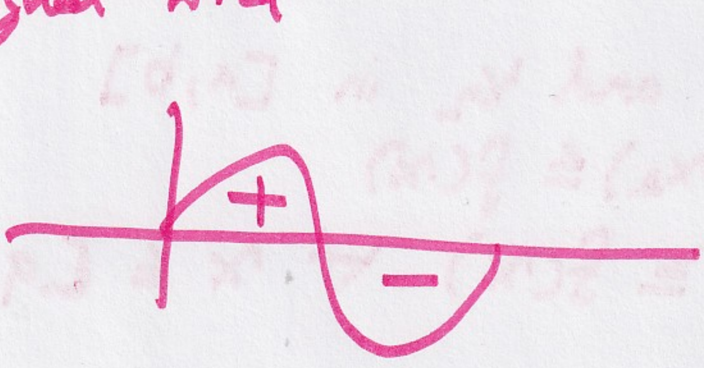
Meeting 17

Σ Area Problems

Partition: $x_0 = \alpha$, $x_n = \beta$

Signed Area



#1)

a) BWT → A bounded sequence contains a convergent subsequence

b) IVT → f cont on $[a,b]$
~~and~~ If $f(x_1) > Y$ and $f(x_2) < Y$ for $x_1, x_2 \in [a,b]$ then $\exists$ $c \in [a,b]$ satisfying $f(c) = Y$

$f(x_2) < f(c) < f(x_1)$

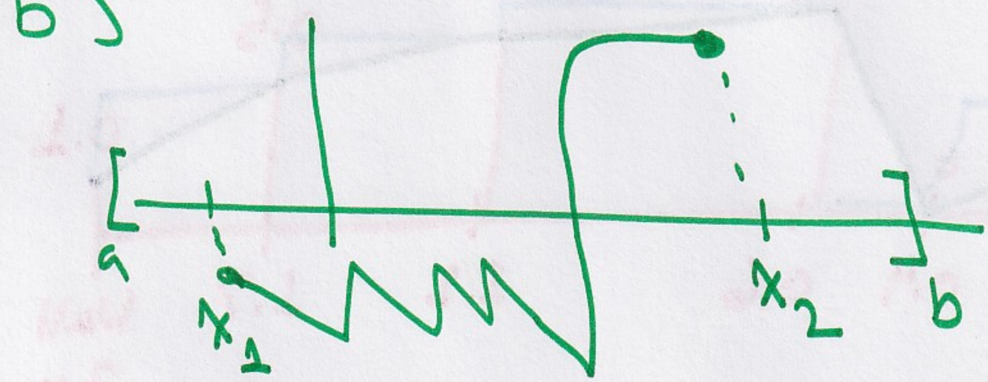
c) EVT → f cont on $[a,b]$
~~Then $\exists$ x_1 and x_2 in $[a,b]$ then~~
~~Y_{min} and Y_{max} s.t.~~

Then $\exists$ x_1 and x_2 in $[a,b]$
s.t. $f(x_1) \leq f(x)$
$f(x_2) \geq f(x)$ $\forall$ $x \in [a,b]$

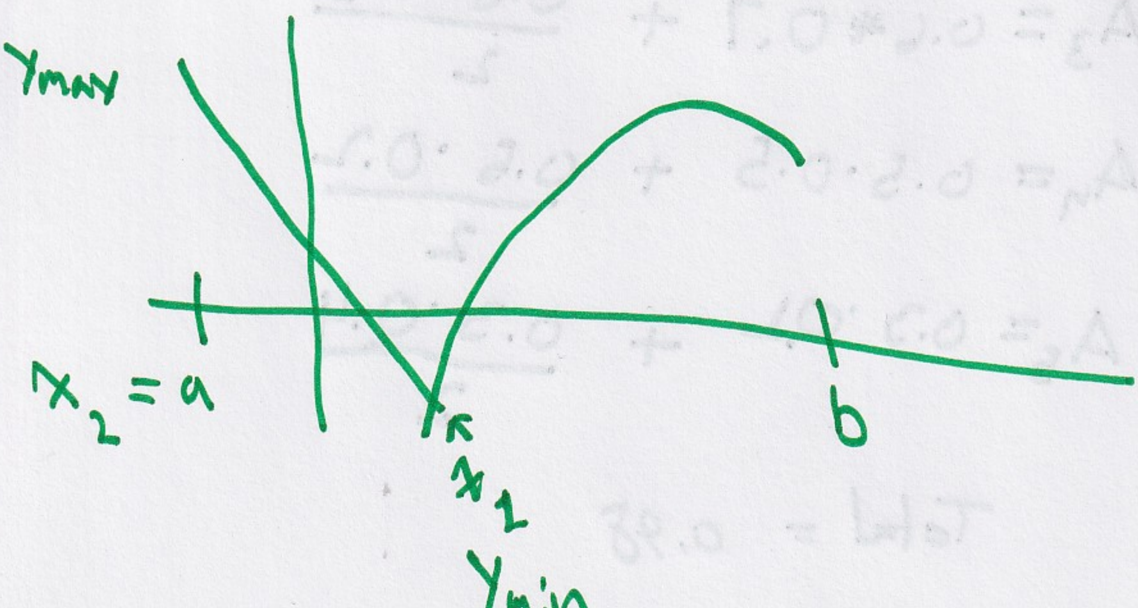
d) MVT → f cont on $[a,b]$, differentiable on (a,b) then $\exists\, c \in (a,b)$ s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

b)

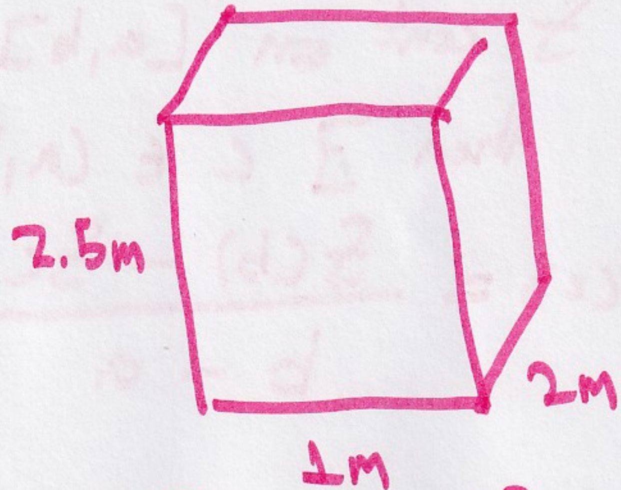


c)

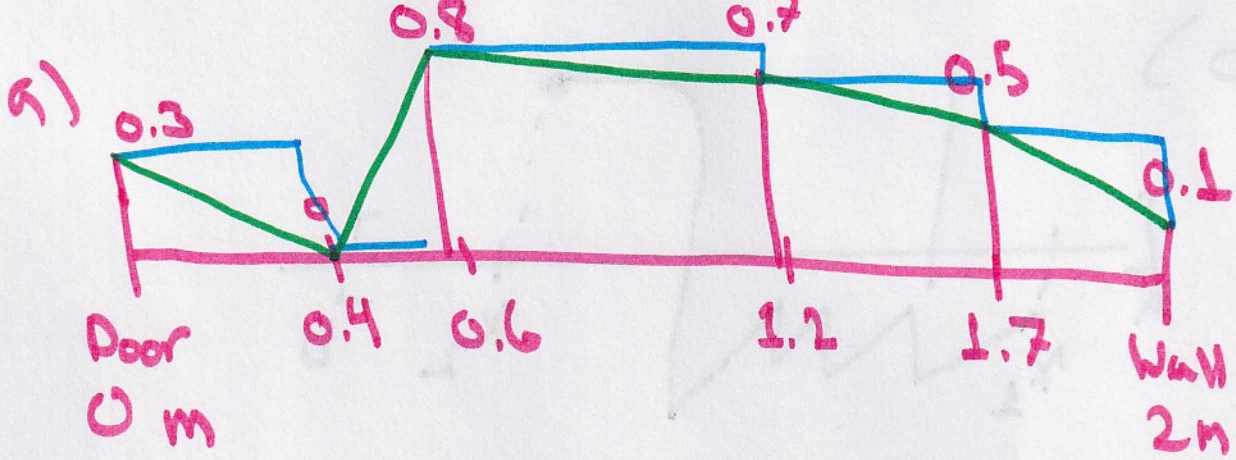


4

#2



a)



$$A_1 = \frac{0.3(0.4}{2}$$

$$A_2 = \frac{0.2 \cdot 0.8}{2}$$

$$A_3 = 0.6 * 0.7 + \frac{0.6 \cdot 0.1}{2}$$

$$A_4 = 0.5 \cdot 0.5 + \frac{0.5 \cdot 0.2}{2}$$

$$A_5 = 0.3 \cdot 0.1 + \frac{0.3 \cdot 0.4}{2}$$

Total = 0.98

Rectangles

$0.4 \times 0.3 + 0 + 0.8 \cdot 0.6 + 0.7 \cdot 0.5 + 0.5 \cdot 0.3$

$\approx \frac{11}{10}$ or $1.1\ m^3$

highest hieght 2m deep

b) $\sup = 0.8 \cdot 2 = 1.6 m^3$

upper bound of what could be there, a crude estimate

c) $\inf = 0 \times 2 = 0 m^3$

#3 (1) $2 \times 4 + \frac{8 \cdot 4}{2} = 24$

(2) $6 \times 4 + \frac{4 \times 4}{2} = 32$

(3) $\frac{2 \cdot 6}{2} - \frac{3 \cdot 1}{2} = 4.5$

(4) $3 \times 4 + \frac{\pi (2)^2}{2} = 12 - 2\pi$

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#4 We partitioned each piece of the curve as if it were piecewise ~~continuous~~

-1 < 3 < 7 < 10 < 14

#5

R = 0 4 5.5 4.5 3 1 0 30.5
W = 1 2 1 2 2 2 1

G = 5 7 7 7.5 6.5 4.5 2
W = 1 2 1 2 2 2 1 65

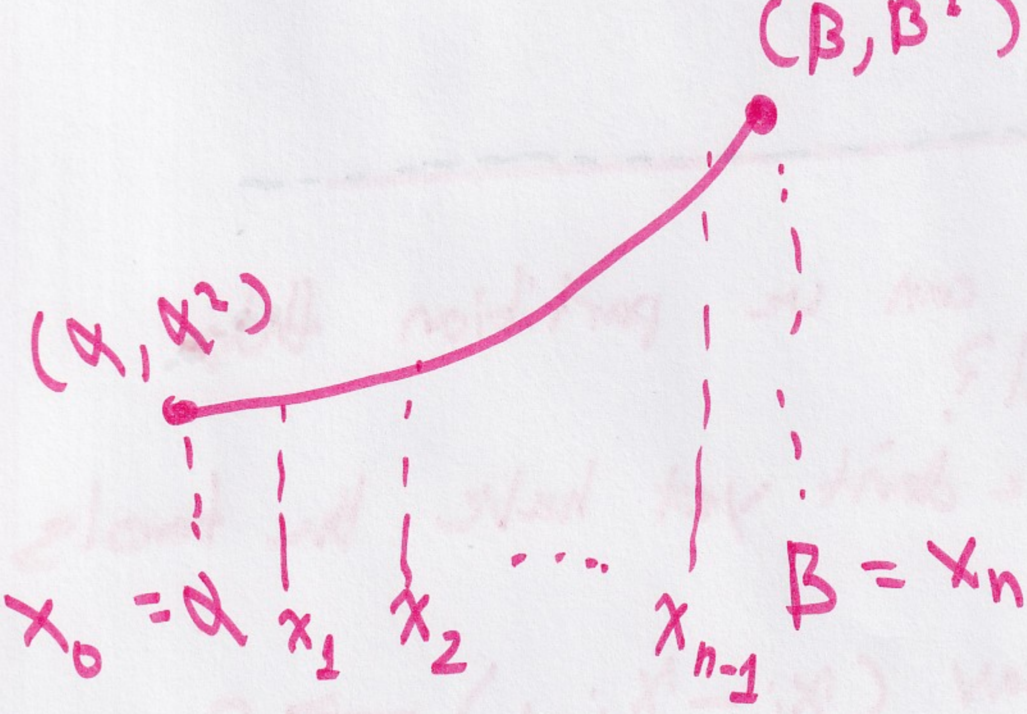
$\boxed{50.42}$ Actual

#6.)

$0 < 1 < 3 < 4 < 6 < 8 < 10 < 11$

we did it for computational ease

#7.) $f(x) = x^2$ $\quad I = [\alpha, \beta]$



Let ~~[illegible]~~

$x_0 = \alpha, x_1, \dots, x_n = \beta$

be a partition of $[\alpha, \beta]$

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$$\text{Area} \approx \sum_{i=1}^{n} (x_i - x_{i-1}) f(x_i)$$

$$f(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{Q}^c \end{cases}$$

How can we partition this interval?

We don't yet have the tools

$$\lim \max (x_i - x_{i-1}) \to 0$$

if $\Delta x_i = x_i - x_{i-1}$

we want $\displaystyle\lim_{\|\Delta x_i\| \to} \sum f(x_i^*) \Delta x_i$