# Reinforcement Learning - Tutorial

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# Markov Decision Processes

# Problem setting [1/2]

## Building blocks of Markov Decision Process

- **State** $s \in \mathcal{S}$: Current system information, needed to compute next state, rewards and actions
- **Action** $a \in \mathcal{A}$: Response to the state, provided by policy $\pi : s \to a$
- **Reward** $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$: Reward or penalty $r(s, a)$ after performing action $a$ in state $s$
- **Transition** $\mathcal{P} : \mathcal{S} \times \mathcal{S} \times \mathcal{A} \to [0, 1]$: Function to guide time step, based on state, action and exogenous information
- **Discount factor** $\gamma \in [0, 1]$: Discount of future rewards, both for convergence and emphasis on present rewards

# Problem setting [2/2]

## Other problem properties

- **Policy** $\pi : s \rightarrow a$: Mapping from state to action, which we must learn
- **Exogenous variable** $W$: Stochastic information revealed over time (not in this environment)
- **Objective function** $J$:

$$J(\theta) = E_{\tau \sim \pi}[R(\tau)]$$

# Dynamic programming solution

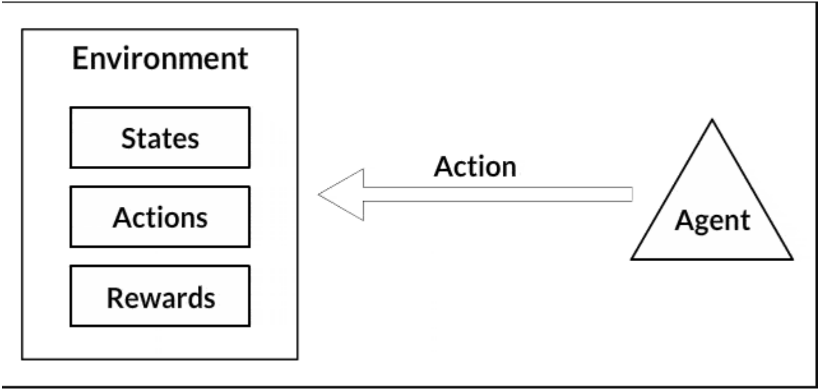
Solve system of Bellman equations:

$$V(s) = \max_a \left( R(s,a) + \gamma \sum_{s'} P(s,a,s')V(s') \right)$$

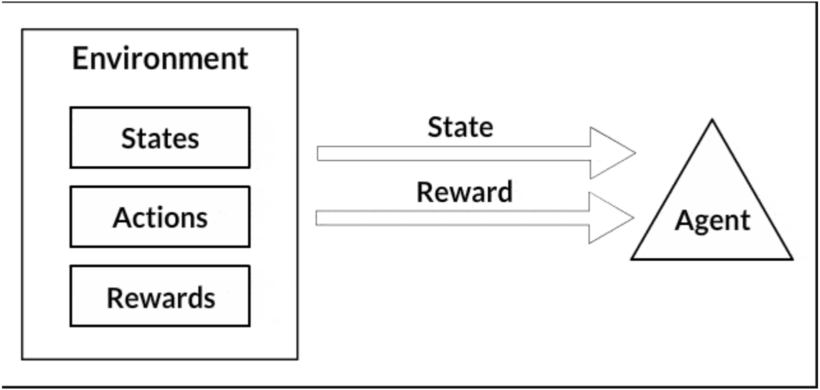
- Can be solved to optimality, but fails for large problems.
- In this tutorial, we try to *learn* the value functions.

# Q-learning

# Model-free learning [1/2]



# Model-free learning [2/2]



# Q-learning algorithm [1/2]

**Algorithm 1:** Q-learning algorithm

Fix parameters: learning rate $\alpha \in (0, 1]$, $\epsilon \in [0, 1]$ discount rate $\gamma \in [0, 1]$;
Initialize $Q(s, a) = 0, \forall s \in \mathcal{S}, a \in \mathcal{A}$;
**foreach** *episode* $n \in N$ **do**
  Initialize state $s$;
  **foreach** *epoch* $t \in \mathcal{T}$ **do**
    Sample $\hat{\epsilon} \in [0, 1]$;
    **if** $\hat{\epsilon} \leq \epsilon$ **then**
      Sample random action $a \in \mathcal{A}$;
    **end if**
    **else if** $\hat{\epsilon} > \epsilon$ **then**
      Select action $a = \arg\max_{a \in \mathcal{A}} r(s, a) + Q(s, a)$;
    **end if**
    $Q(s, a) \leftarrow Q(s, a) + \alpha[r(s, a) + \gamma \max_{a' \in \mathcal{A}} Q(s', a') - Q(s, a)]$;
    Update state $s \leftarrow s'$;
  **end foreach**
**end foreach**

# Q-learning algorithm [2/2]

Update equation:

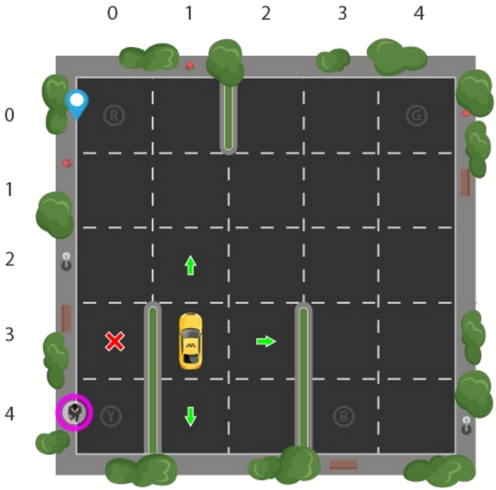
$$Q^{new}(s_t, a_t) \leftarrow \underbrace{Q(s_t, a_t)}_{\text{current value}} + \underbrace{\alpha}_{\text{learning rate}} \cdot \overbrace{\bigg( \underbrace{\underbrace{r_t}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \cdot \underbrace{\max_a Q(s_{t+1}, a)}_{\text{estimate of optimal future value}}}_{\text{new value (temporal difference target)}} - \underbrace{Q(s_t, a_t)}_{\text{current value}} \bigg)}^{\text{temporal difference}}$$

## What is important here?

- We select random action with probability $\epsilon$
- Off-policy: We use max function to find action next time step
  - Lower adverse impact of exploration
- Remember: just a simple weighted average

# Taxi Cab environment

# Taxi Cab Environment



# State space $\mathcal{S}$

**Exercise: setting up the state space**

- What information do we need to put in the state?
- How large is the state space?

# State space $\mathbb{S}$

## State components

- Location of taxi: 5 E 5
- Passenger location: 4 corners + 1 in taxi
- Passenger destination: 4 corners

State representation:

$$S = (\text{x\_pos\_taxi}, \text{y\_pos\_taxi}, \text{passenger\_loc}, \text{passenger\_dest})$$

State space $\mathbb{S}$ comprised of $5 \cdot 5 \cdot (4 + 1) \cdot 4 = 500$ states

# Action space $\mathbb{A}$

## Action

1. Move down
2. Move up
3. Move right
4. Move left
5. Pick up passenger
6. Drop off passenger

- Just one action at a time here, so small action space $\mathbb{A}$.
- Not all actions are feasible!
- We may filter infeasible actions with a mask (Boolean array)

# Reward function $R$

## Reward structure

- Successful drop-off: high positive reward
- Drop-off at wrong location: high negative reward
- Move without drop-off: small negative reward

Why the latter?

# Transition dynamics $\mathbb{P}$

## Direct consequence of actions

- Move to next square
- Passenger picked up
- Passenger dropped off

No stochastic elements in transition!
What uncertainties could we encounter in real-world taxi fleet management?

# Discount rate $\gamma$

## Setting a discount rate

- Set discount rate $\gamma \in [0, 1]$
- Finite horizon problem, so not strictly needed
- To what extent do present actions impact future rewards?

**Time to code!**

# Setting up the notebook

## Setup steps

- Access Notebook via **https://shorturl.at/kG38t**
- Install necessary libraries (uncomment by removing #)
- Press Ctrl+Enter to run cell
- Import libraries

# Sanity check

## Creating our environment

- Run cell with run_animation function (Environment initialization)
- You should see
  - State space: Discrete(500)
  - Action space: Discrete(6)
  - State: [0-499]
  - Action: [0-5]
  - Action mask: Binary vector, e.g., [1, 1, 1, 0, 0, 0, 0]
  - Reward: -1, -10 or +20
  - A frame of the environment

# Simulating with random agent [1/2]

- Agent with random actions
- Starting point for Q-learning, no clue about good actions

## Exercise: complete the code

- Stop loop when successfully completing episode
- Randomly sample an action (hint: use *env.action_space.sample()*)
- Run animation when complete

What do you observe?
What happens when car is standing still?
Who has largest number of epochs?

# Simulating with random agent [2/2]

**Exercise: complete the code**

- Add stop after max. 100 epochs
- Add mask to block infeasible actions (hint: see documentation at top of notebook!)

- Number of epochs likely goes down (but still undirected actions)
- Number of failed dropoffs should get to 0

# Training the agent

- Learn decision-making policy using Q-learning
- Combination of exploitation (taking best action according to Q-table) and exploration (taking random action)

**Exercise: complete the code**

- Set appropriate values for $\epsilon$, $\gamma$, $\alpha$ and $N$
- Design action selection mechanism (you can ignore masks)
- Implement weighted update rule

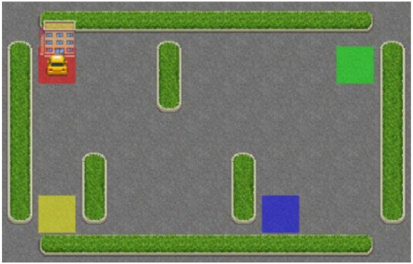
Does the policy converge?
What settings work well?

# Testing the policy

## Exercise: run the policy

- Run the learned policy
- Observe new animations

How does the agent make decisions?



# Wrapping up

# Results


- We learned a good policy for a toy problem
- Note the large number of observations (computational resources) required
- Every time we update the policy (i.e., the Q-table), we must create new observations again

# Possible extensions

- Managing fleet of taxis (send which one where?)
- Multi-agent learning, what do other taxis do?
- Massive increase in state- and action space
- Handling uncertainties (travel times, cancelling passengers, etc.)

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