The Colorability problem on (r,ℓ)-graphs and a few parametrized solutions

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An *(r,ℓ)-graph* is a graph that can be partitioned into r independent sets and ℓ cliques; In the k-Colorability problem we are asked to determine whether a given graph G admits a vertex coloring using at most k colors such that adjacent vertices have different colors.

In this work, we describe a *Poly vs NP-complete* dichotomy of this problem regarding to the parameter r and ℓ of (r, ℓ)-graphs, determining the boundaries of the NP-completenes for such a class. In addition, we analyze the complexity of the problem on (r, ℓ)-graphs under the parametrized complexity perspective.

A parameterized problem (Π, k) is said *fixed-parameter tractable* (FPT) if it can be solved in time $f(k) \times n^{O(1)}$, where f is an arbitrary function, and n is the size of the input.

Using a reduction from k-Colourability on (r,ℓ)-graph to List-Coloring as strategy, we are able to discovery that given a (2,1)-partition of the input graph G, to finding an optimal coloring of G is: W[1]-hard when parametrized by the size of the smallest independet part; Para-NP-complete when parametrized by the size of the complete part; FPT when parametrized by the number of vertices having no neighbors in the complete part; and FPT when the size of the complete part and the size of the smallest independent part are agregated parameters.