

# An Example Paper

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## Abstract

This is a short example to show the basics of using the ENTCS style macro files. Ample examples of how files should look may be found among the published volumes of the series at the ENTCS Home Page <http://www.elsevier.com/locate/entcs>.

*Keywords:* Please list keywords from your paper here, separated by commas.

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## 1 Introduction

## 2 Começou

COLORING( $r, \ell$ ) can be easily solved in polynomial time when  $(r, \ell) \in \{(0, 1), (1, 0), (1, 1), (2, 0)\}$ . Therefore, in order to completely classify the complexity when  $r + \ell \leq 2$  remains to analyse the  $(0, 2)$ -case.

**Lemma 2.1** COLORING( $0, 2$ ) can be solved in polynomial time.

**Proof.**  $(0, 2)$ -graphs, also known as co-bipartite, is a subclass of perfect graphs [?], consequently its chromatic number equals to its clique number, which can be found in polynomial time using a polynomial algorithm for INDEPENDENT SET on bipartite graphs (see [?, ?]).  $\square$

As  $(3, 0)$ -graphs (tripartite graphs) have chromatic number at most three, and to verify if the chromatic number of a graph  $G$  is one or two is trivial, we know that COLORING( $3, 0$ ) can be solved in polynomial time. Next we consider  $(4, 0)$ -graphs.

**Lemma 2.2** COLORING( $4, 0$ ) is NP-Complete.

**Proof.** It is well-known that any planar graph is  $(4, 0)$  (4-colorable) [?], and X and Y provide an  $O(n^2)$  algorithm to find a 4-coloring of a planar graph. In addition, to determine whether a planar graph  $G$  is 3-colorable is NP-complete [?] (even when a  $(4, 0)$ -partition of  $G$  is also provided). Therefore to determine the chromatic number of  $(4, 0)$ -graphs is NP-complete.  $\square$

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Is noticeable that any  $(r, \ell)$ -graph is simultaneously a  $(r + 1, \ell)$ -graph and a  $(r, \ell + 1)$ -graph, since we can choose any vertex to act as both independent set and a clique. Therefore we can state that the problem is NP-Complete for any  $(r, \ell)$ -graphs  $\forall r \geq 4, \ell \geq 0$

The following theorem serves as base for the next demonstrations.

**Theorem 2.3** LIST-COLORING( $r, \ell$ ) is equivalent to COLORING( $r, \ell + 1$ ).

**Proof.** Let  $G$  be the graph of an instance of LIST-COLORING( $r, \ell$ ), and  $C = \{c_1, c_2, \dots, c_n\}$  the color set formed by the lists of colors for each vertex.

For each color  $c_i \in C$  create a vertex  $u_i$  and connects it to all others  $u_j \forall j < i$ . If a vertex  $v \in V(G)$  does not have the color  $c_i$  in its list, then adds an edge from  $v$  to  $u_i$ . This steps generates a new graph  $H$ . It is simple to notice that if  $H$  can be proper colored, then we found a solution to the LIST-COLORING of  $G$ .

The counterpart of the prof can be done by finding the maximum clique  $Q$  of  $H$  and building the color list of the remaining vertices  $v$  as follow:

$\forall u_i \in Q$  if  $(v, u_i) \notin E(H)$  adds  $c_i$  to the list of  $v$ . Note that  $H \setminus Q = G$  and therefore a solution for LIST-COLORING of  $G$  can be easily extended to the COLORING of  $H$ .  $\square$

Based in the previous demonstration we can extract the following corollaries

**Corollary 2.4** COLORING( $1, 2$ ) is NP-Complete.

### 3 Bibliographical references

#### References