Fields and Waves I

L 1

The Wave Equation

4 Parts

1) Transmission Lines
2) Electrostatics
3) Magnetostatics & Magnetodynamics
4) Plane Electromagnetic Waves
(Plane Waves)

Extra

5) Antennas
6) Satelite Comm. Systems and Radars

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Circuit Theory and Electromagnetics

uses simplified (Lumped) Models of components

There are limitations
components are not simply R, L, C

we need details about how they work

Circuit Theory does not include details about
- Distributed properties of the Components
  (Transmission Lines)
- EM Waves ~ μWaves, Radio Waves, Optics
- Capacitive Sensors
- Noise

Linear Circuit Analysis fail in most high-speed circuits. KCL, KVL fail

So we use EM Theory to analyze such Problems

~~Kayfabe~~

OK, Transmission Lines

What's a Field?

$\bar{E}$ electric field   both vectors

$\bar{B}$ magnetic field

- vector that vary w/ position (and time)
  - → direction
  - → magnitude

I → current (total current)

V → voltage

Phasor Analysis

from circuits

$$\mathbb{V} = |V_0| \angle V_0 = |V_0| \angle \Phi_V$$

remember "implied" $e^{j\omega t}$

→ from AC analysis (AC steady state)

$$V = \mathbb{V} e^{j\omega t}$$

$$Re\{V\} = |V| \cos(\omega t + \Phi_V)$$

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Electrostatics is a Big deal in Fields and waves

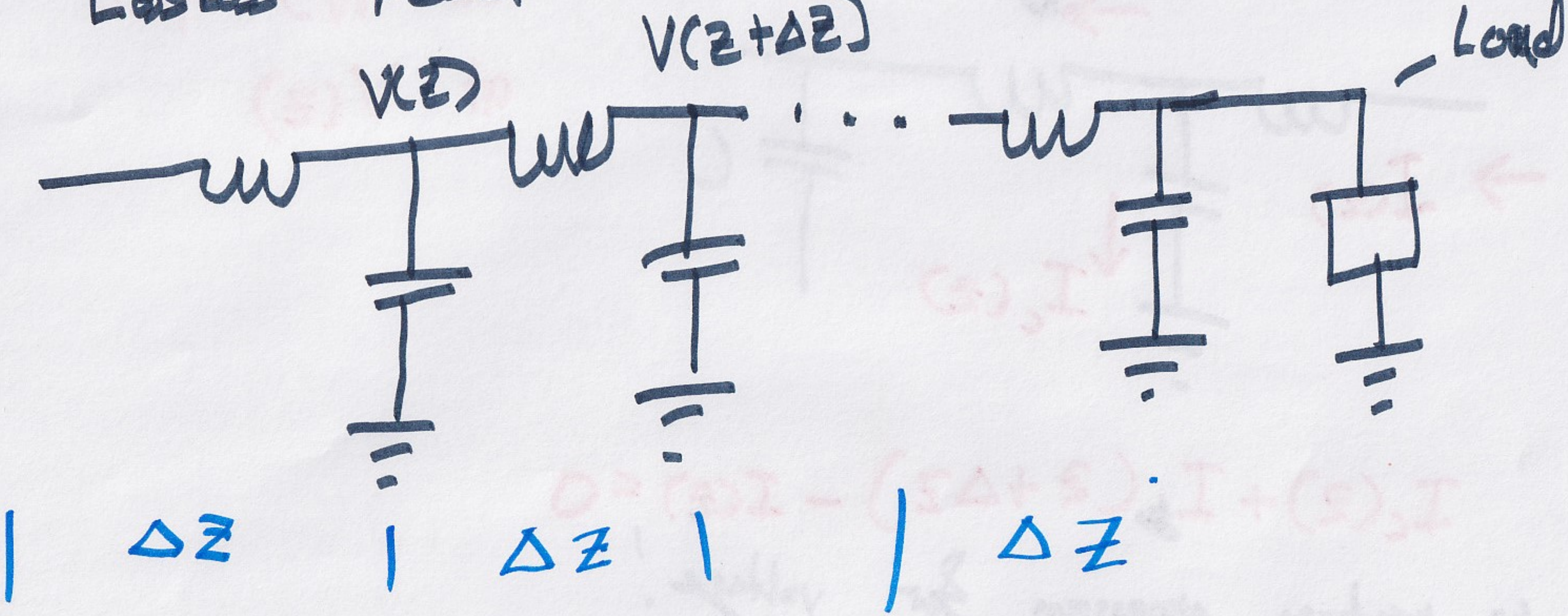
$$\oint \bar{E} \cdot d\bar{l} = -\frac{\partial}{\partial t}\phi = 0$$

→ does not change with time!

Lumped parameter Model

- use capacitive and inductive concepts to model a transmission Line

Lossless Model



Transmission Line Model

Every L, C, section represents ~~some~~ Length of T-Line

— Now use Node Analysis!

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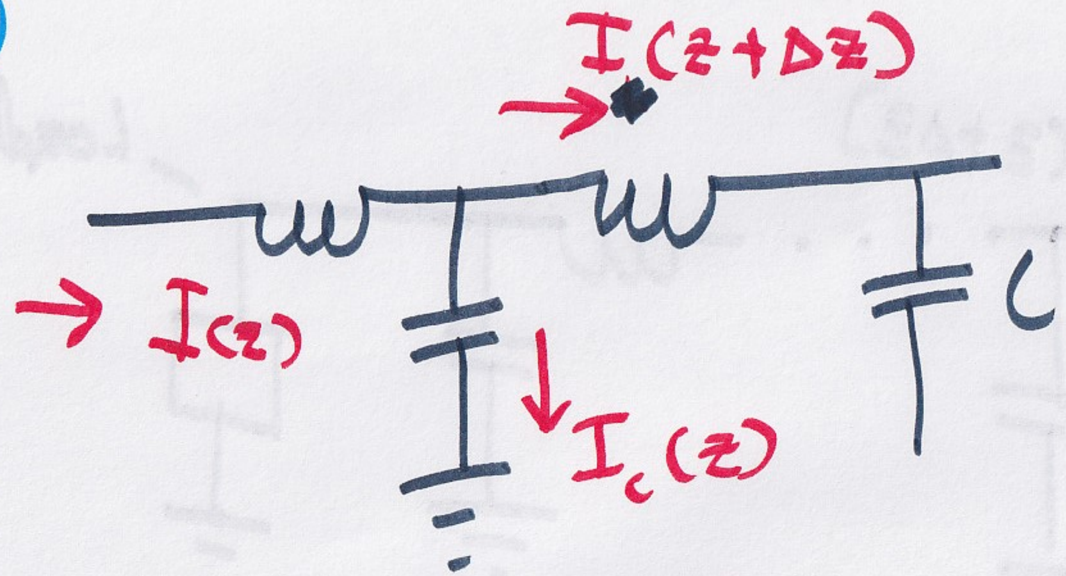

We Apply KCL at V(z)

$$I_c(z) + I(z+\Delta z) - I(z) = 0$$

we need an expression for voltage!

use multivar

$$I_c(z,t) + I(z+\Delta z, t) - I(z,t) = 0$$

current, voltage are a function of both position and time

We know that

$$I_c = C \frac{d}{dt} V_c \qquad V_L = L \frac{d}{dt} I_L$$

So $\Delta V = L \frac{d}{dt} I$ (changing voltage across the inductor)

So we may write (we are going to need this)

$$C \frac{\partial}{\partial t} V_c(z,t) + \Delta z \left(\frac{I(z+\Delta z, t) - I(z,t)}{\Delta z} \right) = 0$$

$$\rightarrow C \cancel{\frac{d}{dt}} V_c(z,t) + \Delta z \frac{\partial}{\partial z} I(z,t) = 0$$

$\frac{\partial}{\partial t}$

Now take $\frac{\partial}{\partial t}$

(10)

$$\frac{\partial}{\partial t}\left[c\frac{\partial}{\partial t}V_c(z,t) + \Delta z \underline{\frac{\partial}{\partial z} I(z+\Delta z, t)} \right] = 0$$

we can also write it like this

(if a dc term exists (a constant) it goes away
(could lead to a problem)

So

$$0 = c\frac{\partial^2}{\partial t^2}V_c(z,t) + (\Delta z)\frac{\partial}{\partial t}\left[\frac{\partial}{\partial z} I(z+\Delta z, t)\right]$$

for some reason, we can do this

$$0 = c\frac{\partial^2}{\partial t^2}V_c(z,t) + (\Delta z)\frac{\partial}{\partial z}\left[\frac{\partial}{\partial t} I(z+\Delta z, t)\right]$$

$$0 = c \frac{\partial^2}{\partial t^2} V_c(z,t) + (\Delta z) \frac{\partial}{\partial z}\left[\frac{-V(z+\Delta z, t) + V(z,t)}{L}\right]$$

$$LC \frac{\partial^2}{\partial t^2} V_c(z,t) - (\Delta z)^2 \frac{\partial^2}{\partial z^2} V(z,t) = 0$$

$$\left(\frac{L}{\Delta z}\right)\left(\frac{C}{\Delta z}\right) \frac{\partial^2}{\partial t^2} V_c(z,t) - \frac{\partial^2}{\partial z^2} V(z,t) = 0$$

$$lc \frac{\partial^2}{\partial t^2} V_c(z,t) - \frac{\partial^2}{\partial z^2} V(z,t) = 0$$

$l = \frac{L}{m}$ $\quad c = \frac{C}{m}$ $\quad$ wave equation!

going back to page 10 [from page 10)

$$\frac{\partial}{\partial t}\left[C \frac{\partial}{\partial t} V_c(z,t) + \Delta z \frac{\partial}{\partial z} I(z,t) \right] = 0$$

$$\rightarrow C \frac{\partial^2}{\partial t^2} V_c(z,t) + [\Delta z] \frac{\partial}{\partial t}\left[\frac{\partial}{\partial z} I(z,t) \right] = 0$$

$$C \frac{\partial^2}{\partial t^2} V_c(z,t) + [\Delta z] \frac{\partial}{\partial z}\left[\frac{\partial}{\partial t} I(z,t) \right] = 0$$

recall $V_L = L \frac{d}{dt} I = \Delta V$ change in voltage across the inductor

So what's the voltage across the Inductor ?

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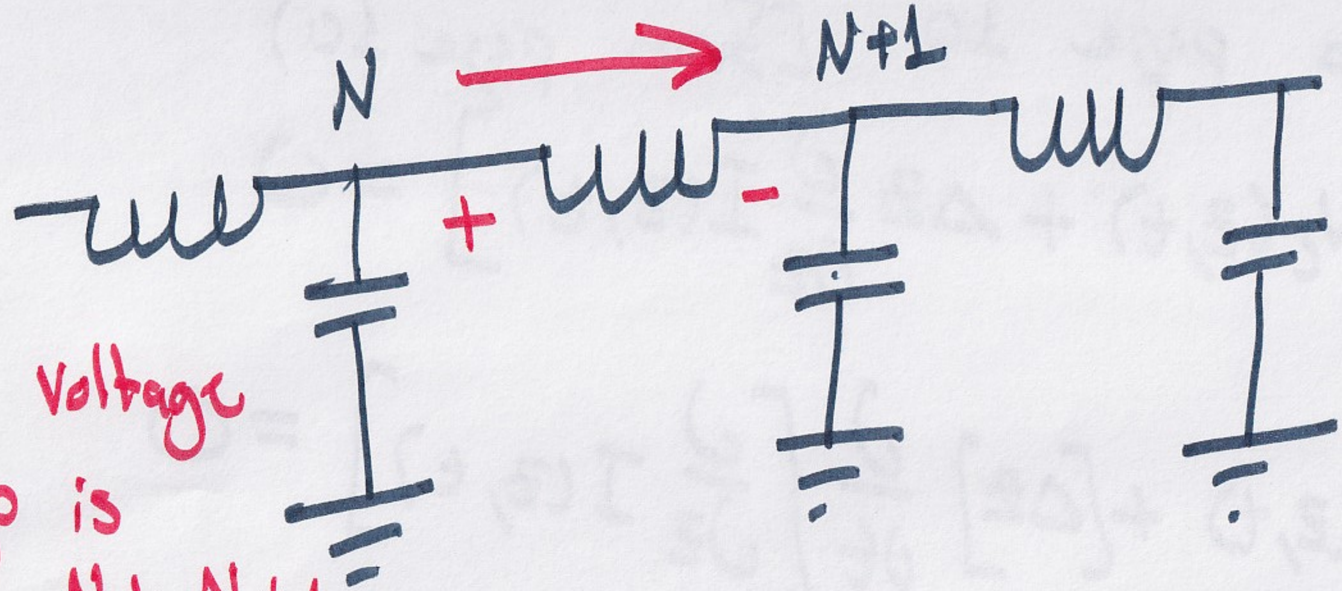

'+' voltage drop is from N to N+1

[Passive sign convention]

$$V_{N+1} - V_N = -V_L \qquad V_{N+1} = V(z+\Delta z, t)$$

$$V_N = V(z, t)$$

So $-\frac{\partial}{\partial t} I(z,t) = \frac{V(z+\Delta z, t) - V(z,t)}{L}$

$$\rightarrow C\frac{\partial^2}{\partial t^2} V_c(z,t) + (\Delta z)\frac{\partial}{\partial z}\left[\frac{-V(z+\Delta z, t) + V(z,t)}{L}\right] = 0$$

$$\rightarrow \quad LC\frac{\partial^2}{\partial t^2}V_c(z,t) - (\Delta z)^2\frac{\partial}{\partial z}\left[\frac{V(z+\Delta z,t)+V(z,t)}{\Delta z}\right] = 0$$

$$LC\frac{\partial}{\partial t^2}V_c(z,t) - (\Delta z)^2\frac{\partial^2}{\partial z^2}V(z,t) = 0$$

$$l = \frac{L}{m} = \frac{H}{m} \qquad c = \frac{C}{m} = \frac{F}{m}$$

$$\left(\frac{L}{\Delta z}\right)\left(\frac{C}{\Delta z}\right)\frac{\partial^2}{\partial t^2}V_c(z,t) - \frac{\partial^2}{\partial z^2}V(z,t) = 0$$

$$lc\frac{\partial^2}{\partial t^2}V_c(z,t) - \frac{\partial^2}{\partial z^2}V(z,t) = 0$$

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Note!

Concerning $\frac{\partial}{\partial t}\left[\frac{\partial}{\partial z} I(z,t)\right] = \frac{\partial}{\partial z}\left[\frac{\partial}{\partial t} I(z,t)\right]$

We can fairly assume (and it is) $I(z,t)$ is of class C^2 (the second partials are continuous)

$$I_{zt} = I_{tz}$$

Basically if the partial derivatives exist in some neighborhood of (z,t) and are continuous you can do this