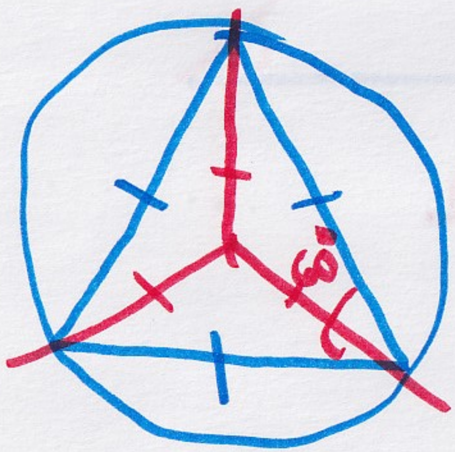
1. Consider an equilateral triangle inscribed in a unit ~~circuit~~ circle. (1)

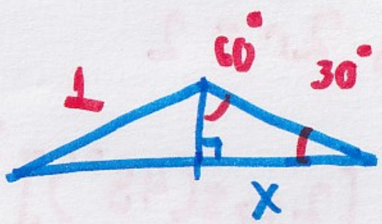
$r = 1$

a) What is the area of the circle? π

b) What is the area of the triangle?

$A = \frac{BH}{2}$





$x = \cos(30) = \frac{\sqrt{3}}{2} = r\cos(30)$

$\cos(\theta) = \frac{\text{Adjacent Leg}}{\text{Hypotenuse}}$

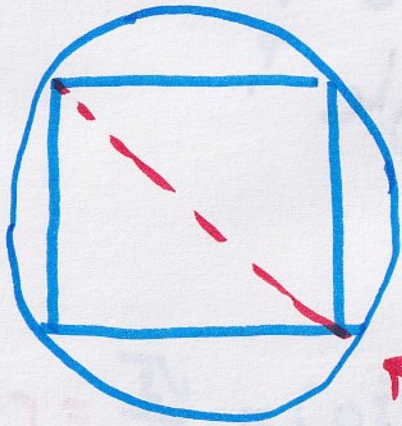
Height is $1 + \sin 30 = r + r\sin(30)$

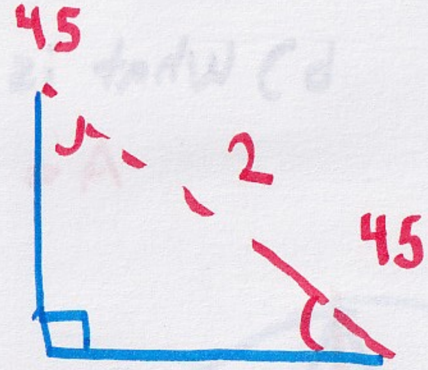
$= \frac{2}{2} + \frac{\sqrt{1}}{2} = \frac{2 + \sqrt{1}}{2} = \frac{3}{2}$

Base is $2x = \sqrt{3}$

$A = \frac{3\sqrt{3}}{4} = \frac{1}{2}[r + r\sin 30][r\cos(30)]$

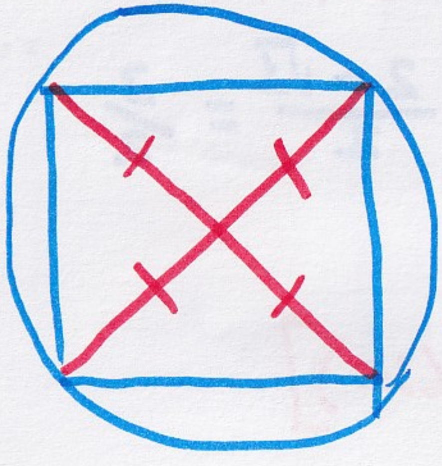
② c) inscribe a square in a unit-radius circle, what is the Area of said square?

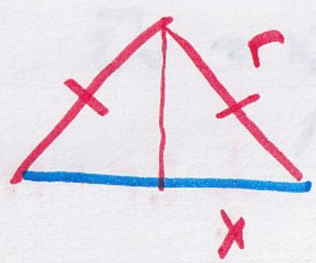




← Length is $2r = 2$

$\cancel{A = co}$ $A = [2\cos(45^\circ)]^2 = 2$



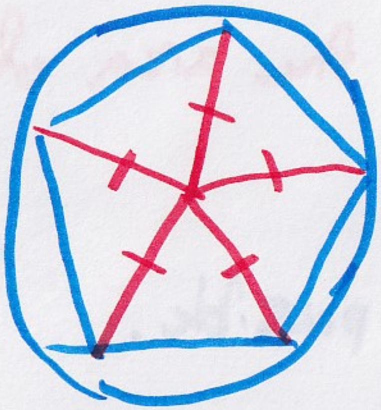


$x = r\cos 45^\circ$

so $A = (2x)^2$

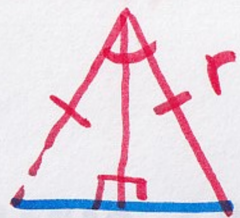
$= (2r\cos 45^\circ)^2 = r^2 2$

d) Now, inscribe a regular pentagon in a unit-radius circle. What is the area of said regular pentagon?



$\frac{360}{n}$ if $n=5$

$\rightarrow 72$



base $= 2r\sin\left(\frac{180}{n}\right)$

Height is $r\cos\left(\frac{180}{n}\right)$

~~Area~~ Area $= r^2\cos\left(\frac{180}{n}\right)\sin\left(\frac{180}{n}\right)$

N-Gon Area $= n r^2\cos\left(\frac{180}{n}\right)\sin\left(\frac{180}{n}\right)$

$= \frac{nr^2}{2}\sin\left(\frac{360}{n}\right)$

e) Investigate the area of a regular 6-gon, 7-gon etc. Inscribed in the same unit-radius circle. What's happening to the area? Why?

As $n \to \infty$, we are approaching the area of the unit circle.

f) Prove your result, being as general as possible.

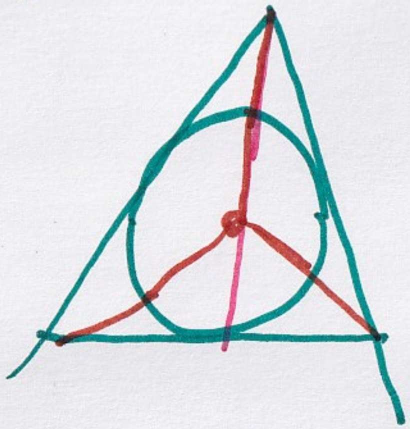
Let $x = \frac{2\pi}{n}$ so

As $\begin{matrix} n \to \infty \\ x \to 0 \end{matrix}$ $\quad \frac{1}{2}\frac{2\pi}{x}\sin(x)$ is N-gon area for $r=1$

$$\lim_{x \to 0} \frac{2\pi \sin(x)}{2x} \to \lim_{x \to 0} \frac{2\pi \cos(x)}{2} \to \pi$$

L'Hospital's rule

2.) Consider an equilateral triangle circumscribed around a unit-radius circle.

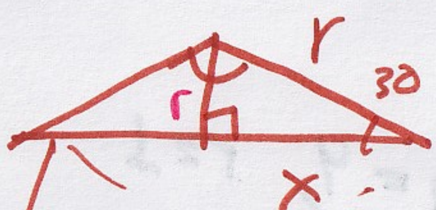


a) What's the area of the circle?

π

b) What's the area of the Triangle?

$$\frac{360}{3 \cdot 2} = 60^\circ = \frac{180}{3} \qquad n = 3$$



the radius is r

$$\cancel{r\cos(30)} \rightarrow \gamma \sin(60^\circ) \qquad r = \gamma\cos(60^\circ)$$

$$= \frac{r \sin 60^\circ}{\cos 60^\circ} \qquad \gamma = \frac{r}{\cos(60^\circ)} = \frac{r}{\cos\left(\frac{180}{n}\right)}, \quad n = 3$$

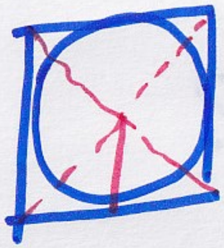
$$\text{Area} = \frac{r^2 \sin\left(\frac{180}{n}\right)}{\cos\left(\frac{180}{n}\right)}$$

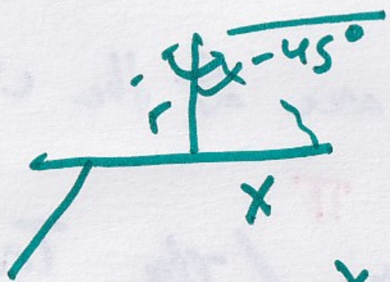
So total Area $n r^2 \tan\left(\frac{180}{n}\right)$

for $n = 3$

⑥

c) Area of a square outside the unit circle





$$\frac{360}{4(2)} = 45$$

$$x = r\tan(45°)$$

Area is $r^2\tan(45°)$

Total Area is $4r^2\tan(45°)$, $r=1$

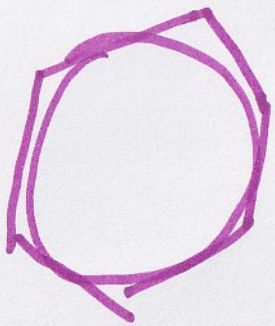
recall $nr^2\tan\left(\frac{180}{4}\right)$ for $n=4$, $r=1$

we have $(4)(1)\tan\left(\frac{180}{4}\right)$

d) Area of a pentagon,

$$(5)(1)^2\tan\left(\frac{180}{5}\right)$$

e) What is happening to the area?



The area of the N-gon is approaching π as $n \to \infty$

f) $n r^2 \tan\left(\frac{\pi}{n}\right)$ Let $x = \frac{\pi}{n}$ as $n \to \infty$, $x \to 0$

$\to \frac{r^2}{x} \pi \overset{\tan(x)}{\cancel{\sec^2(x)}}$

$$\lim_{x \to 0} \frac{r^2 \pi}{x} \tan(x) = \lim_{x \to 0} r^2 \pi \sec^2(x) = \underline{r^2 \pi}$$

Use L'Hospital

⑧

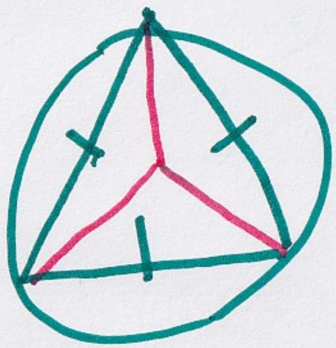
3.) Generalize prompts 1 and 2 to any radius circle.

What's different? What's the same?

→ The Area is now $r^2\pi$.

→ The derivations have r (radius) instead of 1, but they are the same.

4) Now, rework prompt 1, but consider the perimeter of the various polygons.



Angle is $\frac{180}{n}$

Length is $2r\sin(\frac{180}{n})$

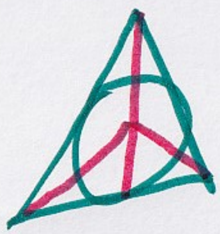
So the perimeter of the Triangle is

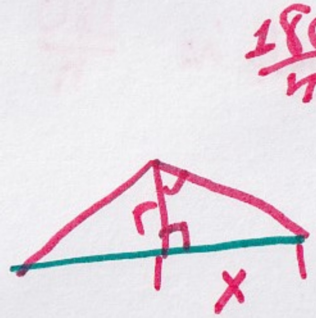
$2nr\sin(\frac{180}{n})$ where $n=3$, $r=1$

We know as $n \longrightarrow \infty$

The perimeter goes to $2r\pi$

5.) Rework prompt 2 similar to prompt 4; use perimeter instead of Area.





$x = r\tan\left(\frac{180}{n}\right)$

So the perimeter of the Triangle is $2nr\tan\left(\frac{180}{n}\right)$

where $n=3$ $r=1$

Let $x = \frac{\pi}{n}$

$$\rightarrow \lim_{x\to 0} \frac{2\pi}{x} r\tan\left(\frac{180}{n}\right) = \lim_{x\to 0} 2\pi r \sec^2(x)$$

$$= 2\pi r$$

$\Rightarrow$ As $n \rightarrow \infty$ the perimeter becomes $2\pi r$

6) Pause and reflect, How is what was just done in prompts 1-5 related to calculus?

we are using Limits which calculus is based upon.

7) Figure 1.1 is a supposed proof without words that $\frac{\pi}{4} = 1$ what's going on?

The perimeter is not well defined.

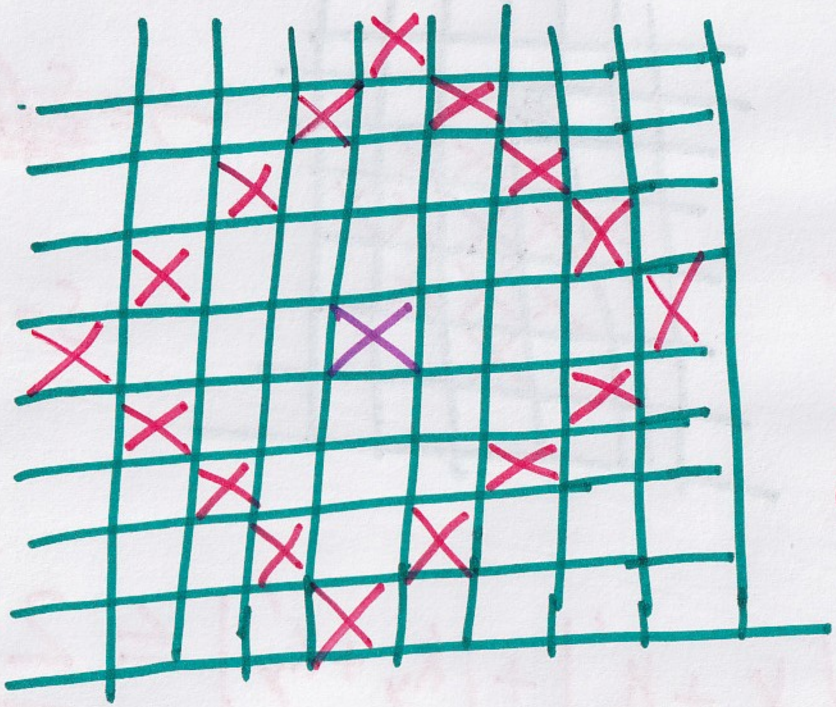
The perimeter seemingly shows that it does not change in length as get a close approximation to $\pi/4$

we know the $\int_0^{1/2} 1 + \frac{x}{(1/4 - x^2)^{1/2}} \, dx = 1$

8.)

8. In the city of Amazingville, city streets are placed in a grid system (i.e., only north, south, east, and west directions are directions of travel), with uniform lengths of each block. Dan is standing at a random spot in Amazingville. Where are all the locations that are three blocks away from Dan? What about n blocks away, for any $n \in \mathbb{Z}$? Describe, in words, all the locations, and also draw a picture!

We are not assuming diagonal travel



If Dan can only travel 3 adjacent spaces, The spaces marked 'X' are out of his reach.

→ All of the 'X' are 4 spaces (adjacent) away from Dan.

→ So for any $n \in \mathbb{Z}$, where n is the number of blocks away from Dan, All of the 'X' are $n+1$ spaces (adjacent) away from Dan

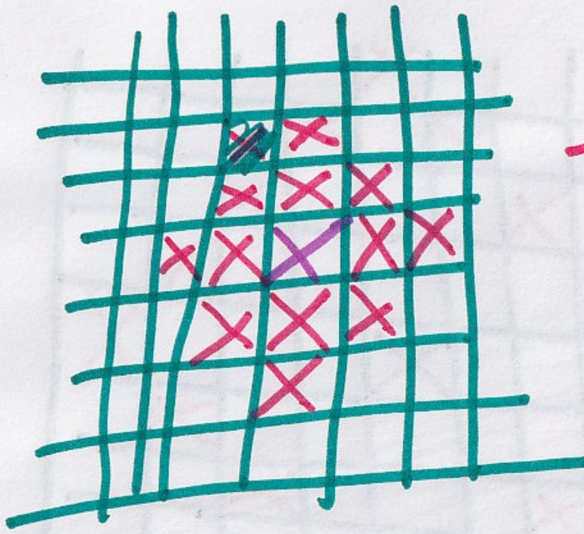
13

9.)

9. A king on a chess board can only move one space. If there were no other pieces on the board, what is the set of all spaces the king could move in two moves. What about in n moves, for any $n \in \mathbb{Z}$? Describe, in words, all the locations, and also draw a picture!

X = King

X, are all the move a king can do in 2 moves.



So the set is

~~$S = \{ (K_x \pm 2 |$~~

$S = \{ K_x \pm 2,$

So the set is

$$S = \{ | K_x + K_x' | + | K_y + K_y' | \leq 2 \}$$

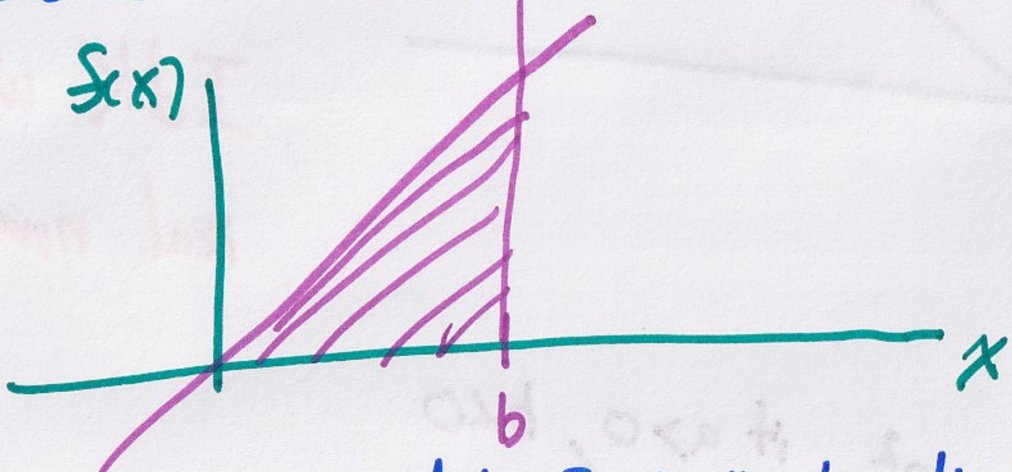
original position

New position

Let 2 be $\underline{n}$ for $\underline{n}$ moves, $n \in \mathbb{Z}$

10.) Consider the function $f(x) = x$ and Let a, b be Parameters that can change.

a) The figure enclosed by $f(x)$, the "positive x-axis" $(x \geq 0)$ and the line $x = b$ has what area?



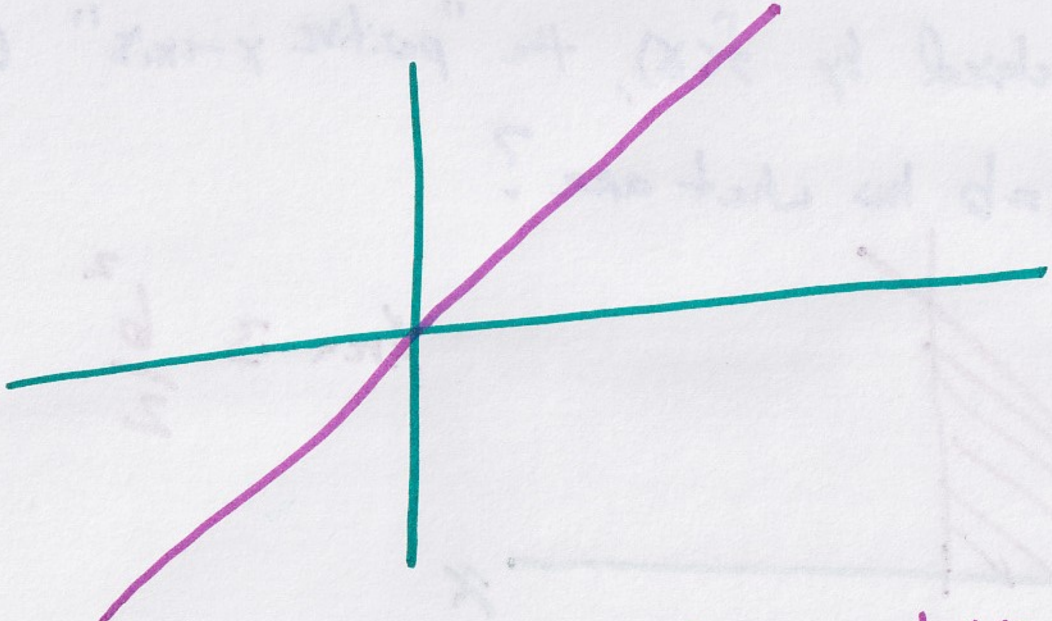
Area is $\frac{b^2}{2}$ for $b > 0$

b) The figure enclosed by $f(x)$, the 'negative x-axis" $(x \leq 0)$ and the line $x = b$ has what area?

$\frac{b^2}{2}$ only for $b < 0$, otherwise we don't have an enclosed area.

(15)

c) The figure enclosed by $f(x)$, the x-axis and two lines $x = a$ and $x = b$ have what area?



11.)

IdK what a real numbers is.

$$\text{Area is } \begin{cases} \frac{b^2}{2} + \frac{a^2}{2} & \text{if } a > 0,\ b < 0 \\ & \text{if } b > 0,\ a < 0 \\ \frac{b^2}{2} - \frac{a^2}{2} & \text{if } b > a > 0 \text{ or } b < a < 0 \end{cases}$$

$$\frac{a^2}{2} - \frac{b^2}{2} \text{ if } a > b > 0 \text{ or } a < b < 0$$