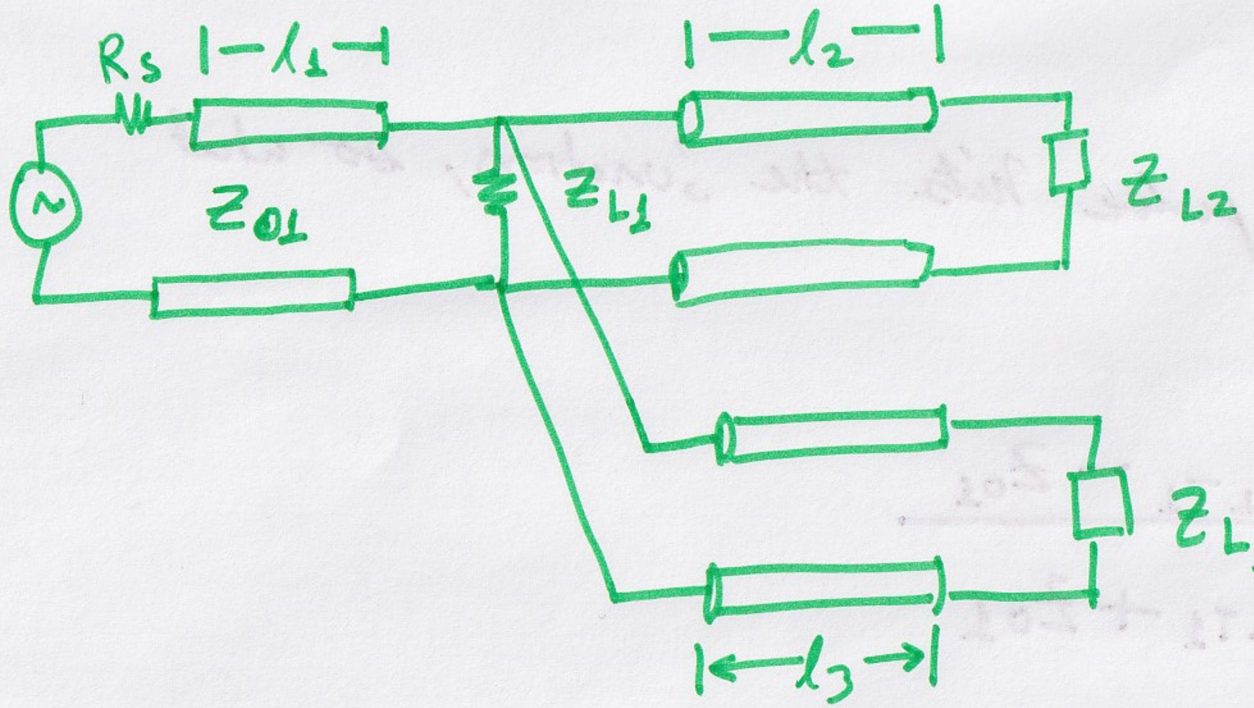
Fields and Waves I   L6



Each Line has their own time delay, td

$l_1 \rightarrow td_1$ (T_1)

$l_2 \rightarrow td_2$ (T_2)

$l_3 \rightarrow td_3$ (T_3)

(2)

So at $t=0$ (No pulse yet)

$$V_0^{1+} = \frac{Z_{01}}{R_s + Z_{01}} V_s$$

at $t = td_1$, the pulse hits the junction, so what bounces back?

$$\Gamma_{LT1} = \frac{Z_{LT1} - Z_{01}}{Z_{LT1} + Z_{01}}$$

What's Z_{LT1}?

Since we are working with pulses, we can determine Z_{LT1} as

$$Z_{LT1} = Z_{L1} \parallel Z_{02} \parallel Z_{03} = \left(\frac{1}{Z_{L1}} + \frac{1}{Z_{03}} + \frac{1}{Z_{02}} \right)^{-1}$$

$$Z_{LT1} = \left(\frac{1}{Z_{L1}} + \frac{1}{Z_{02}} + \frac{1}{Z_{03}} \right) = Z_{L1} \parallel Z_{02} \parallel Z_{03}$$

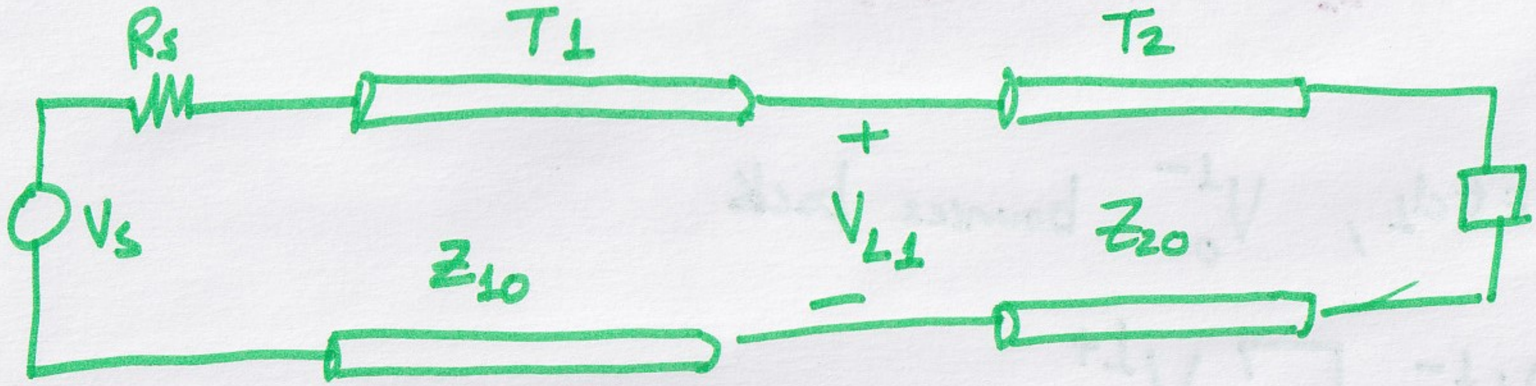
So at $t = td_1$, V_0^{1-} bounces back

$$V_0^{1-} = \Gamma_{LT1} V_0^{1+}$$

So what goes down towards T_2?

$\rightarrow$ A pulse, what's the amplitude?

④ Another Problem



The pulse on T_1 is $V_1^+ = \dfrac{Z_{10}}{Z_{10} + R_s} V_s$

And the reflection coefficient for the first T-Line (T_1) is $\longrightarrow$ $\Gamma_{L1} = \dfrac{Z_{20} - Z_{10}}{Z_{20} + Z_{10}}$

So $V_1^- = \Gamma_{L1} V_1^+$

We may write $V_{L1} = (1 + \Gamma_{L1}) V_1^+$

What's the voltage on T_2? (The transmitted voltage)

We know T_2 is the Load for T_1 and the Load for V_{L1}.

So $V_2^+ = (1 + \Gamma_{L1}) V_1^+$

This is the pulse sent down T_2

Power doesn't increase

Looking at the reflection coefficient, $\frac{Z_L - Z_0}{Z_L + Z_0}$,

Z_L must be larger for increased voltage

⑥

So $\frac{V_L}{Z_L} = I$, if $Z_L \uparrow$, $I \downarrow$

then Power doesn't go up.

Therefore, going back to the previous problem, the ~~amplitude~~ amplitude of the pulse being sent down T_2 and T_3 is

$$V_0^{2,3+} = (1 + \Gamma_{LT_1}) V_0^{1+}$$

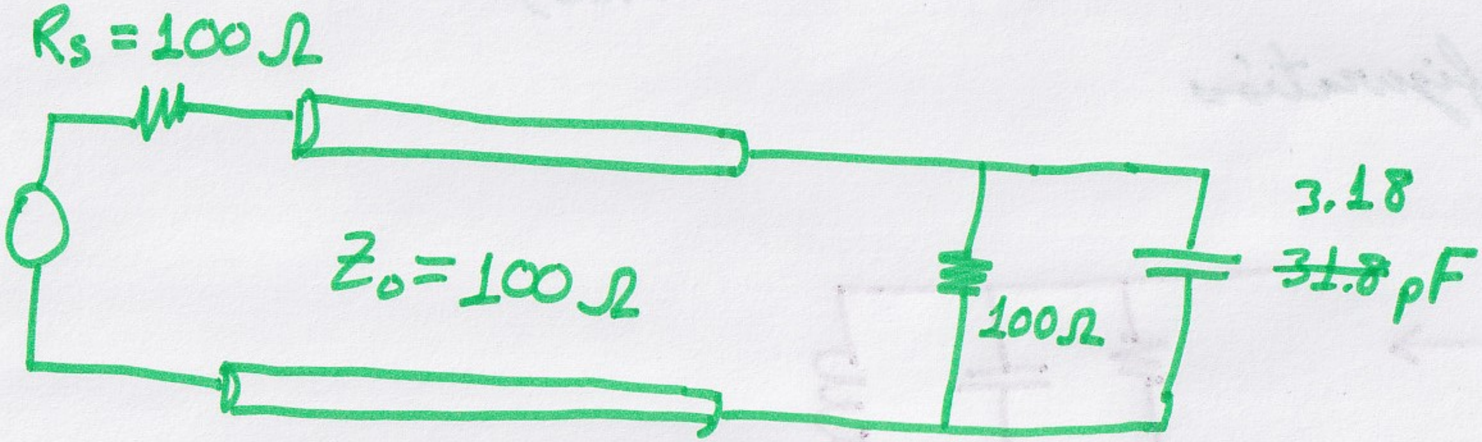
AC Steady State

Impedance Matching

Ideally, we want $\Gamma_L = 0$

which means we have no reflected power

Example)



Vs at 100V at 1 MHz

8

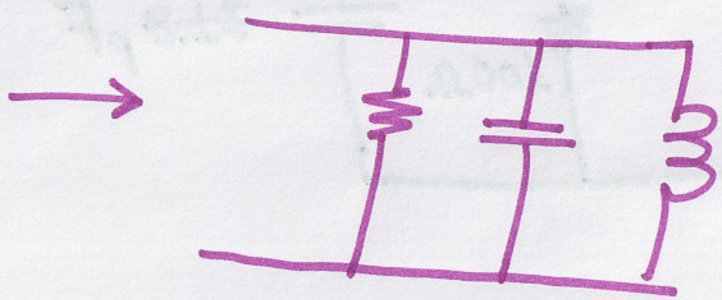
We can see we have a mismatched Load

$$Z_L = R \,||\, \frac{-j}{\omega C} = 100 \,||\, -j50$$

The goal is to reduce the capacitive reactance to zero
(negative reactance)

We do this by adding inductive reactance in <u>parallel</u>
(positive reactance)

for this configuration



if $|Z_L| = |Z_C|$, Then $Z_L = Z_C^*$

equal and opposite

So we need $Y_{Load} = Y_R + Y_L + Y_C$

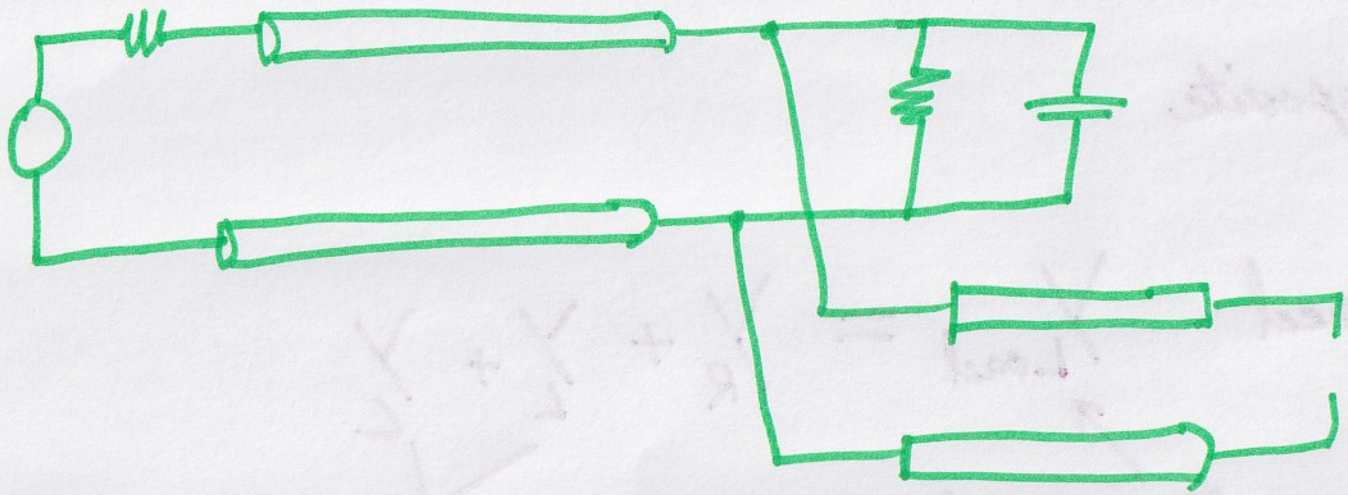
effective admittance (Load) These need to cancel out

$\rightarrow Y_{Load} = Y_R \quad (Z_L = R)$

effective impedance (Load)

When you change the frequency, the reactance of the capacitance and inductance change. So to keep a matched Load we have to design around a given frequency (ω)

10 Here's what we can do



add an open circuit

Zin_{oc} (Z input open circuit)

$$\longrightarrow Zin_{oc} = Z_0\left(\frac{\infty + j Z_0 \tan\beta l}{Z_0 + j\infty\tan\beta l}\right)$$

$$= Z_0\left(\frac{\infty}{j\infty\tan\beta l}\right)$$

$$= -j Z_0 \frac{1}{\tan\beta l}$$

So our effective Load

$$Z_{L\,(\text{effective})} = 100 \,||\, -j50 \,||\, \frac{-jZ_0}{\tan\beta l}$$

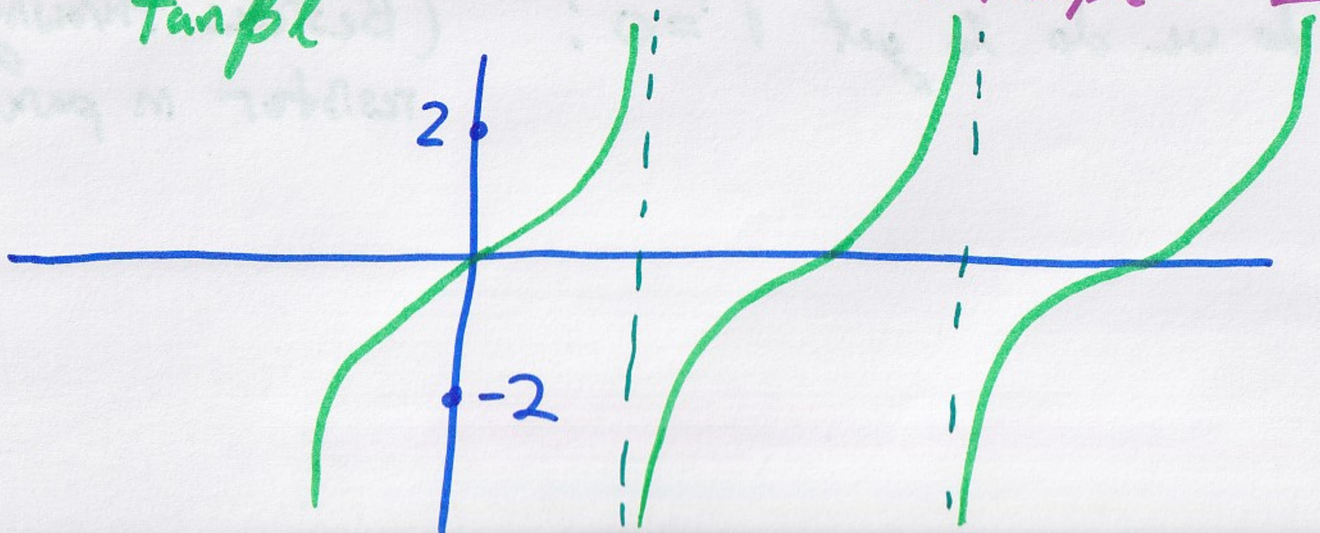
resistor   capacitor   Open circuit stub

We want $Z_{L\,(\text{effective})} = 100$

Stub works if $Z_{in_{oc}} = +j50 \rightarrow j50 = \dfrac{-jZ_0}{\tan\beta l}$ $(Z_0 = 100\,\Omega)$

$\rightarrow \tan\beta l = -2$

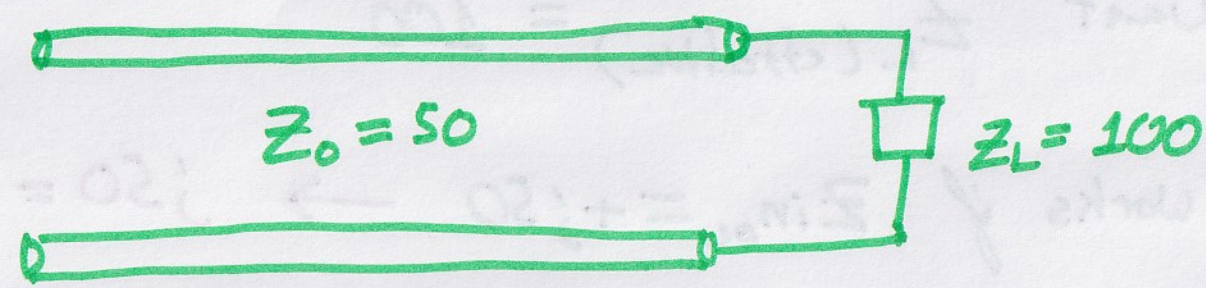
$\tan\beta l$



(12) Find an answer that corresponds to '-2'

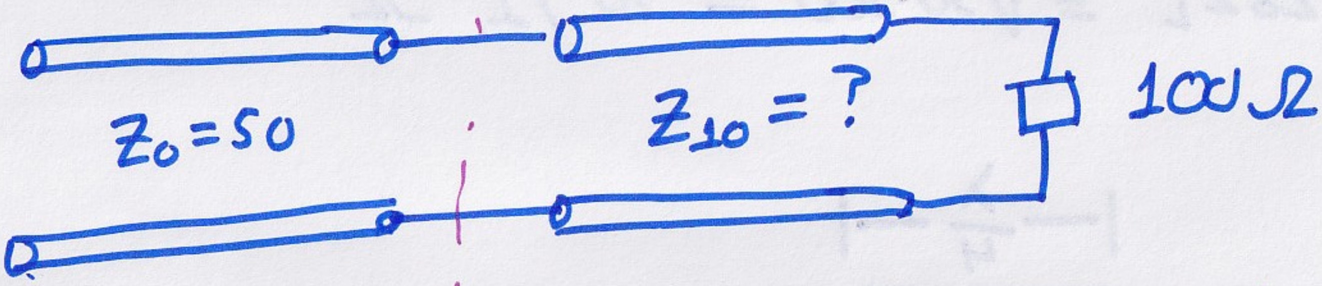
The shortest Line is desired so we don't have So much Loss of Power

Example) What if we have a mismatched resistive Load



What do we do to get $\Gamma = 0$? (Besides throwing a $100\,\Omega$ resistor in parallel)

We can put another Transmission Line in between the Original Transmission Line and the 100 Ω Load.



→ $Z_{in} = 50\,\Omega$ we need 50 Ω to get a matched Load

$$Z_{in} = Z_{10} \frac{Z_L + jZ_{10}\tan(\beta l)}{Z_{10} + jZ_L \tan(\beta l)}$$

We can manipulate Z_{in} by adjusting the Length of the second Transmission Line.

$$l = \frac{\lambda}{2} \longrightarrow Z_{in} = Z_L$$

$$l = \frac{\lambda}{4} \longrightarrow Z_{in} = \frac{Z_{10}^2}{Z_L}$$

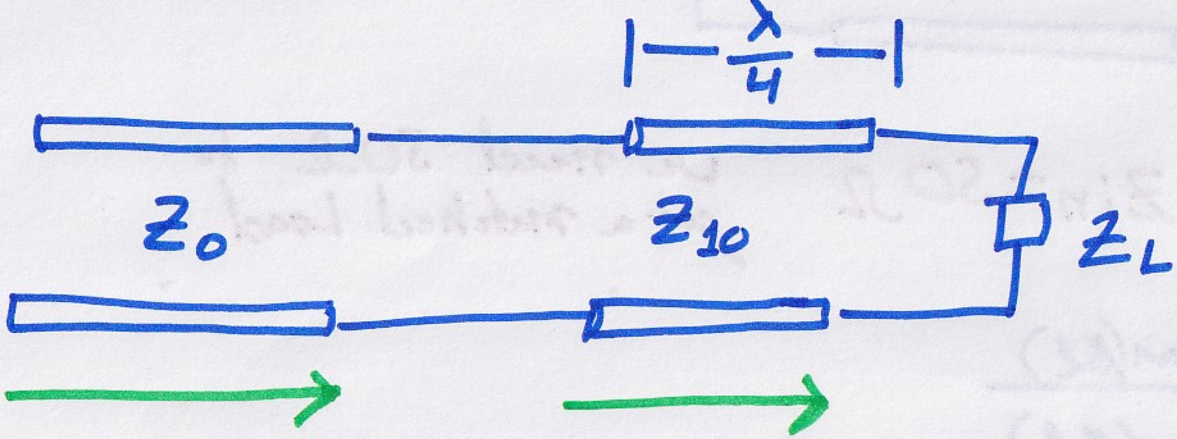
14

We want

$$Z_{in} = Z_0 = \frac{Z_{10}^2}{Z_L}$$

$$Z_{10} = \sqrt{Z_0 Z_L} = \sqrt{50 \cdot 100} = 70.71 \ \Omega$$

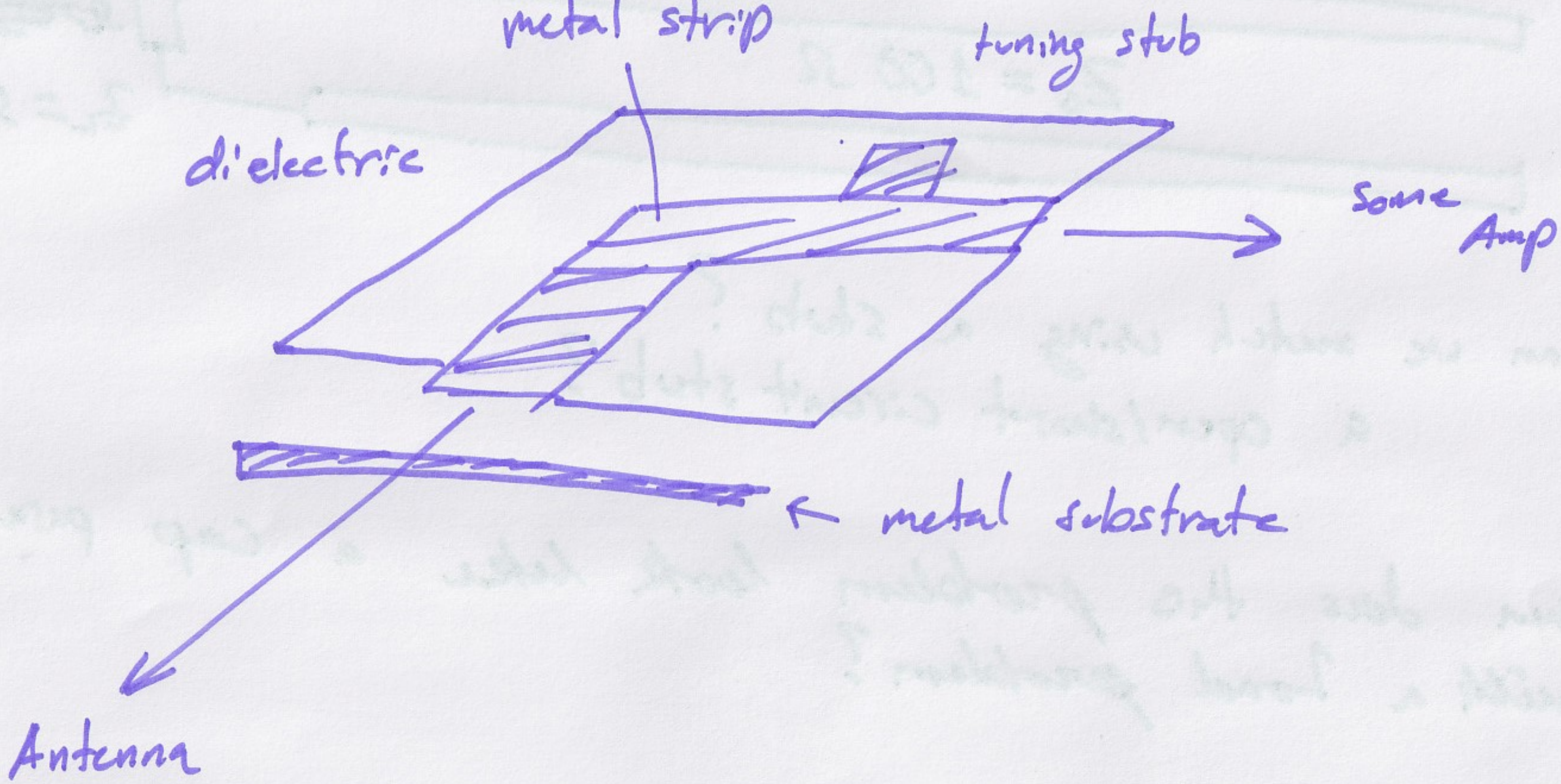
So then



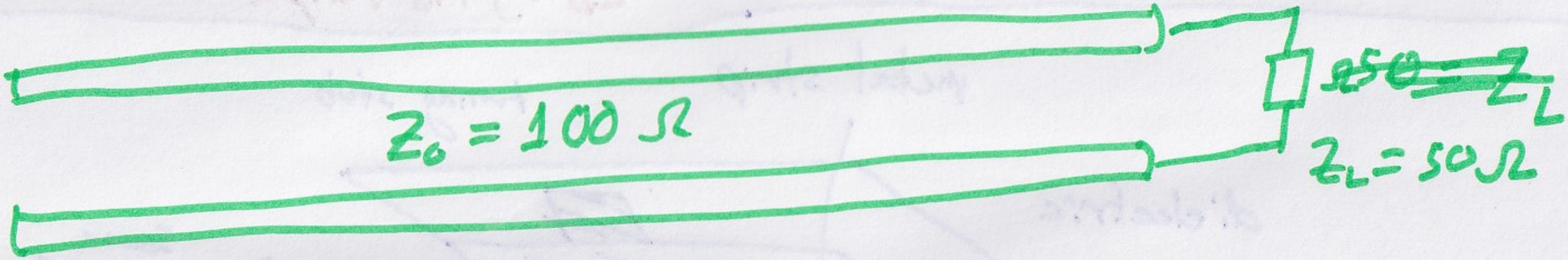
We don't have a matched Load for the second Transmission Line. So there is a reflected wave.

And T_2 (Second T-Line) sees a reflection Coefficient due to T_1 (first T-Line), ~~$Z_{in_{T2}} - Z$~~ $\longrightarrow \dfrac{Z_{in_{T1}} - Z_{10}}{Z_{in_{T1}} + Z_{10}}$

~~Where~~ where $Zin_{T1} = Z_0 \frac{R_s + j Z_0 \tan\beta l}{Z_0 + j R_s \tan\beta l}$



Ex)



Can we match using a stub?

a open/short circuit stub?

When does this problem look like a cap parallel with a load problem?

Think about Z_{in}

$$Z_{in} = Z_0 \frac{Z_L + j Z_0 \tan(\beta \ell)}{Z_0 + j Z_L \tan(\beta \ell)}$$

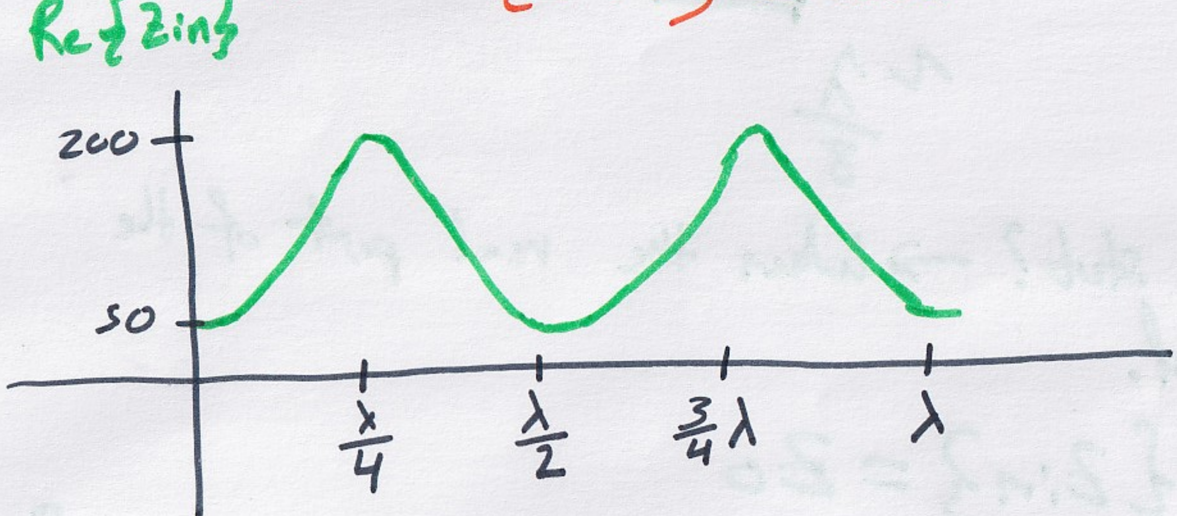
at $l = 0, \frac{\lambda}{2}, \frac{\lambda}{4}, \frac{3}{2}\lambda$

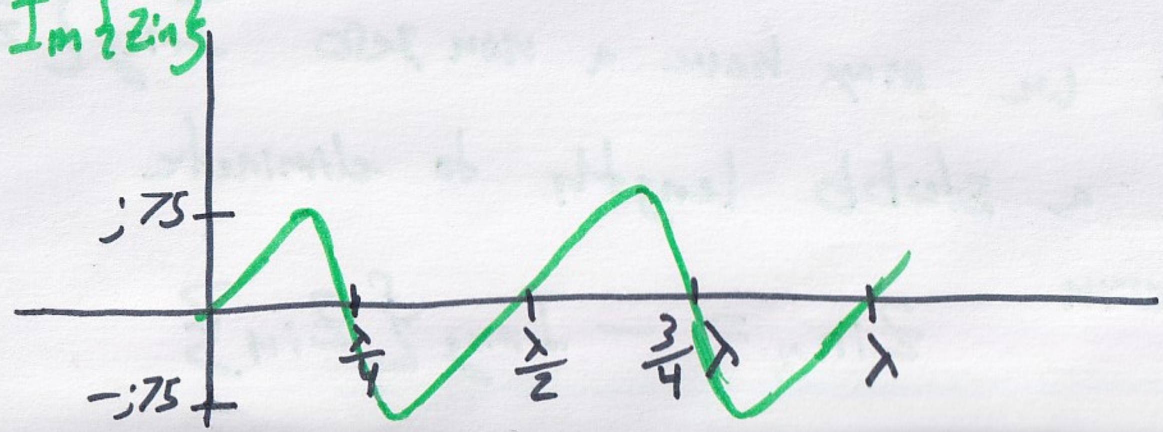
$\rightarrow Z_{in} = 50\ \Omega$

at $l = \frac{\lambda}{4}, \frac{3}{4}\lambda, \frac{5}{4}\lambda$

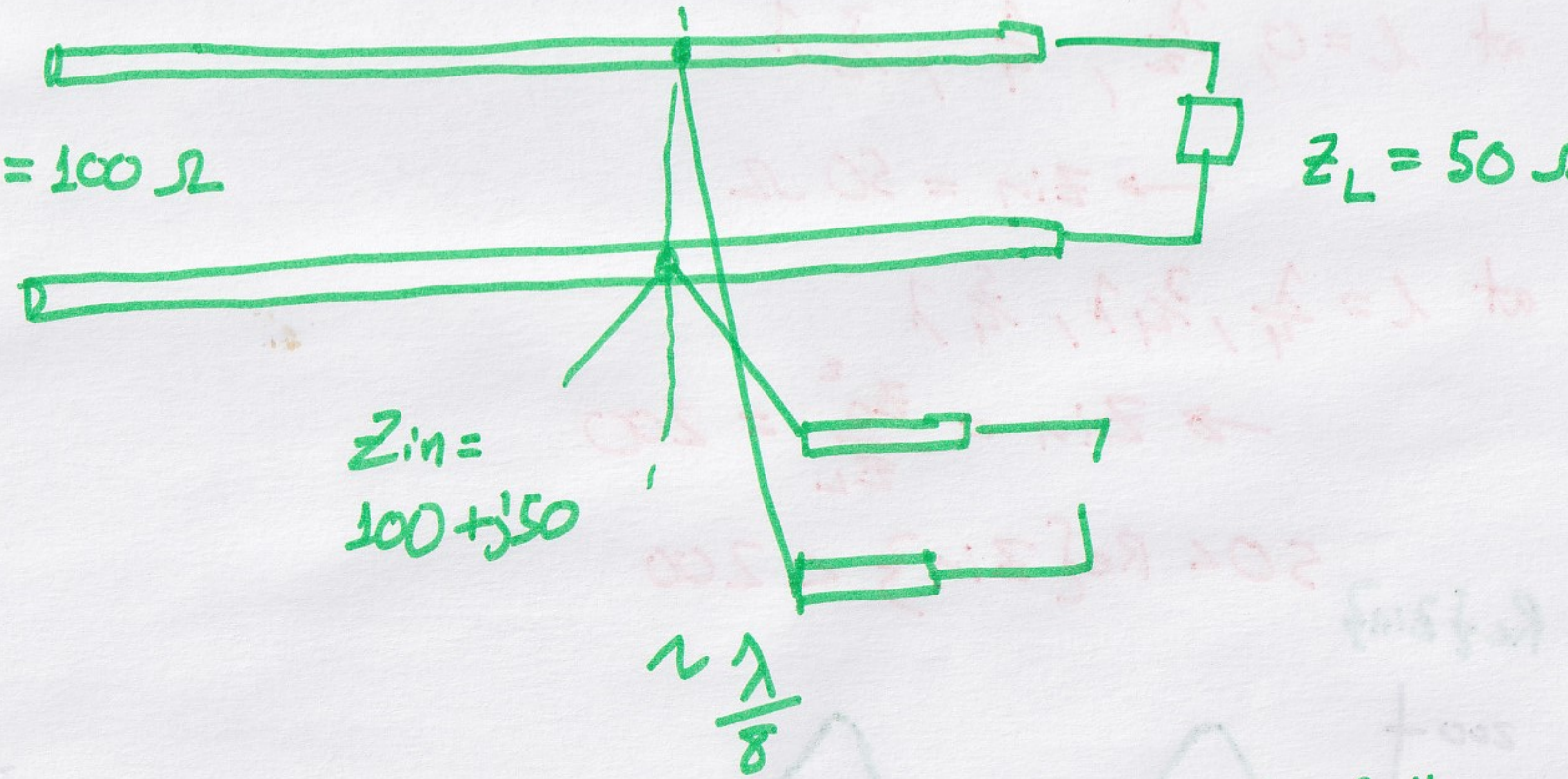
$\rightarrow Z_{in} = \frac{Z_o^2}{Z_L} = 200$

$50 < \mathrm{Re}\{Z_{in}\} < 200$





18



Where do we put this stub? → When the real part of the Impedance is matched.

$$\mathrm{Re}\{Z_{in}\} = Z_0$$

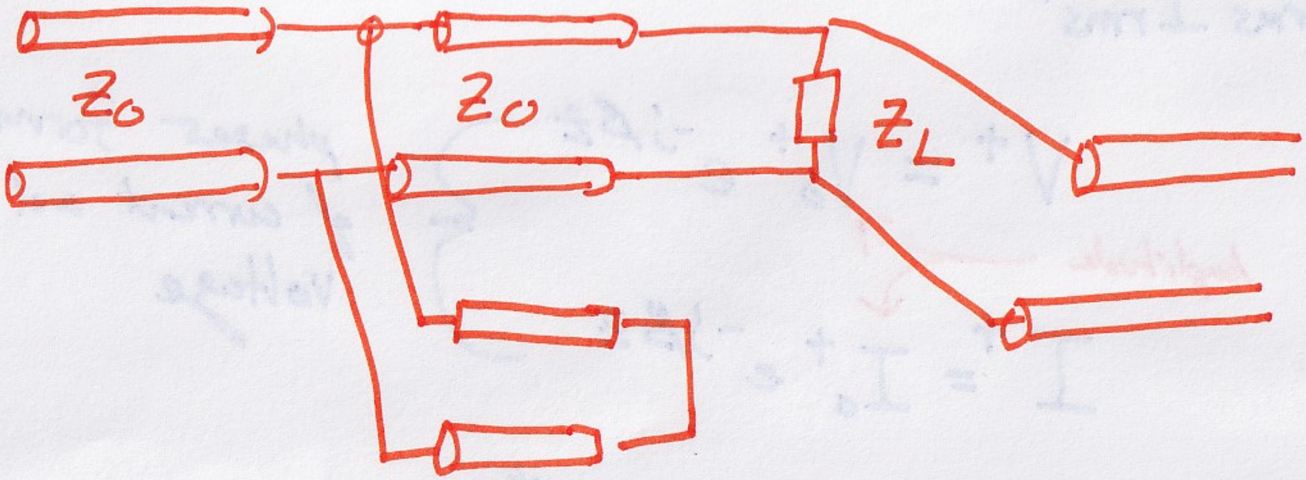
But at this point, we may have a non zero $\mathrm{Imag}\{Z_{in}\}$

→ So use a stubb length to eliminate this term

$$Z_{in_\alpha} = -\,\mathrm{Imag}\{Z_{in}\}$$

Double stub tuning

(open/short ckt)



Use Smith charts!

(20)

$$\text{Power} = \tfrac{1}{2} V I^{*}$$ ← complex conjugate of I

$$= V_{rms} I_{rms}^{*}$$

~~$V_0^+/Z = I_0^+$~~

$$V^{+} = V_0^{+} e^{-j\beta z}$$

Amplitude

$$I^{+} = I_0^{+} e^{-j\beta z}$$

phasor form of current and voltage

$$\text{Power} = \tfrac{1}{2}\left(V_0^{+} e^{-j\beta z}\right)\left(\frac{V_0^{+}}{Z_0} e^{-j\beta z}\right)^{*}$$

$$= \tfrac{1}{2}\left(V_0^{+} e^{-j\beta z}\right)\left(\frac{V_0^{+}}{Z_0} e^{+j\beta z}\right)$$

$$\text{Power} = \tfrac{1}{2}\frac{|V_0^{+}|^2}{Z_0}$$

This is the Power of the forward propagating wave

Reflected wave

$$V^- = V_{OR}^- e^{+j\beta z}$$

$$I^- = I_{OR}^- e^{+j\beta z} = \frac{V_{OR}^-}{Z_0} e^{+j\beta z}$$

$$P_{reflected} = \frac{1}{2} \frac{|V_{OR}|^2}{Z_0} \qquad V_{OR} = \Gamma V_{of}$$

$$= \frac{1}{2} \frac{|\Gamma V_0|^2}{Z_0} = |\Gamma|^2 \left(\frac{1}{2} \frac{|V_0|^2}{Z_0}\right)$$

so

$$P_{reflected} = |\Gamma|^2 P_{forward}$$

reflected power is proportional to $|\Gamma|^2$

(22)

Ex) if $\Gamma = 1/3$, $1/3$ Voltage is reflected

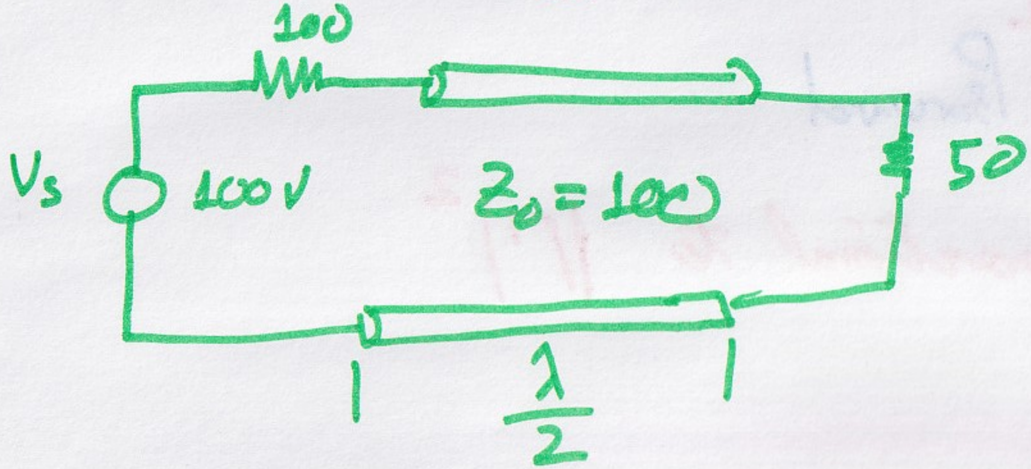
but $1/9$ Power is reflected.

Power delivered to the Load is the Power transmitted?
(Same question)

$$P_{Load} = (1 - |\Gamma_L|^2)\, P_{forward}$$

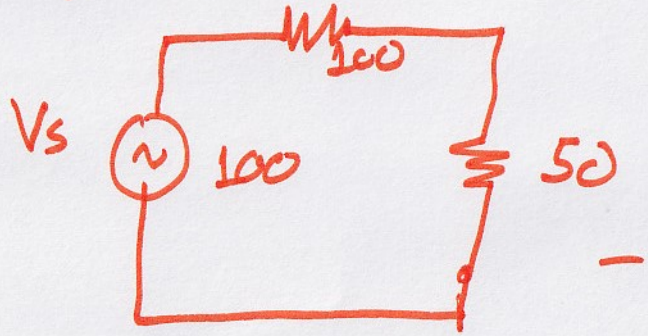
Look at Source resistance.

What's the forward Power?



if $\ell = \frac{\lambda}{2}$, $Z_{in} = 50$

equivalent ckt



$V_{in} = V_L = 66.6\ V$

$$P_L = \frac{1}{2}\,\frac{(66.6)^2}{50} = 44.4\ [W]$$

So Then

$$P_L = (1 - |\Gamma|^2)\,P_{forward}$$
$$= (1 - (\tfrac{1}{3})^2)\,P_{forward}$$

$$\rightarrow P_{forward} = \frac{44.4}{8/9} = 49\ W$$

(24)

So we have about 90% efficiency

what about a capacitive Load?

or inductor?

all power reflected?