Meeting 11

notes

Analysis on the four C's

Summary of 10,

- continuity of a function is connected to the Limit existing and the function being defined at a point
- One-sided Limits (L-H and R-H) can be defined

→ Before You Begin

continuous

#1 $\in$ $f: D \to R$

$g: D \to R$

if we know they are cont. we know

~~about the Limits~~
without Loss of generality

$\lim_{t \to K} f(x) = f(K)$ at all $x = K$ in D

$h(x)$ $h(K)$

~~Lim~~

② Σ Σ of 2 convergent sequences, They add (convergences add)

$a_n \to A$

$b_n \to B$ $\quad a_n + b_n \to A+B$

For two functions $f(x), g(x)$

$$\lim_{x \to K} f(x) + \lim_{x \to K} g(x) =$$

$$|(f(x)+g(x)) - (L_f + L_g)| < \varepsilon$$

$$|f(x) - L_f + g(x) - L_g| < \varepsilon$$

We want this to be true, but we don't know if it is.

$$|f - L_f + g - L_g| \le |f - L_f| + |g - L_g| < \varepsilon$$

well we know $|f - L_f| < \varepsilon$ and $|g - L_g| < \varepsilon$

So ~~theire~~ their additions should be less than ε

So we write this inequality

we know the triangle inequality is true

So write

$$|f - Lf| < \frac{\varepsilon}{2}$$

$$|g - Lg| < \frac{\varepsilon}{2}$$

$$|f - Lf| + |g - Lg| < \varepsilon$$

(one ε only)

we are given 1 ε, use δ in any way

Let $\varepsilon > 0$

choose $\delta = \max(\delta_1, \delta_2)$

because $f(x)$ is continuous,

$$|x - k| < \delta_1 \xrightarrow{\text{implies}} |f(x) - Lf| < \varepsilon/2$$

choose δ_1 to satisfy $\varepsilon/2$

$$|x - k| < \delta_2 \rightarrow |g(x) - Lg| < \varepsilon/2$$

choose δ_2 to satisfy $\varepsilon/2$

Assume

$$|x - k| < \delta \xrightarrow{\text{implies}} |f(x) - Lf| + |g(x) - Lg| <$$

$$\frac{\varepsilon}{2} + \frac{\varepsilon}{2}$$

we know there exists δ_1, δ_2 that satisfies the expression $|g - Lg| < \varepsilon/2$

~~choose δ_1, δ_2~~ to satisfy $|f(x) - Lf| < \varepsilon/2$ and

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$$|f(x) - L_f + g(x) - L_g| \leq$$

$$|f - L_f| + |g - L_g| < \varepsilon$$

Zach disagrees with $\delta = \max(\delta_1, \delta)$

we want a tigth bound around K

we go pass some n, Large n to tight the bound

but we want to choose min, and min is choosen this time to make it tight

So

$$|(f(x) + g(x)) - (L_f + L_g)| < \varepsilon$$

#2)

a) connected and Compact

b) " and not compact

c) connected and not compact

d) not connected and not compact

e) connected, not compact

f) not connected, not compact

g $(1,2)$, connected

$(1,2)\cup(3,4)$, not connected

h) Examples of connected things

assume all x values are within $a \leq x \leq b$

choose Some ~~x~~ = K where K is not within $[a,b]$ $A = (K-\varepsilon, K+\varepsilon)$

$B = (a-\delta, b+\delta)$

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Assume $(a,b) \cup \{K\}$ is connected

Choose some $x = K$ where K is not within $[a,b]$

$A = (K - \varepsilon, K + \varepsilon)$

$B = (a - \delta, b + \delta)$

take a and b and split it, then show some kind of contradiction

assume it's not connected then show it's connected

#3 what conditions where ($f(x)$) are connected?

#5 and #7