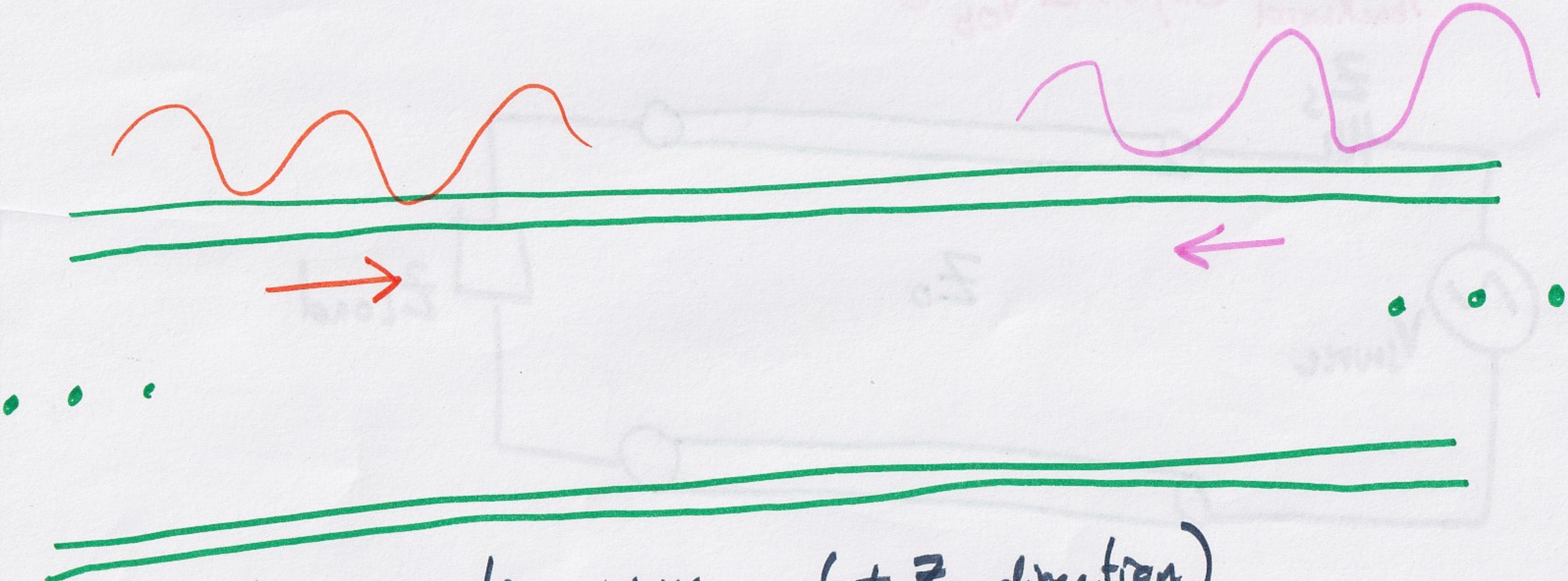
Fields and Waves I   L3

The Reflection Coefficient

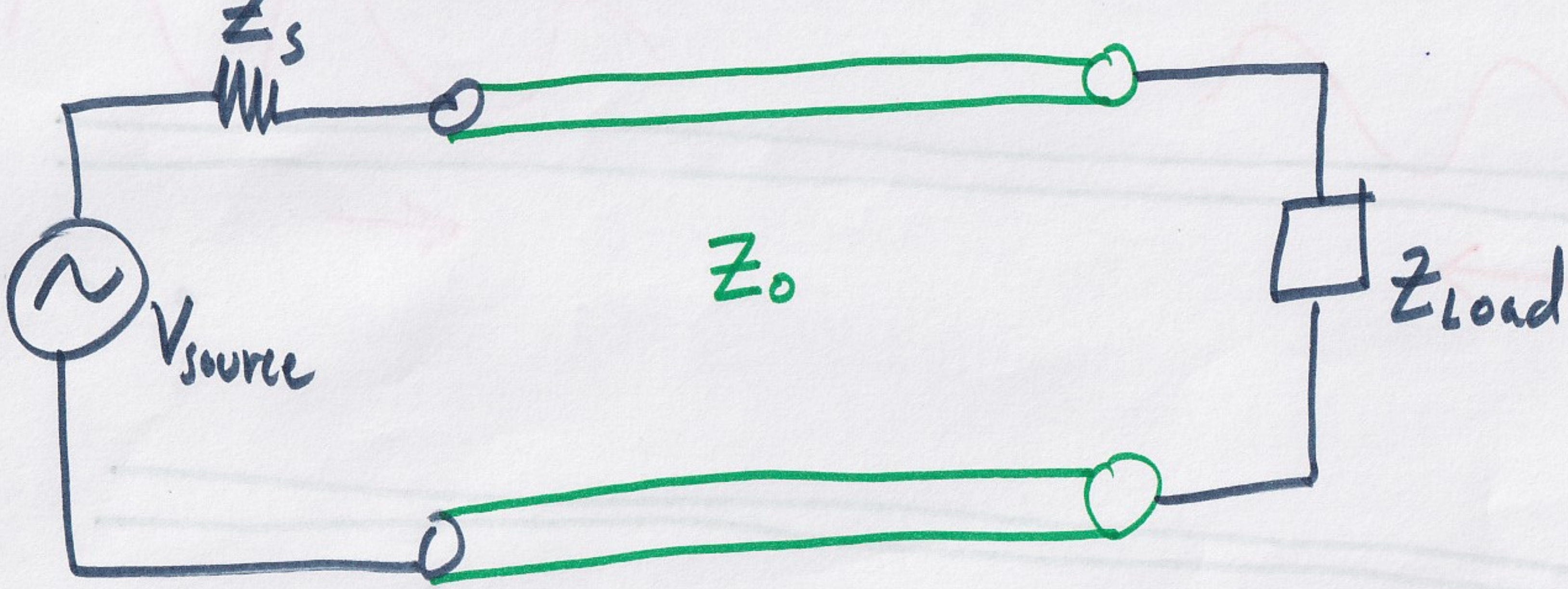


Forward propagating wave (+Z direction)

$$V_{forward}(z,t) = V_{of}\, e^{j(\omega t - \beta z)}$$

(2) backward propagating wave (− Z direction)

$V_{backward}(z,t) = V_{ob}\, e^{j(\omega t + \beta z)}$



source

ckt

$Z_S \rightarrow$ Source impedance

$Z_L \rightarrow$ load impedance

if resistive, They dissipate power

$Z_0 \rightarrow$ characteristic impedance of T-line

if real, Line itself is lossless

Z_0 is not a resistor, ~~just~~ a line of just Transmission

if DC, the ckt is a voltage divider

(4) "Think" waves in a bathtub reflect backwards

Note, we ~~are~~ are dealing with phasors

$V_{of} = V_0 e^{j(wt - B z)} \longleftrightarrow V_0 e^{-jBz}$

phasor

So then,

Forward propagating waves come from the source

Backwards propagating waves come from the Load → reflected waves

Denote Blue as

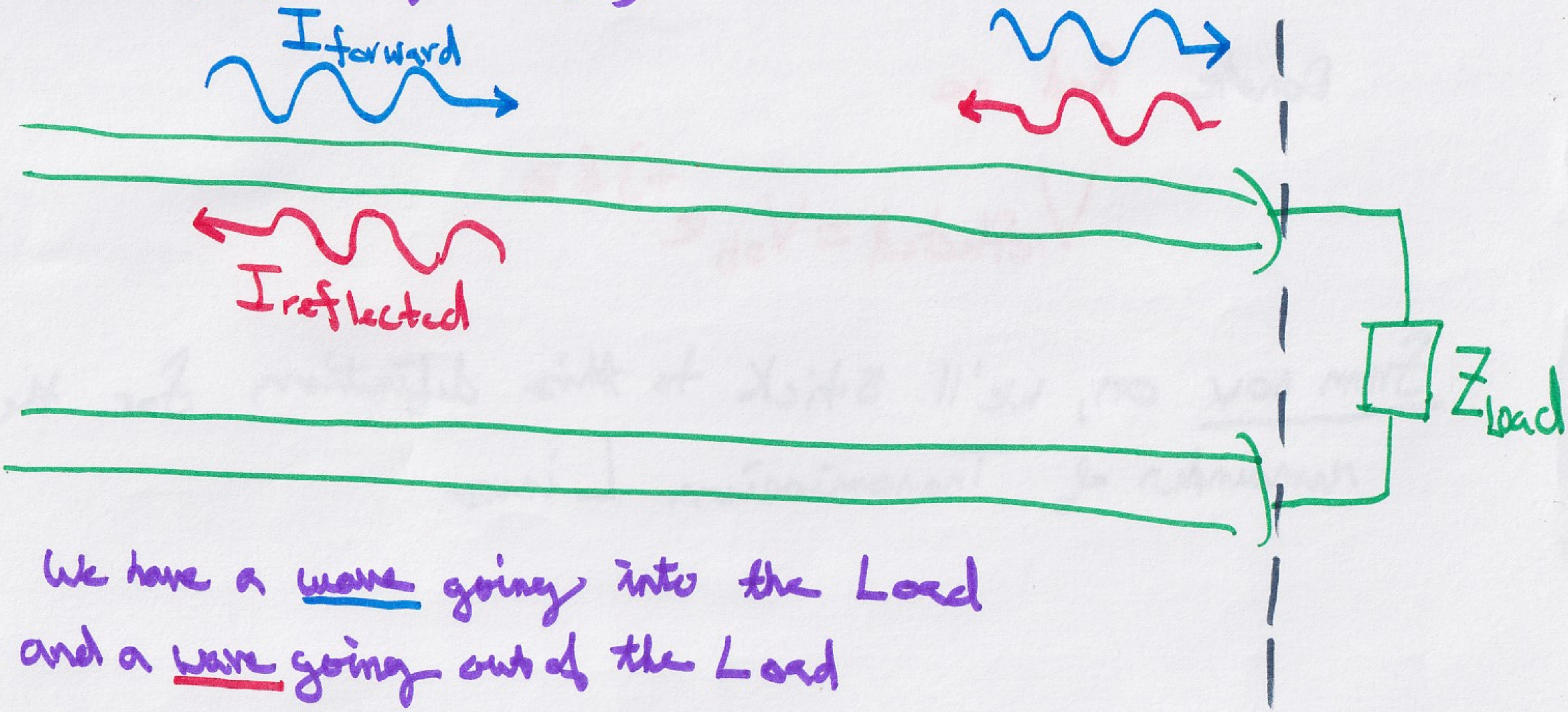
$$V_{forward} = V_{of} e^{-j\beta z}$$

Denote Red as

$$V_{reflected} = V_{ob} e^{+j\beta z}$$

From now on, we'll stick to this definition for the remainder of Transmission Lines

⑥ Assign an origin for coordinate system
(it's just a reference)



We have a wave going into the Load
and a wave going out of the Load

$Z = 0$
(position)

The waves just add

$$V_{total} = V_{forward} + V_{reflected}$$

$$V_{total} = V_{of} e^{-j\beta z} + V_{or} e^{+j\beta z}$$

z = position β is consistant (constant)

$$V_{total}(z=0) = V_{of} + V_{or} = V_{Load}$$

$z=0$, means we are at the Load

$$V_{total}(z=0) = V_{of} e^{-j\beta z} + V_{or} e^{+j\beta z} = V_{Load}$$

for now, Let's say this is partially true

(8) So

V_{0f} is known via source constraint

V_{or}, unknown
V_{Load}, unknown

(2) unknowns
1 equation so far

$I_{forward} = I_{0f} e^{-j\beta z}$

$I_{backward} = I_{or} e^{+j\beta z}$
(reflected)

$I_{forward} = \frac{V_{0f}}{Z_0} e^{-j\beta z}$

$I_{reflected} = \frac{V_{or}}{Z_0} e^{+j\beta z}$

Question!

So do I current waves add?

No! current waves subtract

$$I_{total} = I_{forward} - I_{reflected}$$

$$= \frac{V_{of}}{Z_0} e^{-j\beta z} - \frac{V_{or}}{Z_0} e^{+j\beta z}$$

At $z = 0$ (The Load)

$$I_{total} = \frac{V_{of}}{Z_0} - \underbrace{\frac{V_{or}}{Z_0} = I_{Load} = \frac{V_{Load}}{Z_{Load}}}_{\text{we can do this by KCL}}$$

→ This is our <u>2nd</u> equation

10

$$V_{of} + V_{or} = V_{Load}$$ 2 equations

2 unknowns

$$\frac{V_{of}}{Z_0} - \frac{V_{or}}{Z_0} = \frac{V_{Load}}{Z_{Load}}$$ → we know V_{of}

~~So~~ So, we have a revelation

After a bunch of Algebra,

$$V_{o\,reflected} = \Gamma\, V_{o\,forward}$$

↑

reflection coefficient

we may write

$$V_{of}\,(1+\Gamma) = V_{Load}$$

$$\boxed{V_{or} = \Gamma V_{of}}$$

$$V_{of}\,\frac{(1-\Gamma)}{Z_o} = \frac{V_{Load}}{Z_{Load}}$$

Solve for Γ

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$V_{total} = V_0\left[e^{-j\beta z} + \Gamma e^{+j\beta z}\right]\left\{e^{+j\omega t}\right\}$$

"implied"

$$I_{total} = \frac{V_0}{Z_0}\left[e^{-j\beta z} - \Gamma e^{+j\beta z}\right]$$

(12) So what if $\Gamma = 0$?

→ Nothing is reflected

we dont want waves reflected

Now we ask ourselves, is this ~~always~~ always possible?

Short Circuit

$Z_L = 0$

$$\Gamma_L = \frac{0 - Z_0}{0 + Z_0} = -1$$

What does this mean?

for $\Gamma = -1$

$$V_{or} = (-)V_{of} \longrightarrow |V_{or}| = |V_{of}|$$

Here's a consequence,

if $Z_L = 0$, power to the Load is entirely reflected. (Which is bad)

What's the voltage at the Load?

$$V_{total} = V_0\left[e^{-j\beta z} + \Gamma e^{+j\beta z}\right]\Big|_{z=0}$$

$$= V_0[1 + \Gamma]$$

$$= 0 \quad \text{when } \Gamma = -1$$

14

What's the current at the Load?

$$I_{total} = \frac{V_0}{Z_0}\left[e^{-j\beta z} - \Gamma e^{+j\beta z}\right]\Big|_{z=0}$$

$$= \frac{V_0}{Z_0}\left[1 - \Gamma\right]$$

for a short circuit ($Z_L = 0$)

$$\Gamma = -1 \qquad I_{total}(z=0) = \frac{V_0}{Z_0}(2)$$

⟹ The current waves are adding constructive interferance

But twice the current isn't useful without power

What about an open circuit?

$Z_L \to \infty$, $\Gamma = \dfrac{\infty - Z_0}{\infty + Z_0} \to 1$

So for $\Gamma = 1$

$V_{Load} = 2V_0$ (we are now using $V_{0f} = V_0$)

$I_{Load} = 0$

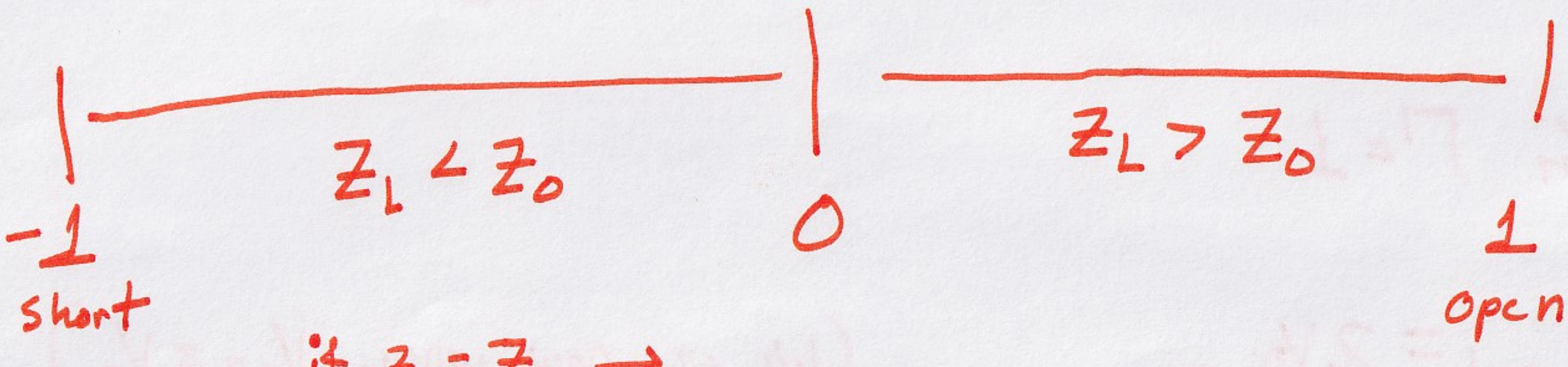
$\left.\begin{matrix} V_{Load} = 2V_0 \\ I_{Load} = 0 \end{matrix}\right\} \underline{P_{Load} = 0}$

And for short circuit

$\left.\begin{matrix} V_{Load} = 0 \\ I_{total} = \frac{V_0}{Z_0}(2) \end{matrix}\right\} P_{Load} = 0$

16

For resistive Loads $\rightarrow \Gamma$ real $\quad -1 < \Gamma < 1$



if $Z_L = Z_0 \rightarrow$ ____________

Note, Amplitude of reflected wave does not exceed the wave from the source

for $\quad -1 < \Gamma < 1$

Amplitude of reflected wave $\leq$ Amplitude of forward wave

Lets Look at Power

Let $Z_o = 100\ \Omega$

$Z_L = 50\ \Omega$

So $\Gamma = \dfrac{50-100}{50+100} = -\frac{1}{3}$ Γ is now negative

$$V_{total} = 10\left(e^{-j\beta z} - \tfrac{1}{3}e^{+j\beta z}\right)$$

remember, this is the phasor form, we don't see the $e^{j\omega t}$. And the measurable wave is a cosine

$$V_{Load}(z=0) = 10\left(1-\tfrac{1}{3}\right) = 6.66\ V$$

Voltage Amplitude

$$\Rightarrow V_{Load} = 6.66\cos(\omega t)$$

(18) Recall ohm's Law, calculate Power

$$P = \frac{V^2}{Z_L} = \frac{(6.66)^2}{50} \approx \frac{400}{450} [W]$$

(Lets call it rms to make us happy)

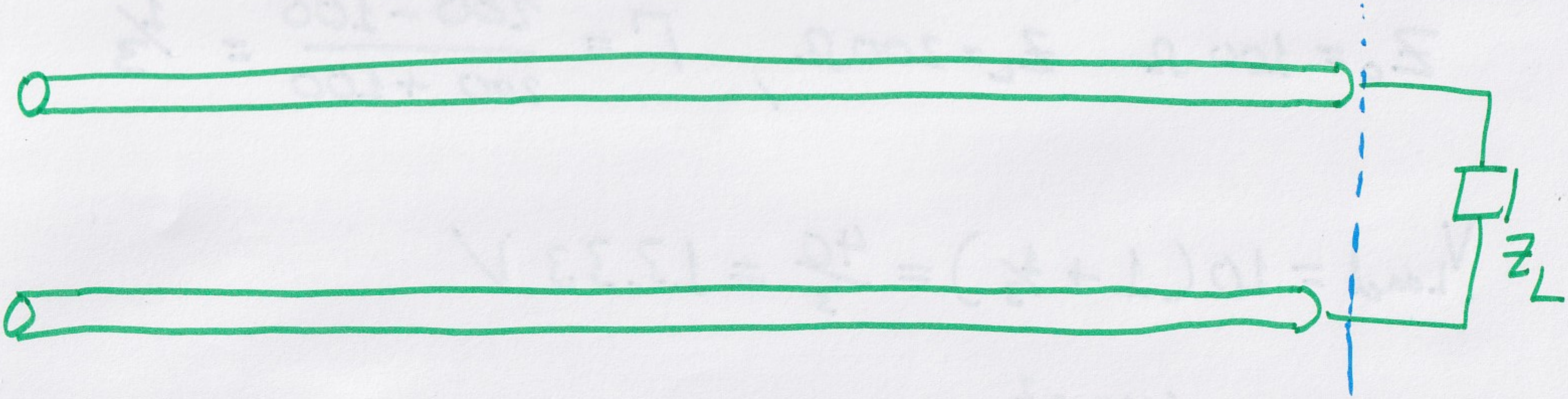
Another Example

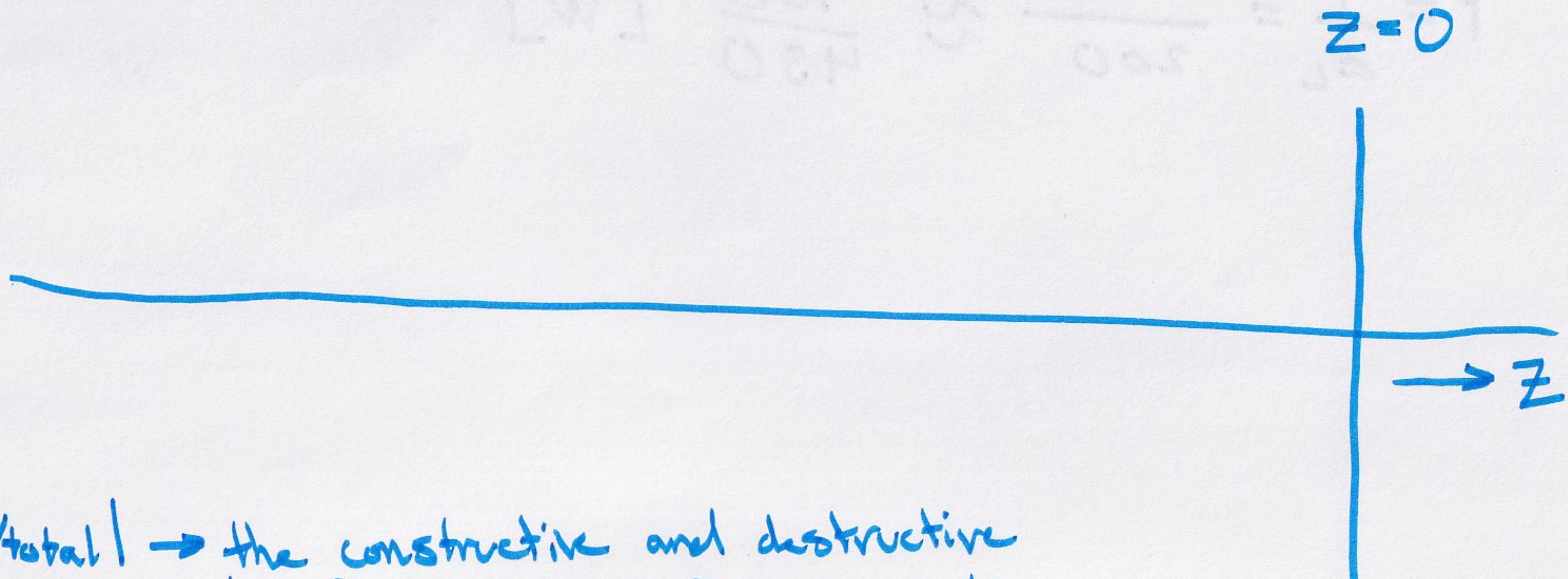
$Z_0 = 100\,\Omega \quad Z_L = 200\,\Omega\,, \quad \Gamma = \frac{200-100}{200+100} = 1/3$

$V_{Load} = 10\left(1 + 1/3\right) = \frac{40}{3} = 13.33\ V$

$P = \frac{V^2}{Z_L} = \frac{(40/3)^2}{200} \approx \frac{400}{450}\ [W]$

Look at the T-Line now





$|V_{total}|$ → the constructive and destructive interference as a function of time

So we can write

$$|V_{total}| = \left|V_0\left(e^{-j\beta z} + \Gamma e^{+j\beta z}\right)\right|$$

$$= |V_0| \left|\left(1 + \Gamma e^{+2j\beta z}\right)\right| \left|e^{-j\beta z}\right|$$

we know $|e^{-j\beta z}| = \underline{1}$

because, $|e^{j\phi}| = |\cos(\phi) + j\sin(\phi)|$

$$= \left(\cos^2(\phi) + \sin^2(\phi)\right)^{1/2} = \underline{1}$$

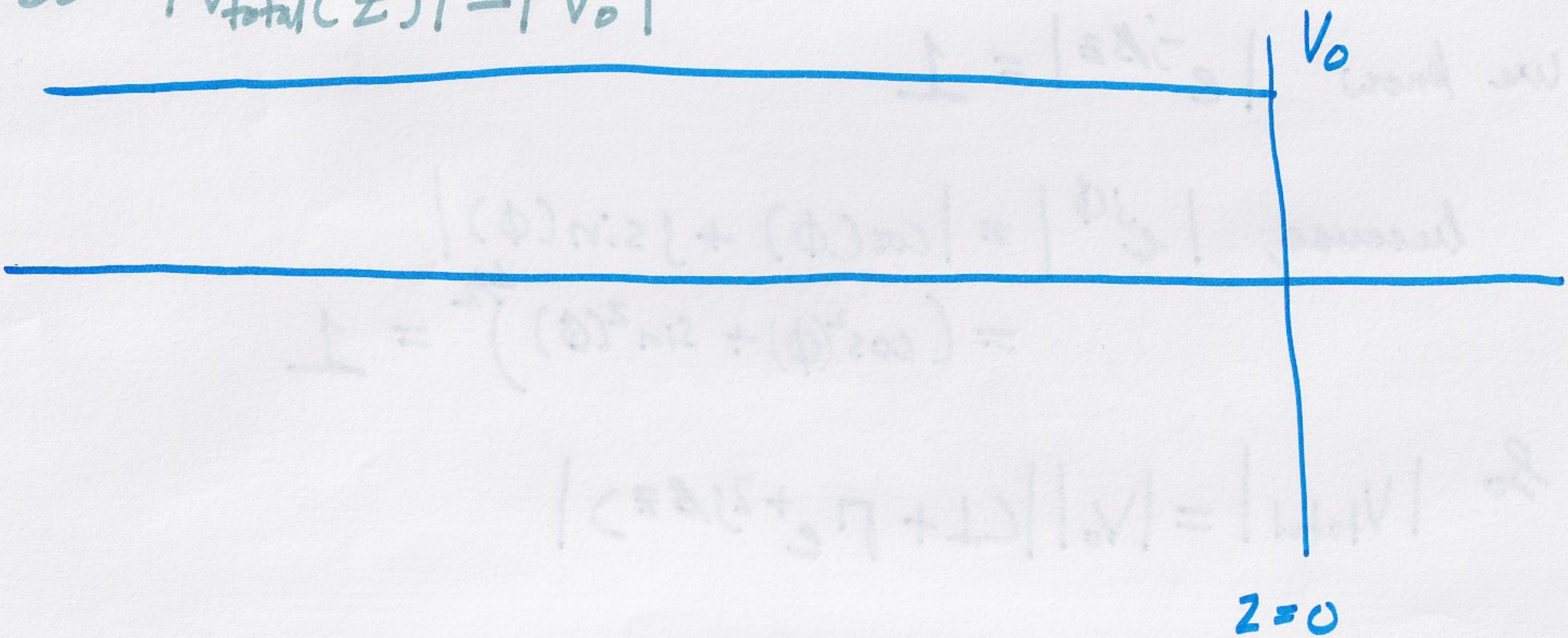
So $|V_{total}| = |V_0| \left|\left(1 + \Gamma e^{+2j\beta z}\right)\right|$

(22) Ex)

$$Z_L = 100\,\Omega \qquad Z_0 = 100\,\Omega$$

$$\Rightarrow \boxed{\Gamma = 0}$$

So $|V_{total}(z)| = |V_0|$



So, what does this mean?

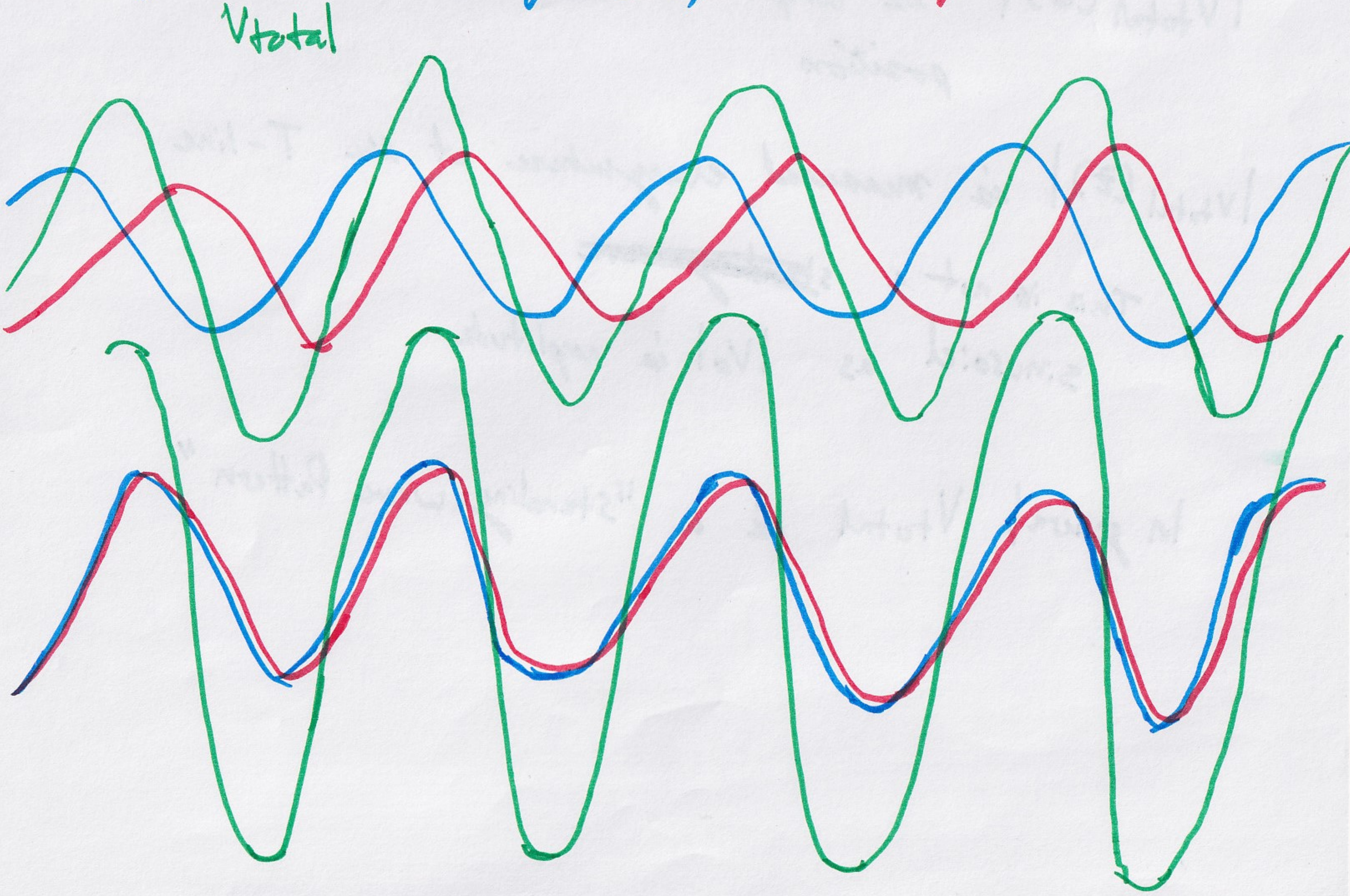
$|V_{total}(z)|$ is amplitude as a function of position

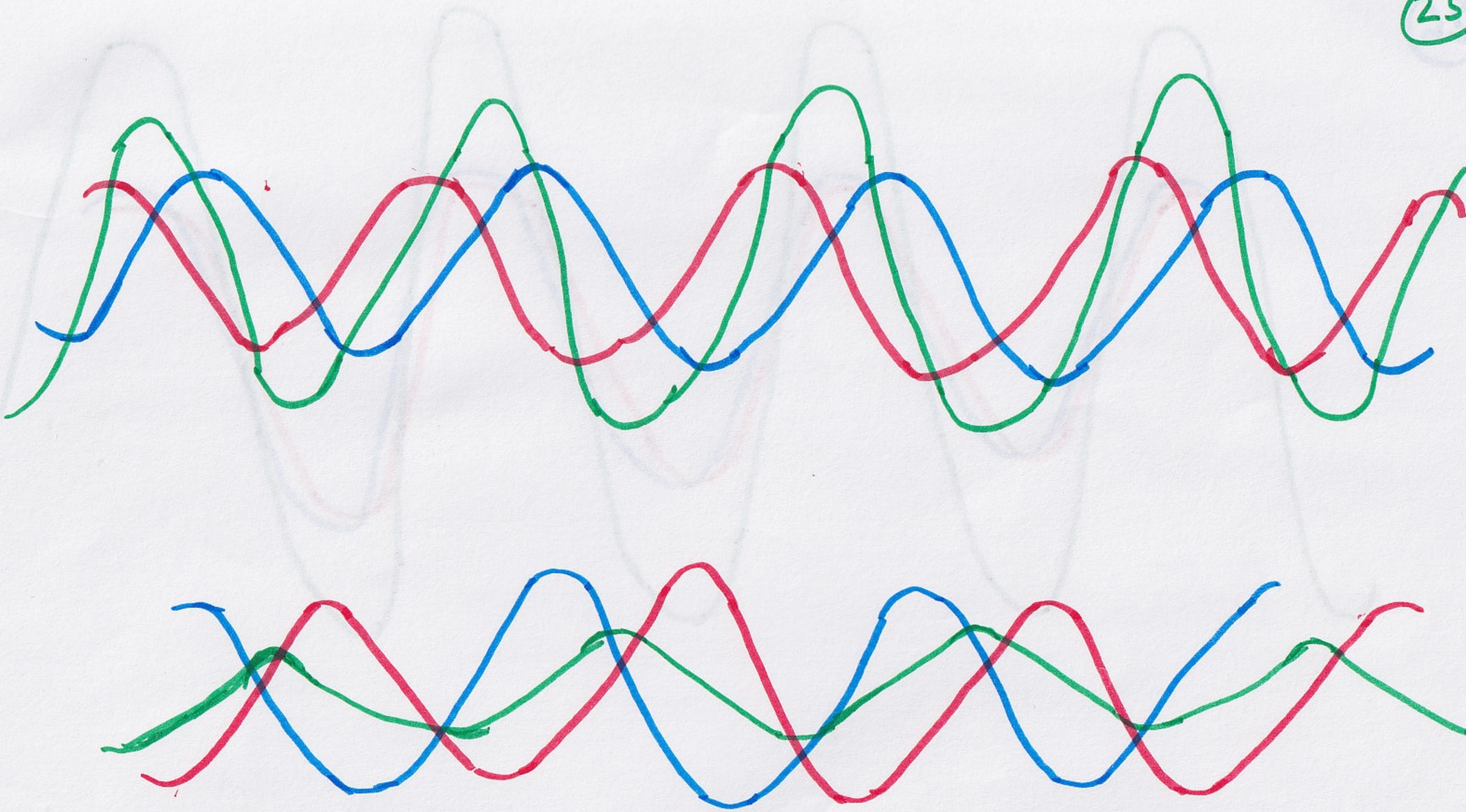
$|V_{total}(z)|$ is measured everywhere at the T-line

This is not a s~~tanding wave~~ sinusoid as $|V_0|$ is amplitude

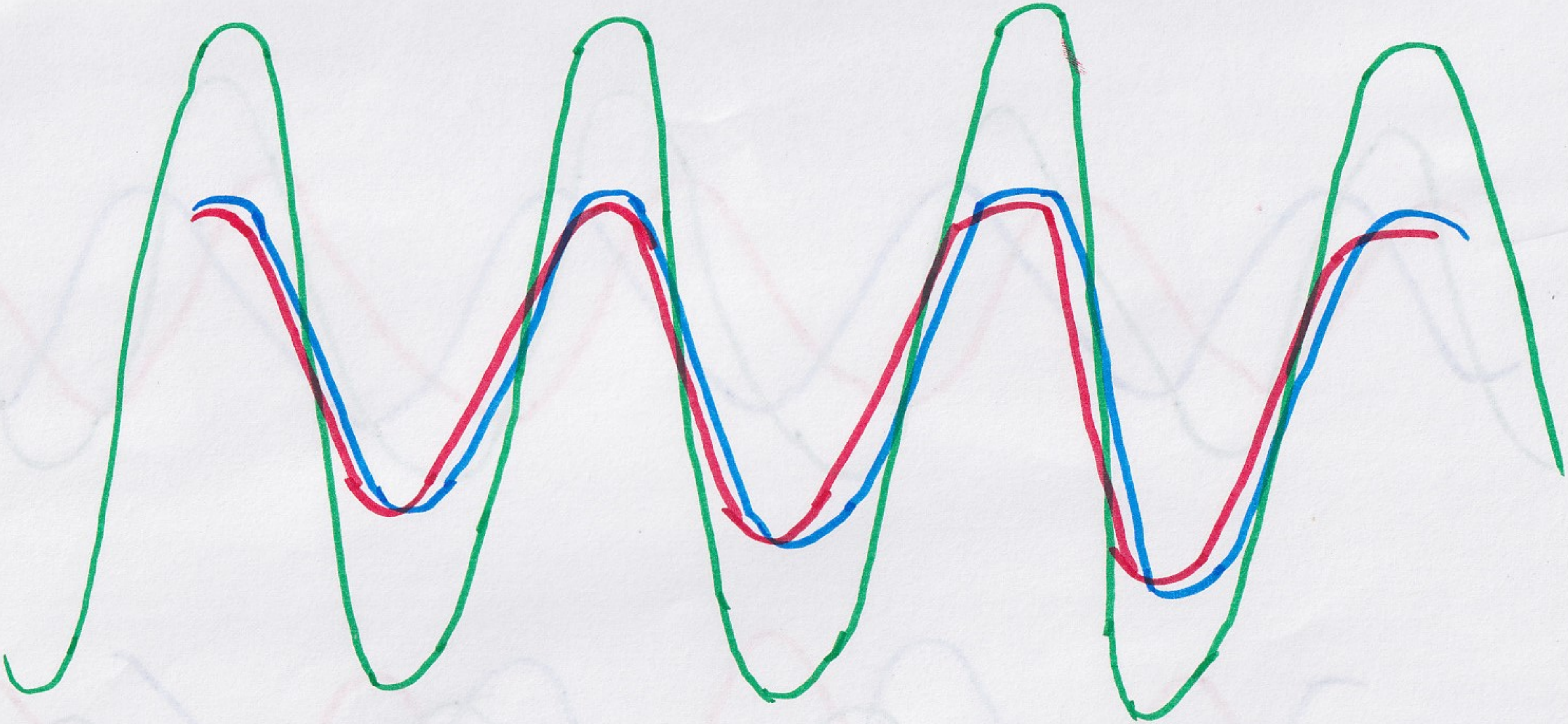
In general, V_{total} is a "standing wave Pattern"

(24) ~~forw~~ blue is forward, red is reflected





26



Ex)

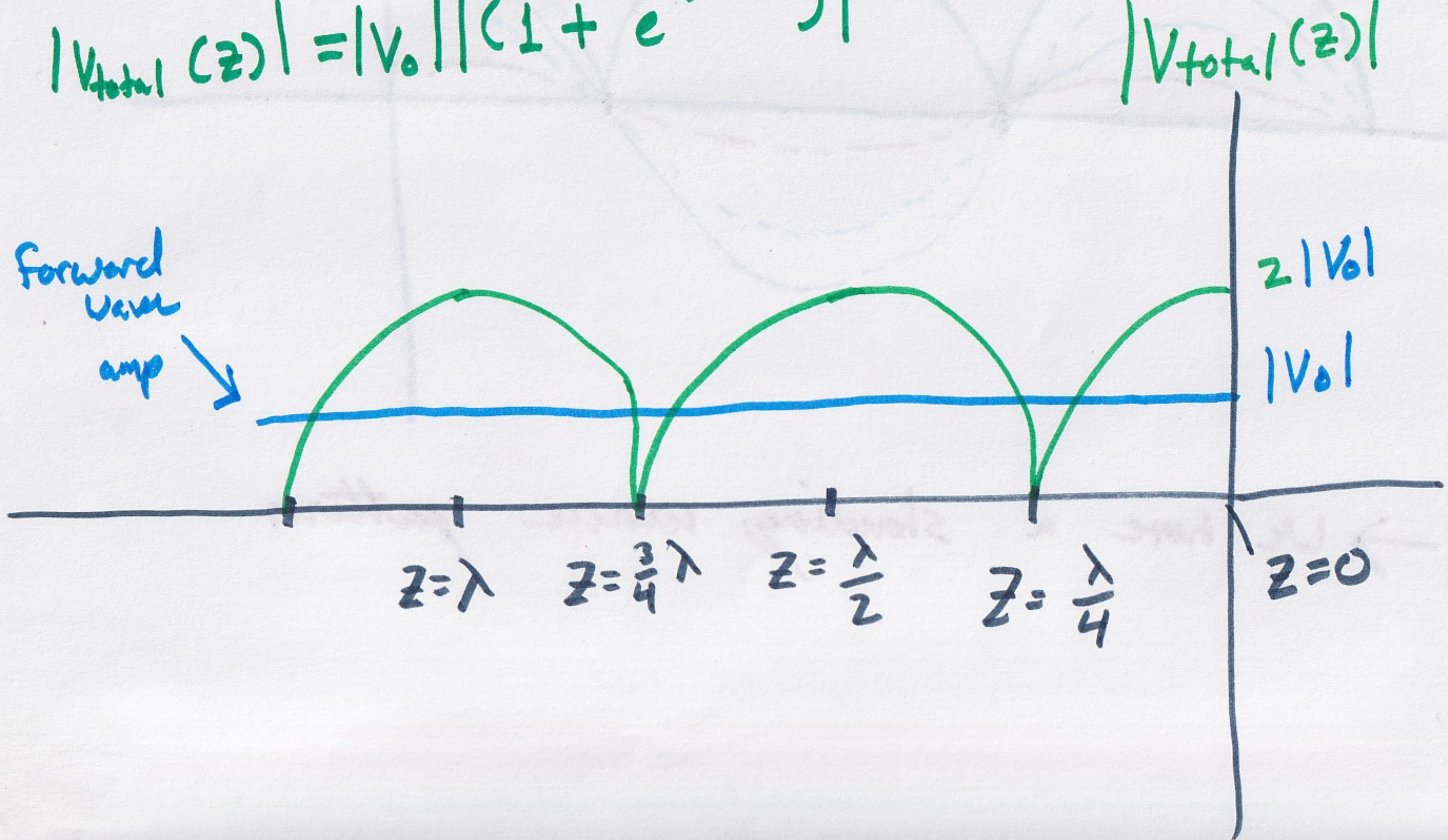
$Z_0 = 100\ \Omega$

$Z_L = \infty$ (open ckt)

recall $\beta = \frac{2\pi}{\lambda}$

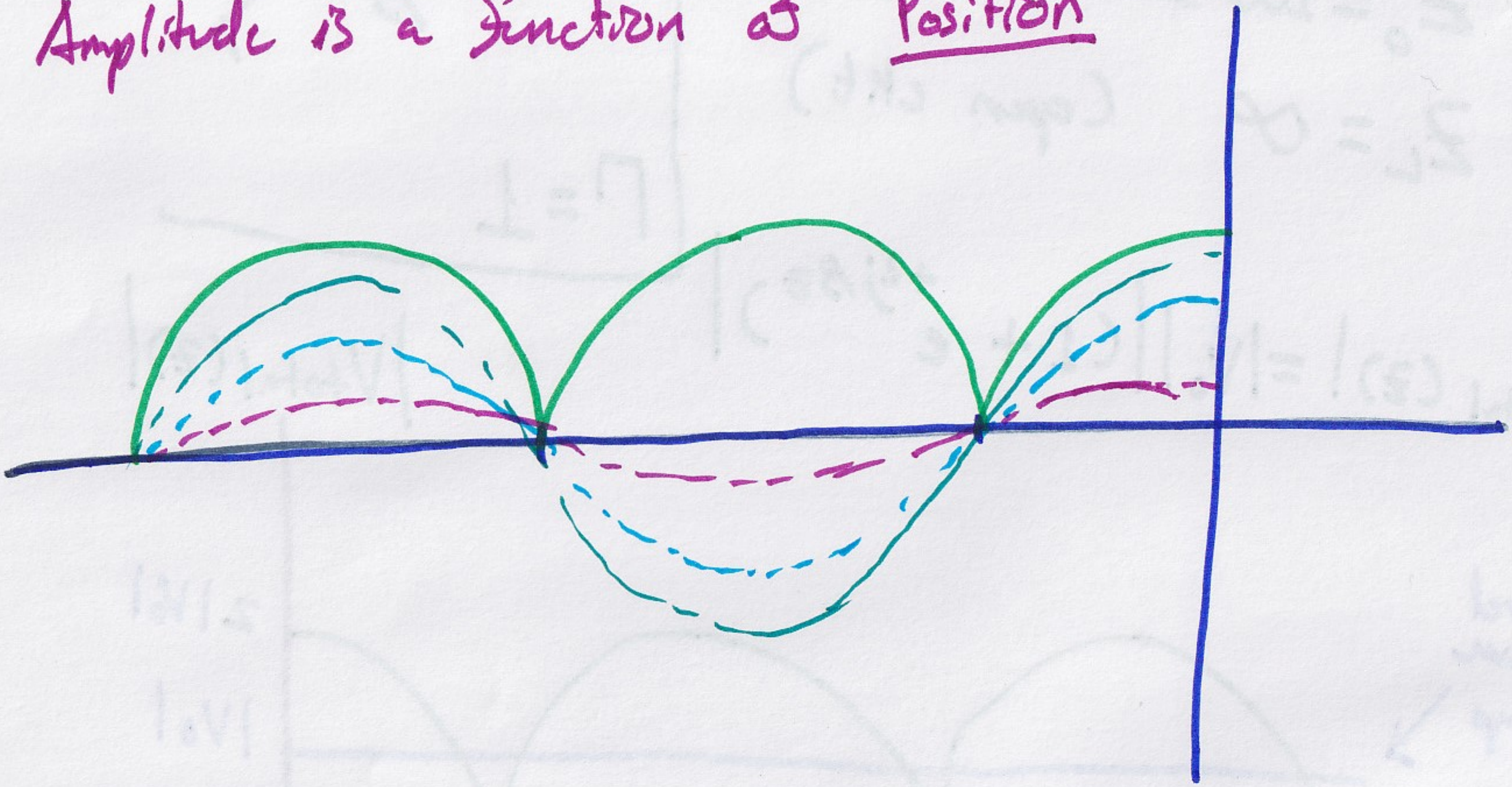
$\Gamma = 1$

$|V_{total}(z)| = |V_0| \, |(1 + e^{+2j\beta z})|$



(28) we put z in terms of λ, makes things easier

Amplitude is a function of Position



→ We have a standing wave pattern

Ex)

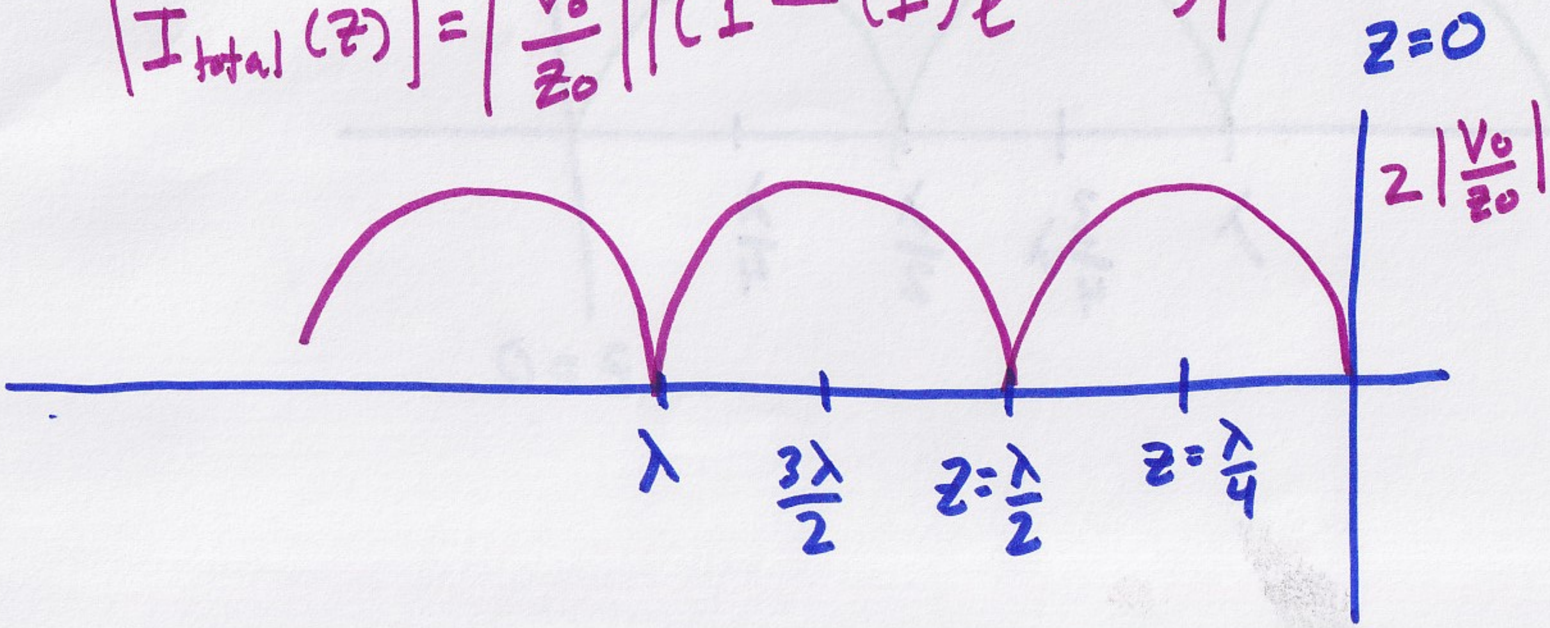
$$|I_{total}(z)| = \left|\frac{V_0}{Z_0}\right| \left|(1 - \Gamma e^{+2j\beta z})\right|$$

→ current standing wave pattern

$Z_0 = 100$ $\quad \Gamma = 1$

$Z_L = \infty$

$$|I_{total}(z)| = \left|\frac{V_0}{Z_0}\right| \left|(1 - (1) e^{-2j\beta z})\right|$$



30

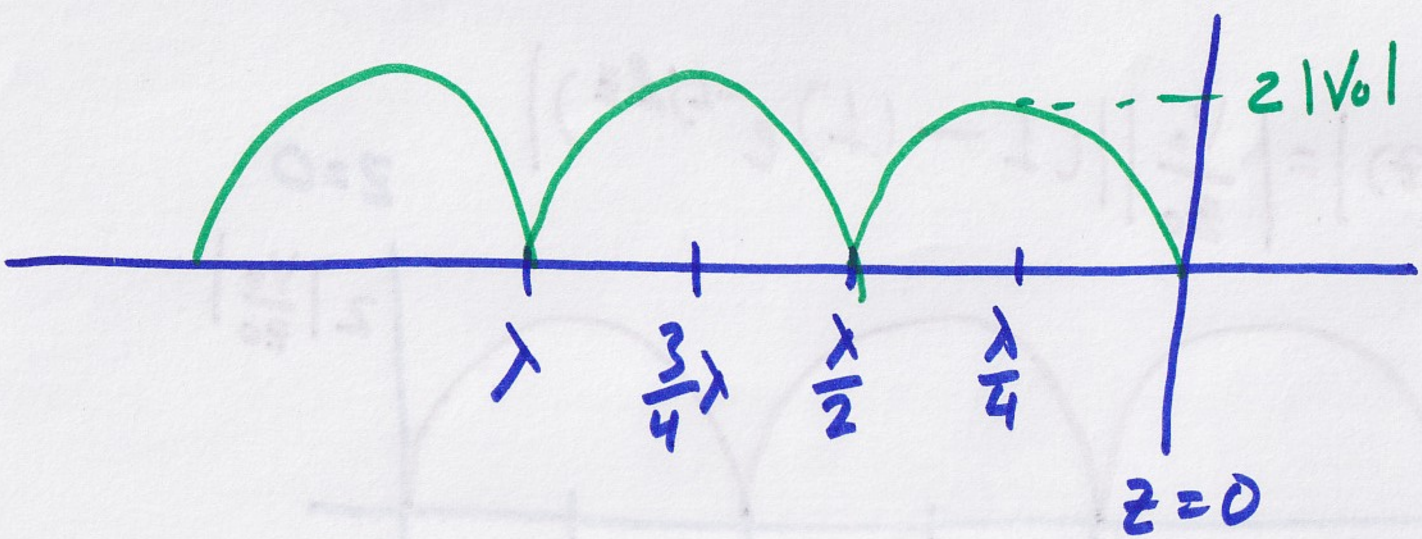
Look at short ckt

Ex)

$Z_0 = 100\,\Omega$

$Z_L = 0$ (short ckt) $\Gamma = -1$

$$|V_{total}(z)| = |V_0|\,|(1 - 1\,e^{+2j\beta z})|$$



Standing wave pattern is shifted by $\frac{\lambda}{2}$

for short ckt