Bloom Filter

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Main Points

- Bloom filters are very small, typically smaller than the data stored
- There is one-sided error. Bloom filters may say yes to x in B? when the answer is no.

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  - In cache rather than RAM
  - In RAM rather than Disc
  - On a local machine rather than a server
  - On a mobile Device

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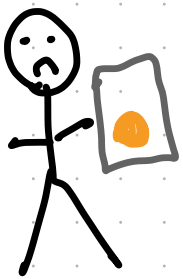
Option 1:



Put the DB on each user's device

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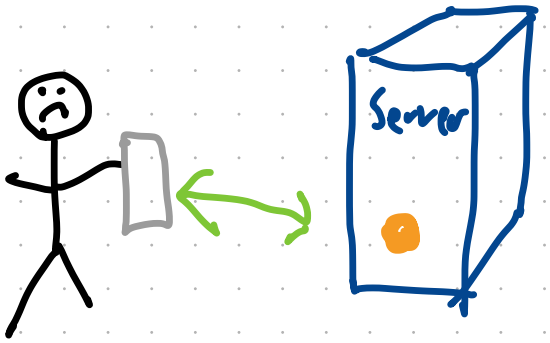

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Option 2:

Put the DB on a server

Option 3:

Put a Bloom filter on the phone and a DB on a server.

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Can we improve?

Bloom Filters, For Real

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- Let k denote the size of the Bitarray
  (usually $k = cn$, for a small c, e.g. 10)
- Let h denote the number of hash functions
- To insert x, set $A[hash_i(x) \bmod k] = 1$
  for all i
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Code

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- How should we choose the right number of hash function
- What is the traveaff between error rate and k (size)?

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(false positive)

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$$P[k \text{ Random Bits one}] \approx$$

with optimal h
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$$P[\underset{\text{with optimal } h}{\underset{\text{(false positive)}}{\text{Random } h \text{ bits one}}}] \approx \left(1-e^{hn/k}\right)^{h} = \left(\underbrace{1-e^{\left(\frac{k}{n}\ln 2\right)\frac{n}{k}}}_{1/2}\right)^{\frac{k}{n}\ln 2} = \left(\frac{1}{2}\right)^{\frac{k}{n}\ln 2}$$

Call this ϵ

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with optimal h

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$$\epsilon = 2^{-\frac{k}{n}\ln 2}$$

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$1/2$

with optimal h (false positive)

Call this ϵ

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$$\ln \epsilon = (\ln 2)\left(-\frac{k}{n}\ln 2\right)$$

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($1-e^{\left(\frac{k}{n}\ln 2\right)\frac{n}{k}} = 1/2$)

with optimal h (false positive)

Call this ϵ

$$\epsilon = 2^{-\frac{k}{n}\ln 2}$$

$$\ln \epsilon = (\ln 2)\left(-\frac{k}{n}\ln 2\right)$$

$$\frac{k}{n} = \frac{-1}{\ln^2 2}\ln \epsilon$$

Summary

Choose $\frac{k}{n} = -\frac{1}{\ln^2 2} \ln \epsilon$

(ϵ: error rate; $\frac{k}{n}$: bits per item)

Choose $h = \frac{k}{n} \ln 2 = -\frac{1}{\ln 2} \ln \epsilon$

(h: number of hash functions)

Summary

error rate

Choose $\frac{k}{n} = -\frac{1}{\ln^2 2} \ln \epsilon$

Bits per item

Choose $h = \frac{k}{n} \ln 2 = -\frac{1}{\ln 2} \ln \epsilon$

number of hash functions

Examples	k/n	h
$\epsilon = 1\%$	10	7
$\epsilon = 0.1\%$	19	13

URL Example

- 1 million URL's, 70 bytes on average
  - 70 MB of data

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- 1 million URL's, 70 bytes on average
  - 70 MB of data
- Bloom Filter 1% error $\frac{k}{n} = 10$
  - 1.2 MB of data