Meeting 20 Notes

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Sums and differences, Part I

Summary of Meeting #19
- The mean value theorem for integrals

Given a continuous, real-valued function $\gamma(x)$ on $[c,d]$, there is a value $a \in [c,d]$ where

$$\int_c^d \gamma(x)\,dx = \gamma(a)(d-c)$$

#1.) $\Sigma(x) = \int_1^x \gamma^2\,d\gamma$

a) $$\frac{\gamma^3}{3}\Big|_1^x = \frac{1}{3}\left[x^3 - 1\right]$$

$$\Sigma(3) = \frac{26}{3} \quad \Sigma(5) = \frac{124}{3}$$

b) $\frac{124}{3} - \frac{26}{3} = \frac{98}{3}$

c)

$$\frac{1}{3}x^3 \Big|_a^b = \frac{1}{3}[b^3 - a^3]$$

so if $\frac{98}{3} = \frac{1}{3}x^3 \Big|_a^b = \frac{1}{3}[b^3 - a^3]$

evinculum

$$98 = b^3 - a^3$$

vinculum

$$b = \sqrt[3]{98 + a^3}$$

As many possibilities that satisfy this

a	b
1	$\sqrt[3]{99}$
2	$\sqrt[3]{106}$
3	$\sqrt[3]{125}$
4	$\sqrt[3]{162}$

$$\int_3^5 x^2 dx = \Sigma(5) - \Sigma(3)$$

$$= \Sigma(\sqrt[3]{99}) - \Sigma(1) = \frac{98}{3}$$

d) does it satisfy the definition of continuity?

$$\left(\frac{1}{3}\lim_{x\to 3} x^3\right) - \frac{1}{3} = \frac{26}{3}$$

we know this exists, we showed this

the Limit of $x^3 \longrightarrow 27$

e) formally define $\frac{d}{dx}\Sigma(x)$

$$\Sigma = \frac{1}{3}(x^3 - 1)$$

$$\Sigma' = \frac{1}{3}3x^2 = x^2$$

we showed this

for x^2 before (that $\frac{d}{dx}x^3$ exists)

This is the same as the gamma γ !

#2) As defined ~~it~~ in the previous prompt, is $\Sigma(x)$ continuous on any intervals? Why or why not?

$$\Sigma(x) = \int_1^x r^2\,dr$$

#58

The darboux sums shows that we can do this (one of the problems said we could do this)

is $\frac{1}{3}[x^3 - 1]$ is cont. on $\mathbb{R}$

Then $\Sigma(x)$ is continuous

#3) You can use darboux sums to integrate the Heaviside function. (or Rieman sums)

$x^2 \rightarrow r(x)$, r is integrable

if cont. then we know it's integrable

if it's integrable, we don't know if it's continuous.

we need continuity to EVT

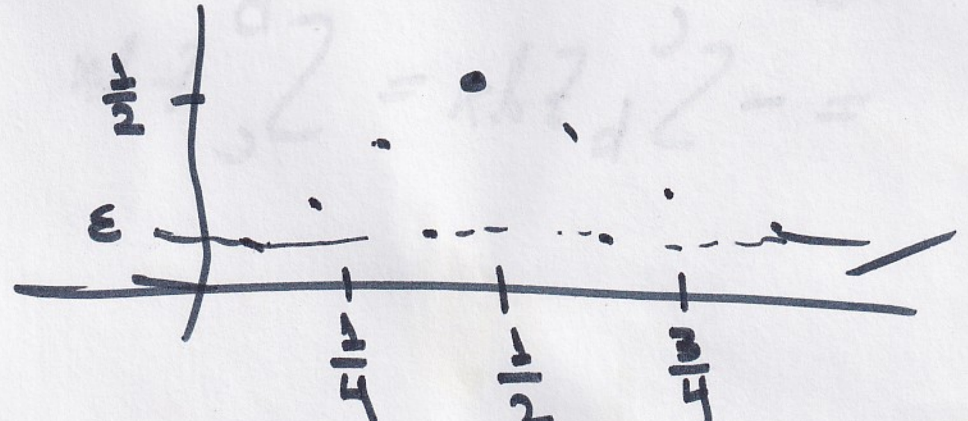
need EVT to bound a function

Saying its continuous is the hand wavy-ness

To get derivatives, we ~~don't~~ need continuity

$$f(x) = \begin{cases} \frac{1}{n} & x = \frac{m}{n} \in \mathbb{Q} \\ 0 & x \in \mathbb{R} - \mathbb{Q} \end{cases}$$

The $\int f = 0$ is for integrating in this specific section



very dense

$\int f = 0$

sup can be the inf if ε is small enough so $\int f = 0$

we can integrate ~~this~~ but it's not continuous.

6. We can make sequences around the points.

x^2 can be changed out for any continuous f in $\mathbb{R}$

The bounds can be anywhere in the domain of the function.

we could also write

$$\int_1^5 f\,dx - \int_1^3 f\,dx$$

$$= \int_3^5 f\,dx$$

$$\int_a^b f\,dx - \int_a^c f\,dx$$

$$= -\int_b^c f\,dx = \int_c^b f\,dx$$

#4) Let $\eta(x)$ be continuous on $[\alpha,\beta]$,
Then Letting $H(x)=\int_\alpha^x \eta(\gamma)d\gamma$,

$$\frac{dH(x)}{dx}=\eta(x) \text{ so long as } x\in[\alpha,\beta]$$

$$\lim_{h\to 0}\frac{H(x+h)-H(x)}{h}=\eta(x)$$

what does $H(x+h)$ mean?

$$\frac{\int_\alpha^{x+h}\eta(r)dr-\int_\alpha^{x}\eta(\gamma)d\gamma}{h}$$

$$=\lim_{h\to 0}\frac{1}{h}\int_x^{x+h}\eta(r)dr$$

$$=\eta(a),\ a\in[x,x+h]$$

$=\eta(x)$ The interval will get small until it converges to x

This is the first part of the fundamental Theorem of calculus.

we used MVT to do the proof

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Consider the function $D : [0, 1] \to \mathbb{R}$

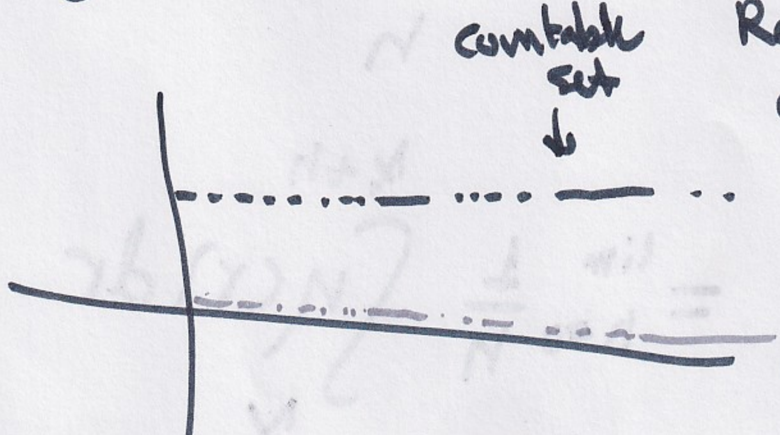
with $\cancel{F(x)}$ $D(x) = \begin{cases} 0 & x \in \mathbb{R} - \mathbb{Q} \\ 1 & \cancel{x} \in \mathbb{Q} \end{cases}$

$$\int_0^1 D(x)\,dx$$

1 for the Rational

0 for the irrational

we can't use continuous conditions here



uncountable set

use the definition of the integral

$\overline{\int} D\,dx = 1$ for every partition

$\underline{\int} D\,dx = 0$ ~~Either 1 or 0~~

recall the definition of the Integral

$$\left|\overline{\int} - \underline{\int}\right| < \varepsilon$$

~~This doesn't exist~~

The Integral doesn't Exist