## 9.3 Dynamic risk measures

In this section, we present the basics for building dynamic carbon metrics that are very useful when defining net-zero investment portfolios and assessing the decarbonization policy of issuers. The main tools are the carbon budget, the carbon trend and the carbon target. By combining these tools, we will be able to present the $\mathcal{PAC}$ framework which is the cornerstone of implied temperature ratings (ITR). It measures the participation, the ambition and the credibility of a company to reduce its carbon emissions.

### 9.3.1 Carbon budget

#### Definition

The carbon budget defines the amount of GHG emissions that a country, a company or an organization produces over the time period $[t_0, t]$. From a mathematical point of view, it corresponds to the signed area of the region bounded by the function $\boldsymbol{\mathcal{CE}}(t)$:

$$\boldsymbol{\mathcal{CB}}(t_0, t) = \int_{t_0}^{t} \boldsymbol{\mathcal{CE}}(s)\, \mathrm{d}s$$

The carbon budget can be computed with other functions than the carbon emissions. For instance, if the reference level is equal to $\boldsymbol{\mathcal{CE}}^\star(t)$ at time $t$, we obtain:

$$\boldsymbol{\mathcal{CB}}^\star(t_0, t) = \int_{t_0}^{t} \boldsymbol{\mathcal{CE}}^\star(s)\, \mathrm{d}s$$

Therefore, we can easily compute the excess (or net) carbon budget since we have:

$$\int_{t_0}^{t} \left(\boldsymbol{\mathcal{CE}}(s) - \boldsymbol{\mathcal{CE}}^\star(s)\right) \mathrm{d}s = \boldsymbol{\mathcal{CB}}(t_0, t) - \boldsymbol{\mathcal{CB}}^\star(t_0, t)$$

If the reference level is constant — $\boldsymbol{\mathcal{CE}}^\star(t) = \boldsymbol{\mathcal{CE}}^\star$, the previous formula becomes:

$$\int_{t_0}^{t} \left(\boldsymbol{\mathcal{CE}}(s) - \boldsymbol{\mathcal{CE}}^\star\right) \mathrm{d}s = \boldsymbol{\mathcal{CB}}(t_0, t) - \boldsymbol{\mathcal{CE}}^\star(t - t_0)$$

**Example 40** *In Table 9.17, we report the historical data of carbon emissions from 2010 to 2020. Moreover, the company has announced its carbon targets for the years until 2050.*

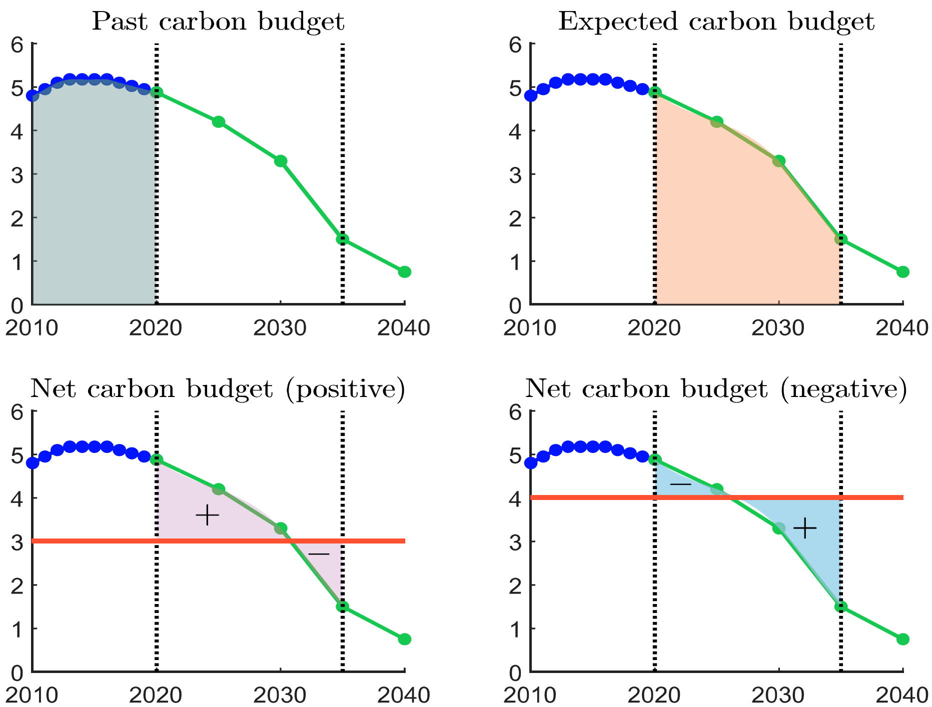
Table 9.17: Carbon emissions in $MtCO_2e$

| $t$ | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 |
|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{\mathcal{CE}}(t)$ | 4.800 | 4.950 | 5.100 | 5.175 | 5.175 | 5.175 | 5.175 | 5.100 |
| $t$ | 2018 | 2019 | 2020 | 2025* | 2030* | 2035* | 2040* | 2050* |
| $\boldsymbol{\mathcal{CE}}(t)$ | 5.025 | 4.950 | 4.875 | 4.200 | 3.300 | 1.500 | 0.750 | 0.150 |

The asterisk * indicates that the company has announced a carbon target for this year.

We consider the carbon pathway given in Example 40 and report different carbon budgets in Figure 9.20. The first panel (top/left) corresponds to the carbon budget that was spent by the company from 2010 to 2020. The second panel (top/right) is the targeted carbon budget that is estimated or planned by the company for the period between 2020 and 2035. The last two panels (bottom/left and bottom/right) considers a constant reference level, which may be for example the average target of the industry. If we assume that the reference level $\mathcal{CE}^{\star}$ is equal to 3 $MtCO_2e$ (bottom/left panel), we notice that the net carbon budget is the difference between two areas. From January 2020 to October 2030, the carbon emissions are greater than $\mathcal{CE}^{\star}$ and this period has a positive contribution to the carbon budget. On the contrary, the period from November 2030 to December 2035 has a negative contribution. On average, the net carbon budget is positive. In the case where the reference level $\mathcal{CE}^{\star}$ is equal to 4 $MtCO_2e$ (bottom/right panel), the excess carbon budget is negative.

Figure 9.20: Past, expected and net carbon budgets (Example 40)




**Computation of the carbon budget (numerical solution)**

We consider the equally-spaced partition $\{[t_0, t_0 + \Delta t], \ldots, [t - \Delta t, t]\}$ of $[t_0, t]$. Let $m = \dfrac{t - t_0}{\Delta t}$ be the number of intervals. We set $\mathcal{CE}_k = \mathcal{CE}(t_0 + k\Delta t)$. The right Riemann approximation is:

$$\mathcal{CB}(t_0, t) = \int_{t_0}^{t} \mathcal{CE}(s)\,\mathrm{d}s \approx \sum_{k=1}^{m} \mathcal{CE}(t_0 + k\Delta t)\,\Delta t = \Delta t \sum_{k=1}^{m} \mathcal{CE}_k$$

If we use the left Riemann sum, we obtain:

$$\mathcal{CB}(t_0, t) \approx \Delta t \sum_{k=0}^{m-1} \mathcal{CE}_k$$

Finally, the midpoint rule is given by:

$$\mathcal{CB}(t_0, t) \approx \Delta t \sum_{k=1}^{m} \mathcal{CE}\left(t_0 + \frac{k}{2}\Delta t\right)$$

In the case of a yearly partition, the previous formulas are simplified since we have $\Delta t = 1$. For instance, the left Riemann sum becomes:

$$\mathcal{CB}(t_0, t) = \sum_{k=0}^{m-1} \mathcal{CE}_k = \mathcal{CE}(t_0) + \ldots + \mathcal{CE}(t-1)$$

If we consider Example 40, the carbon budget from 1st January 2010 to 1st January 2020 is equal to:

$$\begin{aligned} \mathcal{CB}(2010, 2020) &= 4.8 + 4.95 + 5.1 + 5.175 + 5.175 + 5.175 + 5.175 + 5.1 + 5.025 + 4.95 \\ &= 50.625 \text{ MtCO}_2\text{e} \end{aligned}$$

**Remark 107** *Instead of Riemann sums, we can use more sophisticated methods such as trapezoidal and Simpon's rules (Roncalli, 2020a, Section A.1.2.3, pages 1037-1041). We can also interpolate the carbon emissions with spline functions and then implement a Gaussian quadrature.*

### Computation of the carbon budget (analytical solution)

**Constant reduction rate** If we use a constant linear reduction rate $\mathcal{R}(t_0, t) = \mathcal{R}(t - t_0)$, we obtain the following analytical expression:

$$\mathcal{CB}(t_0, t) = \int_{t_0}^{t} (\mathcal{CE}(t_0) - \mathcal{R}(s - t_0)) \, \mathrm{d}s = (t - t_0)\,\mathcal{CE}(t_0) - \frac{(t - t_0)^2}{2}\mathcal{R} \tag{9.6}$$

In the case of a constant compound reduction rate:

$$\mathcal{CE}(t) = (1 - \mathcal{R})^{(t-t_0)}\, \mathcal{CE}(t_0)$$

we obtain:

$$\begin{aligned} \mathcal{CB}(t_0, t) &= \mathcal{CE}(t_0) \int_{t_0}^{t} (1 - \mathcal{R})^{(s-t_0)} \, \mathrm{d}s \\ &= \mathcal{CE}(t_0) \left[ \frac{(1 - \mathcal{R})^{(s-t_0)}}{\ln(1 - \mathcal{R})} \right]_{t_0}^{t} \\ &= \frac{(1 - \mathcal{R})^{(t-t_0)} - 1}{\ln(1 - \mathcal{R})}\mathcal{CE}(t_0) \end{aligned} \tag{9.7}$$

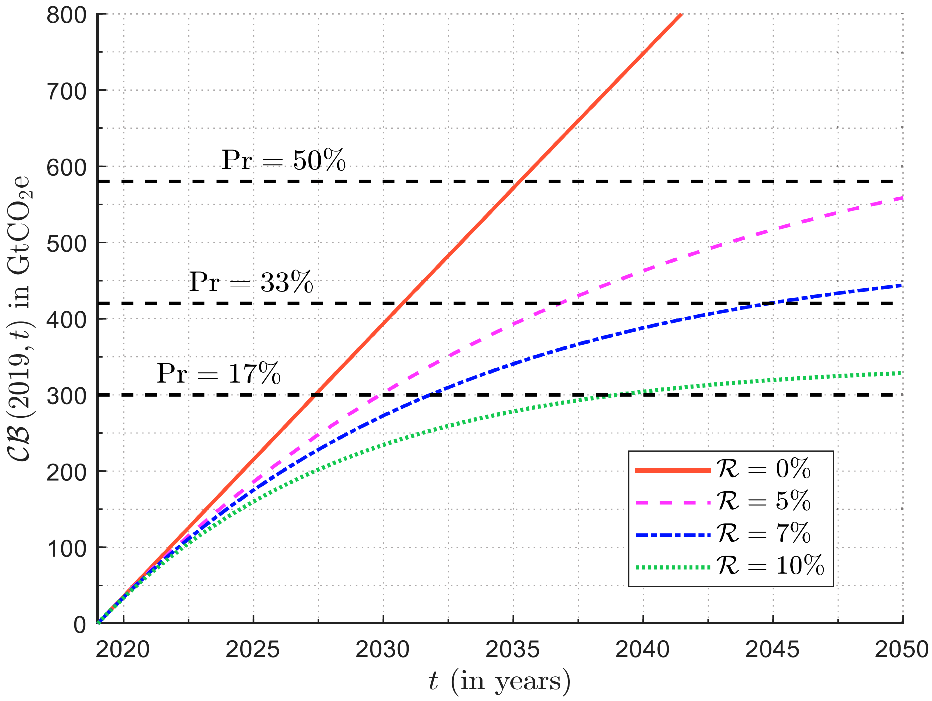
If we assume that $\mathcal{CE}(t) = e^{-\mathcal{R}(t-t_0)}\mathcal{CE}(t_0)$, we have:

$$\mathcal{CB}(t_0, t) = \mathcal{CE}(t_0) \left[ -\frac{e^{-\mathcal{R}(s-t_0)}}{\mathcal{R}} \right]_{t_0}^{t} = \mathcal{CE}(t_0)\, \frac{\left(1 - e^{-\mathcal{R}(t-t_0)}\right)}{\mathcal{R}} \tag{9.8}$$

**Remark 108** *If the carbon emissions increase at a positive growth rate $g$, we set $\mathcal{R} = -g$.*

According to IPCC (2018), the probability that the temperature $\mathcal{T}$ remains below 1.5°C by 2050 depends on a carbon budget. They estimated that the remaining carbon budget $\mathcal{CB}(2019, t)$ is 580 $\text{GtCO}_2\text{e}$ for a 50% probability of limiting warming to 1.5°C, 420 $\text{GtCO}_2\text{e}$ for a 66% probability and 300 $\text{GtCO}_2\text{e}$ for a 83% probability. In Figure 9.21, we have computed $\mathcal{CB}(2019, t)$ by setting $\mathcal{CE}(2019) = 36$ $\text{GtCO}_2\text{e}$ and assuming a constant compound reduction rate $\mathcal{R}$. If nothing is done, the probability to reach 1.5°C by 2035 is close to 50% since we obtain $\mathcal{CB}(2019, 2035) = 571.41$ $\text{GtCO}_2\text{e}$. With a reduction rate of 7%, we have a probability of 65% to limit global warming to 1.5°C by 2050. This explains that many reduction targets are calibrated on this figure, for example the Paris aligned benchmarks (CTB and PAB).

Figure 9.21: Probability to reach 1.5°C




**Linear function** If we assume that $\boldsymbol{\mathcal{CE}}(t) = \beta_0 + \beta_1 t$, we deduce that:

$$\begin{aligned} \boldsymbol{\mathcal{CB}}(t_0, t) &= \int_{t_0}^{t} (\beta_0 + \beta_1 s)\, \mathrm{d}s \\ &= \left[\beta_0 s + \frac{1}{2}\beta_1 s^2\right]_{t_0}^{t} \\ &= \beta_0 (t - t_0) + \frac{1}{2}\beta_1 \left(t^2 - t_0^2\right) \end{aligned} \tag{9.9}$$

We can extend this formula to a piecewise linear function. We assume that $\boldsymbol{\mathcal{CE}}(t)$ is known for $t \in \{t_0, t_1, \ldots, t_m\}$ and $\boldsymbol{\mathcal{CE}}(t)$ is linear between two consecutive dates:

$$\boldsymbol{\mathcal{CE}}(t) = \boldsymbol{\mathcal{CE}}(t_{k-1}) + \frac{\boldsymbol{\mathcal{CE}}(t_k) - \boldsymbol{\mathcal{CE}}(t_{k-1})}{t_k - t_{k-1}} (t - t_{k-1}) \qquad \text{if } t \in [t_{k-1}, t_k]$$

We notice that this equation can be written as:

$$\boldsymbol{\mathcal{CE}}(t) = \underbrace{\frac{t_k}{t_k - t_{k-1}}\boldsymbol{\mathcal{CE}}(t_{k-1}) - \frac{t_{k-1}}{t_k - t_{k-1}}\boldsymbol{\mathcal{CE}}(t_k)}_{\beta_{0,k}} + \underbrace{\frac{\boldsymbol{\mathcal{CE}}(t_k) - \boldsymbol{\mathcal{CE}}(t_{k-1})}{t_k - t_{k-1}}}_{\beta_{1,k}} t$$

We deduce that:

$$\boldsymbol{\mathcal{CB}}(t_0, t) = \sum_{k=1}^{k(t)} \int_{t_{k-1}}^{t_k} \boldsymbol{\mathcal{CE}}(s)\, \mathrm{d}s + \int_{t_{k(t)}}^{t} \boldsymbol{\mathcal{CE}}(s)\, \mathrm{d}s$$

where $k(t) = \{\max k : t_k \leq t\}$. Using Equation (9.9), we conclude that[33]:

$$\mathcal{CB}(t_0, t) = \sum_{k=1}^{k(t)} \beta_{0,k}^{\star}(t_k - t_{k-1}) + \frac{1}{2}\sum_{k=1}^{k(t)} \beta_{1,k}\left(t_k^2 - t_{k-1}^2\right) + \beta_{0,k(t)+1}\left(t - t_{k(t)}\right) + \frac{1}{2}\beta_{1,k(t)+1}\left(t^2 - t_{k(t)}^2\right) \tag{9.10}$$

If we consider Example 40 and assume that the carbon emissions are linear between two consecutive years, the carbon budget from 1st January 2010 to 1st January 2020 is equal to 50.662 $\mathrm{MtCO_2e}$, which is close to the value 50.625 $\mathrm{MtCO_2e}$ obtained previously. When we consider the carbon targets, the Riemann sums are not appropriate because the targets are measured every five or ten years. Since the objective is that the company reduces continuously its carbon emissions, it is better to compute the carbon budget by assuming a piecewise linear function. In our example, we obtain $\mathcal{CB}(2020, 2035) = 53.437$ $\mathrm{MtCO_2e}$with the following decomposition[34]: $\mathcal{CB}(2020, 2025) = 22.687$ $\mathrm{MtCO_2e}$ (42.46%), $\mathcal{CB}(2025, 2030) = 18.750$ $\mathrm{MtCO_2e}$ (35.09%) and $\mathcal{CB}(2030, 2035) = 12.000$ $\mathrm{MtCO_2e}$ (22.46%). We also have $\mathcal{CB}(2020, 2050) = 63.562$ $\mathrm{MtCO_2e}$, implying that the first period 2020–2035 represents 84.07% of the carbon emissions.

Table 9.18: IEA NZE scenario (in $\mathrm{GtCO_2e}$)

| Sector | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 |
|---|---|---|---|---|---|---|---|---|---|---|
| Electricity | 12.4 | 13 | 13.3 | 13.5 | 13.6 | 13.3 | 13.3 | 13.5 | 14 | 13.8 |
| Buildings | 2.89 | 2.81 | 2.78 | 2.9 | 2.84 | 2.87 | 2.91 | 2.95 | 2.98 | 3.01 |
| Transport | 7.01 | 7.13 | 7.18 | 7.37 | 7.5 | 7.72 | 7.88 | 8.08 | 8.25 | 8.29 |
| Industry | 8.06 | 8.47 | 8.57 | 8.71 | 8.78 | 8.71 | 8.56 | 8.52 | 8.72 | 8.9 |
| Other | 1.87 | 1.89 | 1.91 | 1.96 | 1.87 | 1.89 | 1.89 | 1.92 | 1.92 | 1.91 |
| Gross emissions | 32.2 | 33.3 | 33.7 | 34.4 | 34.5 | 34.5 | 34.5 | 35 | 35.9 | 35.9 |
| BECCS/DACCS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Net emissions | 32.2 | 33.3 | 33.7 | 34.4 | 34.5 | 34.5 | 34.5 | 35 | 35.9 | 35.9 |

| Sector | 2020 | 2025 | 2030 | 2035 | 2040 | 2045 | 2050 |
|---|---|---|---|---|---|---|---|
| Electricity | 13.5 | 10.8 | 5.82 | 2.12 | −0.08 | −0.31 | −0.37 |
| Buildings | 2.86 | 2.43 | 1.81 | 1.21 | 0.69 | 0.32 | 0.12 |
| Transport | 7.15 | 7.23 | 5.72 | 4.11 | 2.69 | 1.5 | 0.69 |
| Industry | 8.48 | 8.14 | 6.89 | 5.25 | 3.48 | 1.8 | 0.52 |
| Other | 1.91 | 1.66 | 0.91 | 0.09 | −0.46 | −0.82 | −0.96 |
| Gross emissions | 33.9 | 30.3 | 21.5 | 13.7 | 7.77 | 4.3 | 1.94 |
| BECCS/DACCS | 0 | −0.06 | −0.32 | −0.96 | −1.46 | −1.8 | −1.94 |
| Net emissions | 33.9 | 30.2 | 21.1 | 12.8 | 6.32 | 2.5 | 0.00 |

Source: IEA (2021, Figure 2.3, page 55).

[33] When $t$ belongs to the set $\{t_0, t_1, \ldots, t_m\}$, we can simplify this expression as follows:

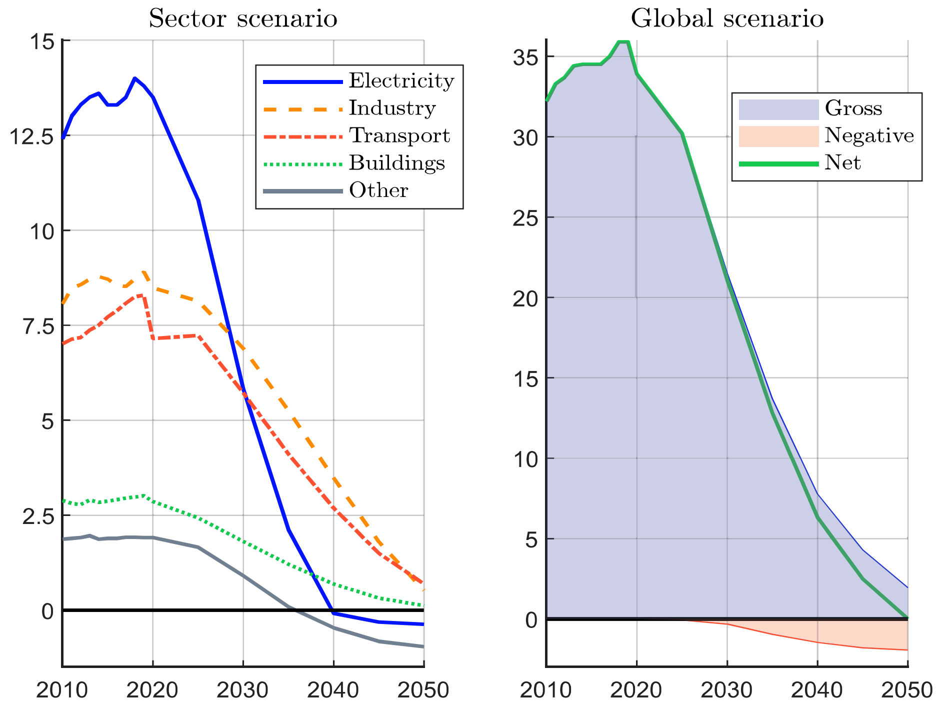
$$\mathcal{CB}(t_0, t) = \sum_{k=1}^{k(t)} \left(\mathcal{CE}(t_{k-1})\, t_k - \mathcal{CE}(t_k)\, t_{k-1}\right) + \frac{1}{2}\sum_{k=1}^{k(t)} \left(\mathcal{CE}(t_k) - \mathcal{CE}(t_{k-1})\right)(t_k + t_{k-1})$$

[34] We use the Chasles property of the Riemann integral:

$$\mathcal{CB}(t_0, t_2) = \int_{t_0}^{t_2} \mathcal{CE}(s)\,\mathrm{d}s = \int_{t_0}^{t_1} \mathcal{CE}(s)\,\mathrm{d}s + \int_{t1}^{t_2} \mathcal{CE}(s)\,\mathrm{d}s = \mathcal{CB}(t_0, t_1) + \mathcal{CB}(t_1, t_2)$$

**Example 41** *We consider the net-zero emissions (NZE) scenario provided by the International Energy Agency (IEA, 2021). We remind that it is a normative scenario that shows a pathway for the global energy sector to achieve net-zero* $CO_2e$ *emissions by 2050. For each important sector, IEA gives the past trajectory of carbon emissions and the decarbonization pathway that could be achievable (Table 9.18). This net-zero scenario has been calibrated with a carbon budget of approximatively* 500 $GtCO_2e$.

Figure 9.22: $CO_2$ emissions by sector in the IEA NZE scenario (in $GtCO_2e$)




Source: IEA (2021) & Author's calculations.

In Figure 9.22, we show the decarbonization pathway of each sector and the global economy. We notice the importance of the technologies bioenergy with carbon capture and storage (BECCS) and direct air carbon capture with carbon storage (DACCS). Indeed, they will help to compensate the remaining 2 $GtCO_2e$ of carbon emissions. In Table 9.19, we have computed the carbon budget $\mathcal{CB}(2019, t)$ for each sector and the global system. We notice that the sectors Electricity, Industry and Transport represent about 85% of the global budget.

Table 9.19: Carbon budget in the IEA NZE scenario (in $GtCO_2e$)

| $t$ | Electricity | Buildings | Transport | Industry | Other | Gross emissions |
|---|---|---|---|---|---|---|
| 2025 | 74.4 | 50.2 | 43.7 | 16.2 | 10.8 | 195.4 |
| 2030 | 115.9 | 87.8 | 76.0 | 26.8 | 17.3 | 324.9 |
| 2040 | 140.9 | 140.0 | 117.6 | 39.1 | 18.8 | 466.6 |
| 2045 | 139.9 | 153.2 | 128.1 | 41.6 | 15.6 | 496.8 |
| 2050 | 138.2 | 159.0 | 133.6 | 42.7 | 11.2 | 512.4 |

Source: IEA (2021) & Author's calculations.

### 9.3.2 Carbon trend

#### Linear trend model

Le Guenedal *et al.* (2022) defined the carbon trend by considering the linear trend model:

$$\boldsymbol{\mathcal{CE}}(t) = \beta_0 + \beta_1 t + u(t) \tag{9.11}$$

where $u(t) \sim \mathcal{N}\left(0, \sigma_u^2\right)$. We estimate the parameters $\beta_0$ and $\beta_1$ with the least squares method and a sample of observations. Therefore, the projected carbon trajectory is given by:

$$\boldsymbol{\mathcal{CE}}^{\mathcal{T}rend}(t) = \widehat{\boldsymbol{\mathcal{CE}}}(t) = \hat{\beta}_0 + \hat{\beta}_1 t \tag{9.12}$$

Let $\{t_{\mathcal{F}irst}, t_{\mathcal{F}irst} + 1, \ldots, t_{\mathcal{L}ast}\}$ be the set of observation dates. We interpret $\boldsymbol{\mathcal{CE}}^{\mathcal{T}rend}(t)$ as follows:

- If $t < t_{\mathcal{F}irst}$, $\boldsymbol{\mathcal{CE}}^{\mathcal{T}rend}(t)$ is the *back-calculated* value of carbon emissions before the first observation date (retropolation);

- If $t_{\mathcal{F}irst} \leq t \leq t_{\mathcal{L}ast}$, $\boldsymbol{\mathcal{CE}}^{\mathcal{T}rend}(t)$ is the *predicted* value of carbon emissions and can be compared with the *observed* value of carbon emissions (out-of-sample estimation);

- If $t > t_{\mathcal{L}ast}$, $\boldsymbol{\mathcal{CE}}^{\mathcal{T}rend}(t)$ is the *forecast* value of carbon emissions and could be compared with the *future* value of carbon emissions when this later will be available (out-of-sample estimation);

This model is very simple and the underlying idea is to extrapolate the past trajectory. Nevertheless, we can derive several metrics that are useful to compare the existing track record of the issuer with its willingness to really reduce its carbon emissions.

Equation (9.12) is not easy to interpret, because the intercept $\hat{\beta}_0$ corresponds to the estimated value $\widehat{\boldsymbol{\mathcal{CE}}}(0)$ at time $t = 0$. Then, it is convenient to use another base year $t_0$, implying that Equation (9.11) becomes:

$$\boldsymbol{\mathcal{CE}}(t) = \beta_0' + \beta_1'(t - t_0) + u(t) \tag{9.13}$$

In this case, the carbon trend is given by:

$$\boldsymbol{\mathcal{CE}}^{\mathcal{T}rend}(t) = \hat{\beta}_0' + \hat{\beta}_1'(t - t_0) \tag{9.14}$$

We can show that the two models (9.12) and (9.14) are equivalent and give the same value $\widehat{\boldsymbol{\mathcal{CE}}}(t)$. Indeed, we have the following relationships:

$$\left\{\begin{array}{l} \beta_0' = \beta_0 + \beta_1 t_0 \\ \beta_1' = \beta_1 \end{array}\right.$$

The new parameterization does not change the slope of the trend, but only the constant $\hat{\beta}_0'$ which is now equal to $\widehat{\boldsymbol{\mathcal{CE}}}(t_0)$.

**Remark 109** *The previous approach can be extended to the carbon intensity measure* $\boldsymbol{\mathcal{CI}}(t)$.

**Example 42** *In Table 9.20, we report the evolution of scope* $1+2$ *carbon emissions for company A.*

Table 9.20: Carbon emissions in $MtCO_2e$ (company $A$)

| Year | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 |
|---|---|---|---|---|---|---|---|
| $\mathcal{CE}(t)$ | 57.8 | 58.4 | 57.9 | 55.1 | 51.6 | 48.3 | 47.1 |
| Year | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
| $\mathcal{CE}(t)$ | 46.1 | 44.4 | 42.7 | 41.4 | 40.2 | 41.9 | 45.0 |

Using the carbon emissions given in Table 9.20, we obtain the following estimates[35]: $\hat{\beta}_0 = 2\,970.43$ and $\hat{\beta}_1 = -1.4512$. If we consider the regression model (9.13), the results become $\hat{\beta}'_0 = 57.85$ and $\hat{\beta}'_1 = -1.4512$ if the base year $t_0$ is set to 2007, and $\hat{\beta}'_0 = 38.99$ and $\hat{\beta}'_1 = -1.4512$ if the base year $t_0$ is set to 2020. We verify that all the figures are coherent:

$$\begin{aligned} \mathcal{CE}^{\mathcal{T}rend}(t) &= 38.99 - 1.4512 \times (t - 2020) \\ &= 2\,970.43 - 1.4512 \times t \end{aligned}$$

We notice that the trend model is more intuitive if we use the base year $t_0 = 2020$. The estimated carbon emissions is equal to 38.99 $MtCO_2e$ in 2020 and we observe a reduction of 1.4512 $MtCO_2e$ every year. For instance, the forecast value for the year 2025 is:

$$\mathcal{CE}^{\mathcal{T}rend}(2025) = 38.99 - 1.4512 \times 5 = 31.73 \text{ MtCO}_2\text{e}$$

We have reported the in-sample estimated values and out-of-sample forecast values in Figure 9.23. When $t$ is set to the year 2020, we observe that there is a gap between the observed value $\mathcal{CE}(t)$ and the estimated value $\widehat{\mathcal{CE}}(t)$ because $\mathcal{CE}(2020) = 45.0 \gg \widehat{\mathcal{CE}}(2020) = 38.99$. We deduce that the current carbon emissions are greater than the figure given by the trend, meaning that the company has made less effort in recent years compared to the past history. We can then rescale the trend model by imposing that the last value $\mathcal{CE}(t_{\mathcal{L}ast})$ is equal to the estimated value $\widehat{\mathcal{CE}}(t_{\mathcal{L}ast})$. We deduce that:

$$\hat{\beta}'_0 + \hat{\beta}'_1 (t_{\mathcal{L}ast} - t_0) = \mathcal{CE}(t_{\mathcal{L}ast}) \Leftrightarrow \hat{\beta}'_0 = \mathcal{CE}(t_{\mathcal{L}ast}) - \hat{\beta}'_1 (t_{\mathcal{L}ast} - t_0)$$

If $t_{\mathcal{L}ast} = t_0$, we obtain $\hat{\beta}'_0 = \mathcal{CE}(t_{\mathcal{L}ast})$. In our example, the rescaled model has the following expression:

$$\mathcal{CE}^{\mathcal{T}rend}(t) = 45 - 1.4512 \times (t - 2020)$$

In Figure 9.23, we verify that the rescaled model has the same slope as previously, but it is now coherent with the last observation of carbon emissions. In the sequel, we will always consider rescaled trend models, because they are more relevant.

**Log-linear trend model**

Instead of a linear model, we can use a log-linear trend model:

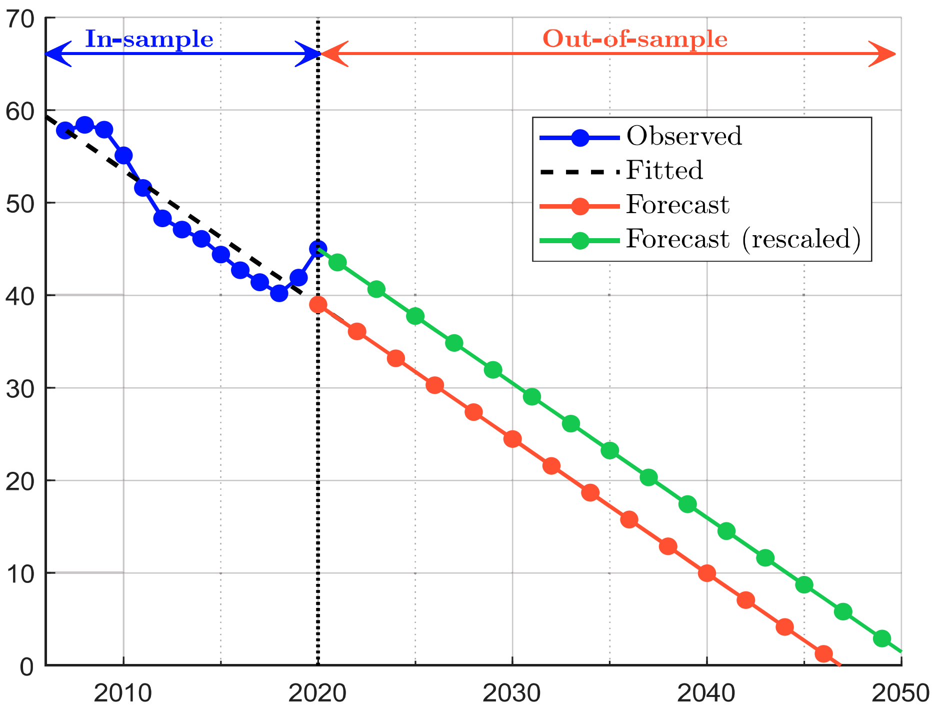
$$\ln \mathcal{CE}(t) = \gamma_0 + \gamma_1 (t - t_0) + v(t) \tag{9.15}$$

where $v(t) \sim \mathcal{N}(0, \sigma_v^2)$. Again, we estimate the parameters $\gamma_0$ and $\gamma_1$ by ordinary least squares. Let $Y(t) = \ln \mathcal{CE}(t)$ be the logarithmic transform of the carbon emissions. We have:

$$\hat{Y}(t) = \hat{\gamma}_0 + \hat{\gamma}_1 (t - t_0)$$

[35] $\hat{\sigma}_u$ is equal to 2.5844.

Figure 9.23: Linear carbon trend (Example 42)




and:

$$\begin{aligned}\widehat{\mathcal{CE}}(t) &= \exp\left(\hat{Y}(t)\right) \\ &= \exp\left(\hat{\gamma}_0 + \hat{\gamma}_1 (t - t_0)\right) \\ &= \widehat{\mathcal{CE}}(t_0) \exp\left(\hat{\gamma}_1 (t - t_0)\right)\end{aligned}$$

where $\widehat{\mathcal{CE}}(t_0) = \exp(\hat{\gamma}_0)$. The estimator (9.15) does not take into account the variance bias of log-linear models. Indeed, the correct value of the mathematical expectation is equal to[36]:

$$\begin{aligned}\mathbb{E}\left[\mathcal{CE}(t)\right] &= \mathbb{E}\left[e^{Y(t)}\right] \\ &= \mathbb{E}\left[\mathcal{LN}\left(\gamma_0 + \gamma_1 (t - t_0), \sigma_v^2\right)\right] \\ &= \exp\left(\gamma_0 + \gamma_1 (t - t_0) + \frac{1}{2}\sigma_v^2\right)\end{aligned}$$

Therefore, we obtain:

$$\begin{aligned}\widehat{\mathcal{CE}}(t) &= \exp\left(\hat{\gamma}_0 + \hat{\gamma}_1 (t - t_0) + \frac{1}{2}\hat{\sigma}_v^2\right) \\ &= \widehat{\mathcal{CE}}(t_0) \exp\left(\hat{\gamma}_1 (t - t_0)\right)\end{aligned}$$

where $\widehat{\mathcal{CE}}(t_0) = \exp\left(\hat{\gamma}_0 + \frac{1}{2}\hat{\sigma}_v^2\right)$. Again, we can rescale the trend model such that $\widehat{\mathcal{CE}}(t_{\mathcal{L}ast}) = \mathcal{CE}(t_{\mathcal{L}ast})$. It follows that:

$$\hat{\gamma}_0 + \frac{1}{2}\hat{\sigma}_v^2 = \ln \mathcal{CE}(t_{\mathcal{L}ast}) - \hat{\gamma}_1 (t_{\mathcal{L}ast} - t_0)$$

[36]We remind that $\mathbb{E}[X] = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$ if $X \sim \mathcal{LN}(\mu, \sigma^2)$ (see Section A.2.1 on page 1063).

If the base year $t_0$ is equal to the last year $t_{\mathcal{L}ast}$, the forecast value is equal to:

$$\mathcal{CE}^{\mathcal{T}rend}(t) = \mathcal{CE}(t_0)\exp(\hat{\gamma}_1(t - t_0)) \tag{9.16}$$

**Remark 110** *While the slope of the trend is measured in* $CO_2$e *in the linear trend model, it is measured in % in the log-linear model. In fact, we estimate an absolute trend in the former model and a relative trend in the later model. From Equation (9.15), we have:*

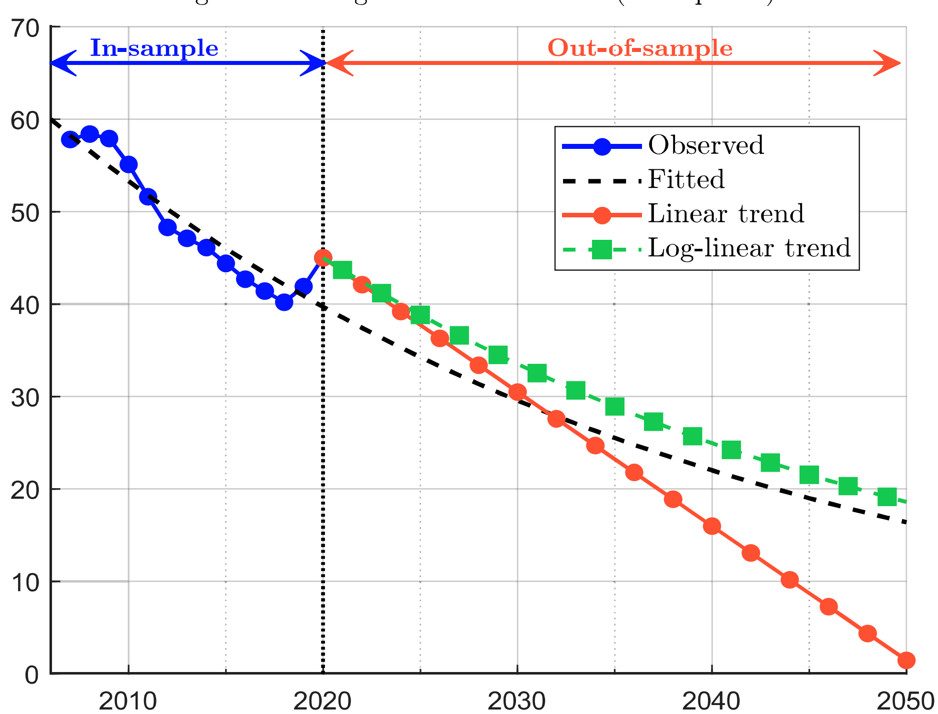
$$\frac{\partial\,\mathcal{CE}(t)}{\partial\,t} = \frac{\exp(\gamma_0 + \gamma_1(t - t_0) + v(t))}{\partial\,t} = \gamma_1\mathcal{CE}(t)$$

*We verify that the slope* $\gamma_1$ *is the relative variation of carbon emissions:*

$$\frac{\dfrac{\partial\,\mathcal{CE}(t)}{\partial\,t}}{\mathcal{CE}(t)} = \frac{\partial\,\ln\mathcal{CE}(t)}{\partial\,t} = \gamma_1$$

Using Example 42 and the 2020 base year, we obtain the following results: $\hat{\gamma}_0 = 3.6800$, $\hat{\gamma}_1 = -2.95\%$ and $\hat{\sigma}_v = 0.0520$. It follows that $\widehat{\mathcal{CE}}(2020)$ takes the value 39.65 $MtCO_2$e without the correction of the variance bias and 39.70 $MtCO_2$e with the correction of the variance bias. Using the parameterization (9.16), we compare the estimated log-linear trend with the estimated linear trend in Figure 9.24. We notice that the log-linear trend is convex and the future reduction rate of carbon emissions are less important than those obtained with the linear model.

Figure 9.24: Log-linear carbon trend (Example 42)

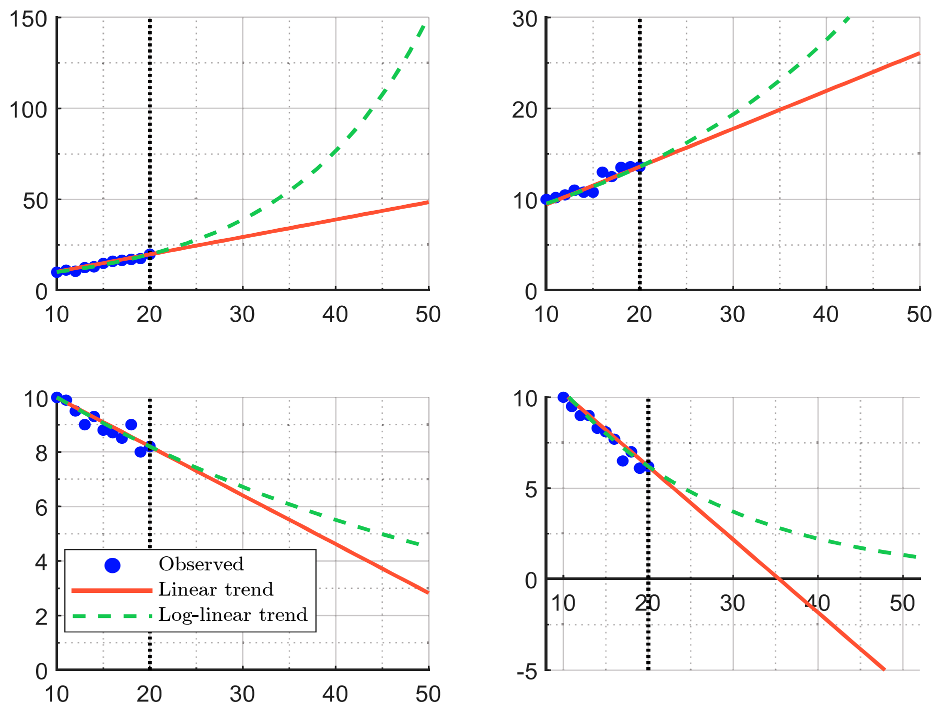


**Example 43** *We consider several historical trajectories of scope* 1 *carbon emissions:*

| # | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10.0 | 11.1 | 10.5 | 12.5 | 13.0 | 14.8 | 16.0 | 16.5 | 17.0 | 17.5 | 19.8 |
| 2 | 10.0 | 10.2 | 10.5 | 11.0 | 10.8 | 10.8 | 13.0 | 12.5 | 13.5 | 13.6 | 13.6 |
| 3 | 10.0 | 9.9 | 9.5 | 9.0 | 9.3 | 8.8 | 8.7 | 8.5 | 9.0 | 8.0 | 8.2 |
| 4 | 10.0 | 9.5 | 9.0 | 9.0 | 8.3 | 8.1 | 7.7 | 6.5 | 7.0 | 6.1 | 6.2 |

For each historical trajectory, we estimate the two models and report the estimated trends in Figure 9.25. When the slope is positive, the log-linear trend is systematically above the linear trend in the long run ($t \to \infty$). When the slope is negative, we observe the oppositive phenomenon. In this last case, the linear trend $\widehat{\boldsymbol{\mathcal{CE}}}(\infty)$ tends to $-\infty$ while the log-linear trend $\widehat{\boldsymbol{\mathcal{CE}}}(\infty)$ tends to 0. The fact that carbon emissions are negative may be disturbing. From a theoretical viewpoint, it is not impossible because of the impact of negative emissions (due to negative emissions, carbon credits or carbon removal methods for instance). Nevertheless, it is extremely rare especially if we take into account scope 3 emissions. From a modeling viewpoint, it is then better to impose that carbon emissions are positive.

Figure 9.25: Log-linear vs. linear carbon trend (Example 43)




It seems then that the log-linear model is more relevant when the trend is negative. If we consider companies with a positive trend, the log-linear model may produce exploding carbon emissions. Let us compute the ratio of expected growth rates between $t_0$ and $t$. For the linear model, the expected growth rate is:

$$\frac{\widehat{\boldsymbol{\mathcal{CE}}}(t) - \boldsymbol{\mathcal{CE}}(t_0)}{\boldsymbol{\mathcal{CE}}(t_0)} = \frac{\boldsymbol{\mathcal{CE}}(t_0) + \hat{\beta}_1 (t - t_0) - \boldsymbol{\mathcal{CE}}(t_0)}{\boldsymbol{\mathcal{CE}}(t_0)} = \frac{\hat{\beta}_1 (t - t_0)}{\boldsymbol{\mathcal{CE}}(t_0)}$$

while the log-linear model gives:

$$\frac{\widehat{\mathcal{CE}}(t)-\mathcal{CE}(t_0)}{\mathcal{CE}(t_0)}=\frac{\mathcal{CE}(t_0)\exp\left(\hat{\gamma}_1(t-t_0)\right)-\mathcal{CE}(t_0)}{\mathcal{CE}(t_0)}=\exp\left(\hat{\gamma}_1(t-t_0)\right)-1$$

Therefore, the ratio of expected growth rates is proportional to:

$$\begin{aligned}\frac{\exp\left(\hat{\gamma}_1(t-t_0)\right)-1}{\hat{\beta}_1(t-t_0)} &\approx \frac{1}{\hat{\beta}_1(t-t_0)}\sum_{n=1}^{\infty}\frac{\hat{\gamma}_1^n(t-t_0)^n}{n!}\\ &= \frac{\hat{\gamma}_1}{\hat{\beta}_1}\sum_{n=0}^{\infty}\frac{\hat{\gamma}_1^n(t-t_0)^n}{(n+1)!}\\ &\approx \frac{\hat{\gamma}_1}{\hat{\beta}_1}\left(1+\frac{1}{2}\hat{\gamma}_1(t-t_0)+\frac{1}{6}\hat{\gamma}_1^2(t-t_0)^2+\frac{1}{24}\hat{\gamma}_1^3(t-t_0)^3+\ldots\right)\end{aligned}$$

We deduce that the exploding effect cannot be avoided. This is why we must be very careful when we consider the log-linear model. One possible solution is to use the linear model when $\hat{\gamma}_1$ is greater than a threshold and the log-linear model otherwise. In Table 9.21, we compute the multiplication factor $M(\gamma_1, t) = e^{\gamma t}$ for different values of the growth rate $\gamma_1$ and different time horizon. For instance, if we target the year 2050, we have $t \approx 30$ years, we notice that carbon emissions are multiplied by a factor greater than 10 if the growth rate is 8%. Since there are many uncertainties about data collection and data computation, an historical growth rate of 8% may be explained by several factors:

- The company has really increased its carbon emissions by 8%;
- The company has underestimated its carbon emissions in the past and is more conservative today;
- The company has changed the reporting perimeter;
- Etc.

Applying a factor of 10 with a 30-year time horizon may then be not realistic. This is why we must be careful when estimating the carbon trend and analyze the outlier companies.

Table 9.21: Multiplication factor $M(\gamma_1, t)$

| $t$ | $\gamma_1$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (in years) | 1% | 2% | 3% | 4% | 5% | 6% | 7% | 8% | 9% | 10% |
| 1 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 | 1.06 | 1.07 | 1.08 | 1.09 | 1.11 |
| 5 | 1.05 | 1.11 | 1.16 | 1.22 | 1.28 | 1.35 | 1.42 | 1.49 | 1.57 | 1.65 |
| 10 | 1.11 | 1.22 | 1.35 | 1.49 | 1.65 | 1.82 | 2.01 | 2.23 | 2.46 | 2.72 |
| 15 | 1.16 | 1.35 | 1.57 | 1.82 | 2.12 | 2.46 | 2.86 | 3.32 | 3.86 | 4.48 |
| 20 | 1.22 | 1.49 | 1.82 | 2.23 | 2.72 | 3.32 | 4.06 | 4.95 | 6.05 | 7.39 |
| 25 | 1.28 | 1.65 | 2.12 | 2.72 | 3.49 | 4.48 | 5.75 | 7.39 | 9.49 | 12.18 |
| 30 | 1.35 | 1.82 | 2.46 | 3.32 | 4.48 | 6.05 | 8.17 | 11.02 | 14.88 | 20.09 |

**Remark 111** *The expected growth rate is related to the concept of carbon momentum that will be presented later on page 927.*

### Stochastic trend model

Following Roncalli (2020a), the linear trend model can be written as:

$$\left\{\begin{array}{l} y(t) = \mu(t) + u(t) \\ \mu(t) = \mu(t-1) + \beta_1 \end{array}\right.$$

where $u(t) \sim \mathcal{N}\left(0, \sigma_u^2\right)$. In this case, we have $y(t) = \beta_0 + \beta_1 t + u(t)$ where $\beta_0 = \mu(t_0) - \beta_1 t_0$. A way to introduce a stochastic trend is to add a noise $\eta(t)$ in the trend equation: $\mu(t) = \mu(t-1) + \beta_1 + \eta(t)$ where $\eta(t) \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$. Let us now assume that the slope of the trend is also stochastic:

$$\left\{\begin{array}{l} y(t) = \mu(t) + u(t) \\ \mu(t) = \mu(t-1) + \beta_1(t-1) + \eta(t) \\ \beta_1(t) = \beta_1(t-1) + \zeta(t) \end{array}\right.$$

where $\zeta(t) \sim \mathcal{N}\left(0, \sigma_\zeta^2\right)$. This model is called the local linear trend (LLT) model (Roncalli, 2020a, page 653). Using the Kalman filter (KF), we can estimate both the stochastic trend $\mu(t)$ and the stochastic slope $\beta_1(t)$.

Let us come back to Example 42. We estimate the parameters $(\sigma_u, \sigma_\eta, \sigma_\zeta)$ by maximizing the Whittle log-likelihood function (Roncalli, 2020a, pages 686-687). We obtain $\hat{\sigma}_u = 0.7022$, $\hat{\sigma}_\eta = 0.7019$ and $\hat{\sigma}_\zeta = 0.8350$. We deduce that the standard deviation of the stochastic slope variation $\beta_1(t) - \beta_1(t-1)$ is equal to 0.8350 $MtCO_2e$. This indicates that there is a high uncertainty in the trend computation. Then, we run the Kalman filter to estimate $\hat{\mu}(t)$ and $\hat{\beta}_1(t)$. In Table 9.22, we report these values and compare them with the estimates using the rolling least squares (RLS). We notice that the time-varying slope produced by the Kalman filter may be very different from the one produced by the method of least squares[37]. In particular, the magnitude of the variability is not the same. This is normal since the rolling least squares estimate a global slope from the beginning of the sample to time $t$ whereas the Kalman filter estimates a local slope for the period $[t-1, t]$. Therefore, $\hat{\beta}_i(t)$ is an estimator of the average slope in the case of the linear trend model and an estimator of the marginal slope in the case of the local linear trend model.

**Remark 112** *With the stochastic trend model, Le Guenedal et al. (2022) introduce the concept of carbon velocity, which measures the normalized slope change between $t-h$ and $t$:*

$$\boldsymbol{v}^{(h)}(t) = \frac{\hat{\beta}_1(t) - \hat{\beta}_1(t-h)}{h}$$

*The rationale for this measure is the following. A commitment to reduce carbon emissions implies a negative trend: $\hat{\beta}_1(t) < 0$. Nevertheless, it can take many years for a company to change the sign of the trend slope if it has a bad track record. Therefore, we can use the velocity to verify that the company is making significant efforts in the recent period. In this case, we must have $\boldsymbol{v}^{(h)}(t) < 0$ for low values[38] of $h$. In the case of the local linear trend model, we notice that the one-step velocity is equal to the innovation of the slope:*

$$\boldsymbol{v}^{(1)}(t) = \hat{\beta}_1(t) - \hat{\beta}_1(t-1) = \hat{\zeta}(t)$$

[37]For instance, we have $\hat{\beta}_1(2020) = -1.4512$ $MtCO_2e$ with the least squares method and $\hat{\beta}_1(2020) = +1.7701$ $MtCO_2e$ with the Kalman filter.

[38]Generally, $h$ is equal to 1, 2 or 3 years.

Table 9.22: Kalman filter estimation of the stochastic trend (Example 42)

| $t$ | $\mathcal{CE}(t)$ | $\hat{\beta}_1(t)$ (RLS) | $\beta_1(t)$ (KF) | $\mu(t)$ (KF) |
|---|---|---|---|---|
| 2007 | 57.80 | | 0.0000 | 57.80 |
| 2008 | 58.40 | | 0.2168 | 58.25 |
| 2009 | 57.90 | 0.0500 | −0.0441 | 58.00 |
| 2010 | 55.10 | −0.8600 | −1.3941 | 55.56 |
| 2011 | 51.60 | −1.5700 | −2.6080 | 52.01 |
| 2012 | 48.30 | −2.0200 | −3.1288 | 48.47 |
| 2013 | 47.10 | −2.0929 | −2.2977 | 46.82 |
| 2014 | 46.10 | −2.0321 | −1.5508 | 45.85 |
| 2015 | 44.40 | −1.9817 | −1.5029 | 44.38 |
| 2016 | 42.70 | −1.9406 | −1.5887 | 42.73 |
| 2017 | 41.40 | −1.8891 | −1.4655 | 41.36 |
| 2018 | 40.20 | −1.8329 | −1.3202 | 40.15 |
| 2019 | 41.90 | −1.6824 | 0.1339 | 41.41 |
| 2020 | 45.00 | −1.4512 | 1.7701 | 44.45 |

### Carbon momentum

Le Guenedal *et al.* (2022) define the long-term carbon momentum as the growth rate of carbon emissions. In the case of the linear trend model, we have:

$$\mathcal{CM}^{\mathcal{L}ong}(t) = \frac{\hat{\beta}_1(t)}{\mathcal{CE}(t)}$$

while it is directly equal to $\hat{\gamma}_1(t)$ in the case of the log-linear trend model:

$$\mathcal{CM}^{\mathcal{L}ong}(t) = \hat{\gamma}_1(t)$$

Le Guenedal *et al.* (2022) also define the short-term carbon momentum as the one-year carbon velocity:

$$\mathcal{CM}^{\mathcal{S}hort}(t) = \frac{\boldsymbol{v}^{(1)}(t)}{\mathcal{CE}(t)}$$

If we apply this concept to the log-linear model, we obtain $\mathcal{CM}^{\mathcal{S}hort}(t) = \boldsymbol{v}^{(1)}(t)$ where:

$$\boldsymbol{v}^{(h)}(t) = \frac{\hat{\gamma}_1(t) - \hat{\gamma}_1(t-h)}{h}$$

In the case of the stochastic trend model, we have $\mathcal{CM}^{\mathcal{S}hort}(t) = \hat{\zeta}(t)$.

**Remark 113** *Carbon momentum plays a key role when we will define net-zero investment portfolios, because it is highly related to the concept of self-decarbonization*[39].

[39] See Section 11.3 on page 993.

### Application

Table 9.23 and 9.24 gives some statistics about carbon momentum. It reproduces the results obtained by Barahhou *et al.* (2022) by considering the issuers of the MSCI World index. Since it is difficult to obtain at least 5-year historical data, we focus on the scopes $\mathcal{SC}_1$, $\mathcal{SC}_{1-2}$ and $\mathcal{SC}_{1-3}^{\mathrm{up}}$, and we do not consider the scope $\mathcal{SC}_{1-3}$. If we use the linear trend model, the median value of $\mathcal{CM}^{\mathcal{L}ong}(t)$ is equal to 0% for scope 1, 1.6% when we include the scope 2, and 2.3% when we add the upstream scope 3. The carbon momentum is negative for only 29.4% of issuers when we consider $\mathcal{SC}_{1-3}^{\mathrm{up}}$. This means that a majority of issuers have a positive carbon trend. For instance, about 10% of issuers have a carbon momentum greater than 10%! If we consider carbon intensity instead of carbon emission, we obtain another story. Indeed, issuers with a negative trend dominate issuers with a positive trend. Therefore, it is easier to build a self-decarbonized portfolio when we consider the carbon intensity measure. If we estimate the carbon momentum with the log-linear trend model, results are slightly different. For instance, 19.2% of issuers have a carbon momentum $\mathcal{SC}_{1-3}^{\mathrm{up}}$ greater than 10% versus 8.0% with the linear trend model.

Table 9.23: Statistics (in %) of carbon momentum $\mathcal{CM}^{\mathcal{L}ong}(t)$ (MSCI World index, 1995 – 2021, linear trend)

| Statistics | Carbon emissions | | | Carbon intensity | | |
|---|---|---|---|---|---|---|
| | $\mathcal{SC}_1$ | $\mathcal{SC}_{1-2}$ | $\mathcal{SC}_{1-3}^{\mathrm{up}}$ | $\mathcal{SC}_1$ | $\mathcal{SC}_{1-2}$ | $\mathcal{SC}_{1-3}^{\mathrm{up}}$ |
| Median | 0.0 | 1.6 | 2.3 | −4.8 | −2.4 | −1.3 |
| Negative | 49.9 | 41.1 | 29.4 | 76.0 | 69.6 | 75.6 |
| Positive | 50.1 | 58.9 | 70.6 | 24.0 | 30.4 | 24.4 |
| $< -10\%$ | 23.4 | 15.8 | 5.8 | 36.0 | 25.0 | 5.7 |
| $< -5\%$ | 32.1 | 22.2 | 10.6 | 48.6 | 36.7 | 13.4 |
| $> +5\%$ | 22.9 | 27.5 | 23.6 | 6.2 | 7.3 | 2.7 |
| $> +10\%$ | 9.2 | 9.5 | 8.0 | 2.3 | 2.6 | 1.0 |

Source: Trucost database (2022) & Barahhou *et al.* (2022).

Table 9.24: Statistics (in %) of carbon momentum $\mathcal{CM}^{\mathcal{L}ong}(t)$ (MSCI World index, 1995 – 2021, log-linear trend)

| Statistics | Carbon emissions | | | Carbon intensity | | |
|---|---|---|---|---|---|---|
| | $\mathcal{SC}_1$ | $\mathcal{SC}_{1-2}$ | $\mathcal{SC}_{1-3}^{\mathrm{up}}$ | $\mathcal{SC}_1$ | $\mathcal{SC}_{1-2}$ | $\mathcal{SC}_{1-3}^{\mathrm{up}}$ |
| Median | −0.1 | 1.7 | 2.8 | −3.6 | −1.9 | −1.2 |
| Negative | 50.6 | 40.3 | 29.0 | 76.3 | 69.0 | 75.8 |
| Positive | 49.4 | 59.7 | 71.0 | 23.7 | 31.0 | 24.2 |
| $< -10\%$ | 13.6 | 8.0 | 2.8 | 20.8 | 12.3 | 2.1 |
| $< -5\%$ | 26.6 | 16.9 | 7.5 | 42.3 | 29.0 | 8.4 |
| $> +5\%$ | 29.8 | 35.9 | 37.1 | 9.0 | 10.1 | 4.0 |
| $> +10\%$ | 16.9 | 19.4 | 19.2 | 4.0 | 4.1 | 1.6 |

Source: Trucost database (2022) & Barahhou *et al.* (2022).

### 9.3.3 Participation, ambition and credibility for an alignment strategy

In this section, we define the three pillars that help to evaluate a company's alignment strategy with respect to a given climate scenario, *e.g.*, the net-zero emissions scenario. These three pillars are participation, ambition and credibility. They form the $\mathcal{PAC}$ framework, and they can be quantified using the tool of carbon budget.

#### Carbon target and decarbonization scenario

In addition to the historical pathway of carbon emissions, the $\mathcal{PAC}$ framework requires two other time series:

- The reduction targets announced by the company;
- The market-based sector scenario associated to the company that defines the decarbonization pathway.

Carbon reduction targets are defined by companies at a scope emissions level with different time horizons[40]. For instance, the issuer can commit to reduce its scope 1 emissions by 50% over a period of 20 years and its scope 3 emissions by 30% over a period of 10 years. Even if the time frame of carbon reduction targets goes to 60 years, most of reduction targets concern the next twenty years. In the CDP database, we observe that most targets are underway or new, and a large proportion of companies set targets to reduce emissions by less than 50% from their base year. We also notice that some targets are reported over multiple scopes[41] and we can have multiple release dates. Therefore, it is important to transform these heterogenous figures into a unique reduction pathway with one base year $t_0$:

$$\mathbb{CT} = \left\{ \boldsymbol{\mathcal{R}}^{\mathcal{T}arget} \left(t_0, t_k\right), k = 1, \ldots, n_T \right\}$$

where $n_T$ is the number of targets and $\boldsymbol{\mathcal{R}}^{\mathcal{T}arget} \left(t_0, t_k\right)$ is the reduction rate between $t_0$ and $t_k$ for the $k^{\text{th}}$ target.

Concerning the market-based scenario, we generally use sector scenarios provided by IPCC, IEA or IIASA. In some circumstances, we can take global scenarios, but only when we do not have the choice because there is no appropriate scenario for the sector. Again, the decarbonization scenario is defined as a set of reduction rates:

$$\mathbb{CS} = \left\{ \boldsymbol{\mathcal{R}}^{\mathcal{S}cenario} \left(t_0, t_k\right), k = 1, \ldots, n_S \right\}$$

where $n_S$ is the number of scenario data points. The reduction rate is calculated as follows:

$$\boldsymbol{\mathcal{R}}^{\mathcal{S}cenario} \left(t_0, t_k\right) = 1 - \frac{\boldsymbol{\mathcal{CE}}^{\mathcal{S}cenario} \left(t_k\right)}{\boldsymbol{\mathcal{CE}}^{\mathcal{S}cenario} \left(t_0\right)}$$

where $t_0$ is the base year and $\boldsymbol{\mathcal{CE}}^{S} \left(t_k\right)$ is the value of carbon emissions at time $t_k$ in the market-based scenario. For instance, if we consider the IEA NZE scenario (see Table 9.18 on page 918), we obtain the results given in Table 9.25. Carbon emissions have been floored at zero in order to verify that the reduction rate is always less than or equal to 100%. We notice that the Electricity sector must decarbonize very quickly: −20% in 2025, −57% in 2030 and −84% in 2035. The carbon emissions reduction of the Industry and Transport sectors is delayed and really begins after 2025. If we consider the global scenario, the reduction rate is set to 10% in 2025 and increases by 5% every year until 2035.

[40] Carbon reduction targets can be found in the CDP database.

[41] For instance, the target can concern only one scope, scope $1 + 2$ or all scopes.

Table 9.25: Reduction rates of the IEA NZE scenario (base year = 2020)

| Year | Electricity | Industry | Transport | Buildings | Other | Global |
|---|---|---|---|---|---|---|
| 2025 | 20.0 | 4.0 | −1.1 | 15.0 | 13.1 | 10.6 |
| 2030 | 56.9 | 18.8 | 20.0 | 36.7 | 52.4 | 36.6 |
| 2035 | 84.3 | 38.1 | 42.5 | 57.7 | 95.3 | 59.6 |
| 2040 | 100.0 | 59.0 | 62.4 | 75.9 | 100.0 | 77.1 |
| 2045 | 100.0 | 78.8 | 79.0 | 88.8 | 100.0 | 87.3 |
| 2050 | 100.0 | 93.9 | 90.3 | 95.8 | 100.0 | 94.3 |

Source: IEA (2021) & Author's calculations.

**Definition of the $\mathcal{PAC}$ framework**

The $\mathcal{PAC}$ framework has been introduced by Le Guenedal *et al.* (2022) and is based on the relative positioning of three carbon trajectories: (1) the historical trajectory and its trend, (2) the carbon targets and (3) the decarbonization scenario. It helps to answer several operational questions.

1. First, is the trend of the issuer in line with the scenario? In this case, we would like to know if the company has already reduced its carbon emissions. While the reduction targets correspond to future intentions, the carbon trend measures the past efforts of the company.

2. Is the commitment of the issuer to fight climate change ambitious? In particular, we would like to know if the target trajectory is above, below or in line with the market-based scenario, which is appropriate for the sector of the issuer. This is an important topic, because achieving the net-zero emissions scenario can only be possible if there are no free riders.

3. Finally, a third question is critical and certainly the most important issue. Is the target setting of the company relevant and robust? Indeed, we may wonder if the target trajectory is a too ambitious promise and a form of greenwashing or, on the contrary, a plausible decarbonization pathway.

Therefore, the assessment of the company's targets has three dimensions or pillars: (historical) participation, ambition and credibility. They form the $\mathcal{PAC}$ framework.

**Example 44** *We consider again Example 42. Company A has announced the following targets:* $\boldsymbol{\mathcal{R}}^{\mathcal{T}arget}(2020, 2025) = 40\%$, $\boldsymbol{\mathcal{R}}^{\mathcal{T}arget}(2020, 2030) = 50\%$, $\boldsymbol{\mathcal{R}}^{\mathcal{T}arget}(2020, 2035) = 75\%$, $\boldsymbol{\mathcal{R}}^{\mathcal{T}arget}(2020, 2040) = 80\%$ *and* $\boldsymbol{\mathcal{R}}^{\mathcal{T}arget}(2020, 2050) = 90\%$. *Since company A is an utility corporation, we propose to use the IEA NZE scenario for the sector Electricity.*

We have reported the different pathways of company $A$ in Figure 9.26. We notice that the announced targets are below the carbon trend except in 2050. A comparison between the targets and the global scenario indicates that company $A$ is more ambitious than the average firm. Nevertheless, the comparison is less favorable when we consider the decarbonization scenario of the corresponding sector. In order to quantify the relative the relative positioning of these trajectories, we compute the carbon budgets with the different pathways. If we consider the time horizon 2035, the carbon budget of the targets is slightly lower than the carbon budget of the decarbonization scenario (388 vs. 407 $\text{MtCO}_2\text{e}$). This indicates a true ambition to reduce its carbon emissions in line with what the market expects. Nevertheless, we observe a high carbon budget based on the trend model (512 $\text{MtCO}_2\text{e}$). This questions the credibility of the targets, even if the company has done some efforts in the past.

Table 9.26: Comparison of carbon budgets (base year = 2020, Example 44)

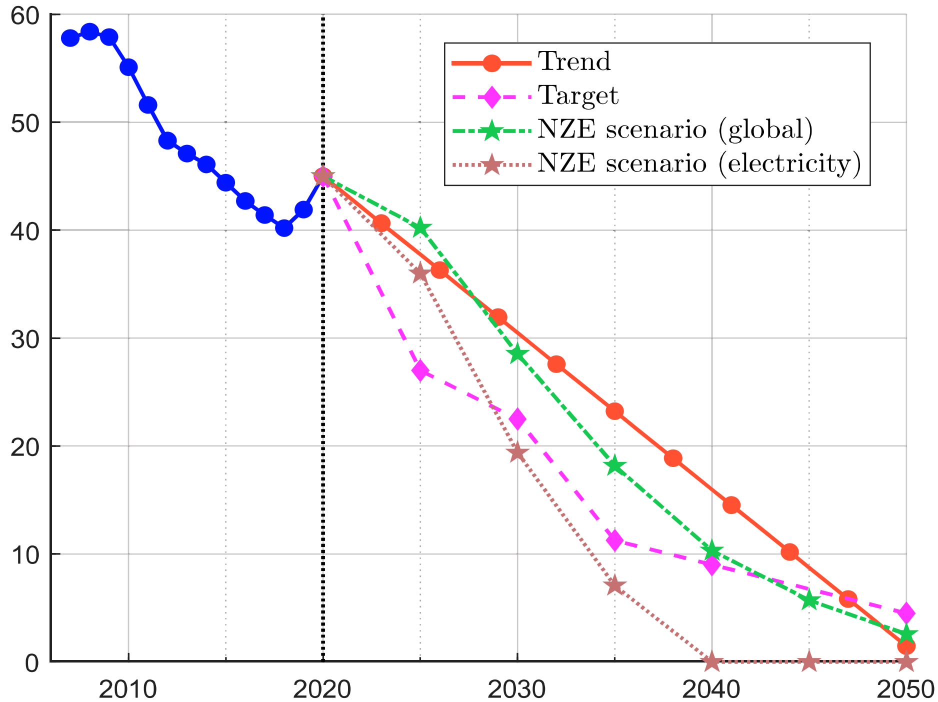
| Year | Trend (linear) | Trend (log-linear) | Target | Scenario (global) | Scenario (electricity) |
|---|---|---|---|---|---|
| 2025 | 207 | 209 | 180 | 213 | 203 |
| 2030 | 377 | 390 | 304 | 385 | 341 |
| 2035 | 512 | 546 | 388 | 502 | 407 |
| 2040 | 610 | 680 | 439 | 573 | 425 |
| 2045 | 671 | 796 | 478 | 613 | 425 |
| 2050 | 697 | 896 | 506 | 634 | 425 |

**Assessment of the $\mathcal{PAC}$ pillars**

These three pillars depend on the carbon trajectories $\boldsymbol{\mathcal{CE}}(t)$, $\boldsymbol{\mathcal{CE}}^{\mathcal{T}rend}(t)$, $\boldsymbol{\mathcal{CE}}^{\mathcal{T}arget}(t)$ and $\boldsymbol{\mathcal{CE}}^{\mathcal{S}cenario}(t)$, where $\boldsymbol{\mathcal{CE}}(t)$ is the time series of historical carbon emissions, $\boldsymbol{\mathcal{CE}}^{\mathcal{T}rend}(t)$ and $\boldsymbol{\mathcal{CE}}^{\mathcal{T}arget}(t)$ are the estimated carbon emissions deduced from the trend model and the targets, and $\boldsymbol{\mathcal{CE}}^{\mathcal{S}cenario}(t)$ is the market-based decarbonization scenario. Generally, the participation only depends on the past observations and corresponds to the track record analysis of historical carbon emissions. The ambition compares the target trajectory on one side and the scenario or the trend on the other side. Indeed, we measure to what extent companies are willing to reverse their current carbon emissions and have objectives that match the scenario. Finally, we can measure the credibility of the targets by comparing the current trend of carbon emissions and the reduction targets or by analyzing the recent dynamics of the track record.

We note $t_{\mathcal{F}irst}$ as the first date, $t_{\mathcal{L}ast}$ as the last reporting date and $t_{\mathcal{S}cenario}$ as the target date of the decarbonization scenario. In Figure 9.27, we illustrate the underlying ideas of the $\mathcal{PAC}$ pillars. Let us consider the first three panels. They show the historical carbon emissions of different companies. It is obvious that the company in the top-left panel has a positive participation to slow global warming, whereas the participation of the company in the top-center panel is negative. In the top-right panel, we give three examples that are mixed. In this case, we do not observe a clear pattern: downward or upward trend of carbon emissions. Therefore, the company's participation can be measured by the metrics that are related to the carbon trend. The next three panels in Figure 9.27 illustrate the ambition pillar. In this case, we directly compare the carbon targets of the company and the market-based risk scenario. The companies belong to the same sector, implying that the decarbonization scenario is the same for the middle-left, middle-center and middle-right panels. The middle-left panel shows an ambitious company since its carbon targets are lower than the market-based scenario. In other words, the company has announced that it will make a greater effort than is expected by the market. On the contrary, the company in the middle-center panel is less ambitious, because it plans to reduce its carbon emissions at a slower pace. Finally, the middle-right panel presents two mixed situations. The first one concerns a company that has high ambitions at the beginning of the period $[t_{\mathcal{L}ast}, t_{\mathcal{S}cenario}]$ but it has not disclosed its ambitions for the end of the period. The company's ambition in the short term is then counterweighted by the absence of ambition in the long run. The second example is about a company that concentrates its ambition in the long run. These two examples question the true willingness of these companies to substantially reduce their carbon emissions. Finally, the credibility pillar is illustrated in the last three panels in Figure 9.27. In this case, we compare the carbon emissions trend and the targets communicated by the company. The bottom-left panel corresponds to a credible company, since it has announced more or less a reduction trajectory that is in line with what it has done in the

Figure 9.26: Carbon trend, targets and NZE scenario of company $A$




Source: IEA (2021) & Author's calculations.

past. This is not the case of the company in the bottom-center panel. Clearly, it has announced a reduction of its carbon emissions, but it has continuously increased them in the past. Again, the bottom-right panel presents a mixed situation. The company has announced a reduction trajectory that is not very far from the past trend, but there are two issues. The first one is that it has increased its carbon emissions in the short term, implying that we can have some doubts about the downward trend. The second issue is that it accelerates its objective of carbon emissions reduction at the end of the period $[t_{\mathcal{L}ast}, t_{\mathcal{S}cenario}]$ in order to meet the requirements of the market-based scenario, but its efforts are not very substantial in the short term.

### Temperature scoring system

Le Guenedal *et al.* (2022) derive many metrics to measure the three dimensions. They can be classified into four main families. The gap metrics measure the differences between two trajectories or carbon budgets[42]. The duration metrics calculate the time which is necessary to achieve a given objective[43]. The velocity metrics assess the short-term dynamics of the carbon emissions. Finally,

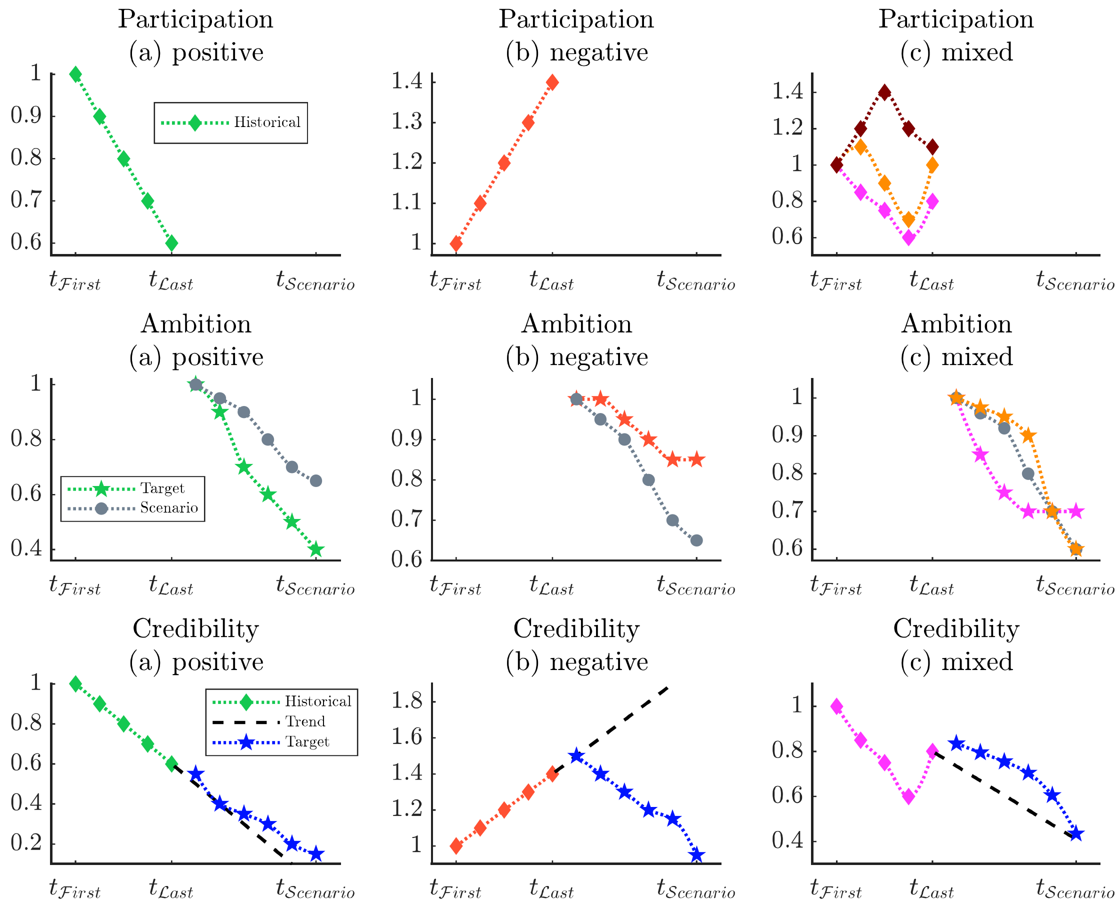
[42] An example is the difference between the trend budget and the scenario budget:

$$\mathcal{G}ap\left(t, t^{Scenario}\right) = \boldsymbol{\mathcal{CB}}^{\mathcal{T}rend}\left(t, t^{Scenario}\right) - \boldsymbol{\mathcal{CB}}^{Scenario}\left(t, t^{Scenario}\right)$$

[43] For instance, the trend duration is the time horizon for achieving zero carbon emissions:

$$\boldsymbol{\mathcal{D}}^{\mathcal{T}rend} = \inf\left\{t : \boldsymbol{\mathcal{CE}}^{\mathcal{T}rend}(t) \leq 0\right\}$$

Figure 9.27: Illustration of the participation, ambition and credibility pillars



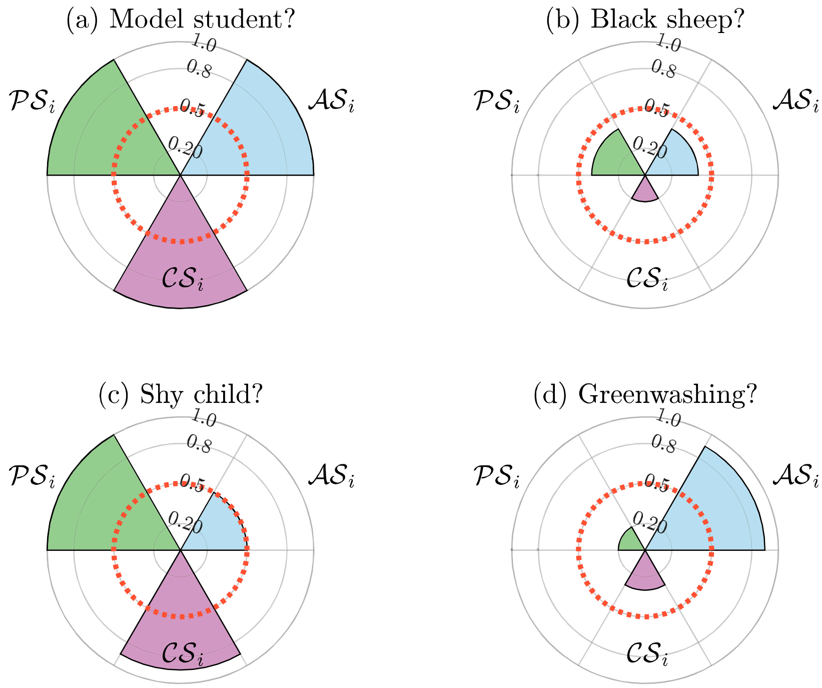

Source: Le Guenedal *et al.* (2022).

the last category computes the short-term reduction that the company must implement to satisfy the objective imposed by the market-based scenario[44]. Each pillar can then be studied using different metrics. Like the ESG risk, we can use a scoring system in order to analyze the $\mathcal{PAC}$ pillars and build three scores: the participation score $\mathcal{PS}_i$, the ambition score $\mathcal{AS}_i$ and the credibility score $\mathcal{CS}_i$. In Figure 9.28, we have represented several configurations of the $\mathcal{PAC}$ scoring system. If the three scores $\mathcal{PS}_i$, $\mathcal{AS}_i$ and $\mathcal{CS}_i$ are high and greater than 0.5 (which is the median value of a $q$-score), the company is both ambitious and credible and has already made some efforts to reduce its carbon emissions (Panel (a)). On the contrary, in Panel (b), we have a company, whose three scores are below the median. These two extreme cases are very frequent. Nevertheless, we can also obtain a more balanced scoring. For instance, Panel (c) corresponds to a company that has substantially reduced its past emissions but has announced weak reduction targets. Therefore, its ambition score is low, but its credibility score is high. It may be a company that does not talk a lot about its climate change policy, but its track record has demonstrated that it is committed. Finally, Panel (d) represents the scoring of a company with very high ambition, but it has continuously increased its carbon emissions in the past. Therefore, we can suspect a type of greenwashing. These examples

[44]The burn-out scenario refers to a sudden and violent reduction of carbon emissions such that the gap is equal to zero.

show that the three dimensions are correlated. For instance, we can assume a positive correlation between participation and credibility, and a negative correlation between ambition and credibility. Indeed, high credibility can only be obtained if participation is high or ambition is weak. Similarly, low credibility can be associated with excessively high ambition or weak participation, implying that the correlation between participation and credibility is unclear.

Figure 9.28: The $\mathcal{PAC}$ scoring system




Source: Le Guenedal *et al.* (2022).

**Remark 114** *While the $\mathcal{PAC}$ framework is the backbone to analyze the decarbonization commitment of the company's climate strategy (like the ESG pillars for analyzing the extra-financial risks of the company), the $\mathcal{PAC}$ scoring system is similar to the ESG scoring system. Most of implied temperature ratings are based on the participation, ambition and credibility pillars*[45].

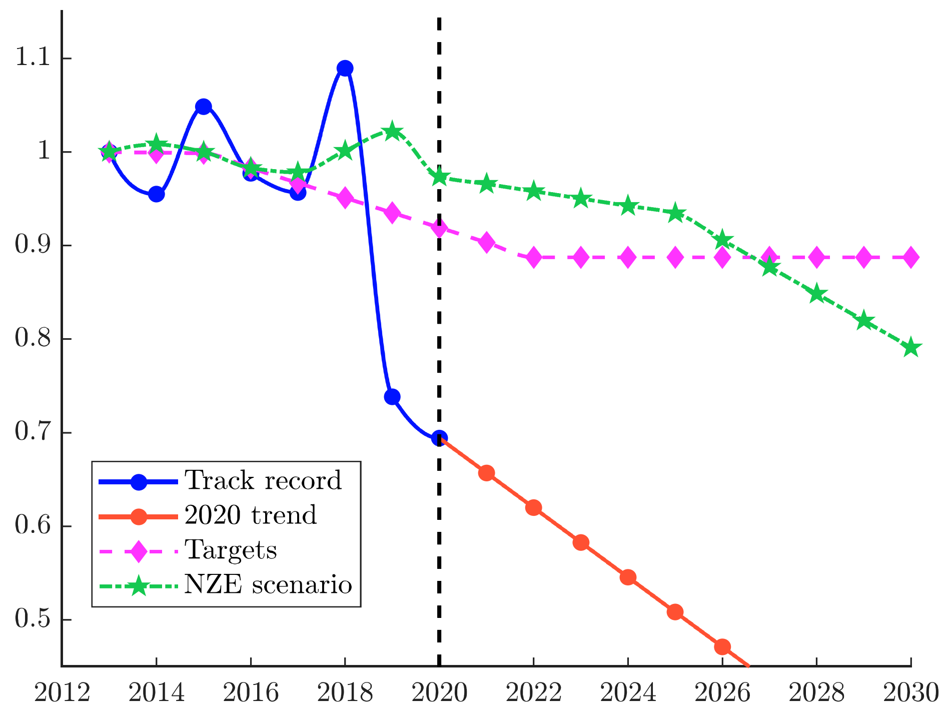
### 9.3.4 Illustration

We consider the analysis done by Le Guenedal *et al.* (2022). The base year is 2013 and they rebase the trajectories by the carbon emissions $\boldsymbol{\mathcal{CE}}$ (2013). The market-based scenario is the IEA NZE scenario. Company $B$ is a US based multinational technology conglomerate. Carbon emissions and targets are reported in Figure 9.29. Company $B$ is a particularly relevant example of the expectations from investors in the NZE context. Indeed, participation switched favorably after 2018 with a significant reduction in the scope 3 emissions from the use of sold products. The credibility is confirmed with the 2020 data point as the duration $\boldsymbol{\mathcal{D}}^{\mathcal{T}rend}$ drops under the duration of the NZE scenario. Figure 9.30 illustrates Company $C$ which is a major US airline. Its participation switched favorably in 2020. The credibility has not switched even with the significant drop in 2020, since the duration $\boldsymbol{\mathcal{D}}^{\mathcal{T}rend}$

[45] See Section 10.5.1 on page 947.

remains larger than the NZE time horizon. In fact, the reduction of $\mathcal{CE}$ (2020) sourced from both scope 1 and scope 3 emissions is related to the drop in activity due to the Covid-19 crisis. Company $D$ is a European multinational company which supplies industrial resources and services to various industries (Figure 9.31). The company has a clear ambition and has embraced the NZE context. However, the metrics indicate that in terms of participation, the trend has not been negative and has deteriorated in previous years. We stress here that although Company $D$ pays attention to its carbon intensity policy, it has not been active on the absolute carbon emissions level.

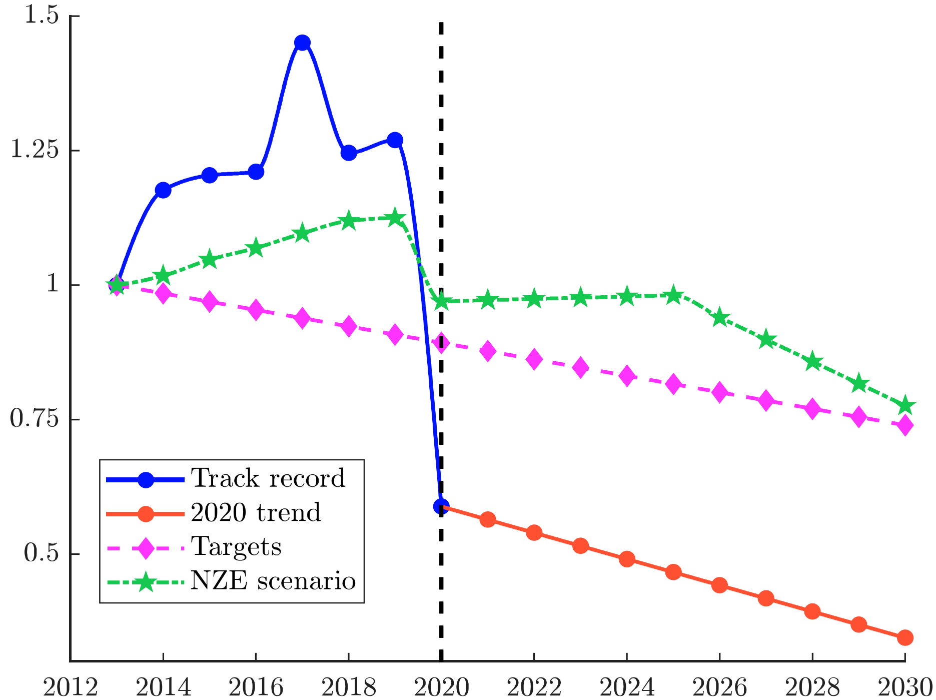
Figure 9.29: Carbon emissions, trend, targets and NZE scenario (Company $B$)




Source: CDP database (2021), IEA (2021) & Le Guenedal *et al.* (2022).

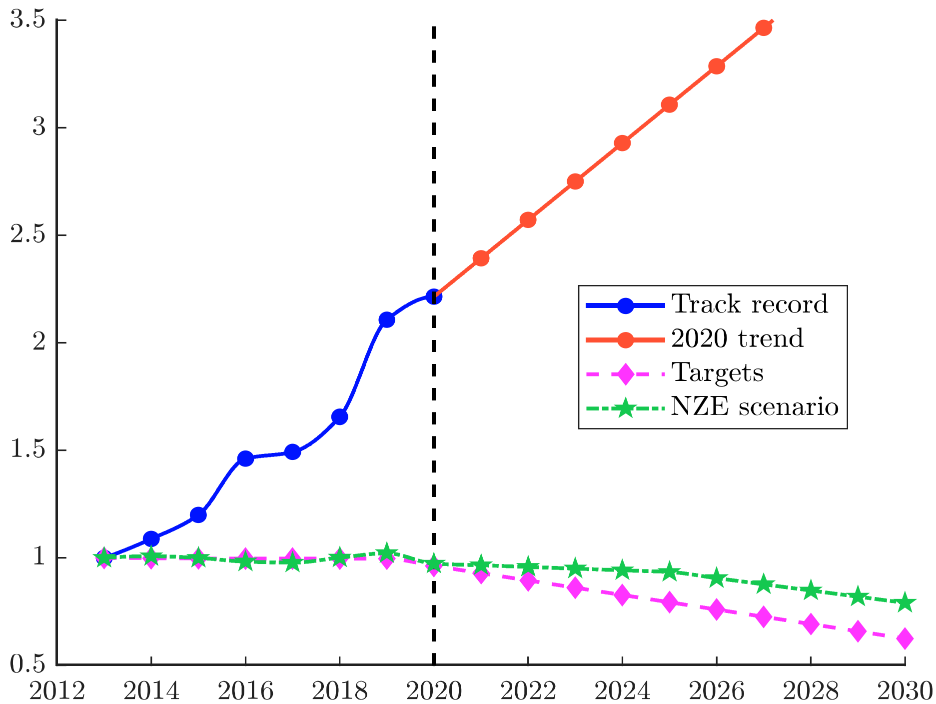
On a global basis, we observe an increase of carbon emissions between 2013 and 2019, and a plateau in 2020, which is probably due to emissions reduction related to the Covid-19 crisis (Figure 9.32). Carbon targets are in line with the NZE scenario until 2025. After this date, we clearly see that the targeted reduction rates are lower than the NZE required reduction rates. We notice that the reduction targets are more or less in line with the NZE scenario. However, and more strikingly, there is a huge gap between the upward trend between 2013 and 2020 and what has been announced by the companies. Figure 9.32 perfectly illustrates the interest of the $\mathcal{PAC}$ framework, since we observe inconsistencies between the ambition of these issuers on one side, and their participation and credibility on another side. The sector analysis is interesting, because we observe some large differences between the sectors in Figure 9.33. First, the trajectory of carbon emissions is highly dependent on the sector. Electricity is the sector that has been making the greatest effort whereas we observe a large increase in carbon emissions for the Industry sector. The impact of the Covid-19 pandemic on the transport sector is particularly striking. Second, there are small differences between the reduction targets, except for the Transport sector that is slightly less ambitious.

Figure 9.30: Carbon emissions, trend, targets and NZE scenario (Company $C$)



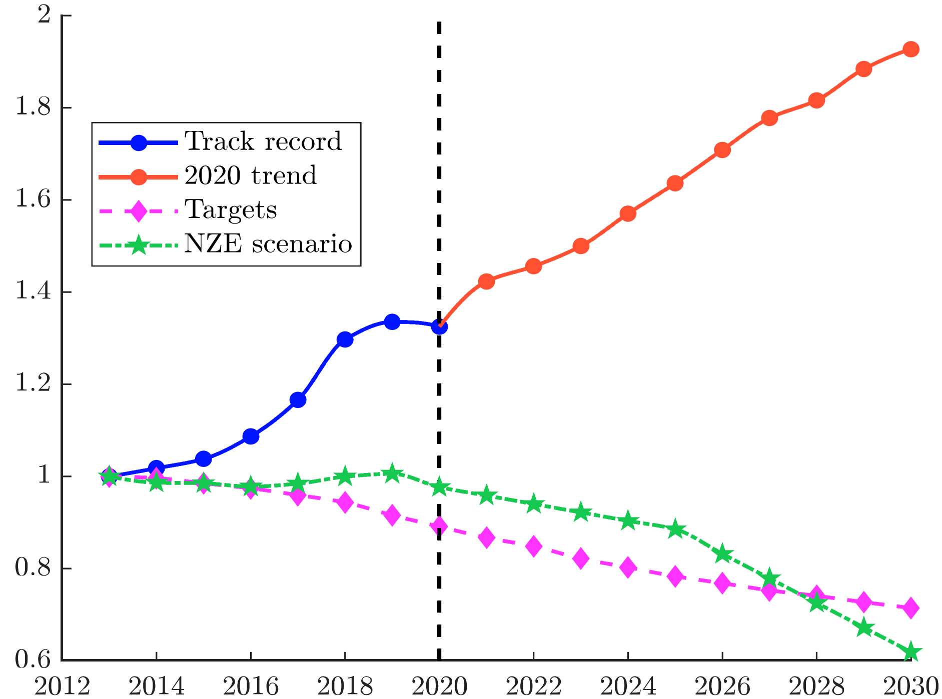

Source: CDP database (2021), IEA (2021) & Le Guenedal *et al.* (2022).

Figure 9.31: Carbon emissions, trend, targets and NZE scenario (Company $D$)



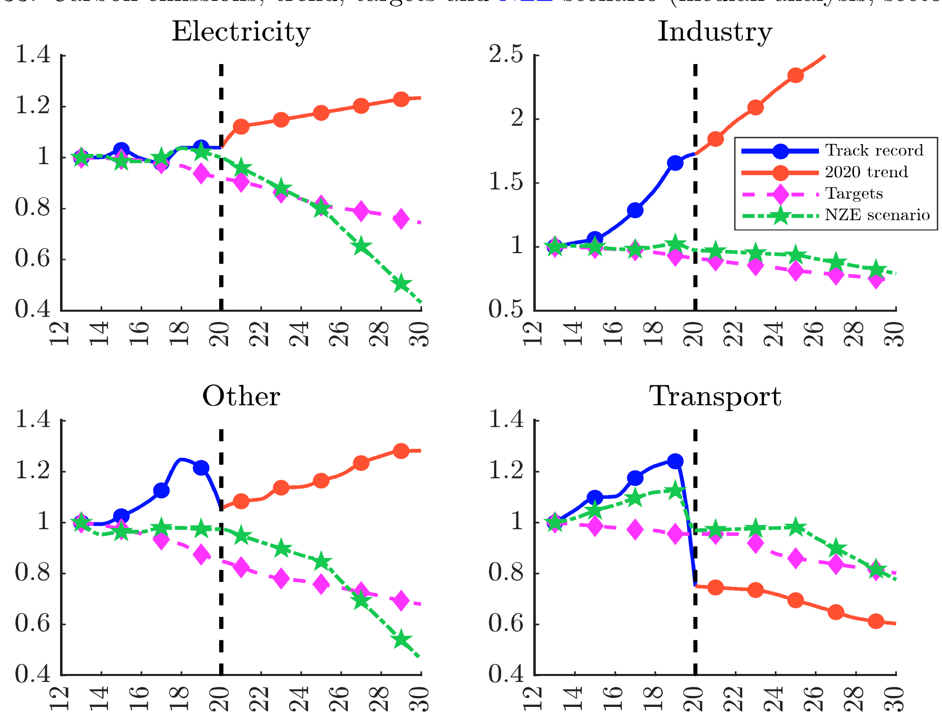

Source: CDP database (2021), IEA (2021) & Le Guenedal *et al.* (2022).

Figure 9.32: Carbon emissions, trend, targets and NZE scenario (median analysis, global universe)




Source: CDP database (2021), IEA (2021) & Le Guenedal *et al.* (2022).

Figure 9.33: Carbon emissions, trend, targets and NZE scenario (median analysis, sector universe)

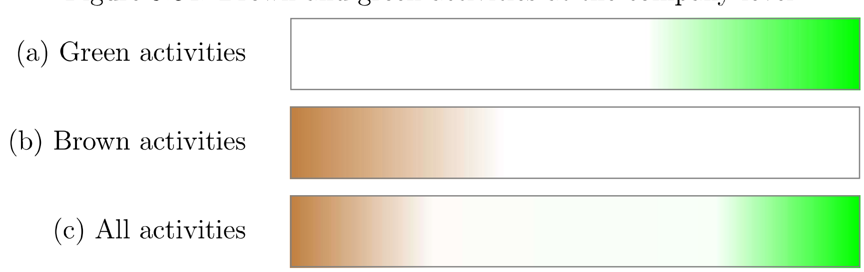



Source: CDP database (2021), IEA (2021) & Le Guenedal *et al.* (2022).

## 9.4 Greenness measures

Until now, we have focused on carbon metrics to measure the "*brownness*" of organizations or products. We now consider "*greenness*" measures, whose objective is to assess the positive contribution to limit global warming. In some sense, brownness and greenness measures are related. Nevertheless, we cannot deduce one measure from another one. Let us define the brown and green intensities of a company as the proportion of brown and green activities. We note them $\boldsymbol{\mathcal{BI}}$ and $\boldsymbol{\mathcal{GI}}$. By construction, we have $\boldsymbol{\mathcal{BI}} \in [0,1]$, $\boldsymbol{\mathcal{GI}} \in [0,1]$ and $0 \leq \boldsymbol{\mathcal{BI}} + \boldsymbol{\mathcal{GI}} \leq 1$. Most of the time, we have $\boldsymbol{\mathcal{BI}} + \boldsymbol{\mathcal{GI}} \neq 1$, meaning that we cannot deduce the green intensity from the brown intensity. While the carbon footprint is a well-defined concept, greenness is then more difficult to assess. In fact, it is a multi-faceted concept. For instance, if one company changes its business model so that its new products are carbon efficient, we can measure the company's greenness based on the avoided emissions generated by the change of the business model. For other companies, the greenness can be evaluated by estimating the R&D amount dedicated to green projects. Therefore, we observe a big difference between carbon and greenness metrics. Indeed, while it makes sense to compute the carbon footprint of all companies, the greenness may be indefinite for some companies, because they have no vocation to participate in the transition to a low-carbon economy. Therefore, Figure 9.34 illustrates that we cannot classifies all activities into these two categories, since there are many activities that are neither brown nor green. Some companies are then neutral and are not exposed to the green business. These remarks argue in favor of considering simple and homogeneous measures of greenness.

Figure 9.34: Brown and green activities at the company level

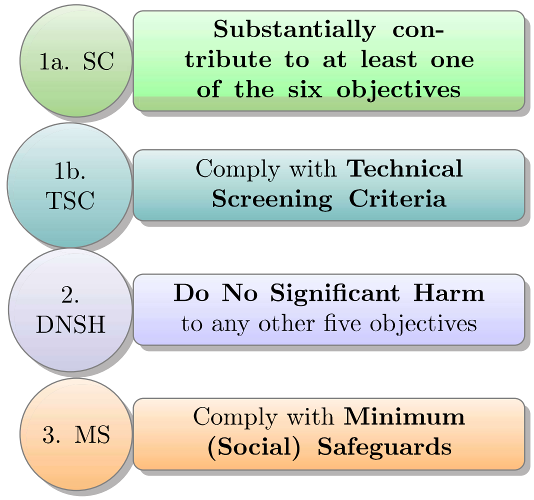



### 9.4.1 Green taxonomy

The purpose of a green taxonomy is to define what is green, and its objective is to inform investors about the greenness of their investments. Therefore, they can evaluate whether these levels satisfy or not their expectations. A green taxonomy is all the more important as we observe a strong development of green sentiment among investors (Brière and Ramelli, 2021). In this context, the investor may want to assess the proportion of his portfolio that is invested in environmentally sustainable assets. Therefore, a green taxonomy is necessary for both asset owners and managers.

The most famous example is the European green taxonomy, which has been already presented on page 33. We remind that the EU taxonomy for sustainable activities is "*a classification system, establishing a list of environmentally sustainable economic activities.*" These economic activities must have a substantive contribution to at least one of the following six environmental objectives: (1) climate change mitigation; (2) climate change adaptation; (3) sustainable use and protection of water and marine resources; (4) transition to a circular economy; (5) pollution prevention and control; (6) protection and restoration of biodiversity and ecosystem. Moreover, a business activity must also meet two other criteria to qualify as sustainable. First, the activity must do no significant harm to the other environmental objectives (DNSH constraint) and second, it must comply with minimum social safeguards (MS constraint). Figure 9.35 summarizes the different steps.

Figure 9.35: EU taxonomy for sustainable activities




**Remark 115** *The EU taxonomy is not finalized and only concerns the first two objectives as of today (January 2023).*

### 9.4.2 Green revenue share

#### Relationship between the green intensity and the green revenue share

There are several ways to compute the green intensity. This is why we observe some significant differences between data providers. One method is to translate the 3-step approach of the EU taxonomy into the following equation:

$$\boldsymbol{\mathcal{GI}} = \frac{\mathcal{GR}}{\mathcal{TR}} \cdot (1 - \mathcal{P}) \cdot \mathbb{1}\left\{\boldsymbol{\mathcal{S}} \geq \boldsymbol{\mathcal{S}}^{\star}\right\}$$

where $\mathcal{GR}$ is the green revenue deduced from the six environmentally sustainable objectives, $\mathcal{TR}$ is the total revenue, $\mathcal{P}$ is the penalty coefficient reflecting the DNSH constraint, $\boldsymbol{\mathcal{S}}$ is the minimum safeguard score and $\boldsymbol{\mathcal{S}}^{\star}$ is the threshold. The first term is a proxy of the turnover KPI and corresponds to the green revenue share:

$$\boldsymbol{\mathcal{GRS}} = \frac{\mathcal{GR}}{\mathcal{TR}}$$

By construction, we have $0 \leq \boldsymbol{\mathcal{GRS}} \leq 1$. This measure is then impacted by the DNSH coefficient. If the penalty coefficient is equal to zero, the green activities of the issuer do not significantly harm the other objectives and we have $\boldsymbol{\mathcal{GI}} = \boldsymbol{\mathcal{GRS}}$. Otherwise, the green intensity satisfies $0 \leq \boldsymbol{\mathcal{GI}} = \boldsymbol{\mathcal{GRS}} \cdot (1 - \mathcal{P}) \leq \boldsymbol{\mathcal{GRS}}$. Finally, the indicator function $\mathbb{1}\left\{\boldsymbol{\mathcal{S}} \geq \boldsymbol{\mathcal{S}}^{\star}\right\}$ is a binary all-or-nothing variable. It is equal to one if the firm complies with minimum social safeguards. Otherwise, the green intensity is equal to zero if the firm doesn't pass this materiality test. It follows that an upper bound of the green intensity is the green revenue share since we have $\boldsymbol{\mathcal{GI}} \leq \boldsymbol{\mathcal{GRS}}$.

Box 9.5: EU green taxonomy

The EU taxonomy is described in the Delegated Act on the climate objectives of 4 June 2021. For each activity, three items are provided: the description of the activity, the technical screening criteria, and the DNSH compliance. Let us consider Transport of $CO_2$ (page 100 of the Delegated Act). We have the following information:

1. Description of the activity
   This concerns the transport of captured $CO_2$ via all modes; the construction and operation of $CO_2$ pipelines and retrofit of gas networks where the main purpose is the integration of captured $CO_2$. The economic activities in this category could be associated with several NACE codes, in particular F42.21 and H49.50. An economic activity in this category is an enabling activity.

2. Technical screening criteria
   This activity has a substantial contribution to climate change mitigation:

   (a) The $CO_2$ transported from the installation where it is captured to the injection point does not lead to $CO_2$ leakages above 0.5% of the mass of $CO_2$ transported.
   (b) The $CO_2$ is delivered to a permanent $CO_2$ storage site that meets the criteria for underground geological storage of $CO_2$ set out in Section 5.12 of this Annex; or to other transport modalities, which lead to permanent $CO_2$ storage site that meet those criteria.
   (c) Appropriate leak detection systems are applied and a monitoring plan is in place, with the report verified by an independent third party.
   (d) The activity may include the installation of assets that increase the flexibility and improve the management of an existing network.

3. Do no significant harm
   Three out of five categories are concerned: (2) climate change adaptation: the activity complies with the criteria set out in Appendix A to this Annex; (3) sustainable use and protection of water and marine resources: The activity complies with the criteria set out in Appendix B to this Annex. (4) transition to a circular economy: N/A; (5) pollution prevention and control: N/A; (6) protection and restoration of biodiversity and ecosystems: the activity complies with the criteria set out in Appendix D to this Annex.

This example shows that the revenues generated by the transport of $CO_2$ are not necessarily green, because the technical screening criteria imply that $CO_2$ leakages must be below 0.5% of the mass of $CO_2$ transported. We also observe that some criteria are generic while others are specific to an activity. For instance, the life cycle GHG emissions from the generation of electricity (whatever the electricity source) must be lower than 100 $gCO_2e$ per kWh. Some criteria also concern the activity efficiency. For example, the power density of the electricity generation facility must be above 5 Watt per $m^2$ for the hydropower sector.

Concerning the DNSH compliance criteria, they may for instance imply that the activity does use persistent organic pollutants, ether, mercury, substances that deplete the ozone layer, certain hazardous substances, etc. In a similar way, an environmental impact assessment (EIA) must be conducted for sites located near biodiversity-sensitive areas.

**Example 45** *We consider a company in the hydropower sector which has five production sites. Below, we indicate the power density efficiency, the GHG emissions, the DNSH compliance with respect to the biodiversity and the corresponding revenue:*

| Site | #1 | #2 | #3 | #4 | #5 |
|---|---|---|---|---|---|
| Efficiency (in Watt per $m^2$) | 3.2 | 3.5 | 3.3 | 5.6 | 4.2 |
| GHG emissions (in $gCO_2e$ per kWh) | 35 | 103 | 45 | 12 | 36 |
| Biodiversity DNSH compliance | ✓ | ✓ | ✓ | ✓ | |
| Revenue (in \$ mn) | 103 | 256 | 89 | 174 | 218 |

The total revenue is equal to:

$$\mathcal{TR} = 103 + 256 + 89 + 174 + 218 = \$840 \text{ mn}$$

We notice that the fourth site does not pass the technical screening, because the power density is above 5 Watt per $m^2$. The second site does not also comply because it has a GHG emissions greater than 100 $gCO_2e$ per kWh. We deduce that the green revenue is equal to:

$$\mathcal{GR} = 103 + 89 + 218 = \$410 \text{ mn}$$

We conclude that the green revenue share is equal to 48.8%. According to the EU green taxonomy, the green intensity is lower because the last site is close to a biodiversity area and has a negative impact. Therefore, we have:

$$\boldsymbol{\mathcal{GI}} = \frac{103 + 89}{840} = 22.9\%$$

**Statistics**

In Table 9.27, we report the descriptive statistics of green revenue share calculated by Barahhou *et al.* (2022) with the MSCI database. For each category[46], they have computed:

- The frequency $\mathbf{F}(x) = \Pr\{\boldsymbol{\mathcal{GRS}} > x\}$;
- The statistical quantile $\mathbf{Q}(\alpha) = \inf\{x : \Pr\{\boldsymbol{\mathcal{GRS}} \leq x\} \geq \alpha\}$;
- The arithmetic average $n^{-1}\sum_{i=1}^{n}\boldsymbol{\mathcal{GRS}}_i$ and the weighted mean $\boldsymbol{\mathcal{GRS}}(b) = \sum_{i=1}^{n} b_i \boldsymbol{\mathcal{GRS}}_i$ where $b_i$ is the weight of issuer $i$ in the MSCI ACWI IMI benchmark.

For instance, 9.82% of issuers have a green revenue share that concerns alternative energy. This figure becomes less than 1% if we consider a green revenue share greater than 50%. The average value is equal to 1.36% whereas the weighted value is equal to 0.77%. This indicates a small cap bias. For energy efficiency, the average is lower than the weighted mean, implying a bias towards big companies. If we consider the total green revenue share, 27.85% have a positive figure and only 3.17% have a figure greater than 50%. The 90% quintile is equal to 11.82%. Therefore, we notice a high positive skewness for the distribution. The green revenue share is then located in a small number of companies.

**Remark 116** *Barahhou et al. (2022) estimated that the green revenue share of the MSCI World index and the Bloomberg Global Investment Grade Corporate Bond index are respectively equal to* 5.24% *and* 3.49% *in June 2022. This is not a high figure, because the economy is today far to be green. These results are confirmed by Alessi and Battiston (2022), who estimated "a greenness of about 2.8% for EU financial markets" according to the existing EU green taxonomy*[47]*.*

[46] The MSCI taxonomy uses 6 categories: (1) alternative energy, (2) energy efficiency, (3) green building, (4) pollution prevention and control, (5) sustainable agriculture and (6) sustainable water.

[47] This concerns the first two categories, which are the most important.

Table 9.27: Statistics in % of green revenue share (MSCI ACWI IMI, June 2022)

| Category | Frequency $\mathbf{F}(x)$ | | | | Quantile $\mathbf{Q}(\alpha)$ | | | | Mean | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 25% | 50% | 75% | 75% | 90% | 95% | Max | Avg | Wgt |
| (1) | 9.82 | 1.47 | 0.96 | 0.75 | 0.00 | 0.00 | 2.85 | 100.00 | 1.36 | 0.77 |
| (2) | 14.10 | 1.45 | 0.65 | 0.31 | 0.00 | 1.25 | 6.12 | 100.00 | 1.39 | 3.50 |
| (3) | 4.84 | 1.68 | 1.02 | 0.31 | 0.00 | 0.00 | 0.00 | 100.00 | 1.16 | 0.51 |
| (4) | 4.79 | 0.30 | 0.10 | 0.06 | 0.00 | 0.00 | 0.00 | 99.69 | 0.32 | 0.22 |
| (5) | 1.00 | 0.39 | 0.20 | 0.09 | 0.00 | 0.00 | 0.00 | 98.47 | 0.26 | 0.10 |
| (6) | 4.75 | 0.28 | 0.11 | 0.05 | 0.00 | 0.00 | 0.00 | 99.98 | 0.29 | 0.14 |
| Total | 27.85 | 5.82 | 3.17 | 1.68 | 0.42 | 11.82 | 30.36 | 100.00 | 4.78 | 5.24 |

Source: MSCI (2022) & Barahhou *et al.* (2022).

### 9.4.3 Green capex

Green capEx refers to investments in physical assets that contribute to environmental sustainability or climate change mitigation. These long-term investments in physical assets help reduce a company's environmental footprint and typically include renewable energy infrastructure, energy-efficient buildings, sustainable manufacturing equipment, heat recovery systems, green manufacturing infrastructure, clean transportation, and water recycling and treatment systems. While green revenue shares reflect a company's current environmental footprint, green capex is generally considered an indicator of a company's future environmental footprint, making it a forward-looking measure of the company's greenness.

Table 9.28: Carbon intensity and green intensity by sector (MSCI EMU, June 2024)

| Sector | Carbon intensity (in $tCO_2e$/\$ mn) | | | | Green intensity (in %) | | |
|---|---|---|---|---|---|---|---|
| | $\mathcal{SC}_1$ | $\mathcal{SC}_{1-2}$ | $\mathcal{SC}_{1-3}^{\text{up}}$ | $\mathcal{SC}_{1-3}$ | Turnover | Capex | Opex |
| Communication Services | 2.6 | 32.2 | 73.6 | 133.4 | 1.1 | 0.5 | 0.2 |
| Consumer Discretionary | 6.7 | 19.5 | 170.9 | 617.7 | 2.0 | 7.9 | 7.4 |
| Consumer Staples | 19.4 | 34.3 | 296.0 | 471.2 | 0.0 | 7.9 | 4.2 |
| Energy | 220.7 | 237.8 | 462.5 | 2 120.9 | 3.9 | 19.1 | 8.7 |
| Financials | 1.2 | 4.7 | 25.8 | 684.5 | 0.1 | 0.1 | 0.1 |
| Health Care | 11.9 | 28.1 | 106.5 | 128.9 | 0.0 | 1.1 | 0.0 |
| Industrials | 20.7 | 30.7 | 147.3 | 3 383.6 | 9.1 | 10.7 | 12.1 |
| Information Technology | 3.6 | 14.8 | 106.5 | 653.2 | 0.3 | 0.4 | 0.3 |
| Materials | 477.4 | 780.9 | 1 026.4 | 1 418.3 | 1.7 | 5.5 | 3.1 |
| Real Estate | 48.1 | 90.7 | 119.3 | 515.0 | 17.5 | 35.0 | 17.2 |
| Utilities | 433.3 | 497.7 | 601.0 | 911.8 | 31.6 | 75.4 | 55.7 |
| MSCI EMU | 65.6 | 95.5 | 209.3 | 1 162.5 | 4.1 | 9.2 | 7.2 |

Source: MSCI, Trucost & Author's calculations.

### 9.4.4 Green intensity versus carbon intensity

Green intensity does not convey the same information as carbon intensity. For example, one might think that companies with a high green intensity have a low carbon intensity. This is not currently the case. Table 9.28 shows the carbon and green intensities of sectors in the MSCI EMU Index at the end of June 2024. We find that high carbon sectors also have high green intensities. On average, utilities stocks in the MSCI EMU stocks have a Scope 1 of 433 tonnes of $CO_2$ per million dollars of revenue, while the green turnover, capex and opex[48] are 31.6%, 75.4% and 55.7%, respectively. These results show that some carbon-intensive sectors also have high green footprints. This is really one of the big issues in implementing a net zero investment policy, because reducing the carbon footprint of a portfolio is not the same as increasing the green footprint of the portfolio or improving the greenness of the economy.

**Remark 117** *Other metrics used to assess a company's green footprint include green R&D investment and the green-to-brown ratio, which compares environmentally beneficial activities to environmentally harmful activities.*

## 9.5 Exercises

[48]We use the three measures of green intensity defined in the EU taxonomy: green turnover, green capex (capital expenditure) and green opex (operating expenditure). The data correspond to the figures reported by the companies.

Chapter 10

# Transition Risk Modeling

## 10.1 A primer on the economic analysis of negative externalities

### 10.1.1 Optimal taxation

### 10.1.2 Economic theory of quotas

### 10.1.3 Game theory

#### Cooperative solution

#### Non-cooperative solution

#### Coordinated solution

### 10.1.4 Uncertainty

McKibbin and Wilcoxen (2002)

### 10.1.5 Irreversibility

### 10.1.6 Cost-benefit analysis versus option theory

## 10.2 Carbon tax and pricing

### 10.2.1 Mathematics of carbon tax

### 10.2.2 Abatement cost

### 10.2.3 Cap-and-trade

## 10.3 Stranded assets

## 10.4 Decarbonization pathway

### 10.4.1 Global analysis

### 10.4.2 Sector analysis

#### Power and electricity

#### Hydrogen

#### Buildings

#### Mobility and transport

#### Materials

#### Industry

#### Water management

#### Waste management and circular economy

## 10.5 Transition risk measures

### 10.5.1 Temperature rating modeling

## 10.6 Exercises

# Chapter 11

# Climate Portfolio Construction

With the 2015 Paris Agreement, the development of ESG investing, and the emergence of net-zero investment policies, climate risk is undoubtedly the most important issue and challenge for asset owners and managers today and in the coming years. Building portfolios to manage climate risk began in 2014, when asset owners AP4 and FRR worked with asset manager Amundi and index provider MSCI to define an investment strategy to help hedge climate risk (Andersson *et al.*, 2016). Together, they defined the concept of a low-carbon portfolio. The 2015–2020 period corresponds to the growth of ESG investing and the adoption of sustainable finance by many asset owners and managers. During this period, climate investing is part of ESG investing, and climate portfolio construction remains relatively marginal, essentially involving passive management. On the contrary, ESG investing has seen a major development with the adoption of ESG scores in active management. Since 2020, we have seen a new trend. The line between ESG investing and climate investing is becoming increasingly blurred, and the two issues are now separate. One of the reasons for this is the emergence of net-zero investment policies, which have profoundly changed the investment decisions of asset owners. This separation has accelerated with COP26. The proliferation of net zero alliances (GFANZ, NZAOA, NZAM, NZBA, etc.), the commitments made by financial institutions (asset managers, banks, pension funds, insurance companies, etc.), and the push for regulation[1] are all contributing to the shift from ESG investing to climate investing.

This chapter is dedicated to portfolio construction when we integrate climate risk measures. It is therefore closely related to Chapter 9, as we use carbon footprint and green footprint metrics. It is also related to Chapter 10 on transition risk, as the goal of climate investing is to reduce the transition risk of investment portfolios. Integrating physical risk is more complicated today, because we do not have the right metrics to assess physical risk at the corporate or security level. Finally, it is related to Chapter 2, which is dedicated to the impact of ESG investing on asset prices and portfolio returns, because we use the same tools and methodologies. We will therefore make extensive use of portfolio optimization. A comprehensive review of portfolio optimization is presented in the first section. We distinguish between allocations to equity and fixed income portfolios because they require two different approaches. The reason is that we generally measure equity risk in terms of volatility risk, while bond risk is multi-dimensional and must at least integrate duration and credit risk. The second section is a guide to building a low-carbon portfolio. While there are several approaches, the choice of Scope emissions is certainly the most important decision and has a major impact on asset allocation. Finally, the last section focuses on net-zero investing and lists many challenges to defining an investment portfolio that is aligned with a net-zero emissions scenario.

[1] Examples include the Net-Zero Industry Act of the European Commission, the work of the NGFS on climate scenarios, or the climate stress tests organized by the ECB.

## 11.1 Portfolio optimization in practice

Before studying portfolio allocation in the context of climate risk, we first begin to remind some basics about portfolio optimization. As mentioned by Perrin and Roncalli (2020), the success of mean-variance optimization is due to the appealing properties of the quadratic utility function, and it is easy to solve numerically quadratic programming problems[2]. This is why most of the portfolio allocation problems that we will encounter in this chapter will be cast into a QP problem, whose standard formulation is[3]:

$$\begin{aligned} x^\star &= \arg\min \frac{1}{2} x^\top Q x - x^\top R \\ & \text{s.t. } \left\{ \begin{array}{l} Ax = B \\ Cx \leq D \\ x^- \leq x \leq x^+ \end{array} \right. \end{aligned} \tag{11.1}$$

where $x$ is a $n \times 1$ vector, $Q$ is a $n \times n$ matrix, $R$ is a $n \times 1$ vector, $A$ is a $n_A \times n$ matrix, $B$ is a $n_A \times 1$ vector, $C$ is a $n_C \times n$ matrix, $D$ is a $n_C \times 1$ vector, and $x^-$ and $x^+$ are two $n \times 1$ vectors. If $n_A = 0$, there is no equality constraints. Similarly, there is no inequality constraints if $n_C = 0$. If there is no lower bounds or/and upper bounds, this implies that $x^- = -\infty \cdot \mathbf{1}_n$ and $x^+ = \infty \cdot \mathbf{1}_n$. From a numerical viewpoint, QP solvers generally replace these bounds by $x^- = -c \cdot \mathbf{1}_n$ and $x^+ = c \cdot \mathbf{1}_n$ where $c$ is a large floating-point number[4] (*e.g.*, $c = 10^{200}$).

### 11.1.1 Equity portfolios

#### Basic optimization problems

We consider a universe of $n$ assets. We note $w$ the vector of portfolio weights. Let $\mu$ and $\Sigma$ be the vector of expected returns and the covariance matrix of asset returns. The long-only mean-variance optimization problem is given by:

$$\begin{aligned} w^\star &= \arg\min \frac{1}{2} w^\top \Sigma w - \gamma w^\top \mu \\ & \text{s.t. } \left\{ \begin{array}{l} \mathbf{1}_n^\top w = 1 \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right. \end{aligned}$$

where $\gamma$ is the risk-tolerance coefficient, the equality constraint is the budget constraint ($\sum_{i=1}^n w_i = 1$) and the bounds correspond to the no short-selling restriction ($w_i \geq 0$). We recognize a QP problem where $Q = \Sigma$, $R = \gamma\mu$, $A = \mathbf{1}_n^\top$, $B = 1$, $w^- = \mathbf{0}_n$ and $w^+ = \mathbf{1}$. In this problem, we have one equality constraint ($n_A = 1$) and zero inequality constraint[5] ($n_C = 1$).

In many problems, we will have to manage the portfolio with respect to a benchmark. In this case, the objective function depends on the tracking error risk variance $\sigma^2(w \mid b) = (w - b)^\top \Sigma (w - b)$ where $b$ is the vector of benchmark weights. In Section 3.1.1 on page 148, we have seen that the tracking error optimization problem is defined as:

$$\begin{aligned} w^\star &= \arg\min \frac{1}{2} w^\top \Sigma w - w^\top (\gamma\mu + \Sigma b) \\ & \text{s.t. } \left\{ \begin{array}{l} \mathbf{1}_n^\top w = 1 \\ \mathbf{0}_n \leq w \leq \mathbf{1}_n \end{array} \right. \end{aligned}$$

[2] See Appendix A.1.2 on page 1054.
[3] The objective function is a quadratic form and is noted $\mathcal{QF}(x; Q, R, \mathbf{0}_n)$.
[4] The largest finite floating-point number in IEEE double precision is equal to $\left(2 - 2^{-52}\right) \times 2^{1023}$.
[5] We do not take into account the bounds.

This is exactly the same QP problem as previously except that $R = \gamma\mu + \Sigma b$. If the objective of the portfolio manager is to minimize the tracking error risk, we obtain $R = \Sigma b$.

**Specification of the constraints**

We can extend the previous framework by considering more constraints. For instance, we consider a sector weight constraint:

$$s_j^- \leq \sum_{i \in \boldsymbol{\mathcal{S}}ector_j} w_i \leq s_j^+$$

We notice that:

$$\sum_{i \in \boldsymbol{\mathcal{S}}ector_j} w_i = \boldsymbol{s}_j^\top w$$

where $\boldsymbol{s}_j$ is the $n \times 1$ sector-mapping vector whose elements are $\boldsymbol{s}_{i,j} = \mathbb{1}\left\{i \in \boldsymbol{\mathcal{S}}ector_j\right\}$. We deduce that the sector constraint can be written as:

$$s_j^- \leq \sum_{i \in \boldsymbol{\mathcal{S}}ector_j} w_i \leq s_j^+ \Leftrightarrow \left\{ \begin{array}{l} s_j^- \leq \boldsymbol{s}_j^\top w \\ \boldsymbol{s}_j^\top w \leq s_j^+ \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} -\boldsymbol{s}_j^\top w \leq -s_j^- \\ \boldsymbol{s}_j^\top w \leq s_j^+ \end{array} \right.$$

It follows that the inequality constraint $Cw \leq D$ is defined by the following system:

$$\underbrace{\begin{pmatrix} -\boldsymbol{s}_j^\top \\ \boldsymbol{s}_j^\top \end{pmatrix}}_{C} w \leq \underbrace{\begin{pmatrix} -s_j^- \\ s_j^+ \end{pmatrix}}_{D}$$

In this case, $C$ is a $2 \times n$ matrix and $D$ is a $2 \times 1$ vector. The previous analysis can be extended when there are many sectors.

We denote by $\boldsymbol{\mathcal{S}}$ a vector of scores (*e.g.*, ESG scores) and we would like to impose that the (linear) score of the portfolio is greater than a threshold $\boldsymbol{\mathcal{S}}^\star$:

$$\sum_{i=1}^n w_i \boldsymbol{\mathcal{S}}_i \geq \boldsymbol{\mathcal{S}}^\star$$

The QP form of this constraint is:

$$-\boldsymbol{\mathcal{S}}^\top w \leq -\boldsymbol{\mathcal{S}}^\star$$

Let us now assume that we would like to apply this constraint to a sector. In this case, we have:

$$\begin{aligned} \sum_{i \in \boldsymbol{\mathcal{S}}ector_j} w_i \boldsymbol{\mathcal{S}}_i \geq \boldsymbol{\mathcal{S}}_j^\star \quad & \Leftrightarrow \quad \sum_{i=1}^n \mathbb{1}\left\{i \in \boldsymbol{\mathcal{S}}ector_j\right\} w_i \boldsymbol{\mathcal{S}}_i \geq \boldsymbol{\mathcal{S}}_j^\star \\ & \Leftrightarrow \quad \sum_{i=1}^n \boldsymbol{s}_{i,j} w_i \boldsymbol{\mathcal{S}}_i \geq \boldsymbol{\mathcal{S}}_j^\star \\ & \Leftrightarrow \quad \sum_{i=1}^n w_i \left(\boldsymbol{s}_{i,j} \boldsymbol{\mathcal{S}}_i\right) \geq \boldsymbol{\mathcal{S}}_j^\star \\ & \Leftrightarrow \quad \left(\boldsymbol{s}_j \circ \boldsymbol{\mathcal{S}}\right)^\top w \geq \boldsymbol{\mathcal{S}}_j^\star \end{aligned}$$

where $a \circ b$ is the Hadamard product: $\left(a \circ b\right)_i = a_i b_i$. The QP form of the sector-specific score constraint is defined by $C = -\left(\boldsymbol{s}_j \circ \boldsymbol{\mathcal{S}}\right)^\top$ and $D = -\boldsymbol{\mathcal{S}}_j^\star$.

**Example 46** *We consider a capitalization-weighted equity index, which is composed of 8 stocks. The weights are equal to 23%, 19%, 17%, 13%, 9%, 8%, 6% and 5%. We assume that the stock volatilities are equal to 22%, 20%, 25%, 18%, 35%, 23%, 13% and 29%. The correlation matrix is given by:*

$$\mathbb{C} = \begin{pmatrix} 100\% & & & & & & & \\ 80\% & 100\% & & & & & & \\ 70\% & 75\% & 100\% & & & & & \\ 60\% & 65\% & 80\% & 100\% & & & & \\ 70\% & 50\% & 70\% & 85\% & 100\% & & & \\ 50\% & 60\% & 70\% & 80\% & 60\% & 100\% & & \\ 70\% & 50\% & 70\% & 75\% & 80\% & 50\% & 100\% & \\ 60\% & 65\% & 70\% & 75\% & 65\% & 70\% & 80\% & 100\% \end{pmatrix}$$

*The ESG score, carbon intensity and sector of the eight stocks are the following:*

| Stock | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 |
|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{\mathcal{S}}$ | −1.20 | 0.80 | 2.75 | 1.60 | −2.75 | −1.30 | 0.90 | −1.70 |
| $\boldsymbol{\mathcal{CI}}$ | 125 | 75 | 254 | 822 | 109 | 17 | 341 | 741 |
| $\boldsymbol{\mathcal{S}}ector$ | 1 | 1 | 2 | 2 | 1 | 2 | 1 | 2 |

The objective function is minimizing the tracking error risk. We deduce that the equivalent QP problem is:

$$\begin{aligned} w^\star &= \arg\min \frac{1}{2} w^\top Q w - w^\top R \\ &\text{s.t.} \left\{ \begin{array}{l} Aw = B \\ Cw \leq D \\ w^- \leq w \leq w^+ \end{array} \right. \end{aligned}$$

where[6]:

$$Q = \Sigma = \begin{pmatrix} 484.00 & 352.00 & 385.00 & 237.60 & 539.00 & 253.00 & 200.20 & 382.80 \\ 352.00 & 400.00 & 375.00 & 234.00 & 350.00 & 276.00 & 130.00 & 377.00 \\ 385.00 & 375.00 & 625.00 & 360.00 & 612.50 & 402.50 & 227.50 & 507.50 \\ 237.60 & 234.00 & 360.00 & 324.00 & 535.50 & 331.20 & 175.50 & 391.50 \\ 539.00 & 350.00 & 612.50 & 535.50 & 1225.00 & 483.00 & 364.00 & 659.75 \\ 253.00 & 276.00 & 402.50 & 331.20 & 483.00 & 529.00 & 149.50 & 466.90 \\ 200.20 & 130.00 & 227.50 & 175.50 & 364.00 & 149.50 & 169.00 & 301.60 \\ 382.80 & 377.00 & 507.50 & 391.50 & 659.75 & 466.90 & 301.60 & 841.00 \end{pmatrix} \times 10^{-4}$$

and:

$$R = \Sigma b = \begin{pmatrix} 3.74 \\ 3.31 \\ 4.39 \\ 3.07 \\ 5.68 \\ 3.40 \\ 2.02 \\ 4.54 \end{pmatrix} \times 10^{-2}$$

[6]We have $\Sigma_{i,j} = \mathbb{C}_{i,j}\sigma_i\sigma_j$.

We assume that the portfolio is long-only. It follows that $w^- = \mathbf{0}_8$ and $w^+ = \mathbf{1}_8$. To satisfy the budget constraint $\sum_{i=1}^{8} w_i = 1$, we have a first linear equation $A_0 w = B_0$ where $A_0 = \mathbf{1}_8^\top$ and $B_0 = 1$. We consider three type of constraints:

- We impose a relative reduction of the benchmark carbon intensity:

$$\boldsymbol{\mathcal{CI}}(w) \leq (1 - \boldsymbol{\mathcal{R}}) \boldsymbol{\mathcal{CI}}(b)$$

where $\boldsymbol{\mathcal{R}}$ is the reduction rate. Since $\boldsymbol{\mathcal{CI}}(w) = \boldsymbol{\mathcal{CI}}^\top w$, we deduce the following inequality constraint $C_1 w \leq D_1$ where $C_1 = \boldsymbol{\mathcal{CI}}^\top$ and $D_1 = (1 - \boldsymbol{\mathcal{R}}) \boldsymbol{\mathcal{CI}}(b)$.

- We impose an absolute increase of the benchmark ESG score:

$$\boldsymbol{\mathcal{S}}(w) \geq \boldsymbol{\mathcal{S}}(b) + \Delta \boldsymbol{\mathcal{S}}^\star$$

Since $\boldsymbol{\mathcal{S}}(w) = \boldsymbol{\mathcal{S}}^\top w$, we deduce the following inequality constraint $C_2 w \leq D_2$ where $C_2 = -\boldsymbol{\mathcal{S}}^\top$ and $D_2 = -(\boldsymbol{\mathcal{S}}(b) + \Delta \boldsymbol{\mathcal{S}}^\star)$.

- We impose the sector neutrality of the portfolio meaning that:

$$\sum_{i \in \boldsymbol{\mathcal{S}}ector_j} w_i = \sum_{i \in \boldsymbol{\mathcal{S}}ector_j} b_i$$

We note:

$$A_1 = \boldsymbol{s}_1^\top = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

and:

$$A_2 = \boldsymbol{s}_2^\top = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

We compute $B_1 = \boldsymbol{s}_1^\top b = \sum_{i \in \boldsymbol{\mathcal{S}}ector_1} b_i$ and $B_2 = \boldsymbol{s}_2^\top b = \sum_{i \in \boldsymbol{\mathcal{S}}ector_2} b_i$. The sector neutrality constraint can be written as:

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} w = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

Let us now combine the different constraints. For that, we use the block matrix notation, which is particularly convenient when manipulating nested QP problems. The set #1 of constraint corresponds to the reduction of the carbon intensity, the set #2 corresponds to the ESG score improvement, the set #3 combines the two constraints and we add the sector neutrality in the set #4 of constraints.

| Set of constraints | Carbon intensity | ESG score | Sector neutrality | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|---|---|---|
| #1 | ✓ | | | $A_0$ | $B_0$ | $C_1$ | $D_1$ |
| #2 | | ✓ | | $A_0$ | $B_0$ | $C_2$ | $D_2$ |
| #3 | ✓ | ✓ | | $A_0$ | $B_0$ | $\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$ | $\begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$ |
| #4 | ✓ | ✓ | ✓ | $\begin{bmatrix} A_0 \\ A_1 \\ A_2 \end{bmatrix}$ | $\begin{bmatrix} B_0 \\ B_1 \\ B_2 \end{bmatrix}$ | $\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$ | $\begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$ |