Meeting 15 Notes

Averages, Secants, and Tangents

#1) a) The Bolzano-Weierstrass Theorem

Notion of convergent subsequence

define convergence, first we need sequences

b) sequences are many to $\mathbb{R}$, connected

c) maxima, minima on an I

We want to maintain a "niceness" of the real numbers.

We expect the properties of ~~$\mathbb{R}$~~ $\mathbb{R}$ to have "niceties"

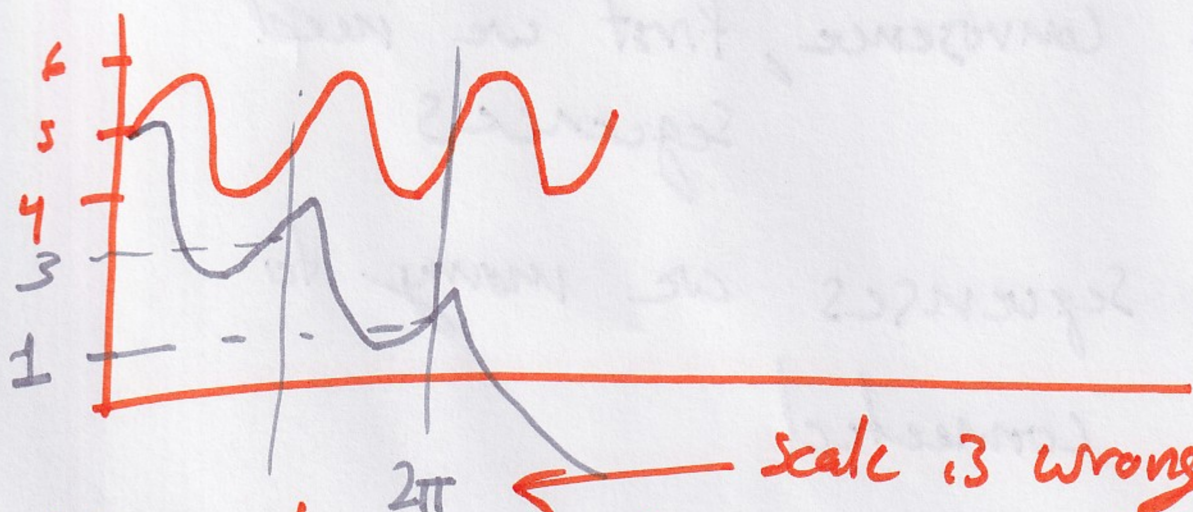
→ complete ness, , continuity, intervals

②

#2) $\alpha(x) = \sin(x) + 5$

$\beta(x) = \sin(x) + 5 - \frac{2}{\pi}x$

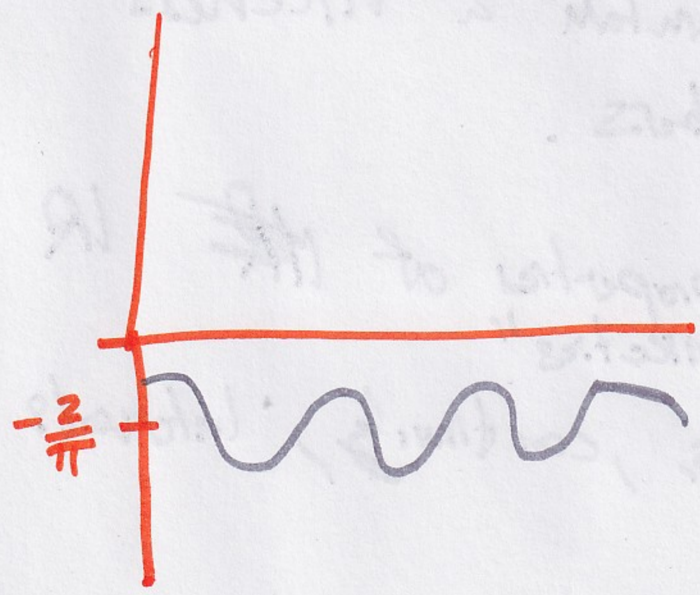
$I = (0, \frac{\pi}{2})$



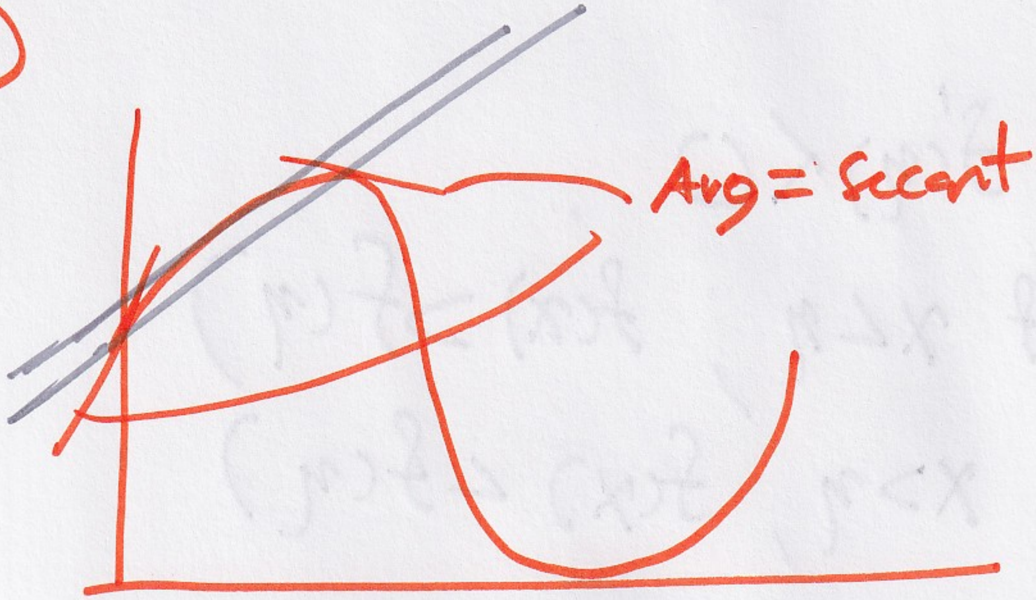
scale is wrong!

b) yes! Individually they are continuous

c) $\beta'(x) = \cos(x) - \frac{2}{\pi}$



d)



Continuous differentiability is important!

secant is the avg of the tangents of the two intersecting points of the secant

~~#2~~ #3)

$f(x)$ $\quad I = (\eta - \varepsilon, \eta + \varepsilon)$

$f(\eta) > 0$ then $f(x) < f(\eta)$

$x < \eta$

$f(x) > f(\eta) \quad x > \eta$

$$f'(x) = \lim_{x \to \eta} \frac{f(\eta) - f(x)}{\eta - x}$$

consider the signs '+' or '−'
if $f(\eta) > f(x)$
we must have $\eta > x$
and vice versa

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#4) $f'(\eta) < 0$

if $x < \eta$, $f(x) > f(\eta)$

" $x > \eta$, $f(x) < f(\eta)$

so "$-|f'(\eta)|$" for a negative value

$$f'(x) = \lim_{x \to \eta} \frac{-f(\eta) + f(x)}{\eta - x}$$

#5)

"The derivative of a f at that point"

We shown EVT, we know on I,

A maximum is somewhere

$\exists M$, where M is a maximum

where $M = f(x_0) \in (a,b)$

We know this is true from EVT (5)

Case 1: constant function $\rightarrow f'(x) = 0 \ \forall \ x \in I$

$x_0 = a$ or $x_0 = b$, constant f

Case 2: $x_0 \in I$

What's the $f'(x_0)$?

$$\lim_{x \to x_0^+} \frac{f(x) - f(x_0)}{x - x_0} \leq 0$$

The denominator will be < 0 (negative)

$$\lim_{x \to x_0^-} \frac{f(x) - f(x_0)}{x - x_0} \geq 0$$

What's the general Limit?

If both are true, this suggest the general derivative is zero, so rolles is correct!

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = 0$$

(b) ~~A its true even~~ we didn't say $f(a)=f(b)=0$!

So we made a general statement.

6 $f: I \to \mathbb{R}$

a) monotone but with functions!

↓

f is an increasing function if

$\forall x \in I$, $f'(x)>0 \to f(a)>f(x)$ if $a>x$

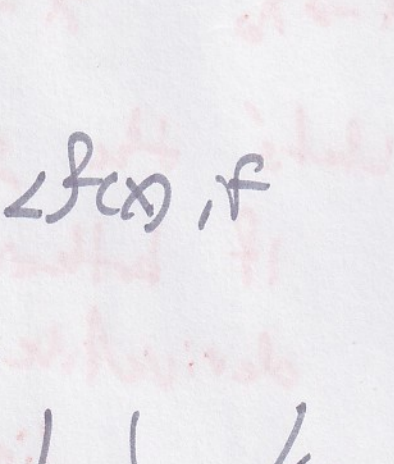
$|a-x|<\varepsilon$, the function is strictly increasing

b) drop a negative, done

$\forall x \in I, f'(x) <$ so $f(a) < f(x)$, if

$a > x$ and $|a-x|<\varepsilon$

c) If $\lim L^{+} \to 0, L \geqq 0$



$f(x) = x^3$

$f'(x) = 3x^2$

zeros at inflections
and extreme values

x^3 at $x=0$, $f'(x)=0$

but $f(0)$ is not a max/min