Homework 7

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Question 1

1.

when $N = 3$

$W = \exp\left[\dfrac{-2\pi i}{N}\right]$

$= \exp\left[\dfrac{-2\pi i}{3}\right]$

$= -0.5 - 0.866i$

$\therefore F_1 = \begin{bmatrix} W^{0\times 0} & W^{0\times 1} & W^{0\times 2} \\ W^{1\times 0} & W^{1\times 1} & W^{1\times 2} \\ W^{2\times 0} & W^{2\times 1} & W^{2\times 2} \end{bmatrix} \times \dfrac{1}{3}$

$= \begin{bmatrix} 0.3333 & 0.3333 & 0.3333 \\ 0.3333 & -0.1667 - 0.2887i & -0.1667 + 0.2887i \\ 0.3333 & -0.1667 + 0.2887i & -0.1667 - 0.2887i \end{bmatrix}$

Question 2

when $N = 5$

$W = \exp\left[\dfrac{-2\pi i}{N}\right]$

$= \exp\left[\dfrac{-2\pi i}{5}\right]$

$= 0.309 - 0.9511i$

$\therefore F_2 = \begin{bmatrix} W^{0\times 0} & W^{0\times 1} & W^{0\times 2} & W^{0\times 3} & W^{0\times 4} \\ W^{1\times 0} & W^{1\times 1} & W^{1\times 2} & W^{1\times 3} & W^{1\times 4} \\ W^{2\times 0} & W^{2\times 1} & W^{2\times 2} & W^{2\times 3} & W^{2\times 4} \\ W^{3\times 0} & W^{3\times 1} & W^{3\times 2} & W^{3\times 3} & W^{3\times 4} \\ W^{4\times 0} & W^{4\times 1} & W^{4\times 2} & W^{4\times 3} & W^{4\times 4} \end{bmatrix} \times \dfrac{1}{5}$

$= \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.0618 - 0.1902i & -0.1618 - 0.1176i & -0.1618 + 0.1176i & 0.0618 + 0.1902i \\ 0.2 & -0.1618 - 0.1176i & 0.0618 + 0.1902i & 0.0618 - 0.1902i & -0.1618 + 0.1176i \\ 0.2 & -0.1618 + 0.1176i & 0.0618 - 0.1902i & 0.0618 + 0.1902i & -0.1618 - 0.1176i \\ 0.2 & 0.0618 + 0.1902i & -0.1618 + 0.1176i & 0.1618 - 0.1176i & 0.0618 - 0.1902i \end{bmatrix}$

Question 3 (a)

(a) $f = \{1, 1, 5, 2, 8\}$

$$F = \frac{1}{\sqrt{5}} f = \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.0618 - 0.1902i & -0.1618 - 0.1176i & -0.1618 + 0.1176i & 0.0618 + 0.1902i \\ 0.2 & -0.1618 - 0.1176i & 0.0618 + 0.1902i & 0.0618 - 0.1902i & -0.1618 + 0.1176i \\ 0.2 & -0.1618 + 0.1176i & 0.0618 - 0.1902i & 0.0618 + 0.1902i & -0.1618 - 0.1176i \\ 0.2 & 0.0618 + 0.1902i & -0.1618 + 0.1176i & -0.1618 - 0.1176i & 0.0618 - 0.1902i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 5 \\ 2 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2.4 \\ 0.3264 + 0.9788i \\ 0.8236 + 1.3935i \\ 0.8236 - 1.3935i \\ 0.3264 - 0.9788i \end{bmatrix}$$

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Question 3 (b)

(b) $(\text{new } \omega) = \exp\left[\frac{2\pi i}{N}\right] = \bar{\omega}$

$F = \frac{1}{\sqrt{5}} f \Rightarrow f = \frac{1}{\sqrt{5}} F$

$$= \frac{1}{\sqrt{5}} \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.0618 + 0.1902i & -0.1618 + 0.1176i & -0.1618 - 0.1176i & 0.0618 - 0.1902i \\ 0.2 & -0.1618 + 0.1176i & 0.0618 - 0.1902i & 0.0618 + 0.1902i & -0.1618 - 0.1176i \\ 0.2 & -0.1618 - 0.1176i & 0.0618 + 0.1902i & 0.0618 - 0.1902i & -0.1618 + 0.1176i \\ 0.2 & 0.0618 - 0.1902i & -0.1618 - 0.1176i & -0.1618 + 0.1176i & 0.0618 + 0.1902i \end{bmatrix} \begin{bmatrix} 2.4 \\ 0.3264 + 0.9788i \\ 0.8236 + 1.3935i \\ 0.8236 - 1.3935i \\ 0.3264 - 0.9788i \end{bmatrix}$$

$$= \begin{bmatrix} 1.0 + 0.0i \\ 1.0 + 0.0i \\ 5.0 + 0.0i \\ 2.0 + 0.0i \\ 8.0 + 0.0i \end{bmatrix}$$

∴ 結果與原輸入 $f = \{1, 1, 5, 2, 8\}$ 相同.

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