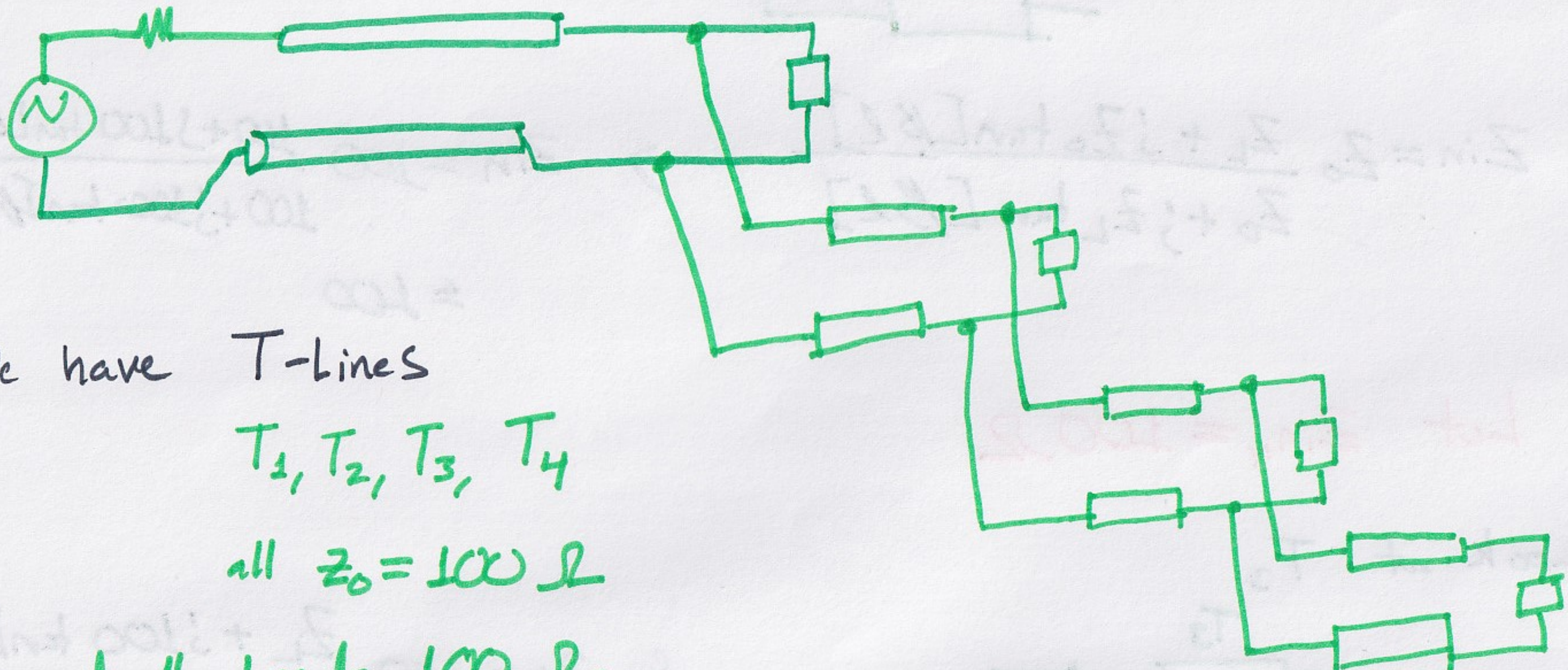
# Fields and Waves I    L5



## The Lattice Diagram



We have T-Lines

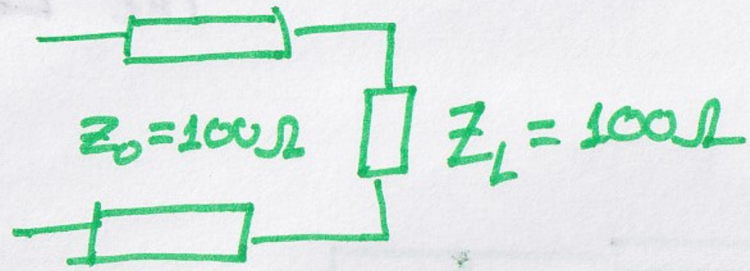
$T_1, T_2, T_3, T_4$

all $Z_0 = 100\ \Omega$

and all Loads $100\ \Omega$

So what are the Input Impedances?

(2) Look at $T_4$

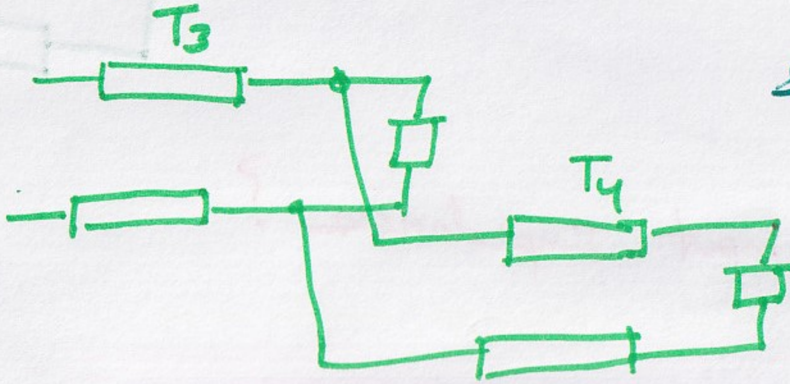



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan[\beta l]}{Z_0 + jZ_L \tan[\beta l]} \rightarrow Z_{in} = 100 \frac{100 + j100 \tan[\beta l]}{100 + j100 \tan[\beta l]}$$

$$\doteq 100$$

Let $Z_{in_4} = 100\,\Omega$

Look at $T_3$




$$\text{So } Z_{in_3} = 100 \frac{Z_L + j100 \tan[\beta l]}{100 + jZ_L \tan[\beta l]}$$

$$Z_L = 100 \| 100 = 50\ \Omega = Z_{in_4} \| 100$$

~~$= Z_{in_4} \| 100$~~

The Load for $T_3$ is $Z_{in_4}$ in parallel with a $100\ \Omega$ Load.

So whats $Z_{in_3}$?

if we Let $\beta l = 0, 2\pi, 4\pi \ldots$ etc

$$Z_{in_3} = 100 \frac{50\Omega + j100\,(0)}{100 + j\,50\Omega\,(0)} = 50\ \Omega$$

So Looking at $T_2$, the effective Load for $T_2$ is

$$Z_L = Z_{in_3} \| 100\ \Omega = 50 \| 100 \approx 33\ \Omega$$

So then

$$Z_{in_2} = 100 \frac{33 + j\,100 \cancel{\tan(\beta l)}^{\,0}}{100 + j\,33 \cancel{\tan(\beta l)}^{\,0}} = 33\ \Omega$$

(4) So for $T_1$, the effective load is

$$Z_{in_2} \parallel 100\,\Omega = 33 \parallel 100 \simeq 25\,\Omega$$

If we keep adding Loads, eventually the effective Loads become

effective Loads = 0

so then $\Gamma = 1$

Basically, theres a limit to how many people can be on the line.

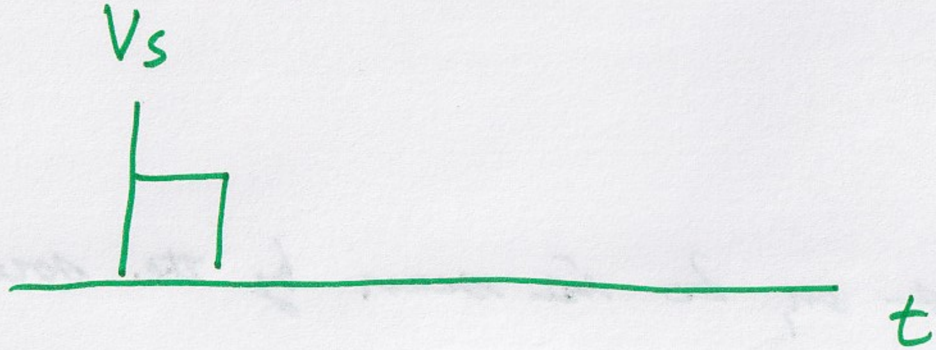
Let's talk about ~~short~~ short pulses

we have done AC Steady state problems

- Standing waves, SWR
- Input Impedance, Zin

These are not used for pulse Problems

Short pulses



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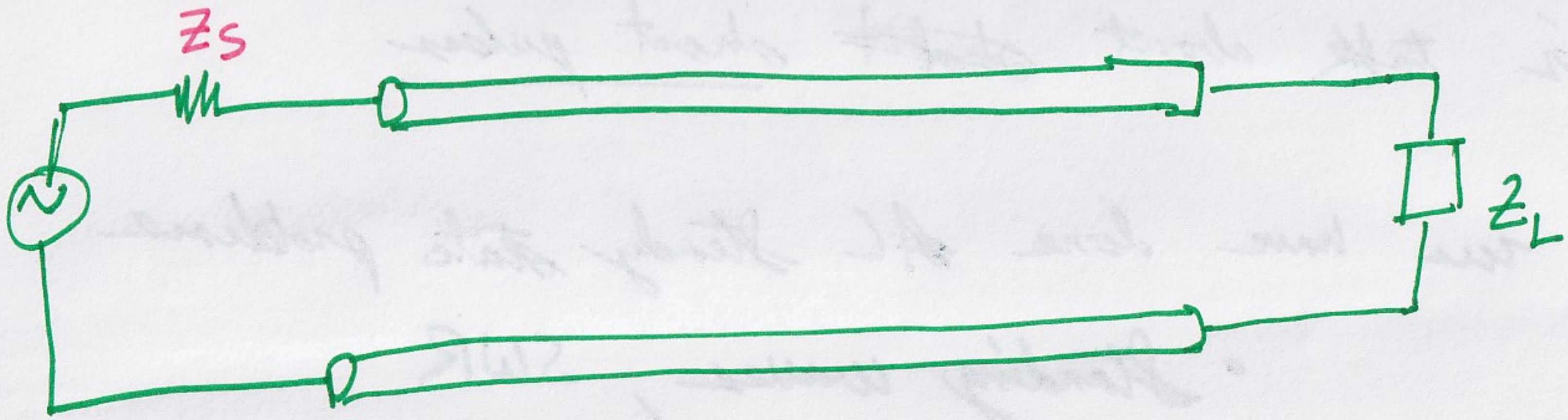
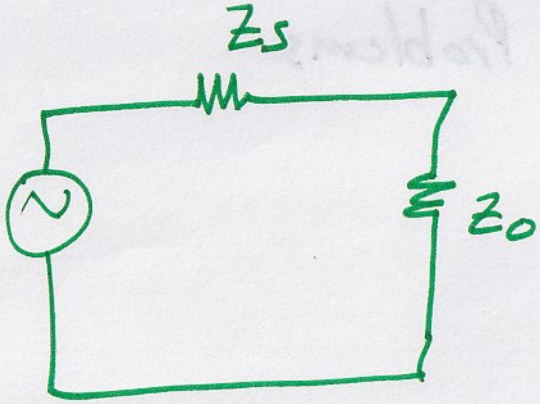



At $t=0$, what circuit does the source see?




$Z_S$ is the internal Impedance

$Z_o$ is the characteristic Impedance

→ When the source turns on, $Z_o$ is seen by the source circuit

A pulse propagates down the line

$V_1^+ \leftarrow$ forward propagating pulse

Ex)

$V_s = 20\,V$ Short Pulse (from the generator)

$Z_L = Z_0 = Z_s = 50\,\Omega$

So then

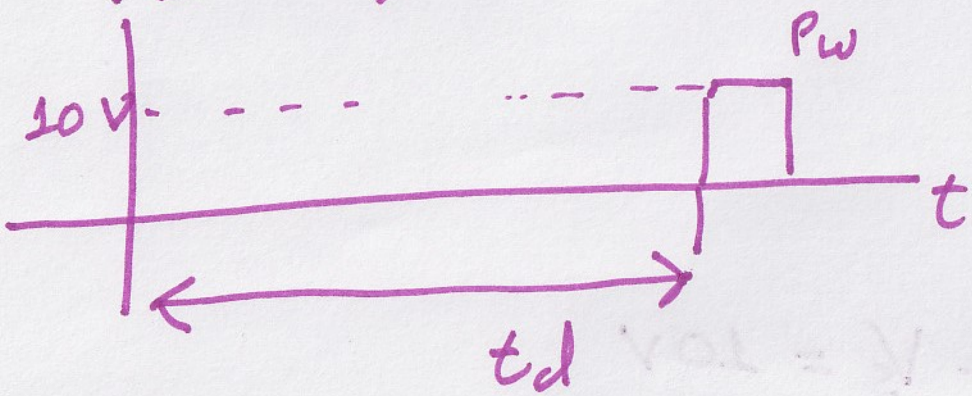
$$V_1^+ = \frac{Z_0}{Z_0 + Z_S} V_S = 10\,V$$

What happens when the pulse reaches the Load?

⑧

Vin $\left( V_{in} = \frac{Z_0}{Z_0 + Z_{in}} V_S \right)$ (Vin has the Amplitude of $V_1^+$)


$V_L$ (voltage at the Load, $V_L$)



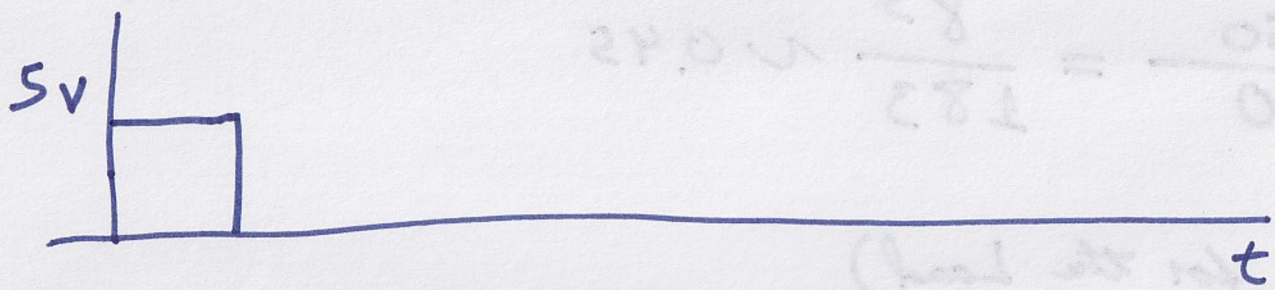

We see that the pulse takes td time to reach the Load. So it's important to note,

a short pulse implies → pulsewidth << td

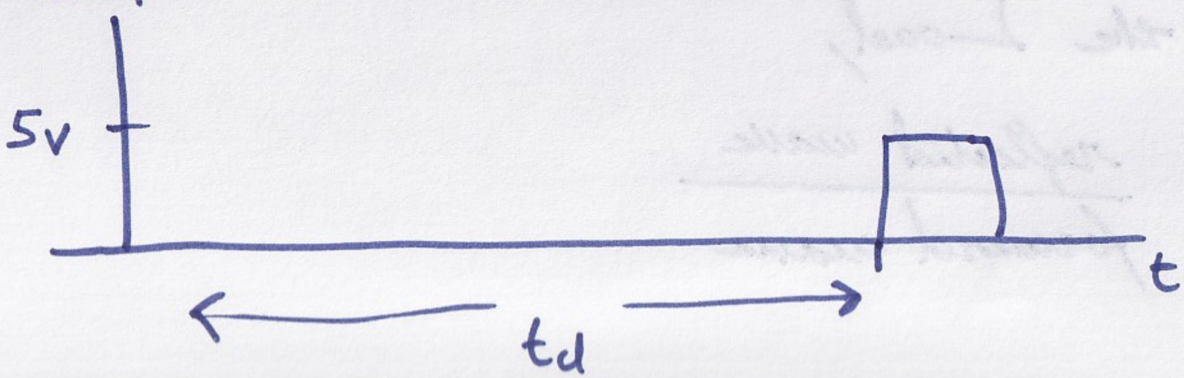
Ex) $Z_S = 150\ \Omega \qquad Z_0 = Z_L = 50\ \Omega$

$$V_1^+ = \frac{50}{50 + 150}(20\ V) = 5\ V$$

$V_{source}$ ($V_{in}$) ($V_1^+$)



$V_{Load}$



(10)

Ex) $Z_L = 133\ \Omega$ $Z_0 = Z_s = 50\ \Omega$ $V_s = 20\text{v}$

at $t=0$

$$V_1^+ = \frac{50}{50+50}(20) = 10\text{v}$$

$$\Gamma_L = \frac{133-50}{133+50} = \frac{83}{183} \approx 0.45$$

(reflection coefficient for the Load)

When the pulse hits the Load,

$$\Gamma_L = \frac{|V-|}{|V+|} \qquad \frac{\text{reflected wave}}{\text{forward wave}}$$

$V_1^+ \longrightarrow$ first forward pulse

$V_1^- \longrightarrow$ first reflected pulse

$V_L = V_1^+ + V_1^- = V_1^+(1+\Gamma)$

We know $V_1^- \sim 4.5V$
(from the $\Gamma_L$)

So then we can say $V_{L_1} = 14.5\ V$

The first observation at the Load

We have constructive interferance at the Load for $+\Gamma$

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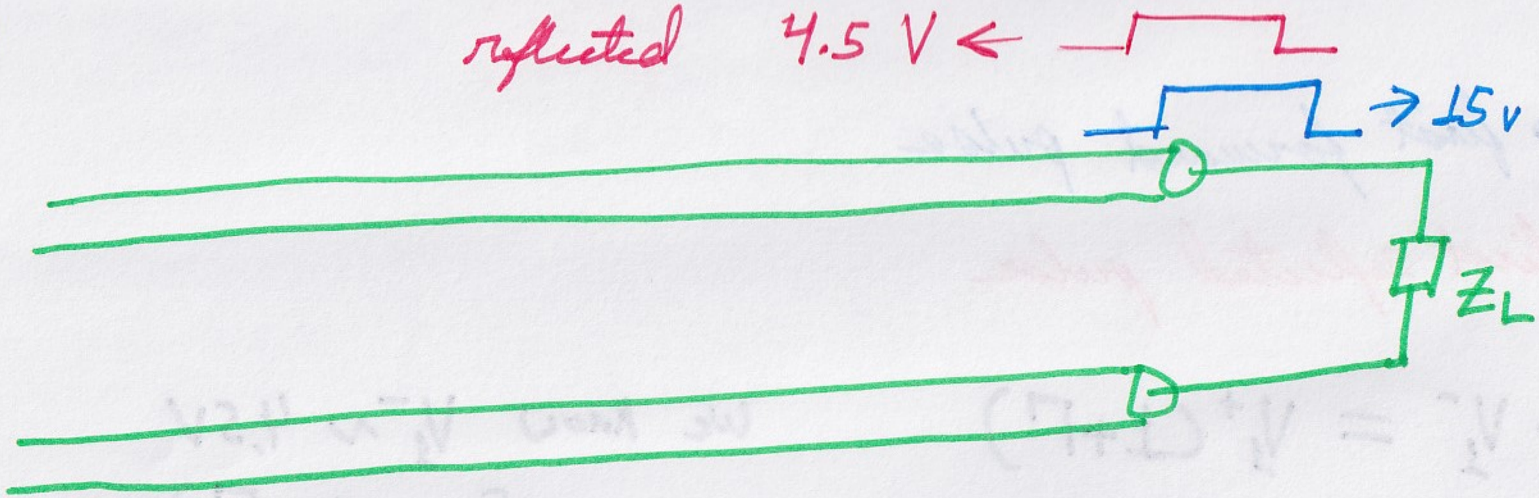
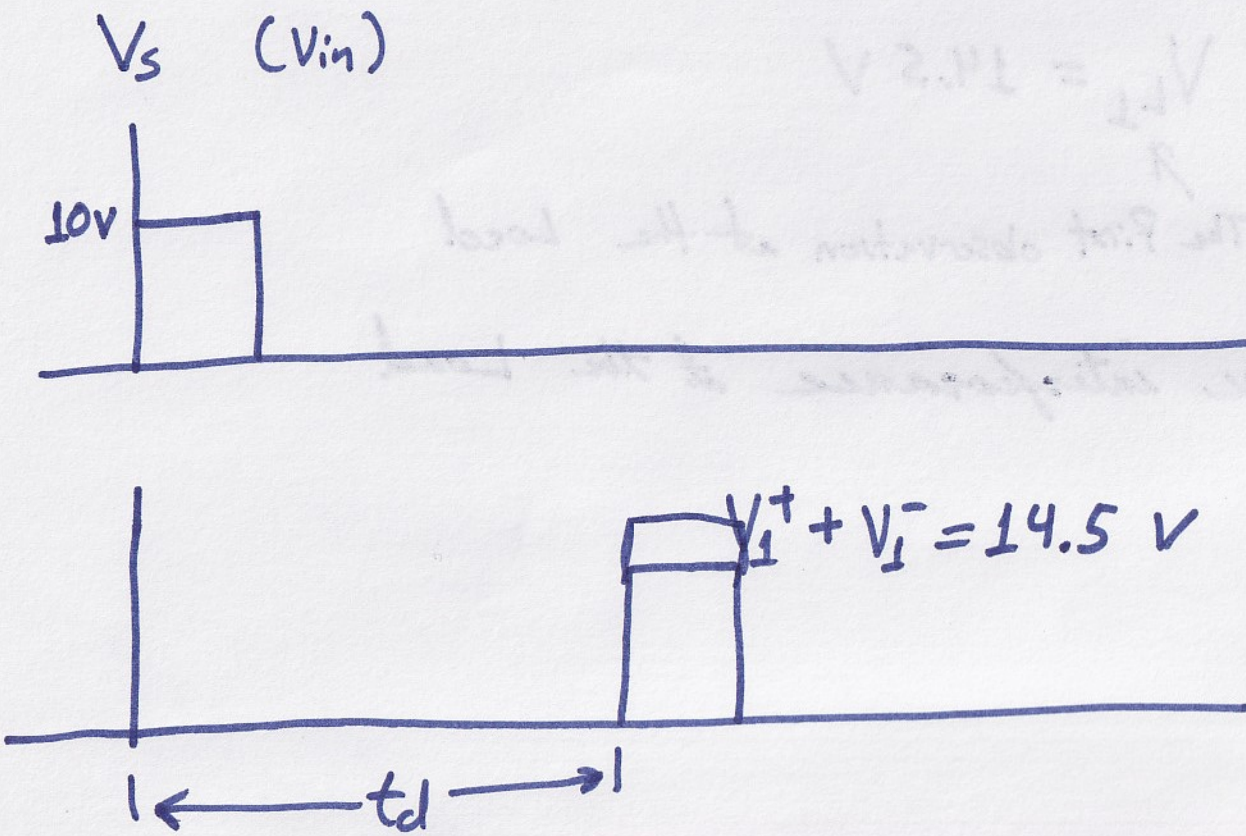
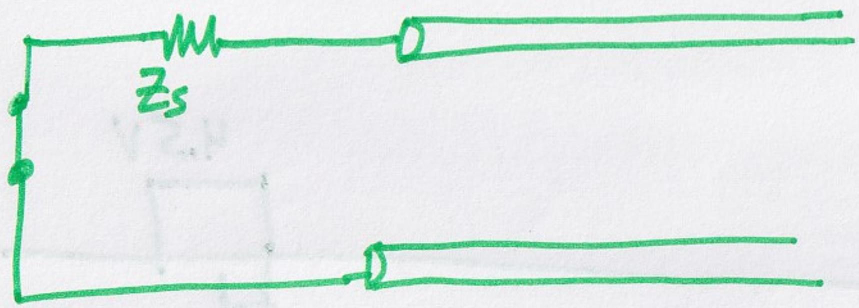



4.5 V reflected is added to the Voltage at the Load. At 2tds, ~~14.5V will~~ 4.5V goes back to the source

So for $t >> td$




well now, $\Gamma_S$ this is the reflection coefficient at the source end of the Line

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

$Z_0 = Z_S$ we have a 'Load match' for this problem

so $\Gamma_S = 0$

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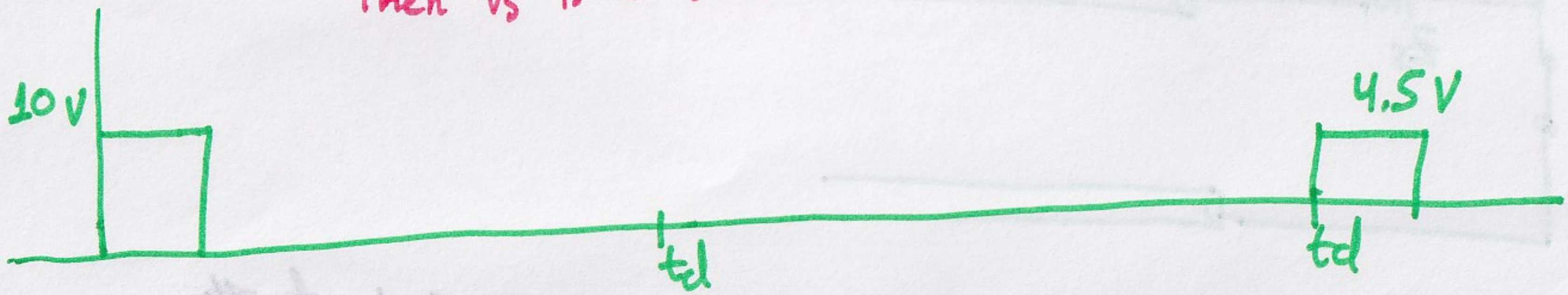
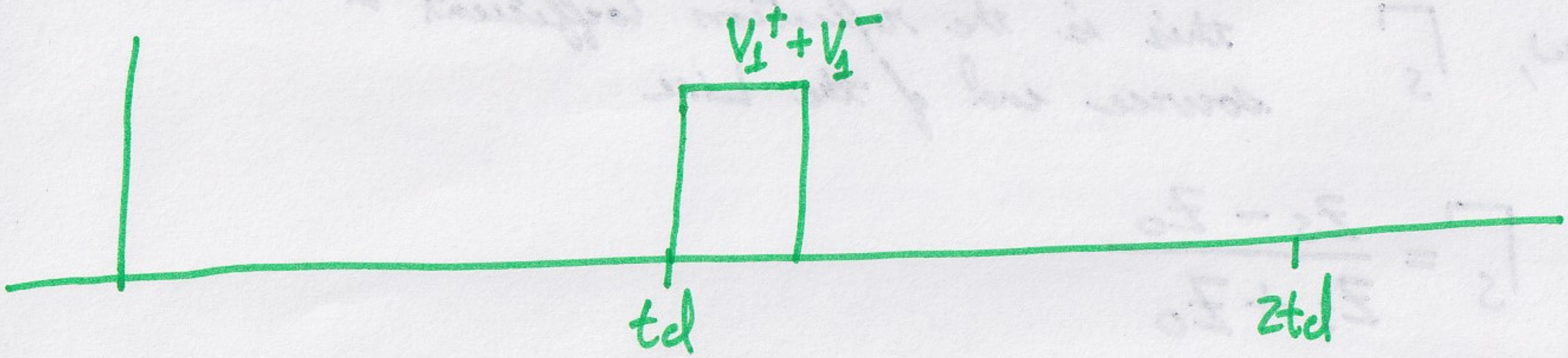
$V_s$ (Vin) Vin. ($V_s$) is the Voltage going into the T-Line
Then $V_s$ is as follows




The reflected pulse comes back to hit the source at 2td
(4.5V ~~reflected~~ reflected to the source)

as $\Gamma_s = 0$ there is no reflected Voltage to the Load


$V_1^+ = \frac{Z_0}{Z_0 + Z_S} V_S$

$V_1^- + V_2^+ = \Gamma_L V_1^+ + \Gamma_S V_1^-$

$= \Gamma_L V_1^+ + \Gamma_S (\Gamma_L V_1^+)$

$= (\Gamma_L + \Gamma_S \Gamma_L) V_1^+$

$V_{L1} = V_1^+ + V_1^- = (1 + \Gamma_L) V_1^+$


$V_2^+ + V_2^-$

(16)

# ~~A~~nother Example

$Z_0 = 50\,\Omega$, $Z_L = 83\,\Omega$, $Z_S = 25$

$V_S = 15V$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{83-50}{83+50} = \frac{1}{4}$$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{25-50}{25+50} = -\frac{1}{3}$$

~~$V_1^+ = \frac{50}{25+50} =$~~

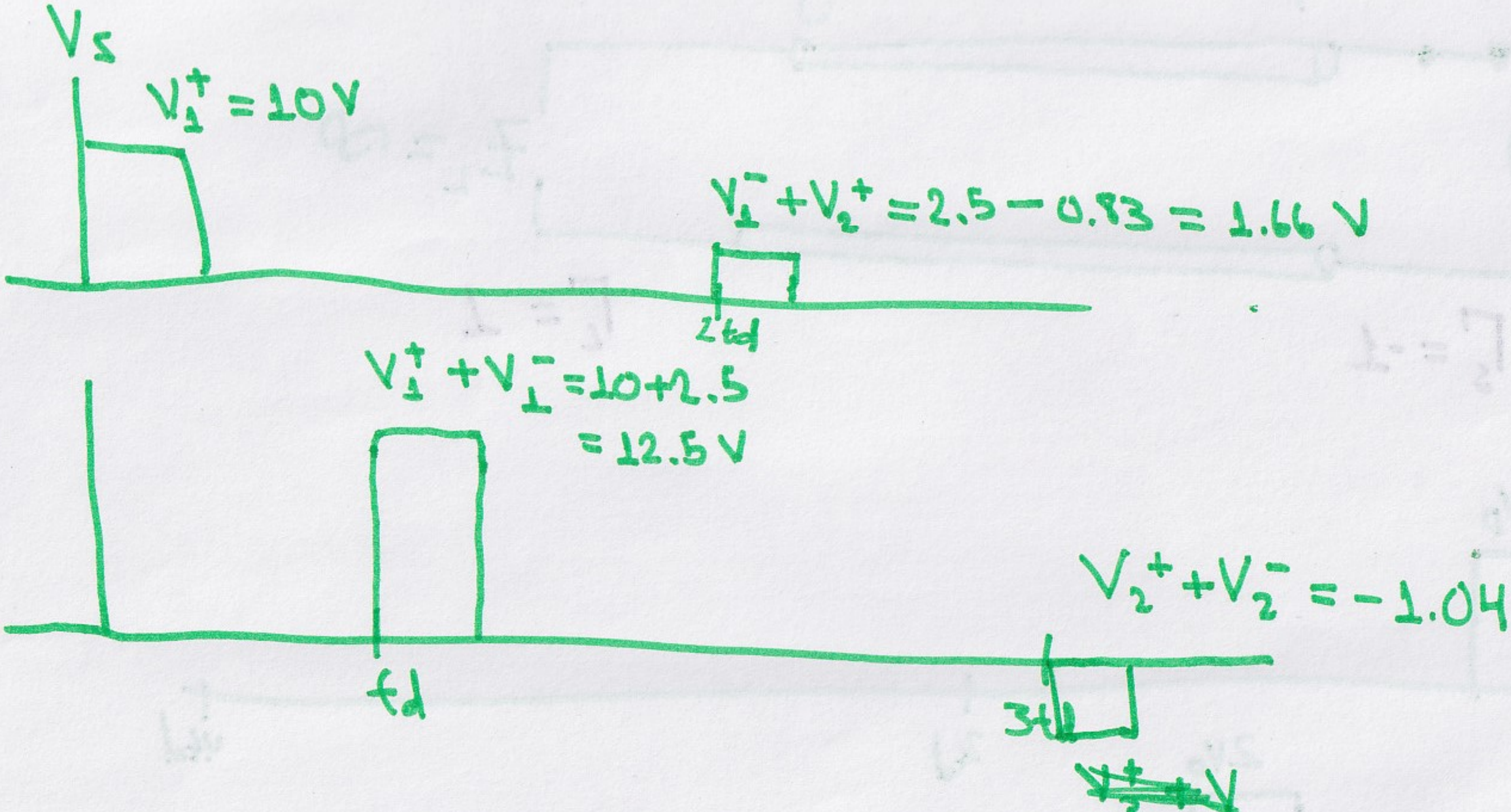
$$V_1^+ = \frac{Z_0}{Z_S + Z_0} V_S = \frac{50}{25+50}(15)$$

$= 10\,V$ This is our first forward wave

at $t_d$ $V_1^- = \Gamma_L V_1^+ = \frac{1}{4}(10) = 2.5V$ This is the first reflected wave

at $2t_d$ $V_2^+ = \Gamma_S V_1^- = \left(-\frac{1}{3}\right)(2.5) \simeq -0.83$ This is the 2nd forward wave

at $3t_d$ $V_2^- = \Gamma_L V_2^+ = \left(\frac{1}{4}\right)(-0.83) \simeq -0.21$ This is the 2nd reflected wave


Vs
V1+ = 10 V
V1- + V2+ = 2.5 − 0.83 = 1.66 V
2td
V1+ + V1- = 10+2.5
= 12.5 V
td
V2+ + V2- = −1.04
3td
V3+ + V

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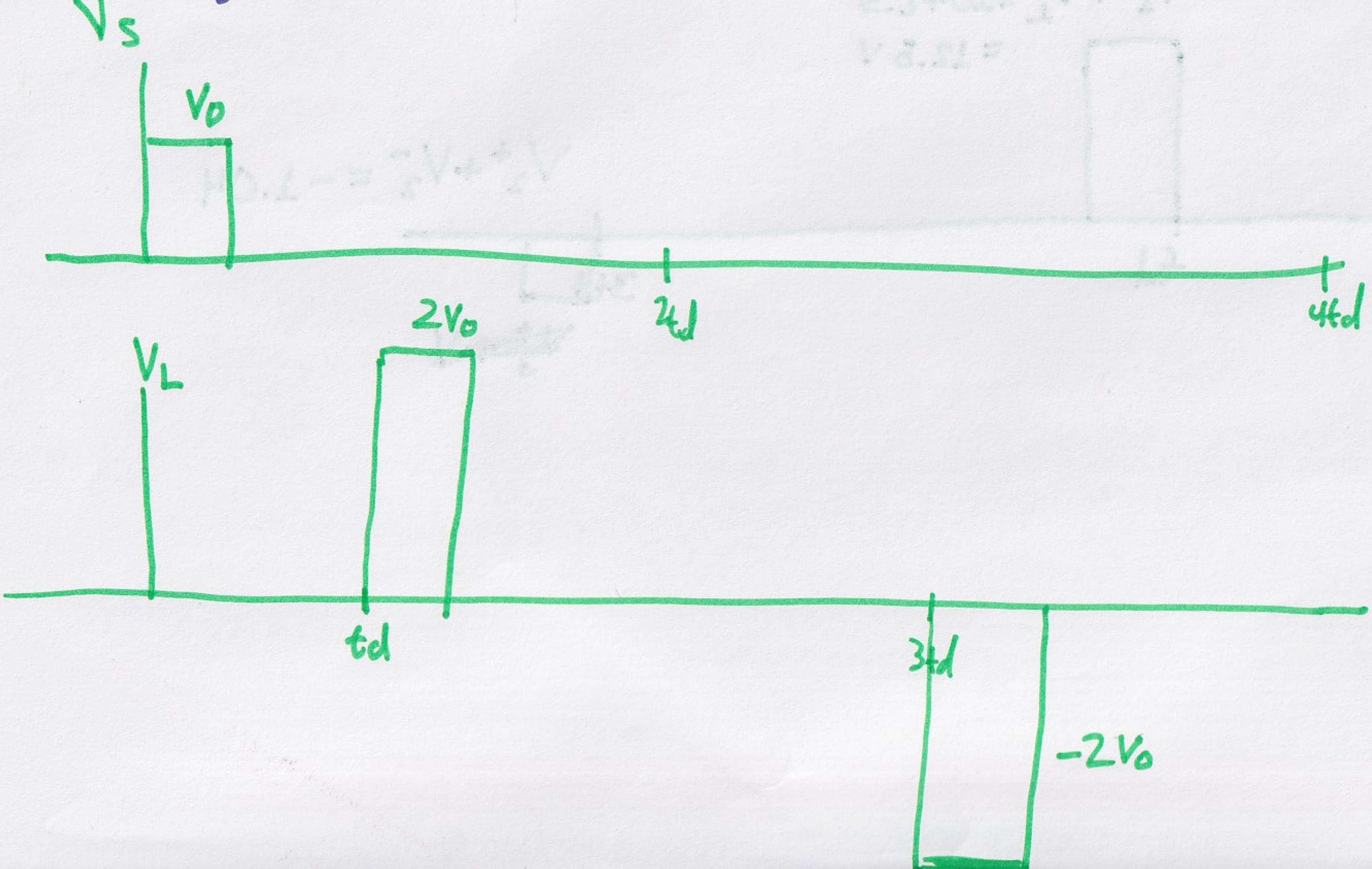
# Goofy Case Study

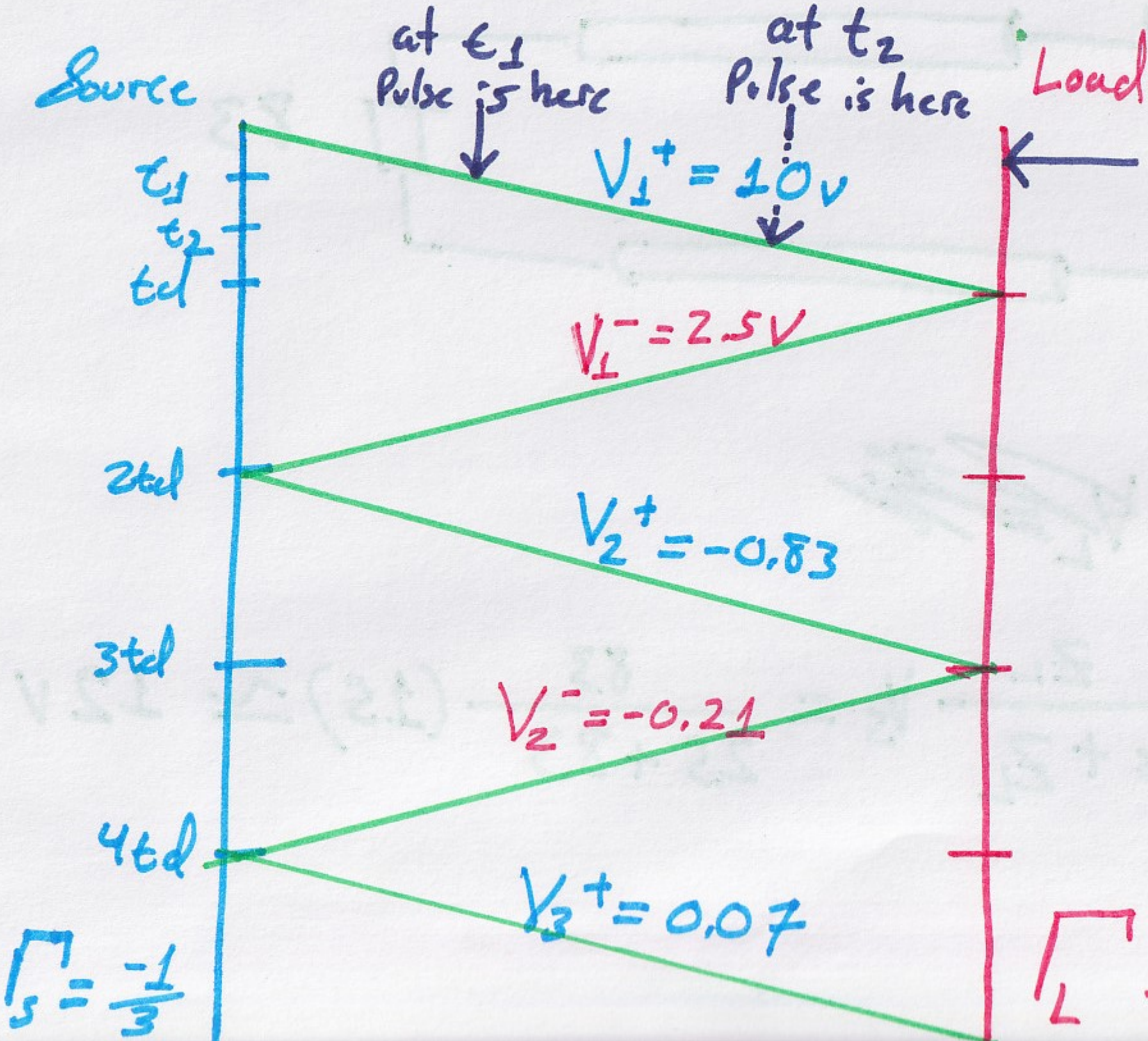

$\Gamma_S = -1$ $\Gamma_L = 1$



# Bounce Diagram (Lattice Diagram)

$Z_0 = 50$, $Z_L = 83$, $Z_S = 25$ $\quad V_S = 15$ V pulse

$\Gamma_L = \frac{1}{4}$ $\quad \Gamma_S = -\frac{1}{3}$ $\quad$ We have another way to Look at This




Analyzing voltage at $t_1 \to t_2$

In this window, what is $V_L$?

$V_L = 0$

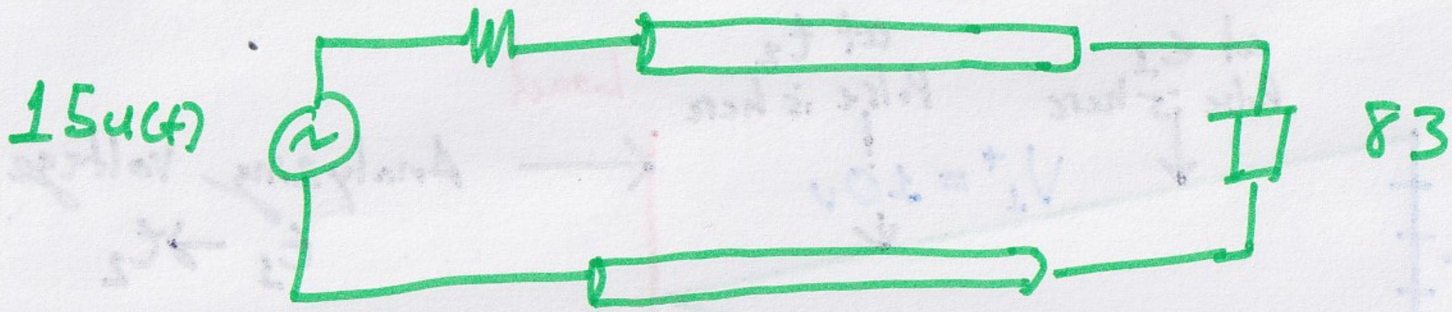
where is the pulse?

# Long Pulse

Example is step function

→ turn a switch on and Leave it on.

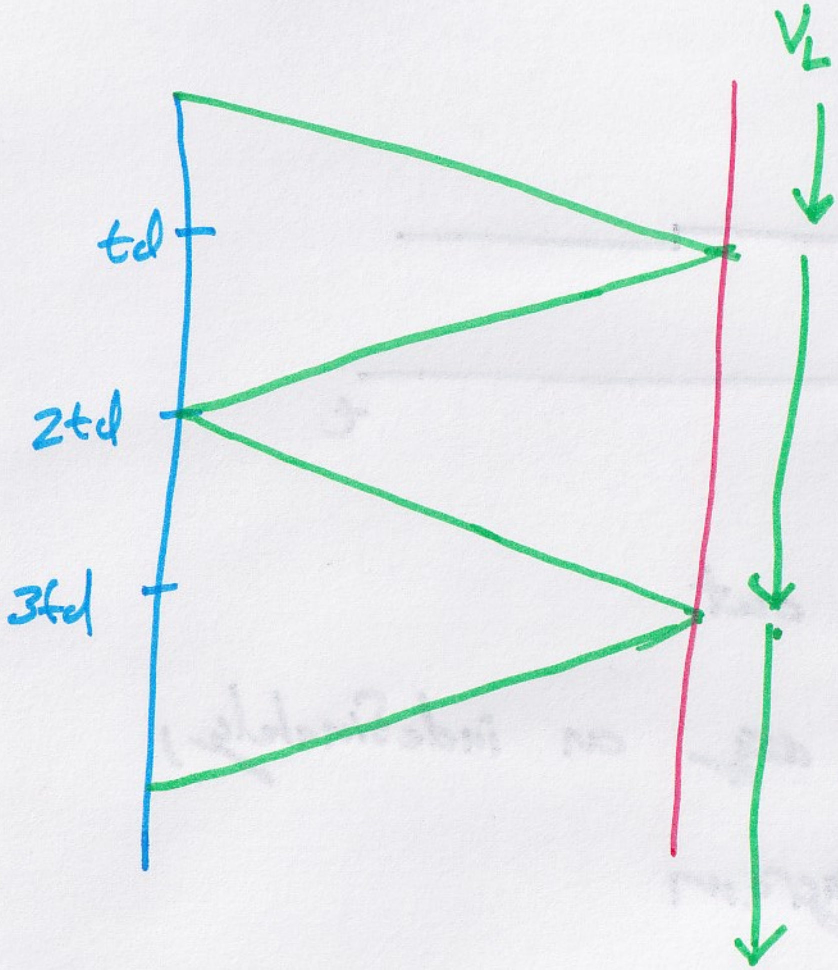
$V_s = 15\,V$, turned on at $t=0$ (stays on)




as $t \to \infty$

$$\rightarrow V_L = \frac{Z_L}{Z_s + Z_L} V_s = \frac{83}{25+83}(15) \simeq 12\,V$$

Here's how this happens




$V_1^+ = 10V$

0V

12.5V

$$\underbrace{12.5 - 1.01}_{V_2^+ + V_2^-} = 11.4V$$

$$11.4 + \cancel{0.9}\,\overset{0.09}{} = 11.49$$

<u>eventually</u> 11.528 V

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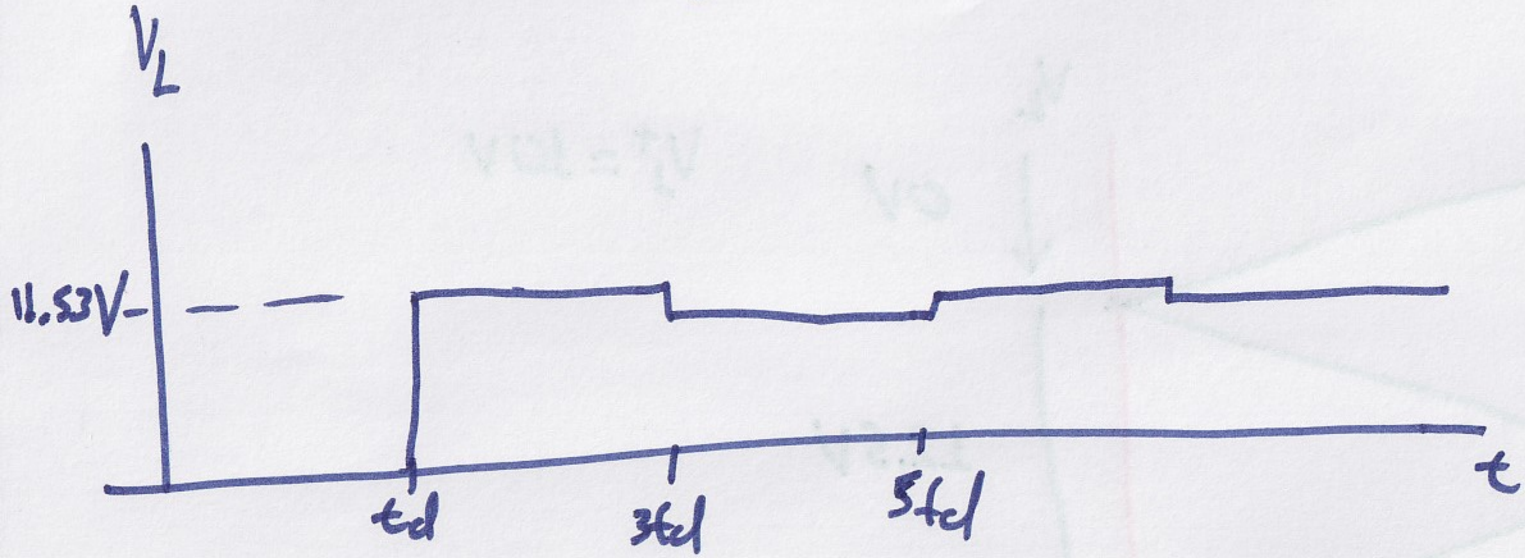
Eventually we converge to DC value




The reflection waves balance out

For a Long Pulse not turned ~~off~~ on indefinately,

use the Lattice Diagram

if $\Gamma$ was Large, then it could take a Long time to balance out.