Practice 5

Consider two small food services at an airport. Assume that both of them have a customer arrival stream with an average interarrival time of 4 min and follows a Poisson arrival pattern. The processing time is 3 min per customer and the service time follows an exponential distribution.

(a) What is the long run average waiting time for each food service?

a = 4min Cva=1, p = 3min, CVp = 1, m= 1

(b) Now two food services want to collaborate with each other. The capacity of the pooled services is the sum of each food service. (Note that the service time for a single customer remains unchanged.) The customer arrivals follow the original distribution but they will wait in a single queue to be served. What is the long run average wait time? Rp = 2/3 cust/min, p = 3min, Ra = 2 * ¼ = ½ /min, a = 1/Ra = 2min, m = 2

UCL
SCHOOL OF
MANAGEMENT

MSIN0095: Operations Analytics

Class 1-4: Process Analysis

Class 5, 7: Waiting Time Analysis

Class 6: Inventory Management I: Newsvendor Model

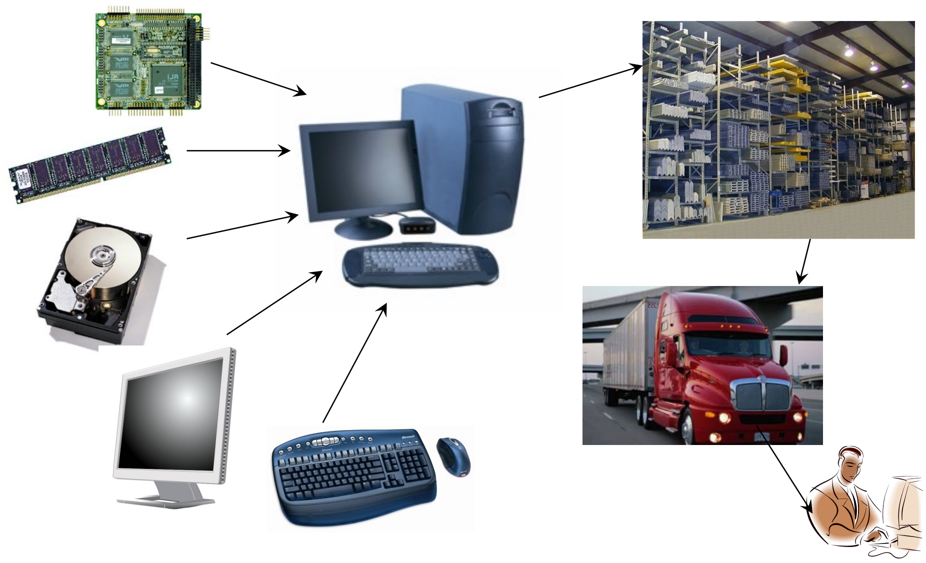
Learning Objectives

- **Understand the costs associated with and the strategic role of inventory**
- Understand the “Newsvendor Logic” and apply it to inventory management

Inventory

- Definition: The **stock** of any item or resource used in an organization
- In the form of
  - Raw materials & component parts
  - Work in process (WIP)
  - Finished goods (FG)
  - Replacement parts, tools, & supplies
  - Goods-in-transit to warehouses or customers

Inventory Example



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Inventory in the Value Chain

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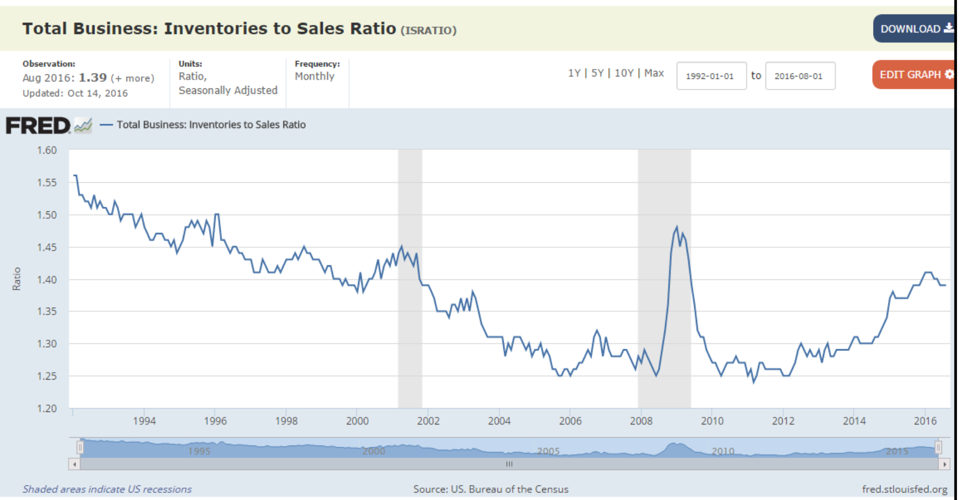
Inventory in the Value Chain



Types of Inventory/Reasons to Hold Inventory

1. Safety stock
   - Due to random variation
2. Seasonal inventories
   - Due to seasonal variation
3. Pipeline inventories
   - Inventory in transit
4. Speculative inventories
   - e.g., Drug hoarding to hedge against increase prices
5. Work in process inventories

Service level is defined as in-stock probability.

Reasons to Not to Hold Inventory

1. Opportunity costs of capital
2. Physical holding costs
   - Warehousing, labor, insurance
3. Obsolescence, spoilage
4. Hide problems
   - e.g. Toyota holds low inventory in order to let the problems surface

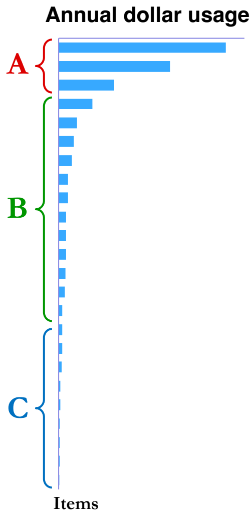
Cost of Holding Inventories

- Annual holding cost of inventory is 30 to 35% of its value.
- With $1.8 Trillion total business inventory in 2016
  - U.S. total inventory holding cost = $ 1.8 Trillion × 30% = $ 540 Billion !!

ABC Classification of Inventory

Rank items according to **annual dollar usage (Annual demand ∗ cost)**

- **Class A**: top 5-10% of items that constitute 50% or more of total annual dollar usage
  - Use most sophisticated inventory tools, plus possibly individualized attention
- **Class B**: next 50-70% of items that account for most of the remaining dollar usage
  - Use formal tools, but not individualized management
- **Class C**: remaining 20-40% of items that represent only a minor portion of total dollar usage
  - Use simple tools
  - Avoid disruptions due to stock-outs



Learning Objectives

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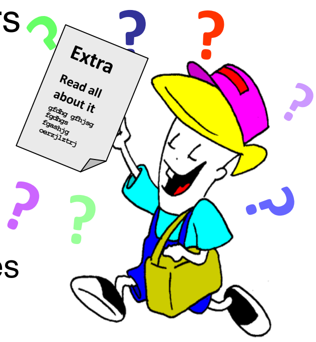
- **Understand the “Newsvendor Logic” and apply it to inventory management**

Example: Newsvendor Problem

- A newsvendor stocks newspapers to sell that day
- **How many newspapers to stock?**
- **<u>Tradeoffs:</u>**
  - If stocks <u>too few</u> newspapers, misses potential sales.
  - If stocks <u>too many</u> newspapers, money wasted on unsold newspapers.



"Rocky"

Newsvendor model: Rocky wants to know…

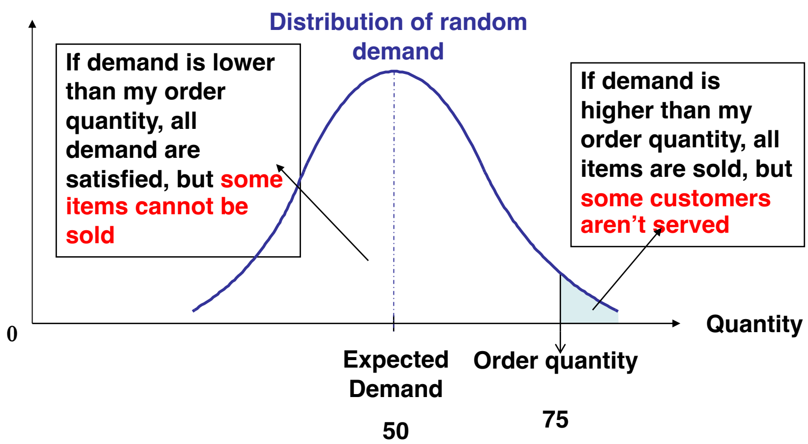
- What is the best service level?
- How much to order/produce?

Newsvendor Problem: Key Features

- Demand is uncertain
- Stock before knowing the demand
- Unmatched demand will be lost
  - Customers are unwilling to wait!
- Leftover inventory has no value (or reduced value)
- “One period”
- Maximize total expected profit

How to Think About the Problem?

= 50?	> 50?	< 50?	Not enough information?



Solving the Newsvendor Problem

- **Rocky pays \$0.5 for each paper, and sells for \$1.5**
- **Daily newspaper demand distribution:**

Demand:	**87**	**88**	**89**	**90**	**91**	**92**	**93**
Probability:	**0.03**	**0.07**	**0.2**	**0.4**	**0.2**	**0.07**	**0.03**

- **If Rocky buys 87 papers, profit = \$_87____**
- **Should Rocky buy the 88^{th}? Let's see…**

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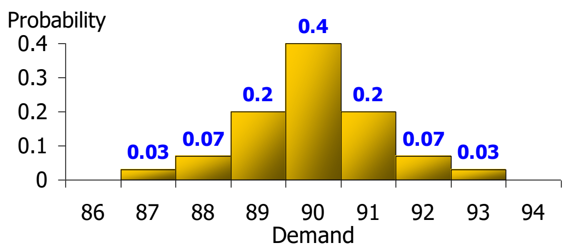
Marginal Analysis: 88^{th} paper

- With probability _0.03__, the 88^{th} paper will not be sold, and it costs Rocky \$_0.5___
- With probability _0.97__, the 88^{th} paper will be sold and brings Rocky profit of \$_1____
- Cost <> Benefit

  0.03 × \$0.5 < 0.97 × \$1
- Should Rocky buy the 88^{th} paper? Absolutely



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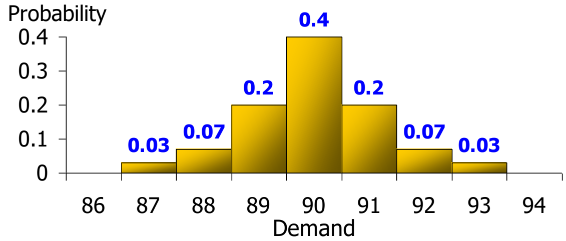
Marginal Analysis: 89th paper

- With probability __0.1__, the 89th paper will not be sold, and it costs Rocky $ 0.5__
- With probability __0.9__, the 89th paper will be sold and brings Rocky profit of $__1__
- Cost <> Benefit

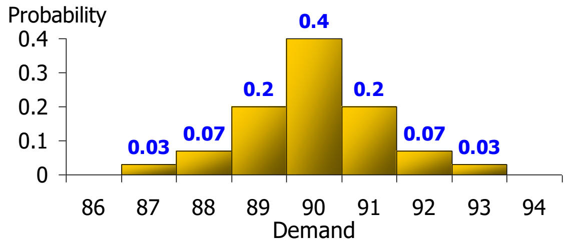
  0.1 × $0.5 < 0.9 × $1
- Should Rocky buy the 89th paper? Definitely yes.



Marginal Analysis: 90th paper

- With probability __0.3__, the 90th paper will not be sold, and it costs Rocky $ 0.5__
- With probability __0.7__, the 90th paper will be sold and brings Rocky profit of $__1__
- Cost <> Benefit

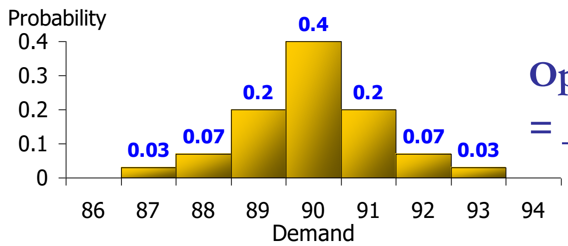
  0.3 × $0.5 < 0.7 × $1
- Should Rocky buy the 90th paper? He certainly should.



Marginal Analysis: 91st paper

- With probability 0.7, the 91st paper will not be sold, and it costs Rocky $ 0.5
- With probability 0.3, the 91st paper will be sold and brings Rocky profit of $ 1
- Cost <> Benefit

  0.7 × $0.5 > 0.3 × $1
- Should Rocky buy the 91st paper? Of course not.



Optimal quantity = 90

Marginal Analysis: Generalized

Notation:

C_o = Cost of over-stocking one unit "Overage"

C_u = Cost of under-stocking one unit "Underage"

- Do not buy Q+1st unit, as long as its cost ≥ benefit:

Expected Marginal cost of not selling the Q+1th unit	Expected Marginal profit of selling the Q+1th unit

$P(D \le Q)\ C_o \ge$

or equivalently: $P(D \le Q) \ \ge\ C_u / (C_u+C_o)$

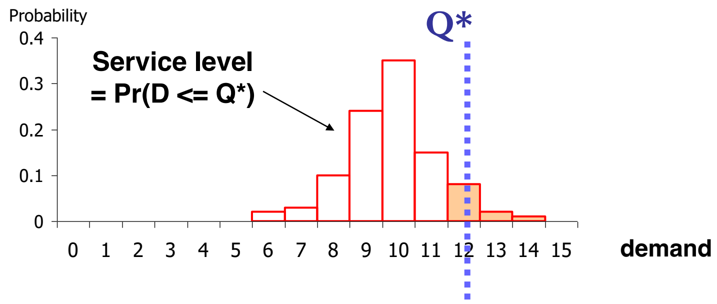
- Q* is the smallest quantity such that

  $SL \ge C_u / (C_u+C_o)$

The "Critical Ratio" = Best Service Level (SL*)

How to Compute Q* Given Critical Ratio

- **With discrete demand,**

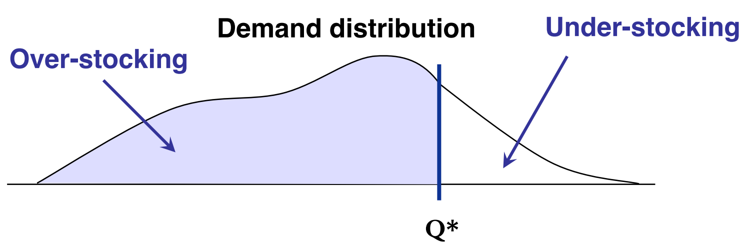


Q* = smallest quantity such that SL ≥ $C_u / (C_u+C_o)$

Demand with Continuous Distribution

- **With continuous demand,**
- The best Q* can be found directly from

$$P(D \leq Q^*) \quad = \quad C_u / (C_u+C_o)$$



How to Compute Q* Given Critical Ratio

- **With discrete demand,**

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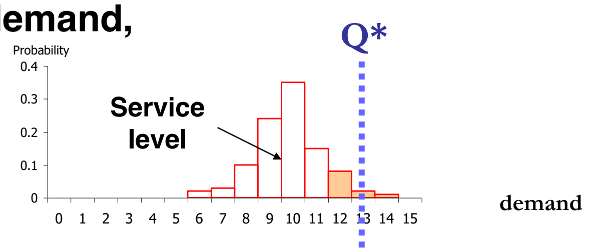
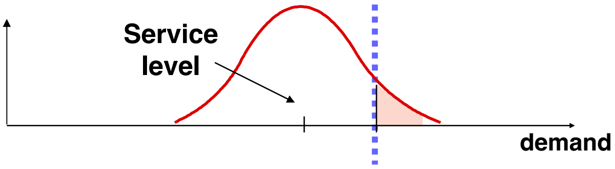


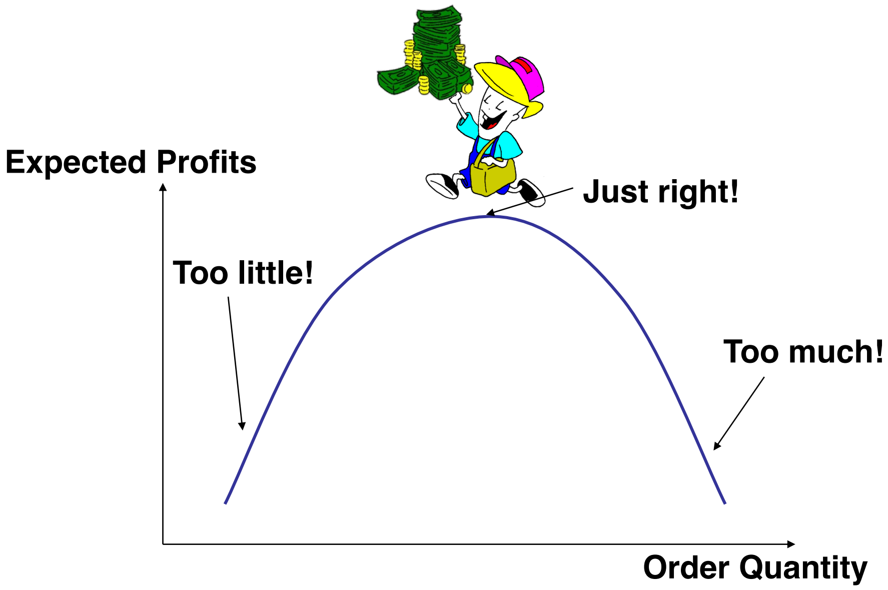
- **With normal demand N(μ, σ)**

Q* = the quantity such that SL = C_u / (C_u+C_o)



Step 1: find z, z = NORMSINV(C_u / (C_u+C_o)), or use z table
Step 2: compute Q*, Q* = μ + z σ

The Newsvendor's Profit Curve



Next Class

- Class 7: Logan Airport Case
- Class 8: Inventory models II: EOQ and continuous review