Vrms, 60 Hz voltage source. Find the complex power delivered to the load and the load power factor. State whether the power factor is leading or lagging.

The source angular frequency is $\omega = 2\pi 60 \approx 377$ rad/s

$$Z_L = R \| Z_C = \frac{R\left(\frac{1}{j\omega C}\right)}{R + \frac{1}{j\omega C}} = 549 - j1035\ \Omega$$

$$|\mathbb{I}| = \frac{|\mathbb{V}|}{|Z_L|} = \frac{440}{1171.6} = 0.375\ A\ rms$$

$$S_L = |\mathbb{I}|^2 Z_L = (0.375)^2 (549 - j1035)$$

$$= 77.4 - j146\ VA$$

$$= \underset{\text{Avg Power}}{P_L} - j\underset{\text{reactive power}}{Q_L}$$

$$pf = \frac{P_L}{|S_L|} = 0.469 \qquad -Q_L \rightarrow \text{Leading}$$

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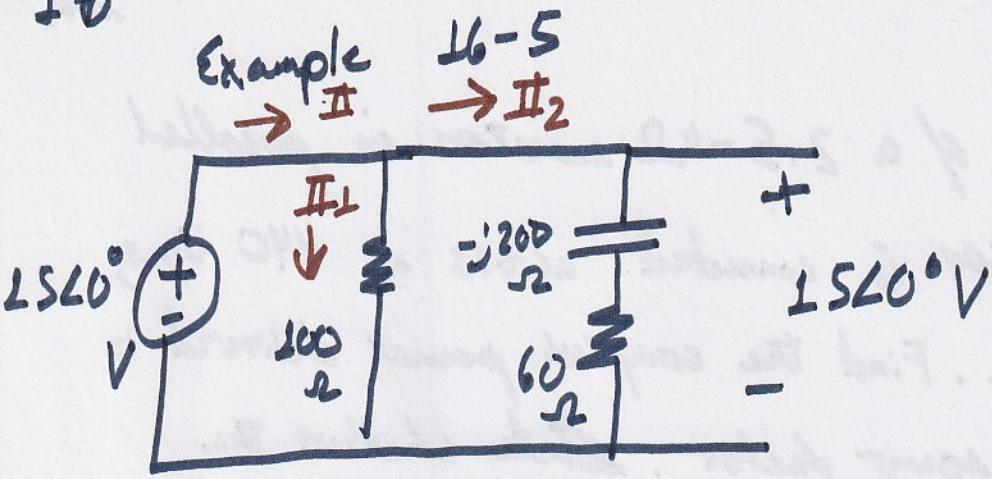
Example 16-5



(a) find the complex power delivered to each load.

(b) Find the complex power produced by the source.

$$\mathbb{I}_1 = \frac{15\angle 0^\circ}{100} = 0.15\angle 0^\circ \text{ A rms}$$

$$\mathbb{I}_2 = \frac{15\angle 0^\circ}{60 - j200} = 0.0718\angle 73.3^\circ \text{ A rms}$$

$$S_1 = |\mathbb{I}_1|^2 Z_1 = (0.15)^2(100) = 2.25 \text{ VA}$$

$$S_2 = |\mathbb{I}_2|^2 Z_2 = (0.0718)^2(60 - j200) = 0.309 - j1.03 \text{ VA}$$

As $\mathbb{I} = \mathbb{I}_1 + \mathbb{I}_2$, $S = \mathbb{V}\mathbb{I}^*$

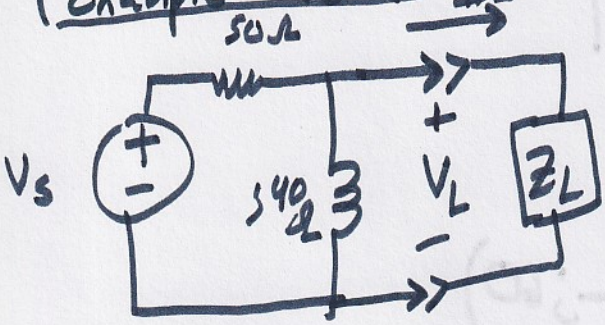
→ Due to the conservation of complex power

The power "produced by" the source is the sum of the powers "delivered to" the Loads

$$S = S_1 + S_2 = P + jQ = 2.56 - j1.03 \text{ VA}$$

$pf = \frac{P}{|S|} = 0.928$, $Q = -|Q| \rightarrow$ leading

Example 16-6



The load Z_L is a 100-Ω resistor in series with a capacitor whose reactance is −60 Ω. The source voltage is 880 V rms. Find the complex power delivered to the load and the load power factor.

Use Node Voltage

$$-\frac{V_A - V_s}{50} - \frac{V_A}{j40} - I_L = 0 \qquad V_A = V_L$$

$$-\frac{V_L - V_s}{50} - \frac{V_L}{j40} - \frac{V_L}{Z_L} = 0$$

$V_s = 880\angle 0^\circ$ V , $Z_L = 100 - j60\ \Omega$

$$V_L\left(\frac{1}{50} + \frac{1}{j40} + \frac{1}{Z_L}\right) = \frac{V_s}{50}$$

$V_L = 411 + j309$ Vrms

to get $\mathbb{I}_L$

$$|\mathbb{I}_L| = \frac{|\mathbb{V}_L|}{|Z_L|} = \frac{|411 + j309|}{|100 - j60|} = 4.41 \text{ A rms}$$

The power delivered to the Load

$$S_L = |\mathbb{I}_L|^2 Z_L = 4.41^2 (100 - j60)$$

$$= (1.95 - j1.17) \times 10^3 \text{ VA}$$

The power factor

$$pf = \frac{P_L}{|S_L|} = \frac{1.95 \times 10^3}{|S_L|} = 0.857$$

$Q = -|Q|$, Leading

Evaluation Example 16-7

→ On another document