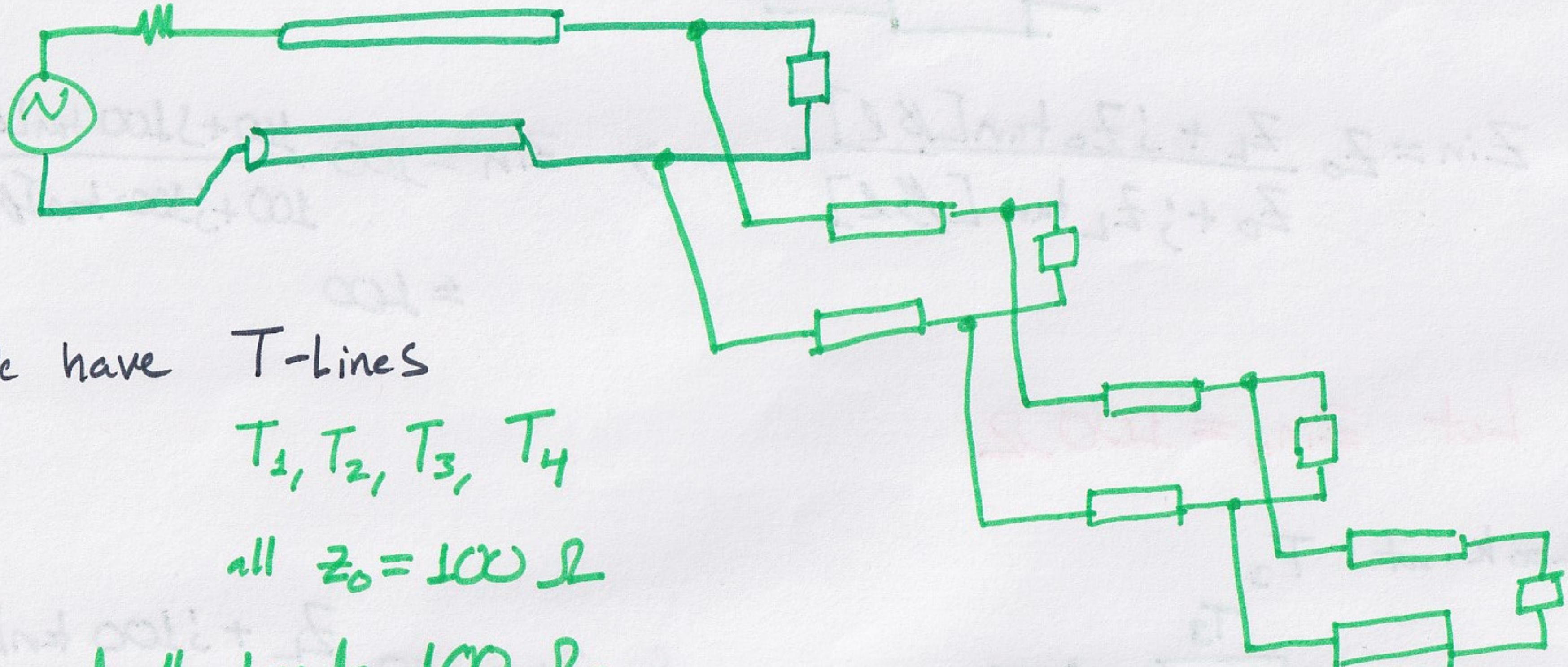
Fields and Waves I L5

The Lattice Diagram



We have T-Lines

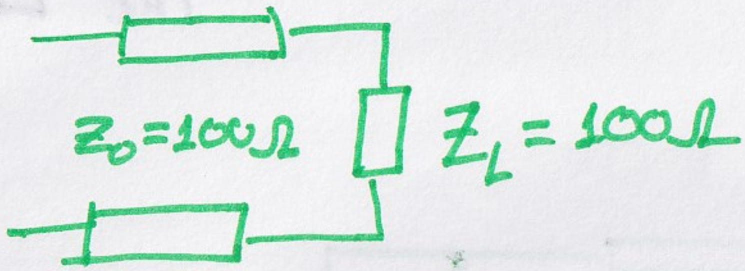
T_1, T_2, T_3, T_4

all $Z_0 = 100\ \Omega$

and all Loads $100\ \Omega$

So what are the Input Impedances?

(2) Look at T_4

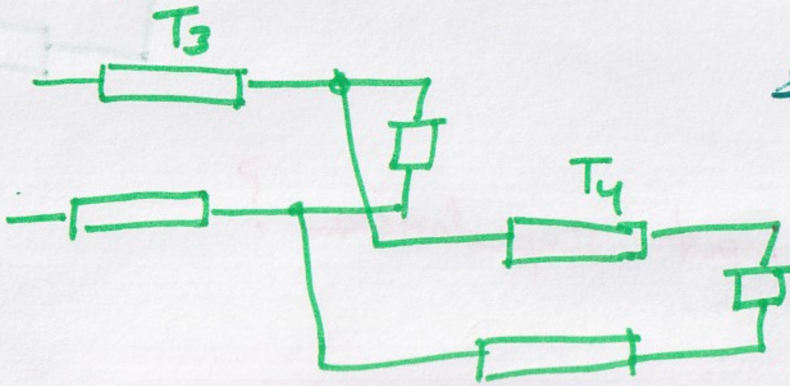


$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan[\beta l]}{Z_0 + jZ_L \tan[\beta l]} \rightarrow Z_{in} = 100 \frac{100 + j100 \tan[\beta l]}{100 + j100 \tan[\beta l]}$$

$$\doteq 100$$

Let $Z_{in_4} = 100\,\Omega$

Look at T_3



$$\text{so } Z_{in_3} = 100 \frac{Z_L + j100 \tan[\beta l]}{100 + jZ_L \tan[\beta l]}$$

$$Z_L = 100 \| 100 = 50\ \Omega = Z_{in_4} \| 100$$

$$\cancel{= Z_{in_4} \| 100}$$

The Load for T_3 is Z_{in_4} in parallel with a $100\ \Omega$ Load.

So what's Z_{in_3}?

if we Let $\beta\ell = 0, 2\pi, 4\pi \ldots$ etc

$$Z_{in_3} = 100 \frac{50\Omega + j100(0)}{100 + j50\Omega(0)} = 50\ \Omega$$

So Looking at T_2, the effective Load for T_2 is

$$Z_L = Z_{in_3} \| 100\ \Omega = 50 \| 100 \approx 33\ \Omega$$

So then

$$Z_{in_2} = 100 \frac{33 + j100 \cancelto{0}{\tan(\beta\ell)}}{100 + j33 \cancelto{0}{\tan(\beta\ell)}} = 33\ \Omega$$

④ So for T_1, the effective load is

$$Z_{in_2} \parallel 100\,\Omega = 33 \parallel 100 \simeq 25\,\Omega$$

If we keep adding Loads, eventually the effective Loads become

effective Loads $= 0$

so then $\Gamma = 1$

Basically, theres a limit to how many people can be on the line.

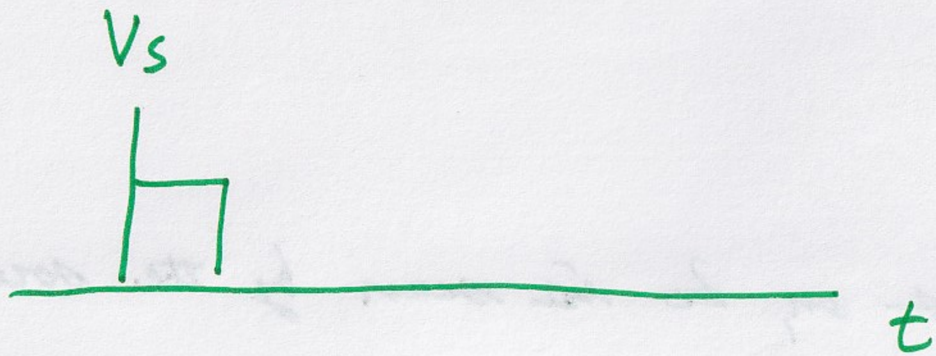
Let's talk about ~~shot~~ short pulses

we have done AC Steady state problems

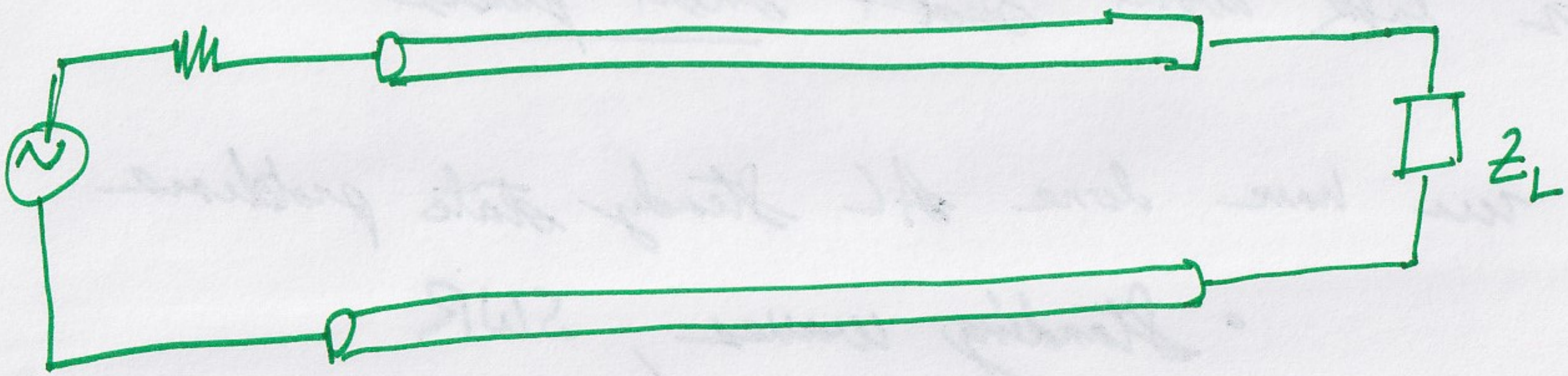
- Standing waves, SWR
- Input Impedance, Z_{in}

These are not used for pulse Problems

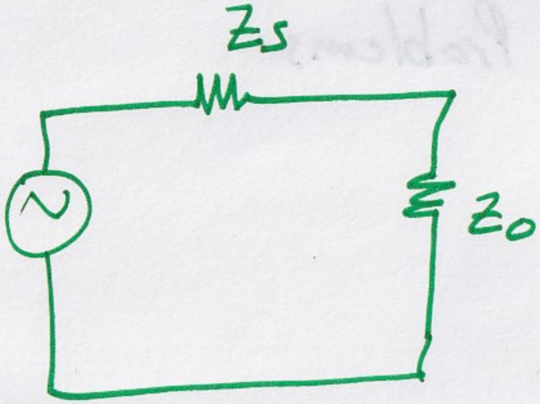
Short pulses



⑥



At $t=0$, what circuit does the source see?



Z_S is the internal Impedance

Z_0 is the characteristic Impedance

→ When the source turns on, Z_0 is seen by the source circuit

A pulse propagates down the line

$V_1^+ \leftarrow$ forward propagating pulse

Ex)

$V_s = 20\ V$ Short Pulse (from the generator)

$Z_L = Z_0 = Z_s = 50\ \Omega$

So then

$$V_1^+ = \frac{Z_0}{Z_0 + Z_s} V_s = 10V$$

What happens when the pulse reaches the Load?

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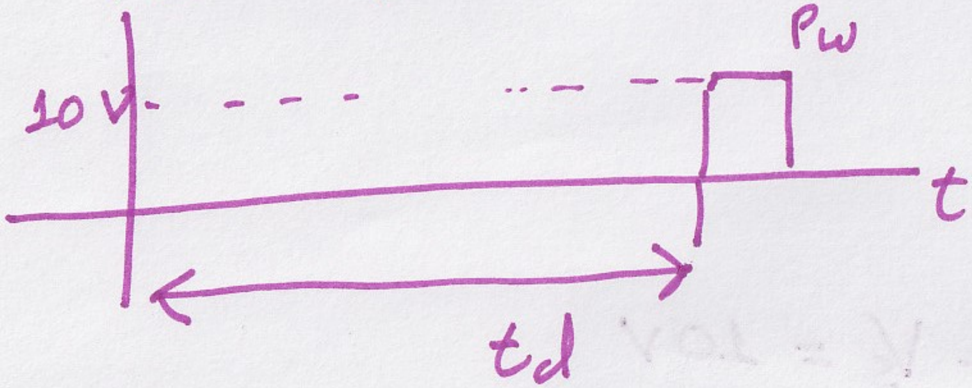
V_{in} ($V_{in} = \frac{Z_0}{Z_0 + Z_{in}} V_S$) (V_{in} has the Amplitude of V_1^+)

V_L (Voltage at the Load, V_L)

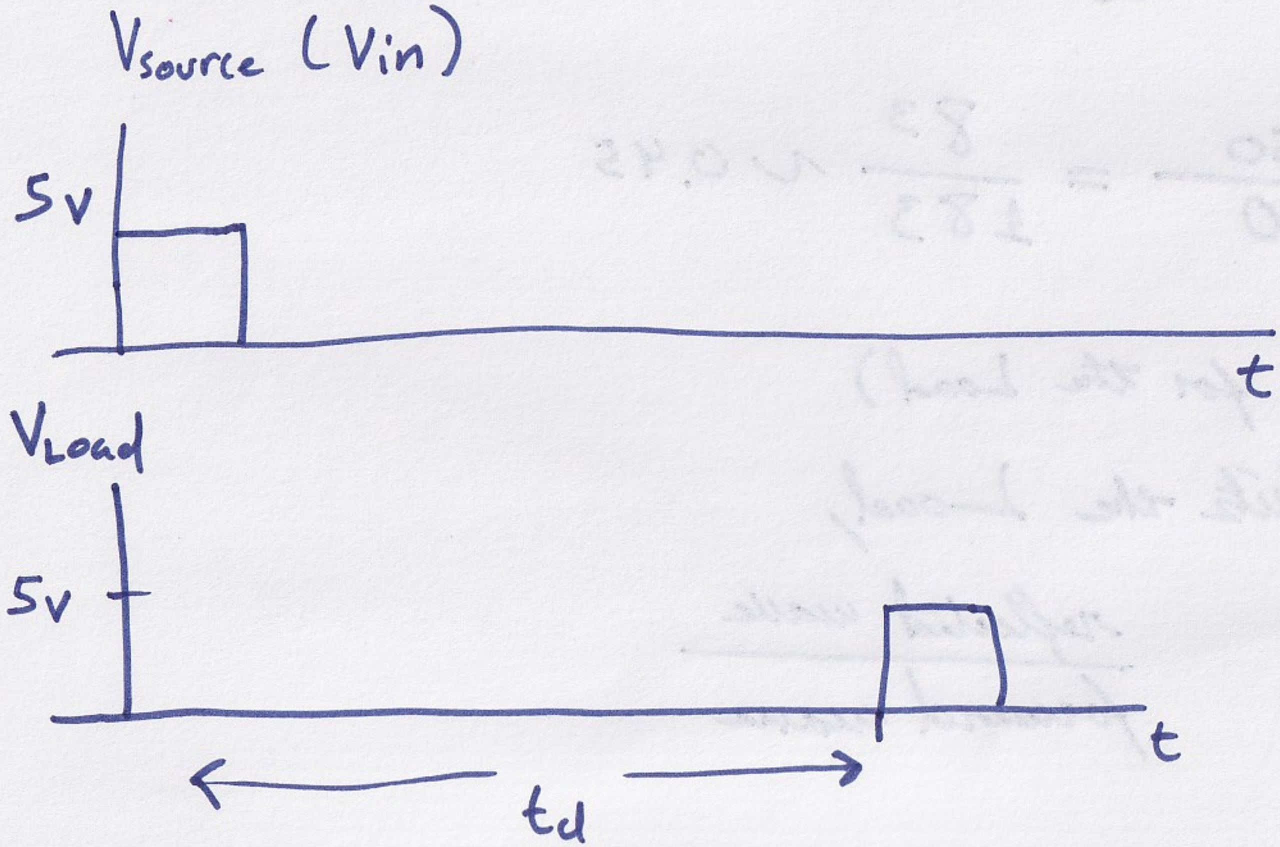


We see that the pulse takes td time to reach the Load. So it's important to note,

a short pulse implies $\rightarrow$ pulsewidth $<<$ td

Ex) $Z_s = 150\ \Omega \qquad Z_0 = Z_L = 50\ \Omega$

$$V_1^+ = \frac{50}{50 + 150}(20\ V) = 5\ V$$



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Ex) $Z_L = 133\ \Omega$ $Z_0 = Z_s = 50\ \Omega$ $V_s = 20\,V$

at $t=0$

$$V_1^+ = \frac{50}{50+50}(20) = 10\,V$$

$$\Gamma_L = \frac{133-50}{133+50} = \frac{83}{183} \sim 0.45$$

(reflection coefficient for the Load)

When the pulse hits the Load,

$$\Gamma_L = \frac{|V^-|}{|V^+|} \qquad \frac{\text{reflected wave}}{\text{forward wave}}$$

$V_1^+ \longrightarrow$ first forward pulse

$V_1^- \longrightarrow$ first reflected pulse

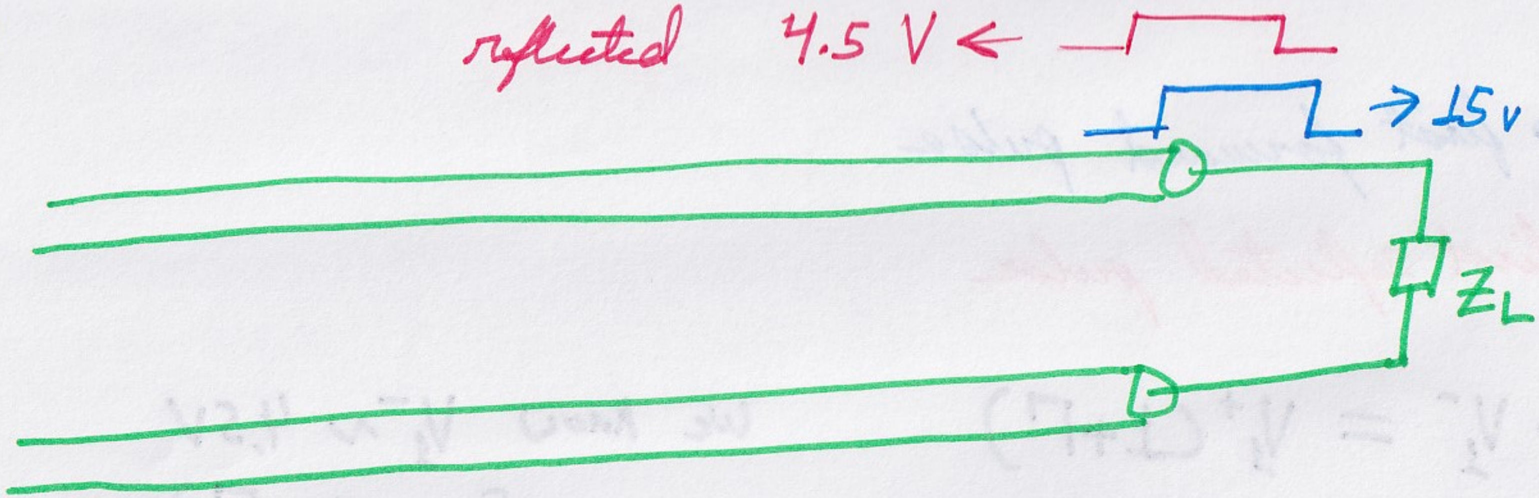
$V_L = V_1^+ + V_1^- = V_1^+(1+\Gamma)$

We know $V_1^- \sim 4.5V$ (from the Γ_L)

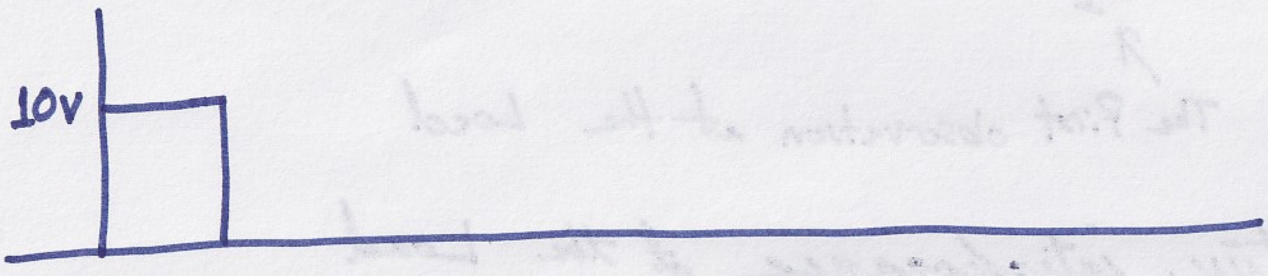
So then we can say $V_{L_1} = 14.5\ V$

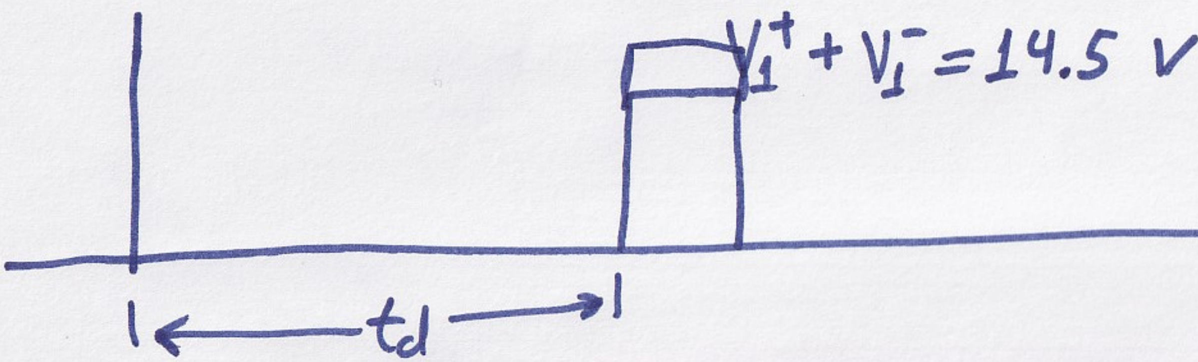
The first observation at the Load

We have constructive interferance at the Load for $+\Gamma$



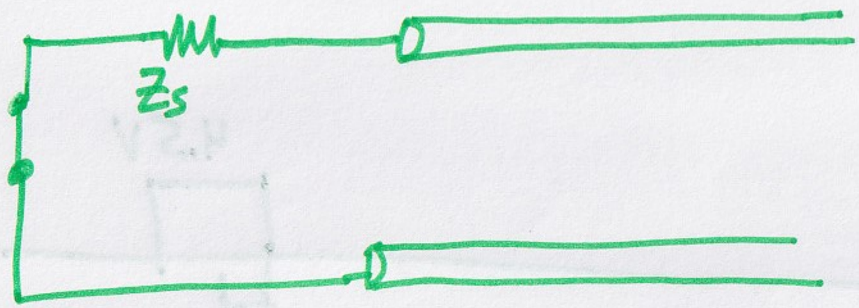
Vs (Vin)





4.5 V reflected is added to the Voltage at the Load. At 2tds, ~~14.5V will~~ 4.5V goes back to the source

So for $t >> td$



well now, Γ_s this is the reflection coefficient at the source end of the Line

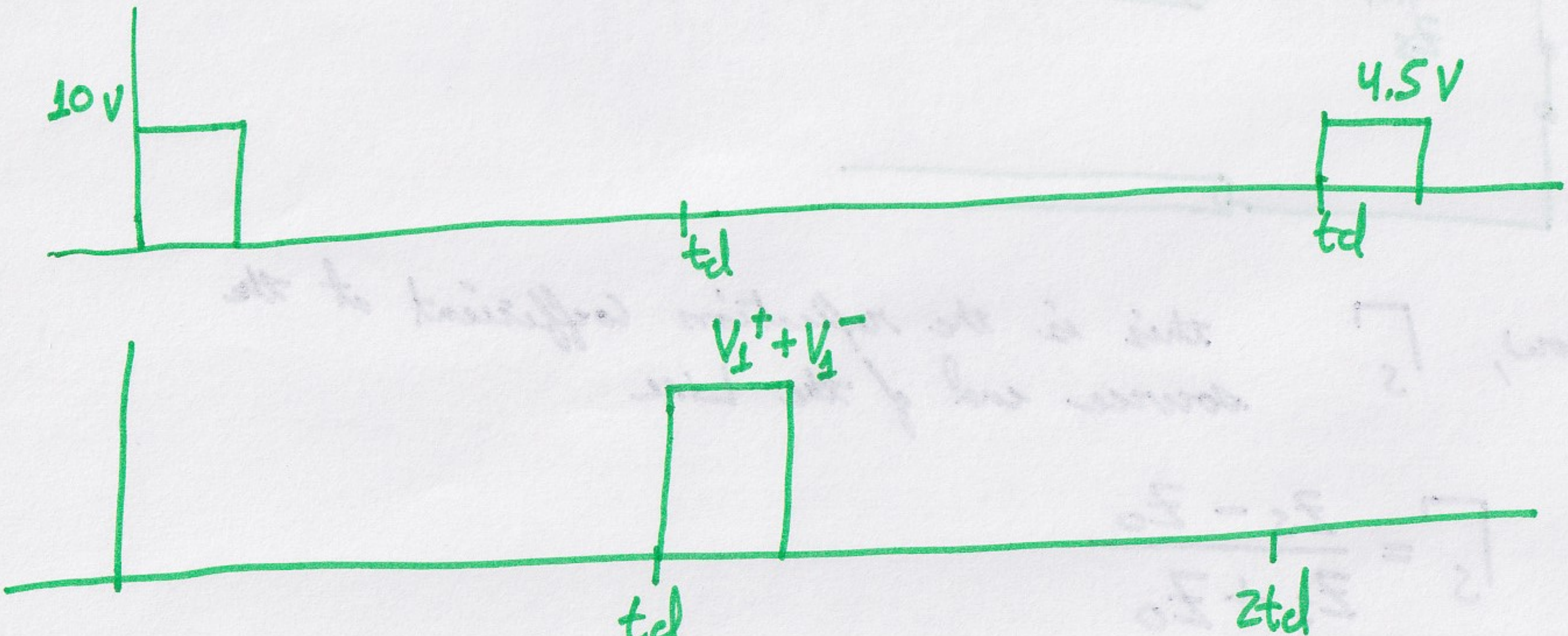
$$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o}$$

$Z_o = Z_s$ we have a 'Load match' for this problem

so $\Gamma_s = 0$

L4

V_s (Vin)

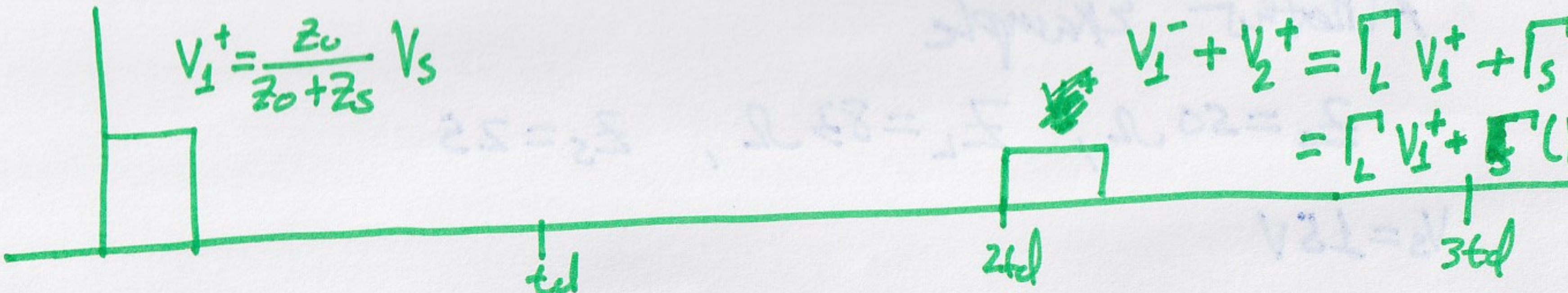


The reflected pulse comes back to hit the source at 2td
(4.5V ~~reflected~~ reflected to the source)

as $\Gamma_s = 0$ there is no reflected Voltage to the Load

$$V_1^+ = \frac{Z_0}{Z_0 + Z_S} V_S$$

$$V_1^- + V_2^+ = \Gamma_L V_1^+ + \Gamma_S V_1^-$$
$$= \Gamma_L V_1^+ + \Gamma_S (\Gamma_L V_1^+)$$



$$= (\Gamma_L + \Gamma_S \Gamma_L) V_1^+$$

$$V_{L1} = V_1^+ + V_1^- = (1 + \Gamma_L) V_1^+$$

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Another Example

$Z_0 = 50\,\Omega, \quad Z_L = 83\,\Omega, \quad Z_S = 25$

$V_S = 15\,V$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{83-50}{83+50} = \frac{1}{4}$$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{25-50}{25+50} = -\frac{1}{3}$$

$\cancel{V_1^+ = \frac{50}{25+50} = }$

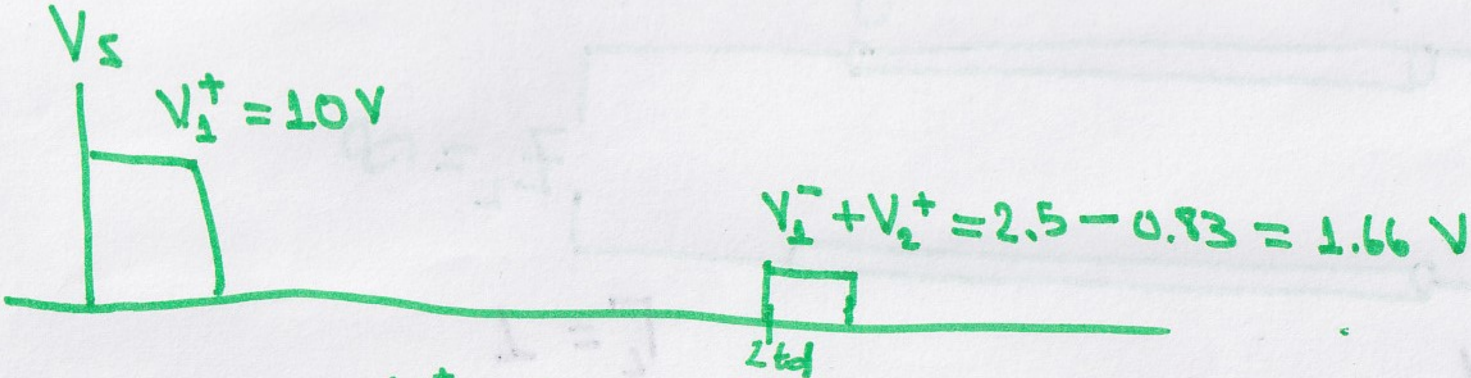
$$V_1^+ = \frac{Z_0}{Z_S + Z_0} V_S = \frac{50}{25+50}(15)$$

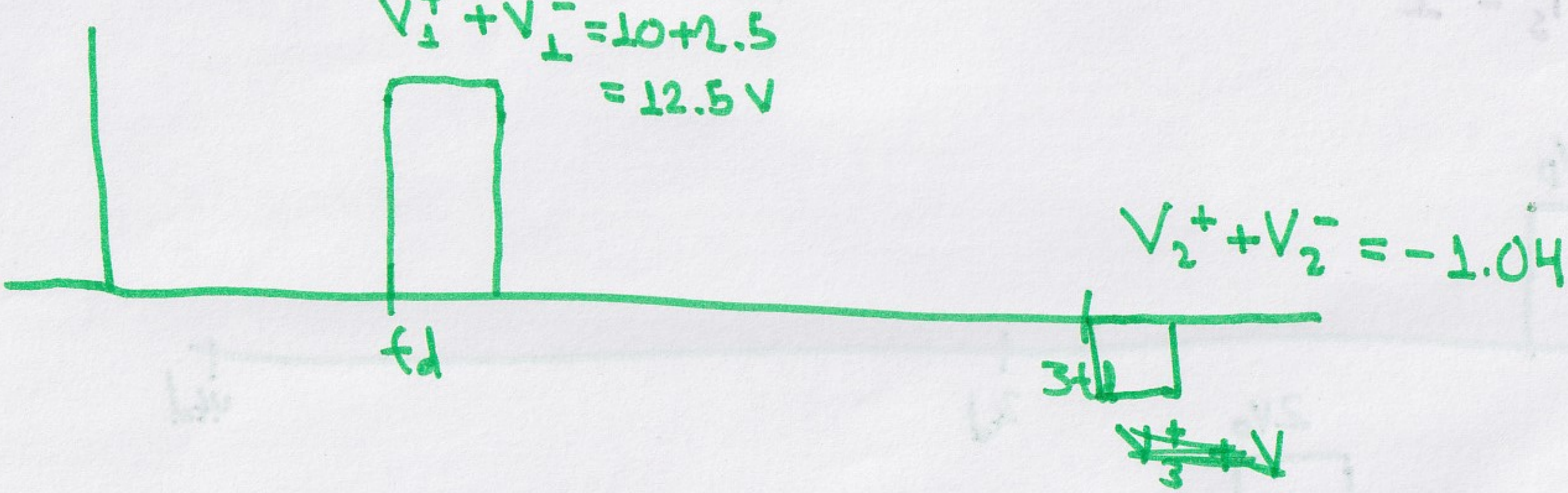
$= 10\,V$ This is our first forward wave

at t_d $V_1^- = \Gamma_L V_1^+ = \frac{1}{4}(10) = 2.5\,V$ This is the first reflected wave

at $2t_d$ $V_2^+ = \Gamma_S V_1^- = \left(-\frac{1}{3}\right)(2.5) \simeq -0.83$ This is the 2nd forward wave

at $3t_d$ $V_2^- = \Gamma_L V_2^+ = \left(\frac{1}{4}\right)(-0.83) \simeq -0.21$ This is the 2nd reflected wave


V_s
$V_1^+ = 10\,V$
$V_1^- + V_2^+ = 2.5 - 0.83 = 1.66\,V$
$2t_d$


$V_1^+ + V_1^- = 10 + 2.5$
$= 12.5\,V$
t_d
$V_2^+ + V_2^- = -1.04$
$3t_d$
$\cancel{V_3^+ + V}$

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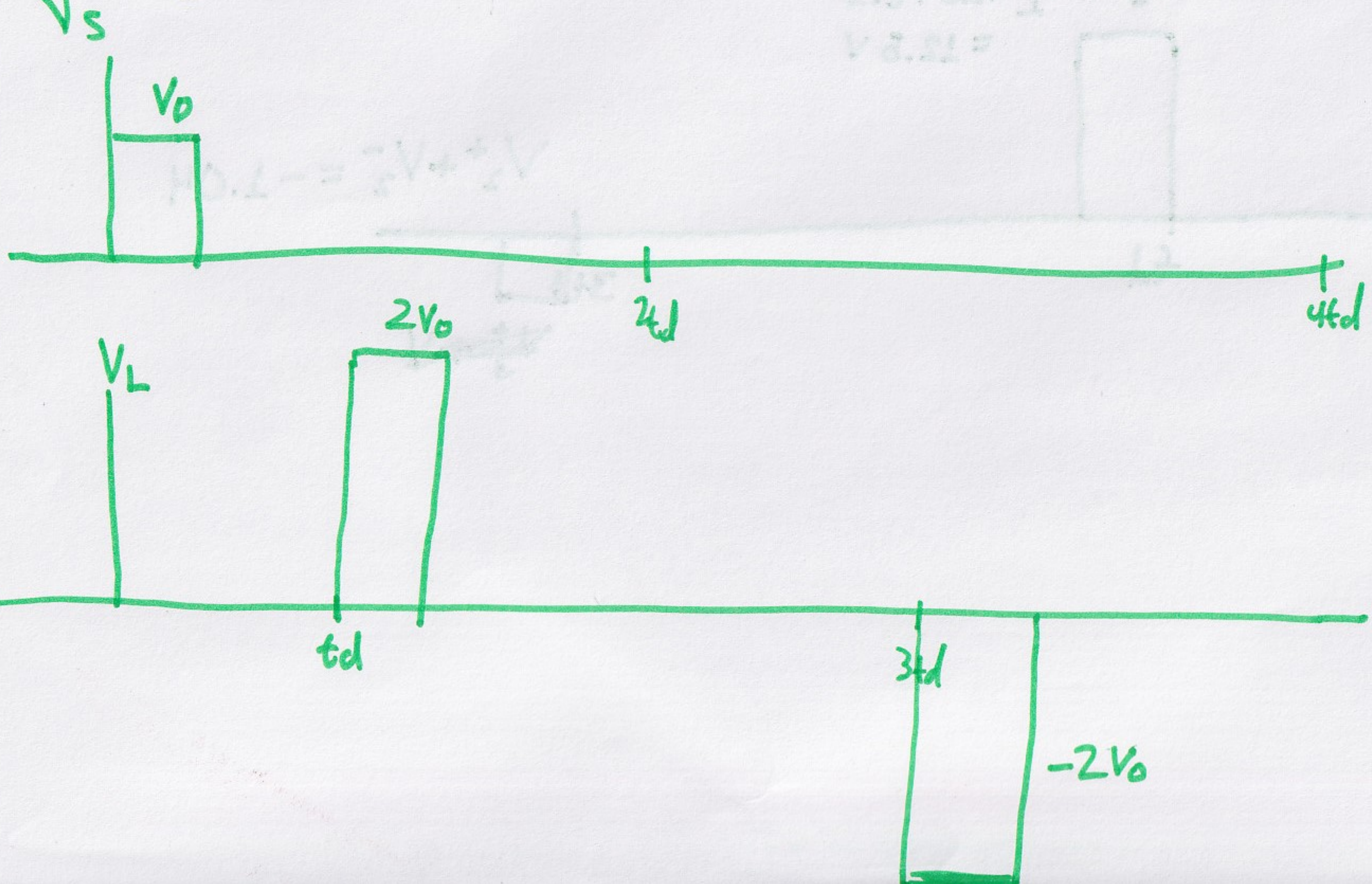
Goofy Case Study

$\Gamma_s = -1$ $\Gamma_L = 1$

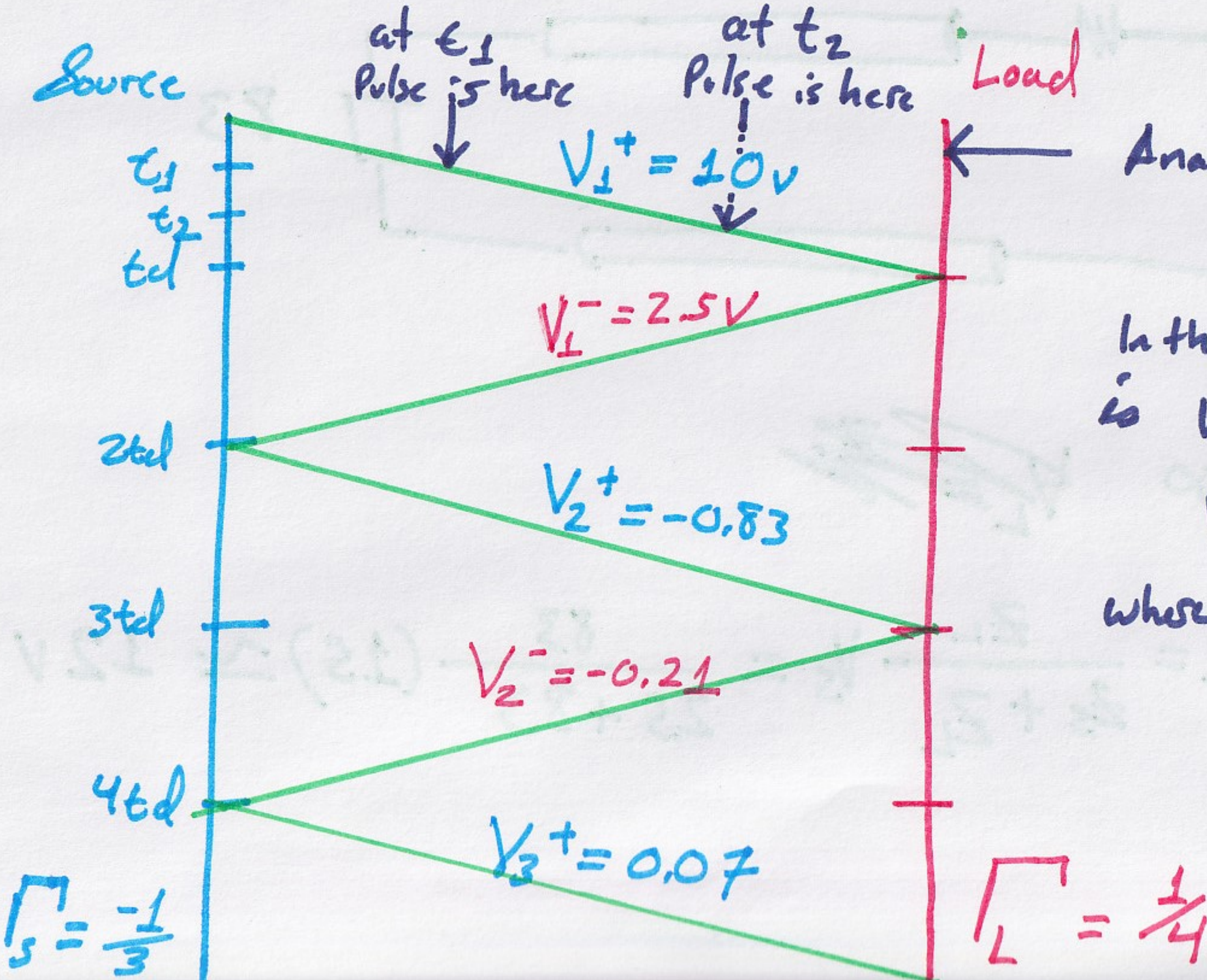


Bounce Diagram (Lattice Diagram)

$Z_0 = 50$, $Z_L = 83$, $Z_S = 25$ $\quad V_S = 15\ V$ pulse

$\Gamma_L = 1/4 \quad \Gamma_S = -\frac{1}{3}$

We have another way to Look at This



Analyzing voltage at $t_1 \rightarrow t_2$

In this window, what is V_L?

$V_L = 0$

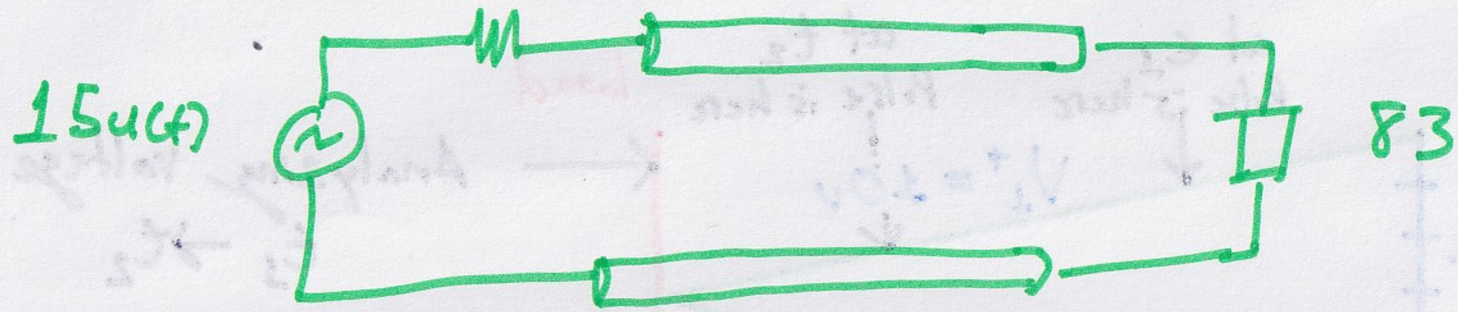
where is the pulse?

Long Pulse

Example is step function

→ turn a switch on and Leave it on.

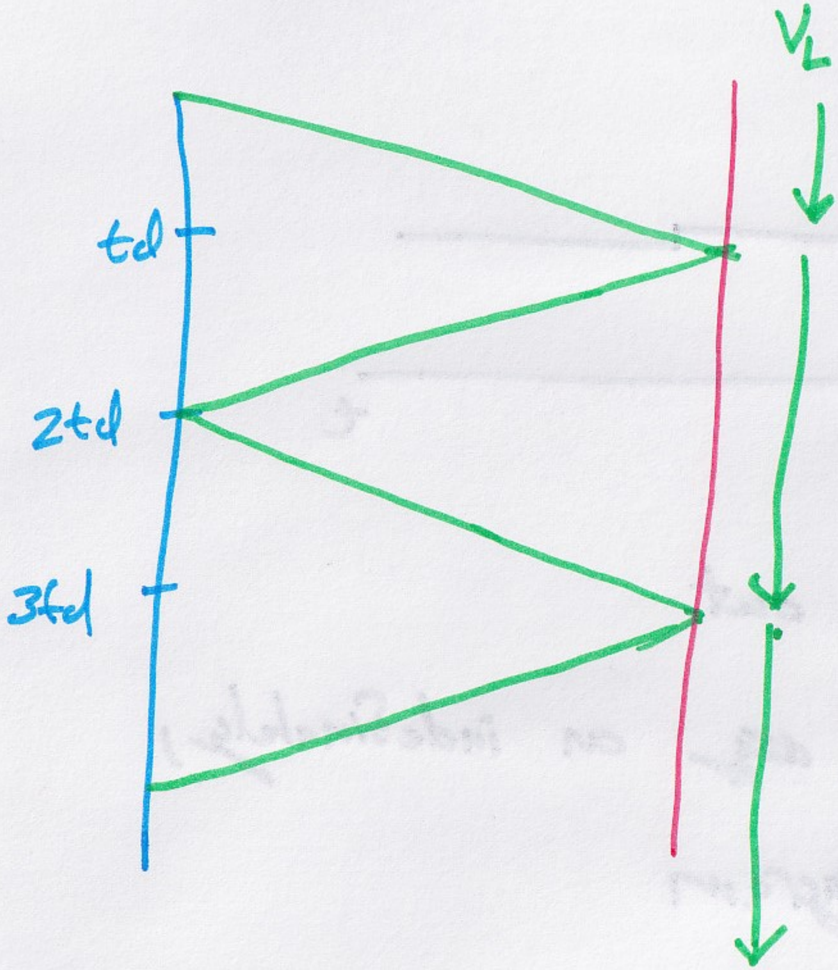
$V_s = 15\,V$, turned on at $t=0$ (stays on)



as $t \to \infty$

$$\rightarrow V_L = \frac{Z_L}{Z_s + Z_L} V_s = \frac{83}{25+83}(15) \simeq 12\,V$$

Here's how this happens



$V_1^+ = 10\,V$

0V

12.5V

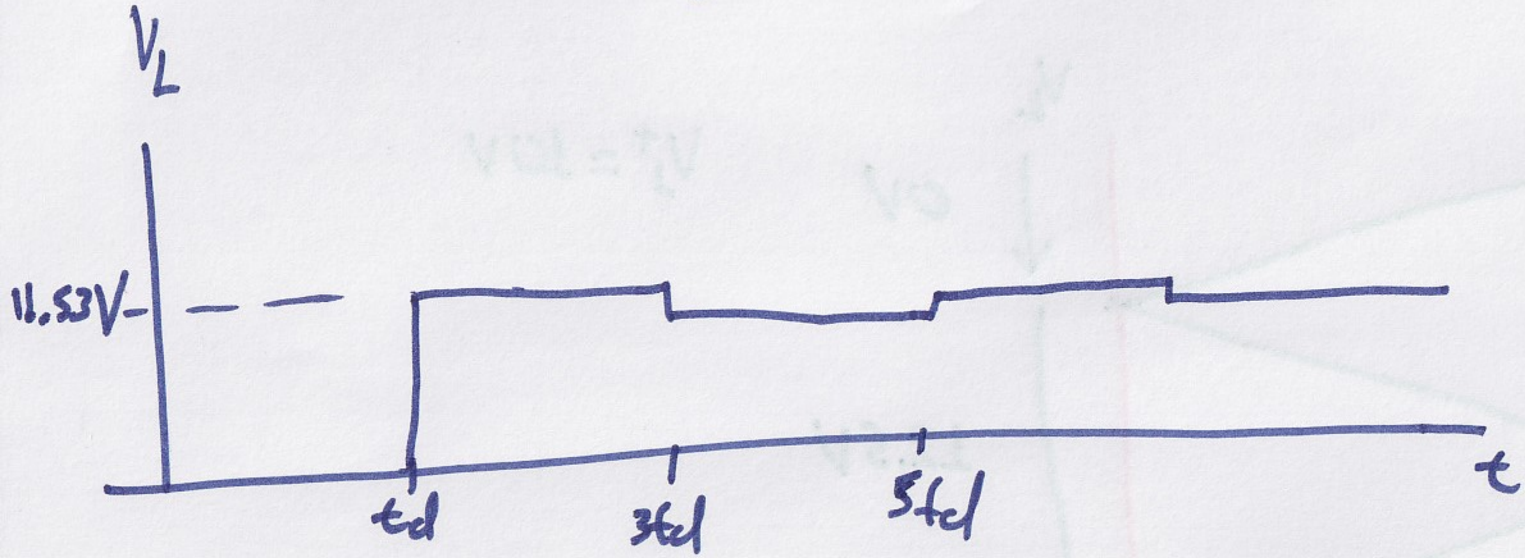
$12.5 - \underbrace{1.01}_{V_2^+ + V_2^-} = 11.4\,V$

$11.4 + \overset{0.09}{\cancel{0.9}} = 11.49$

<u>eventually</u> $11.528\,V$

(22)

Eventually we converge to DC value



The reflection waves balance out

For a Long Pulse not turned ~~off~~ on indefinately,

use the Lattice Diagram

if Γ was Large, then it could take a Long time to balance out.