Climate Risk Measuring and Analysis

based on Thierry Roncalli's book "Handbook of Sustainable Finance"

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Overview

- Climate risk metrics are crucial for understanding environmental impact.
- Focus on carbon footprint metrics.
- Topics included:
  1. Measuring Carbon Emissions
  2. Carbon Intensity

Figure: Concept of carbon footprint: seems easy to understand, but difficult to define precise

Overview

1. Carbon Emissions
   - Three emission scopes
   - Use of CO_2e unit (carbon dioxide equivalent)
   - Based on global warming potential (GWP)
   - Two approaches: Activity-based, Energy-based

Overview

2. Carbon Intensity
   - Normalizes emissions for comparison
   - Physical Intensity Ratios: Emissions per km traveled; Emissions per capita
   - Monetary Intensity Ratios: Emissions per unit of revenue; Emissions per unit of GDP
   - Important for finance and portfolio management

Overview

Moving further: Dynamic Risk Measures

- Beyond static metrics
- Based on: Carbon budget, Emission trends over time
- Help assess future risks and trajectories

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Carbon emissions

What are carbon emissions?

- Carbon emissions refer to the release of carbon, especially carbon dioxide (CO_2), into the atmosphere
- CO_2 is a greenhouse gas that traps heat and contributes to global warming
- Emissions can be both natural and human-induced

What are main sources of carbon emissions?

- Burning fossil fuels – for electricity, transportation, and industry
- Deforestation – fewer trees to absorb CO_2
- Industrial processes – such as cement and steel production

It matters because (a) carbon emissions are a key driver of climate change, (b) reducing emissions is essential to limit global warming, and (c) policy, technology, and behavior all play a role in mitigation.

Carbon emissions

Carbon footprint is a generic term used to define the total greenhouse gas (GHG) emissions caused directly and indirectly by a given system, activity, company, country, or region. Usually measured in units of carbon dioxide equivalents (CO_2e). Helps quantify environmental impact and identify ways to reduce emissions.

CO_2e is measured using formula

$$\text{mass of } CO_2e = \text{mass of greenhouse gas} \times \text{GWP}$$

where GWP - Global Warming Potential of gas relative to CO_2.

For instance, 1 kg of methane corresponds to 28 kg of CO_2 and 1 kg of nitrous oxide corresponds to 265 kg of CO_2 (IPCC, 2014a).

Carbon emissions

- To compute the carbon footprint of a system that is made up of several gases, the sum formula is applied

$$m = \sum_{i=1}^{n} m_i \cdot gwp_i$$

where m is the mass of CO_2 equivalent, m_i and gwp_i are the mass and the global warming potential of the ith gas, and n is the number of gases. m and m_i have the same mass unit (e.g., kilogram or kg, tonne or t, kilotonne or kt, megatonne or Mt, gigatonne or Gt, i.e. $kgCO_2e$, tCO_2e, $ktCO_2e$, $MtCO_2e$, or $GtCO_2e$).

Mass of Carbon Emissions

Examples

Let consider a company A that emits 3 017 tonnes of CO_2, 10 tonnes of CH_4 and 1.8 tonnes of N_2O. For the company B, the GHG emissions are respectively equal to 2302 tonnes of CO_2, 32 tonnes of CH_4 and 3.0 tonnes of N_2O.
The mass of CO_2 equivalent for companies A and B is equal to:

$$m_A = 3017 \times 1 + 10 \times 28 + 1.8 \times 265 = 3774 \text{ tCO2e}$$

$$m_B = 2302 \times 1 + 32 \times 28 + 3.0 \times 265 = 3993 \text{ tCO2e}$$

Notably, company B emits more carbon emissions than company A when they are measured in CO_2equivalent.

Global warming potential

Global Warming Potential (GWP) measures how much energy a greenhouse gas traps in the atmosphere over a period of time compared to carbon dioxide (CO_2).

- Since each gas differs in their capacity to trap the heat (radiative efficiency) and how long it stays in the atmosphere (lifetime), its impact on global warming depends on these two factors.
- The higher radiative efficiency is, the more warming that gas causes. The longer lifetime of a greenhouse gas is, the more it contributes to long-term persistent warming.
- GWP tells: "If I emit 1 kg of gas x, how much total warming will it cause compared to 1 kg of CO_2 over the next T years?

Global warming potential

Comparison of common gases:

Gas	Lifetime	GWP (100 yrs)	Comment
CO_2	300–1000+ yrs	1	Longest-lived, sets the baseline
CH_4	~12 yrs	~27–30	Short-lived but very potent
N_2O	~120 yrs	~273	Both long-lived and very potent

Global warming potential

- The mathematical definition of the global warming potential is:

$$gwp_i(t) = \frac{\int_0^t A_i(s)\mathbf{S}_i(s)\,ds}{\int_0^t A_0(s)\mathbf{S}_0(s)\,ds} = \frac{\int_0^t RF_i(s)\,ds}{\int_0^t RF_0(s)\,ds} = \frac{Agwp_i(t)}{Agwp_0(t)}, \tag{1}$$

where $A_i(t)$ is the radiative efficiency value of gas i (or the radiative forcing increase per unit mass increase of gas i in the atmosphere), $\mathbf{S}_i(t)$ is the decay function (or the fraction of gas i remaining in the atmosphere after t years following an incremental pulse of the gas) and $i = 0$ is the reference gas (e.g, CO_2). The radiative forcing is $RF_i(t) = A_i(t) \cdot \mathbf{S}_i(t)$, which is the product of the radiative efficiency and the decay function, whereas the absolute global warming potential $Agwp_i(t) = \int_0^t \mathrm{RF}_i(t)\,dt = \int_0^t A_i(t) \cdot \mathbf{S}_i(t)\,dt$ is the cumulative radiative forcing of the gas i between 0 and t, where t- the time horizon (usually 20, 100, or 500 years).

Global warming potential

- It is generally accepted to describe the decay function (or impulse response function) by exponential functions (Joos et al., 2013):

$$\mathbf{S}_i(t) = \sum_{j=1}^{m} a_{i,j} e^{-\lambda_{i,j} t} \tag{2}$$

  where $\sum_{j=1}^{m} a_{i,j} = 1$.
- Once the radiative efficiency function $A_i(t)$ and the set of parameters $\{(a_{i,j}, \lambda_{i,j}), j = 1, \ldots, m\}$ of the decay function are defined, Equation 1 is computed using numerical integration.
- In the case where $A_i(t)$ and $A_0(t)$ are constant, GWP is given by:

$$gwp_i(t) = \frac{A_i \sum_{j=1}^{m} a_{i,j} \lambda_{i,j}^{-1} \left(1 - e^{-\lambda_{i,j} t}\right)}{A_0 \sum_{j=1}^{m} a_{0,j} \lambda_{0,j}^{-1} \left(1 - e^{-\lambda_{0,j} t}\right)}$$

Global warming potential

- Under assumption that the radiative intensity is constant, the following values are used: Carbon dioxide $A_{CO_2} = 1.76 \times 10^{-18}, W \cdot m^2$/molecule and methane $A_{CH_4} = 2.11 \times 10^{-16}, W \cdot m^2$/molecule.
- In IPCC (2013), the decay functions were estimated by least squares and the following approximated curve was found:

$$\mathbf{S}_{CO_2}(t) = 0.2173 + 0.2240 \cdot \exp\left(-\frac{t}{394.4}\right) + \\ + 0.2824 \cdot \exp\left(-\frac{t}{36.54}\right) + 0.2763 \cdot \exp\left(-\frac{t}{4.304}\right)$$

and

$$\mathbf{S}_{CH_4}(t) = \exp\left(-\frac{t}{12.4}\right)$$

Global warming potential

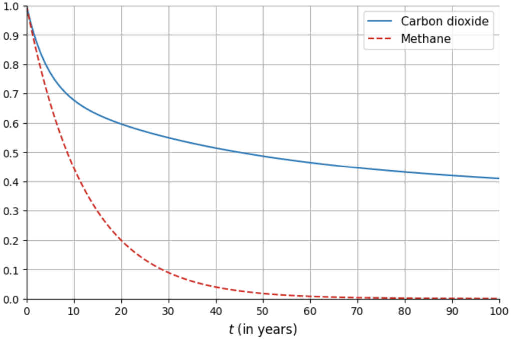

Figure: Fraction of gas remaining in the atmosphere. Source: Kleinberg(2020) & Roncalli (2024)

We can interpret them as survival functions, meaning that the density function can be computed as $f_i(t) = -\partial_t \mathbf{S}_i(t)$.

Global warming potential

Remark about exponential distribution

- In the case of the exponential distribution $\mathcal{E}(\lambda)$, we have $\mathbf{S}_i(t) = e^{-\lambda t}$.
- The density function is equal to $f_i(t) = -\partial_t \mathbf{S}_i(t)$. In the case of the exponential distribution $f_i(t) = \lambda e^{-\lambda t}$.
- Let τ_i be random time that the gas remains in the atmosphere. In the case of the exponential distribution $\mathcal{E}(\lambda)$, we have $\mathbb{E}[\tau_i] = \frac{1}{\lambda}$

The survival function of the CH_4 gas is exponential with a mean time equal to 12.4 years ($\lambda = 1/12.4$).

Global warming potential

- In the case of Equation 2, the probability density function is equal to:

$$f_i(t) = -\partial_t \mathbf{S}_i(t) = \sum_{j=1}^{m} a_{i,j} \lambda_{i,j} e^{-\lambda_{i,j} t}$$

- The mean time $\mathcal{T}_i$ is given by:

$$\mathcal{T}_i = \mathbb{E}[\mathcal{T}_i] = \int_0^\infty s f_i(s) ds = \sum_{j=1}^{m} a_{i,j} \int_0^\infty \lambda_{i,j} s e^{-\lambda_{i,j} s} ds = \sum_{j=1}^{m} \frac{a_{i,j}}{\lambda_{i,j}}$$

Remark. We have $\mathcal{T}_{\mathsf{CH_4}} = 12.4$ years.

DIGITAL

Global warming potential

Remark. $\mathcal{T}_{CO_2} = \infty$.

- The reason lies in the complex and incomplete removal processes for CO_2.
- Unlike gases like methane (CH_4) or nitrous oxide (N_2O), which are removed relatively quickly by well-defined chemical or biological reactions, CO_2 interacts with multiple Earth systems, some of which operate over centuries to millennia — and some never fully remove the carbon.

Mathematically,

$$\int_0^\infty \left(0.2173 + 0.2240\, e^{-t/394.4} + 0.2824\, e^{-t/36.54} + 0.2763\, e^{-t/4.304}\right) dt = \infty$$

and mean lifetime $\mathcal{T} = \int_0^\infty S(t)\, dt = \infty$.

DIGITAL

Global warming potential

Another way to find this result is to notice that $f_i(t)$ is an exponential mixture distribution where m is the number of mixture components. $\mathcal{E}(\lambda_{i,j})$ is the probability distribution associated with the j^{th} component and $a_{i,j}$ is the mixture weight of the j^{th} component.

- We have

$$\mathcal{T}_i = \mathbb{E}[\mathcal{T}_i] = \sum_{j=1}^{m} a_{i,j}\mathbb{E}[\mathcal{T}_{i,j}] = \sum_{j=1}^{m} a_{i,j}\mathcal{T}_{i,j}$$

For the CO_2 gas, the exponential mixture distribution is defined by the following parameters:

j	1	2	3	4
$a_{i,j}$	0.2173	0.2240	0.2824	0.2763
$\mathcal{T}_{i,j}$ (in years)	∞	394.4	36.54	4.304

Global warming potential

$\text{Agwp}_{\text{CO}_2}(t)$ and $\text{Agwp}_{\text{CH}_4}(t)$ are reported in Figure.

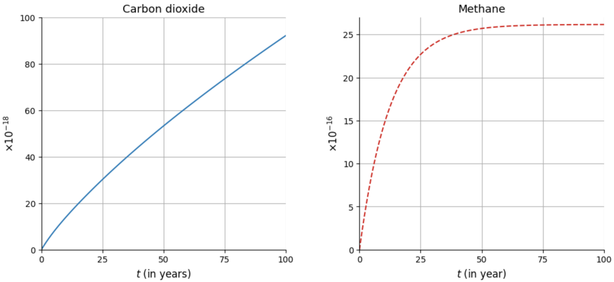

Figure: Absolute global warming potential. Source: Kleinberg(2020) & Roncalli (2024)

$\text{Agwp}_{\text{CO}_2}(\infty) = \infty$.
For the methane, $\text{Agwp}_{\text{CH}_4}$ reaches an upper bound $=$
$\text{Agwp}_{\text{CH}_4}(\infty) = A_{\text{CH}_4} \times \mathcal{T}_{\text{CH}_4} \propto 2.11 \times 12.4 = 26.164$.

Global warming potential

- The instantaneous GWP of the methane is equal to:

$$\mathsf{gwp}_{\mathsf{CH}_4}(0) = \frac{A_{\mathsf{CH}_4}}{A_{\mathsf{CO}_2}} = \frac{2.11 \times 10^{-16}}{1.76 \times 10^{-18}} \approx 119.9$$

- After 100 years, GWP value is (IPCC, 2014):

$$\mathsf{gwp}_{\mathsf{CH}_4}(100) = 28.3853.$$

- Because of the persistent regime of the carbon dioxyde, we have $\mathsf{gwp}_{\mathsf{CH}_4}(\infty) = 0$.

Remark. GWP values could be found in IPCC reports.

Global warming potential

The GHG protocol considers the 6 gases listed in the Kyoto Protocol: carbon dioxide, methane, nitrous oxide, sulphur hexafluoride, hydrofluorocarbons, and perfluorocarbons. Table gives their GWP values according to the different IPCC reports. We notice that they have continuously changed.

Name	Formula	AR2	AR4	AR5	AR6
Carbon dioxide	CO_2	1	1	1	1
Methane	CH_4	21	25	28	27.9
Nitrous oxide	N_2O	310	298	265	273
Sulphur hexafluoride	SF_6	23 900	22 800	23 500	25 200
Hydrofluorocarbons (HFC)	CHF_3	11 700	14 800	12 400	14 600
	CH_2F_2	650	675	677	771
	Etc.				
Perfluorocarbons (PFC)	CF_4	6 500	7 390	6 630	7 380
	C_2F_6	9 200	12 200	11 100	12 400
	Etc.				

Figure: GWP values for 100-year time horizon

Scope 1, 2 and 3 of carbon emissions

GHG Protocol (www.ghgprotocol.org/corporate-standard)

- Scope 1 denotes direct GHG emissions occurring from sources that are owned and controlled by the issuer.
- Scope 2 corresponds to the indirect GHG emissions from the consumption of purchased electricity, heat or steam.
- Scope 3 are other indirect emissions (not included in scope 2) of the entire value chain. They can be divided into two main categories:
  - Upstream scope 3 emissions are defined as indirect carbon emissions related to purchased goods and services.
  - Downstream scope 3 emissions are defined as indirect carbon emissions related to sold goods and services.

Unlike scope 1 and 2, scope 3 is an optional reporting category.

Scope 1 emissions

- Typically, the company lists all the activities that result in a GHG emission, and allocates them to the three scopes.
- Then, the emission to each activity and each gas is computed:

$$E_{g,h} = A_h \cdot \mathcal{EF}_{g,h}$$

where A_h is the h^{th} activity rate and $\mathcal{EF}_{g,h}$ is the emission factor for the h^{th} activity and the g^{th} gas. Here, A_h can be measured in volume, weight, distance, duration, surface, etc., $E_{g,h}$ is expressed in tonne, $\mathcal{EF}_{g,h}$ is measured in tonne per activity unit.

Scope 1 emissions

- For each gas, the total emissions is calculated:

$$E_g = \sum_{h=1}^{n_A} E_{g,h} = \sum_{h=1}^{n_A} A_h \cdot \mathcal{EF}_{g,h}$$

where n_A is the number of activities.

- Finally, we estimate the carbon emissions by applying the GWP:

$$\mathcal{CE} = \sum_{g=1}^{n_G} gwp_g \cdot E_g$$

where n_G is the number of gases [1]

- The compact formula is:

$$\mathcal{CE} = \sum_{g=1}^{n_G} gwp_g \left(\sum_{h=1}^{n_A} A_h \cdot \mathcal{EF}_{g,h} \right)$$

[1] n_G is equal to six in the GHG Protocol.

Scope 1 emissions

- The carbon footprint of the company is computed:

$$\mathcal{CE} = \sum_{h=1}^{n_A} \left(A_h \cdot \sum_{g=1}^{n_G} gwp_g \cdot \mathcal{EF}_{g,h} \right) = \sum_{h=1}^{n_A} A_h \cdot \mathcal{EF}_h = \sum_{h=1}^{n_A} \mathcal{CE}_h$$

where $\mathcal{EF}_h$ and $\mathcal{CE}_h$ are the global emission factor and carbon emissions related to the hth activity.

- Several activities could be also aggregated:

$$\mathcal{CE}_A = \sum_{h \in \mathcal{A}} \mathcal{CE}_h = \sum_{h \in \mathcal{A}} A_h \cdot \mathcal{EF}_h$$

where $\mathcal{A}$ is the set of activities.

Scope 1 emissions

- It may happen that some emission factors are defined without a reference to a specific gas (e.g., CO2 or CH4). In this case, the emission factor is a synthetic measure which already take into account the GWP of the gases:

$$\mathcal{EF}_h = \sum_{g=1}^{n_G} gwp_g \cdot \mathcal{EF}_{g,h}$$

- The expression of the carbon footprint becomes:

$$\mathcal{CE} = \sum_{h \in \mathcal{A}_1} A_h \left(\sum_{g=1}^{n_G} gwp_g \cdot \mathcal{EF}_{g,h} \right) + \sum_{h \in \mathcal{A}_2} A_h \cdot \mathcal{EF}_h$$

where $\mathcal{A}_1$ and $\mathcal{A}_2$ are the sets of activities without and with synthetic emission factors.

Scope 1 emissions

Tier methods for emission factors. The choice of data inputs is codified by IPCC (2019):

- Tier 1 methods use global default emission factors;
- Tier 2 methods use country-level or region-specific emission factors;
- Tier 3 methods use directly monitored or site-specific emission factors.

$\Longrightarrow$ IPCC Emission Factor Database, National Inventory Reports (NIRs), country emission factor databases, etc. In the US, the emission factors are calculated by the Environmental Protection Agency (US EPA). In France, the database is managed by ADEME (Agence de l'Environnement et de la Maîtrise de l'Energie) and contains about 5 300 validated emission factors.

Scope 1 emissions

Reporting of scope 1 emissions. GHG inventory document of Enel [2].

	CO_2	CH_4	N_2O	NF_3	SF_6	HFCs	Total
Electricity power generation	50 643.54	385.25	98.14	0.014	31.15	10.22	51 168.32
Electricity distribution	208.33	0.24	0.45		111.62		320.64
Real estate	79.87	0.22	1.24				81.30
Total	50 931.72	385.71	99.83	0.014	142.77	10.22	51 750.26

The scope 1 emissions of Enel is equal to 51.75 $MtCO_2e$. The contribution of CO_2 is the most important since it represents 98.4% of the total emissions, implying that the other gases have a small impact. In terms of activities, GHG emissions are mainly located in the electricity power generation. Buildings has a contribution of 0.2%.

[2] Enel (2022). Quantification and Reporting of Greenhouse Gas Emissions in Accordance with the Corporate GHG Protocol, `www.enel.com/investors/sustainability`

Scope 2 emissions

GHG Protocol, 2015

Scope 2 is "an indirect emission category that includes GHG emissions from the purchased or acquired electricity, steam, heat, or cooling consumed":

- Electricity. People use electricity for operating machines, lighting, heating, cooling, electric vehicle charging, computers, electronics, public transportation systems, etc.
- Steam. Industries use steam for mechanical work, heating, propulsion, driven turbines in electric power plants, etc.
- Heat. Buildings use heat to control inside temperature and heat water, while the industrial sector uses heat for washing, cooking, drying, etc. Heat may be produced from electricity, solar heat processes or thermal combustion.
- Cooling.It is produced from electricity or though the processes of forced air, conduction, convection, etc.

Scope 2 emissions

Scope 2 includes indirect emissions from generation only. For instance, the distribution of energy within a grid is tracked in Scope 3.

Examples

If the consumed electricity comes from owned/operated equipment, no Scope 2 emissions are reported. If the consumed electricity comes from a direct line transfer or the grid, the consumer of the energy reports the emissions in Scope 2.

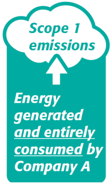

Figure: Energy production and consumption from owned/operated generation

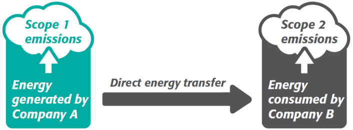

Figure: Direct line energy transfer

Scope 2 emissions

Scope 2 emissions are calculated using activity data and emission factors[3] expressed in MWh and tCO_2e/MWh:

$$\mathcal{CE} = \sum_s A_s \cdot \mathcal{EF}_s$$

where:

- A_s is the amount of purchased electricity for the energy generation source s.
- $\mathcal{EF}_s$ is the emission factor of the source s. The source can be an electricity supplier, a specific or country grid, a specific power station, etc.

[3] This approach is also known as the emission rate approach.

Scope 2 emissions

About units:

- Megawatt-hour is the common billing unit for electrical energy delivered to consumers.
- 1 000 MWh is equivalent to 3.6 TeraJoule (TJ). The TJ unit is used by IPCC (2006).
- A third energy unit is also used to define emissions factors in North America (Canada and the US) and the United Kingdom: the British thermal unit or Btu (1 Btu is equivalent to 1.0551 KJ or 0.2931 Wh).

Scope 2 emissions

Examples

We consider a company, whose electricity consumption is equal to 2000 MWh per year. The electricity comes from two sources: 60% from a direct line with an electricity supplier (source S_1) and 40% from the country grid (source S_2). The emission factors are respectively equal to 200 and 350 $\text{gCO}_2\text{e}/\text{kWh}$.

- We deduce that the carbon emissions from this source is:

$$\mathcal{CE}(S_1) = (1.2 \times 10^6) \times 200 = 240 \times 10^6 \, \text{gCO}_2\text{e} = 240 \, \text{tCO}_2\text{e}$$

- For the second source, we obtain:

$$\mathcal{CE}(S_2) = (0.8 \times 10^6) \times 350 = 280 \times 10^6 \, \text{gCO}_2\text{e} = 280 \, \text{tCO}_2\text{e}$$

- So, the Scope 2 carbon emissions of this company is equal to $520 \, \text{tCO}_2\text{e}$.

Scope 2 emissions

Two main methods are available for accounting scope 2 emissions:

- **Location-based** method. In this approach, the company uses the average emission factor of the region or the country. For instance, if the electricity consumption is located in France, the company can use the emission intensity of the French energy mix;
- **Market-based** method.This approach reflects the GHG emissions from the electricity that the company has chosen in the market. This means that the scope 2 carbon emissions will depend on the Scope 1 carbon intensity of the electricity supplier. Under the market-based method, an emission factor is associated to each electricity contract.

Scope 2 emissions

The location-based method depends on the emission factor of the regional, subnational or national grid. Its value highly depends on the energy mix and the grid infrastructure.

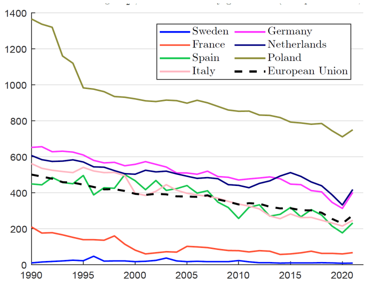

Figure: Emission factor in gCO_2e/kWh of electricity generation (EU, 1990 – 2022)

Scope 2 emissions

Emission factors for several region and countries:

Table: Emission factor in gCO_2e/kWh of electricity generation in the world

Region	$\mathcal{EF}$	Country	$\mathcal{EF}$	Country	$\mathcal{EF}$	Country	$\mathcal{EF}$
Africa	484	Australia	531	Germany	354	Portugal	183
Asia	539	Canada	128	India	637	Russia	360
Europe	280	China	544	Iran	492	Spain	169
North America	352	Costa Rica	33	Italy	226	Switzerland	47
South America	204	Cuba	575	Japan	479	United Kingdom	270
World	442	France	58	Norway	26	United States	380

Two countries which are geographically close may have different emission factors.

Source: `https://ourworldindata.org/grapher/carbon-intensity-electricity`

Scope 2 emissions

Examples

Demonstration of location-based method.
We consider a French bank, whose activities are mainly located in France and the Western Europe. Below, we report the energy consumption (in MWh) by country:

Belgium	125 807	France	1 132 261
Germany	71 890	Ireland	125 807
Italy	197 696	Luxembourg	33 069
Netherlands	18 152	Portugal	12 581
Spain	61 106	Switzerland	73 148
UK	124 010	World	37 742

Scope 2 emissions

- If we consider a Tier 1 approach, we can estimate the scope 2 emissions of the bank by computing the total activity data and multiplying by the global emission factor.
- Since we have twelve sources, we obtain:

$$A = \sum_{s=1}^{12} A_s = 125\,807 + 1\,132\,261 + \cdots + 37\,742 = 2\,013\,269 \text{ MWh}$$

and:

$$\begin{aligned} \mathcal{CE} &= A \cdot \mathcal{EF}_{\text{World}} \\ &= (2\,013\,269 \times 10^3) \times 442 \\ &= 889\,864\,898\,000 \text{ gCO}_2\text{e} \\ &= 889.86 \text{ ktCO}_2\text{e} \end{aligned}$$

Scope 2 emissions

- Another Tier 1 approach is to consider the emission factor of the European Union, because the rest of the world represents less than 2% of the electricity consumption.
- Using $\mathcal{EF}_{EU} = 275$, we obtain $\mathcal{CE} = 553.65\,\mathrm{ktCO_2e}$.

Scope 2 emissions

- The third approach uses a Tier 2 method by considering the emission factor of each country. In this case, the data should be collected. Using figures from the example, i.e. Belgium (143); Ireland (402); Luxembourg (68) and Netherlands (331), the results are as follows:

$$\begin{aligned}\mathcal{CE} &= \sum_{s=1}^{12} A_s \cdot \mathcal{EF}_s \\ &= (125\,807 \times 143 + 1\,132\,261 \times 58 + \ldots \\ &\quad + 124\,010 \times 270 + 37\,742 \times 442) \times \frac{10^3}{10^9} \\ &= 278.85\,\mathrm{ktCO_2e}\end{aligned}$$

$\Longrightarrow$ Notably, the estimated scope 2 emissions of this bank are sensitive to the approach.

Scope 3 emissions

Scope 3 emissions are all the indirect emissions in the company's value chain, apart from indirect emissions which are reported in scope 2.
They are divided into fifteen categories of emissions: 8 upstream categories and 7 downstream categories.
All these categories share the principle that there is no double counting of emissions between the scopes.

Upstream

1. Purchased goods and services
2. Capital goods
3. Fuel and energy related activities
4. Upstream transportation and distribution
5. Waste generated in operations
6. Business travel
7. Employee commuting
8. Upstream leased assets
9. Other upstream

Downstream

1. Downstream transportation and distribution
2. Processing of sold products
3. Use of sold products
4. End-of-life treatment of sold products
5. Downstream leased assets
6. Franchises
7. Investments
8. Other downstream

Figure: The scope 3 carbon emissions categories. Source: Roncalli (2024)

Scope 3 emissions

For instance, the transport categories do not concern vehicles and facilities owned, controlled or operated by the company, because their GHG emissions are already reported in scope 1 and 2.
This means that the transport of employees with a company's vehicle is reported in scope 1 and 2, but not in scope 3.
On the contrary, the public transport of employees is reported in scope 3.

Scope 3 emissions

Vehicle type	CO_2 (kg/unit)	CH_4 (g/unit)	N_2O (g/unit)	Unit
Passenger car	0.332	0.0070	0.0070	vehicle-mile
Light-duty truck	0.454	0.0120	0.0090	vehicle-mile
Motorcycle	0.183	0.0700	0.0070	vehicle-mile
Intercity rail (northeast corridor)	0.058	0.0055	0.0007	passenger-mile
Intercity rail (other routes)	0.150	0.0117	0.0038	passenger-mile
Intercity rail (national average)	0.113	0.0092	0.0026	passenger-mile
Commuter rail	0.139	0.0112	0.0028	passenger-mile
Transit rail (subway, tram)	0.099	0.0084	0.0012	passenger-mile
Bus	0.056	0.0210	0.0009	passenger-mile
Air travel (short haul, $<$ 300 miles)	0.207	0.0064	0.0066	passenger-mile
Air travel (medium haul, 300-2300 miles)	0.129	0.0006	0.0041	passenger-mile
Air travel (long haul, $>$ 2300 miles)	0.163	0.0006	0.0052	passenger-mile

Figure: Scope 3 emission factors for business travel and employee commuting (United States). These factors are intended for use in the distance-based method defined in the scope 3 calculation guidance. If fuel data are available, then the fuel-based method should be used.

Carbon emissions of investment portfolios

Carbon emissions of investment portfolios refer to the greenhouse gas emissions associated with the assets (companies, projects, bonds, etc.) held in a portfolio, typically expressed in CO_2e. It is measured due:

- Climate impact: Understand how much your investments contribute to global warming.
- Risk management: High-carbon companies may face regulatory, market, or reputation risks.
- ESG strategy: Supports responsible investing and sustainability goals.
- Disclosure standards: Required for climate reporting under TCFD, SFDR, ISSB, etc.

In finance, the most commonly reported are Scope $1 + 2$, but Scope 3 is gaining attention.

Carbon emissions of investment portfolios

Key metrics used:

- Financed emissions attributed to your investment

$$\text{Financed Emissions} = \text{Company Emissions} \times \frac{\text{Investment Value}}{\text{Company EVIC}}$$

where EVIC= Market Capitalization (Equity)+ Total Debt+ (Cash and Cash Equivalents). Financed Emissions reported in tons of CO_2e.

Carbon emissions of investment portfolios

Formally,

- Let W be the wealth invested in the company, the financed emissions are equal to:

$$\mathcal{FE}(W) = \frac{W}{\text{EVIC}} \cdot \mathcal{CE}$$

- In the case of a portfolio $(W_1, \ldots, W_n)$ where W_i is the wealth invested in company i, we have:

$$\mathcal{FE}(W) = \sum_{i=1}^{n} \mathcal{FE}_i(W_i) = \sum_{i=1}^{n} \frac{W_i}{\text{EVIC}_i} \cdot \mathcal{CE}_i \tag{3}$$

where EVIC_i and $\mathcal{CE}_i$ are the enterprise value and carbon emissions of company i. It follows that $\mathcal{FE}(W)$ is expressed in tCO_2e.

Carbon emissions of investment portfolios

Remark. For equity portfolio, to compute the market value (or the total market capitalization), the following approximation method could be used:

$$MV = \frac{MC}{\mathcal{FP}}$$

where MC and $\mathcal{FP}$ are the free float market capitalisation and percentage of the company.

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Carbon emissions of investment portfolios

Examples

Consider an investment portfolio with two equities:

Company	**Investment Value**	**Total Emissions (CO_2)**	**Market Value**
Company A	\$5,000,000	100,000	\$100,000,000
Company B	\$2,000,000	50,000	\$200,000,000

$$\text{Financed Emissions}_A = 100{,}000 \times \left(\frac{5{,}000{,}000}{100{,}000{,}000}\right) = 5{,}000 \text{ tCO}_2\text{e}$$

$$\text{Financed Emissions}_B = 50{,}000 \times \left(\frac{2{,}000{,}000}{200{,}000{,}000}\right) = 500 \text{ tCO}_2\text{e}$$

Carbon emissions of investment portfolios

$$\text{Total Financed Emissions} = 5{,}000 + 500 = 5{,}500 \text{ tCO}_2\text{e}$$

Portfolio Carbon Footprint (per \$1M invested):

$$\frac{5{,}500\text{tCO}_2}{\text{7 million USD}} \approx 785.7 \text{ tCO}_2\text{e per \$1M invested}$$

Negative emissions, avoided emissions, and carbon offsetting

Definition

Negative emissions, also known as carbon dioxide removal (CDR) is the process of removing CO_2 from the atmosphere

There are two main categories of negative emissions:

- Natural climate solutions. Examples include forest restoration and afforestation, reducing soil disturbance, etc.
- Negative emission technologies (NET). Examples are direct air capture with carbon storage (DACCS), bioenergy with carbon capture and storage (BECCS), enhanced weathering, ocean fertilization, etc.

Negative emissions, avoided emissions, and carbon offsetting

Definition

Avoided emissions are the estimated reductions in future GHG emissions resulting from using a low-carbon solution instead of a conventional one.

- Avoided emissions refer to the greenhouse gas (GHG) emissions that do not occur because a product, service, or activity replaces a more carbon-intensive alternative.
- This is the difference between the total, attributional, life-cycle GHG inventories of a company's product (the assessed product) and an alternative (or reference) product that provides an equivalent function:

$$\mathcal{AE} = \mathcal{CE}\ \text{(reference product)} - \mathcal{CE}\ \text{(assessed product)}$$

Negative emissions, avoided emissions, and carbon offsetting. Avoided emissions

Green Solution	Replaces	Emissions Avoided By...
Electric vehicles (EVs)	Gasoline cars	Avoiding tailpipe CO_2
Renewable electricity	Coal or gas-based power	Avoiding fossil fuel combustion
LED bulbs	Incandescent bulbs	Reducing energy consumption

Figure: Examples of Avoided Emissions

Negative emissions, avoided emissions, and carbon offsetting

Definition

Carbon offsetting allows an individual, company, or organization to "neutralize" their emissions by paying for projects that reduce or remove emissions elsewhere.

- Carbon offsetting includes a third concept: carbon credits.
- Carbon credits are transferable financial instruments that represent one tonne of carbon dioxide or another greenhouse gas.
- They are traded on carbon markets where companies, governments and individuals can buy and sell credits to meet their emission reduction targets.
- The price of carbon credits can vary depending on supply and demand, as well as the type of project and the region in which it is located.

Negative emissions, avoided emissions, and carbon offsetting

To sum up:

- When you compute carbon footprint, you start with gross emissions (from your investments).
- Then, you subtract negative emissions and avoided emissions to find net emissions.
- You can then apply carbon offsets to balance out remaining emissions if you want to report a net-zero or lower footprint

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Carbon intensity

- Carbon emissions = absolute carbon footprint in an absolute value
- Carbon intensity = relative carbon footprint
- Carbon emissions are normalised by a size or activity unit

For instance, we can measure the carbon footprint of countries by tCO_2e per capita, watching television by CO_2e emissions per viewer-hour, washing machines by $kgCO_2e$ per wash, cars by $kgCO_2e$ per kilometer driven, etc.

Two types of carbon intensity

Carbon intensities whose activity units are physical.
Carbon intensities whose activity units are monetary.

Physical intensity ratios

The product carbon footprint measures the relative carbon emissions of a product throughout its life cycle, which could be assessed using two distinguished methods:

- Cradle-to-gate refers to the carbon footprint of a product from the moment it is produced (including the extraction of raw materials) to the moment it enters the store
- Cradle-to-grave covers the entire life cycle of a product, including the use-phase and recycling

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Physical intensity ratios

Product	Category	Cradle-to-gate	Cradle-to-grave
Screen	21.5 inches	222	236
	23.8 inches	248	265
Computer	Laptop	156	169
	Desktop	169	189
	High performance	295	394
Smartphone	Classical	16	16
	5 inches	33	32
Oven	Built-in electric	187	319
	Professional (combi steamer)	734	12 676
Washing machine	Capacity 5kg	248	468
	Capacity 7kg	275	539
Shirt	Coton	10	13
	Viscose	9	12
Balloon	Football	3.4	5.1
	Basket-ball	3.6	5.9

Figure: Examples of product carbon footprint (in kgCO_2e per unit). Source: Lhotellier et al (2018, Annex 4, pages 212-215)

Physical intensity ratios

The previous analysis can be extended to corporate carbon footprint.

Sector	Unit	Description
Transport sector (aviation)	CO_2e/RPK	Revenue passenger kilometers
Transport sector (shipping)	CO_2e/RTK	Revenue tonne kilometers
Industry (cement)	CO_2e/t cement	Tonne of cement
Industry (steel)	CO_2e/t steel	Tonne of steel
Electricity	CO_2e/MWh	Megawatt hour
Buildings	CO_2e/SQM	Square meter

Figure: Physical carbon intensity per production unit

For instance, in the airline sector, the main traffic metric is the revenue passenger kilometers (RPK), which is calculated by multiplying the number of paying passengers by the distance traveled.

Monetary intensity ratios

- From a financial point of view, it does not make sense to compare and aggregate the carbon emissions of a large cap company with the carbon emissions of a small cap company.
- Nevertheless, the physical intensity ratios are not relevant when we consider a portfolio that is invested in several sectors?

Monetary intensity ratios are a class of carbon intensity metrics that express greenhouse gas (GHG) emissions relative to monetary values such as revenue, market capitalization, enterprise value, assets under management.

Monetary intensity ratios

Portfolio managers use monetary intensity ratios, which are defined as:

$$\mathcal{CI} = \frac{\mathcal{CE}}{Y}$$

where $\mathcal{CE}$ is the company's carbon emissions and Y is a monetary variable measuring its activity. For instance,

Revenue	Sales	EVIC	MV
$\mathcal{CI}^{\text{Revenue}} = \frac{\mathcal{CE}}{\text{Revenue}}$	$\mathcal{CI}^{\text{Sales}} = \frac{\mathcal{CE}}{\text{Sales}}$	$\mathcal{CI}^{\text{EVIC}} = \frac{\mathcal{CE}}{\text{EVIC}}$	$\mathcal{CI}^{\text{MV}} = \frac{\mathcal{CE}}{\text{MV}}$

Remark. The previous carbon emission metrics based on EVIC (market value) can be viewed as carbon intensity metrics.

Monetary intensity ratios

- In case of EVIC-based approach, the carbon intensity of the portfolio:

$$\begin{aligned}
\mathcal{CI}^{\mathsf{EVIC}}(w) &= \frac{\mathcal{CE}^{\mathsf{EVIC}}(W)}{W} \\
&= \frac{1}{W}\sum_{i=1}^{n}\frac{W_i}{\mathsf{EVIC}_i}\cdot\mathcal{CE}_i \\
&= \sum_{i=1}^{n}\frac{W_i}{W}\cdot\frac{\mathcal{CE}_i}{\mathsf{EVIC}_i} = \sum_{i=1}^{n} w_i\cdot\mathcal{CI}_i^{\mathsf{EVIC}}
\end{aligned}$$

where $w = (w_1, \ldots, w_n)$ is the vector of portfolio weights. We notice that the carbon intensity satisfies the additivity property.

Monetary intensity ratios

For the portfolio, we use Weighted Average Carbon Intensity, which measures a portfolio's exposure to carbon-intensive companies based on their Scope 1 and and Scope2 GHG emissions relative to revenue, weighted by the size of the investment.

$$\text{WACI} = \sum_{i=1}^{n} \left(\frac{V_i}{V_{\text{portfolio}}} \times \frac{E_i}{R_i} \right)$$

Where:

- V_i = Value of investment in company i
- $V_{\text{portfolio}}$ = Total value of the portfolio
- E_i = Scope 1 and 2 Emissions of company i (in tCO_2e)
- R_i = Revenue of company i (in million USD)
- $\frac{E_i}{R_i}$ = Carbon intensity of company i

Units: tCO_2e per million USD revenue

Monetary intensity ratios

Key differences:

Aspect	EVIC-based Approach	WACI
Denominator for intensity	Enterprise Value Including Cash (EVIC)	Revenue
Weights companies by:	Proportion of EVIC in portfolio	Proportion of market value in portfolio
Reflects:	Company value from all capital providers (equity + debt)	Revenue-based carbon intensity; focus on operational emissions efficiency
Used for:	More comprehensive valuation including debt, useful for credit investors	Commonly used for equity portfolios and benchmarking carbon footprint

Monetary intensity ratios

Suppose you have an investment portfolio with 3 companies:

Company	Investment Value	Carbon Intensity
X	100,000	250
Y	50,000	150
Z	150,000	100

* Carbon Intensity in tons CO_2e / millionU EUR revenue

Monetary intensity ratios

Step 1: Calculate total portfolio value

$$\text{Total Portfolio Value} = 100,000 + 50,000 + 150,000 = 300,000 \text{ EUR}$$

Step 2: Calculate the weight of each company in the portfolio

$$\text{Weight}_i = \frac{\text{Investment Value}_i}{\text{Total Portfolio Value}}$$

Company	Investment Value	Weight
X	100,000	$\frac{100,000}{300,000} = 0.3333$
Y	50,000	$\frac{50,000}{300,000} = 0.1667$
Z	150,000	$\frac{150,000}{300,000} = 0.5$

Table: Weights of each company in the portfolio

Monetary intensity ratios

Step 3: Multiply each company's carbon intensity by its weight

$$\text{Weighted Carbon Intensity}_i = \text{Weight}_i \times \text{Carbon Intensity}_i$$

Company	Weight	Carbon Intensity	Weighted Carbon Intensity
X	0.3333	250	$0.3333 \times 250 = 83.33$
Y	0.1667	150	$0.1667 \times 150 = 25.00$
Z	0.5	100	$0.5 \times 100 = 50.00$

Table: Weighted carbon intensities

Step 4: Sum the weighted carbon intensities

$$\text{WACI} = 83.33 + 25.00 + 50.00 = 158.33 \text{ tons } CO_2e \text{ per million EUR revenue}$$

Result: The weighted average carbon intensity (WACI) of the portfolio is 158.33 tons CO_2e per million WACI revenue.

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To sum up:

- Climate risk assessment requires multiple metrics
- Carbon footprint and intensity are foundational
- Definitions and methodologies continue to evolve

Climate Risk Measuring and Analysis

based on Thierry Roncalli's book "Handbook of Sustainable Finance"

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