Meeting 12 @MRcmark

#1) $f : [0, 2\pi] \longrightarrow \mathbb{R}$

a) $f(x) = \sin(x) + \cos(x)$

$\frac{3}{4}\pi$ and $\frac{7}{4}\pi$

b) if $f(x) = x^2 + 1$, no place to find zero

Restrictions (these need to be satisfied to find zeroes)

~~f is~~ f is continuous on a domain

$f(x) < 0$ for some $x \in D$

$f(x) > 0$ for some $x \in D$

$\frac{1}{x}$ has a disconnected domain

cosecant is continuous on its domain, but not over all intervals

(2)

Restriction: f is continuous on interval $[a,b]$

$f(x) < 0$ for some $x \in I$

$f(x) >$ for some $x \in I$

In the literature, $f : D \to \mathbb{R}$

This D is implied to be the domain of f.

So $D \neq I$ (interval)

if ~~x~~ f is continuous on I

then I is connected? → Yes! because that's what an interval is (interval has no holes)

Is this an Interval? $[1,3) \cup (4,5]$

$a < x < b$

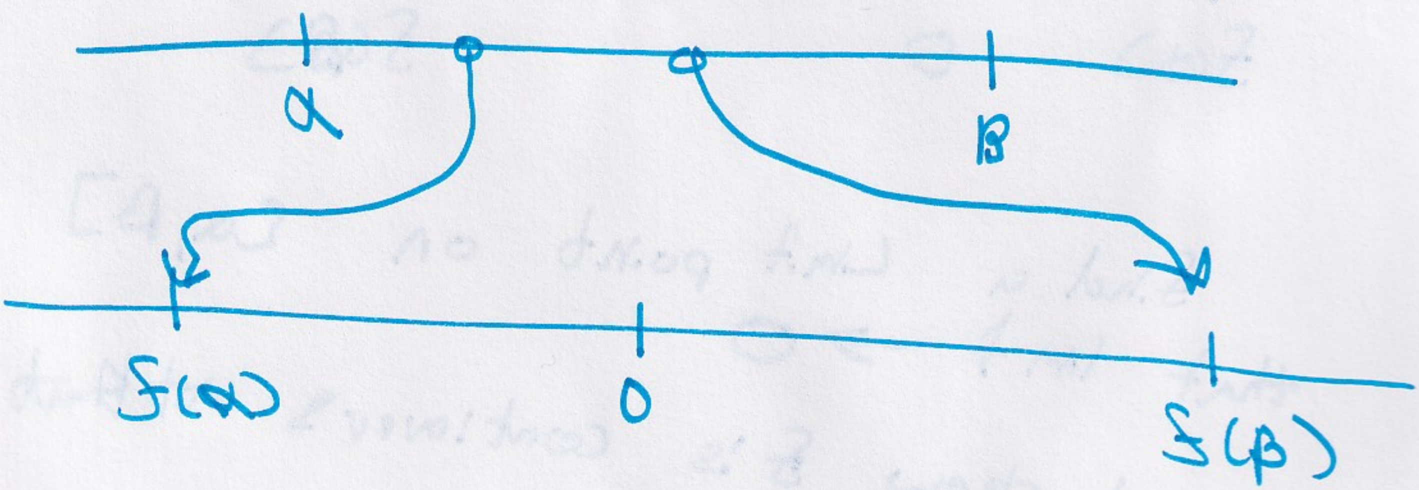
This is how we want to write an Interval

→ A circle Isn't an Interval

(a set in $\mathbb{R}^2$)

#2) f $f(\alpha) < 0 < f(\beta)$

$I = [\alpha, \beta]$

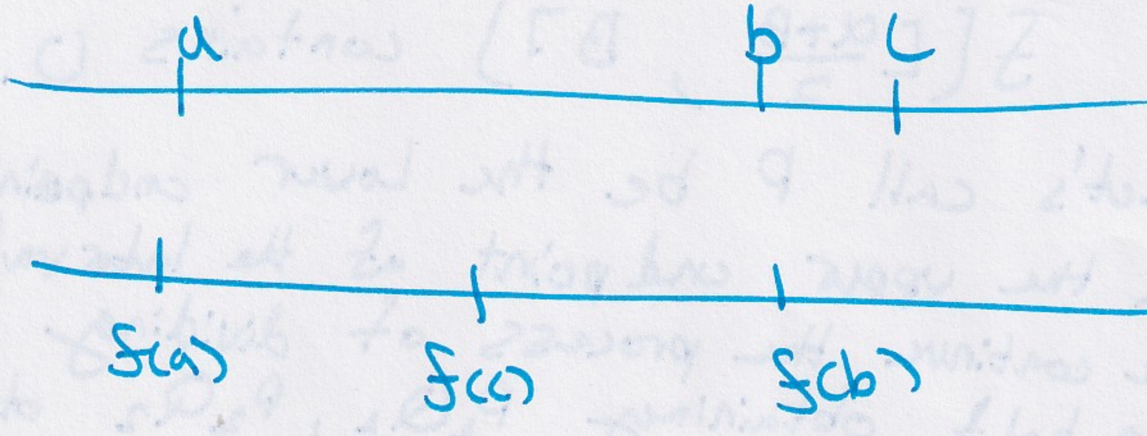


$f([\alpha, \beta]) = [f(\alpha), f(\beta)]$

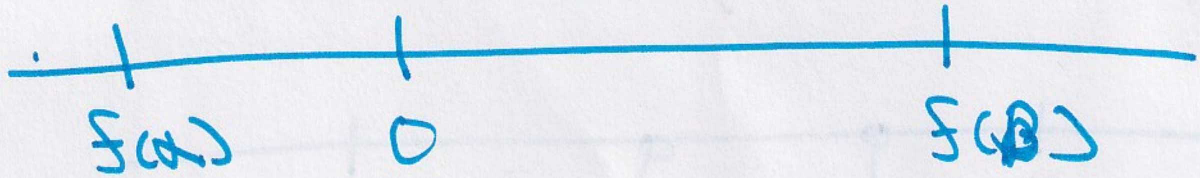
for $x = a, b$

$f(a) < f(c) < f(b)$

Assume there's no c s.t. the above inequality is true?



④



find a Limit point on $[\alpha, \beta]$ that $\lim f \to 0$

and show f is continuous at that Limit point

Proof:

WLOG we can assume

$f([\alpha, \frac{\alpha+\beta}{2}])$ or

$f([\frac{\alpha+\beta}{2}, \beta])$ contains 0.

Let's call P be the Lower endpoint and Q the upper endpoint of the Interval. We continue the process of dividing PQ in half, obtaining P_1Q_1, P_2Q_2 etc. Both sides of our interval tend to a Limit point over time

So $P_n Q_n \to \{x\}$ satisfying

$f(x)=0$

Note: each $f(P_i)<0$ and $f(Q_i)>0$

#3

P = Lower endpoint

Q = upper endpoint

$\gamma - P < 0$

$Q - \gamma > 0$

$\gamma - P < 0 < Q - \gamma$

$h(\alpha) < \gamma < h(\beta)$

$\underbrace{h(\alpha)-\gamma}_{f(\alpha)} < 0 < \underbrace{h(\beta)-\gamma}_{f(\beta)}$

$f(\alpha) < 0 < f(\beta)$

If we can find $f(x)=0$

then $f(x)=h(x)-\gamma=0$

$h(x)=\gamma$

Zach says this ok to say what we did before is good enough don't need to rewrite it all

6

we could call this a

$\boxed{\text{Corollary}}$ (common are corollarys in the Literature)

#4

Continuous:

$\lim_{x \to c} f(x) = f(c) \ \forall c \in D$

$\lim_{n \to ?} a_n$

$\forall \varepsilon > 0 \ \exists \delta > 0$ s.t.

$|x - c| < \delta \Rightarrow |f(x) - f(c)| < \varepsilon$

~~think of $a_n : \mathbb{N} \to \mathbb{R}$ $n \in \mathbb{N}$~~

~~$(a_n) = (K)$~~

~~Let $\varepsilon > 0$ choose N~~

think of $f: N \to R$ s.t.

$f \to L$ as $n \to \infty$

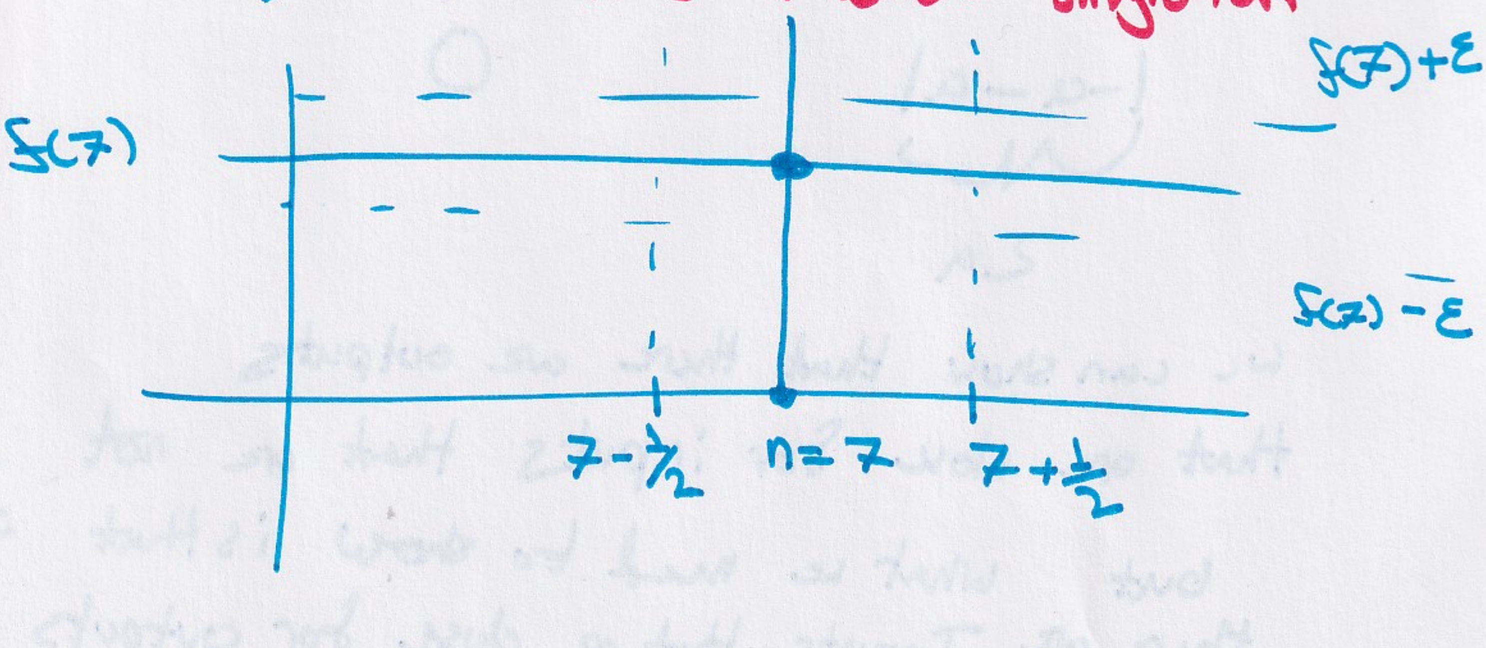
Let $\varepsilon > 0$ choose $\delta = ?$

$$|n - c| < \delta \overset{?}{\Longrightarrow} |f(n) - f(c)| < \varepsilon$$

we could just write 0.5 instead of δ

a sequence is a function whose domain is $\mathbb{N}$ (natural numbers) That maps to $\mathbb{R}$

The Limit exists at each singleton



8

think of $f: \mathbb{N} \to \mathbb{R}$ st

$f(n) \to f(c)$ as $n \to \infty$

Let $\varepsilon > 0$, choose $\delta = 1/2$

Then $|n - c| < \delta \Rightarrow |f(n) - f(c)| < \varepsilon$

---

5 $f: I \to R$

a) $\delta > 0$ s.t. $|x-y| < \delta \Rightarrow$

$I = [-2, 2]$

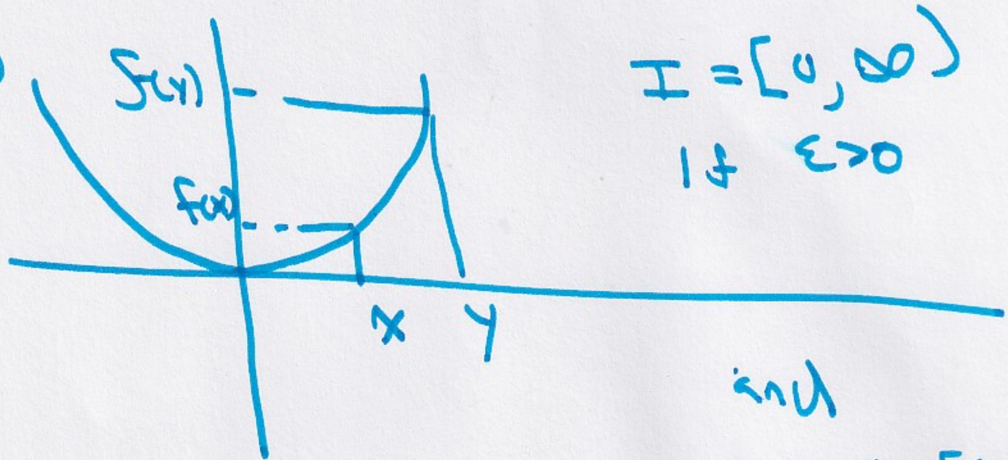
$\varepsilon =$ —

$\underbrace{|f(-a) - f(a)|}_{0}$

$\underbrace{|-a-a|}_{2a}$

we can show that there are outputs that are close for inputs that are not

but what we need to show is that there are Inputs that are close for outputs that are not a)



$I = [0, \infty)$

If $\varepsilon > 0$

and

$x, y \in [0, \infty)$

Then for some fixed $\delta > 0$

$|x-y| < \delta$ doesn't imply $|f(x) - f(y)| < \varepsilon$

Ex)

$$\left.\begin{matrix} x = 1 \\ y = 2 \end{matrix}\right\} \rightarrow \left.\begin{matrix} f(1) = 1 \\ f(2) = 4 \end{matrix}\right\} 3$$

$\delta = 1$ $\qquad$ $\varepsilon = 3$

$$\left.\begin{matrix} x = 4 \\ y = 5 \end{matrix}\right\} 1 \longrightarrow \left.\begin{matrix} f(4) = 16 \\ f(5) = 25 \end{matrix}\right\} 9$$

$\delta = 1$ $\qquad$ $\varepsilon = 9$

x^2 ~~that~~ is not uniformly continuous on D

← D is the domain of any f