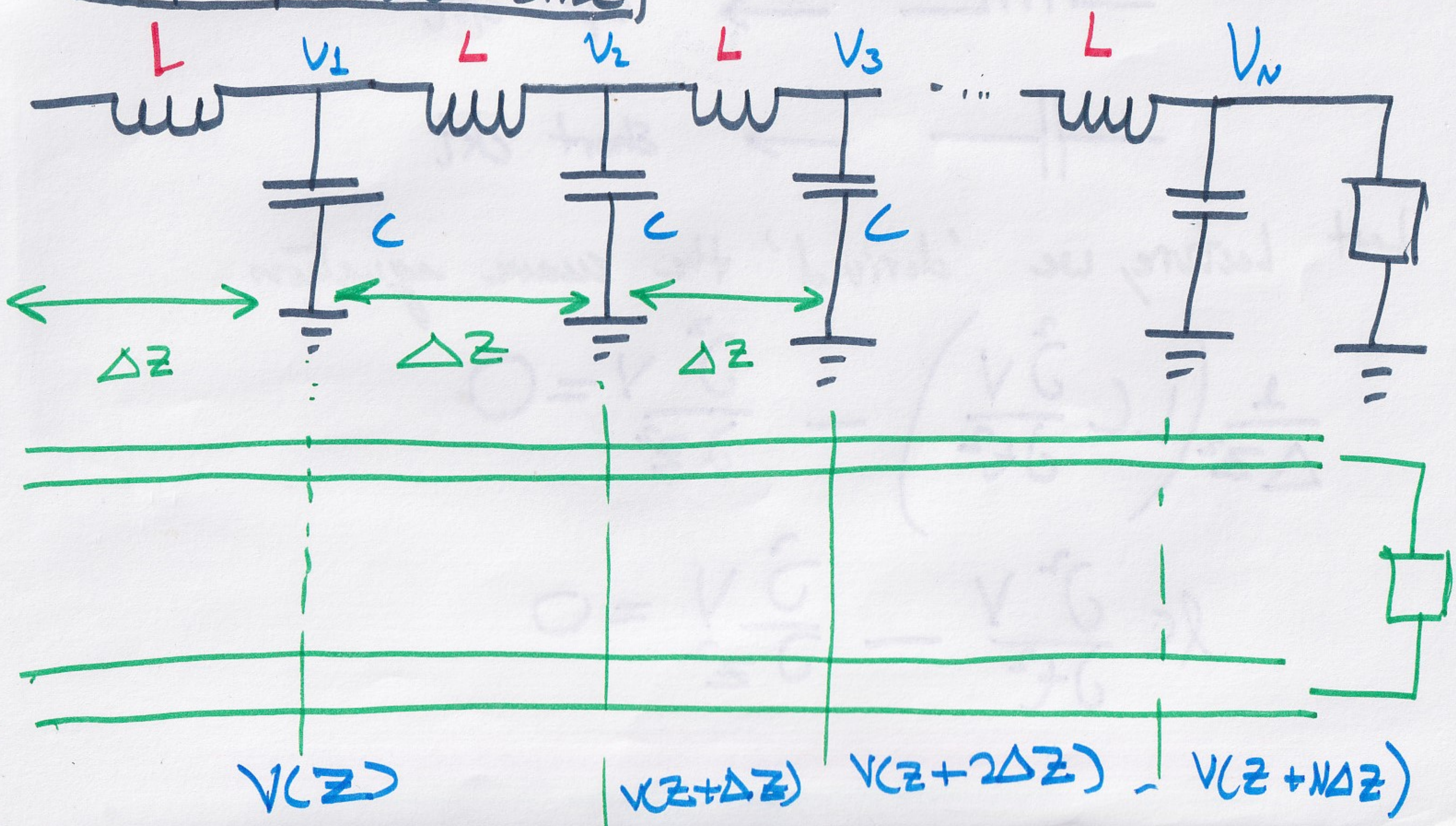
Fields and Waves I L2 1

Transmission Lines

The Travelling wave

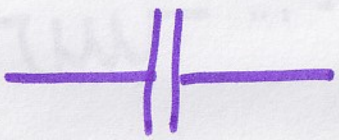
$\boxed{\text{Lumped parameter Line}}$



(2) recall from circuits

as frequencies increase (goes really high)

 → open ckt

 → short ckt

Last Lecture, we 'derived' the wave equation

$$\frac{1}{\Delta z^2}\left(LC\frac{\partial^2 V}{\partial t^2}\right) - \frac{\partial^2}{\partial z^2}V = 0$$

$$lc\frac{\partial^2 V}{\partial t^2} - \frac{\partial^2 V}{\partial z^2} = 0$$

$\ell \longrightarrow \frac{H}{m}$ $\qquad$ $C \longrightarrow \frac{F}{m}$

We can say

$$V(z,t) = f\left(t \pm \tfrac{1}{u} z\right)$$

u is a constant

$f(\ )$ is a function

$$\text{"} \Rightarrow \text{"} = f\left(t + \tfrac{1}{u} z\right)$$

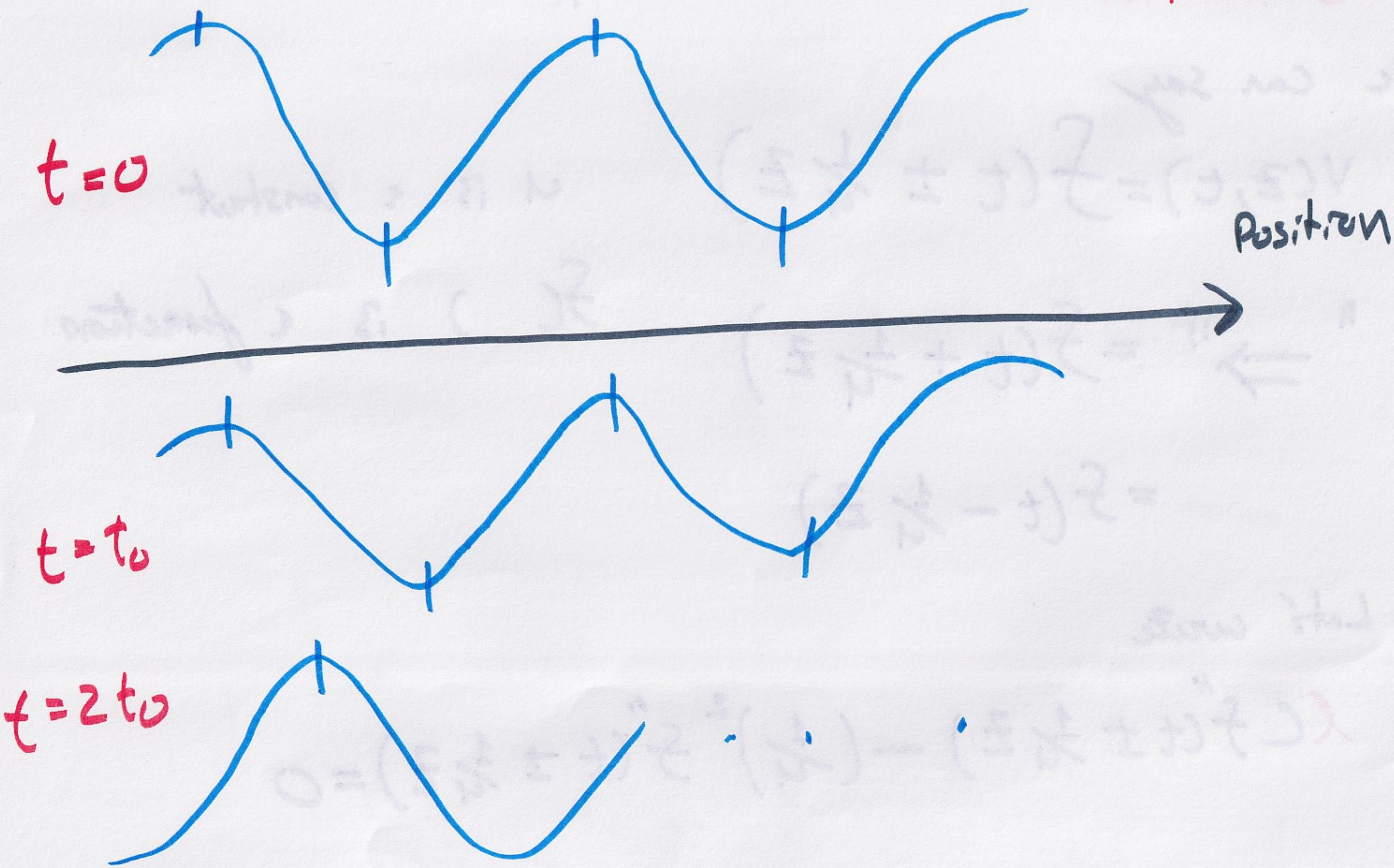
$$= f\left(t - \tfrac{1}{u} z\right)$$

So Let's write

$$\ell C f''\left(t \pm \tfrac{1}{u} z\right) - \left(\tfrac{1}{u}\right)^2 f''\left(t \pm \tfrac{1}{u} z\right) = 0$$

4 Solutions are traveling waves

"Snapshot in time"



as time changes, position changes

So the function evaluates at ______ at a position in time

If you follow a peak, the function $f(t - \frac{1}{u}z)$ is constant with changing t, z

$\Longrightarrow$ the argument of f must be constant

$$t - \frac{1}{u} z = \text{constant}$$

$$z = -u \text{ constant} + ut$$

⑥

$$\frac{\partial}{\partial t} z = \frac{\partial}{\partial t}(ut + u\,\text{constant})$$

$$\frac{\partial}{\partial t} z = u \quad \rightarrow \text{velocity of the wave}$$

$V_p \Rightarrow$ velocity of propagation

In vacuum $\quad V_p = c = 3 \times 10^8\ m/s$

"Air-filled Transmission Lines can go the speed of c"

$$u = \frac{1}{\sqrt{lC}} \qquad l = \frac{H}{m} \qquad C = \frac{F}{m}$$

$$V_p = \frac{1}{\sqrt{lC}}$$ Velocity is inversely proportional to capacitance and inductance

$$V_p = \frac{c}{\sqrt{\varepsilon_r}}$$ ← relative permittivity

Impedance

$Z \Rightarrow$ impedance, Ω

(Same characteristic, different physical interpretation)

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Voltage wave expression

$$LC \frac{\partial^2}{\partial t^2} V - \frac{\partial^2}{\partial z^2} V = 0$$

Same solution space, different coefficient

Current wave equation

$$\sim \quad \frac{\partial^2}{\partial t^2} I - \frac{\partial^2}{\partial z^2} I = 0$$

$$V(z,t) = f\left(t \pm \frac{1}{u} z\right)$$

$$V = V_0 \cos(\omega t - \beta z)$$

$$= V_0 \cos\left(\omega\left(t - \frac{\beta}{\omega} z\right)\right)$$

This is a wave
" a wiggle

what does the current ~~to~~ look like?

$$I = I_0 \cos(\omega t - \beta z)$$

$$I = \frac{V_0}{Z}\cos(\omega t - \beta z) \rightarrow Z = \frac{V_0}{I}\cos(\omega t - \beta z)$$

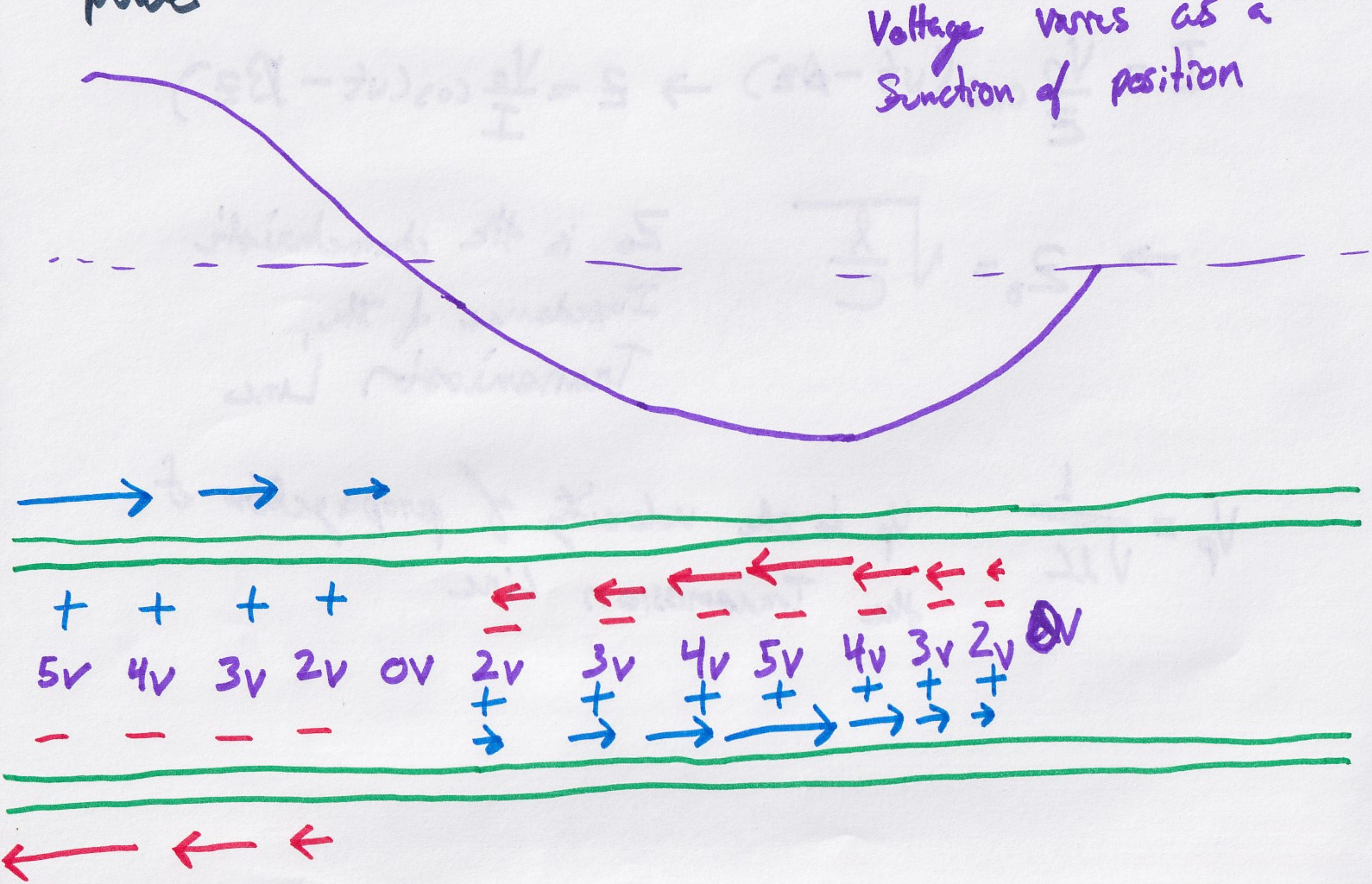
$$\rightarrow Z_0 = \sqrt{\frac{\ell}{C}}$$

Z_0 is the characteristic Impedance of the Transmission Line

$$V_p = \frac{1}{\sqrt{\ell C}}$$

V_p is the velocity of propagation of the Transmission Line

(10) A lossless Transmission Line doesn't dissipate power



Voltage varies as a function of position, moving with position and changing with time – it's dynamic

General function

$$V(z,t) = f(t \pm \frac{1}{u} z)$$

Sinusoidal Travelling wave

$$V(z,t) = V_0 \cos(\omega t \pm \beta z)$$

Will a square wave propagate?

→ Yes! Because it's made of sinusoids

(12)

→ put in any type of signal, it works!

(Fourier Series)

Look at

$$V_p = \frac{1}{\sqrt{\ell c}}$$

These guys are frequency dependent

Does changing the applied frequency change V_p?

Sinusoidal Travelling Wave

$$V(z,t) = V_0 \cos(\omega t \pm \beta z)$$

ω - radial frequency $\quad 2\pi f = \omega$

β - propagation constant $\quad \frac{2\pi}{\lambda} = \beta$

And $\lambda f = V_p$

$\lambda \rightarrow$ wave Length m

$f \rightarrow$ frequency Hz

$V_p \rightarrow$ velocity m/s

$$\frac{2\pi}{2\pi}\lambda f = V_p \frac{2\pi}{2\pi}$$

$$2\pi f\left(\frac{\lambda}{2\pi}\right) = V_p = \omega\frac{1}{\beta} = V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

As frequency goes up

sL $\quad \frac{1}{sC}$

$f\uparrow \quad \lambda\downarrow \quad L\uparrow \quad C\downarrow$

As frequency goes down

$f\downarrow \quad \lambda\uparrow \quad L\downarrow \quad C\uparrow$

frequency affects The Impedance of L, C

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From this, we can say,

$\Delta z << \lambda$ for valid lump parameter equivalence, the effective length of an L-C section ~~much~~ must be much smaller than a wave length.

~~The Role of Wavelength~~

~~So we wonder the speed of current.~~

~~Perhaps it can be said that current moves near the speed of light. C is the speed of EM waves~~

$\rightarrow$ L, C are determined by Δz

and so L, C must be a certain value for the solution of the wave eqn.

So we have

$$V(z,t) = V_0 \cos(\omega t \pm \beta z)$$

$\Rightarrow$ Real and measurable wave

$$\mathrm{Re}\left\{ V(z,t) = V_0 e^{j(\omega t \pm \beta z)} \right\}$$

$\rightarrow$ complex exponential wave solution

Phasor form of wave expression

$$V(z) = V_0 e^{\pm j\beta z} \{ e^{j\omega t} \}$$

implied oscillation in time

what does '$\pm$' mean?

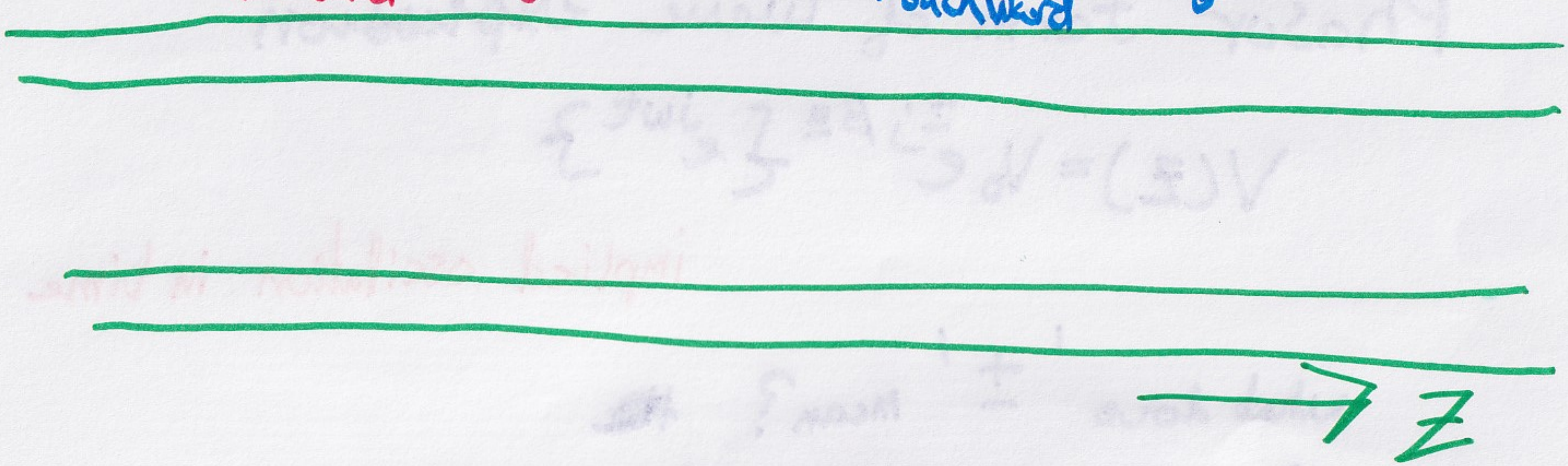
$\rightarrow$ The direction of propagation

$$\left.\begin{matrix}+\\-\end{matrix}\right\} \equiv \begin{cases}\text{propagation is in } -z \text{ direction}\\ \text{propagation is in } +z \text{ direction}\end{cases}$$

$+ \rightarrow$ left

$- \rightarrow$ right

$V_{forward} = V_f e^{-j\beta z}$ $\qquad$ $V_{backward} = V_b e^{+j\beta z}$



To Note

$$1\ \text{Henry} = \frac{1\ \text{weber}}{1\ \text{ampere}} = \frac{1\ \text{volt-second}}{1\ \text{ampere}} = 1\ \text{ohm-Second}$$

$$\left(1\ \Omega = 1\ \frac{V}{A} \left(\frac{\text{Volts}}{\text{Amperes}} \right) \right)$$

→ 1 henry is the amount of inductance necessary to induce one volt when the current in coil changes at a rate of one ampere per second.

$$\frac{\text{Volt}}{\text{Ampere}/\text{second}} = \frac{V\text{-}s}{A}$$

$$1\ \text{Farad} = \frac{\text{Coulomb}}{\text{Volt}} = \frac{\text{Seconds}}{\text{ohm}} = \frac{s}{\Omega}$$

$H = s\text{-}\Omega$ $(\Omega\text{-}s)$ $F = \frac{s}{\Omega}$ $c = \frac{1}{Z_0 V_p}$ (F/m) $l = \frac{Z_0}{V_p}$ (H/m)