Meeting #7 (1)

1.) a) "if bounded, we have a sup and infimum"

$$|A+B| \le |A| + |B|$$

b) An open interval has no max or min but has an inf or sup

Is this correct? Do we need boundedness?

No but the right is a property

~~a closed interval if it has a max or min will necessarily contain the sup or inf~~ ~~will contain it limit points (inf and sup)~~ ~~will contain both~~ ~~its~~ inf ~~and sup~~

an open interval will not contain its inf or sup → don't contain end points (a,b)

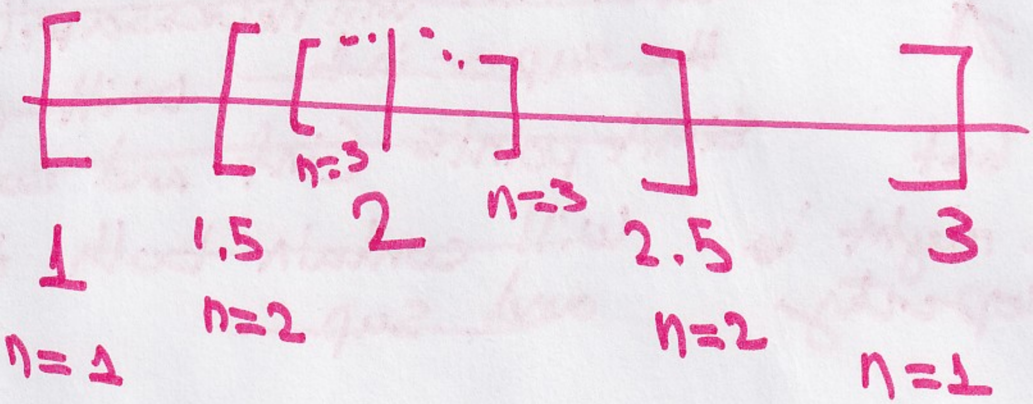
a closed interval will contain both its inf and sup → contain endpoints [a,b]

②

A compact interval $[3,4]$
inf sup

Intervals that are not compact, $(3,4)$ $(3,\infty)$, $[3,4)$

2.)



Sequence of intervals

$$\underbrace{\bigcap_{n=1}^{\infty}\left[2-\tfrac{1}{n}, 2+\tfrac{1}{n}\right]}_{} = [2-1, 2+1] \cap \left[2-\tfrac{1}{2}, 2+\tfrac{1}{2}\right.$$

An infinite intersection of closed intervals

$2-\frac{1}{n} \rightarrow$ a sequence of lower endpoints

$2+\frac{1}{n} \rightarrow$ a sequence of upper endpoints

#2

$H_n = \left(\frac{n-1}{n}\right)$ converges to 1 (never reaching it)

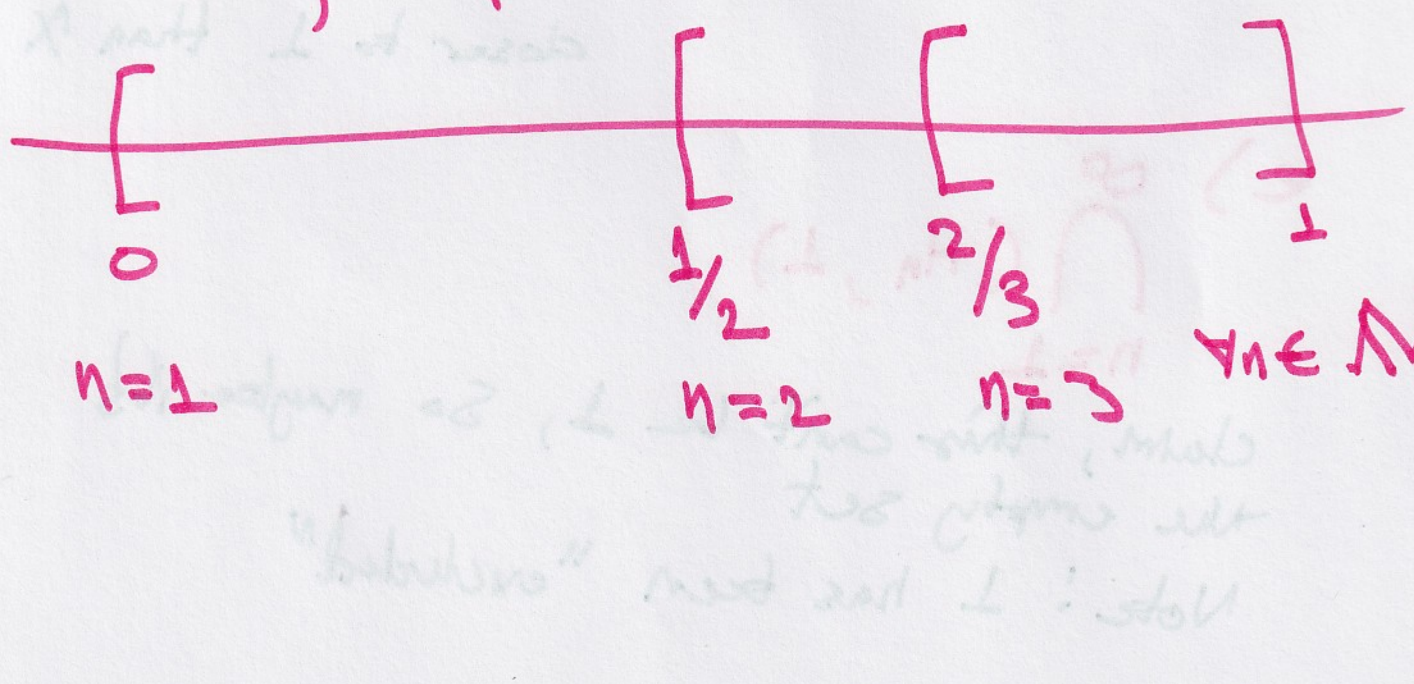
$(H_n) = \left(0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\right)$

$a_n \geq a_{n-1}$, so increasing and monotone

d) $\bigcap_{n=1}^{\infty} [H_n, 1]$ $\quad H_n = \frac{n-1}{n}$

what is the intersection?. Why?

Note, $\sup H_n = 1$



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claim: $\bigcap_{n=1}^{\infty} [H_n, 1] = \{1\}$

Can we convince ourselves that 1 is in every $[H_n, 1]$ and nothing else?

Yes, the Left side can be made <u>arbitrarily close</u> to 1

$[1-\epsilon, 1] \quad \forall \epsilon > 0$

for any x

[we can find this, 1]

and its closer to 1 than x

e) $\bigcap_{n=1}^{\infty} (H_n, 1)$

claim, this can't be 1, So maybe it's the empty set

Note: 1 has been "excluded"

so $\bigcap_{n=1}^{\infty} (H_n, 1) = \emptyset$

#3)

a)

$W_n \to 1$ monotone decreasing

$K_n \to \frac{1}{2}$

$\sigma_n \to$ No

eventually $\frac{(-1)^n}{n} \to 0$

alternates $-2, 2$ No max or min

inf sup

No convergence

b) Do subsequences converge?

W_n, K_n subsequences converge

σ_n, choose a subsequence that converges

could converge to $(+2, -2)$

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c) Infinite portion

where things tend to go after Large n

for σ_n, we see that 2 subsequences have a "infinite portion"

σ_n ~~has~~ '2' Infinite portions but not for the original σ_n

d) $1 < w_n \leq 1 + \frac{1}{2}$

$-\frac{1}{2} \leq K_n \leq 1$

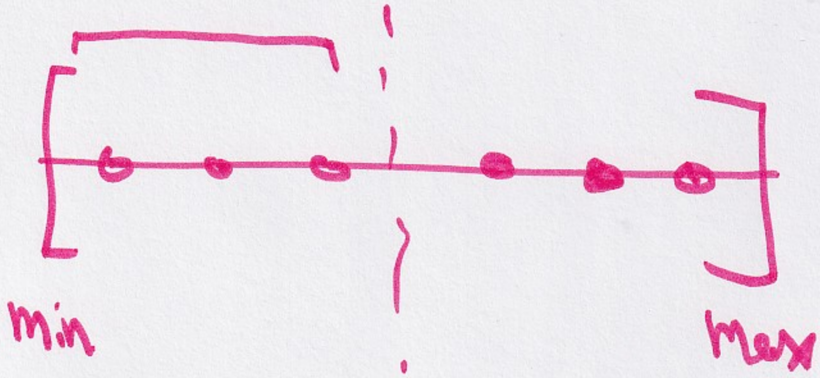
$-2 < \sigma_n < 2$

* all are bounded

* all have subsequences that converge

#4)

claim: All bounded sequences have a convergent subsequence



If we can make a monotone sequence bounded, we can show it converges