Meeting 10 Notes

Continuous Improvement

As $n \to 5$,

$|a_n - \cancel{L_a}^{\,5}| < \varepsilon_1$ $\quad$ $|b_n - \cancel{L_B}^{\,10}| < \varepsilon_2$

The ε should be different

$|a_n - 5| < \varepsilon_1$ $\quad$ $f(a_n) = 2a_n$

$|f(a_n) - 10| < \varepsilon_2$

$|2a_n - 10| < \varepsilon_2$

$|2(a_n - 5)| < \varepsilon_2$

$|2||a_n - 5| < \varepsilon_2$

$2|a_n - 5| < \varepsilon_2$

$|a_n - 5| < \frac{\varepsilon_2}{2}$

Danny's

Logic thinking

1 $f: R \to R,\ f(x) = x^2$

a) $\mathbb{R}$

b) $\mathbb{R}$

c) $R^+ \cup \{0\}$

d) $[0, 9]$

codomain is the canvas we want to paint, throw a bunch of paint from a can, then the range is what we actually get.

e) $+\sqrt{2}, -\sqrt{2}$

2 $\varepsilon > 0$ $\quad \delta = \frac{\varepsilon}{6}$

$|6x - 5 - 13| < \varepsilon$

$|6||x - 3| < \varepsilon$

$\varepsilon > 0$ choose $\delta = \frac{\varepsilon}{6}$

$|x-3| < \delta$

$|x-3| < \frac{\varepsilon}{6}$

$|6x-5-13| < \varepsilon$

Jeffry method (above) (its a joke)

Hazel method

we know δ first (full proof)

#3) ~~Proof~~ Prove

$f(x) = 6x - 5$ cont. $\forall x \in \mathbb{R}$

given $c \in \mathbb{R}$, $\lim x \to c = f(c) = 6c - 5$

Take $\delta = \varepsilon$

Jeff ~~$\exists$~~ $\varepsilon > 0$

$|(6x-5)-(6c-5)| < \varepsilon$

$|6x-6c| < \varepsilon$

$|6(x-c)| \rightarrow |6||x-c| < \varepsilon$

$|x-c| < \frac{\varepsilon}{6}$

(4)

$$\delta = \frac{\varepsilon}{6}$$

$$|x - c| < \delta$$

$$|x - c| < \frac{\varepsilon}{6}$$

$$6|x - c| < \varepsilon$$

$$|6x - 6c| < \varepsilon$$

$$|6x - 5 - 6c + 5| < \varepsilon$$

$$|(6x - 5) - (6c - 5)| < \varepsilon$$

$$|f(x) - L| < \varepsilon$$

$x = c$ ← General

$(6x - 5)$

$|f(x) - L|$ ← what we think the Limit will be

we know it was 13 before

Limit and Y value are the same? (Limit point)?

Limit and y are the same if the function is continuous

#4) $M(x) = \dfrac{x^2 - 6x}{x^2 - 8x + 12} = \dfrac{x(x-6)}{(x-6)(x-2)}$

$36 - 48 + 12$

$\dfrac{2x-6}{2x-8} = \dfrac{6}{4}$

This is what the Limit is

The Limit Value exists)

c) The Limit exist at 6

but the function cannot be evaluated at 6

so its not continuous in $\mathbb{R}$

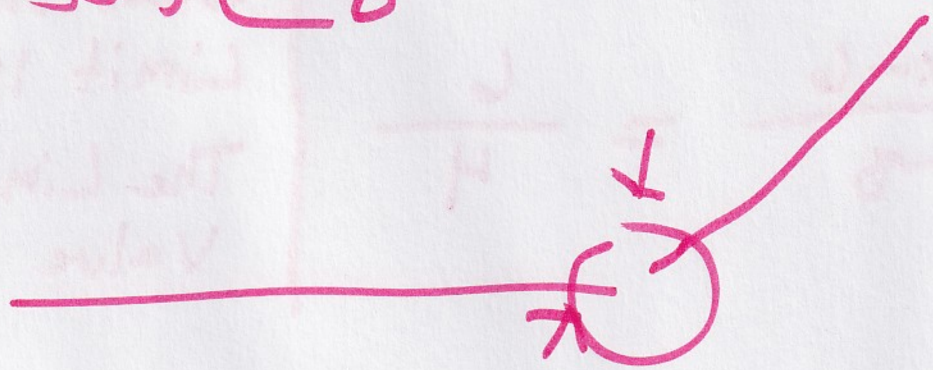
(because it its not continuous at 6)

→ also 2

(6) Clearly we are distinguishing between The Limit of a function and evaluating the function

$f(x) = x \{$

$$f(x) \begin{cases} x & x > 1 \\ 0 & x \leq 1 \end{cases}$$



We have 2 Limits and $f(x)$ evaluates to 1 value

Not continuous in $\mathbb{R}$

cont. in $\mathbb{R}^+$?, $\mathbb{R}^-$?

#5

a) Yes! at those examples

b) Depends on continous over stated domain vs continuous everywhere.

It's continuous for it's stated domain

Yes it's continuous!

(Not continuous everywhere)

#6

$$f(x) = \begin{cases} 2 & x < 3 \\ \pi & x = 3 \\ 1/2 & x > 3 \end{cases}$$

a)

$|x - K| < \delta$

$-\delta < x - K < \delta$

for $\varepsilon > 0, \delta > 0$

$\forall x$ s.t. if $-\delta < x - K < 0$, then

$|f(x) - W| < \varepsilon \iff \lim_{x \to K^-} f(x) = W$

This is 0 because we are approaching from the negative side

8 " if $|x-k|<\delta$,

we may indicate we don't know what we are doing "

b) $\lim_{x \to 3^-} f(x) = 2$ (So the Limit is 2)

c) would showing that the "Left piece" is continuous prove that the Left side Limit = 2 ?

~~Yes but...~~

~~we need~~

$|f(x) - W| < \varepsilon$ $\quad -\delta < x - c < 0$

$|2 - 2| < \varepsilon$ $\quad -\delta < x - 3 < 0$

choose $\delta > 0$ Any δ

even $\delta = \varepsilon$

~~and~~ we want $\varepsilon > 0$, it works

because we don't care about δ if $\delta = \varepsilon$

Since δ doesn't remain in our result, we conclude that we may choose Any δ and we can be sure that the Limit inequality will be satisfied.

for $\varepsilon > 0$, $\delta > 0$ Choose any δ

$\forall x$ s.t. $-\delta < x - K < 0$, then $|f(x) - W| < \varepsilon$

$$\Longleftrightarrow \lim_{x \to K^-} f(x) = W$$

$$|f(x) - W| < \varepsilon$$

$$|2 - 2| < \varepsilon$$

Or

for $\varepsilon > 0$ $\delta > 0$

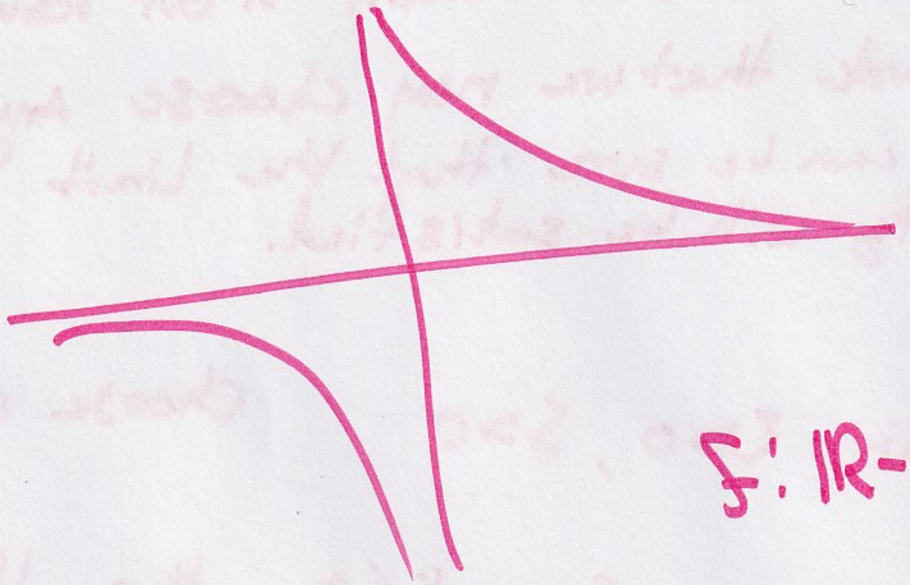
$$|f(x) - W| < \varepsilon$$

$$|2 - 2| < \varepsilon$$

if this inequality is true, the Limit exist

It's true instantly cuz δ can be anything

(10)



$f: \mathbb{R} - \{0\} \to \mathbb{R}$

$f(x) = \frac{1}{x}$

this is continuous !