Meeting #9 Notes

$\sqrt{2}$

#1) elements of a sequence

a) are very close to each other (eventually)

b) make sense

c) sequences are ∞ as $n \in \mathbb{N}$

$[1, 2]$

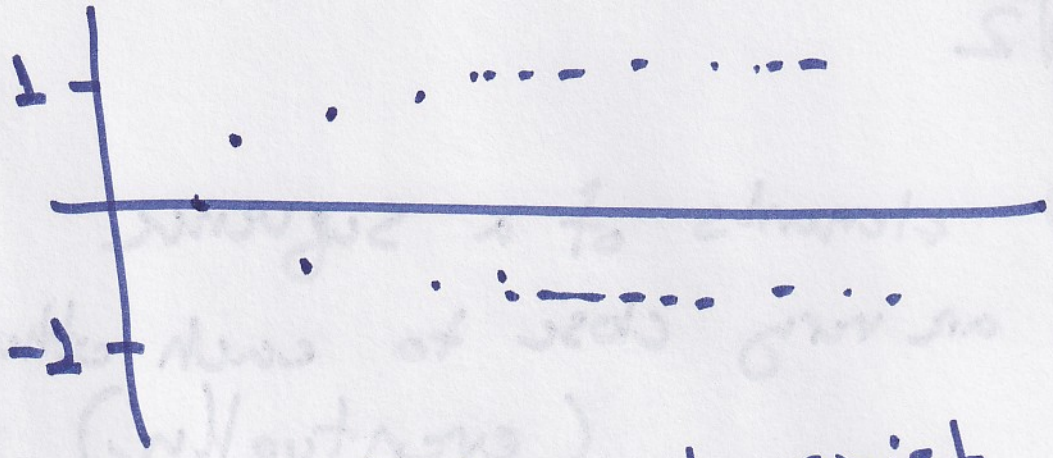
as long we have a sequence

a limit of a sequence is unique

~~$a_n = \frac{2n(-1)^n}{n}$~~ does not converge

2 Limit points 2, -2

② $a_n = (-1)^n + \frac{1}{n}$



Limit does not exist

2 points of accumulation

we can make a set out of the sequence

#2) $|x-4| < 1$

$-1 + 4 < x < 1 + 4$

$3 < x < 5$

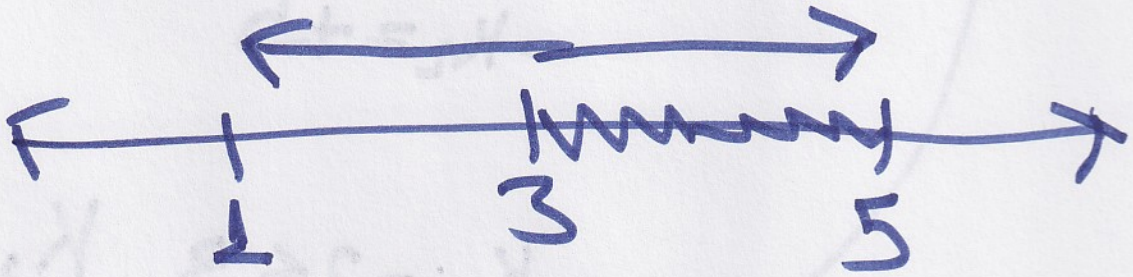
$x \in (3, 5)$

b) open because $x \in V_\varepsilon$ at either endpoint is an element of the set, but endpoints themselves

$x \in (3,5)$

c) $|x-3| < K_1$

$-K_1 + 3 < x < K_1 + 3$



for $K_1 = 2$

LHS $K_2 \geq 0$

RHS $K_1 \geq 2$

$\longrightarrow K_1 \geq 2$

what values of K_1, K_2 cover the interval in part (a)

(4)

$-K_2 - 2 < x < K_2 - 2$

$K_2 - 2 \geq 5$ — we want to surround

$K_2 = 7$?

$-K_2 - 2 \leq 3$ → we want to cover 3

$-K_2 = +5$

$-K_2 - 2 \leq 3, \quad K_2 \geq 5$

Choose $K_2 \geq 7$ (more restrictive inequality)

d) We want to cover

$|x^2 - 7x + 12| < V$

$|x-4| < 1$

$|x-3| < 2$ (from K_1)

$|x-4||x-3| < 2$

if $x = 3$ → 0

if $x = 5$ → 2

$\min(V) = 5$

(3,5) is correct

We wont get more than 2

c) $|x^3 - 5x^2 - 2x + 24| < V$

$(x^2 - 7x + 12)(x + 2)$

$x^3 - 7x^2 + 12x + 2x^2 - 14x + 24)$

$(x^3 - 5x^2 - 2x + 24)$

$|x-4| < 1$

$|x-3| < 2$

$|x+2| < 7$

$\min(14)$

$|x-4||x-3||x+2| < 14$

$(1 \cdot 2 \cdot 7)$

(6)

#3

a) $a_n = 5 + \frac{1}{n}$

$b_n = 5 - \frac{1}{n}$

$c_n = 5 + \frac{3}{n}$

b) $(6, 5.5, 5+\frac{1}{3}, \cdots$

(

They all converge to the same

#

Notation a_n, $f(a_n)$

$\forall \varepsilon > 0 \ \exists n > N$ $a_n, 2a_n$

$a_n \to 5$, $f(a_n) = 2a_n$

$\longrightarrow f(a_n) \to 2(5) = 10$

$|a_n - 5| < \delta$

$|f(a_n) - 10| < \varepsilon$

↘ This decreases by a different amount, so we can't use the same ε

$|x - 5| < \delta$

$|f(x) - 10| < \varepsilon$

↓

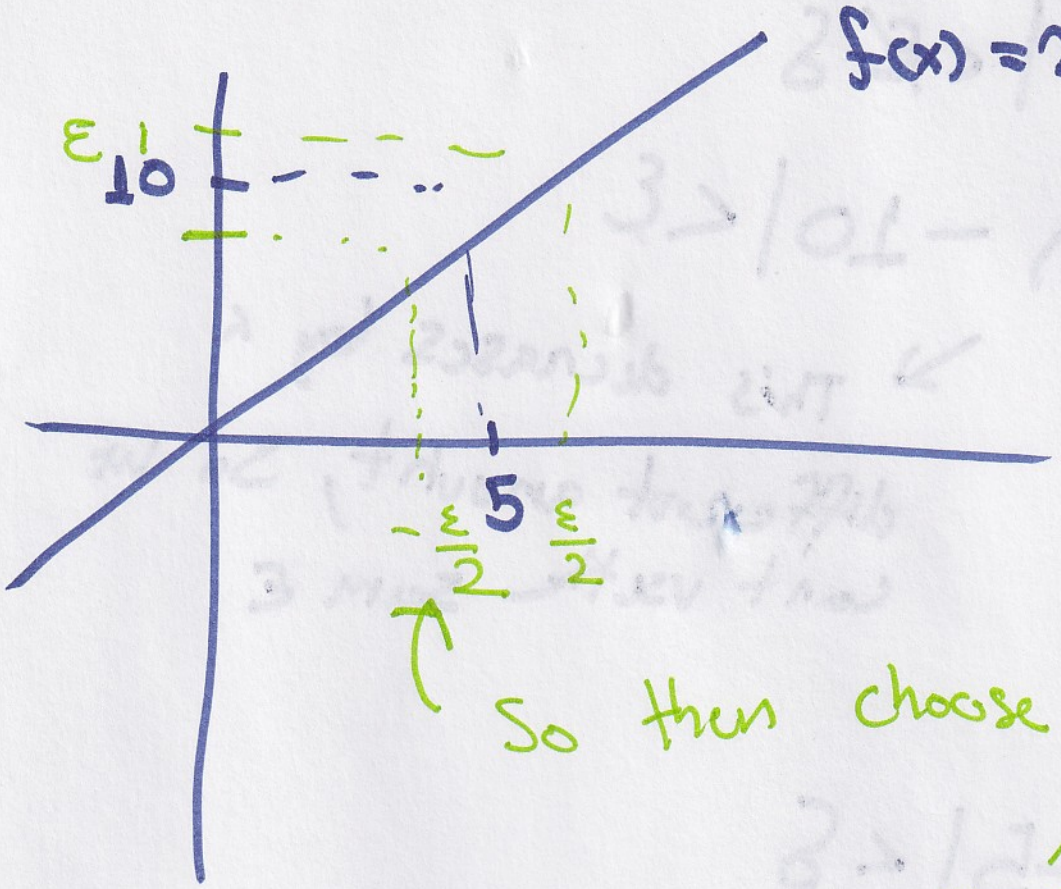
$|2x - 10| < \varepsilon$

How to connect these statements

does $\varepsilon = 2\delta$ →

write $\varepsilon = 2\delta$ → $\delta = \frac{\varepsilon}{2}$ we want this

(8)



So then choose $\delta = \frac{\varepsilon}{2}$

We write it like THIS

(v) $\varepsilon = 2\delta$ doesn't make sense with the picture in mind

Prove $\lim_{x \to 5} 2x = 10$

Given $\varepsilon > 0$, choose $\delta = \frac{\varepsilon}{2}$ then

$$|x - 5| < \delta$$

$$\rightarrow |x - 5| < \frac{\varepsilon}{2}$$

$$\rightarrow 2|x - 5| < \varepsilon$$

$$\rightarrow |2x - 10| < \varepsilon$$

5, 6, Look at #2
you will need it