Fourier Series Stream

Youtube -@MRemark

Notes from Fourier Series by Georgi P. Tolstov

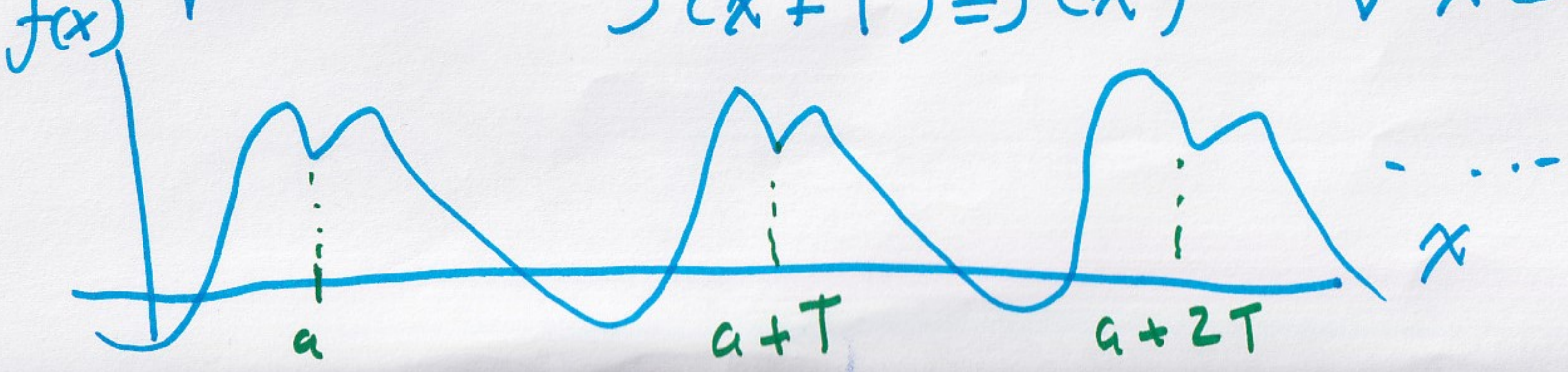
Part 1 Trigonomic Fourier Series

1) Periodic functions

$f: \mathbb{R} \to \mathbb{R}$

f is periodic if $\exists\ T > 0$ $T \in \mathbb{R}$

for which $f(x+T) = f(x)$ $\forall x \in \mathbb{R}$



②

~~*~~ if T is a period of f, $2T, 3T, 4T, \ldots$ are also periods of f

$$f(x) = f(x+T) = f(x+2T) = f(x+3T)$$

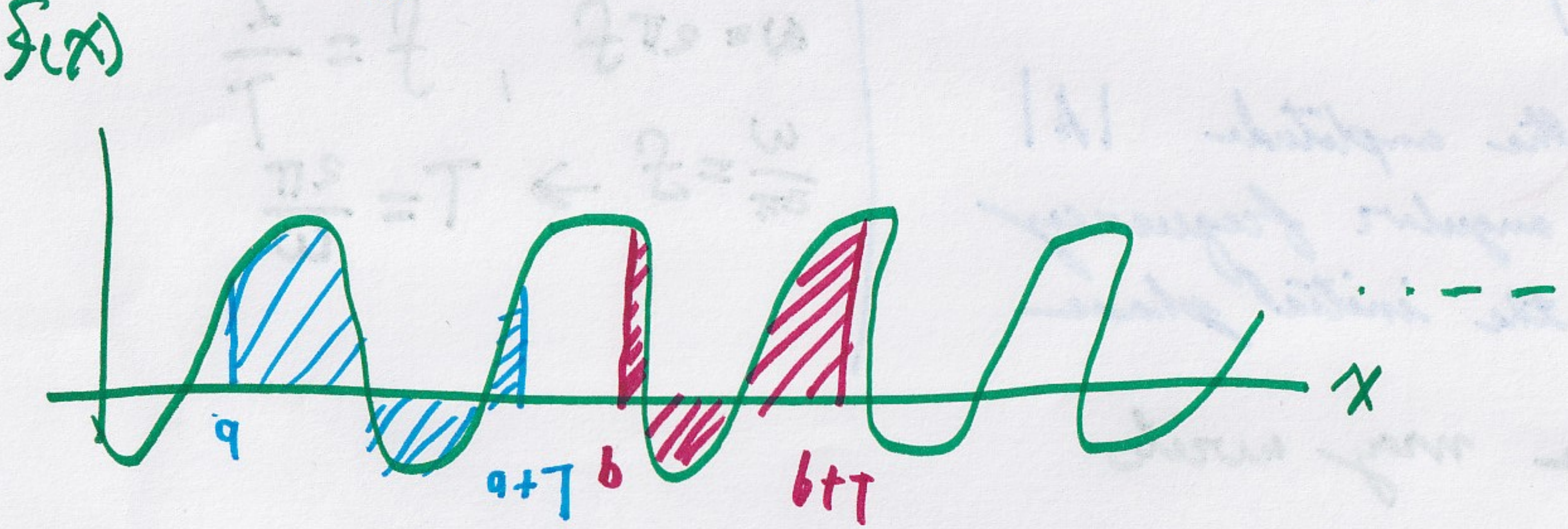
kT is a period $\quad k \in \mathbb{N}$

~~if a period exists~~ if $\exists\, T$, it is not unique

③

if $f(x)$ is integrable on any interval of Length T Then

$$\int_a^{a+T} f(x)\,dx = \int_b^{b+T} f(x)\,dx \quad \forall\, a, b \in \mathbb{R}$$



The areas are the same!

T is a fixed interval

4 2) Harmonics

$$f(x) = A\sin(\omega x + \phi)$$

A, ω, and ϕ are constants

A is the amplitude $|A|$

ω is angular frequency

ϕ is the initial phase

what is the period of this harmonic?

$\omega = 2\pi f$, $f = \frac{1}{T}$

$\frac{\omega}{2\pi} = f \rightarrow T = \frac{2\pi}{\omega}$

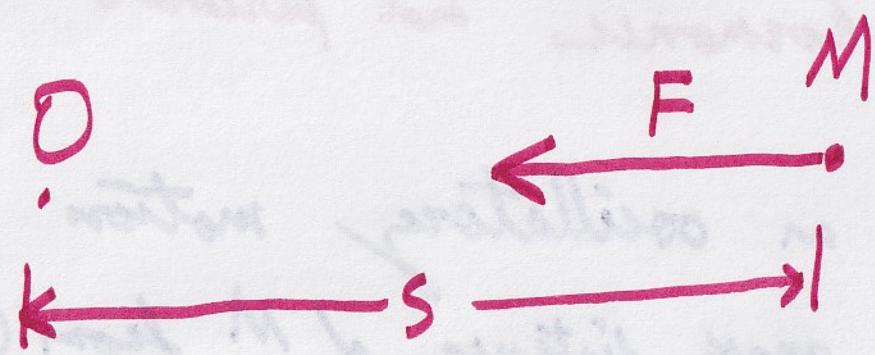
So we may write

$$A\sin\left(\omega\left(x + \frac{2\pi}{\omega}\right) + \phi\right) = A\sin(\omega x + 2\pi + \phi)$$
$$= A\sin(\omega x + \phi)$$

Simple Harmonic Motion

point mass M, with mass m

moves along a straight line under the action of restoring force F ~~with~~ which is proportional to the distance of M from a fixed origin O. That force is directed at O



$F = -ks$, $k > 0$ and is a constant of proportionality

$$F = ma \longrightarrow m\frac{d^2 s}{dt^2} = -ks \longrightarrow \frac{d^2 s}{dt^2} + \frac{k}{m}s = 0$$

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The way ~~authors~~ authors put it

$\omega^2 = \frac{K}{M}$, so $\omega = \sqrt{K/M}$

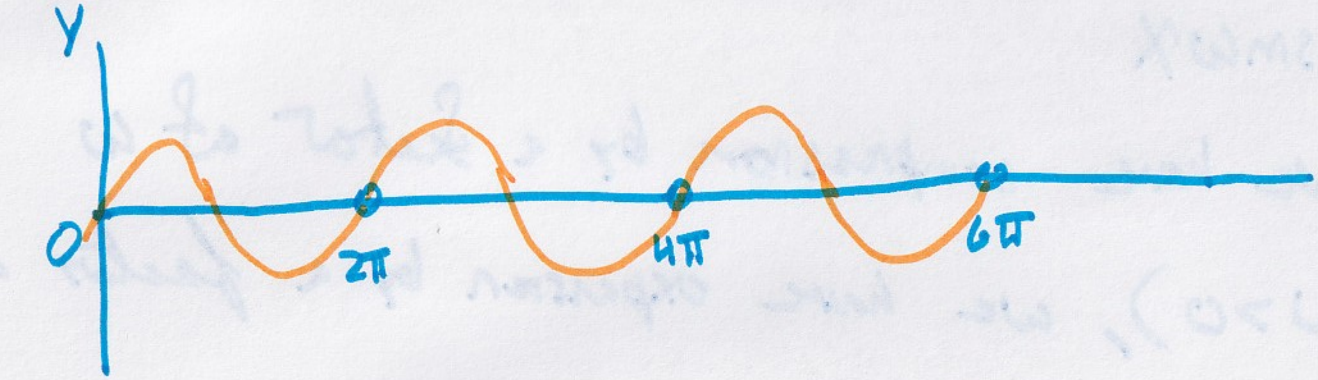
the solution is $s = A\sin(\omega t + \phi)$

A, ϕ can be known from I.Cs (position and velocity)

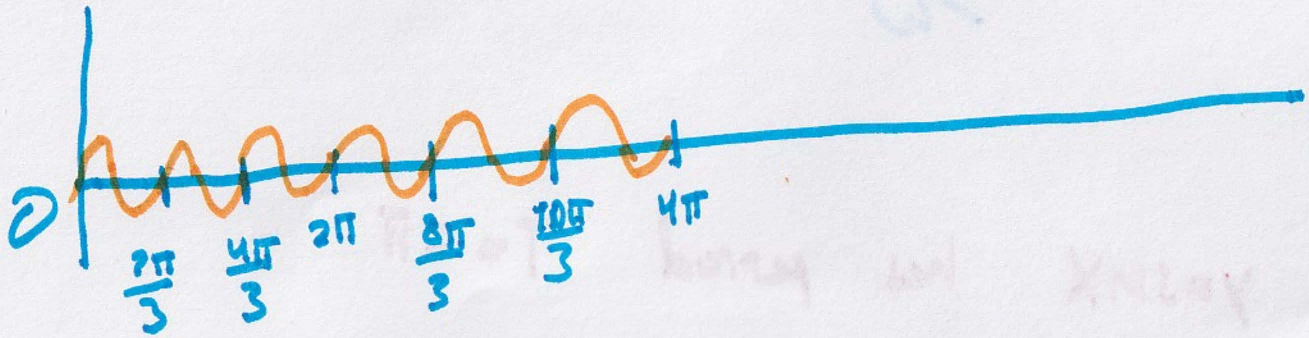
the solution is harmonic and periodic $\left(T = \frac{2\pi}{\omega}\right)$

M is undergoing an oscillatory motion

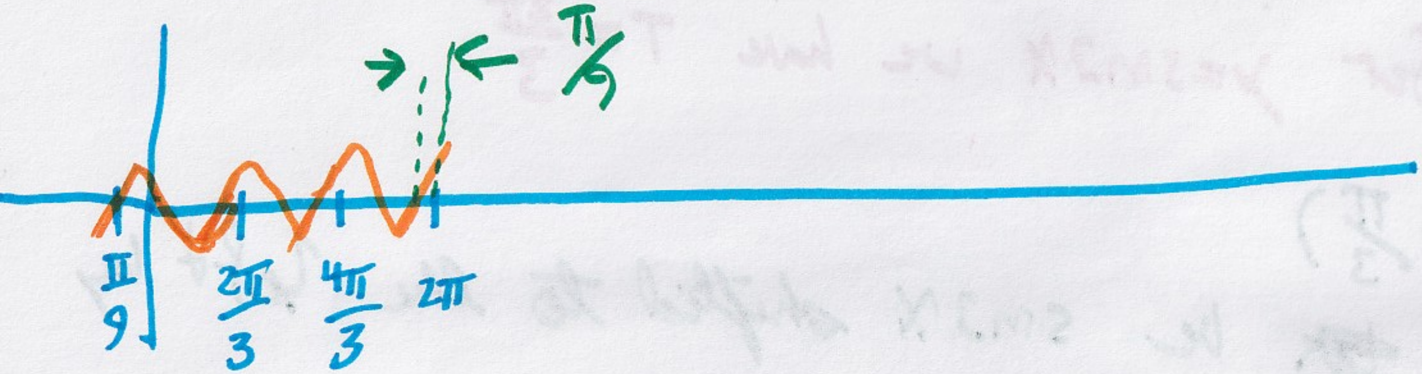
the value A is the max distance of M from O



$y = \sin x$



$y = \sin 3x$



$y = \sin(3x + \frac{\pi}{3})$

$= \sin(3(x + \frac{\pi}{9}))$

(8) consider $y = \sin \omega x$

for $\omega > 1$, we have compression by a factor of ω

for $\omega < 1$ $(\omega > 0)$, we have expansion by a factor of $1/\omega$

plot $y = \sin 3x$

we know that $y = \sin x$ has period $T = 2\pi$

$T = \frac{2\pi}{\omega}$ so for $y = \sin 3x$ we have $T = \frac{2\pi}{3}$

plot $y = \sin(3x + \frac{\pi}{3})$

Answer: it will ~~to~~ be $\sin 3x$ shifted to the left by $\pi/9$

$\sin(3x + \frac{\pi}{3})$

Think $\delta(x) \begin{cases} 1 & x = 0 \\ 0 & \forall x \neq 0 \end{cases}$

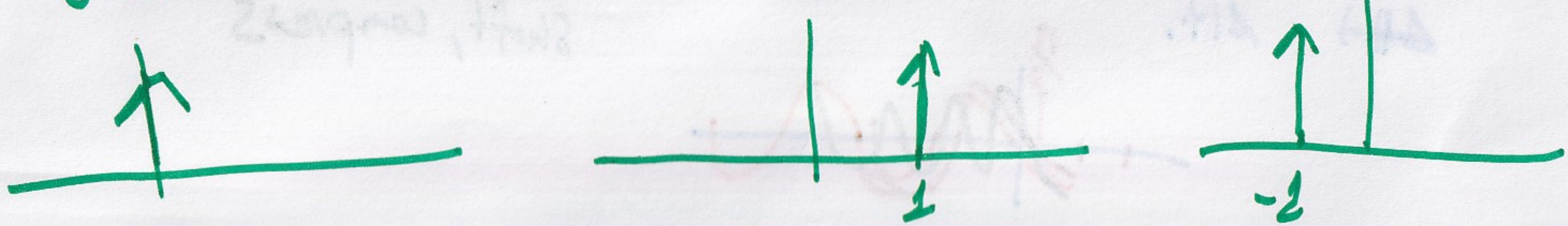
$\delta(x-1) \begin{cases} 1 & x = 1 \\ 0 & \forall x \neq 1 \end{cases}$

$\delta(0) = 1$

$\underset{\uparrow}{\sim\sim\sim} = 0$, $x - 1 = 0$?, $x = 1$

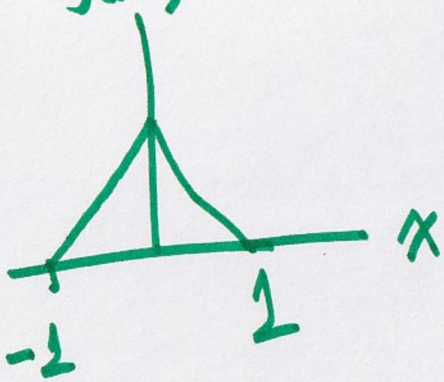
$\delta(x+1) = \begin{cases} 1 & x = -1 \\ 0 & \forall x \neq -1 \end{cases}$

$\delta(x)$ $\delta(x-1)$ $\delta(x+1)$



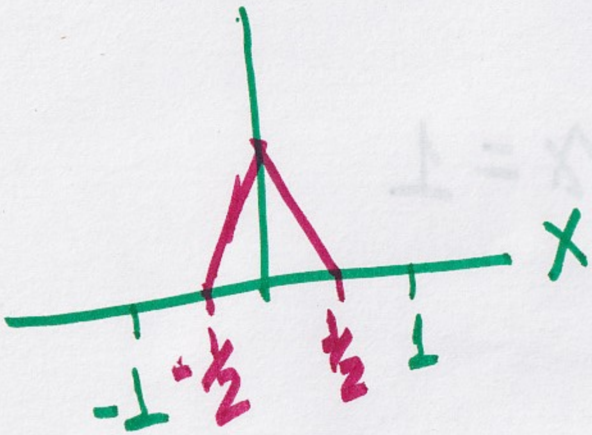
(10)

$f(x)$

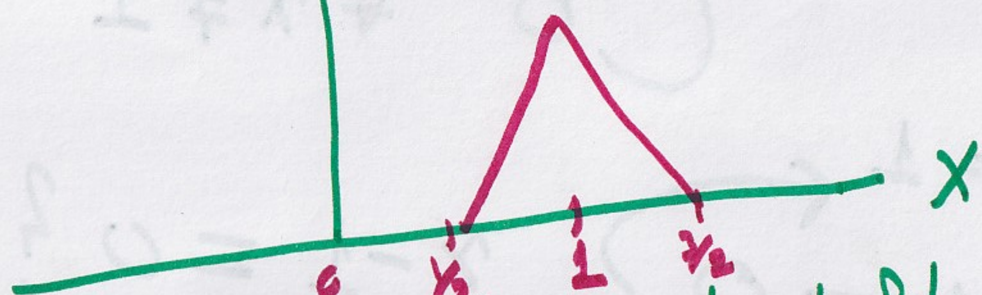


$f(x-1)$

$f(2x)$



$f(2(x-1))$



full proof

Scale, shift, flip at shift

$u[-k+n]$

$\sin(3x + \frac{\pi}{3}) = \sin(3[x + \frac{\pi}{9}])$

~~Alt~~ Alt.

shift, compress

plot $y = 2\sin(3x + \pi/3)$

its just $\sin(3x + \pi/3)$ w/ Amplitude of 2

from trigonometry

$$A\sin(\omega x + \phi) = A\left[\cos(\omega x)\sin\phi + \sin(\omega x)\cos\phi\right]$$

$$a = A\sin\phi, \quad b = A\cos\phi$$

So we say: Every harmonic can be represented in the form

$$a\cos(\omega x) + b\sin(\omega x)$$

from $a\cos\omega x + b\sin\omega x$, find A

$$A = \sqrt{a^2+b^2}$$

$$\sin\phi = \frac{a}{A} = \frac{a}{\sqrt{a^2+b^2}}$$

$$\cos\phi = \frac{b}{A} = \frac{b}{\sqrt{a^2+b^2}}$$

what's a and b for $2\sin(3x+\frac{\pi}{3})$?

$$a = 2\sin(\pi/3)$$

$$2\left(\frac{\sqrt{3}}{2}\right)$$

$$= \sqrt{3}$$

$$b = 2\cos(\pi/3)$$

$$= 2\left(\frac{\sqrt{1}}{2}\right)$$

$$= 1$$

so $\sqrt{3}\cos(\omega x) + \sin(\omega x)$ $\qquad \omega = 3$