#5

a function $f: N \to R$ (1)

1.) a) a function whose inputs are natural numbers whose outputs are real numbers.

→ A function whose domain is the natural number

b) $\frac{1}{n}$ doesn't have $n=0$ because that's not an input

bounded below → it has a minimum

bounded above → it has a maximum

c) Define what it means for a sequence to be convergent

$$\forall \epsilon > 0, \exists N \text{ s.t. } \forall n \geq N, |a_n - L| < \epsilon$$

$\iff$ a_n converges to the limit L

②

d) Does $((-1)^n)$ converge or diverge? Why?

No, It oscillates

$(-1)^1, 1, -1, 1, \ldots$

for Now, we will say something diverges if it ~~does~~ doesn't converge.

→ If you jam it thru c) we can easily see that it doesn't converge, No N

L isn't unique.

take $\epsilon = 0.5$, $\forall N$ assume L exists.

if $|a_n - L| < 0.5$, $|a_{n+1} - L| = |a_n - L| + 1 \not< \epsilon$

e) $a_n = n$ Monotonic increasing

$a_n = -n$

$a_n = \frac{1}{n}$ Monotonic decreasing

f) trian

"good for proofs, can take complicated sums and put them together to fit inequalities for example"

Limit proofs, $a_n - b_n \to L$

triangle Inequality is used to split up the terms

its nice to take a distance of a sum to a sum of distances.

④

2.) Consider the following sets   upper and Lower

a) and b) are asking the boundaries

~~A~~ $\Lambda =$ a) -3 c) $[-3,3]$

b) 3

$-3, 3$ are elements in Λ

b) $-3 + \frac{2}{1}$

Ω a) no minimum

$\rightarrow \Omega \rightarrow L$, $L = -3$

c) Lower bound of Ω is 3

5

$-5 \le \Lambda < -3$ $\quad$ $-3 < \Omega \le -1$

min: -5, max: none $\quad$ min: none, max: -1

for Π, even term monotonically decreasing

odd term " "

$\cancel{?} \le \Pi \le \frac{1}{4}$ $\quad$ min:

-1

$-3 \le \Lambda \le 3$ $\quad$ $-3 < \Omega \le -1$

min: -3, max: 3 $\quad$ min: none, max: -1

Σ $\quad$ No max or min

we don't have a min or max, so we are calling it Supremum

$-1 < \Sigma < 1$

Least upper bound

~~Least~~ Greatest Lower bound / infimum

⑥

Prompt 3 is a rehash of prompt 2

#4)

a) are all bounded sequences convergent?

No $a_n = (-1)^n$ is a counter example

→ to show, run an thru the definition of convergence, then show where it fails

def of convergence, normally we have

L must be unique, ~~but~~

→ but doesn't say we have to have

say L is unique

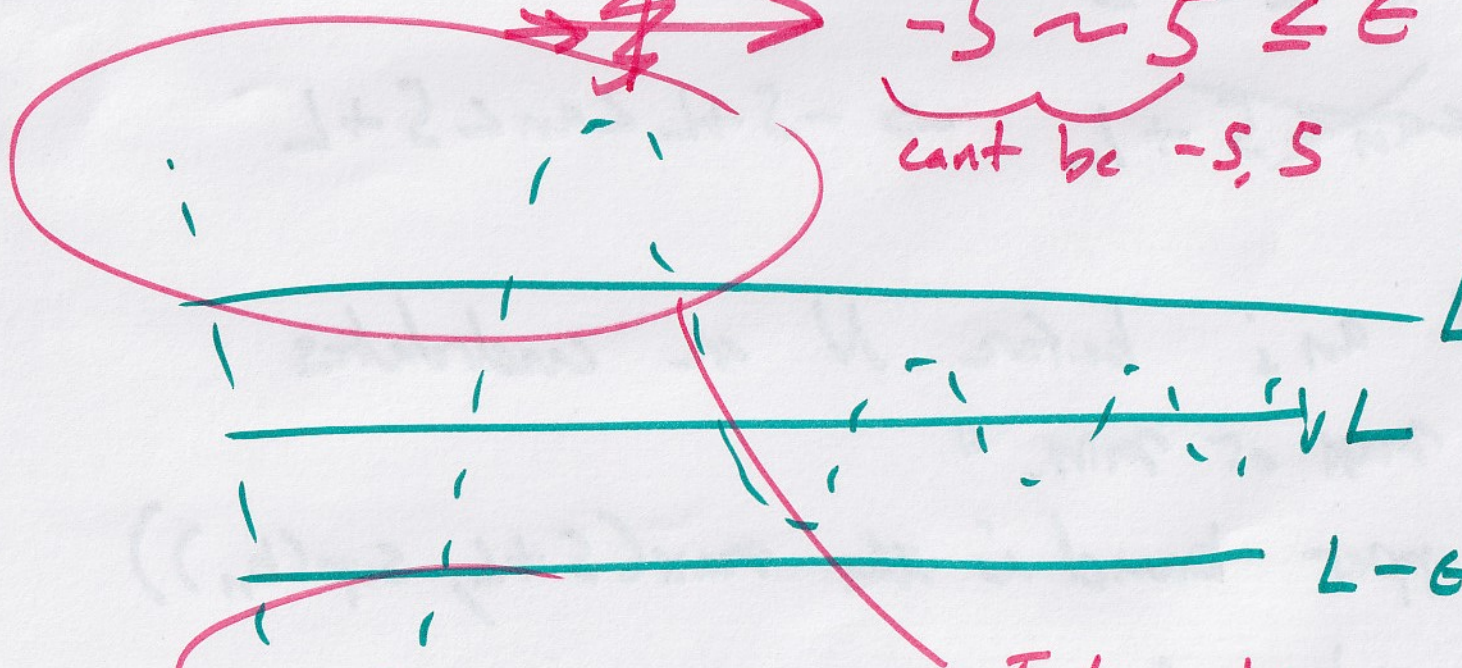
b) are all convergent sequences bounded?

we may have a convergent sequence that is not bounded

$$|a_n - L| < \epsilon$$

if $\epsilon = 5$

$$\not\Rightarrow \underbrace{-5 \sim 5}_{\text{cant be } -5,\, 5} \le \epsilon$$



Take the max outside things as the bound (Supremum)

infimum

(8)

~~So~~ So, we know

define $b_n = \{a_n / n < N\}$

finite, therefore bounded

1.) The limit is convergent

There exist N, s.t. $|a_n - L| < \cancel{\epsilon} \; 5$

we ~~should~~ find an N to make this work

so

$$-5 < a_n - L < 5$$

$$\rightarrow -\cancel{5 < a_n <} 5 + L \rightarrow -5 + L < a_n < 5 + L$$

" therefore

all a_n's before N are candidates

for the max or min. "

Therefore the upper bound is the $\max(5 + L, \sup(b_n))$

and the Lower bound is the $\min(-5 + L, \inf(b_n))$

1) The limit is convergent, there exist N,
define $b_n := \{a_n \mid n < N\}$ (finite, therefore bounded)
st $|a_n - L| < 5$, $-5+L < a_n < 5+L$

therefore the upper bound is the max$(5+L, \sup(b_n))$
and the lower bound min$(-5+L, \inf(b_n))$

Yes, all convergent sequences are bounded

10

5.)

$$|a_n b_n - A b_n + A b_n - AB|$$

$$(a_n - A)(b_n - B)$$ Expand this expression to make things easier!

Zach says this makes this less helpful