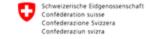
DIGITAL FINANCE

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Explainable AI in Finance XAI Methods (SHAP, LIME) Deep Dive



Prof. Dr. **Branka** Hadji Misheva
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The original Shapley values article by Lloyd Shapley (1953) is **foundational for explainable AI (XAI)**, as it provides a game-theoretic framework that underpins many of today's most widely used model explanation methods.

Shapley, L. (1953) *A Value for n-Person Games*. In: Kuhn, H. and Tucker, A., Eds., Contributions to the Theory of Games II, Princeton University Press, Princeton, 307-317. https://doi.org/10.1515/9781400881970-018 A VALUE FOR n-PERSON GAMES 1

L. S. Shapley

§ 1. INTRODUCTION

At the foundation of the theory of games is the assumption that the players of a game can evaluate, in their utility scales, every "prospect" that might arise as a result of a play. In attempting to apply the theory to any field, one would normally expect to be permitted to include, in the class of "prospects," the prospect of having to play a game. The possibility of evaluating games is therefore of critical importance. So long as the theory is unable to assign values to the games typically found in application, only relatively simple situations -- where games do not depend on other games -- will be susceptible to analysis and solution.

In the finite theory of von Neumann and Morgenstern² difficulty in evaluation persists for the "essential" games, and for only those. In this note we deduce a value for the "essential" case and examine a number of its elementary properties. We proceed from a set of three axioms, having simple intuitive interpretations, which suffice to determine the value uniquely.

Our present work, though mathematically self-contained, is founded conceptually on the von Neumann-Morgenstern theory up to their introduction of characteristic functions. We thereby inherit certain important underlying assumptions: (a) that utility is objective and transferable; (b) that games are cooperative affairs; (c) that games, granting (a) and (b), are adequately represented by their characteristic functions. However, we are not committed to the assumptions regarding rational behavior embodied in the von Neumann-Morgenstern notion of "solution."

We shall think of a "game" as a set of rules with specified players in the playing positions. The rules alone describe what we shall

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²Reference [3] at the end of this paper. Examples of infinite games without values may be found in [2], pp. 58-59, and in [1], p. 110. See also Karlin [2], pp. 152-153.

(Mathematical) Background

 Shapley (1953) provides a framework for a fair distribution of payout in a collaborative game where players work together for a common goal but maybe do not contribute equally.

What properties would a fair distribution of payouts have?





(Mathematical) Background

Efficiency

The sum of all players' contribution must be equal to the payout

Additivity

In a game with multiple subgames, each having a separate payout, the contribution of a player to the combined game is equal to the sum of contributions to each individual subgame.

Null Player

If a player does not contribute to any coalition, their share of the playout is 0.

Symmetry

If two players contribute the same to all coalitions, they should receive equal payout



$$\varphi_i(v) = \sum_{S \subseteq N\{i\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S))$$



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$$\varphi_i(v) = \sum_{S \subseteq N\{i\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S))$$

Shapley value for a given feature *i*

We calculate the contribution of each feature to a prediction



$$\varphi_i(v) = \sum_{S \subseteq N\{i\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S))$$

Sum over all possible coalitions that do not contain i

The Shapley value aims to measure the average contribution of feature i to the prediction, **considering all** possible scenarios where i could join a coalition



$$\varphi_i(v) = \sum_{S \subseteq N\{i\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S))$$

Coalition without feature i



$$\varphi_i(v) = \sum_{S \subseteq N\{i\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S))$$

Coalition with feature i



$$\varphi_i(v) = \sum_{S \subseteq N\{i\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S))$$

Marginal contribution of *i* to the coalition

Marginal change in the model's score after adding feature *i*





$$\varphi_i(v) = \sum_{S \subseteq N\{i\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S))$$

Weighting the contributions of i by its share in the number of total coalitions

- ISI is the size of the coalition S
 (excluding feature i)
- *n* is the total **number of feature**



Let's calculate manually ...

Let's imagine a case in which we are combining different drugs (A, B and C), and we want to calculate the contribution to each drug on the likelihood of surviving.

All together

When Drug A, B and C are given together, this leads to a survival likelihood of 90%.

Separately

Drug A: 40% Drug B: 50% Drug C: 60%

Pair-wise coalitions

Drug A and B: 70% Drug A and C: 65% Drug B and C: 80%



Let's calculate manually ...

HINT: Start by identifying all coalitions to which **Drug A** can can be added. Then for each, apply the formula: $\phi_i(v) = \sum_{S \subseteq N\{i\}} \frac{|S|!(n-|S|-1)!}{n!} (v(S \cup \{i\}) - v(S))$

What is the fair drug contribution of **Drug A** to the triple combination likelihood of 90%?





Coalition size of S	Drug A's marginal contributions	Drug B's marginal contributions	Drug C's marginal contributions
0	$V(A) - V(\emptyset) = 40$	$V(B) - V(\emptyset) = 50$	$V(C) - V(\emptyset) = 60$
1	V(B,A) - V(B) = 70 - 50 = 20	V(A,B) - V(A) = 70 - 40 = 30	V(A,C) - V(A) = 65 - 40 = 25
	V(C,A) - V(C) = 65 - 60 = 5	V(C,B) - V(C) = 80 - 60 = 20	V(B,C) - V(B) = 80 - 50 = 30
2	V(B,C,A) - V(B,C) = 90 - 80 = 10	V(A,C,B) - V(A,C) = 90 - 65 = 25	V(A,B,C) - V(A,B) = 90 - 70 = 20
Shapley values	$\frac{1}{3}\cdot 40 + \frac{1}{6}\cdot 20 + \frac{1}{6}\cdot 5 + \frac{1}{3}\cdot 10 =$ 20.83%	$\frac{1}{3}.50 + \frac{1}{6}.30 + \frac{1}{6}.20 + \frac{1}{3}.25$ = 33.33%	$\frac{1}{3}.60 + \frac{1}{6}.25 + \frac{1}{6}.30 + \frac{1}{3}.20$ = 35.83%



- The literature offers many attempts to use **Shapley values for the purpose of fairly** quantifying the contribution of features to a prediction task.
- Let's consider a simple case: a multivariate regression case!

$$y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \dots + \beta_n X_{ni} + e_i$$



What are we usually interested in besides prediction accuracy?



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How is predictor importance usually measured?



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How is predictor importance usually measured?

Coefficients β_1 , t - stat, R^2 ; $R^2_{inc} = R^2_{full} - R^2_{without X}$



- The literature offers many attempts to use **Shapley values for the purpose of fairly** quantifying the contribution of features to a prediction task.
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What do you think happens to regression coefficients when predictors are highly correlated?



 $y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \dots + \beta_n X_{ni} + e_i$

In linear regression, the solution for the coefficients ß can be written as:

$$b = C^{-1}r$$

 $C = X^t X$ is the **correlation (or covariance) matrix** of the predictors $r = X^t y$ is the vector capturing the **relationship between** predictors and the response variable.

If the matrix is illconditioned then its inverse C^{-1} becomes numerically unstable.



What do you think happens to regression coefficients when predictors are highly correlated?



$$y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \dots + \beta_n X_{ni} + e_i$$

If the matrix is illconditioned then its inverse C^{-1} becomes numerically unstable.

Even tiny changes or noise in the data can cause large swings in the values of the regression coefficients.



What do you think happens to regression coefficients when predictors are highly correlated?



$$y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \dots + \beta_n X_{ni} + e_i$$

- One might get negative coefficient due to redundancy in shared variance.
- If $Corr(X_1, X_2) = 0.9$, the inversion of C becomes numerically unstable, and the coefficient may flip sign arbitrary with small data changes.



Suppose X1 and X2 are both positively correlated with Y, but also highly correlated with each other. What might happen if we include both in a regression?



$$y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \dots + \beta_n X_{ni} + e_i$$



If we can't trust the signs or magnitude of coefficients, how can we trust our variable importance ranking?





Lipovetsky and Conklin (2001): propose using **Shapley values to compute each predictor's** average marginal contribution to \mathbb{R}^2 over all possible subsets:

$$\varphi_i = \sum_{S \subseteq N\{i\}} \frac{|S|!(n-|S|-1)!}{n!} (R^2(S \cup \{i\}) - R^2(S))$$

- We average out the collinearity effects
- Differences in R^2 are more numerically stable
- If two variables share explanatory power (highly correlated), their combined contribution is split fairly, avoiding over- or under-representation
- Unlike classical net effects (which can be negative), Shapley Values are **always positive** and **sum to total** \mathbb{R}^2



In practice ...

Suppose you have predictors A, B, and C.

STEP 1

Calculate marginal contributions

- Subsets: A, B, C, AB, AC, BC, ABC
- For each subset, compute R2 of the model.
 - A: The marginal contribution of A to the subset BC, for example, is: R²(ABC) – R²(BC). You then average A's marginal contribution across all such subsets where A appears. That gives A's Shapley Value. Repeat for all variables.

STEP 2

Propose adjusted coefficients

- After computing Shapley Values for each variable, calculate adjusted coefficients for each feature so that they reflect these stable net effects.
- Assume standardized data (so each predictor and the target has mean 0 and SD 1), then define:

$$a_j = \frac{SV_j}{r_j}$$

where SV_j is the Shapley-based net effect & r_j is the correlation between x_j and y



Shapley Values for ML models: early work

Sampling-based approximation

- Sample random **feature orderings** (permutations).
- Sample background instances from the data.
- Compute **differences in predictions** when feature *i* is added to partial subsets.
- Use Monte Carlo estimation to average contributions.

Enhancements:

- Adaptive sampling: Focus sampling effort on high-variance features.
- Quasi-random sampling: Use Sobol sequences for faster convergence.

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REGULAR PAPE

Explaining prediction models and individual predictions with feature contributions

Erik Štrumbelj · Igor Kononenko

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One of the early contributions toward SHAP (SHapley Additive exPlanations)

Abstract We present a sensitivity analysis-based method for explaining prediction models that can be applied to any type of classification or regression model. Its advantage over existing general methods is that all subsets of input features are perturbed, so interactions and redundancies between features are taken into account. Furthermore, when explaining an additive model, the method is equivalent to commonly used additive model-specific methods. We illustrate the method's usefulness with examples from artificial and real-world data sets and an empirical analysis of running times. Results from a controlled experiment with 122 participants suggest that the method's explanations improved the participants' understanding of the model.

 $\textbf{Keywords} \quad \text{Knowledge discovery} \cdot \text{Data mining} \cdot \text{Visualization} \cdot \text{Interpretability} \\ \text{Decision support}$

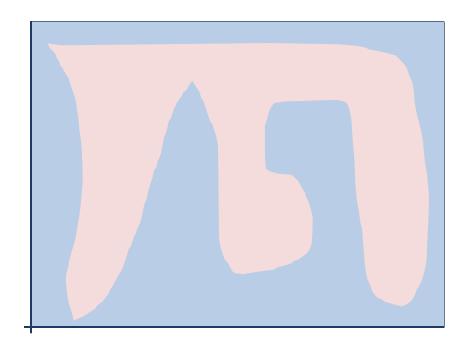
1 Introduction

Prediction models are an important component of decision support systems. Applications range from credit scoring [11] and fraud detection [5] to financial auditing [4] and efficiency analysis [18]. In such applications, model interpretability is often as important if not more important than prediction accuracy.

Some more difficult to interpret models require additional post-processing to (a) obtain a better understanding of the model and (b) increase the end-user's level of trust in the model. The latter is especially important in risk-sensitive domains such as finance and medicine, where experts are reluctant to trust prediction model's predictions without an additional explanation.



LIME



"Why Should I Trust You?" Explaining the Predictions of Any Classifier

Marco Tulio Ribeiro University of Washington Seattle, WA 98105, USA marcotcr@cs.uw.edu Sameer Singh University of Washington Seattle, WA 98105, USA sameer@cs.uw.edu Carlos Guestrin University of Washington Seattle, WA 98105, USA guestrin@cs.uw.edu

ABSTRACT

Despite widespread adoption, machine learning models remain mostly black boxes. Understanding the reasons behind predictions is, however, quite important in assessing trust, which is fundamental if one plans to take action based on a prediction, or when choosing whether to deploy a new model. Such understanding also provides insights into the model, which can be used to transform an untrustworthy model or prediction into a trustworthy one.

In this work, we propose LIME, a novel explanation technique that explains the predictions of any classifier in an interpretable and faithful manner, by learning an interpretable model locally around the prediction. We also propose a method to explain models by presenting representative individual predictions and their explanations in a non-redundant way, framing the task as a submodular optimization problem. We demonstrate the flexibility of these methods by explaining different models for text (e.g. random forests) and image classification (e.g. neural networks). We show the utility of explanations via novel experiments, both simulated and with human subjects, on various scenarios that require trust: deciding if one should trust a prediction, choosing between models, improving an untrustworthy classifier, and identifying why a classifier should not be trusted.

1. INTRODUCTION

Machine learning is at the core of many recent advances in science and technology. Unfortunately, the important role of humans is an oft-overlooked aspect in the field. Whether humans are directly using machine learning classifiers as tools, or are deploying models within other products, a vital concern remains: if the users do not trust a model or a prediction, they will not use it. It is important to differentiate between two different (but related) definitions of trust: (1) trusting a prediction, i.e. whether a user trusts an individual prediction sufficiently to take some action based on it, and (2) trusting a model, i.e. whether the user trusts a model to behave in reasonable ways if deployed. Both are directly impacted by

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how much the human understands a model's behaviour, as opposed to seeing it as a black box.

Determining trust in individual predictions is an important problem when the model is used for decision making. When using machine learning for medical diagnosis [6] or terrorism detection, for example, predictions cannot be acted upon on blind faith, as the consequences may be catastrophic

Apart from trusting individual predictions, there is also a need to evaluate the model as a whole before deploying it "in the wild." To make this decision, users need to be confident that the model will perform well on real-world data, according to the metrics of interest. Currently, models are evaluated using accuracy metrics on an available validation dataset. However, real-world data is often significantly different, and further, the evaluation metric may not be indicative of the product's goal. Inspecting individual predictions and their explanations is a worthwhile solution, in addition to such metrics. In this case, it is important to aid users by suggesting which instances to inspect, especially for large datasets.

In this paper, we propose providing explanations for individual predictions as a solution to the "trusting a prediction" problem, and selecting multiple such predictions (and explanations) as a solution to the "trusting the model" problem. Our main contributions are summarized as follows.

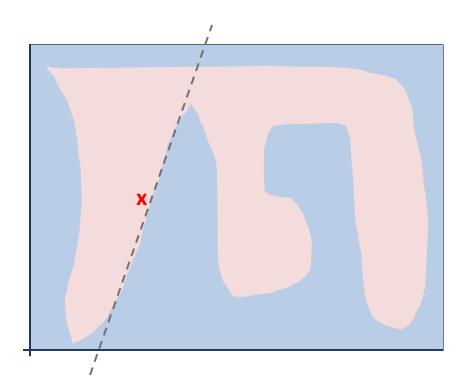
- LIME, an algorithm that can explain the predictions of any classifier or regressor in a faithful way, by approximating it locally with an interpretable model.
- SP-LIME, a method that selects a set of representative instances with explanations to address the "trusting the model" problem, via submodular optimization.
- Comprehensive evaluation with simulated and human subjects, where we measure the impact of explanations on trust and associated tasks. In our experiments, non-experts using LIME are able to pick which classifier from a pair generalizes better in the real world. Further, they are able to greatly improve an untrustworthy classifier trained on 20 newsgroups, by doing feature engineering using LIME. We also show how understanding the predictions of a neural network on images helps practitioners know when and why they should not trust a model.

2. THE CASE FOR EXPLANATIONS

By "explaining a prediction", we mean presenting textual or visual artifacts that provide qualitative understanding of the relationship between the instance's components (e.g. words in text, patches in an image) and the model's prediction. We argue that explaining predictions is an important aspect in



LIME



"Why Should I Trust You?" Explaining the Predictions of Any Classifier

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In this paper, we propose providing explanations for individual predictions as a solution to the "trusting a prediction" problem, and selecting multiple such predictions (and explanations) as a solution to the "trusting the model" problem. Our main contributions are summarized as follows

- LIME, an algorithm that can explain the predictions of any classifier or regressor in a faithful way, by approximating it locally with an interpretable model.
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LIME: Formally

$$\xi(x) = \operatorname{argmin}_{g \in G} L(f, g, \pi_x) + \Omega(g)$$

- $\xi(x)$ is the explanation function.
- f is the black-box model we want to explain.
- $g \in G$ represents the set of **interpretable models** (e.g., linear regression, decision trees).
- $L(f, g, \pi_x)$ is the loss function.
- π_x is the proximity function.
- $\Omega(g)$ is a complexity penalty.





LIME: Formally

$$\xi(x) = \operatorname{argmin}_{g \in G} L(f, g, \pi_x) + \Omega(g) \tag{1}$$

$$L(f, g, \pi_x) = \sum_i \pi_x(x_i) (f(x_i) - g(x_i))^2$$
 (2)

- We want to ensure that the interpretable model g approximates the black-box model f locally. The typical choice is the **weighted squared error**.
- x_i are the perturbed samples around x.
- $\pi_x(x_i)$ are their proximity weights.





LIME: Formally

$$\xi(x) = \operatorname{argmin}_{g \in G} L(f, g, \pi_x) + \Omega(g) \tag{1}$$

- Complexity parameter.
- Prevents the local model g from being too complex.
- Encourages simpler explanations (e.g., fewer features in a linear model).
 - Example: If g is a linear model, $\Omega(g)$ could be the number of non-zero coefficients.



```
Algorithm 1 Sparse Linear Explanations using LIME

Require: Classifier f, Number of samples N

Require: Instance x, and its interpretable version x'

Require: Similarity kernel \pi_x, Length of explanation K

\mathcal{Z} \leftarrow \{\}

for i \in \{1, 2, 3, ..., N\} do

z'_i \leftarrow sample\_around(x')

\mathcal{Z} \leftarrow \mathcal{Z} \cup \langle z'_i, f(z_i), \pi_x(z_i) \rangle

end for

w \leftarrow \text{K-Lasso}(\mathcal{Z}, K) \Rightarrow \text{with } z'_i \text{ as features, } f(z) \text{ as target return } w
```

Step 1. Initialize an empty dataset



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Step 1. Initialize an empty dataset

Step 2. For each of the N samples:

- Generate perturbed sample z_i around x^i using sample_around(x^i)
- Get:
 - The prediction
 - The similarity $\pi_x(z_i)$



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Step 3. Fit the weighted linear model using K-Lasso

- Use z_i as features
- Use $f(z_i)$ as target
- Weigh each sample using the $\pi_{\chi}(z_i)$
- Restrict to k features



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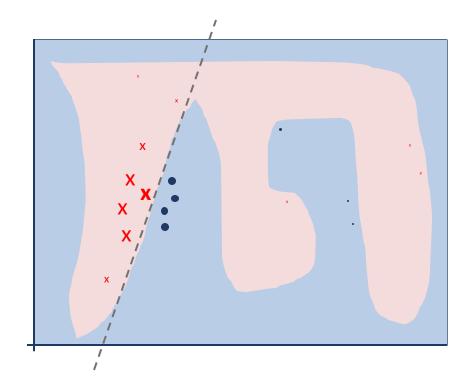
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Step 4. Return w: weights i.e. local explanations



LIME: So, what are all the steps?



Pick an observation, **create and permute** data

Calculate similarity between the original observations and the permutations

Make predictions on new data using your black box

Fit a simple model to the permuted data with n features and similarity scores as weights

Coefficients from the simple model serve as an explanation of the model behavior at the local level



LIME: So, what are all the steps?

Sampling Step

Pick an observation, **create and permute** data

Weighting Step

Calculate similarity between the original observations and the permutations

Local Model Step

Make predictions on new data using your black box

Fit a simple model to the permuted data with n features and similarity scores as weights





lime/lime tabular.py

 LIME samples each feature independently from a normal distribution centred at the (instance's) feature value

```
if sampling_method == 'gaussian':
    data = self.random_state.normal(0, 1, num_samples * num_cols
                                    ).reshape(num_samples, num_cols)
    data = np.array(data)
elif sampling_method == 'lhs':
    data = lhs(num_cols, samples=num_samples
              ).reshape(num_samples, num_cols)
    means = np.zeros(num_cols)
    stdvs = np.array([1]*num_cols)
    for i in range(num_cols):
        data[:, i] = norm(loc=means[i], scale=stdvs[i]).ppf(data[:, i])
    data = np.array(data)
else:
   warnings.warn('''Invalid input for sampling_method.
                     Defaulting to Gaussian sampling.''', UserWarning)
    data = self.random_state.normal(0, 1, num_samples * num_cols
                                    ).reshape(num_samples, num_cols)
    data = np.array(data)
if self.sample_around_instance:
    data = data * scale + instance_sample
else:
    data = data * scale + mean
```





- LIME samples each feature independently from a normal distribution centred at the (instance's) feature value
- Steps:
 - Draws independent standard
 Gaussian noise for each feature

```
if sampling_method == 'gaussian':
    data = self.random state.normal(0, 1, num samples * num cols
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       data[:, i] = norm(loc=means[i], scale=stdvs[i]).ppf(data[:, i])
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    data = self.random_state.normal(0, 1, num_samples * num_cols
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    data = np.array(data)
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   data = data * scale + instance sample
else:
    data = data * scale + mean
```





- LIME samples each feature independently from a normal distribution centred at the (instance's) feature value
- Steps:
 - Draws independent standard
 Gaussian noise for each feature
 - The sampled values are multiplied by the standard deviation of each feature

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if sampling_method == 'gaussian':
    data = self.random_state.normal(0, 1, num_samples * num_cols
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   data = data * scale + instance_sample
else:
    data = data * scale + mean
```





- LIME samples each feature independently from a normal distribution centred at the (instance's) feature value
- Steps:
 - Draws independent standard
 Gaussian noise for each feature
 - The sampled values are multiplied by the standard deviation of each feature
 - Then the noise is added to either:
 - the instance value (instance_sample) → if sample_around_instance=True

```
if sampling_method == 'gaussian':
    data = self.random state.normal(0, 1, num samples * num cols
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   data = np.array(data)
if self.sample around instance:
   data = data * scale + instance sample
else:
    data = data * scale + mean
```





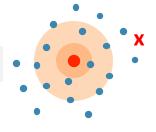
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- Steps:
 - Draws independent standard
 Gaussian noise for each feature
 - The sampled values are multiplied by the standard deviation of each feature
 - Then the noise is added to either:
 - the instance value (instance_sample) → if sample_around_instance=True
 - the feature mean (mean) → if sample_around_instance=False





Why the options?

sample_around_instance=False



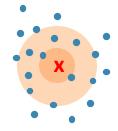
Simpler coverage - samples explore the overall data distribution.

More stable if the instance is an outlier. Avoids extrapolation into sparse or unseen areas.

Not truly local - perturbations may be far from the instance.

Violates the initiation of LIME





Perturbations cluster near the instance - more faithful local surrogate.

Aligns with LIME's core idea

If the instance is near the edge of the data distribution, sampled points may fall in low-density or unrealistic regions

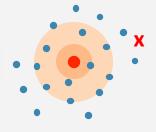


What is better?



Why the options?

sample_around_instance=False



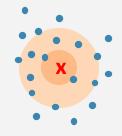
Simpler coverage - samples explore the overall data distribution.

More stable if the instance is an outlier. Avoids extrapolation into sparse or unseen areas.

Not truly local - perturbations may be far from the instance.

Violates the initiation of LIME





Perturbations cluster near the instance - more faithful local surrogate.

Aliane with LIME's core idea

The "right" sampling radius depends on the local shape of the model.

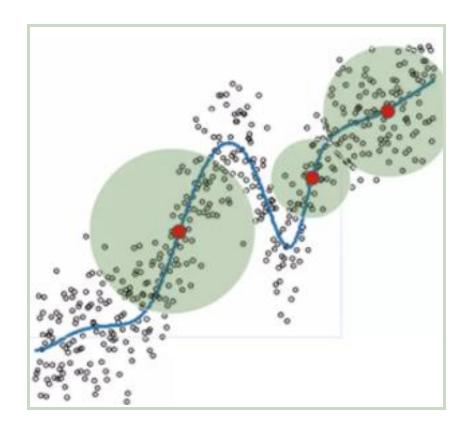
data low-

density or unrealistic regions



What is better?

(some) Guidelines



Ideally, sampled points should lie in a **meaningful neighbourhood** around the instance.

But how big should that neighbourhood be? That's tricky.

Proper size ... depends on the reference point

The best neighbourhood is not fixed but it depends on how curvy the model is nearby.

Near flat regions \rightarrow you can sample wider.

i.e. if the function around the point is flat, a
larger neighbourhood can still be wellapproximated linearly.

Near sharp bends \rightarrow you must stay narrow to preserve locality.

• i.e., if the function around the point has high curvature (nonlinear), a **small neighbourhood** is needed to get a linear fit.





Weighting step

$$\xi(x) = \operatorname{argmin}_{g \in G} L(f, g, \pi_{\chi}) + \Omega(g) \tag{1}$$

$$L(f,g,\pi_x) = \sum_i \pi_x(x_i) \big(f(x_i) - g(x_i) \big)^2 \quad (2)$$

$$\pi_{x}(x_{i}) = \exp\left(\frac{-D(x, x_{i})^{2}}{\sigma^{2}}\right)$$
(3)

- Controls which points are considered more relevant for the explanation.
- $D(x,x_i)^2$ is the **Euclidean distance** between the perturbed point x and the original instance.
- σ controls the **scale of locality** (how fast weights decrease as distance increases).



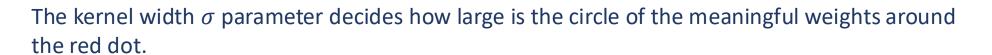
Weighting step

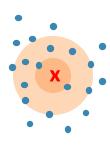
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$$\pi_{x}(x_{i}) = \exp\left(\frac{-D(x, x_{i})^{2}}{\sigma^{2}}\right)$$
(3)

The proximity parameter attributes a value in the range [0, 1], the higher the closer to the reference point.





Local model step

- As the last step, LIME uses a surrogate model to approximate the ML model in the small region around our reference red dot, determined by the weights.
- We may choose any kind of explainable model for the approximation (Decision Trees, Logistic Regression, GLM, GAM, etc.)
- The default surrogate model in LIME's Python implementation is Ridge Regression



lime/lime tabular.py

model_regressor: sklearn regressor to use in explanation. Defaults
 to Ridge regression in LimeBase. Must have
 model_regressor.coef_ and 'sample_weight' as a parameter
 to model_regressor.fit()

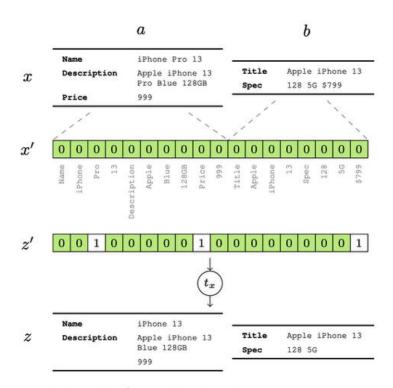


Ridge Regression is a type of linear regression that adds L2 regularization to prevent overfitting by penalizing large coefficients.



Detour: What about text?

 Text perturbation in LIME - LIME perturbs text by randomly removing words from the original instance.



Tokenize words and generate binary vectors

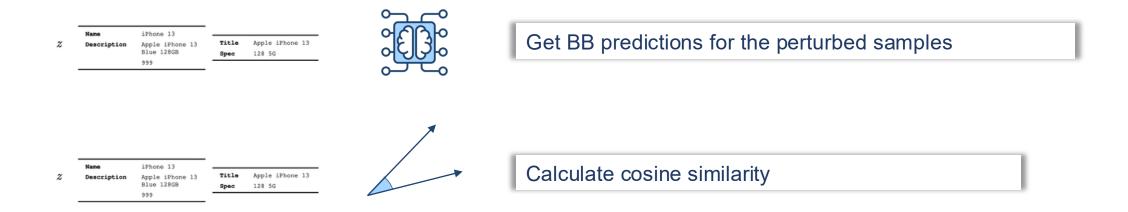
Create perturbed samples (by masking certain words)





Detour: What about text?

 Text perturbation in LIME - LIME perturbs text by randomly removing words from the original instance.



Train the surrogate model \rightarrow a weighted linear regression is trained on the binary vectors and their corresponding predictions.



- DeepLIFT (Deep Learning Important FeaTures), a method for decomposing the output prediction of a neural network on a specific input by backpropagating the contributions of all neurons in the network to every feature of the input.
- Other methods for explaining the output of NNs often rely of gradients.
 - Q: How sensitive is the output to a small change in input?

Saliency	∂Output/∂Input	
Integrated gradients	Gradients + Integrated path	
•••	••••	More on this later!

• Instead of looking only at the gradients, **DeepLIFT** compares the activation of each neuron for a given input to the activation for a reference input (like a baseline).



- Let's say you care about a specific output of the model, like a classification score. Call that output t.
- Next, we define $\Delta t = t t_0$, where t_0 is the model's output for the reference input.
- DeepLIFT assigns scores to each input feature (or neuron in an intermediate layer) to explain this Δt .
- It ensures that:

$$\sum_{i=1}^{n} C_{\Delta x_i \Delta t} = \Delta t$$

• This is called the **summation-to-delta** property. It means: all the contribution scores add up exactly to the output difference from reference.



Gradients can be **zero** even when a feature matters (due to ReLU or saturation). DeepLIFT doesn't suffer from this; it gives non-zero contributions by using **differences instead of derivatives**.

STEP 1

Pick a reference input (e.g., an all-zeros image or the mean input). This is chosen by the user.

STEP 2

Compute the output difference between your actual input and this reference.

STEP 3

Backpropagate the contribution of that output difference through the network, assigning *blame* to each neuron along the way.

STEP 4

Each input feature gets **a score** telling you how much it contributed to the difference from the reference.



Gradients can be **zero** even when a feature matters (due to ReLU or saturation). DeepLIFT doesn't suffer from this; it gives non-zero contributions by using **differences instead of derivatives**.

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STEP 4

Each input feature gets **a score** telling you how much it contributed to the difference from the reference.



Multipliers & the Chain Rule

- Let's say you're tracking how a change in input x led to a change in output t.
- We define a multiplier:

$$m_{\Delta \mathbf{x} \to \Delta \mathbf{t}} = \frac{C_{\Delta \mathbf{x} \to \Delta \mathbf{t}}}{\Delta \mathbf{x}}$$

Think of it like a finite difference version of a partial derivative:

Partial derivative: $\frac{\partial t}{\partial x}$ (infinitesimal change)

Multiplier: $\frac{\Delta t}{\Delta x}$ (finite change)

Where:

$$\Delta x = x - x^{ref}$$

$$\Delta t = t - t^{ref}$$

 $C_{\Delta x \to \Delta t}$ is the contribution of Δx to Δt

• Next, we want to trace how input x_i affects output t, **through** hidden neurons y_i . We apply a chair rule:

$$m_{\Delta \mathbf{x} \to \Delta \mathbf{t}} = \sum_{j} m_{x_i \to y_i} * m_{y_i \to t}$$



Extensions ...

- Layer-Wise Relevance Propagation (LRP)
- The LRP method also aims to interpret the predictions of NNs.
- As noted by <u>Shrikumar et al. (2017)</u> this method is equivalent to DeepLIFT with the reference activations of all neurons fixed to zero.
- Thus, $x = h_x(x)$ converts binary values into the original input space, where 1 means that an input takes its original value, and 0 means an input takes the 0 value.
- Why are we discussing these methods?
 - DeepLIFT & its extensions like the LRP are additive feature attribution methods!



 This brings us to the first contribution in the paper published by Lundberg and Lee (2017)

A unified approach to interpreting model predictions

SM Lundberg, SI Lee - Advances in neural information ..., 2017 - proceedings.neurips.cc

... as crucial as the prediction's accuracy in many applications. However, the highest accuracy for large modern datasets is often achieved **by** complex models that even experts struggle to ...

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A Unified Approach to Interpreting Model Predictions

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Abstract

Understanding why a model makes a certain prediction can be as crucial as the prediction's accuracy in many applications. However, the highest accuracy for large modern datasets is often achieved by complex models that even experts struggle to interpret, such as ensemble or deep learning models, creating a tension between accuracy and interpretability. In response, various methods have recently been proposed to help users interpret the predictions of complex models, but it is often unclear how these methods are related and when one method is preferable over another. To address this problem, we present a unified framework for interpreting predictions, SHAP (SHapley Additive exPlanations). SHAP assigns each feature an importance value for a particular prediction. Its novel components include: (1) the identification of a new class of additive feature importance measures, and (2) theoretical results showing there is a unique solution in this class with a set of desirable properties. The new class unifies six existing methods, notable because several recent methods in the class lack the proposed desirable properties. Based on insights from this unification, we present new methods that show improved computational performance and/or better consistency with human intuition than previous approaches.



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Q. Can we take the theoretical elegance of Shapley values and use it to build a **practical**, **consistent**, **and unified framework for explaining predictions** even for complex, black-box models?



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Contribution 1

The authors propose that many explanation methods can be described by a **common form** i.e. an explanation model that is a linear function of binary variables:

$$f(x) \approx g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$

They show that **all well-known explanation methods** are all **additive** in this way.



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$$f(x) \approx g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$



$$f(x) \approx g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$

The underlining complex model



$$f(x) \approx g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$

Local explanation model

An interpretable model that approximates the behaviour of a complex model locally



$$f(x) \approx g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$

$$x = h_x(z')$$

Perturbation function h_x that maps simplified input z^r to the original input space x



$$f(x) \approx g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$

Base value, i.e., the model's expected output when no features are present



$$f(x) \approx g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$

Number of features



$$f(x) \approx g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$

Attribution for feature *i*



$$f(x) \approx g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$

Coalition $Z_i' \in \{0,1\}^M$



United in the additive nature ...

• In LIME, we have:

$$\xi(x) = \operatorname{argmin}_{g \in G} L(f, g, \pi_x) + \Omega(g)$$

• and g follows:

$$g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$

Additive feature attribution method



DeepLIFT uses a "summation-to-delta" property that states:

$$\sum_{i=1}^{n} C_{\Delta x_i \Delta t} = \Delta t$$

• As noted by Shrikumar et al., the LRP method is equivalent to DeepLIFT with the reference activations of all neurons fixed to zero.

Additive feature attribution method



Additive feature attribution method







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Contribution 2

A surprising attribute of the class of additive feature attribution methods is the presence of a single unique solution in this class with desirable properties (!)

Efficiency

The **sum of attributions = model output**

Consistency

Consistency says if a feature's **contribution increases** in a newer model, its attribution should not decrease

Null Player

If a feature is **not present** in any coalition, it should get **0 contribution**

Only **Shapley values satisfy all properties**. Methods not based on Shapley values violate local accuracy and/or consistency



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LIME & the Properties



It prevents misleading explanations and guarantees that no part of the prediction is left "unexplained."

Efficiency

The **sum of attributions = model output**

- Remember: Lime uses a local surrogate model (linear regression)
- The weights assigned to perturbed samples are calculated using a distancebased kernel, not a theoretically grounded one like SHAP
- Because of these heuristic weights, the surrogate model is not required to exactly match the model's output at the original input x, that is $g(1,1,...1)\neq f(x)$
- As a result, the sum of the feature attributions from the surrogate model may not equal the model's prediction



A Unified Approach to Interpreting Model **Predictions**

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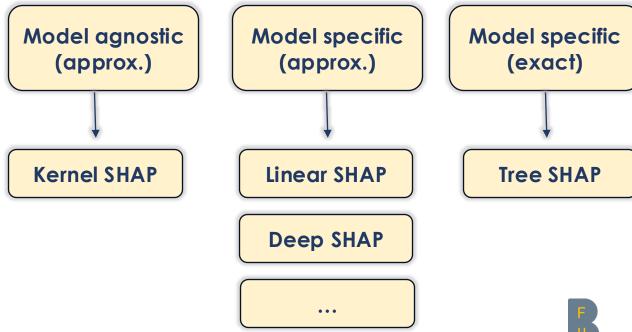
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Abstract

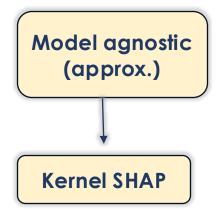
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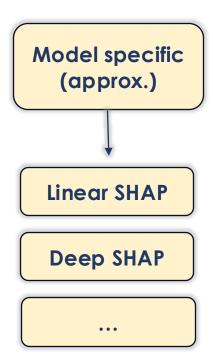
Contribution 3

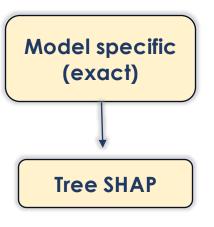
The authors also propose practical SHAP algorithms that can approximate Shapley values



SHAP Implementations



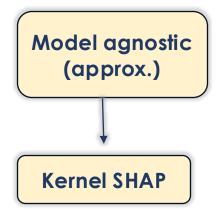


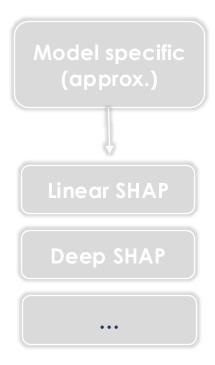


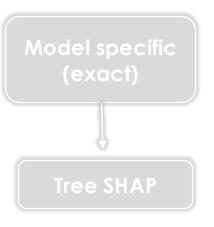


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SHAP Implementations









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KernelSHAP: Steps

Steps:

01. Sample coalitions (random – chain of 0s and 1s)

02. **Get predictions** from the BB model for each coalition

03. Compute the weights for each coalition

04. Fit a weighted linear model

05. **Return SHAP** values (coefficients)

For example, the vector of (1,0,1,0,0,1) means that we have a coalition of the first, third and sixth feature.

Coalitions
$$\xrightarrow{h_x(z')}$$
 Feature values

Instance
$$x$$
 $x' = \frac{Age}{1} \frac{Weight}{1} \frac{Color}{1}$ $x = \frac{Age}{0.5} \frac{Weight}{20} \frac{Color}{Blue}$

Instance with "absent" features



KernelSHAP: Step 1 - Sampling

- 1. Kernel SHAP samples binary vectors z'. These represent feature subsets (coalitions in Shapley terms).
- 2. Reconstruct Inputs with Only Present Features
- For each sample z', Kernel SHAP calls a **mapping function** $h_x(z')$ to generate a **full input vector** $x_{z'}$, where:
 - Features marked as 1 are taken from x,
 - Features marked as 0 are replaced.



How do you think the replacement carried out?



KernelSHAP: Step 1 - Sampling

- 1. Kernel SHAP samples binary vectors z'. These represent feature subsets (coalitions in Shapley terms).
- 2. Reconstruct Inputs with Only Present Features
- For each sample z', Kernel SHAP calls a **mapping function** $h_{x}(z')$ to generate a **full input vector** $x_{z'}$, where:
 - Features marked as 1 are taken from x,
 - Features marked as 0 are replaced.
 - Marginal sampling: sample missing features from their marginal distribution (i.e., from the data directly, regardless of the present features).
 - Conditional sampling: model the conditional distribution $P(x_{missing} \mid x_{present})$ (using k-nearest neighbours, generative models, or copulas).
 - Fixed baseline values: replace missing features with global baseline values (mean, mode, etc.)



KernelSHAP: Step 2 - Evaluate the model & fit weighted linear model

Evaluate:

For each sample z':

 $y_{z'} = f(h_x(z') - i.e.$ the model output when only those features are used.

Fit:

$$g(z') = \phi_o + \sum_{i=1}^{M} \phi_i Z'_i$$

Using weighted least squares, where the weights for each sample z' is:

$$\pi(z') = \frac{(M-1)}{\binom{M}{|z'|} * |z'| * (M-|z'|)}$$



KernelSHAP: Step 2 - Evaluate the model & fit weighted linear model

$$\pi(z') = \frac{(M-1)}{\binom{M}{|z'|} * |z'| * (M-|z'|)}$$

Shapley kernel (!)

- Higher weights to medium size coalitions
- Infinite weights to:
 - z' = [0,0,...,0]
 - z' = [1,1,...,1]



Is the local accuracy satisfied?

LIME

$$\pi_{X}(z) = \exp\left(\frac{-D(x,z)^{2}}{\sigma^{2}}\right)$$

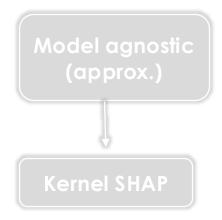
- D(x,z) = distance between the original input x and the perturbed input z
- σ = kernel width (a hyperparameter). Essentially a Gaussian kernel.

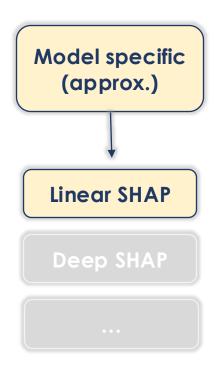
SHAP

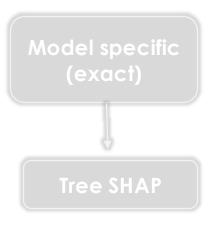
$$\pi(z') = \frac{(M-1)}{\binom{M}{|z'|} * |z'| * (M-|z'|)}$$

- Not heuristic (!)
- Local accuracy → ensured

SHAP Implementations









LinearSHAP

- A closed-form method to compute SHAP values for linear models
- Assumes:
 - **Linearity** in features
 - Feature independence
 - Background dataset to define reference values
- Goal: Assign SHAP values ϕ_i such that:

$$f(x) = \phi_o + \sum_{j=1}^m \phi_j$$

where,

 $\phi_o = E[f(x)]$ is the expected output of the model over the background data (i.e. the base value)

 ϕ_i is the marginal contribution of feature j to the derivation from the expected value.



LinearSHAP

• Given our assumptions, we can directly compute:

$$\phi_o = b + \sum_{j=1}^{M} w_j * E[x_j]$$

$$\phi_j = w_j * (x_j - E[x_j])$$

This gives us:

$$f(x) = \phi_o + \sum_{j=1}^m \phi_j$$

which is the SHAP decompensation.



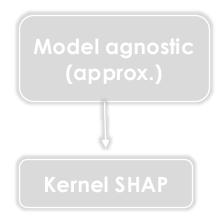
LinearSHAP

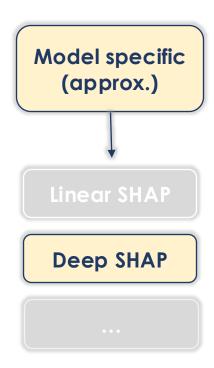
$$f(x) = \phi_o + \sum_{j=1}^m \phi_j$$

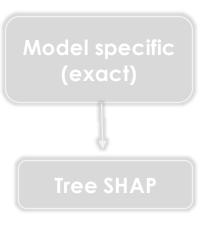
- ϕ_j is the change in prediction caused by moving from the expected value of the feature to the actual value of the feature.
- The base value ϕ_o is the model prediction if you know nothing (i.e. all features at their expected value)
- For each ϕ_j we answer: How much does this particular feature deviate from the average, weighted by its importance in the model?



SHAP Implementations









DeepSHAP

- Builds on **DeepLIFT** (as DeepLIFT already efficiently backpropagates contribution scores through a deep network using reference values)
- You pick an input x, and a **background input** (reference), say x^{ref} usually the average input from the training set
- It computes the difference in model output between your actual input and the reference:

$$\Delta f = f(x) - f(x^{ref})$$

• Then, DeepSHAP attributes this difference backward to each input feature



How are the **feature attributions** calculated?

- At each layer, we treat it as a little sub-model, and compute SHAP values just for that layer, using its own inputs and outputs (and their reference values).
- We apply the following:
 - SHAP multiplier for each neuron:

$$m_{\mathbf{x} \to \mathbf{y}} = \frac{\phi_{\mathcal{Y}}}{\mathbf{x} - E[x]}$$

Chain rule (to propagate from output all the way to input:

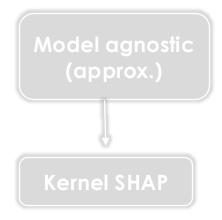
$$\phi_{x} = m_{x \to h} * m_{h \to v} * (x - E[x])$$

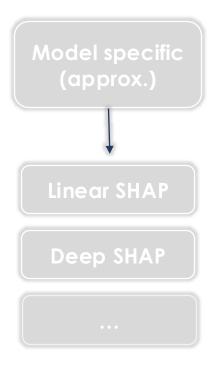
This rule makes sure the final attribution for x is composed of the **SHAP values along the** path from output \rightarrow hidden \rightarrow input.

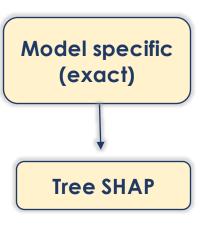




SHAP Implementations





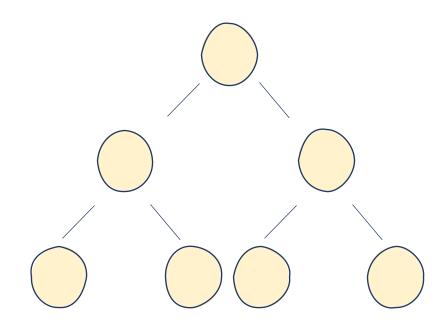




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TreeSHAP

- Each input x follows a unique path from the root to a leaf.
- Each leaf has a prediction value (e.g., probability or score).
- In Tree SHAP, the question becomes:
 - What would happen to that path if a feature was unknown (missing)?
- Some branches may no longer be taken
- The model must marginalize over those missing decisions



Tree SHAP calculates the **expected output** when a feature is known vs. when it is missing.



TreeSHAP: Steps

Let's imagine a decision tree with internal nodes splitting on features A, B, and C.

- An instance x = (A = 1, B = 0, C = 1)
- You want to explain the impact of feature A on the prediction f(x)





TreeSHAP: Steps

Let's imagine a decision tree with internal nodes splitting on features A, B, and C.

- An instance x = (A = 1, B = 0, C = 1)
- You want to explain the impact of feature A on the prediction f(x)

Step 1. Tree SHAP will consider **all subsets of features** (coalitions) that do *not* include A $S = \emptyset; S = B; S = C; S = BC$

Step 2. For each S, we compute: $\Delta f = f(x_{S \cup \{A\}}) - f(x_S)$

- x_S : The input where we **know only the features in S**
- For the others (like A), we treat them as **unknown**

Tree SHAP does not sample, but rather marginalizes over the unknowns using the training data distribution encoded in the tree



TreeSHAP: Steps

Step 3: Tree Traversal with Known/Missing Features

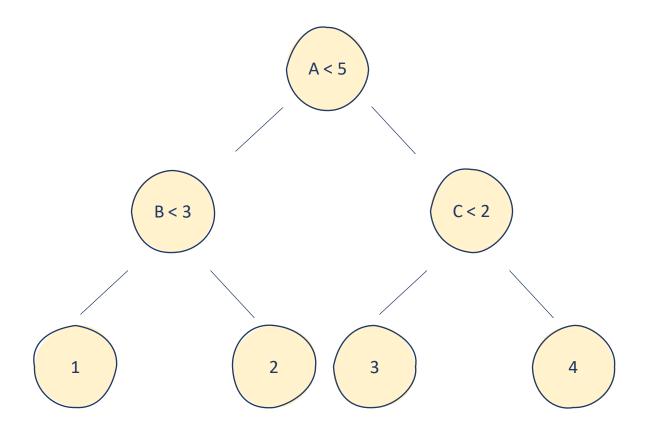
- At each node in the tree:
 - If the split is on a **known** feature (e.g., feature in $S \cup \{A\}$, follow the path based on x's value.
 - If the split is on a **missing** feature (not in S), **take both branches**, and **weight them** by the proportion of training data that went each way.
 - \rightarrow This gives you the **expected model output** under the subset $S \cup \{A\}$, and under just S.

Step 4: Compute weighted average of differences

$$Weight = \frac{|S|!(2-|S|)!}{N!}$$

S = number of features in the coalition N = number of features

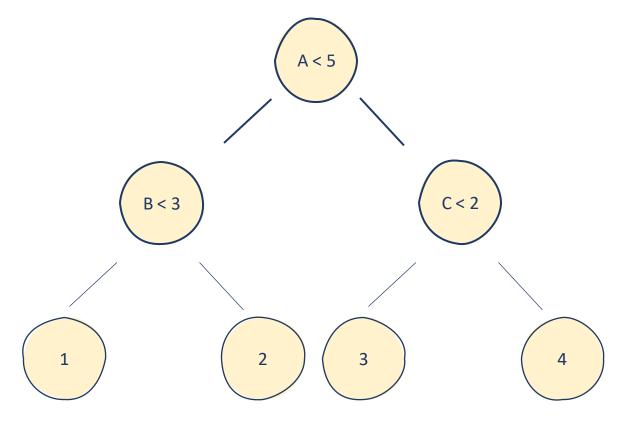




Let's assume the training data distribution is:

- At A < 5: 60% of data went left and 40% went right
- At B < 3: 50%/50%
- At C < 2: 20% left, 80% right





Input instance: $x = \{A = 6, B = 2, C = 3\}$

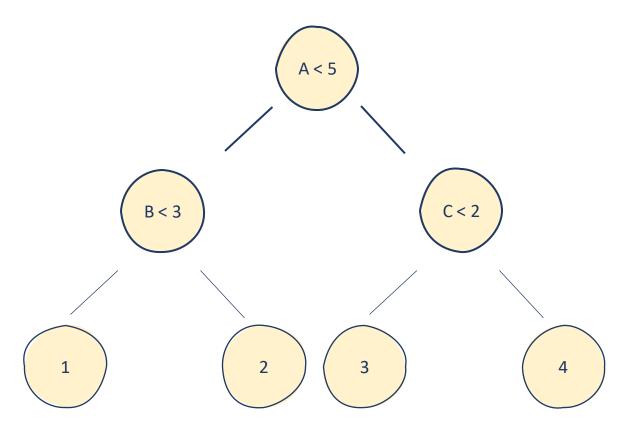
Q. How do we compute the expected outcome when a feature is unknown?

A. We take **both left and right** branches and weight by training data: Left (A < 5): 60%; Right $(A \ge 5)$: 40%

We compute:

E[f(x)|A unknown]= 0.6 * Left SubTree Prediction + 0.4 * Right SubTree Prediction





Input instance: $x = \{A = 6, B = 2, C = 3\}$

We compute:

$$E[f(x)|A unknown]$$
= 0.6 * Left SubTree Prediction + 0.4
* Right SubTree Prediction

Next, we evaluate both subtrees using known features (B and C)

- SubTree (A<5) --> B=2, value = 1
- SubTree (A>5) --> C=3, value = 2

Combine:

$$E[f(x)|A unknown] = 0.6 * 1 + 0.4 * 4 = 2.2$$

Expected model prediction when A is unknown but B and C are known.





Input instance: $x = \{A = 6, B = 2, C = 3\}$

$$E[f(x)|A \ unknown] = 0.6 * 1 + 0.4 * 4 = 2.2$$



Are you done? Is this the contribution of A to the prediction?

To compute the SHAP value for feature A, you'll repeat this logic for all subsets of features that do not include A, and then compare what happens when A is added.

And with that ...

LET'S SEE IT!



