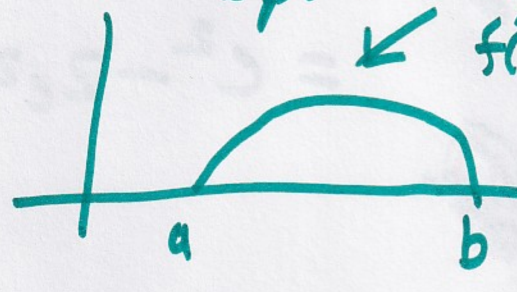
Meeting 16 Notes

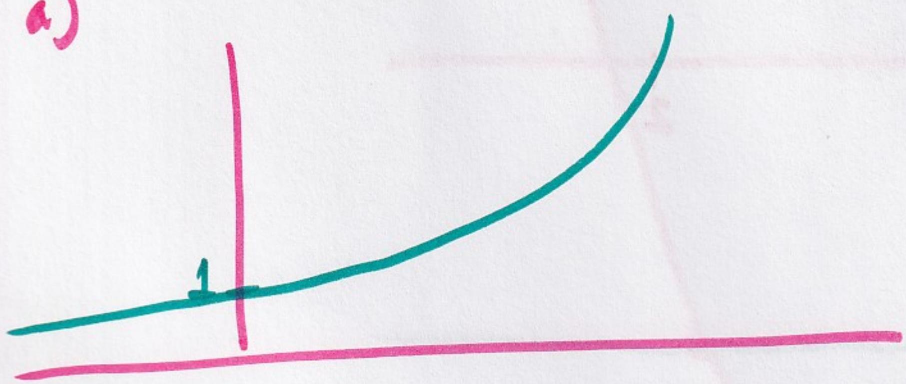
Averages, Secants, and Tangents,

Part II



#1) $f(x) = e^x$

a)



b) $\frac{e^2 - e^1}{1} \rightarrow$ slope $\quad m = \frac{f(2) - f(1)}{2-1}$

c) $g(x) = f(x) - (e^2 - e^1)x$, how was $g(x)$ made? (what does $g(x)$ do?)

We are subtracting the slope of the secant

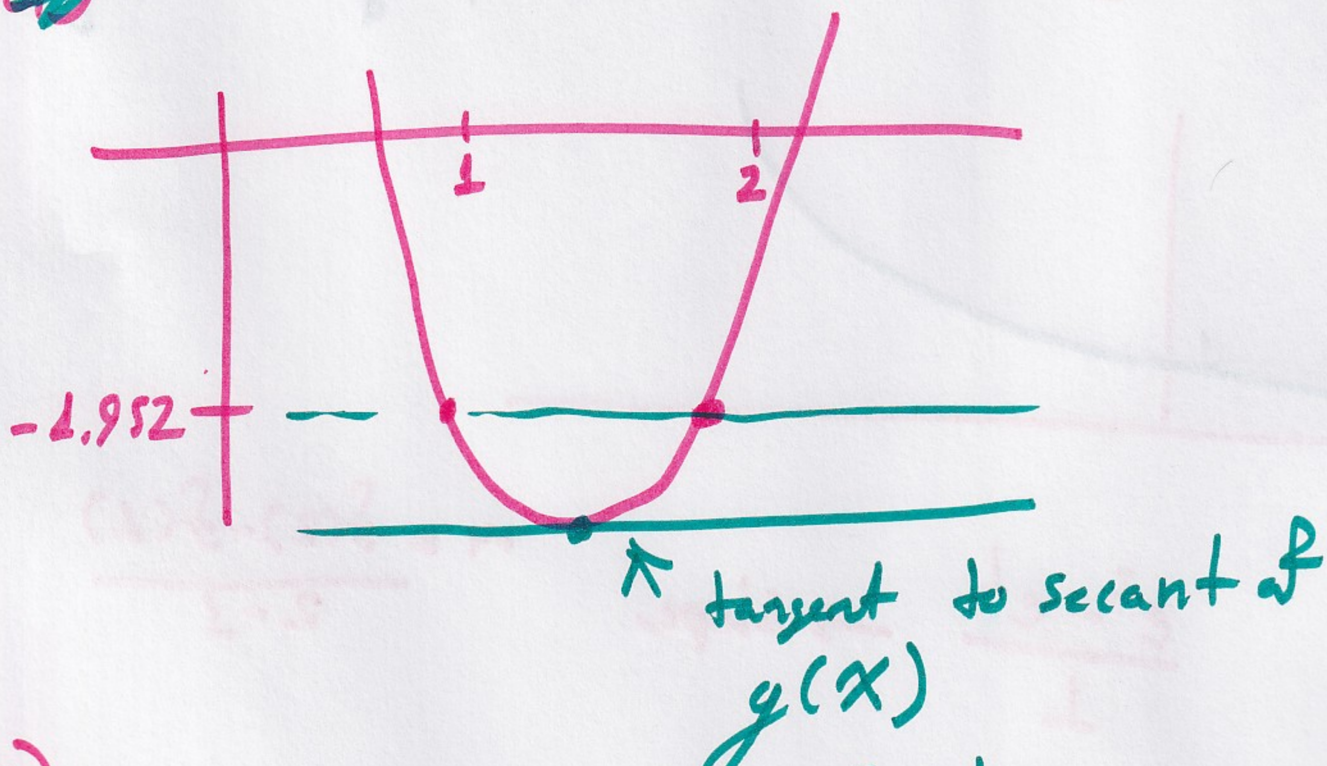
$$g(x) = e^x - (e^2 - e)x$$

(2) (d)

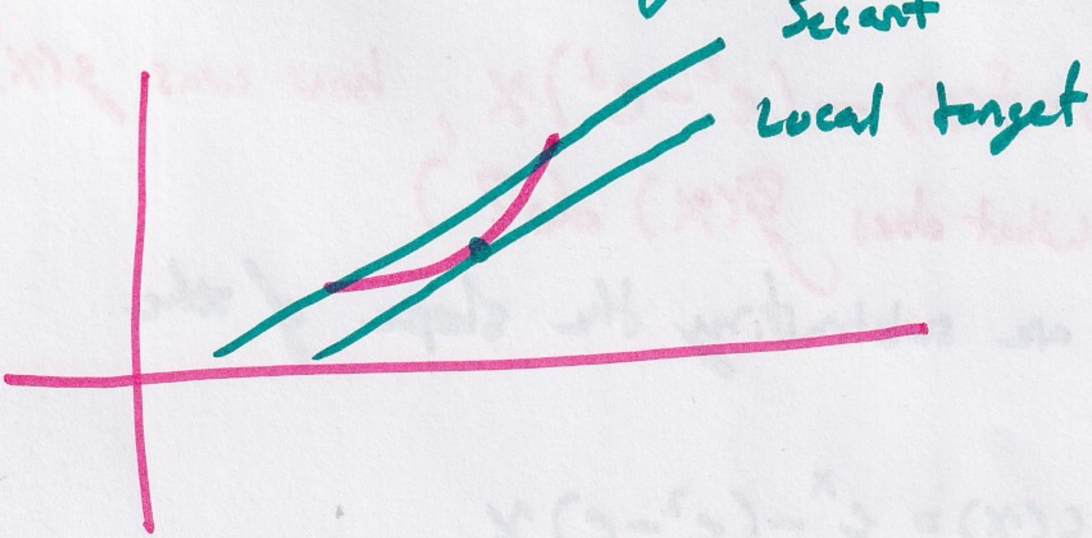
$$g(1) = e - e^2 + e = 2e - e^2$$

$$g(2) = e^2 - (e^2 - e)(2)$$

$$= e^2 - 2e^2 + 2e = 2e - e^2$$



e)



We want x s.t. $f'(x) = e^2 - e$,
note that this is equivalent to $g'(x) = 0$
since $g'(x) = f'(x) - (e^2 - e)$

We want to solve $g'(x) = f'(x) - (e^2 - e)$

$$\cancel{= e^x}\; e^2$$

$$= e^x - e^2 + e = 0$$

Solution is at $x = \ln(e^2 - e)$

So X value shares the same location?

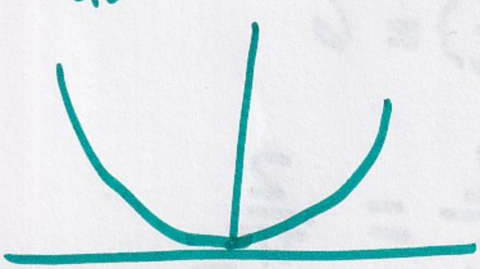
#2

a) $c(x) = x^2$, $[9, 100]$

b) $d(x) = \sin(x) + 5$, $[0, \frac{\pi}{2}]$

a)

a)



b $c(x) = x^2$

$c(9) = 81$ $c(100) = 10000$

c) $g(x) = x^2 - \frac{10^4 - 81}{100 - 9} x$

$g(9) = 81 - 109(9) = -900$

$g(100) = 10^4 - 109(100) = -900$

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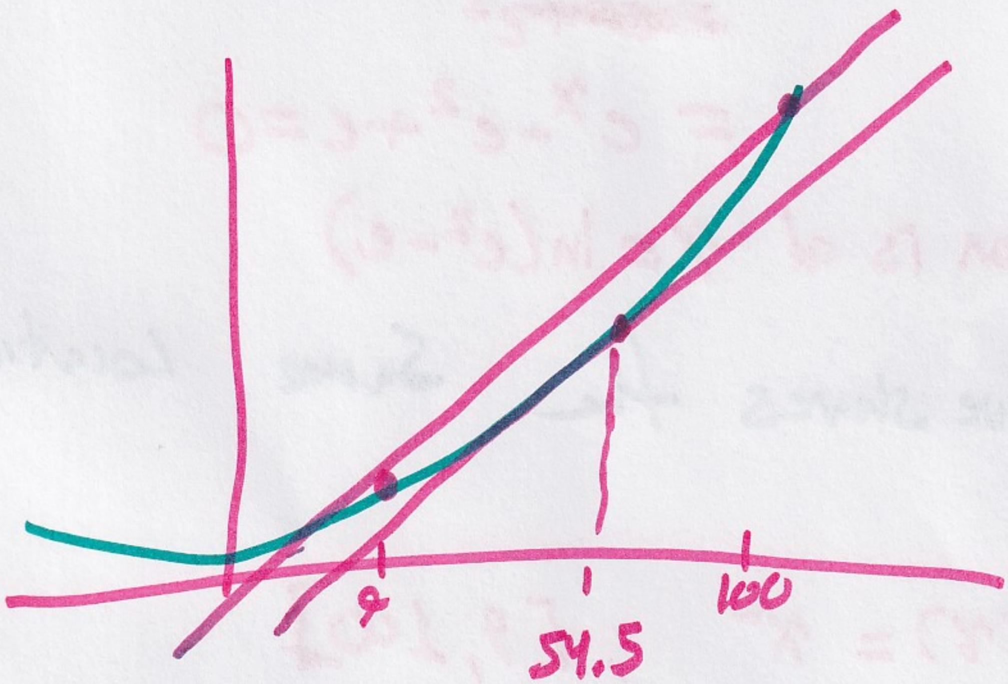
$g'(x) = 2x - 109 = 0$

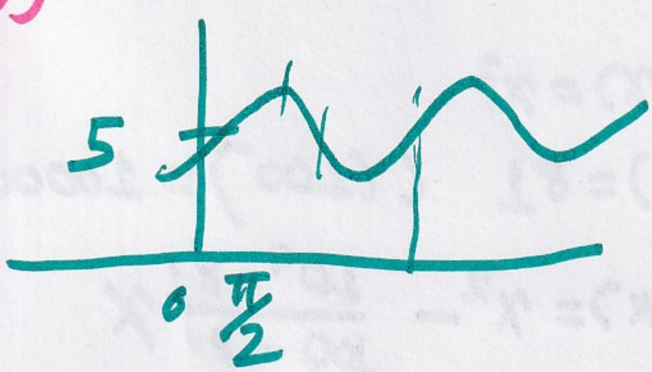
$2x = 109 \quad x = 54.5$

$f'(x) = 2x = 109$

$x = 54.5 \leftarrow$ checking where the tangent is the same



b) a)



b) $d(x) = \sin x + 5$

$d(0) = 5$

$d(\frac{\pi}{2}) = 6$

$$\frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$$ $\leftarrow$ slope

$$g(x) = \sin x + 5 - \frac{6-5}{\frac{\pi}{2}-0}x$$

$$= \sin x + 5 - \frac{2}{\pi}x$$

Notice that $g(0) = 5$ $g(\pi/2) = 1 + 5 - 1 = 5$

and $g'(x) = \cos x - \frac{2}{\pi} = 0$

$$x = \cos^{-1}(2/\pi)$$

and $f'(x) = \cos x = \cancel{4}\ 2/\pi$

$x = \cos^{-1}(2/\pi)$ ← location of local tangent

#3

"we have $f(x)$"

we want to investigate the general Royce theorem, so we dont need the y values to be the same.

#4) MVT

Given $f: I \to \mathbb{R}$, with $I = (\alpha, \beta) \subset \mathbb{R}$. If f is cont. on $\overline{I}$ and f is differentiable on I, then there is a $\gamma \in I$ s.t.

$$f'(\gamma) = \frac{f(\beta) - f(\alpha)}{\beta - \alpha}$$

Proof: Let $(\alpha, \beta) \subset \mathbb{R}$ with f continuous on $[\alpha, \beta]$ and differentiable on (α, β)

consider the function

$$g(x) = \theta(x) - \frac{\theta(\beta) - \theta(\alpha)}{\beta - \alpha} x$$

notice that $g(\beta) = \theta(\beta) - \dfrac{\theta(\beta) - \theta(\alpha)}{\beta - \alpha} \beta$

$$= \frac{(\beta - \alpha)\theta(\beta)}{\beta - \alpha} - \frac{\beta\theta(\beta) - \beta\theta(\alpha)}{\beta - \alpha}$$

$$= \frac{-\alpha\theta(\beta) + \beta\theta(\alpha)}{\beta - \alpha}$$

and $g(\alpha) = \theta(\alpha) - \dfrac{\theta(\beta) - \theta(\alpha)}{\beta - \alpha}\,\alpha$

$$= \frac{(\beta-\alpha)\theta(\alpha)}{\beta-\alpha} - \frac{\alpha\theta(\beta) - \alpha\theta(\alpha)}{\beta - \alpha}$$

$$= \frac{\beta\theta(\alpha) - \alpha\theta(\beta)}{\beta - \alpha}$$

so $g(\alpha) = g(\beta)$ by Rolle's Thm, we know $\exists$ $\gamma \in (\alpha, \beta)$ s.t. $g'(\gamma) = 0$

$$g'(x) = \theta'(x) - \frac{\theta(\beta) - \theta(\alpha)}{\beta - \alpha}$$

$$g'(\gamma) = \theta'(\gamma) - \frac{\theta(\beta) - \theta(\alpha)}{\beta - \alpha}$$

$$0 = \theta'(\gamma) - \frac{\theta(\beta) - \theta(\alpha)}{\beta - \alpha}$$

$$\frac{\theta(\beta) - \theta(\alpha)}{\beta - \alpha} = \theta'(\gamma)$$

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#5 $f(x) = \sin(\frac{1}{x})$

a)

b) $(0, \infty)$

continuous

$[0, \infty)$

→ issue at $x = 0$

→ not continuous

c) continuity: $\lim_{x \to a} f(x) = f(a)$

$\lim_{n \to \infty} a_n \to a \Rightarrow \lim_{n \to \infty} f(a_n) = f(a)$

* true for all subsequences of a_n

Is it possible to find sub sequences an so that $\sin(\frac{1}{x}) \to 1$ or -1?

consider the following subsequences

(1) $\sin(\frac{1}{x}) \to \frac{1}{x} = \frac{\pi}{2} + 2\pi n$

$$= \frac{\pi}{2} + \frac{4\pi n}{2} = \frac{\pi + 4\pi n}{2}$$

$$(x_n) = \left(\frac{2}{\pi + 4\pi n}\right) \quad n \in \mathbb{Z}^+$$

(2) $\frac{1}{x} = \frac{3\pi}{2} + 2\pi n$

$$\frac{1}{x} = \frac{3\pi + 4\pi n}{2}$$

$$(x_m) = \left(\frac{2}{3\pi + 4\pi m}\right) \quad m \in \mathbb{Z}^+$$

we know $(x_n) \to 0$ as $n \to \infty$

and $(x_m) \to 0$ as $m \to \infty$

but $(f(x_n)) \to 1$

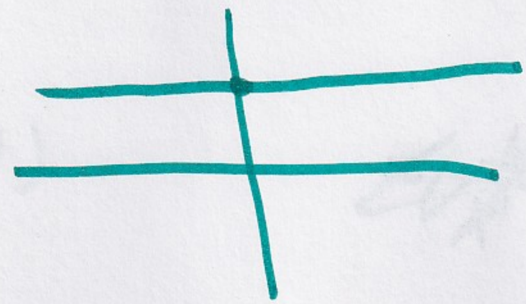
and $(f(x_m)) \to -1$

which is a contradiction

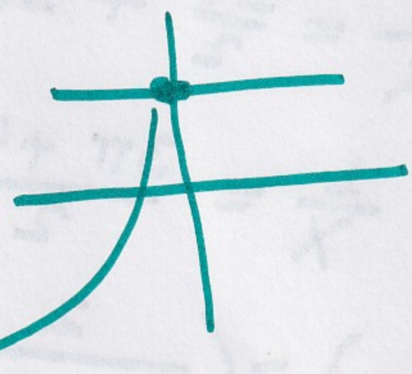
So $f(x)$ is not continuous at $x=0$

Removeable discontinuity, add a point to make that function continuous.

$\frac{x}{x} = 1 \quad (x \neq 0)$



but $\begin{cases} \frac{x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$



fills in hole

as we have -1 and 1 as $n \to \infty$ from LHS and RHS, we can't use a Removeable discontinuity