UCL
SCHOOL OF
MANAGEMENT

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MSIN0095: Operations Analytics

Class 1-4: Process Analysis

Class 5,7: Waiting Time Analysis

Class 6: Inventory Management I: Newsvendor Model

Class 8: Inventory Management II: Newsvendor 2 and Replenishable Inventory

Class 9: Inventory Management III: Replenishable Inventory 2

Replenishable inventory models

Continuous review

- Place an order from the supplier <u>at any time</u> if inventory is running low
- Event-triggered restocking
- Requires continuous monitoring of inventory levels

Periodic review

- Place an order from the supplier <u>only during scheduled times</u> (e.g. every 3 days or weekly).
- Time-triggered restocking
- Only requires to review inventory periodically

How much toilet paper to buy?

- How close is TESCO? (fixed setup cost) → **Not to buy too few each time**
- How big is your car? (constraints)
- Can you use the TP money on your Netflix subscription? (opportunity costs)
- How much will you need? (demand rate)
- How much do you already have? (inventory-status)

Not to buy too many each time

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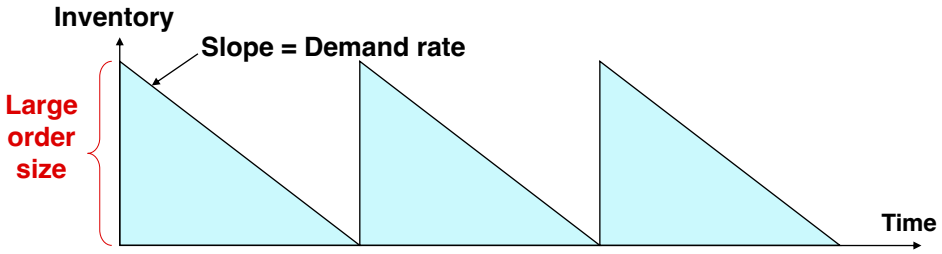
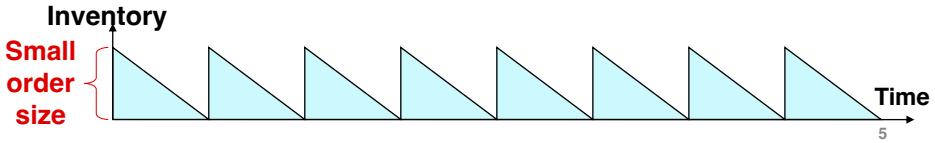
How much to order each time?



	Holding cost	Order setup cost	Purchase cost
Large order size	**High**	**Low**	**Same**
Small order size	**Low**	**High**	**Same**



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Economic Order Quantity (EOQ) Model

- Key Assumptions:
  - All aspects are **known with certainty**
    - Constant demand stream (No demand variability)
    - Constant setup cost per order
    - Constant annual holding cost per unit
    - No lead time (can be relaxed later)
  - No backorders are allowed
- When to order/produce (assuming zero lead time)?
  - When your inventory reaches zero
- How much to order/produce?
  - Let's see…

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Economic Order Quantity (EOQ) Model

- Demand:

  D = Demand rate (units / yr)

- Costs:

  C = Cost of purchasing or producing a unit ($ / unit)

  S = Setup cost or per order or per production run ($)

  H = Annual holding cost per unit of inventory ($ / (unit•yr))

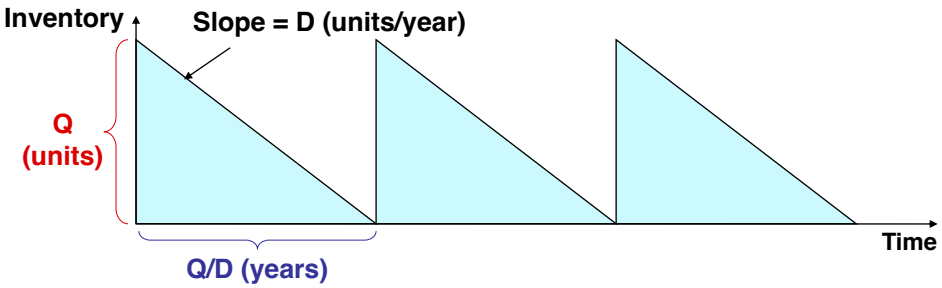
  H is often taken as a percentage of the unit cost:

  $H = i\,C$, where *i* is *annual* percentage holding cost

- Objective: Minimize total annual cost by choosing the order size

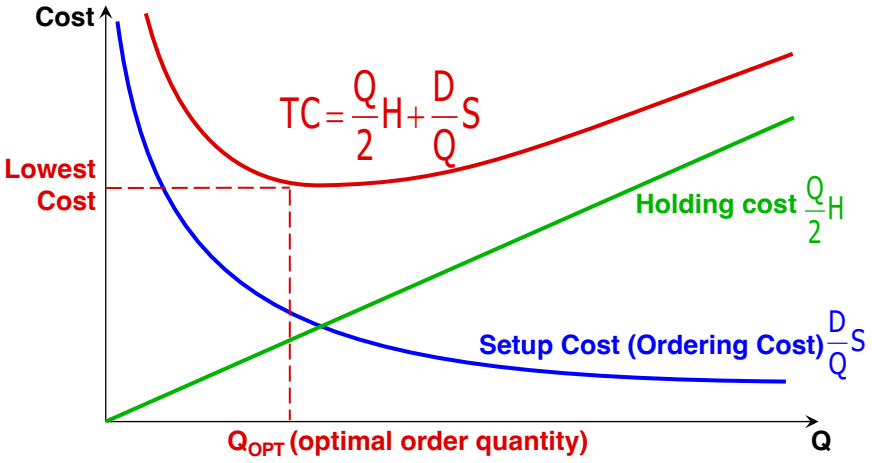
  Q = Quantity of an order (units)

Total Cost if order size is Q



Number of orders per year	=	D / Q	(/ yr)
Annual fixed **(setup) cost**	=	(D / Q) × S	($ / yr)
Average inventory	=	Q / 2	(units)
Annual **holding cost**	=	(Q / 2) × H	($ / yr)
Annual **purchase cost**	=	C × D	($ / yr)

Finding the Optimal Q (EOQ)



$$Q_{OPT} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(\text{Demand Rate})(\text{Order Setup Cost})}{\text{Holding Cost Rate}}}$$

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Practice: EOQ for the ATM

John has a savings account at First Wolverine Bank that pays him 4% interest annually on his deposits. John has $20 per week in expenses (for food, gas, tasty beverages, etc.) and insists on paying cash for everything. Every time John takes money out of the ATM his bank charges him $0.50 in fees.

How much $ should John take out of the ATM at a time?

How often will John have to visit the ATM?

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Practice Problem 1: EOQ for the ATM

- D = \$20 / week = \$1040 / year
- H = \$0.04 / year (holding cost of one dollar)
- S = \$0.50

Keep time units consistent!

- Optimal amount of \$ to withdraw:

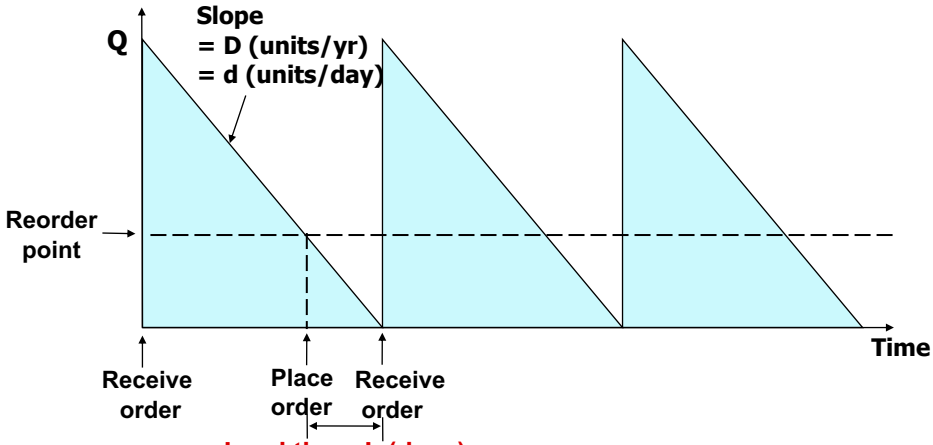
$$Q_{OPT} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 1040 \times .50}{0.04}} = 161.25$$

- John will visit the ATM approximately every 8 weeks (~6.5 times per year).

What if we have lead time?

- It takes L days to deliver.
- What is the Reorder Point?
  - ROP = DL.

What Happens When Lead Time > 0?



Reorder Point: ROP = d×L

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EOQ is powerful and…beautiful

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1. Sensitivity of EOQ Model to Quantity

- Why EOQ is powerful?
  - $EOQ = \sqrt{\frac{2SD}{H}}$, plugging EOQ to total cost
  - Total cost = $\sqrt{2SDH}$

    = H * EOQ = 2 * SD / EOQ
- If we use a wrong ordering quantity Q' (possibly because of wrong estimation of H, S)
  - Total actual cost C'= HQ' / 2 + SD / Q'
  - $\frac{C'}{optimal\ cost} = \frac{\frac{HQ'}{2}+\frac{SD}{Q'}}{\sqrt{2SDH}} = \frac{1}{2}[\frac{Q'}{EOQ} + \frac{EOQ}{Q'}]$

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Sensitivity of EOQ Model to Quantity

- Example: If EOQ = 110 and I use Q' = 100, then the ratio of the actual to optimal cost is (1/2)[1.1 + 1 / 1.1] = 1.004545

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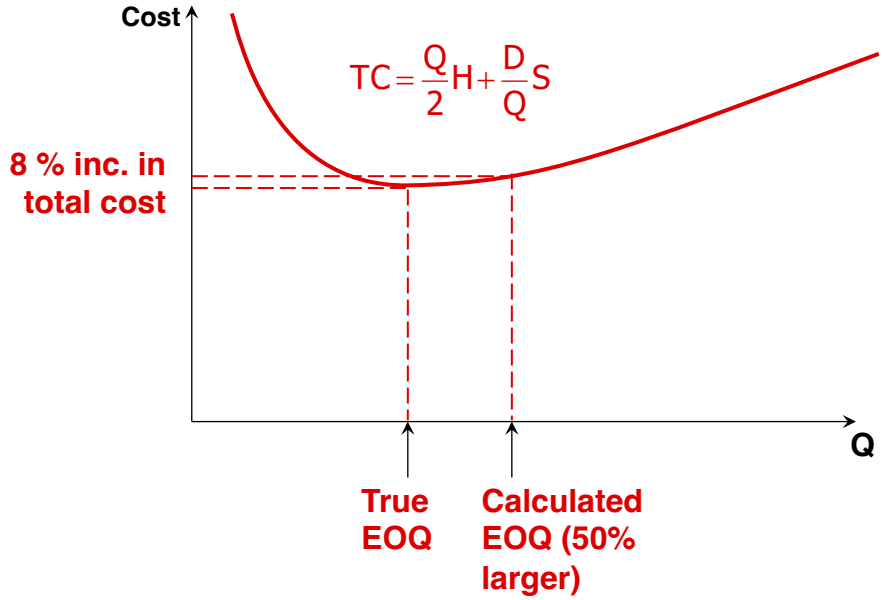
Robustness of EOQ



2. Sensitivity of EOQ Model to Order Interval

- Order Interval: Let T represent time (in years) between orders
  - T = Q / D
  - Optimal Order Interval: $T^* = \frac{Q^*}{D} = \frac{\sqrt{\frac{2SD}{H}}}{D} = \sqrt{\frac{2S}{HD}}$
- If we use T' instead of T*
  - $\frac{(annual\ cost\ under\ T')}{annual\ cost\ under\ T^*} = \frac{1}{2}\left[\frac{T'}{T^*} + \frac{T^*}{T'}\right]$

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3. What if S aren't just handed to us?

- Suppose your client does not have a good estimate of its setup cost S
- You want to demonstrate cost reduction opportunities.
- But client tells you: how many orders they placed last year and how much inventory was held.

Example: Unknown S

- Information:
  - D = 500 units/year,
  - H = $40 /(unit*year),
  - Frequency = 44 orders/year,
  - Inventory holding cost last year = $950K
- **Identify Improvement Opportunities**

Example: Unknown S

- For various hypothetical values for S, calculate

S	Q*	Order freq. D/Q*	Holding cost HQ*/2
1	5.0	100.0	100.0
2	7.1	70.7	141.4
5	11.2	44.7	223.6
20	22.4	22.4	447.2
50	35.4	14.1	707.1
100	50.0	10.0	1000.0
200	70.7	7.1	1414.2

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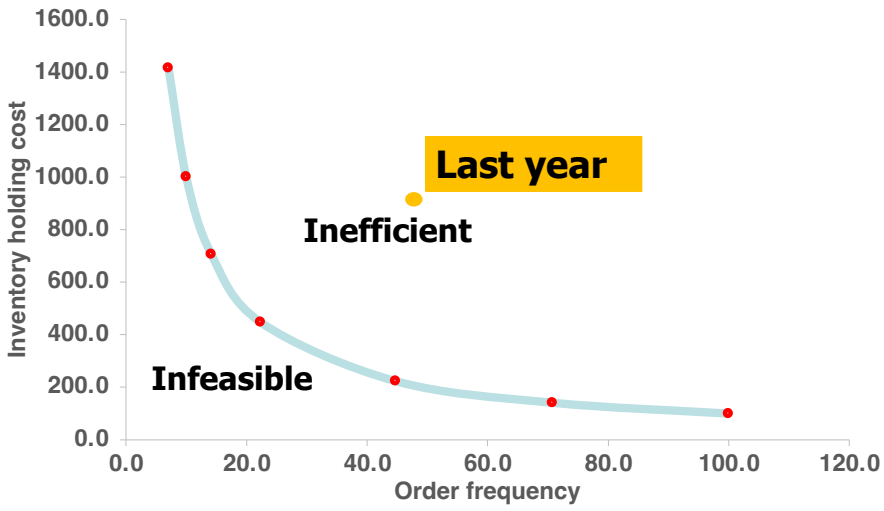
Example: Unknown S



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Summary: Replenishable inventory

- Economic order quantity (EOQ) model
  - Can place an order at any time
  - Trade-off: Fixed ordering cost vs. Holding cost
- Periodic inventory management
  - Can place an order periodically (e.g. every 7 days)
  - Trade-off: Overstocking vs. Understocking
  - Target Stock Level = Mean Exposure Period Demand + Safety Stock
  - Uncertain demand requires firms to carry safety stock