Meeting 13

whats the formal proof of the Intermediate value Theorem

$f(x) = \sin(x)$

1) a) $\sup_{x \in I} f(x)$ $\quad I = (-\infty, \infty)$

~~$\sup_{x \in I} f(x) \leq I$~~

$\sup_{x \in I} f(x) = 1$

b) $f : (0, \infty) \to R$ $\quad f(x) = \frac{1}{x}$

why is continuous?

$|x - k| < \delta \to |f(x) - f(k)| < \varepsilon$

$\forall \varepsilon > 0 \ \exists \delta > 0$ s.t. for $k > 0$

we have

$|x - k| < \delta \to |f(x) - f(k)| < \varepsilon$

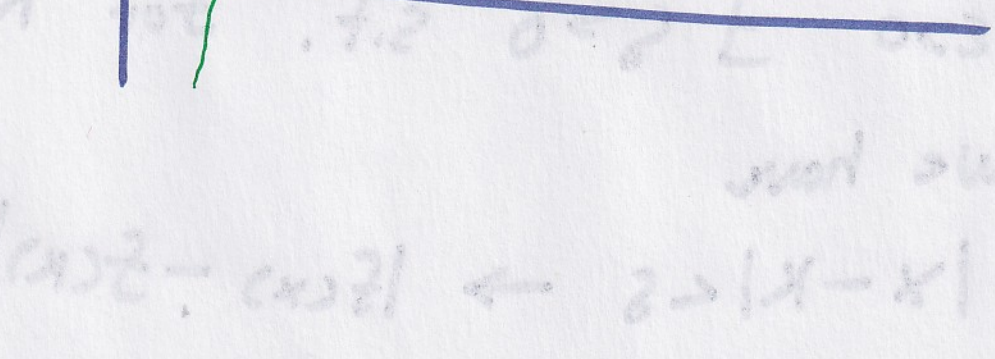
c) Let (a_n) be a sequence if $N \leq a_n \leq m$ Then for all $n \in \mathbb{N}$, (a_n) is bounded

d) A set that is closed has all its Limit points

$[e, \pi]$

#2) $h(x) = 2 - \frac{1}{x}$

a) $\sup_{x \in (0, \infty)} h(x) = 2$



b) $K(x) = \frac{1}{2-h(x)}$

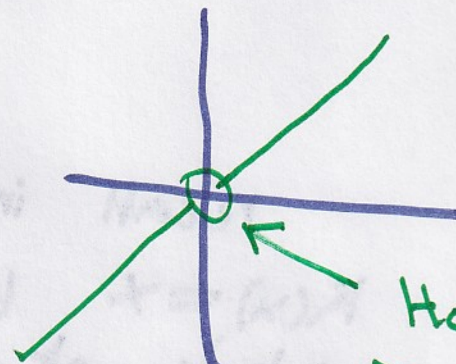
$= \frac{1}{2-(2-\frac{1}{x})}$

$= x \quad (x \neq 0)$

$2-2+\frac{1}{x}=0$

$2x-2x+1=0$

$1=0$



Hole

J. doll parts

$K(2) = \frac{1}{2-h(2)}$

$= \frac{1}{2-(2-\frac{1}{2})}$

$= \frac{1}{1/2} = 2$

cannabal shin Lebovfe?,

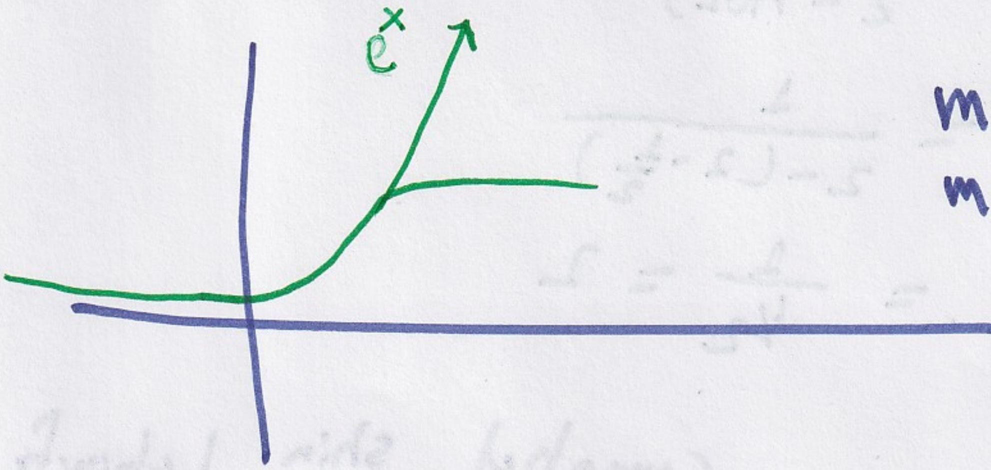
3 min, max, inf, sup

a) $f(x) = \frac{e^{-x}}{1+e^{-x}}$ $D = (-\infty, 0]$

$\frac{e^{-x}}{1+e^{-x}} \cdot \frac{e^x}{e^x}$ $e^x \neq 0$

or ~~else~~
Else we are multiplying $\frac{0}{0}$ by $f(x)$

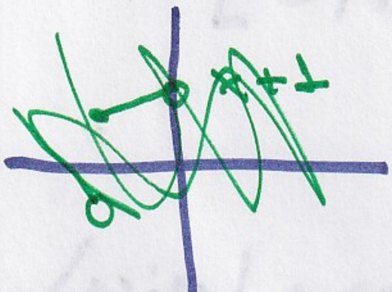
recall in #2, we have
$K(x) = x$ but not really cuz we have a hole at $\underline{x=0}$, be carefull manipulating the function

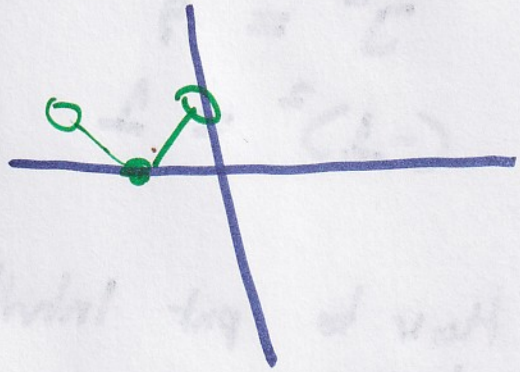


min: ~~None~~ $\frac{1}{2}$
max: None
inf $\rightarrow \frac{1}{2}$
sup: 1

b) $g(x) = \begin{cases} x+1 & x \in [-1,0) \\ -x-1 & x \in (-2,-1] \end{cases}$

$D = (-2,0)$





max → None   min → 0

inf → 0   sup → 1

The function approaches the sup twice, but only has one sup

c) $z(x) = x^2$ $D = [-1,3]$

min 0

Max: 9

Sup: 9

inf: 0

(we don't have a Absolute max for Sin) (cos)

1 max, may hit multiple times

Extreme value theorem

[-1,3]

$(0)^2 = 0$

$3^2 = 9$

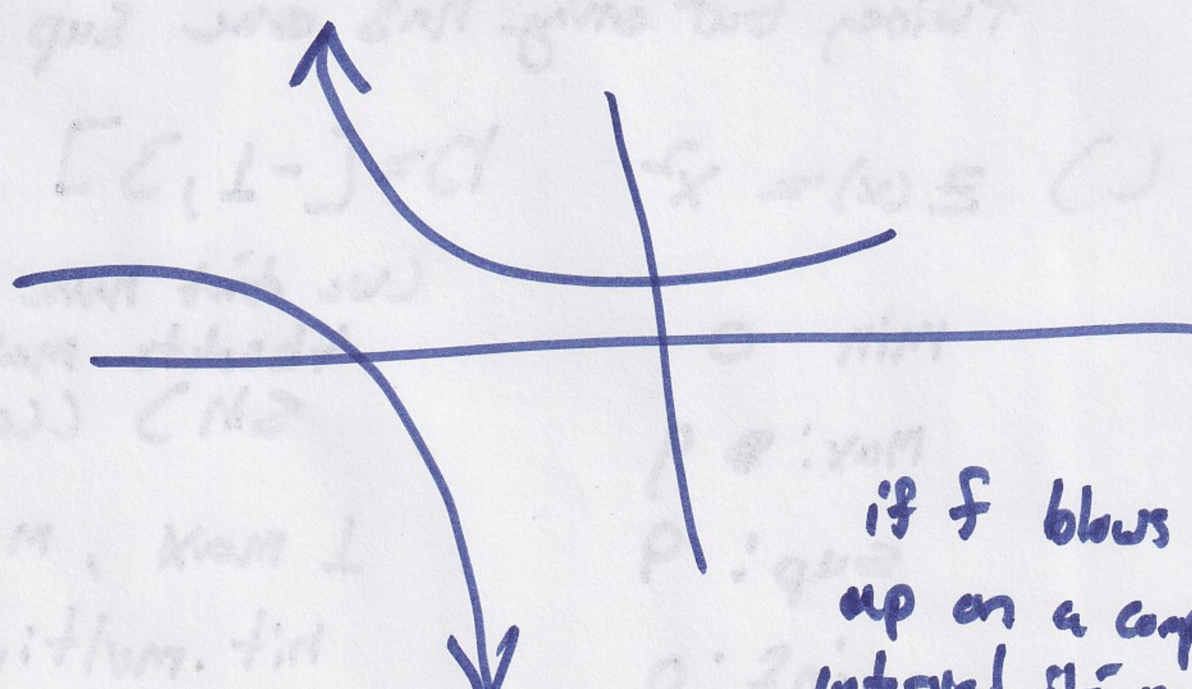
$(-1)^2 = 1$

[-4,3]

How to put Intuition on a Logical footing

$D = [a,b]$, $f(x)$ on D

#4 can $f(x)$ "blow up" and still be continuous?



if f blows up on a compact interval, its not continuous

→ unbounded when $x \to \pm\infty$
(Like x^2)
↳ not a compact interval

→ VA when $x \to a$
↳ not continuous at $x=a$

Feels Like Yes

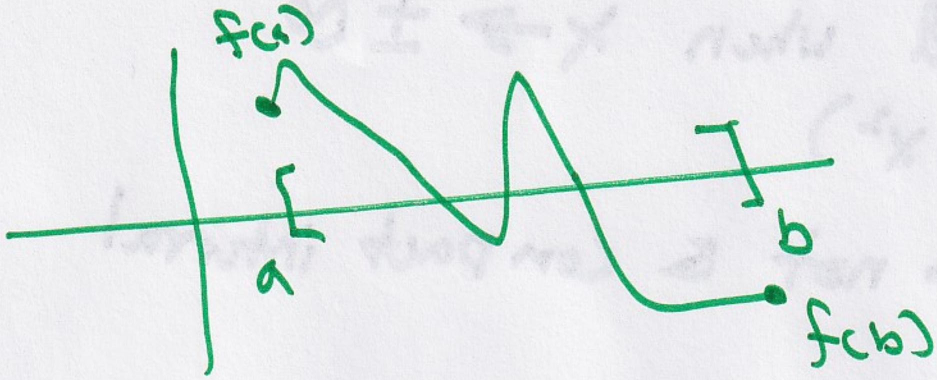
<u>Claim</u>: if $f(x)$ is continuous on $[a,b] \subseteq \mathbb{R}$ then $f(x)$ is bounded on $[a,b]$

<u>Thought</u> Assume by way of contradiction that $f(x)$ is continuous on $[a,b] \subseteq \mathbb{R}$ and $f(x)$ is not bounded on $[a,b]$

$$\neg(A \to B) = A \wedge \neg B$$

↑
if we can show this is false
This implies $A \to B$ is true

⑧



$x \in [a,b]$ s.t. $f(x)$ is not bounded

1 $x=a$

↳ compact $\rightarrow \lim_{x\to a^+} f(x) = f(a)$

2 $x=b$

↳ compact $\rightarrow \lim_{x\to b^-} f(x) = f(b)$

3 $a<x<b$

↳ $f(x)$ is continuous

$|f(x) - f(1<)| < \varepsilon$

if $f(x)$ not bounded, where is it not bounded?
but I is compact

EVT : Extreme Value Theorem

is there a max or min? EVT can attest if they exist