Mark's check-in: I have been working on computing wave constants for solutions of genus 3 or less of

$$\frac{3}{4}u_{yy} = \frac{\partial}{\partial x}(u_t - \frac{1}{4}(6uu_x + u_{xxx}))$$

from *Three-Phase of the Solutions of Kadomtsev-Petviashvili Equation* and from *Theta Functions and Nonlinear equations*. My solutions are of the form

$$u(x,y,t) = 2\frac{\partial^2}{\partial x^2}\log\theta(Ux + Vy + Wt)$$

but it should be easy to convert such a solution into a solution with nonzero mean value c. In this situation, Hirota's Bilinear Form (HBF) reduces to

$$\theta_{xxxx}\theta - 4\theta_{xxx}\theta_x + 3\theta_{xx}^2 + 4\theta_x\theta_t - 4\theta_{xt}\theta + 3\theta_{yy}\theta - 3\theta_y^2 + 8d\theta^2 = 0.$$

I currently have two ways to obtain the wave constants, but they are essentially the same:

1) From a curve $F(x,y) = 0$ and the appropriate parameters, obtain U,V,W, and d.
2) From a Riemann Matrix B and the appropriate parameters, obtain U,V,W, and d.

I have ways in which I can verify my results, but there is more I need to understand about some of the verifications.

I also do not yet have pictures and numbers that match the results in literature or that cross over to the work that Charles has done.

Currently, I am working to ensure that given real parameters and a purely imaginary Riemann Matrix I can produce real, **verifiable** solutions. The need for a fundamental region is also important for us. I believe that the Siegel command (something analogous in Sage?) may be helpful here.

1. Verification

There are two types of errors that we tried to mitigate: code correctness and accuracy. Here are ways I tried mitigation:

- Creation of a universal code so that code correctness is less dependent on genus.
- Use of odd characteristics and their behavior.
  - Since calculations using odd characteristics are, in some sense, independent from the calculations from the even characteristics used in the solution I use it to check the correctness of code. For example, verification of the vanishing at $z = 0$ implies portions of the code are functional enough to imitate parity. That is, it is some justification that the code is correct.
  - The odd theta functions satisfy

    $$(-4\theta_{xxx}\theta_x + 4\theta_x\theta_t - 3\theta_y^2)|_{z=Ux+Vy+Wt=0} = 0.$$

    Thus once U and V are found, W can be found via these relations. We call this $W_{expected}$. We compare this to the W obtained in the classical approach to verify accuracy. If there is error in our wave constants, it is reasonable to assume that W has the most error as it is dependent on U, V and B. **NOTE:** There seems to be times for which certain choices of odd characteristics result in W values that are wrong. For instance, when $\theta_x = 0$, W is not in this equation.
- **The most important error check**: HBF is used to see if $\theta(Ux + Vy + Wt|B)$ is accurate.
  - We compute the left-hand-side of HBF for a given set of parameters and graph a region of the (x,y,t) plane. In the examples I have attempted, I witnessed error in magnitude on the order of the number of digits specified globally. The computation is so long that it is not feasible.
  - An alternative approach I used is to incorporate error estimates in the computations of the parameters as described below.
    - I chose the $2N$ values of x, y, t at random: N of pairs the form

      $$\{\langle s_i^1, 0, s_i^2\rangle,\ \langle 0, s_i^1, s_i^2\rangle\ : s_i^1, s_i^2 \in \mathbb{R},\ \text{for } i = 1,..,N\}.$$

      and then I take the value of HBF at these points. I also check $(0,0,0)$.
    - My program checks the maximum absolute value of these $2N + 1$ complex numbers and compares the magnitude of this maximum against the global specification of digits.
- Finally, I also use other theoretical results as further verification:

- Compatibility Condition: There is a non-singular, square $\frac{g(g+1)}{2}+1$ submatrix of the matrix of theta constants independent of U. This implies indecomposability.
- For $g=3$, The Sign Condition: During the theoretical derivation of the wave constants square roots are taken. The sign of each square root can be found provided U satisfies the determinant condition.
- A choice of real input and a purely imaginary Riemann Matrix B should produce real wave solutions.