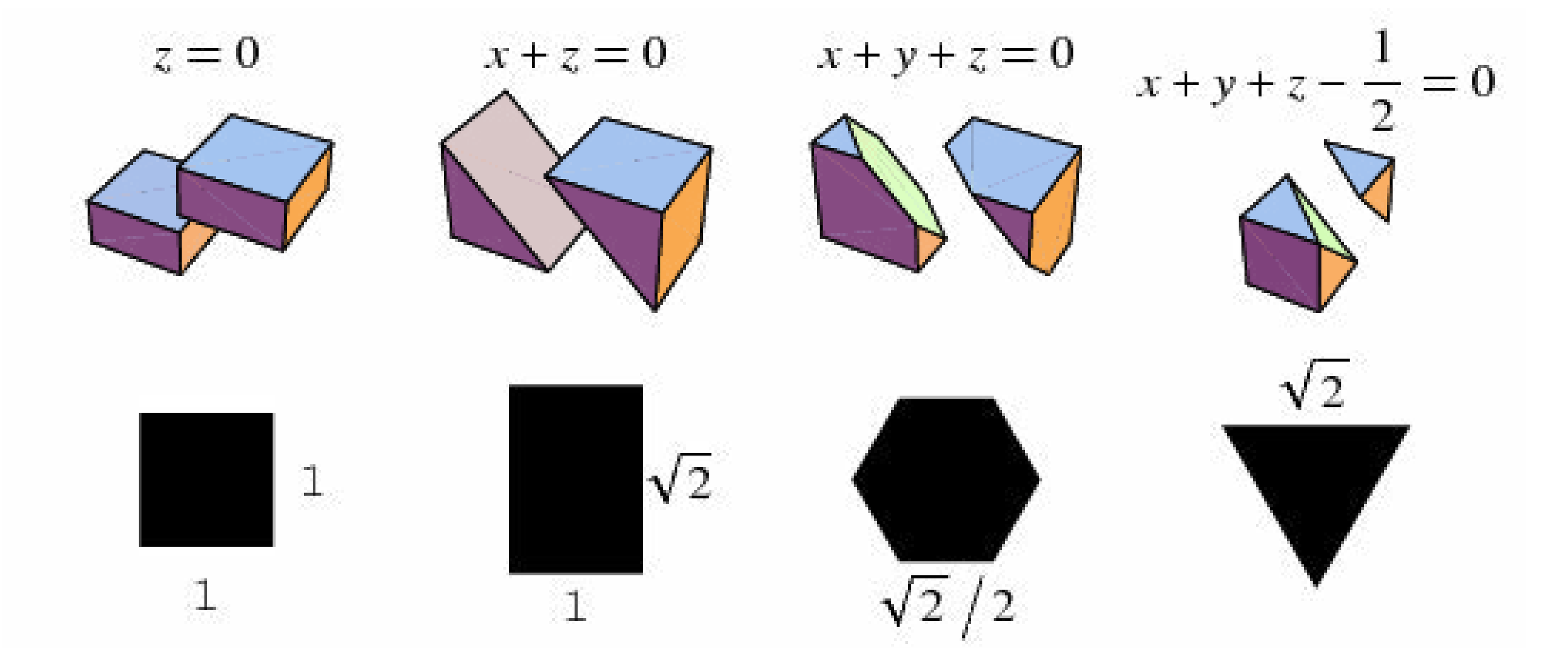
THE CUBE PROBELM

Seeing vs. Believing: What can students' approaches to the cube problem tell us about their attitudes toward modeling as a form of mathematical problem solving?

Morgan Rae Reschenberg | Education 130

ABSTRACT

When I presented my lesson on the cube problem, I was surprised to see that none of the students reached for the modeling materials and very few wanted to discuss their approaches with their group (unless to settle a dispute over the "correct" way to solve the problem). Students didn't try to visualise the problem and instead tended toward algebraic and geometric representations, often ignoring or struggling to conceptualise simpler facets of the problem.



PROBLEM OVERVIEW

The cube problem presented students with two questions regarding the shape of the cross section of a cube in different orientations.

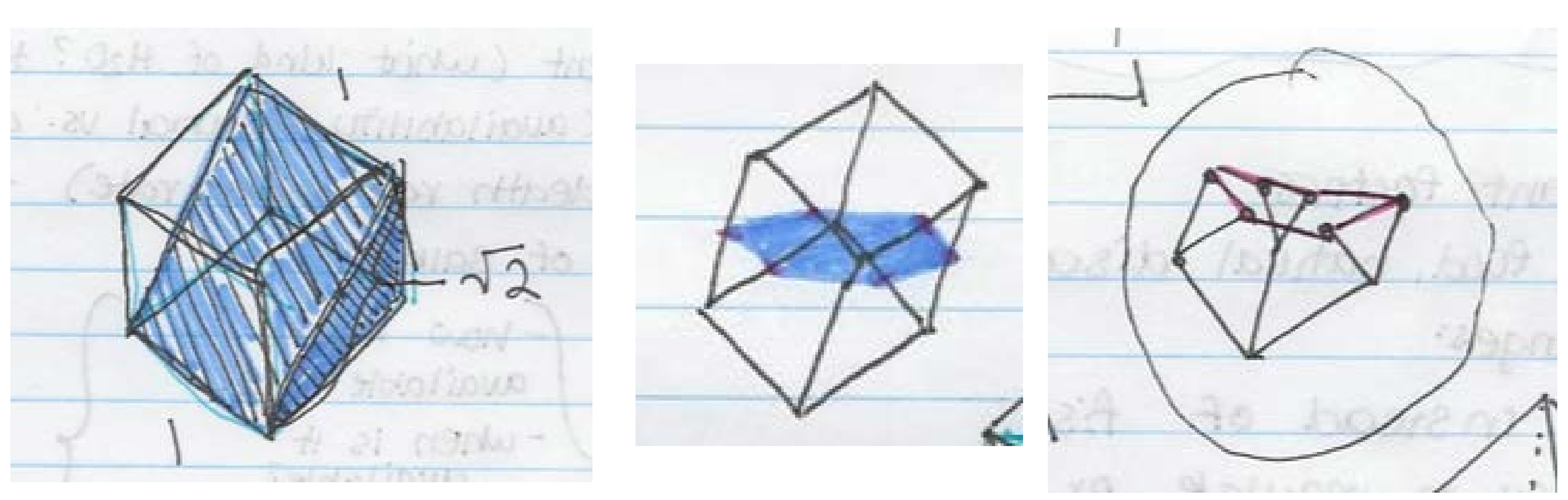
"Imagine a cube half filled with water lying flat on a table: Tilt the cube so it rests on one of its edges. Working with your group, try to determine what shape is made by the surface of the water."

"Now, tilt the cube so it is balancing on one point. What shape is made by the water?"

I chose this problem because my students had been working with finding conic sections and representing them algebraically. I presented this problem in the context of "cubic sections" and expected students to use some of their prior knowledge from conics to solve this problem.

MATERIALS

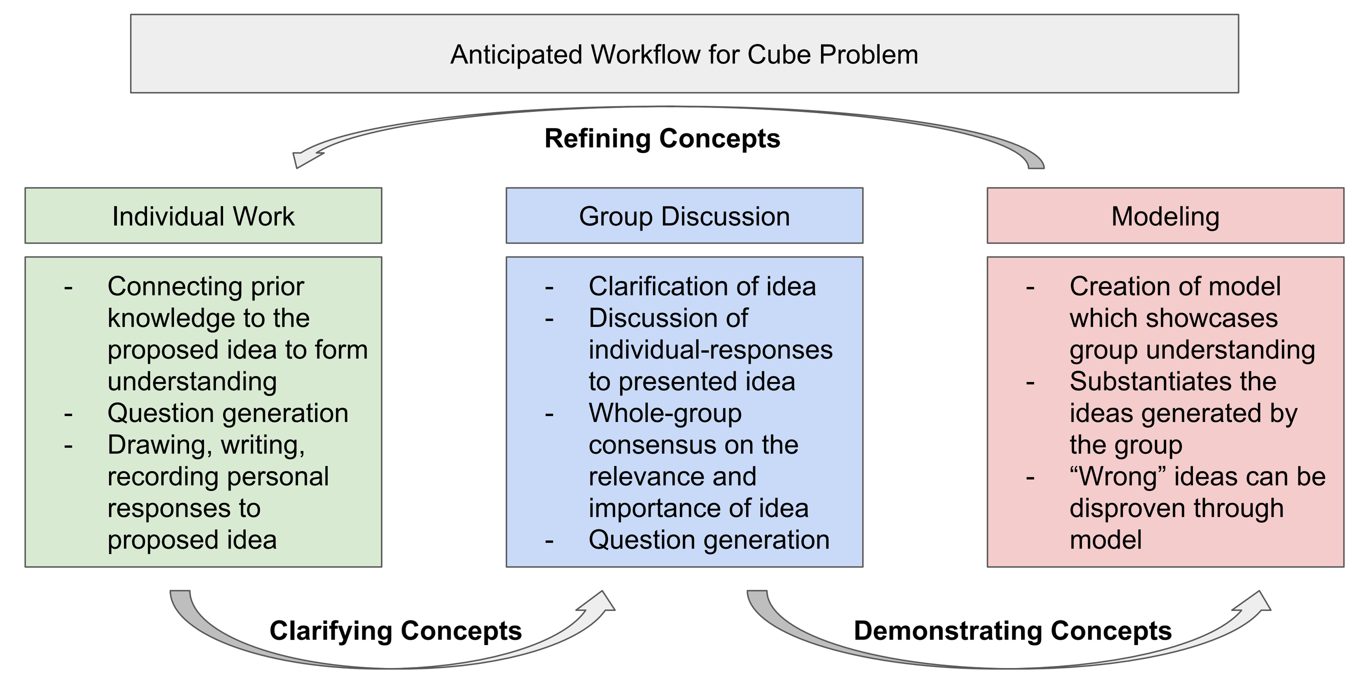
Playdough, paper, pens/pencils, plastic knives, string, rubber bands, wooden cube models



[Examples of expected student drawings during question one (left) and question two (centre, right)]

PROBLEM PRESENTATION & PROCEDURES

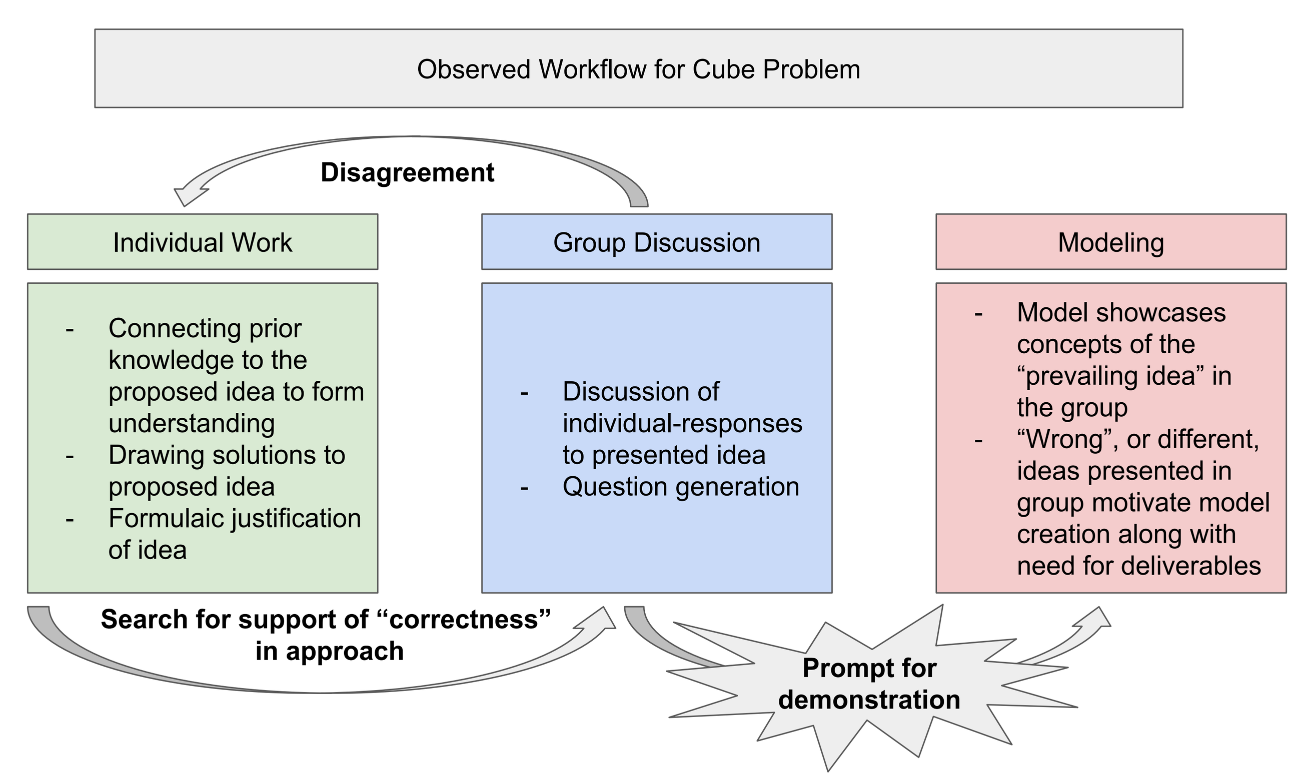
After discussing, drawing, and solving the first problem on their own, students presented their solutions in groups on the board. I read the second question aloud and instructed students to discuss. While probing for guesses from the groups, I passed out wooden cubes to help the students visualise the problem. We regrouped as a whole class and I gave each group access to a variety of materials letting the students know that they could choose to use, or not use, whatever they liked. Groups who worked quickly were asked to find the area of the shape or to demonstrate their solution another way. Because this lesson was given in the context of cubic sections, students were also challenged to discover other cubic sections. At the end of the lesson, groups each elected a representative to talk about the way they approached the problem and the conclusions they were able to draw.



EXPECTED OUTCOMES

Because the students' work with cubic sections had relied on models their teacher had brought in, I also expected they'd gravitate towards the available materials to demonstrate their thinking. In the process of solving, I expected to see students first draw out their solutions while discussing as a group before moving on to more complex modeling. With the materials I brought, I was anticipating a lot of 3D cube modeling, cutting away of cubes (playdough cubes, pipe-cleaner cubes), and attempts to "fit" paper or other materials into the cube models to find the cross section shape.

I expected students to correctly conclude that, in the case of the edge-balanced cube, the water creates a rectangle with side lengths 1 and sqrt(2) for a total area of sqrt(2). In the second problem, I also expected students to correctly conclude that the shape generated was a regular hexagon, but I did not expect students to have enough time to work out the area of the hexagon--my class period was shorter than usual and I anticipated having to deal with a lot of classroom management issues with regards to chatty students.

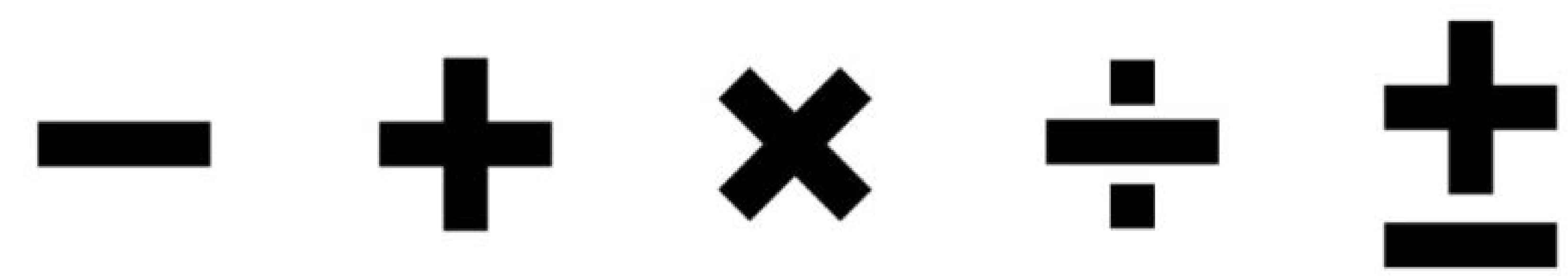


RESULTS

Studentsmade no initial moves to use the materials provided. I had to go around to each group and ask them what they'd like to use (rather than if they'd like to use materials, as I'd previously done). I was interested to see how the materials students chose affected the conclusions they drew, but no one was interested in using materials at all. Instead, they relied heavily on the dimensions of the cube and wrote equations, used special triangles, and altogether relied very heavily on their math backgrounds to solve the problem rather than the visualisation and modeling techniques they'd used in class before.

Students gave complex mathematical justification before solving more rudimentary parts of the problem. When I went around to collect guesses from each group, students were very hesitant to respond. I couldn't get any group to give me a guess--they all wanted more time to discuss and work out the problem. Instead of focusing on figuring out the shape generally, they focused on finding the area of the shape. In one case, a group found the area before determining what the shape was. Because the first problem we went over together resulted in a rectangle, the group assumed I'd set them up to work with a similar shape in this problem.

Students were more satisfied with their algebraic results than my presentation or "visual proof". Though the demonstration with the water cube at the end was interesting to some students, it failed to engage those who had already, by their standards, sufficiently proven that the cross section of the cube was a hexagon. Again, students approached the problem from a math context and assumed the proof I asked for had to be rigorous and algebraic or geometric rather than visual.



CONCLUSION AND REFLECTION

Despite introducing materials for modeling and connecting the lesson to prior knowledge in which modeling was used, students were more comfortable relying on algebraic and geometric representations. From my observation data, it is apparent that there is not enough integration of visualisation and non-formulaic proofs in the classroom to make students comfortable employing modeling as a mathematical problem solving technique. The observations made in my classroom replicate the way many believe math is taught as a whole: without emphasis on student-driven exploration of concepts, and instead with a focus on formulaic explanation.

In the case of the group which determined the area of the cross section before the shape of the cross section, students demonstrated they have a propensity for "doing school" rather than "doing math". That is, they know the problem has classroom context: it was preceded by other problems which framed the steps they should take and clearly highlighted a particular end goal. When I asked why the group had concluded that the cross-section in the second problem was a square, they referenced the previous problem. They approached this problem within the context of a math class; they were primed for doing math in their classroom's context and modeling did not fit into that conception.

In the future, to promote discussion, modeling, and critical thinking, I will give a preamble to each problem question and try to situate it in a story-based context rather than a pure-math context. The connections to conic sections was helpful for some students as a reference point for approaching the problem, but overall I believe this tie-in and the conception of learning in their classroom environment contributed to the results I saw: lack of modeling, lack of productive discussion, and tendency to justify formulaically rather than visually.