UCL
SCHOOL OF
MANAGEMENT

UCL

MSIN0095: Operations Analytics

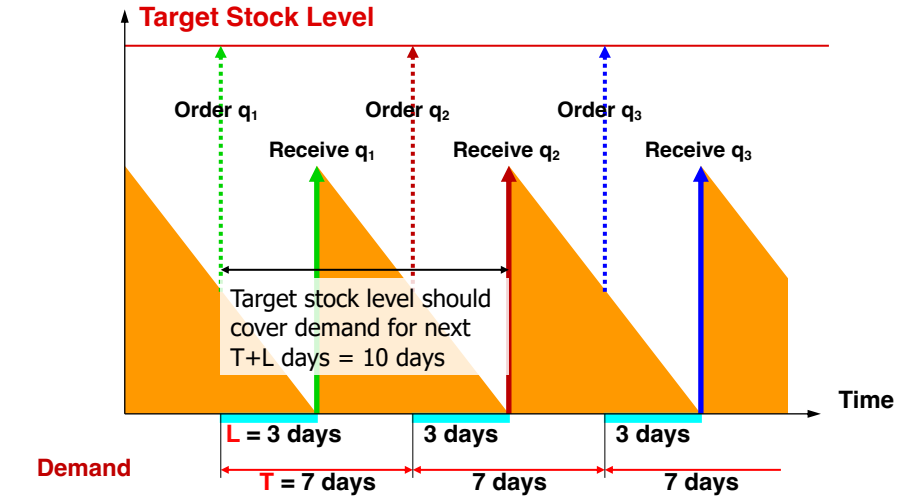
Class 1-4: Process Analysis

Class 5,7: Waiting Time Analysis

Class 6: Inventory Management I: Newsvendor Model

Class 8: Inventory Management II: Newsvendor 2 and Replenishable Inventory

Periodic Review: Deterministic Demand

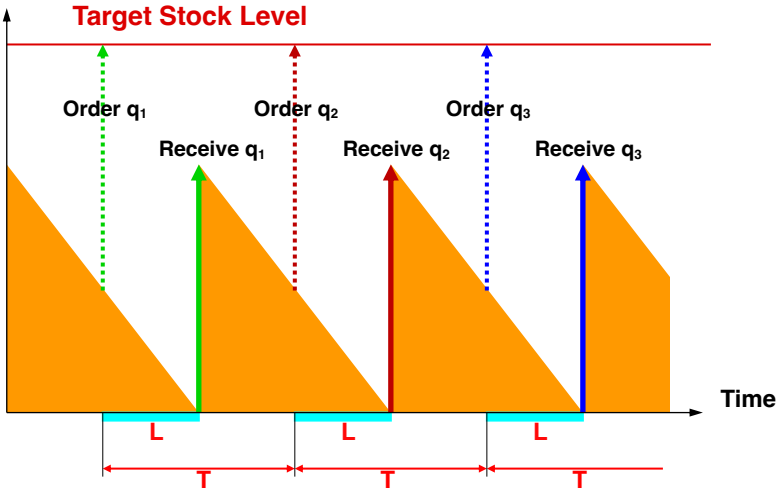


Demand
= 10 units/day

What is the best Target Stock Level?

(a) 30 (b) 70 (c) 100 ✓

Periodic Review: Deterministic Demand



If demand is certain, Target Stock Level =

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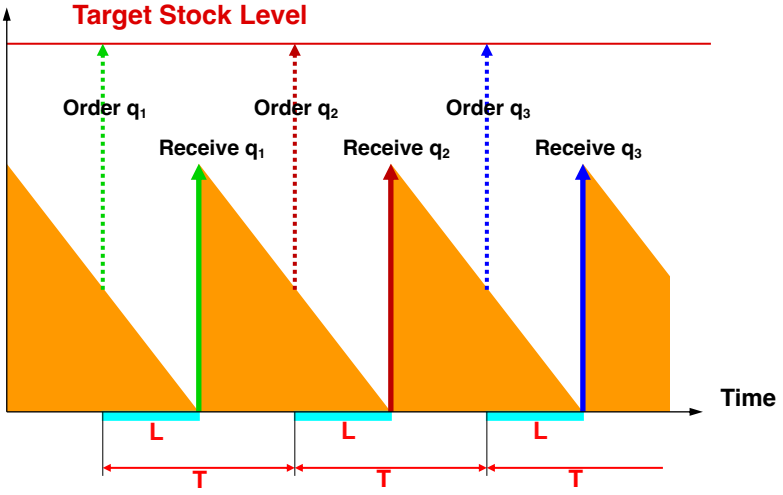
Periodic Review: Deterministic Demand



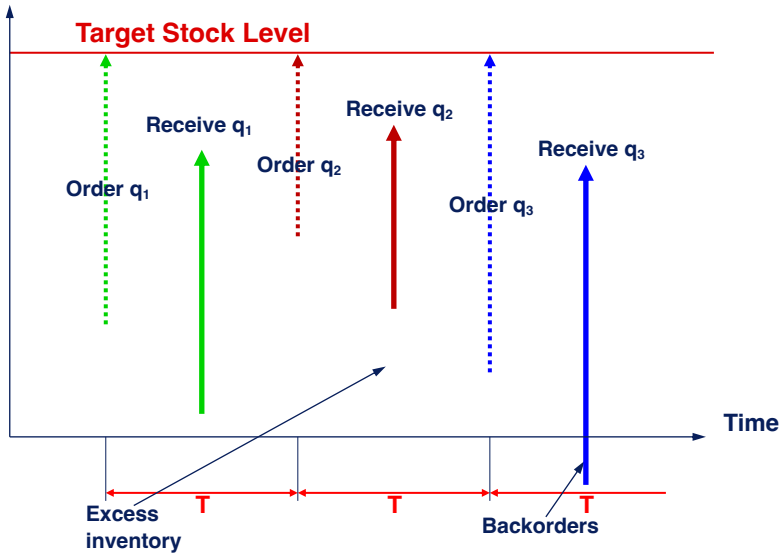
If demand is certain, Target Stock Level = Demand during T+L

4

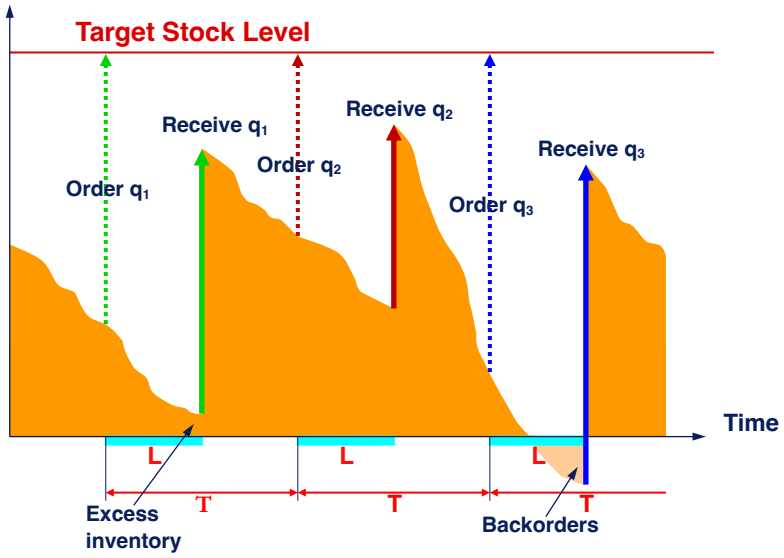
Periodic Review Model

- What if demand is **random**?
  - At the time of review, you may have either **stockouts** or **excess inventory**
  - Excess inventory carried over to the next period (pay holding cost)
  - If stockouts occur, demand is backordered and met when the next shipment comes in (pay backorder cost)
- How to set the right **target stock level**?

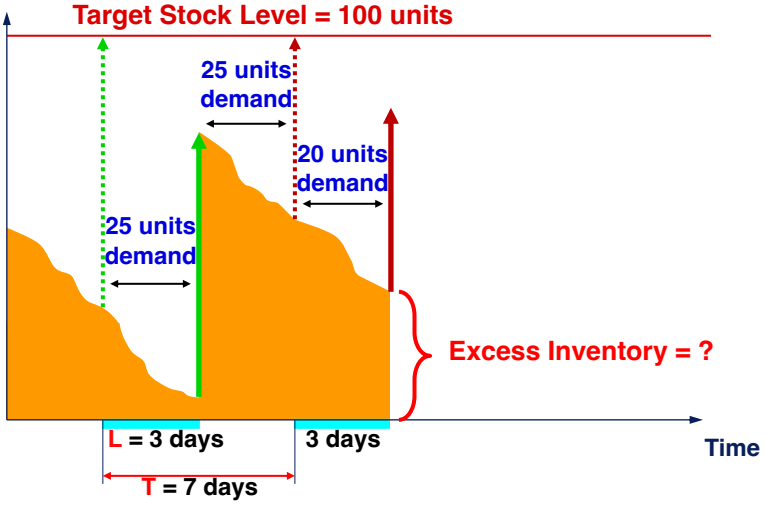
Periodic Review Illustration



Periodic Review Illustration

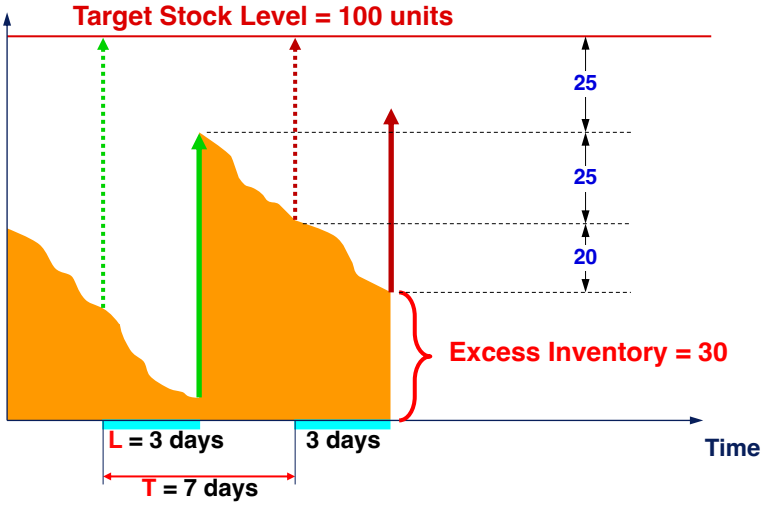


Periodic Review: Overstocking Example



(a) 25 (b) 30 (c) 50

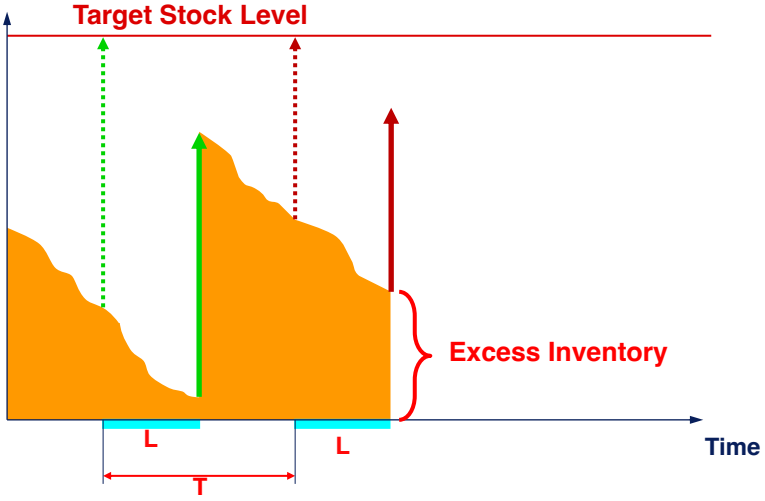
Periodic Review: Overstocking Example



(a) 25 (b) 30 ✓ (c) 50

9

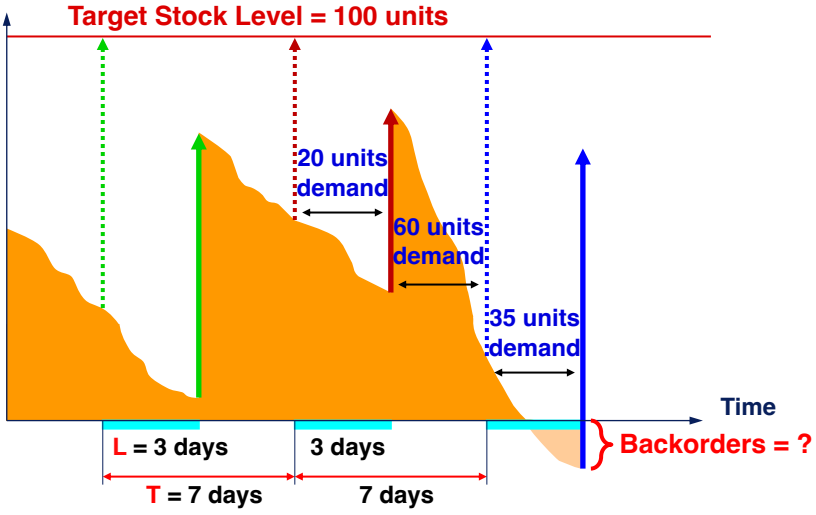
Periodic Review: Overstocking



Excess Inventory = Target Stock Level – Demand during T+L

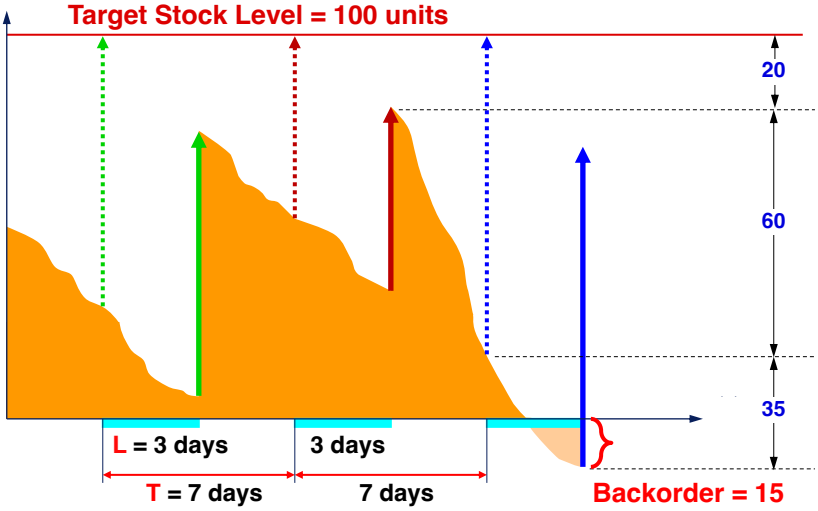
10

Periodic Review: Understocking Example



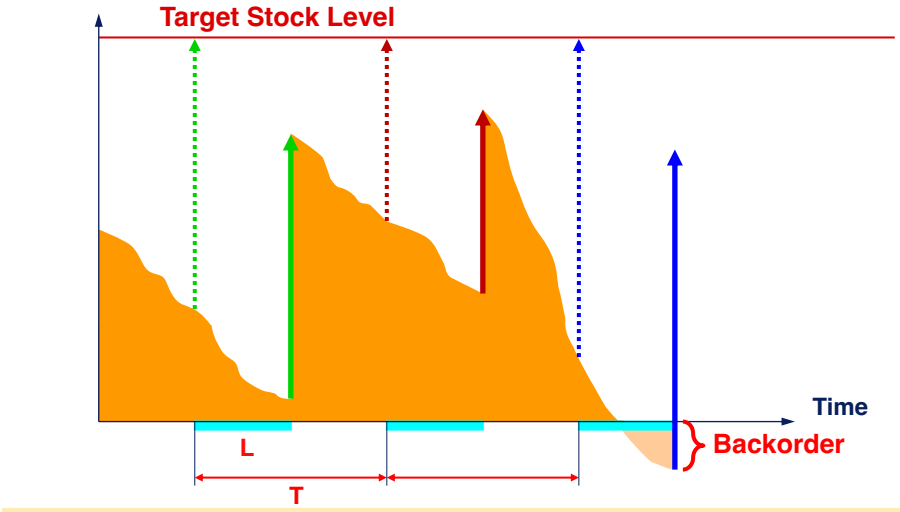
(a) 15 (b) 25 (c) 35

Periodic Review: Understocking Example



(a) 15 ✓ (b) 25 (c) 35

Periodic Review: Understocking



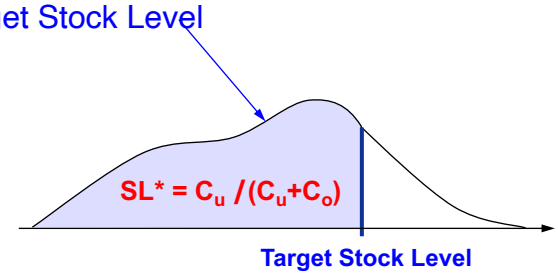
Backorders = Demand during T+L – Target Stock Level

Extending Newsvendor Logic

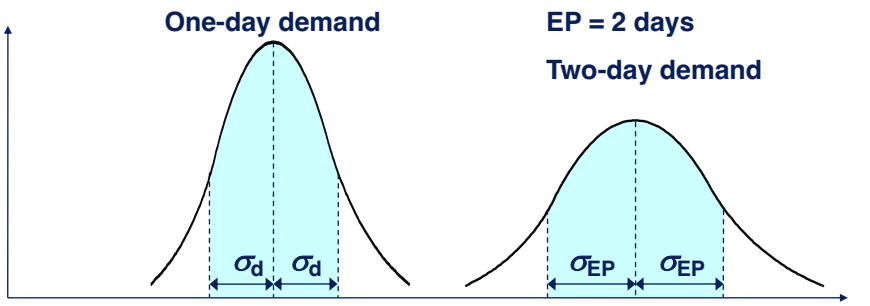
- Overstocking: **Excess Inventory!**
  Target Stock Level > Demand during T+L
- Understocking: **Backorders!**
  Target Stock Level < Demand during T+L
- Analogous to the Newsvendor Model:
  - Target Stock Level ~ Rocky's starting inventory
  - Demand during T+L ~ Daily newspaper demand
  - Newsvendor chooses starting inventory to serve demand during a day ~ In periodic review, you select Target Stock Level to serve demand during **T+L** (**Exposure Period**)

Applying Newsvendor Logic

- C_o = Cost of overstocking a unit
  = Cost of holding one unit over review period T
  C_u = Cost of understocking a unit
  = Backorder cost, goodwill loss, etc.
- Service level (Critical ratio)= $C_u / (C_u+C_o)$
- Find distribution of demand during Exposure Period T+L
- Find the best Target Stock Level



Finding Distribution of D_{EP}



$\sigma_d = 3$

$\sigma_{EP} = 6$? No

$\sigma_{EP} = 3\sqrt{2}$

Daily demand variability σ_d
Exposure period EP (days)
Assume independent daily demand

$\Rightarrow$ $\sigma_{EP} = \sigma_d\sqrt{T+L}$

Motivation Example

- Zara orders a particular clothing item every Friday morning and it is delivered from a warehouse Monday morning
- Unmet demand is lost
- Mean daily demand is 10, standard deviation is 9.45
- Daily demand are independently and identically distributed
- Wholesale cost is $10 each, retail price is $25 each
- Holding cost has been set at $0.5 per week for each item (to reflect obsolescence, damage, etc.)
- **Question: How should Zara set order amounts?**

Look at demand during T+L (Exposure period)

Safety Stock
= Target stock level – average demand during T+L 17

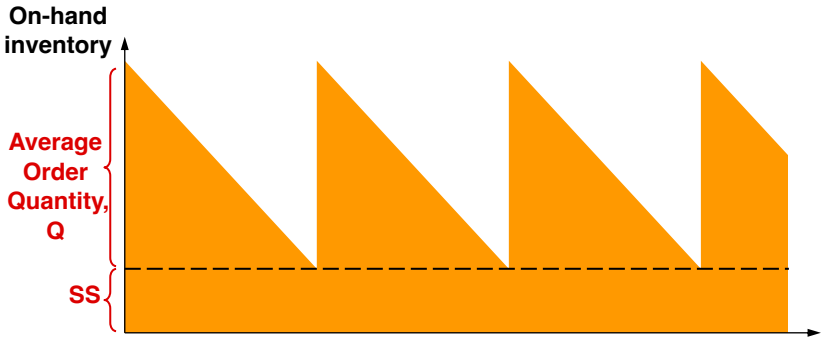
17

Target Stock Level and Safety Stock

	D_{EP} is normally distributed, with Mean=μ_{EP}, Stdev=σ_{EP}	D_{EP} has discrete distribution
Target Stock level	$\mu_{EP} + z\sigma_{EP}$, where z= **NORMSINV** $\left(\frac{C_u}{C_o+C_u}\right)$	Lowest inventory such that: $P(D_{EP} \leq \text{Target}) \geq \frac{C_u}{C_o + C_u}$
Average Safety Stock	$z\,\sigma_{EP}$	Target stock level $-\ \mu_{EP}$
Order Quantity	= Target stock level – Net inventory level = Total demand of last review period (T days) Where Net Inventory Level = on−hand inventory + pipeline inventory – backorders	

18

In the Long Run…Average Inventory Profile



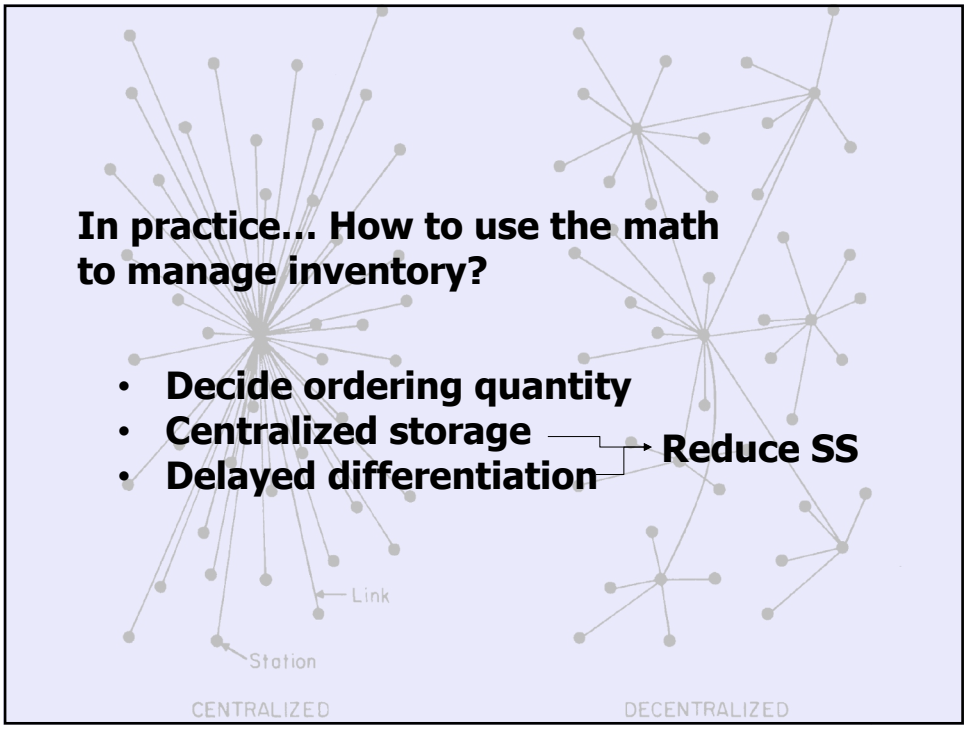
Average Inventory = Average Cycle Stock + Safety Stock (+ Pipeline inventory)* = Q/2 + SS (+ mu$_D$ L)*

- **Periodic review system: Q = mu$_D$ T**

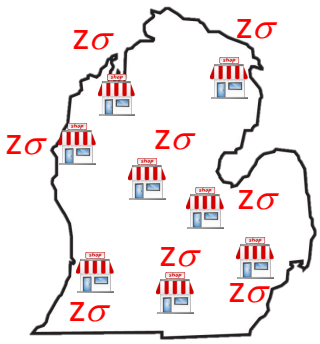
* **Include pipeline inventory if you own it**

Average inventory profile

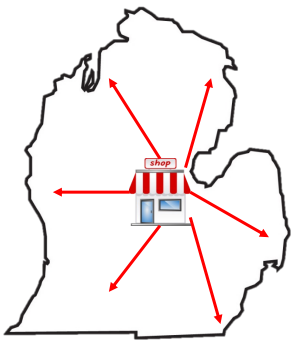
Inventory Type	Definition	Average amount
Order quantity	Placed every T periods. On average, firm needs to replace the inventory sold during the last review period (T).	$\mu_D T$
Pipeline inventory	Items in transit. Owned by firm only if supplier is paid when placing order.	$\mu_D L$
Cycle inventory	Portion of on-hand inventory the firm cycles through to satisfy regular demand (excluding safety stock)	$\frac{1}{2}\mu_D T$
Safety stock	Extra stock maintained to mitigate risk of stockouts	SS
Total on-hand Inventory	Inventory located physically within the firm	$\frac{1}{2}\mu_D T+\text{SS}$
Total inventory	*Includes pipeline inventory only if firm owns it	$\frac{1}{2}\mu_D T+\text{SS}+(\mu_D L)*$



Reducing safety stock by Inventory Pooling



k*zσ



z*std? $=z\sqrt{k}\,\sigma$

Practice: Centralized Storage

- (Decentralized System) Four locations, each experiences daily demand N(100, 50). Inventory is reviewed at each location every T = 6 days. L = 3 days. What is the safety stock and target stock level? (Use z=2 for 98% SL)

- (Centralized System) If you serve demands from a central warehouse. What is the safety stock and target stock level?

After-class practice: Centralized Storage

- (Decentralized System) Four locations, each experiences daily demand N(100, 50). Inventory is reviewed at each location every T = 6 days. L = 3 days. What is the safety stock and target stock level? (Use z=2 for 98% SL)

Safety stock at each location: $SS = z * \sigma_{L+T} = 2 * 50 * \sqrt{3+6} = 300$

Target at each location = d * (L+T) + SS = 100 * 9 + 300 = 1200

Total SS = 4 * 300 = 1200

Total Target = 4 * 1200 = 4800

- (Centralized System) If you serve demands from a central warehouse. What is the safety stock and target stock level?

Avg. total daily demand = 4 * 100 = 400

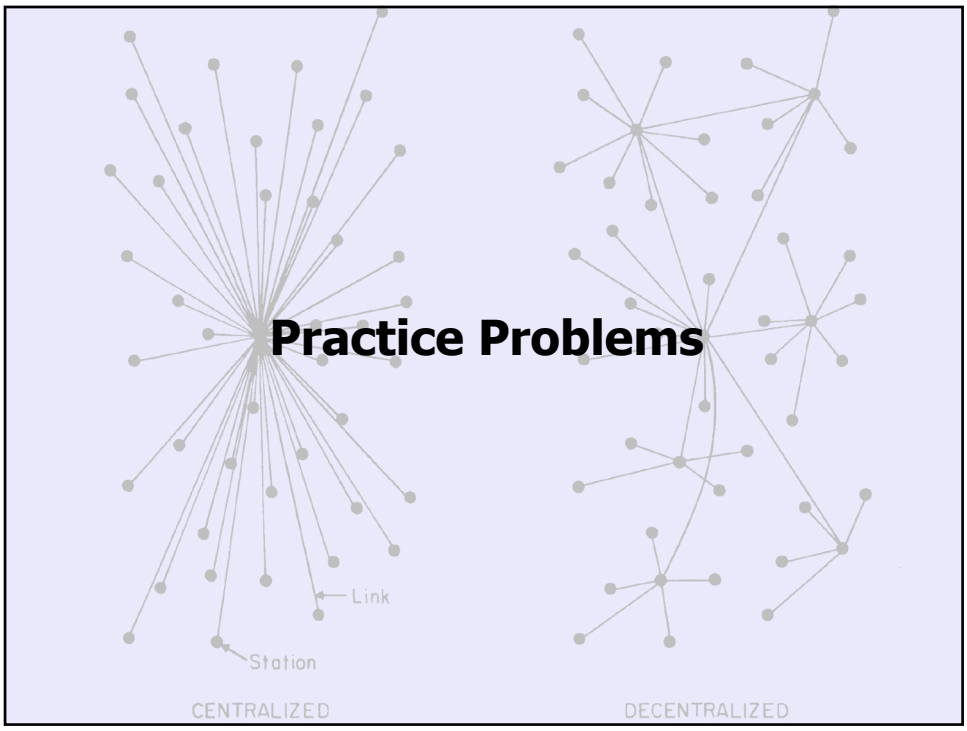
Std.dev. of total daily demand $= 50 * \sqrt{4} = 100$

$SS = z * \sigma_{L+T} = 2 * 100 * \sqrt{3+6} = 600$

Total Target = d * (L+T) + SS = 400 * 9 + 600 = 4200

Reducing safety stock by Delayed Differentiation

- **Situation**:
  - *k* products with independent demands
  - Each product is a variant of the same model (e.g. color). Each product has EP demand = Normal (m, σ)
- **Option 1**: Keep inventory of final products
  - Safety stock for each product = zσ
  - Total safety stock for *k* products = $k\,z\sigma$
- **Option 2**: Keep inventory of base model, then postpone final differentiation
  - Total EP demand is normal ($km, \sqrt{k}\sigma$)
  - Safety stock for base model (for same service level) $= \sqrt{k}\,z\sigma$



Practice Problem 1: Periodic Review

Ann Arbor Automotive sells the popular EX3 model. AAA receives a shipment at the start of each month. Lead time is negligible. Monthly demand for the EX3 is distributed as N(60, 15). The average cost of holding an EX3 for one year is $1500. In case of a shortage, customers are willing to wait, but there is a cost of extra bookkeeping of $75 per customer and a loss-of-goodwill estimated to be $600 per customer. How many cars should AAA order up to every month (i.e., Target Stock Level)?

c_u = 600+75=675

c_o = 1500/12=125

SL* = 675 / (675 + 125) = 0.8438

z = NORMSINV(0.8438) = 0.9945

Target Stock Level = 60 + 15×0.9945 = 75 = Avg EP demand + SS

Practice Problem 2: Periodic Review

The exposure period (T+L) demand for microwave ovens is distributed according to the following probability distribution:

Demand	6	7	8	9	10	11	12	13	14
Probability	0.02	0.03	0.1	0.24	0.4	0.15	0.03	0.02	0.01

Using a periodic review model and a 95% service level, what should the target stock level be? What is the safety stock?

Prob (Demand during T+L ≤ 13) = 0.99 ← Unnecessarily high

Prob (Demand during T+L ≤ 12) = 0.97← Best Target stock level =12

Prob (Demand during T+L ≤ 11) = 0.94 ← Cannot meet service level

Avg. exposure period demand = 6*0.02+7*0.03+…+14*0.01 = 9.7

SS = Target stock level – Avg. exposure period demand = 12 – 9.7 = 2.3

Practice Problem 3

Suppose Benetton sells 16 different colors of a particular sweater, each of which is forecasted to have the same demand. Currently, they carry 1,000 sweaters of safety stock for each color. Total safety stock is 16,000 sweaters. If they were to move to a dye-to-order strategy, so that stock is held in the form of undyed sweaters, how much safety stock would they need to achieve the same level of service level?

Current (per color):

$SS = z\sigma_{\text{dyed}} = 1000$

Delayed Diff. (total): $\sigma_{\text{undyed}} = \sqrt{k}\sigma_{\text{dyed}}$

$SS = z\sigma_{\text{undyed}} = z\sqrt{k}\sigma_{\text{dyed}}$

$= \sqrt{16}\times 1000 = 4000$

Next class

- Beer Game. Don't miss it!