The Correlation Between Specific Humidity and Potential Vorticity in the Atmosphere*

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ROSSBY'S POTENTIAL VORTICITY theorem (1) suggests the existence of a new characteristic property of air masses (2). Other characteristic properties have been studied, but the number of such properties is unknown. It is of interest to determine what relationships exist between any two or more of these quantities. The purpose of this study is to investigate the correlation between specific humidity and potential vorticity.

THE POTENTIAL VORTICITY THEOREM

The circulation theorem for a small horizontal area, A, in an autobarotropic fluid may be written, neglecting friction,

$$\frac{dC}{dt} + 2\,\omega\frac{d\,(A\sin\phi)}{dt} = 0\,.$$

where C is the circulation relative to the earth, ϕ is the latitude and ω is the angular velocity of the earth about its axis. On integration this gives

$$C + 2\,\omega\,A\sin\phi = \text{constant} = K.$$

Since A is very small,

$$\frac{C}{A} = \xi\,,$$

where ξ is the vertical component of the vorticity relative to the earth. Then, writing f for $2\,\omega\sin\phi$, the vertical component of the absolute vorticity of the earth,

$$\xi + f = \frac{K}{A}\,.$$

If A is the cross-sectional area of a cylindrical, autobarotropic fluid slab, ΔP the difference in pressure between the upper and lower surfaces of the slab and M the mass of the fluid cylinder,

$$A\,\Delta P = M\,g\,,$$

where g is gravity.
Thus

$$\xi + f = \frac{K}{M\,g}\,\Delta P$$

and

$$\frac{\xi + f}{\Delta P} = \frac{K}{M\,g} = \text{constant},$$

if the variation of gravity is neglected.
Then

$$\frac{\xi + f}{\Delta P} = \frac{\xi_0 + f_0}{\Delta P_0} = \text{constant},$$

or,

$$\xi_0 = \frac{\xi + f}{\Delta P/\Delta P_0} - f_0\,,$$

where f_0 is the value of f at an arbitrary standard latitude, ΔP_0 an arbitrary standard pressure difference and ξ_0 the vertical component of relative vorticity of the slab under these standard conditions. Rossby has called this quantity, ξ_0, the potential vorticity, i.e., the vorticity an autobarotropic fluid slap would have, if it were brought to a standard latitude and stretched or shrunk to a standard "pressure thickness" without change of mass. It is apparent that the potential vorticity of an autobarotropic fluid is constant in the absence of friction although the relative vorticity will vary with latitude and divergence.

APPLICATION OF THE THEOREM TO THE ATMOSPHERE

Following Rossby (1) and Starr and Neiburger (2), layers of the atmosphere bounded by potential temperature surfaces four Centigrade degrees apart are used as an approximation to autobarotropic slabs. The specific humidity and potential vor-

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ticity are computed for the potential tenperature surface halfway between the bounding surfaces.

Choice of Data.—A triangle of stations at which regular pilot-balloon and radiosonde ascents were made during the year 1940 was chosen for the computation of the desired quantities. The choice of stations was determined by four primary considerations:—

1. The triangle should be as nearly equilateral as possible.
2. It should lie in a region that experiences invasions of both polar and tropical air.
3. It should be free of important topographic influences.
4. The triangle should be fairly small. The triangle chosen is formed by Nashville, Tenn., Charleston, S. C., and Washington, D. C. Although these stations do not satisfy ideally all the requirements, they were the best available. Their main disadvantages are:

1. The triangle is rather large, thus introducing perhaps important errors in the approximation of graphical differentiation.
2. The Appalachian mountain range lies directly across the triangle. This fact probably introduces the most serious errors.
3. The Washington soundings, taken at the Naval Air Station, were generally made about 0400 E.S.T., whereas the soundings at Charleston and Nashville were made about 0100 E.S.T.
4. The pilot-balloon ascents, which were made about 2300 E.S.T., were not synchronous with the soundings.

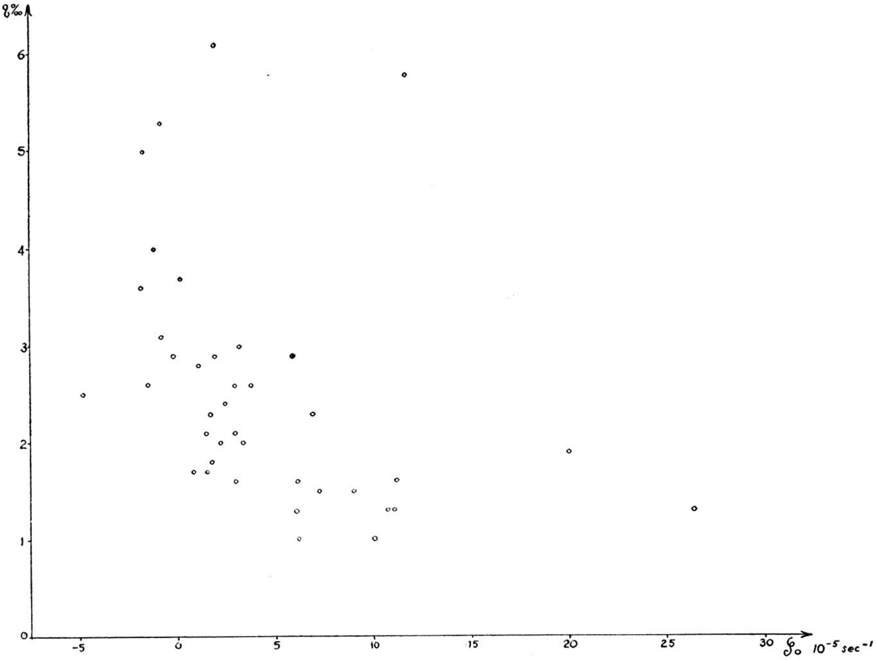
Procedure.—

1. From the original graphed pilot-balloon data was found the maximum height reached for the run at each station at or about 2300 E.S.T.
2. The limits of potential temperature observed at each station were noted using the maximum height of the pilot balloon run at each station as the upper limit. The original Weather Bureau adiabatic charts of the early morning soundings were used.
3. Potential temperature surfaces common to all three stations within the noted limits of each station were chosen for the computations. As many surfaces were used as could be found not less than four degrees apart.
4. The height of each potential temperature surface at each station was obtained from the pressure-height curve drawn on the Weather Bureau adiabatic charts.
5. The wind velocity at the height of the potential temperature surface was obtained from the graphed pilot balloon data at each station and plotted as a vector on a prepared plotting diagram (not shown). This consists of three polar diagrams, one for each station of the triangle, drawn so that the angles bear true direction with respect to a set of coordinate axes. The coordinate axes are the straight line joining Nashville and Washington on the conformal conic projection having standard parallels at 30°N and 60°N, and the perpendicular to this line on the same projection. The purpose in using these coordinate axes was to facilitate the graphical determination of vorticity.
6. Vorticity is defined by the equation

$$\xi = \frac{\partial v}{\partial x} - \frac{\partial v}{\partial y} ,$$

where u and v are the velocity components along the x- and y-axes. On the assumption of a linear velocity distribution, this may be written

$$\xi = \frac{(v_1 - v_2)}{(x_1 - x_2)} - \frac{(u_3 - u_4)}{(y_3 - y_4)} ,$$



SPECIFIC HUMIDITY(q) AGAINST POTENTIAL VORTICITY ($\mathcal{S}_o$) IN ISENTROPIC SURFACES

$$\xi = \frac{v_1}{\Delta x} - \frac{v_2}{\Delta x} - \frac{u_3}{\Delta y} + \frac{u_4}{\Delta y} .$$

Scales were drawn so that the components of the velocity vectors could be read immediately as component parts of the last expression for vorticity.

Δx is the length of the line joining Washington to Nashville and Δy is the distance from this line to Charleston along the perpendicular. Positive vectors are taken northward along y and eastward along x. Point 1 is Nashville, point 2 is Washington, point 3 is Charleston and point 4 is the intersection of the perpendicular from Charleston to the line joining Washington and Nashville is assumed line, which falls halfway along Δx. Thus the mean of the u values at Washington and Nashville is assured for point 4. The relative vorticity was then determined from the last equation.

7. The value of f at the center of the triangle (8.5×10^{-5} sec^{-1}) was added to the relative vorticity.

8. The mean value of ΔP for the three stations was divided by ΔP_o, which was taken as 100 mb following Starr and Neiburger (2). The quotient was then divided into the absolute vorticity determined above.

9. f_o, taken as 10^{-4} sec^{-1}, following Starr and Neiburger (2), was subtracted from the result of step 8 to obtain the potential vorticity.

10. The specific humidity was obtained graphically and the average of the specific humidity values for the three stations was determined.

11. The potential vorticity was plotted against the average specific humidity for each layer examined.

Results

Unfortunately, the results obtained are not very extensive. All of the available layers were examined for

TABLE OF RESULTS

Date (Raob)	Potential Temperature (° Abs)	Specific Humidity ‰	Potential Vorticity (10^{-5} sec^{-1})
1/2/40	277	1.3	10.9
1/3/40	280	1.0	10.2
1/4/40	280	1.6	11.3
1/10/40	280	2.3	7.0
1/16/40	278	1.7	0.9
1/20/40	280	1.9	20.2
1/21/40	274	1.3	26.6
1/22/40	280	1.5	7.2
1/23/40	280	2.1	1.5
1/28/40	276	1.0	6.2
1/29/40	280	1.5	9.1
1/30/40	276	1.8	1.7
2/1/40	280	2.1	3.0
2/1/40	284	1.6	6.2
2/1/40	288	1.6	3.0
2/3/40	274	1.3	6.2
2/3/40	278	1.3	11.1
2/4/40	280	2.6	3.8
2/4/40	284	2.1	2.7
2/4/40	290	2.0	3.3
2/12/40	284	3.1	–0.8
2/12/40	288	2.9	6.0
2/12/40	292	2.6	3.0
2/13/40	288	5.3	–0.8
2/13/40	292	3.6	–1.9
2/16/40	284	1.7	1.6
2/17/40	284	3.7	0.2
2/17/40	288	2.8	1.1
2/17/40	292	2.9	–0.2
2/17/40	296	3.0	3.2
2/23/40	280	2.0	2.2
2/24/40	280	2.9	2.0
2/24/40	284	2.4	2.5
2/24/40	288	2.3	1.7
2/27/40	284	2.6	–1.5
3/18/40	290	5.8	11.9
3/18/40	294	5.0	–1.7
3/18/40	298	2.5	–4.9
3/20/40	288	4.0	–1.1
3/22/40	288	6.1	2.0

the months of January, February and March of 1940. A total number of 40 cases was obtained. The small number is due to the fact that the pilot balloon runs often failed to reach sufficient height or were not made at all. The results are given in the table.

CONCLUSIONS

The correlation between potential vorticity and specific humidity is shown in the accompanying figure in which potential vorticity is plotted against specific humidity as a scatter diagram. All results are plotted on one diagram regardless of differences in potential temperature. A rough inverse correlation is observed between the two quantities.

In the absence of condensation or evaporation, specific humidity is a conservative property of air masses and may be used as a characteristic property since low specific humidity is characteristic of polar air masses and high specific humidity identifies tropical air masses (excepting continental tropical air). Furthermore, the air in polar regions has a higher vertical component of absolute vorticity than tropical air due to the fact that the earth's vorticity increases from zero at the equator to a maximum at the poles. Therefore, if potential vorticity is conservative, it is expected that air with high specific humidity should have low potential vorticity and vice versa. The inverse correlation found between specific humidity and potential vorticity may, therefore, be considered evidence of the conservation of potential vorticity.

REFERENCES

(1) Rossby, C.-G., Planetary Flow Patterns in the Atmosphere., *Quart. Journ. Roy. Meteorol. Soc.*, 1940, Vol. 66, Supplement, p. 68.

(2) Starr, V. P. and Neiburger, M., Potential Vorticity as a Conservative Property., *Journal of Marine Research*, 1940, Vol. III, p. 202.

Effects of Altitude

According to the *Journal of the American Medical Association* of October 24, 1942, a law was recently passed by the Senate of Argentina for the creation of a committee to supervise studies on the effects of altitude. The committee, which will consist of medical men, chemists, physicists and biologists, will form a branch of the Ministry of Internal Affairs. The field covered by the committee will be (1) biological problems in relation to the most adaptable human biotypes; (2) working capacity, feeding, housing and climatological, hydrological, geological, zoological, botanical and physiochemical factors in relation to the life of normal men in various altitudes; (3) pathology of altitudes, studies of diseases which may be improved by hypsotherapy, and the establishment of sanatoria and hospitals and proper altitudes for the cure of certain diseases; (4) creation of portable laboratories and establishment of experimental hypsological centers; and (5) studies of the animals and plants of different regions, the constitution of the soil and meteorological phenomena.—From *Nature*, v. 151, no. 3824, p. 193, Feb. 13, 1943.

Correlation Between Atmospheric Potential Gradient Anomalies and Solar Eruptions, by Frank L. Cooper, *American Journal of Science*, Vol. 240, August, 1942, p. 584-593.—*Abstract:* A series of observations of the atmospheric potential gradient, continuing a previous series taken for other purposes, has been made over a period of five years, 1936-40. The time of the occurrence of maxima and minima of the potential gradient has been correlated with the time of eruptions on the sun. It is found that: (a) Large disturbances of the potential gradient and also the storms occur at the same time that a solar eruption occurs near the central meridian of the sun. (b) Large disturbances of the potential gradient sometimes occur simultaneously with a solar eruption, although the sunspot is not near the central meridian.

It is believed that the sun sends us charged particles, P, and an ionizing radiation, W (ultraviolet radiation), and that the large disturbances in (a) are associated with P while W is responsible for the simultaneous disturbances in (b).