

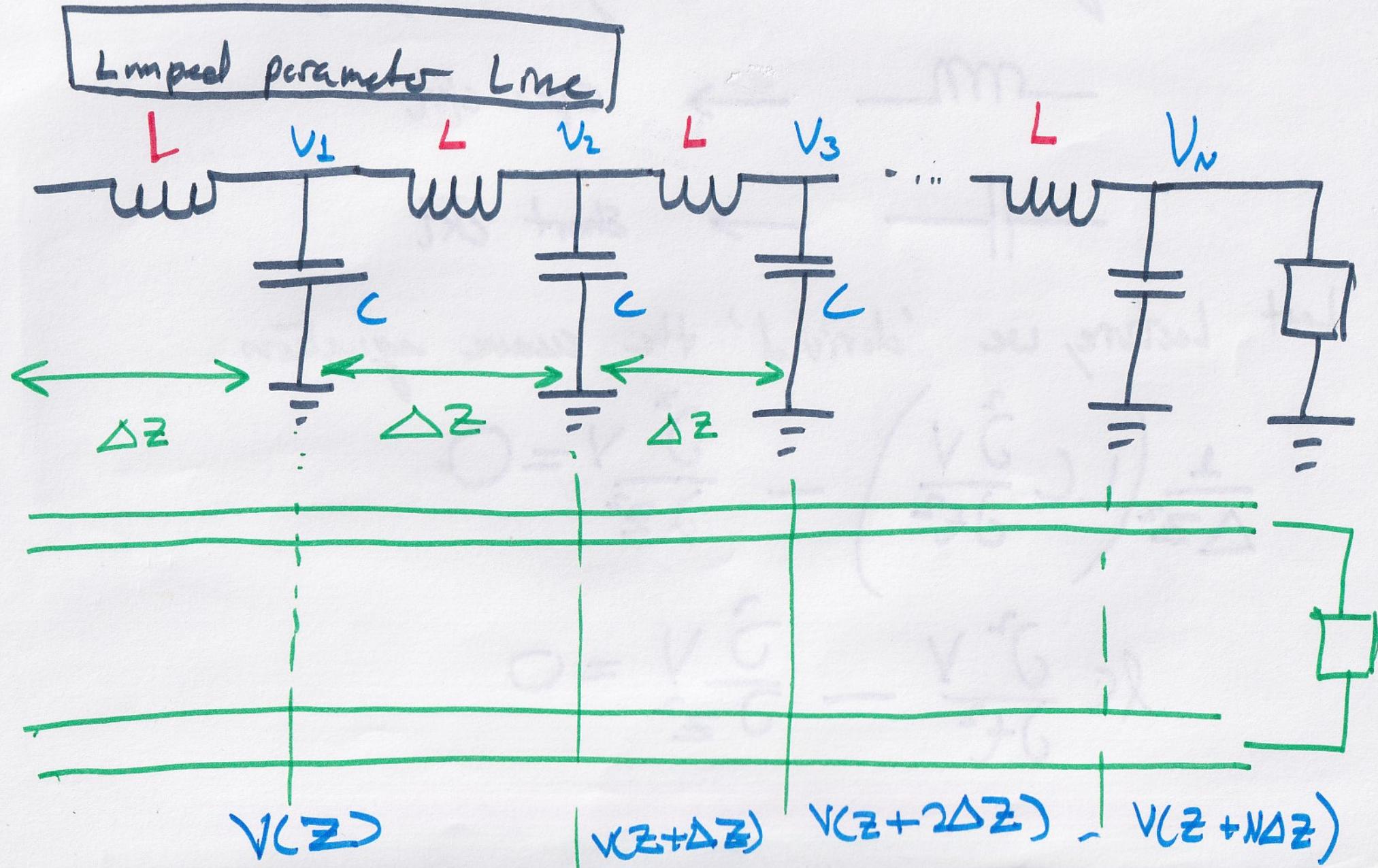
Fields and Waves I

L2

①

Transmission Lines

The Travelling Wave



(2)

Recall from circuits

as Frequencies increase (goes really high)

M → open ckt

|| → short ckt

Last Lecture, we 'derived' the wave equation

$$\frac{1}{\Delta z^2} \left(L C \frac{\partial^2 V}{\partial t^2} \right) - \frac{\partial^2 V}{\partial z^2} = 0$$

$$LC \frac{\partial^2 V}{\partial t^2} - \frac{\partial^2 V}{\partial z^2} = 0$$

(3)

$$l \rightarrow \frac{H}{m} \quad C \rightarrow \frac{F}{m}$$

We can say

$$V(z, t) = f(t \pm \frac{1}{u} z) \quad u \text{ is a constant}$$

$$\begin{aligned} " \Rightarrow " &= f(t + \frac{1}{u} z) \quad f(t) \text{ is a function} \\ &= f(t - \frac{1}{u} z) \end{aligned}$$

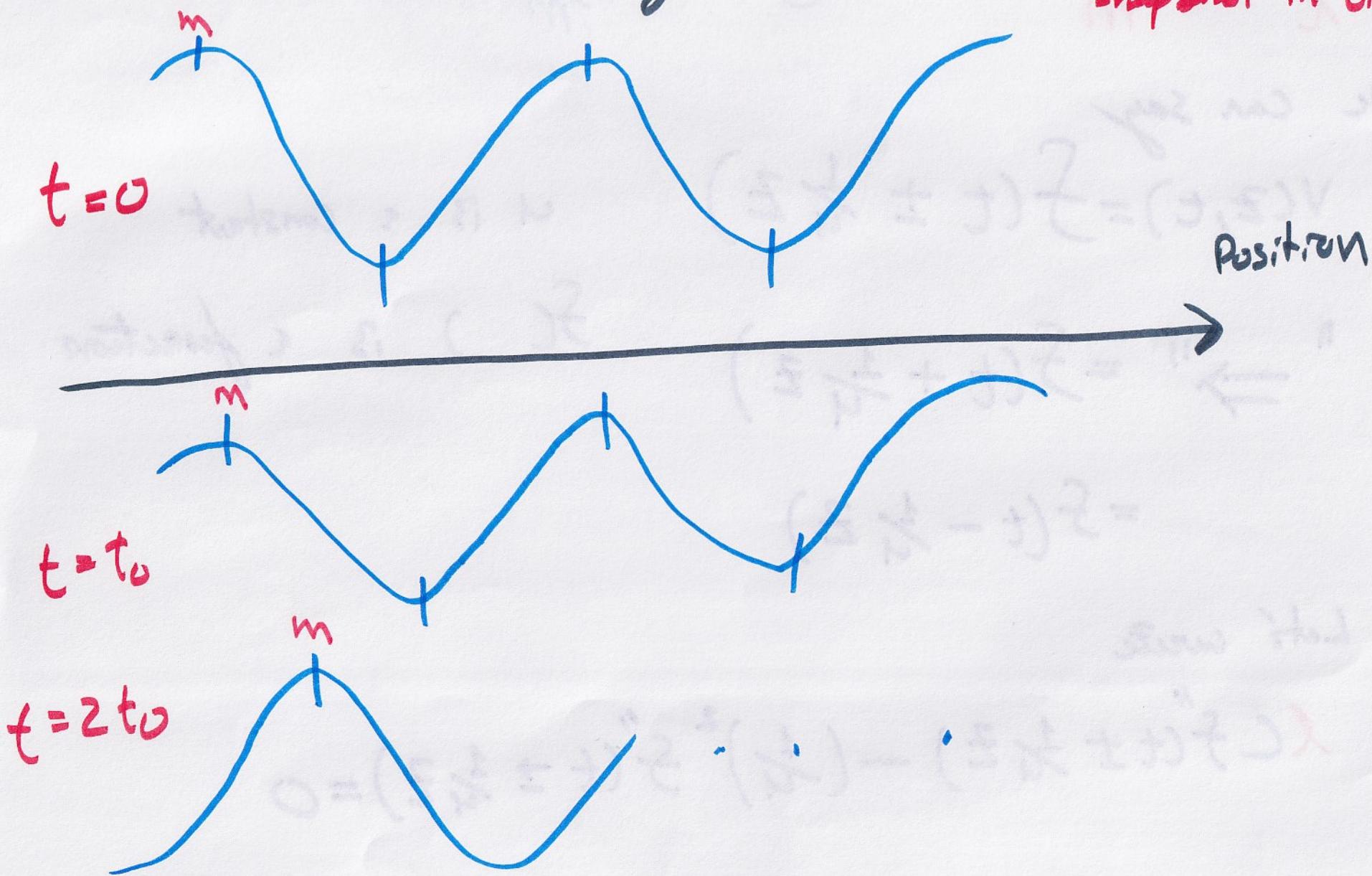
So Let's write

$$LC f''(t \pm \frac{1}{u} z) - \left(\frac{1}{u}\right)^2 f''(t \pm \frac{1}{u} z) = 0$$

④

Solutions are traveling waves

"snapshot in time"



(5)

as time changes, position changes

so the function evaluates at m at a position in time

If you follow a peak, the function $f(t - \frac{1}{u}z)$ is constant with changing t, z

\Rightarrow the argument of f must be constant

$$t - \frac{1}{u}z = \text{constant}$$

$$z = -u \text{constant} + ut$$

⑥ $\frac{\partial}{\partial t} Z = \frac{c}{\rho t} (ut + u \text{ constant})$

$$\frac{\partial}{\partial t} Z = u \quad \rightarrow \text{velocity of the wave}$$

$v_p \Rightarrow$ velocity of propagation

In Vacuum $v_p = c = 3 \times 10^8 \text{ m/s}$

"Air-filled Transmission Lines can go the speed of c "

$$u = \frac{1}{\sqrt{LC}}$$

$$l = \frac{H}{m} \quad C = \frac{F}{m}$$

(7)

$$V_p = \frac{l}{\sqrt{LC}}$$

Velocity is inversely proportional to
capacitance and inductance

$$V_p = \frac{c}{\sqrt{\epsilon_r}}$$

\leftarrow relative permittivity

Impedance

$Z \Rightarrow$ impedance, Ω

(same characteristic, different physical interpretation)

⑧

Voltage wave expression

$$LC \frac{d^2}{dt^2} V - \frac{V^2}{Z^2} = 0$$

same solution
space, different
coefficients

Current wave equation

$$\sim \frac{J^2}{Zt^2} I - \frac{J^2}{Z^2} I = 0$$

$$V(z, t) = f(t \pm \frac{L}{c} z)$$

$$V = V_0 \cos(\omega t - \beta z)$$

$$= V_0 \cos(\omega t - \frac{\beta}{\omega} z)$$

This is a wave
"a wiggly

what does the current ~~to~~ look
like?

(9)

$$I = I_0 \cos(\omega t - \beta z)$$

$$I = \frac{V_0}{Z} \cos(\omega t - \delta z) \rightarrow Z = \frac{V_0}{I} \cos(\omega t - \beta z)$$

$$\rightarrow Z_0 = \sqrt{\frac{L}{C}}$$

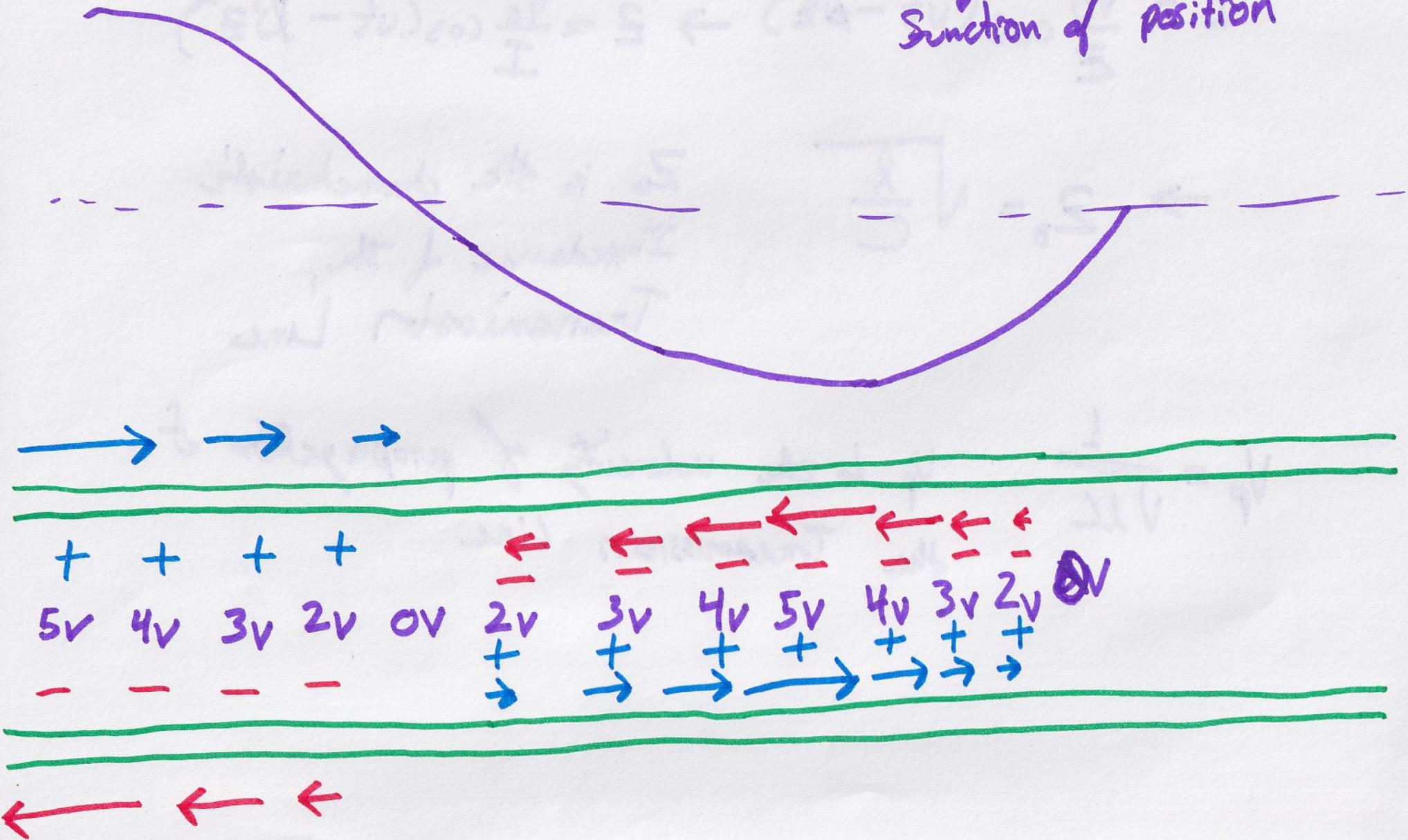
Z_0 is the characteristic
Impedance of the,
Transmission Line

$$V_p = \frac{1}{\sqrt{LC}}$$

V_p is the velocity of propagation of
the Transmission Line

⑩ A lossless Transmission Line doesn't dissipate power

Voltage varies as a function of position



11

Voltage varies as a function of position, moving with position and changing ~~not~~ time - it's dynamic with

General function

$$V(z, t) = f(t \pm \frac{1}{u} z)$$

Sinusoidal Travelling wave

$$V(z, t) = V_0 \cos(\omega t \pm \beta z)$$

Will a square wave propagate?

→ Yes! Because it's made of sinusoids

(12) → put in any type of signal, it works!

(Fourier Series)

Look at

$$V_p = \frac{1}{\sqrt{LC}}$$

Does changing the applied frequency change V_p ?

These guys are frequency dependent

Sinusoidal Travelling Wave

$$V(z,t) = V_0 \cos(\omega t \pm \beta z)$$

ω - radial Frequency

$$2\pi f = \omega$$

β - propagation constant

$$\frac{2\pi}{\lambda} = \beta$$

(13)

And $\lambda f = V_p$

$\lambda \rightarrow$ wave length m
 $f \rightarrow$ frequency Hz
 $V_p \rightarrow$ velocity m/s

$$\frac{2\pi}{\lambda} f = V_p \frac{2\pi}{\lambda}$$

$$2\pi f \left(\frac{\lambda}{2\pi}\right) = V_p = \omega \frac{1}{\beta} = V_p = \frac{\omega}{B} = \frac{1}{\sqrt{LC}}$$

As Frequency goes up

$f \uparrow$	$\lambda \downarrow$	$L \uparrow$	$C \downarrow$
		$\frac{1}{sL}$	$\frac{1}{sC}$

As Frequency goes down

$f \downarrow$	$\lambda \uparrow$	$L \downarrow$	$C \uparrow$
----------------	--------------------	----------------	--------------

frequency affects
The impedance of
 L, C

(14)

From this, we can say,

$$\Delta z \ll \lambda$$

For valid lump parameter

equivalence, the effective length of an L-C section must be much smaller than a wave length.

The Role of Wavelength

So we consider the speed of current.

Perhaps it can be said that current moves near the speed of light. C is the speed of EM waves

$\rightarrow L, C$ are determined by Δz

and so L, C must be a certain value for the solution of the wave eqn.

Do we have

$$V(z, t) = V_0 \cos(\omega t \pm \beta z)$$

\Rightarrow Real and measurable wave

$$\text{Re} \left\{ V(z, t) = V_0 e^{j(\omega t \pm \beta z)} \right\}$$

\rightarrow complex exponential wave solution

Phasor form of wave expression

$$V(z) = V_0 e^{\pm j \beta z} \{ e^{j \omega t} \}$$

implied oscillation in time

what does ' \pm ' mean?

\rightarrow The direction of propagation

16

$+ \rightarrow$ propagation is in $-z$ direction
 $- \rightarrow$ propagation is in $+z$ direction

$+$ \rightarrow left

$-$ \rightarrow right

$$V_{\text{forward}} = V_f e^{-jBz}$$

$$V_{\text{backward}} = V_b e^{+jBz}$$

$\rightarrow z$

To Note

$$1 \text{ Henry} = \frac{1 \text{ Weber}}{1 \text{ ampere}} = \frac{1 \text{ volt-second}}{1 \text{ ampere}} = 1 \text{ ohm-second}$$

$$(1 \Omega = 1 \frac{V}{A} \left(\frac{\text{Volts}}{\text{Amperes}} \right))$$

\rightarrow 1 henry is the amount of inductance necessary to induce one volt when the current in coil changes at a rate of one ampere per second.

$$\cancel{\frac{\text{Volt}}{\text{Ampere}}} \frac{\text{second}}{\text{second}} = \frac{\text{V-S}}{\text{A}}$$

$$1 \text{ Farad} = \frac{\text{Coulomb}}{\text{Volt}} = \frac{\text{Seconds}}{\text{Ohm}} = \frac{s}{\Omega}$$

$$H = s-L \\ (\Omega - s)$$

$$F = \frac{s}{L}$$

$$c = \frac{1}{Z_0 V_p} \\ (E_m)$$

$$l = \frac{Z_0}{V_p} \\ (H_m)$$