Matheus S. D'Andrea Alves

Recognition of (r, ℓ)-partite Graphs

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Abstract

The complexity to recognize if a graph has an (r, ℓ)-partition, i.e. if it can be partitioned into r cliques and ℓ independent sets, is well defined(1). However, as we will demonstrate, the literature-stabilished values for those on the P class can be improved. The following work provides a set of strategies and algortihms that pushes the previous results for the $(2, 1)$-partite (from n^4 to $n * m$), $(1, 2)$-partite (from n^4 to $n * m$) and $(2, 2)$-partite (from n^{12} to $n^2 * m$) recognition.

Keywords— (r, ℓ)-graphs, (r, ℓ)-partitions

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1 Introduction

1.1 Current results

In this section we will explore the current state of the complexity analysis as r and ℓ grows. As we fullfill the following table we expose the strategies and how those can be used to enlight the more complex results.

The most trivial result is the recognition of the $(1, 0)$-graphs, as in order to recognize it we just need to know if $|E(G)| > 0$. Therefore it's complexity is $\mathcal{O}(1)$.

r \ ℓ	0	1	2	3	4	...
0	-	?	?	?	?	...
1	$\mathcal{O}(1)$	?	?	?	?	...
2	?	?	?	?	?	...
3	?	?	?	?	?	...
4	?	?	?	?	?	...
⋮	⋮	⋮	⋮	⋮	⋮	⋱

Table 1 – Incomplete complexity analysis of the (r, ℓ)-partite recognition problem

1.1.1 m-bounded results

r \ ℓ	0	1	2	3	4	...
0	-	$\mathcal{O}(m)$	$\mathcal{O}(m)$	?	?	...
1	$\mathcal{O}(1)$	$\mathcal{O}(m)$	?	?	?	...
2	$\mathcal{O}(m)$	?	?	?	?	...
3	?	?	?	?	?	...
4	?	?	?	?	?	...
⋮	⋮	⋮	⋮	⋮	⋮	⋱

Table 2 – Incomplete complexity analysis of the (r, ℓ)-partite recognition problem

1.1.2 NP-Complete results

asdjkashjfasdhgdjasd asdjkasfakjh asdjkasfakjhasda

asdjkashjfasdhgdjasd

asdjkashjfasdhgdjasd

r \ ℓ	0	1	2	3	4	...
0	-	$\mathcal{O}(m)$	$\mathcal{O}(m)$	NPc	NPc	...
1	$\mathcal{O}(1)$	$\mathcal{O}(m)$	?	NPc	NPc	...
2	$\mathcal{O}(m)$	?	?	NPc	NPc	...
3	NPc	NPc	NPc	NPc	NPc	...
4	NPc	NPc	NPc	NPc	NPc	...
⋮	⋮	⋮	⋮	⋮	⋮	⋱

Table 3 – Incomplete complexity analysis of the (r, ℓ)-partite recognition problem

1.1.3 Frontier results

r \ ℓ	0	1	2	3	4	...
0	-	$\mathcal{O}(m)$	$\mathcal{O}(m)$	NPc	NPc	...
1	$\mathcal{O}(1)$	$\mathcal{O}(m)$	$\mathcal{O}(n^4)$	NPc	NPc	...
2	$\mathcal{O}(m)$	$\mathcal{O}(n^4)$	$\mathcal{O}(n^{12})$	NPc	NPc	...
3	NPc	NPc	NPc	NPc	NPc	...
4	NPc	NPc	NPc	NPc	NPc	...
⋮	⋮	⋮	⋮	⋮	⋮	⋱

Table 4 – Current complexity analysis of the (r, ℓ)-partite recognition problem

Bibliography

1 BRANDSTÄDT, A. *Partitions of graphs into one or two independent sets and cliques.* 1984.