Lecture 1 Practical: Dilemma or Trilemma?

Summer School in International Finance

August 26, 2021

Review of Two Stage Least Squares

Consider the model

$$y_t = \beta x_t + \epsilon_t,$$

with x_t endogenous, i.e. $\mathbb{E}[x_t \epsilon_t] \neq 0$. Then OLS is inconsistent ($\hat{\beta}^{OLS} \overset{p}{\not\to} \beta$).

Suppose there exists a variable ('instrument variable', IV) $\mathcal{Z}_t$ such that

$$\mathbb{E}[\mathcal{Z}_t x_t] = \Psi \neq 0 \text{ (relevance)}$$
$$\mathbb{E}[\mathcal{Z}_t \epsilon_t] = 0 \text{ (validity)},$$

and consider the system of equations

$$y_t = \beta x_t + \epsilon_t$$
$$x_t = \mathcal{Z}_t \pi + e_t,$$

where $x_t = \mathcal{Z}_t \pi + e_t$ is a *projection*, i.e. $\mathbb{E}[\mathcal{Z}_t e_t] = 0$ by definition.

Then β can be identified by TSLS, i.e. $\hat{\beta}^{TSLS} = (\hat{x}'\hat{x})^{-1}\hat{x}'y$, where $\hat{x} = \mathcal{Z}\hat{\pi}$, $\hat{\pi} = (\mathcal{Z}'\mathcal{Z})^{-1}\mathcal{Z}'x$, and $y, x, \mathcal{Z}$ are the stacked $T \times 1$ vectors of $y_t, x_t, \mathcal{Z}_t$, respectively.

Review of Two Stage Least Squares

	TSLS	Proxy SVAR
Structural relationship	$y_t = \beta x_t + \epsilon_t$ $\mathbb{E}[x_t \epsilon_t] \neq 0$	
Identifying Moments	$\mathbb{E}[\mathcal{Z}_t x_t] = \Psi \neq 0$ $\mathbb{E}[\mathcal{Z}_t \epsilon_t] = 0$	
Estimator	$\hat{\beta}^{TSLS} = (\hat{x}'\hat{x})^{-1}\hat{x}'y$ $\hat{x} = \mathcal{Z}\hat{\pi}$, $\hat{\pi} = (\mathcal{Z}'\mathcal{Z})^{-1}\mathcal{Z}'x$	

Proxy SVARs

Suppose you have a three-variable, one-lag VAR,

$$\underbrace{\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix}}_{y_t} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{3,t-1} \end{bmatrix}}_{y_{t-1}} + \underbrace{\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}}_{\varepsilon_t},$$

so that the reduced-form residuals u_t are given by $u_t = B\varepsilon_t$.

Task 1 (see board)

1. *Show that*

$$u_{it} = \zeta_i u_{1t} + \nu_{it},$$

   for $\zeta_i = \frac{b_{i1}}{b_{11}}$, $\nu_{it} = (b_{i2} - \frac{b_{i1}b_{12}}{b_{11}})\varepsilon_{2t} + (b_{i3} - \frac{b_{i1}b_{13}}{b_{11}})\varepsilon_{3t}$ *and* $i = 2, 3$.

2. *Show that*

$$\mathbb{E}[u_{1t}\nu_{it}] = (b_{i2} - \frac{b_{i1}b_{12}}{b_{11}})b_{12} + (b_{i3} - \frac{b_{i1}b_{13}}{b_{11}})b_{13} \neq 0 \text{ (typically)} .$$

Note: *these derivations do not depend on the lag structure.*

Proxy SVARs

Takeaway: relationship between reduced-form residuals is an endogenous linear regression!

	TSLS	Proxy SVAR
Structural relationship	$y_t = \beta x_t + \epsilon_t$ $\mathbb{E}[x_t \epsilon_t] \neq 0$	$u_{it} = \zeta_i u_{1t} + \nu_{it}$ $\mathbb{E}[u_{1t}\nu_{it}] \neq 0$
Identifying Moments	$\mathbb{E}[\mathcal{Z}_t x_t] = \Psi \neq 0$ $\mathbb{E}[\mathcal{Z}_t \epsilon_t] = 0$	
Estimator	$\hat{\beta}^{TSLS} = (\hat{x}'\hat{x})^{-1}\hat{x}'y$ $\hat{x} = \mathcal{Z}\hat{\pi}$, $\hat{\pi} = (\mathcal{Z}'\mathcal{Z})^{-1}\mathcal{Z}'x$	

Proxy SVARs

Suppose you have a variable Z_t such that

$$\mathbb{E}\left[Z_t \varepsilon_{1t}\right] = \Phi.$$
$$\mathbb{E}\left[Z_t \varepsilon_{it}\right] = 0.$$

Task 2 (see board)

1. *Show that*
$$\mathbb{E}[Z_t u_{1t}] = b_{11}\Phi \neq 0$$
2. *Show that*
$$\mathbb{E}[Z_t \nu_{it}] = 0, \;\; \textit{for } i \neq 1.$$

$\Longrightarrow$ *exactly the relevance and validity conditions of TSLS!*

To identify ζ_i, consider

$$u_{it} = \zeta_i u_{1t} + \nu_{it}$$
$$u_{1t} = Z_t \xi + r_t,$$

where $\mathbb{E}[Z_t r_t] = 0$ by definition.

Estimate ζ by $\hat{\zeta}_i^{TSLS} = (\hat{u}_1' \hat{u}_1)^{-1} \hat{u}_1' u_i$, where $\hat{x} = \mathcal{Z}\hat{\pi}$, $\hat{\pi} = (\mathcal{Z}'\mathcal{Z})^{-1}\mathcal{Z}'x$, $\hat{u}_1 = Z\hat{\xi}$, $\hat{\xi} = (Z'Z)^{-1}Z'u_1$ and u_i, u_1, and Z are the stacked $T \times 1$ vectors of u_{1t}, u_{1t} and Z_t, respectively.

Proxy SVARs

	TSLS	Proxy SVAR
Structural relationship	$y_t = \beta x_t + \epsilon_t$ $\mathbb{E}[x_t \epsilon_t] \neq 0$	$u_{it} = \zeta_i u_{1t} + \nu_{it}$ $\mathbb{E}[u_{1t} \nu_{it}] \neq 0$
Identifying Moments	$\mathbb{E}[\mathcal{Z}_t x_t] = \Psi \neq 0$ $\mathbb{E}[\mathcal{Z}_t \epsilon_t] = 0$	$\mathbb{E}[Z_t \varepsilon_{1t}] = \Phi \neq 0$ $\mathbb{E}\left[Z_t \varepsilon_{it}\right] = 0$ for $i \neq 1$
Estimator	$\hat{\beta}^{TSLS} = (\hat{x}'\hat{x})^{-1}\hat{x}'y$ $\hat{x} = \mathcal{Z}\hat{\pi}$, $\hat{\pi} = (\mathcal{Z}'\mathcal{Z})^{-1}\mathcal{Z}'x$	$\hat{\zeta}_i^{TSLS} = (\hat{u}_1'\hat{u}_1)^{-1}\hat{u}_1' u_i$ $\hat{u}_1 = Z\hat{\xi}$, $\hat{\xi} = (Z'Z)^{-1}Z'u_1$

Proxy SVARs

- Note that we do not observe u_{it}; we instead use the plug-in reduced form residuals $\tilde{u}_{ti} = y_t - \hat{A}^{OLS} y_{t-1}$
  - This is valid by the CMT, since $\hat{A}^{OLS} \xrightarrow{p} A$
- Note that $\zeta_i = \frac{b_{i1}}{b_{11}}$ identifies the first column of B *up to scale*. b_{11} can be separately identified, but this is beyond the scope of this introduction
- Once you have ζ_i, IRFs (and their confidence intervals) can be computed as they would be using Choleski identification

Proxy SVARs

Task 3

Complete the code in `reduced_form.m` in the `ProblemSet` folder (see `reduced_form.m` for detailed instructions)

Task 4

Complete the code in `ivsvar.m` in the `ProblemSet` folder (see `ivsvar.m` for detailed instructions)

Check your code by running the file `rey_replication.m` in the `ProblemSet` folder.

Solutions can be found in the `Solutions` folder (you can also replace `addpath('ProblemSet')` in the file `rey_replication.m` with `addpath('Solutions')` to run the suggested answer).