Waiting Time Analysis Practice Problems

Practice 1

Suppose AirOM passengers arrive to the check-in desk every 100 seconds (on average). The desk is staffed by a single ticketing agent, who takes 90 seconds (on average) to process a customer. Assume that $CV_a = CV_p = 1$.

(a) What is the check-in desk's utilization?

(b) What is a passenger's expected waiting time (time in line waiting to reach the check-in desk)?

(a) $a = 100 \text{ sec}, \quad p = 90 \text{ sec}, \quad m = 1.$

$$u = \frac{p}{m \cdot a} = \frac{90}{1 \times 100} = 0.9$$

(b) $$T_q = \frac{CV_a^2 + CV_p^2}{2} \cdot \frac{u}{1-u} \cdot p = \frac{1^2 + 1^2}{2} \cdot \frac{0.9}{1-0.9} \cdot 90 = 810 \text{ sec} = 13.5 \text{ min}$$

The FAA decides to lengthen the "did you pack your own bag" question, to also ask "did you or someone you know pick the outfit you are wearing, it looks great". This new question adds 5 seconds to the average service time, but CV_p still equals 1.

(c) After this change, what is the expected waiting time experienced by customers?

$a = 100 \text{ sec}, \quad p = 95 \text{ sec}, \quad m = 1$

$$u = \frac{p}{m \cdot a} = \frac{95}{1 \times 100} = 0.95$$

$$T_q = \frac{CV_a^2 + CV_p^2}{2} \cdot \frac{u}{1-u} \cdot p = \frac{1^2 + 1^2}{2} \cdot \frac{0.95}{1-0.95} \cdot 95 = 1805 \text{ sec}$$

$$= 30 \text{ min } 5 \text{ sec}$$

Practice 2

During MSIN0095 office hours, students arrive at an average rate of 12 per hour. Assume Poisson arrivals. The MSIN0095 professor can help students at an average rate of 20 per hour. Assume the time each student takes is exponentially distributed.

(a) What is the average number of students waiting?

$a = \frac{1}{12} \text{ hr} = 5 \text{ min}, \quad p = \frac{1}{20} \text{ hr} = 3 \text{ min}, \quad m = 1, \quad u = \frac{p}{m \cdot a} = \frac{3}{1 \times 5} = 0.6$

$CV_a = 1, \quad CV_p = 1, \quad T_q = \frac{u}{1-u} \cdot p = \frac{0.6}{1-0.6} \times 3 = 4.5 \text{ min}$

$I_q = \frac{T_q}{a} = \frac{4.5}{5} = 0.9$ students waiting on average

(b) The professor is concerned that students wait too long. As a remedy, she spends exactly 3 minutes with each student. What is the average number of students waiting?

$u = 0.6$ as before. $\quad CV_a = 1, \quad CV_p = 0$

$$T_q = \frac{CV_a^2 + CV_p^2}{2} \cdot \frac{u}{1-u} \cdot p = \frac{1^2 + 0^2}{2} \cdot \frac{0.6}{1-0.6} \times 3 = 2.25 \text{ min}$$

$I_q = \frac{T_q}{a} = \frac{2.25}{5} = 0.45$ students waiting on average

Practice 3

On Taco Tuesday, hungry customers arrive at the *Los Compadres* taco truck in a Poisson pattern with an average rate of 25 customers per hour. It takes 2 minutes on average to serve a customer, with service time following an exponential distribution. One customer is served at a time.

(a) How long does a customer wait on average before being served?

$R_a = 25/hr \rightarrow a = \frac{60}{25}$ min $= 2.4$ min ; $p = 2$ min $\quad m=1 \quad CV_a = CV_p = 1 \quad u = \frac{p}{ma} = 0.83$

$$T_q = \frac{CV_a^2 + CV_p^2}{2} \times \frac{u}{1-u} \times p = \boxed{9.76 \text{ min}}$$

(b) What is the average number of customers at the taco truck?

$T_{sys} = p + T_q = 2 + 9.76 = 11.76$ min

Little's Law: $I_{sys} = R_a \times T_{sys} = \frac{25}{60} \frac{\text{customers}}{\text{min}} \times 11.76 \text{ min} = \boxed{4.9 \text{ customers}}$

(c) What is the probability that a customer is served immediately?

P(customer is served immediately) = P(server is not busy)

$= 1 -$ P(server is busy) $= 1 - u = \boxed{17\%}$

(d) What is the probability of three or more customers in line?

P(3 or more in line) = P(3 or more in line AND 1 being served)

= P(4 or more in system) $= u^4 = 0.83^4 = \boxed{47.5\%}$

Practice 4

Preceding a Football Game, Jo plans to set up a stand just outside the UCL Stadium to give a 1-minute hug for $5. Demand is overwhelming, and customers arrive at an average rate of 50 per hour, (the arrivals tend to follow a Poisson process). Jo soon discovers that not every hug takes exactly 1 minute: instead, the service time is distributed exponentially, with an average of 1 minute per customer. What is the average time that a customer spends in the queue and in the system?

$p = 1$ min, $\quad a = \frac{1}{50}$ hr $= 1.2$ min, $\quad m = 1, \quad u = \frac{p}{m \cdot a} = \frac{1}{1.2} = \frac{5}{6}$

$T_q = \frac{u}{1-u} \cdot p = \frac{5/6}{1-5/6} \times 1 = 5$ min $\qquad T_{sys} = T_q + p = 5 + 1 = 6$ min

After the first day, Jo started receiving complaints from impatient customers about the long waiting time. Fortunately, Jo's friend, Lo, was available to work at the stand, and could serve customers at the same rate as Jo. The only question was whether to have **two separate queues** (one each for Jo and Lo), or a **single queue** of customers. With two separate queues, the average arrival rate to each queue is half (25/hr) and follows Poisson process. No jockeying between lines is permitted. Which system is better in terms of average waiting time?

Two separate queues:

For each queue: $p = 1$ min, $a = \frac{1}{25}$ hr $= 2.4$ min, $\quad u = \frac{p}{m \cdot a} = \frac{1}{2.4} = \frac{5}{12}$

$T_q = \frac{5/12}{1 - 5/12} \times 1 = \frac{5}{7} = 0.714$ min

One single queue:

$p = 1$ min, $a = 1.2$ min, $m = 2$, $\quad u = \frac{p}{m \cdot a} = \frac{1}{2 \times 1.2} = 0.417$

$$T_q = \frac{u^{\sqrt{2(m+1)}-1}}{m(1-u)} \times p = \frac{0.417^{\sqrt{6}-1}}{2(1-0.417)} = 0.241 \text{ min}$$