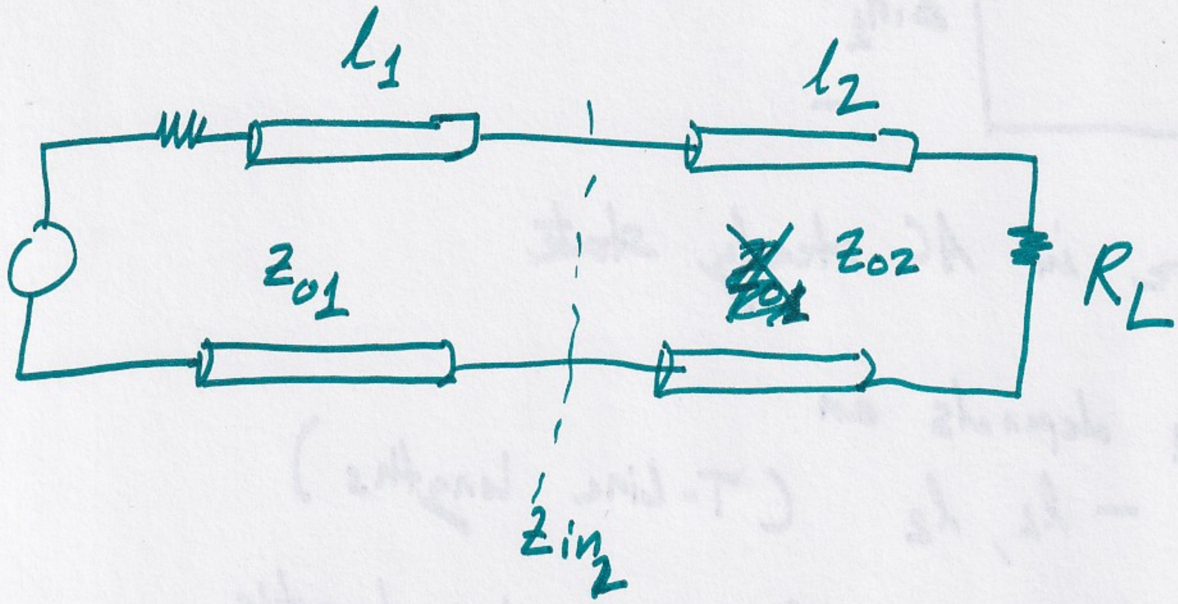
Fields and Waves I L7

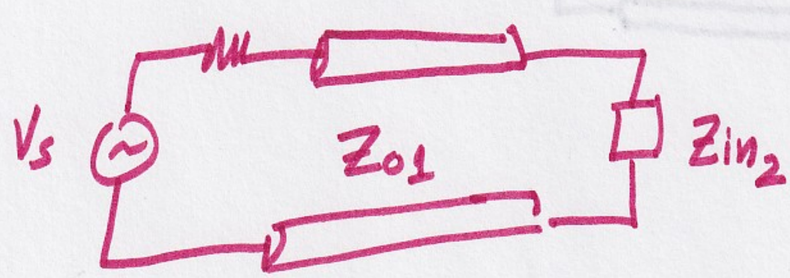
Lossy Transmission Lines



we pay attention to the AC steady state point of view

$$Z_{in_2} = Z_{02} \frac{Z_L + jZ_{02}\tan\beta l_2}{Z_{02} + jZ_L \tan\beta l_2}$$

So then we have

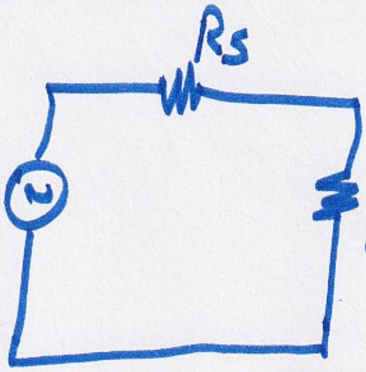


$$Z_{in_1} = Z_{01} \frac{Z_{in_2} + jZ_{01}\tan\beta l_1}{Z_{01} + jZ_{in_2}\tan\beta l_1}$$

The effective load for Line 1

2

So then we have

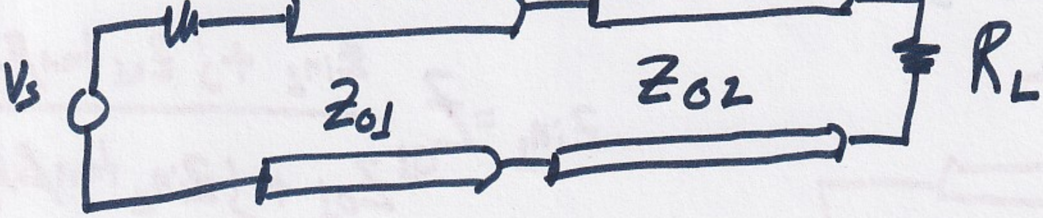


+ Vin$_1$

−

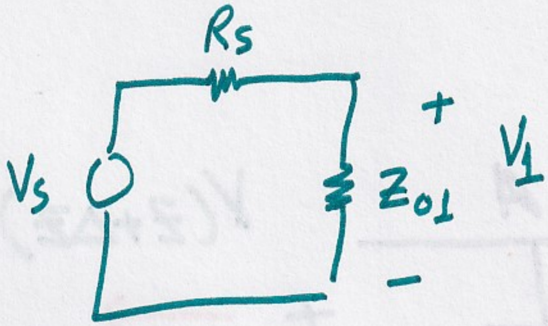
we are in AC steady state

Vin$_1$ depends on
- l_1, l_2 (T-line lengths)
- Z_{01}, Z_{02} (T-line characteristic Impedence)
- Z_L (Load)



Pulse Analysis

V_s turns on and sends a pulse

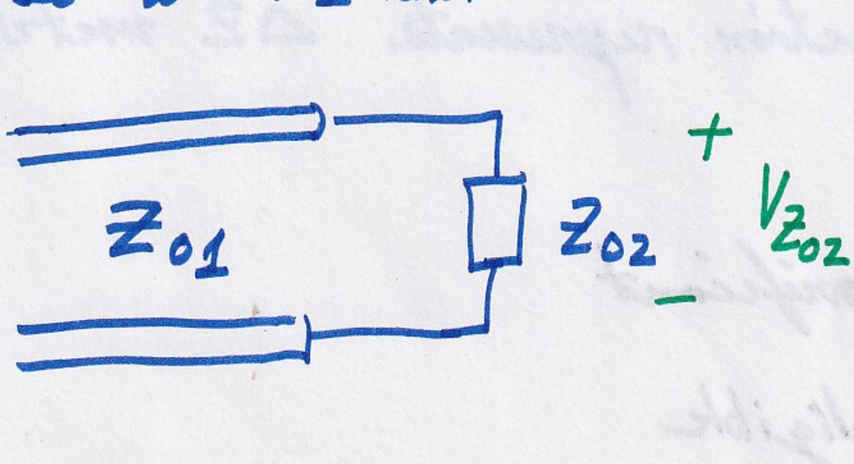


The Impedance seen is the Characteristic Impedance
(All that matters is the characteristic Impedance)

So the pulse hits $Z_{01} - Z_{02}$ junction
→ Then you apply reflection concepts!

For T1 (T-line #1), the load is $\underline{Z_{02}}$

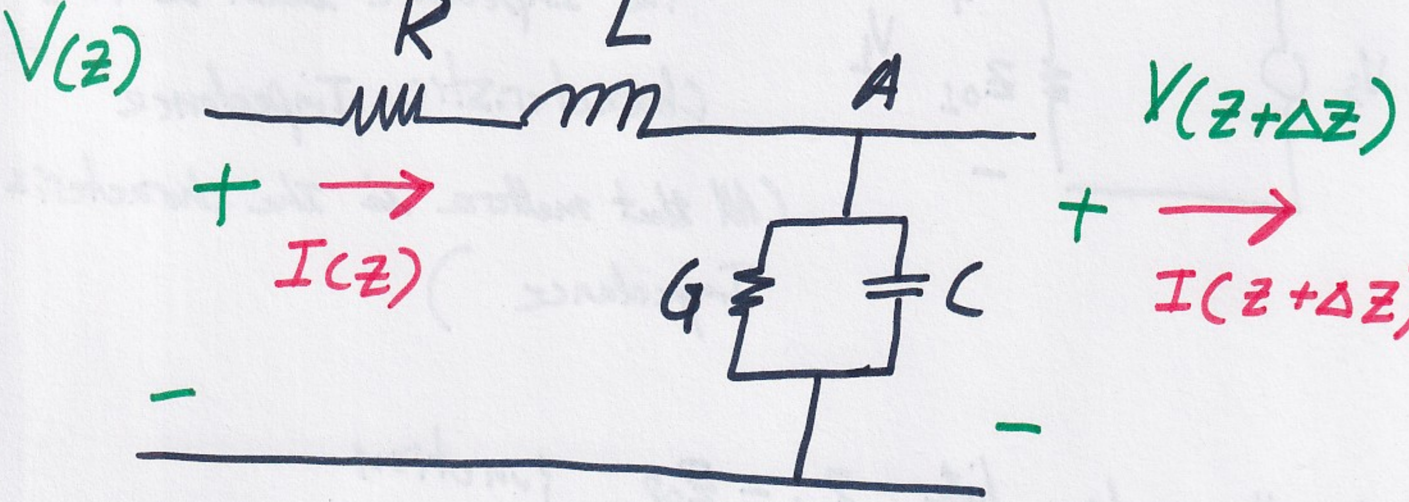
→ So at T1 end



V_{Z02} gets sent down T2
(we will have reflection if $\Gamma \neq 0$)

Lossy Lines

Lumped Parameter Model



$V = IR$ (Ω) R → resistance of the wires

$VG = I$ $\left(\frac{1}{\Omega}\right)$ G → Leakage current between wires

siemen

one section represents Δz meters

typically,

R is significant

G is negligible

How does R, and G affect the wave equation?

KCL at A

$$\frac{1}{\Delta z}\left[I(z+\Delta z) - I(z) = c\frac{d}{dt}V(z+\Delta z) + VG \right]$$

current flows into A

current flows out of A into capacitor

current flowing into the conductance

(or the current is actually into A, may not matter)

Let $\Delta z \rightarrow$, this becomes a derivative

(This is the 'Idea')

$$\frac{d}{dz} I = c\frac{d}{dt}V + gV$$

$\frac{C}{m}$

Capacitance per meter

G/m ↖ conductance per meter

Lets do KVL

$$\frac{1}{\Delta z}\left[V(z+\Delta z) - V(z) = V_L + V_R \right.$$

$$\left. = L\frac{dI(z)}{dt} + I(z)R \right]$$

Take $\Delta z \to 0$

$$\frac{d}{dz}V(z) = l\frac{d}{dt}I(z) + rI(z)$$

L/m Inductance per meter

R/m Resistance per meter

Take the derivative

$$\frac{d^2}{dz^2}V = l\underbrace{\frac{d}{dz}\frac{d}{dt}}I + r\frac{d}{dz}I$$

$$l\frac{d}{dt}\frac{d}{dz}I$$ (we can do this, see Lecture 1)

$$lc\frac{d^2}{dt^2}V + gl\frac{d}{dt}V$$

$$rc\frac{d}{dt}V + rgV$$

so the Lossy wave equation

$$\frac{d^2}{dz^2}V = lc\frac{d^2}{dt^2}V + (rc+gl)\frac{d}{dt}V + rgV$$

So, skipping a Lot of steps, we eventually get

$$V = V_0 e^{j(\omega t \pm \beta z)}$$ A function of t and z

could have oscillation

This is the assumed solution to the wave equation.

Plug in the assumed "$e^{j\omega t}$" into the differential equation.

$$\frac{d^2}{dz^2}V = -\omega^2 V + j\omega(rc+gl)V + rgV$$

let $\gamma = \sqrt{(r+j\omega l)(g+j\omega c)}$

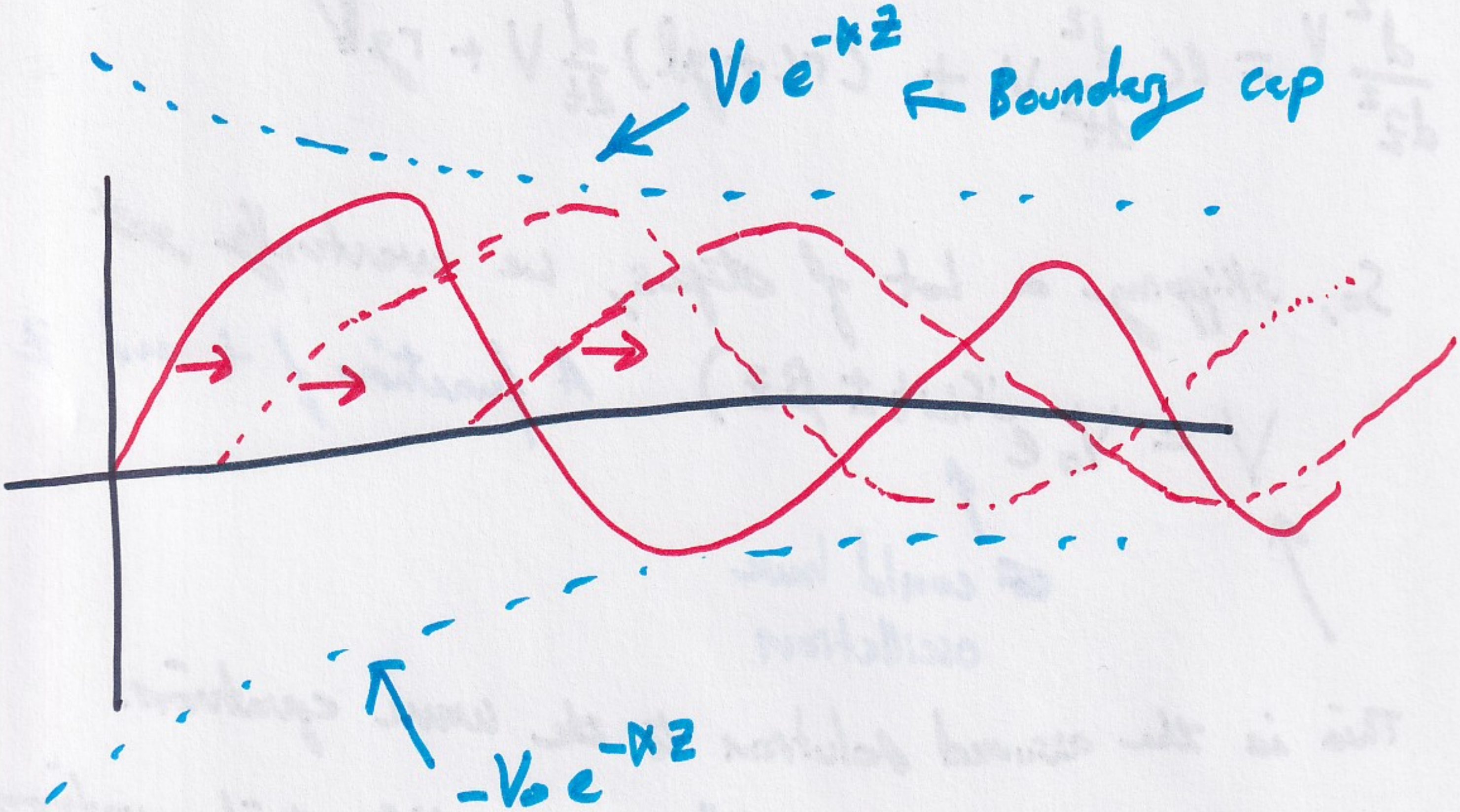
$= \alpha + j\beta$ ← propagation constant $\frac{2\pi}{\lambda}$

attenuation constant →

In phasor form

$$V_{forward} = V_0 e^{-\alpha z} e^{-j\beta z}$$

V_0 = amplitude of wave at $z=0$



for the most part, $G \rightarrow 0$ (assumed)
and ~~R<<jωL~~ $R << j\omega L$

→ we have a <u>Low loss Line</u> for these conditions

So as a consequence,

$$\gamma = \sqrt{(r + j\omega l)(g + j\omega C)}$$

$$\rightarrow \sqrt{(r + j\omega L)(j\omega C)}$$

After some Algebra

$$\gamma = \left(\sqrt{(j\omega C)(j\omega l)}\right)\sqrt{\left(1 + \frac{r}{j\omega l}\right)}$$

$$= \pm j\beta\left(1 + \frac{r}{2j\omega l}\right)$$

recall $\sqrt{1+x} \approx 1 + \frac{x}{2}$ for $x \ll 1$

So we can write

$$\gamma = \pm j\beta\left(1 + \frac{r}{2j\omega l}\right) = \pm(j\beta + \alpha)$$

So $\alpha = \frac{\beta r}{2\omega l}$, $V = V_0 e^{-\alpha z} e^{-j\beta z}$

So what does this mean in terms of Power?

$$P = \frac{1}{2} V I^*$$

$$= \frac{1}{2}\left[V_0 e^{-\alpha z} e^{-j\beta z}\right]\left[I_0 e^{-\alpha z} e^{+j\beta z}\right]$$

$$= \frac{1}{2} V_0 I_0 e^{-2\alpha z}$$

$$= \frac{1}{2} \frac{|V_0|^2}{Z_0} e^{-2\alpha z}$$

(We are ignoring the fact that Z_0 may not be real) → complex?

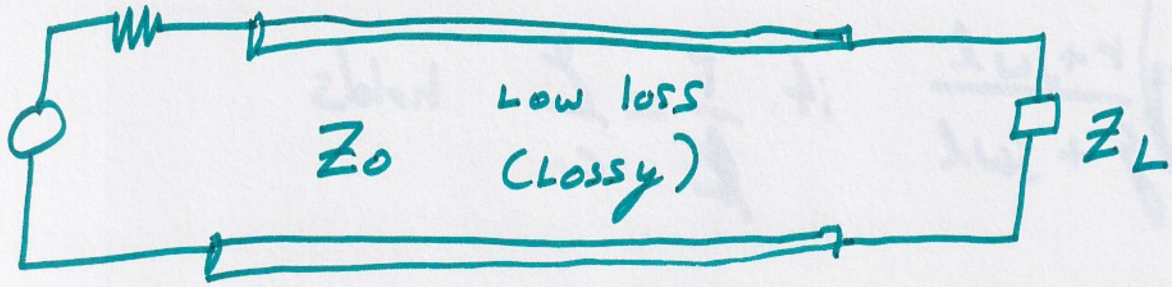
$$= P_{forward}$$

Lossy

$$Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega c}}$$

Lossless

$$Z_0 = \sqrt{\frac{l}{c}}$$



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$ This is valid for Lowloss

$$V = V_0 e^{-\alpha z} e^{-j\beta z}$$

$$P_{forward} = \frac{1}{2} \frac{|V_0|^2}{|Z_0|} e^{-2\alpha z}$$

if we say $2\alpha z = 10$ this is large and T-line is Long

so for mismatched Load
the attenuation may greatly Lower
the power reflected.

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$$Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega l}}$$ if $\frac{r}{l} = \frac{g}{c}$ holds

$Z_0 \to$ real

T-Line is Lossy so α varies with freq