National institute of Technology (Hamirpur)

Updated On:- 01/03/2021

Subject:- Failure Analysis

Course-code:- MSD-323

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FAILURE ANALYSIS

Failure → Material
Failure → Component
Failure → structure → Bridges
structure → Highway
⇓
Material Failure

Definition

Failure of a material component is the loss of ability to function normally or to perform the intended job.

There are 3 general ways for a material to fail.

① Excessive Elastic Deformation :- Eg Buckling.

Controlled by design and elastic modulus of material

② Excessive Plastic Deformation :- Controlled by yield strength of material. Eg loss of shape, creep or stress rupture at elevated temp.

③ Fracture :- It involves complete disruption of continuity of a component under static

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load. These can be brittle or ductile fractures.

Under cyclic load/ fluctuating load: Fatigue.

Fracture Defination: It is defined as the separation or fragmentation of solid body to 2 or more parts under the action of stress.

Fracture can be classified based on several characteristic features:-

Characteristics	Terms Used	
Strain to fracture	Ductile	Brittle
Crystallographic mode	Shear	Cleavage
Appearance	Fibrous	Granular
Crack propogation	Along grain boundries	Through grains

Ductile V/s Brittle Fracture

Parameter	Ductile Fracture	Brittle Fracture
→ Strain Energy Required	Higher	Lower.
→ Stress during Cracking (True)	Increasing	Constant
→ Crack Propagation	Slow	Fast
→ Warning Sign	Plastic deformation.	None
→ Deformation	Extensive	Little
→ Necking	Yes	No
→ Fracture Surface	Rough and dull	Smooth and bright.
→ Type of materials	Most metals (At Room Temp.)*	Ceramics, glasses, etc.

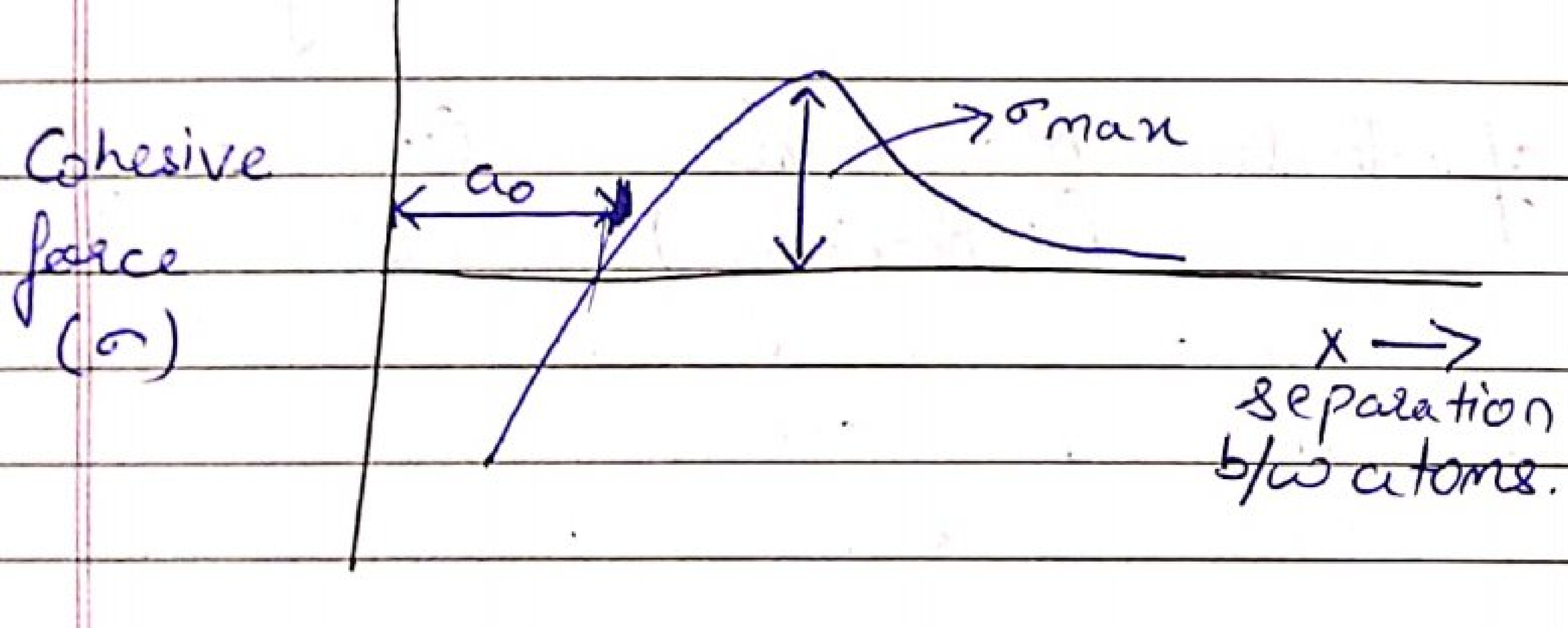
THEORETICAL COHESIVE STRENGTH OF METALS

→ Metals usually posses high strength combined with a certain measure of plasticity

→ The strength in metals is due to Cohesive force / forces b/w the atoms.

→ High cohesive forces are related to large elastic constants ; high M.P. and small coefficients of thermal expansion

→ The variation of cohesive force b/w 2 atoms is a function of separation b/w the two.



→ This curve is the resultant of attractive and repulsive forces b/w the atoms

→ The interatomic spacing of the atoms is the unstrained condition is indicated by a_0.

→ If the crystal is subjected to a tensile load, the separation b/w atoms will be increased

→ The repulsive force decreases more rapidly with increased separation than the attractive force, so that a net force b/w atoms balances the tensile load.

→ As the tensile load, is increased still further, the repulsive force continues to decrease and a point is reached where the repulsive force is negligible and the attractive force is decreasing because of the increased separation of the atoms.

→ This point corresponds to maximum in the curve which is equal to theoretical cohesive strength of the material.

→ A good approximation to the theoretical cohesive strength can be obtained if it is assumed that the cohesive force curve can be representated by a sine curve

$$\boxed{\sigma = \underbrace{\sigma_{max}}_{\text{Theoretical cohesive Strength}} \sin \frac{2\pi x}{\underbrace{\lambda}_{\text{lattice wave length}}}} \quad x \rightarrow a - a_0$$

→ For small displacements $(\sin x = x)$

$$\therefore \quad \sigma = \sigma_{max} \frac{2\pi x}{d} \quad \text{———} (1)$$

→ If we restrict consideration to a brittle elastic solid, then from hook's law

$$\sigma = E e = \frac{E x}{a_0} \quad \text{———} (2)$$

From (1) & (2)

$$\boxed{\sigma_{max} = \frac{dE}{2\pi a_0}}$$

→ Interatomic (equilibrium spacing) = $a_0 = \frac{d}{2}$

$$\therefore \quad \boxed{\sigma_{max} = \frac{E}{\pi}}$$

→ When fracture occurs in a brittle solid, all of the work is expended in producing the fracture surface goes into creating the two new surfaces.

→ Each of these surfaces has a 'surface energy' of γ_s J/m^2

→ The work done per unit area of surface in creating the fracture is the area under the stress displacement curve.

$$U_0 = \int_0^{d/2} \sigma_{max} \sin \frac{2\pi x}{d} = \frac{d\, \sigma_{max}}{\pi}$$

→ This energy is equal to the energy required to create new fracture surfaces

$$\boxed{\frac{d\, \sigma_{max}}{\pi} = 2 \gamma_s}$$

Substituting for d, we get

$$\boxed{d = \frac{2\pi \gamma_s}{\sigma_{max}}}$$

Or $$\boxed{\sigma_{max} = \left(\frac{E \gamma_s}{a_0}\right)^{1/2}}$$

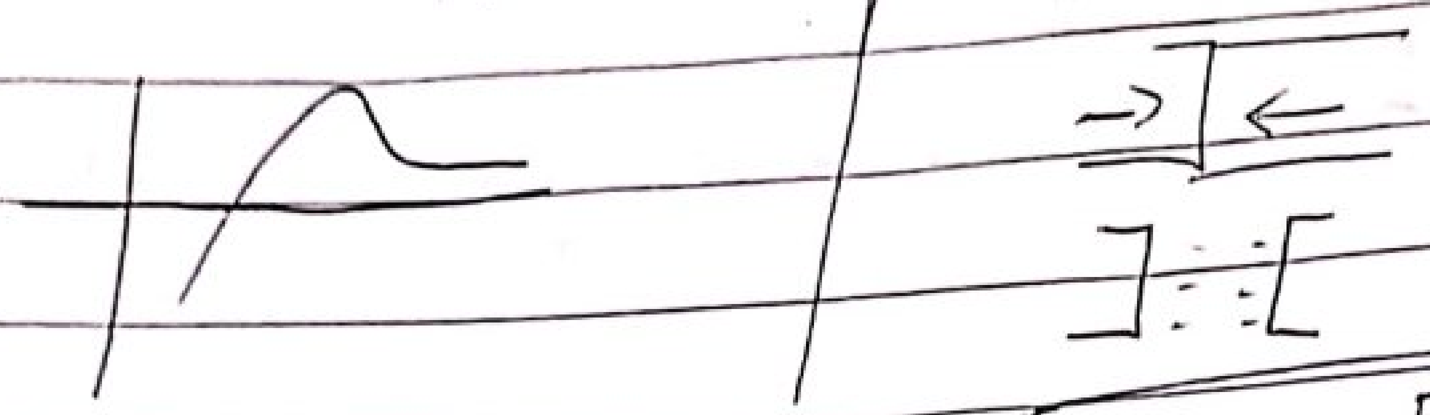
↓

Theoretical cohesive strength.

Stress req. to create fracture

Maximum Theoretical cohesive strength
2 separate criteria for derivation:

Atomistic approach | energy Approach



$$\boxed{\sigma_{max} = \frac{E}{\pi}}$$

$$\boxed{\sigma_{max} = \sqrt{\frac{E\gamma_s}{a_o}}}$$

From energy criteria

$$\sigma_{TFS} = \sigma_{max} = \sqrt{\frac{E\gamma_s}{a_o}} \quad -(1)$$

Theoretical fracture strength

for most metals $\boxed{\gamma_s \simeq \frac{Ea_o}{100}}$

Put in (1)

$$\Rightarrow \sigma_{max} = \sqrt{\frac{E.Ea_o}{a_o 100}}$$

$$\boxed{\sigma_{max} = \frac{E}{10}}$$

⇒ Theoretical fracture strength of a perfect (defect free) metal is in range of E/10

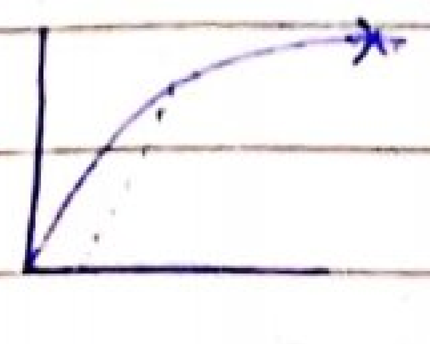
A → Dislocation leads to plastic deformation
→ Cracks leads to fracture

real fracture strength

→ But, in reality the strength of metals is of the order of $E/100$ to $E/1000$.

→ This may be attributed to presence of defect of real material; one of which includes tiny cracks.

→ Cracks play the same role in fracture (of weakening) as dislocations play for plastic deformation.



FRACTURE

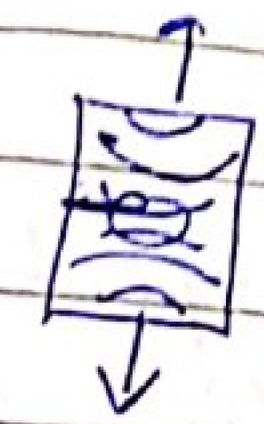
→ Fracture is related to propogation of cracks leading to failure of material or component.

→ If there are no preexisting cracks, then a crack needs to nucleate before propagation to failure

→ Crack nucleation typically requires higher stress levels than crack propagation.

Crack definition
→ A crack is typically a "sharp" void in a material, which acts like a stress concenterator or stress amplifier

- Crack is amplifier of a "far field mean stress".



⇒ Cracks in general may have several geometries. Even a circular hole can be considered as a very "blunt crack".

⇒ A crack may lie fully enclosed by the material or may have crack faces connected to the surface.

⇒ Fracture mechanics is the subject of study wherein the materials resistance to fracture is characterised.

⇒ Crack propagation can be steady (i.e. slowly increasing crack length with time or load) or it can be catastrophic (unsteady crack propagation, leading to failure of the material).

⇒ The subject of fracture mechanics has its origin in the failure of WW II liberty ships. In one of the cases, the ship virtually broke into 2 with a loud sound, not in fighting mode.

⇒ This was caused by lack of fracture toughness at the weld joints, resulting in propagation of brittle cracks.

⇒ It was observed that welding was done for faster production, but this resulted in micro-cracks and residual stresses, which lead to "brittle crack propagation."

⇒ Due to cold sea waters, the ships were harboured and the hull material underwent a "ductile to brittle transitions." (DBT)

CHARACTERISTICS OF A CRACKS

⇒ Cracks can be characterised looking into the following aspects.

★ Its connection with the external free surface

- → Completely internal
- → Internal cracks with connection to the outer surfaces
- → Surface cracks.

⇒ Cracks with some contact with external surfaces ~~...~~ may be prone to oxidation and corrosion.

→ Crack length

It is the defect length in 1 dimension which extends through the material.

→ Crack Orientation :- w.r.t geometry and loading, there are 3 modes of crack opening.

→ Three ideal cases of loading of a cracked body can be considered, which are called the modes of deformation.

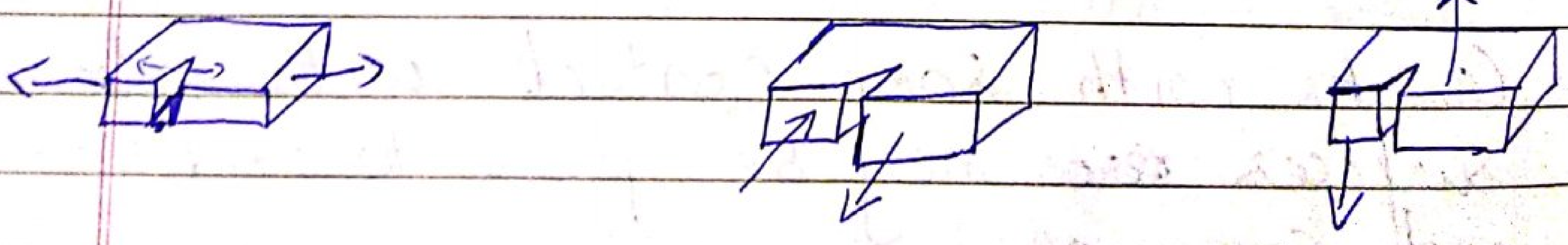
Mode I : opening mode
Mode II : Sliding mode
Mode III : Tearing Mode.

Crate → In the general case (for a crack in an arbitrarily shaped body, under a arbitrary loading), the mode is not pure (i.e. is mixed mode)

Crate

Mode of deformation / Fracture of a Cracked body

- Mode I
- Mode II
- Mode III



FRACTOGRAPHY

→ Considerable amount of information can be gathered regarding the origin and nature of fracture by studying the fracture surface.

→ The fracture surface has to be maintained in a pristined manner (i.e. oxidation, corrosion should be prevented) to get meaningful information from fractography.

→ Fracture can be classified based on :-
a) Crystallographic mode
b) Appearance of fracture surface
c) Strain to fracture
d) Path of crack.

FRACTURE MECHANICS

→ One of the goals of fracture mechanics is to derive material property (fracture toughness), which can characterize the mechanical behaviour of a material with cracks in it.

→ The field of fracture mechanics can be be divided to 2 time periods:

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(i) Primitive or Initial days of Fracture Mechanics
(ii) Modern Fracture mechanics

Early Fracture Mechanics

→ C.F. Inglis (1913) ⇒ stress based criteria for crack growth (Local)

→ Griffith Criteria (1920) ⇒ Energy based criteria for crack growth (global)

Crack growth and failure for brittle Materials

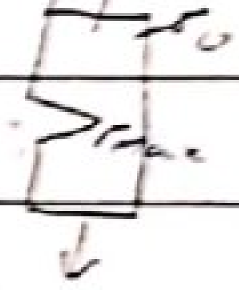
→ For brittle materials, the cracks have sharp tip and there is very little plastic deformation
(Linear elastic fracture Mechanics) (LEFM)

Stress Based Criteria for Crack Propagation

(Inglis Criteria)

→ In 1913, Inglis observed that the stress concentration around a hole (or a notch) depended on the radius of curvature of notch i.e far field stress (σ_s) is amplified near the hole.

$$\boxed{\frac{\sigma_{max}}{\sigma_0} \simeq K} \rightarrow \text{stress intensity factor}$$



$$\sigma_{max} = \sigma_0 \left[1 + 2\sqrt{\frac{c}{\rho}} \right]$$

← found by stress modelling

σ_0 = Applied at field stress
σ_{max} = stress at hole/crack tip
ρ = hole/crack tip radius
c → length of hole/crack

∴ Sharper the crack (smaller the ρ); more the stress amplification (high value of σ_{max})

Eg circular hole has a stress conc. factor of 3

$$\sigma_{max} = \sigma_0 \left[1 + 2\sqrt{\frac{c}{\rho}} \right]$$

$$c = \rho = r$$

$$\therefore \frac{\sigma_{max}}{\sigma_0} = 3$$

for sharper crack

$$\sigma_{max} \simeq 2\sigma_0 \sqrt{\frac{c}{\rho}}$$

c very large
ρ very small

$\therefore \sqrt{\frac{c}{\rho}} >> 1$

→ From Inglis's formula, it can be observed that the ratio of crack length to crack tip radius is important and not just the length of the crack.

→ For the crack to propagate the crack-tip stresses have to do work to break the bonds at crack tip.

→ This implies that the "theoretical cohesive force" of the material has to be overcome.

→ If the applied tensile stress/load is below cohesive force, still the crack can propagate if the local stress conc. become greater than cohesive strength.

→ The fracture stress (applied) is calculated by Inglis criteria ~~as~~

$$\sigma_f = \sqrt{\frac{E \gamma_s}{4 a_0 c}}$$

E = elastic modulus
γ = surface energy
ρ = rad of curvature of notch
a_0 = interatomic spacing

$$\boxed{\sigma_f = \sqrt{\frac{E\gamma\rho}{4a_0c}}} = \sqrt{\frac{E\gamma}{4c}}$$

for sharp cracks $\rho = a_0$

Shortcommings

→ We can observe that the crack length does not independently appear in the Inglis criteria

→ Intutively we can feel that, longer cracks should be more damaging than smaller cracks.

→ One another assumption in Inglis's approach is the implicit understanding that sufficient energy is available in body to propagate the crack. (Suppose, if there is no crack).

Energy based Criteria for Crack Propagation

GRIFFITH CRITERIA (1920)

Keeping some of these factors in view Griffith proposed conditions for crack propagation.

(i) Bonds at the crack tip must be stresses to point of failure (as in Inglis criteria)

(ii) The amount of strain energy released (by slight condition of unloading of the body due to crack extension) must be greater than or equal to surface energy of the crack faces created.

⇒ The second criteria can be written as:-

$$\frac{dU_s}{dc} \geq \frac{dU_r}{dc}$$

$U_s \rightarrow$ strain energy
$U_r \rightarrow$ surface energy
$dc \rightarrow$ infitesimal inc in crack length (c)

where, a_0 & c is given then Inglis criteria
where only c is given then Griffith's criteria

Q Calculate the fracture stress of a brittle material with following prop:-

$E = 100\ GPa$

$\gamma_s = 1\ J/m^2$

$a_0 = 2.5 \times 10^{-10}\ m$

$c = 10^4 . a_0$

A $\sigma_f = \sqrt{\dfrac{E\gamma s}{4 a_0 c}}$

for sharp cracks.

$$\sigma_f = \sqrt{\frac{E\gamma}{4c}}$$

$$= \sqrt{\frac{100 \times 10^9 \times 1}{4 \times 10^4 \times 2.5 \times 10^{-10}}}$$

$$= \sqrt{\frac{10^{17}}{4 \times 2.5}} = \sqrt{\frac{10^{16}}{4} \times \frac{10^2 \times 2}{8}}$$

$$\sigma_f = 10^8$$

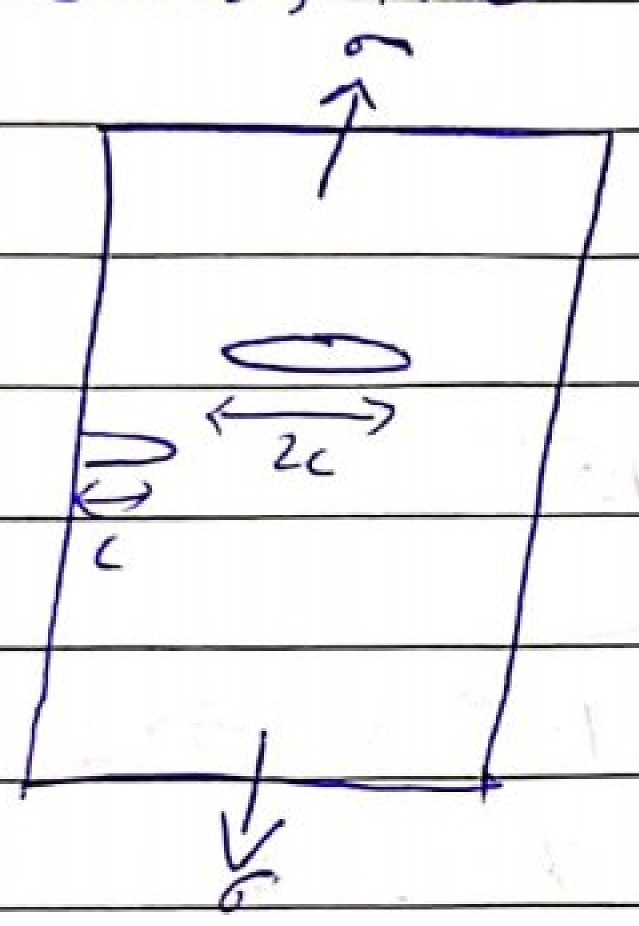
GRIFFITH THEORY

STATEMENT

→ A crack will propogate when the decrease in elastic strain energy is at least equal to the energy required to create new crack surface.

→ This criteria can be used to determine the magnitude of tensile stress which will just cause a crack of a certain size to propagate as brittle fracture.

Consider, the crack model shown below



Plane ~~stress~~ strain (2D)

→ The thickness of plate is negligible ∴ the problem can be treated as in plane stress.

→ The cracks are assumed to have an elliptical shape.

→ For a crack at the interior, the length is '2c', while for an edge crack it is 'c'

→ The effect of both types of crack on fracture behaviour is the same.

→ The stress distribution for an elliptical crack was determined by Inglis

→ A decrease in strain energy results from the formation of a ~~crack~~ crack.

→ The elastic strain energy per unit of plate thickness can be written as:-

$$\boxed{U_E = \frac{-\pi c^2 \sigma^2}{E}}$$

σ → Tensile stress acting normal to the crack length '2c'.

→ ⊖ve sign is used because growth of crack releases elastic strain energy.

→ The surface energy due to presence of crack is

$$\boxed{U_s = 4c\gamma_s}$$

2c
2c
∴ 4c = 2c+2c
Total surface

The total change in Pot. energy resulting from creation of crack is

$$\boxed{\Delta U = U_s + U_E}$$

→ Acc. to Griffiths criteria, the crack will propogate under a const. applied stress 'σ' if an incremental increase in crack length produces no change in the total energy of the system.

$$\frac{d\Delta U}{dc} = 0 = \frac{d}{dc}\left(4cV_s - \frac{\pi c^2 \sigma^2}{E}\right)$$

$$\Rightarrow \quad 4V_s - \frac{2\pi c\sigma^2}{E} = 0$$

$$\therefore \quad \boxed{\sigma = \left(\frac{2EV_s}{\pi c}\right)^{1/2}}$$

⇒ The equation, gives the stress required to propagate a crack in a brittle material as function of size of microcrack.

→ This eq. indicates that the fracture stress is inversely proportional to the sq. root of crack length.

→ For a plate with thickness comparable to length of the crack (plane strain), the Griffith theory can be written as:-

$$\boxed{\sigma = \left[\frac{2E\gamma_s}{(1-\nu^2)\pi c}\right]^{1/2}}$$

ν = Poisson's Ratio.

→ The Griffith's eq. show a strong dependance of fracture strength on crack length.

→ It predicts satisfactorily, the fracture strength of a completely brittle material such as glass.

→ Griffiths eq. for fracture doesnot apply for metals.

→ One way of realising that the fracture stress of material which undergoes plastic deformation before fracture is greater than truly brittle material.

→ One modification of Griffith's equation was made by "Orowan" in 1952 by including a term 'γ_p' expressing the plastic work required to extend the crack wall.

$$\sigma_f = \left[\frac{2E(\gamma_s+\gamma_p)}{\pi c}\right]^{1/2}$$

$$\approx \left[\frac{2E\gamma_p}{\pi c}\right]^{1/2}$$

imp. Surface crack length $= c$
Internal crack length $= 2c$

Q. The length of internal cracks in 2 samples of the same glass is measured to be $c_1 = 0.5\,mm$ & $c_2 = 2\,mm$. The ratio (σ_1/σ_2) of fracture strength of 2 samples is ___

A.
$$\sigma = \left(\frac{2 E \gamma_p}{\pi c}\right)^{1/2}$$

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{c_2}{c_1}\right)^{1/2}$$

$$= \left(\frac{2}{0.5}\right)^{1/2} = 2 \text{ A}$$

Q. A brittle material having (Youngs modulus $= 60\,GPa$) & surface energy of $0.5\,J/m^2$ has a surface crack of length $2\,\mu m$. The fracture strength in MPa of this material is ___

A.
$c = 2\,\mu m = 2 \times 10^{-6}\,m$
$E = 60 \times 10^9\,Pa$
$\gamma_s = 0.5\,J/m^2$

$$\sigma_f = \left[\frac{2 \times 60 \times 10^9 \times 0.5}{\pi \times 2 \times 10^{-6}}\right]^{1/2}$$

$$= \left[\frac{30}{\pi} \times 10^{15}\right]^{1/2}$$

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$= 9.77 \times 10^{8}$

$= 97.7\,MPa$

Q. A glass plate has 2 ∥ cracks, one of them is an int. crack of length 5 μm and other is a surface crack of length 3 μm. A tensile stress is applied ⊥ surfaces (crack). The fracture stress in MPa is ____

$E = 70\,GPa$

$\gamma_s = 1\,J/m^2$

(~~whichever~~ is less is fracture stress.)

A

$2c = 5\,\mu m = 5\times10^{-6}\,m$

$\sigma_{int} = \left(\frac{2E\gamma_s}{\pi c}\right)^{1/2}$

$c' = 3\,\mu m = 3\times10^{-6}\,m$

$\sigma_s = \left(\frac{2E\gamma_s}{\pi c}\right)^{1/2}$

$\cancel{\sigma_{net} = \left[\frac{2E\gamma_s}{\pi}\left(\frac{1}{c_1}+\frac{1}{c_2}\right)\right]^{1/2}}$

$\cancel{= \left[\frac{2\times70\times10^{9}\times1}{\pi\times10^{-6}}\left[\frac{2}{5}+\frac{1}{3}\right]\right]^{1/2}}$

$\cancel{= \left[\frac{140\times10^{9}\times7}{10^{-6}\times22}\left[\frac{11}{15}\right]\right]^{1/2}}$

$\cancel{= \left[\frac{49\times10^{10}}{15\times10^{-6}}\right]^{1/2}}$

~~= 0.057 MPa~~

$\cancel{= 57.15\,MPa}$

$$\sigma_{int} = \left[\frac{2 \times 70\times10^{9} \times 1 \times 2}{\pi \times 5\times10^{-6}}\right]^{1/2}$$

$$= 133 \text{ MPa}$$

$$\sigma_s = \left(\frac{2 \times 70\times10^{9} \times 1}{\pi \times 3 \times10^{-6}}\right)^{1/2}$$

$$= 121.8 \text{ MPa}$$

$\therefore$ fracture stress = 121.8 MPa

FRACTURE IN SINGLE CRYSTAL

→ The brittle fracture of single crystal is considered to be related to resolved normal stress on the cleavage plane.

→ Sohncke's law states that fracture occurs when the resolved normal stress reaches a criticle value.

→ Considering the situation, the component of tensile force which acts normal to cleavage plane is $P\cos\phi$, where ϕ is ∠ b/w tensile axis and normal to the plane.

→ The criticle normal stress for brittle fracture is

$$\sigma_c = \frac{P\cos\phi}{A/\cos\phi} = \frac{P}{A}\cos^2\phi$$

→ The cleavage planes for certain metals and values of the critical normal stresses are given below.

Metal	Lattice	Cleavage Plane	Criticle normal stress (Kg/mm²)
→ Iron	bcc	(100)	26
→ Zinc	hcp	(0001)	0.19
→ Bismuth	Rhombohedral	(111)	0.32
→ Antimony	"	(11$\bar{1}$)	0.66

MODERN FRACTURE MECHANICS

→ Historically, in old times (1910-20), fracture was studied using the Inglis and Griffith criteria, wherein fracture stress (σ_f) was calculated either using a global or local criteria.

→ The birth of fracture mechanics (~1950) lead to the concept of stress intensity factor (K) and energy released rate (G)

→ These concepts worked well in the domain of Linear elastic fracture mechanics (LEFM) for brittle materials.

→ In the presence of crack tip plasticity some of the concepts where extended if the crack tip plasticity was small. (Small scale yielding)

→ In ductile materials with large plastic deformation at the crack tip, concept of J-integral was evolved, wherein, stress fields ahead of crack tip are termed as 'HRR' fields and J-integral characterizes the field intensity of crack tip

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→ The concept of crack tip opening displacement (CTOD) was also proposed to characterize the cracks in ductile material.

• Concept of Energy Release Rate (G)

→ 'G' is defined as total potential energy decrease during unit crack extension (dc).

→ 'G' is also referred to as the crack extension force and is given by :-

$$\boxed{G = -\frac{d\Pi}{dc}} \quad ; \ \Pi \text{ is P.E.}$$

Pot. energy.

→ The potential energy in the absence of external tractions can be visualised as being equal to strain energy released.

$$\boxed{\Pi = U_s}$$

as we know

$$U_s = -\frac{\pi c^2 \sigma_0^2}{E}$$

$$U_s = \Pi \qquad\qquad G = -\frac{d\Pi}{dc}$$

$$\therefore \boxed{G = \frac{2\pi c \sigma_0^2}{E}}$$

$G =$
Crack growth occurs if 'G' exceeds or at least becomes equal to critical value G_c, which is fracture toughness of material

$$\boxed{G = G_C}$$

For perfectly brittle material

$$\boxed{G_C = 2\gamma_C}$$

RELATION B/w K & G

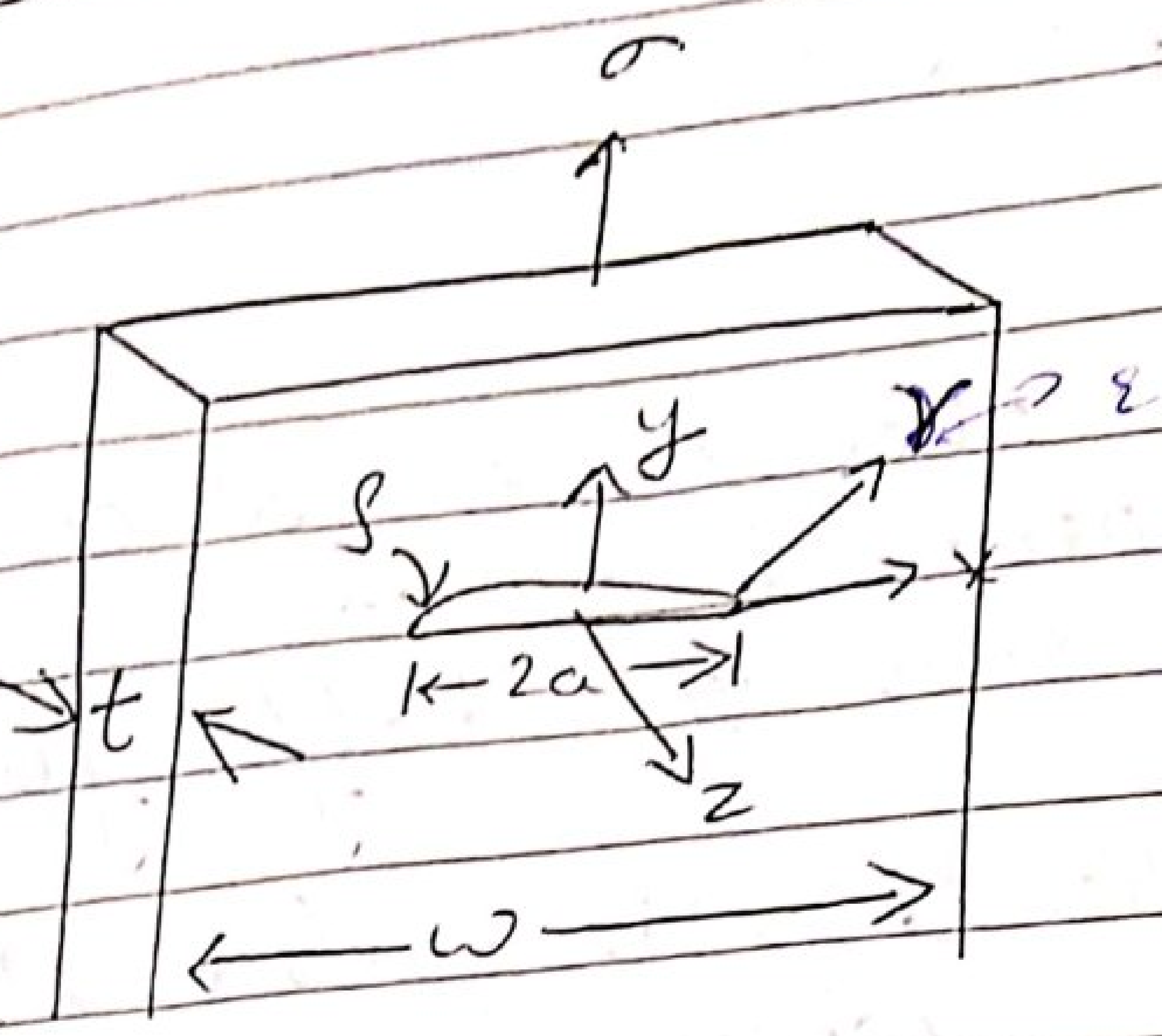
Inspite of fact 'G' has more direct physical interpretation for crack growth process, usually we work with 'K' as it is more amenable to theoretical computations.

'K' can be related to 'G' using following equations.

Plane stress condition. $$\boxed{K^2 = GE}$$

Plane strain condition $$\boxed{K^2 = \frac{GE}{(1-\nu^2)}}$$

Stress Strain Factor



$$\sigma_x = \sigma\left(\frac{a}{2r}\right)^{1/2}\left[\cos\frac{\theta}{2}\left(1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right)\right]$$

$$\sigma_y = \sigma\left(\frac{a}{2r}\right)^{1/2}\left[\cos\frac{\theta}{2}\left(1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right)\right]$$

$$\tau_{xy} = \sigma\left(\frac{a}{2r}\right)^{1/2}\left[\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right]$$

σ = Gross nominal stress $\quad a > r > \rho$

Thee stress distribution at the crack tip in a thin plate for an elastic solid interms of coordinates shown above is:-

for crack $\theta \simeq 0 \longrightarrow$ shap elastic const

$$\sigma_x = \sigma_y = \sigma\left(\frac{a}{2r}\right)^{1/2} \text{ and } \tau_{xy} = 0.$$

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$$\boxed{\sigma_x = \sigma_y = \sigma\left(\frac{a}{2r}\right)^{1/2}}$$

$$\boxed{\tau_{xy} = 0}$$

→ The local stresses near a crack depend on the product of nominal stress σ and sq. root of half-flaw length.

→ This relationship is called stress intensity factor 'K', where for a sharp elastic crack in an infinitely wide plate is defined as:-

$$\boxed{K = \sigma\sqrt{\pi a}}$$

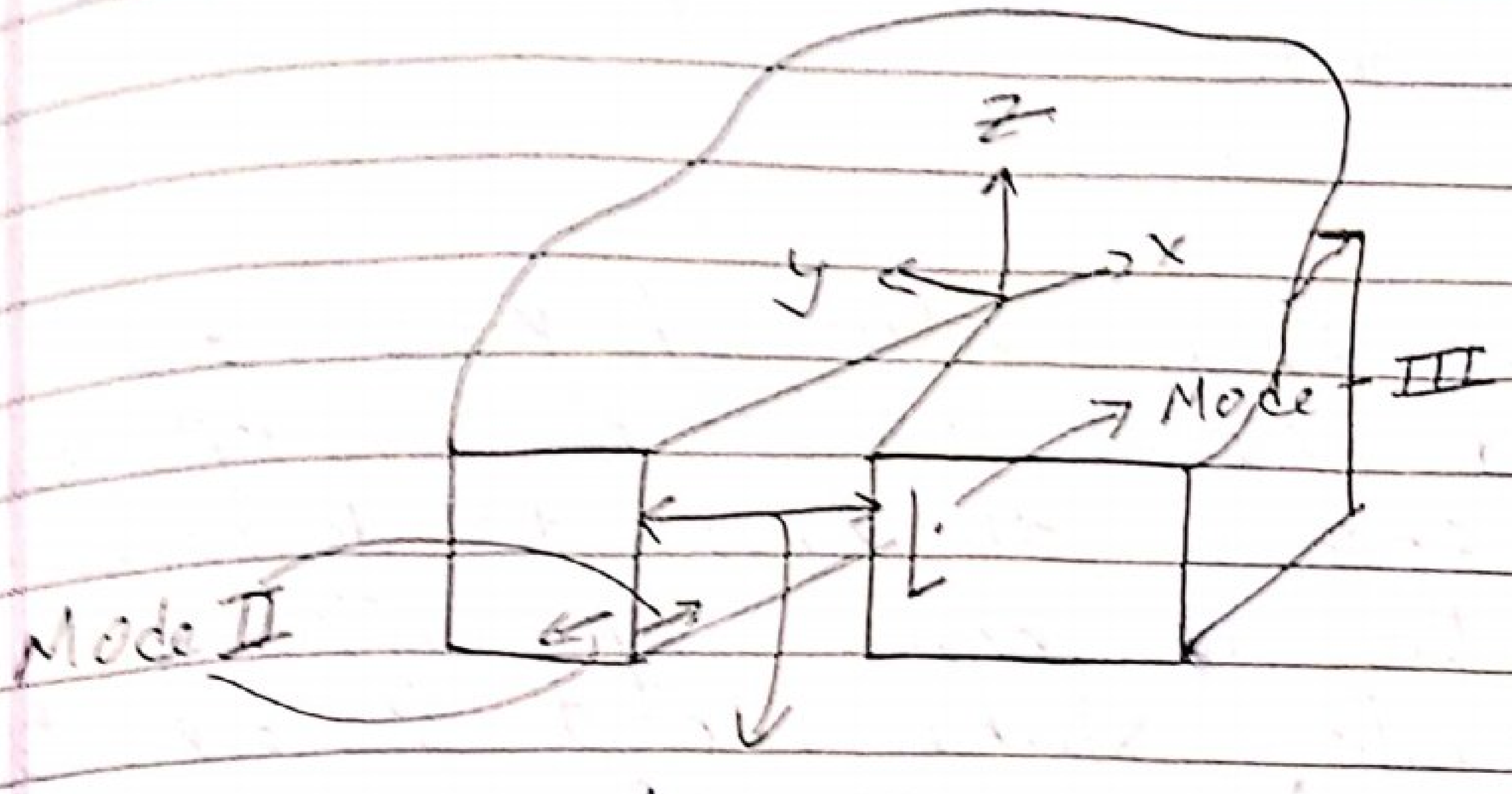
$$\sigma_x = \sigma_y = \sigma\left(\frac{a}{2r}\right)^{1/2}$$

Using this 'K' value the stress intensity factor can be rewritten as

→ Dealing with stress intensity factor, there are several modes of deformation that could be applied to the crack.

→ They can be classified as:
Mode I
Mode II
Mode III

Mode I, crack opening mode. refers to a tensile stress applied in y direction normal to faces of crack.



Mode I

→ Mode I is the usual mode for fracture toughness tests and a critical value of stress intensity (K) determined for this mode would be designated as K_{Ic}

→ There are 2 extreme cases for Mode I loading.

→ With thin-plate-type specimen, the stress state is plane stress while thick specimen there is plane strain

Condition.

→ The plane - strain condition represents the more severe stress state and the values of K_c are lower than for plane - stress specimen.

→ Plane - strain values of K_{Ic} are valid material properties, independent of specimen thickness to describe fracture toughness of strong materials like, heat - treated steel, aluminium and Ti alloys.

→ A properly determined value of K_{IC} represents the fracture toughness of material independent of crack length, geometry or loading system.

→ It is a material property in same sense that ~~material~~ yield strength is material prop.

→ Basic eq. for fracture toughness illustrates the design trade off that is inherent in fracture mechanics design.

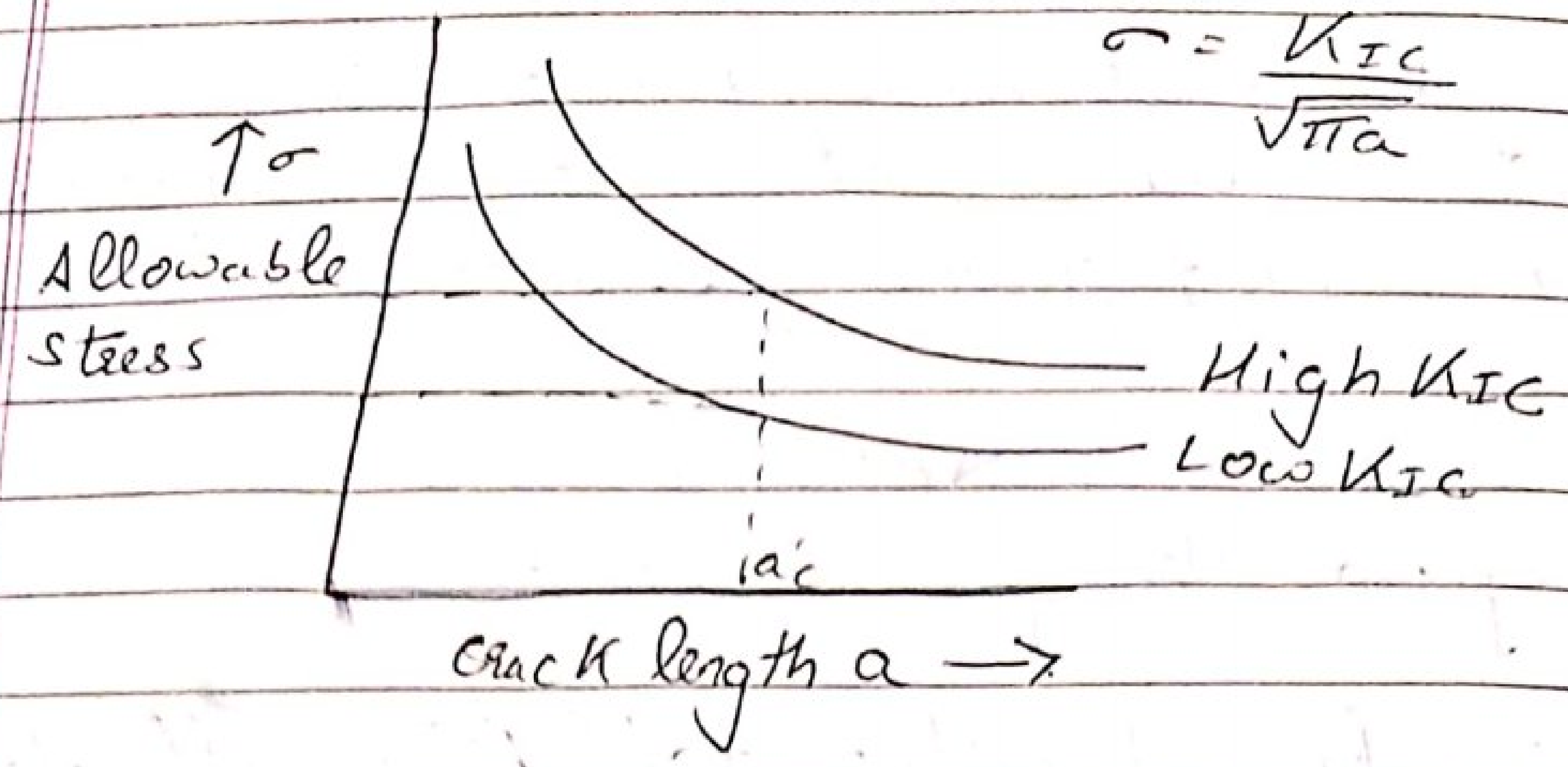
$$K_{IC} = \sigma\sqrt{\pi a}$$

→ If material is selected, K_{IC} is fixed

→ Further if we allow for the presence of a relatively large crack, then design stress is fixed and must be less than K_{IC}

Typical values of K_{IC}

Material	Yield Strength MPa	Fracture toughness K_{IC} ($MPa\sqrt{m}$)
4340 steel	1470	46
Maraging steel	1730	90
Ti-6Al-4V	900	57
2024-T_3 Al alloy	385	26
7075-T_6 Al alloy	500	24



Q The stress intensity for a partial through thickness is given by

$$K = \sigma \sqrt{\pi a} \sqrt{\sec \frac{\pi a}{2t}}$$ where

a is depth of peneteration of the flaw through wall thickness 't'. If the flaw is 5mm deep in a wall 0.5 inches thick. Determine whether the wall will support a stress of 25000 psi (Pounds) if it is made of 7075 - T_6 Al alloy.

K_{IC} (7075-T6 Al Alloy) = 24 MPa $\sqrt{m}$.

A
$$K = \sigma \sqrt{\pi a} \sqrt{\sec \frac{\pi a}{2t}} \quad \text{rad.}$$

σ = 25000 psi = 172.37 MPa
$a = 5 \times 10^{-3}$ m
$t = 1.27 \times 10^{-2}$ m.

$$K = \sigma \sqrt{\pi \, 5 \times 10^{-3}} \sqrt{\sec \frac{\pi (5 \times 10^{-3})}{2 \times 1.27 \times 10^{-2}}}$$

~~$K =$~~

$$24 \times 10^6 = \sigma \; 0.125 \sqrt{\sec \frac{\pi (5 \times 10^{-3})}{2 \times 1.27 \times 10^{-2}}}$$
$$= \sigma \; 0.125 \times 1.107$$

$$\sigma_c = 173.44 \text{ MPa}$$

$\therefore$ nearly equal to applied
$\therefore$ not support.

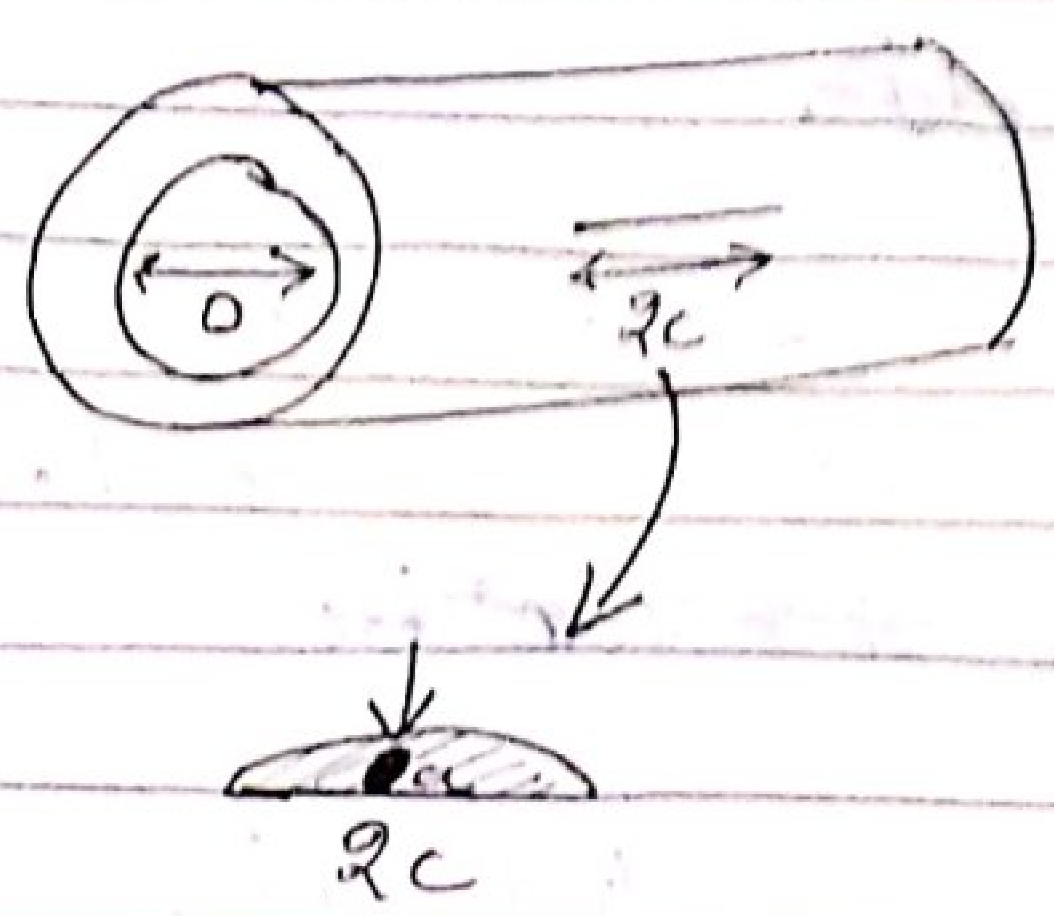
Q A thin wall pressure vessel is made from $T_i - 6Al - 4V$ with $K_{IC} = 57 \text{ MPa}\sqrt{m}$ and $\sigma_0 = 900 \text{ MPa}$. The int. pressure produces a circumferential hoop stress of 360 MPa. The crack is semi-elliptical surface crack oriented with the major plane of the crack $\perp$ to the uniform tensile stress. For this type of loading K_I is given by

$$K_I^2 = \frac{1.21 \sigma^2 a \pi}{Q}$$

where a is surface crack depth
σ = applied nominal stress
$$Q = \phi^2 - 0.212 \left(\frac{\sigma}{\sigma_0}\right)^2$$

Find out criticle crack length for fracture.

$K_I \to K_{IC}$
$a_c = ?$



Assume $2a = 2c$.

A

$\sigma_0 = 900\,MPa$

$\sigma = 360\,MPa$

$K_{IC} = 57\,MPa$.

$$Q = \phi^2 - 0.212\left(\frac{\sigma}{\sigma_0}\right)^2$$

$$Q = \phi^2 - 0.212\left(\frac{360}{900}\right)^2$$

$$Q = \phi^2 - 0.03392$$

$$K_I^2 = \frac{1.21\,\sigma^2 a \pi}{Q}$$

$$(57\times10^6)^2 = \frac{1.21\times(360\times10^6)^2\times a\times 3.14}{\phi^2 - 0.03392}$$

$$\frac{a}{\phi^2 - 0.03392} = 6.598\times10^{-3}$$

PLASTICITY CORRECTIONS

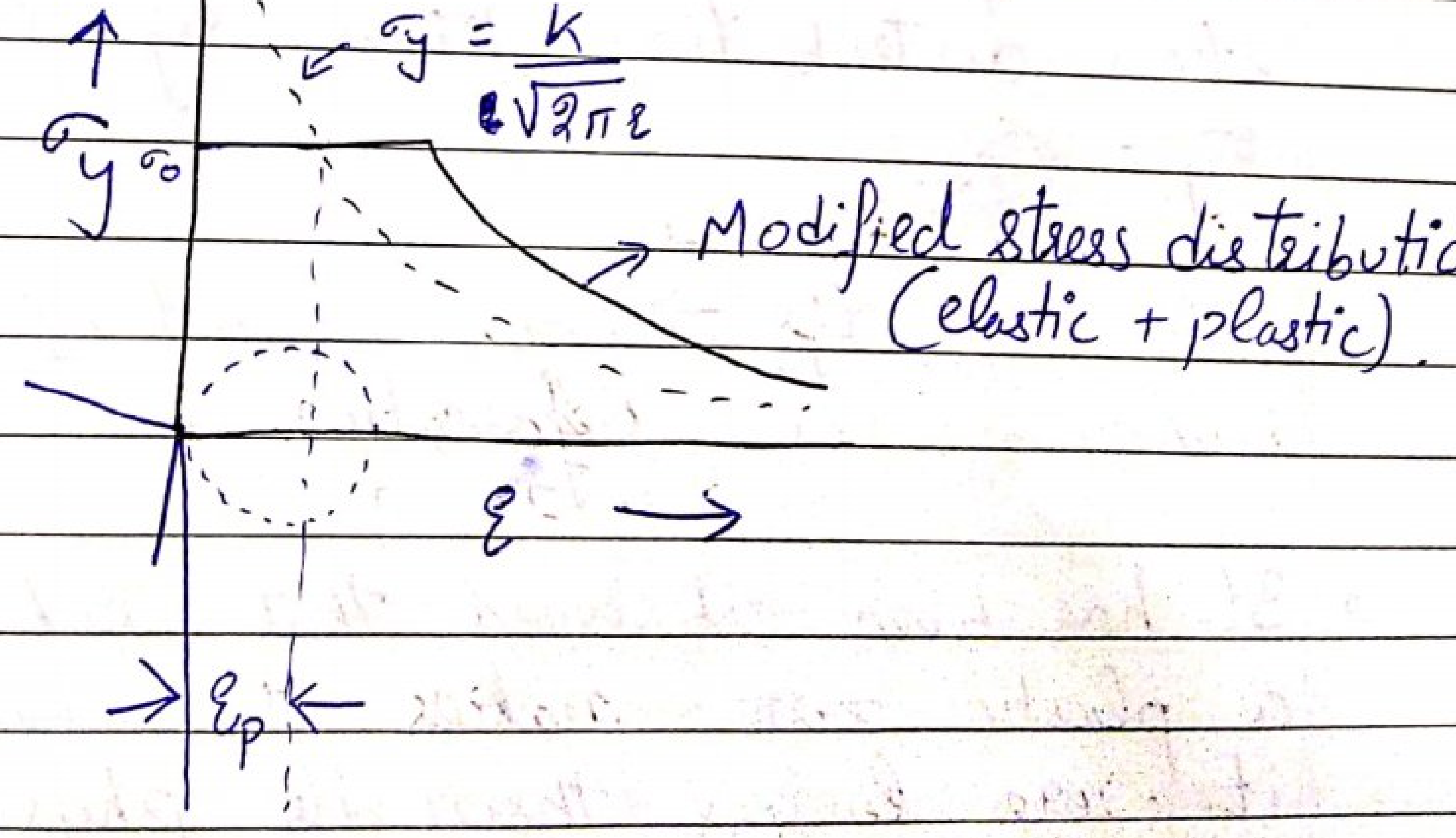
The expressions for the elastic stress field at a crack is described by equation:-

$$\sigma_x = \frac{K}{\sqrt{2\pi r}}\left[\quad " \quad\right]$$

$$\sigma_y = \frac{K}{\sqrt{2\pi r}}\left[\quad " \quad\right]$$

$$\sigma_z = \frac{K}{\sqrt{2\pi r}}\left[\quad " \quad\right]$$

⇒ In reality, metals will yield when $\sigma = \sigma_0$ and a plastic zone will exist at crack tip.



→ In reality metals will yield when $\sigma = \sigma_0$ and a plastic zone will exist at the crack tip.

→ Out to distance $r = r_p$, the elastic stress σ_y is greater than the yield stress σ_0.

→ Approximately, the dist. r_e, is the size of the plastic zone.

For $\theta = 0$, $r = r_p$

$$\sigma_y = \frac{K}{\sqrt{2\pi r_p}} = \sigma_0$$

$$\therefore \boxed{r_p = \frac{K^2}{2\pi \sigma_0^2} = \frac{\sigma^2 a}{2\sigma_0^2}}$$

$K = \sigma\sqrt{\pi a}$

→ It is evident that the plastic zone must be larger than r_p, as it don't allow for yielding caused by elastic stress distribution from $\sigma_y = \sigma_0$ out to $\sigma_y = \sigma_{max}$.

$r_p \rightarrow$ Lower limit of plastic deformation.

→ It has been observed that exchistance of a plastic zone makes the crack act as if it were longer than its physical size.

→ As a result of crack tip plasticity, the displacements are larger and the stiffness is lower than for strictly elastic situation.

→ The usual correction is to assume that the effective crack length is the actual crack length plus radius of plastic zone.

$$a' = a_{eff} = a + r_p$$

Irwin model for plasticity correction

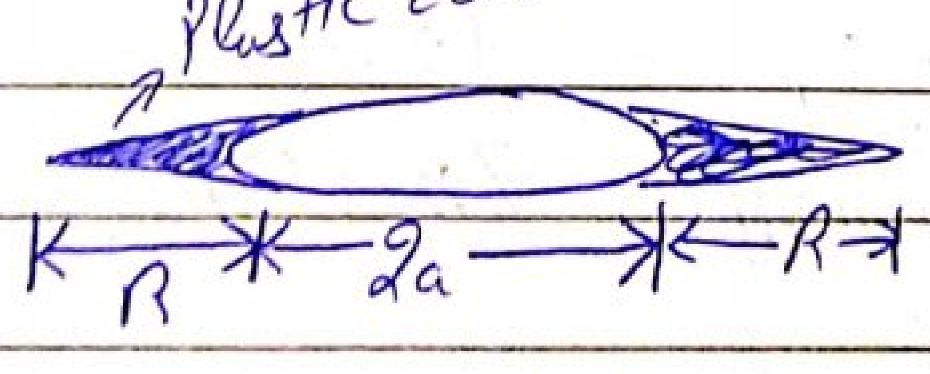
$$\boxed{r_p \simeq \frac{1}{2\pi}\frac{K^2}{\sigma_0^2}}$$ (for 2D plane stress)

$$\boxed{r_p \simeq \frac{1}{6\pi}\frac{K^2}{\sigma_0^2}}$$ (for 3D plane strain)

Dugdale Model

⇒ Another model for plastic zone at a crack tip was proposed by Dugdal, for case of plane stress.

→ He considered that plastic zone takes the form of narrow stripes extending a distance from each crack tip.



→ The analysis is conducted by assuming, there is an elastic crack of length $2(a+R)$ and that ~~that~~ the region of length 'R' is closed up by applying compressive stresses.

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$$\boxed{\begin{aligned} R &= \frac{\pi^2 \sigma^2 a^2}{8\sigma_0^2} \\ \text{or } R &= \frac{\pi K^2}{8\sigma_0^2} \end{aligned}}$$

Q A steel plate with a through thickness crack of length $2a = 20$ mm is subjected to a stress of 400 MPa normal to crack. If yield strength of steel is 1500 MPa, what is plastic zone size and stress intensity factor for the crack. Assume that plate is ∞ wide.

A $K_{eff} = \sigma\sqrt{\pi a_{eff}}$ $\qquad a_{eff} = a + r_p$

$$r_p = \frac{\sigma^2 a}{2\pi\sigma_0^2} = \frac{1}{2\pi}\frac{\sigma^2 a}{\sigma_0^2}$$

$$= \frac{(400\times10^6)^2 \times 10\times10^{-3}}{\pi\times2\times(1500\times10^6)^2}$$

$$= 1.13\times10^{-4}$$

$r_p = 0.113$ mm

$a_{eff} = a + r_p = 10.355$ mm

$\cancel{K_{eff} = \sigma\sqrt{\pi a_{eff}} = 72.15 \text{ MPa}\sqrt{m}}$

$$K_{eff} = \sigma\sqrt{\pi}\sqrt{a + r_p} = \sigma\sqrt{\pi}\sqrt{a + \frac{K^2}{2\pi\sigma_0^2}}$$

$$= \sigma\sqrt{\pi}\sqrt{a + \frac{\sigma^2\pi a}{2\pi\sigma_0^2}}$$

$$= \sigma\sqrt{\pi a}\sqrt{1 + \frac{\sigma^2}{2\sigma_0^2}}$$

$$= 400\sqrt{\pi \times 10 \times 10^{-3}} \times \sqrt{1 + \frac{160000}{2(2250000)}}$$

$$= 72.2\ MPa\sqrt{m}$$

CRACK OPENING DISPLACEMENT

(COD) or (CTOD)

$2\pi r_p$

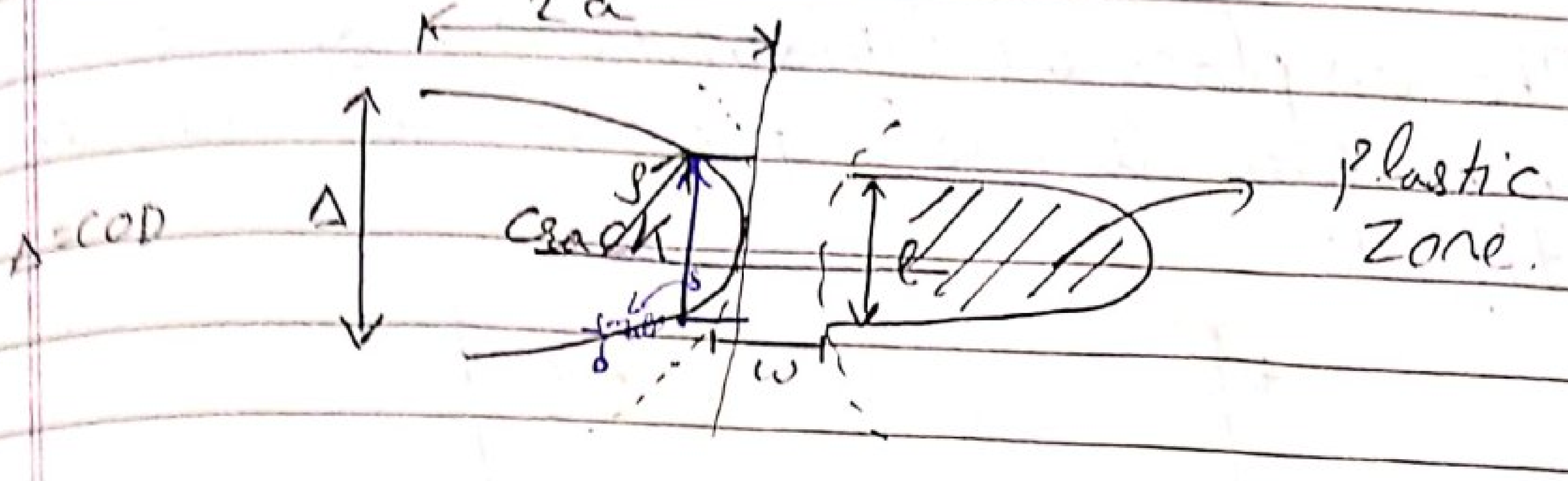
→ The linear elastic fracture mechanics (LEFM) works well for high strength materials ($\sigma_0 > 200000$ Psi) but it is less universally applicable for low strength structural material.

→ There is a limit to extent to which 'K' can be adjusted for crack tip plasticity by the method of previous section (r_p).

→ When r_p become appriciable fraction of 'a' other approaches become necessary.

→ The CTOD concept considers that the material ahead of the crack contains a series of miniature tensile specimen having a gauge length 'l' & width 'w'.

→ The length of the sample is determined by the ~~root~~ radius of the crack 'ρ' and the width is limited by those microstructural factors which control the ductility.



→ In this model crack growth occurs when the specimen adjacent to the crack is fractured.

→ When the failure occur of the 1st specimen adjacent to the crack tip, it immidiately causes the next specimen to fail and so on the overall fracture process is unstable and crack propagation occurs under decreasing stress.

→ When each specimen does not fail immidiately in turn, we have a situation of slow crack growth where the applied stress must be increased for stable crack growth to continue.

→ If a thick plate is loaded in tension (mode I) so that plane strain condition prevail, the plastic deformation at crack tip is confined to narrow band with a thickness of the order of diameter of

Crack tip '2ρ'

→ The displacement (deformation) of the miniature tensile specimen at crack tip is

$$\boxed{\delta = \varepsilon l = \varepsilon 2\rho}$$

→ Unstable fracture occurs when the strain in the specimen adjecent to the crack tip reaches the tensile ductility of the specimen ε_f. Thus the fracture criteria is:

$$\boxed{\delta_c = 2\rho\,\varepsilon_f}$$ plain strain

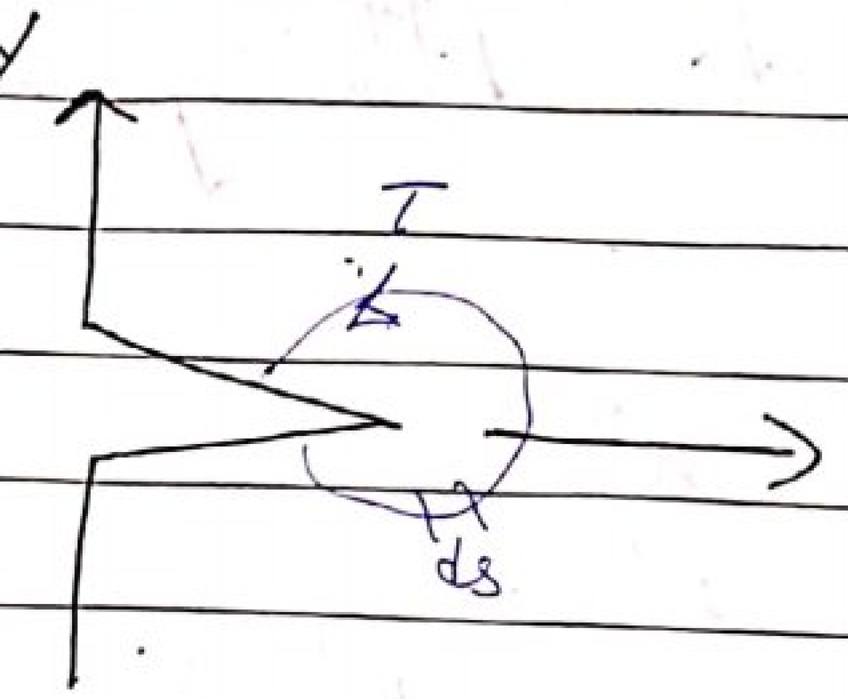
→ In plane-stress (thin plate) tensile loading the strains at the crack tip are distributed over a distance of the order of sheet thickness t.

$$\boxed{\delta = \varepsilon l = \varepsilon t}$$ plane stress

$$\boxed{\delta_c = \varepsilon_f\, t}$$

J - INTEGRAL

A more comprehensive approach to the fracture mechanics of lower - strength ductile materials is provided by J-integral.



Γ → Path of integral
T → outward traction vector occuring on stress contour.

→ The line integral related to energy in vicinity of a crack can be used to solve 2D crack problems in the presence of plastic deformation.

→ Fracture occurs when the J integral reaches a criticle value. J has units MN/m or in-lb/in

$$J = \int_{\Gamma} \left(w\,dy - T \frac{\partial u}{\partial x} ds \right)$$

$W = \int \sigma_{ij}\, d\varepsilon_{ij}$ is the strain energy /unit vol.

U = disp. vector
ds → Increment of contour path.

$\frac{\partial u}{\partial x} ds$ → Rate of work input from stress field into area enclosed by Γ.

→ The J integral is path-independent. Therefore it can be determined from a stress analysis where σ & ε are established by finite element analysis around contour of the crack.

→ The J integral can be interpreted as the potential energy difference b/w 2 identically loaded specimen having slightly diff. crack length.

→ $J = \frac{\partial U_o}{\partial a} = \frac{K^2}{E'}$

$E' = E$ plane stress
$E' = E(1-\nu^2)$ plane strain

R – CURVE

→ The R-curve characterizes the resistance to fracture of a material during slow & stable crack propagation as the plastic zone grows as the crack extends from a sharp notch.

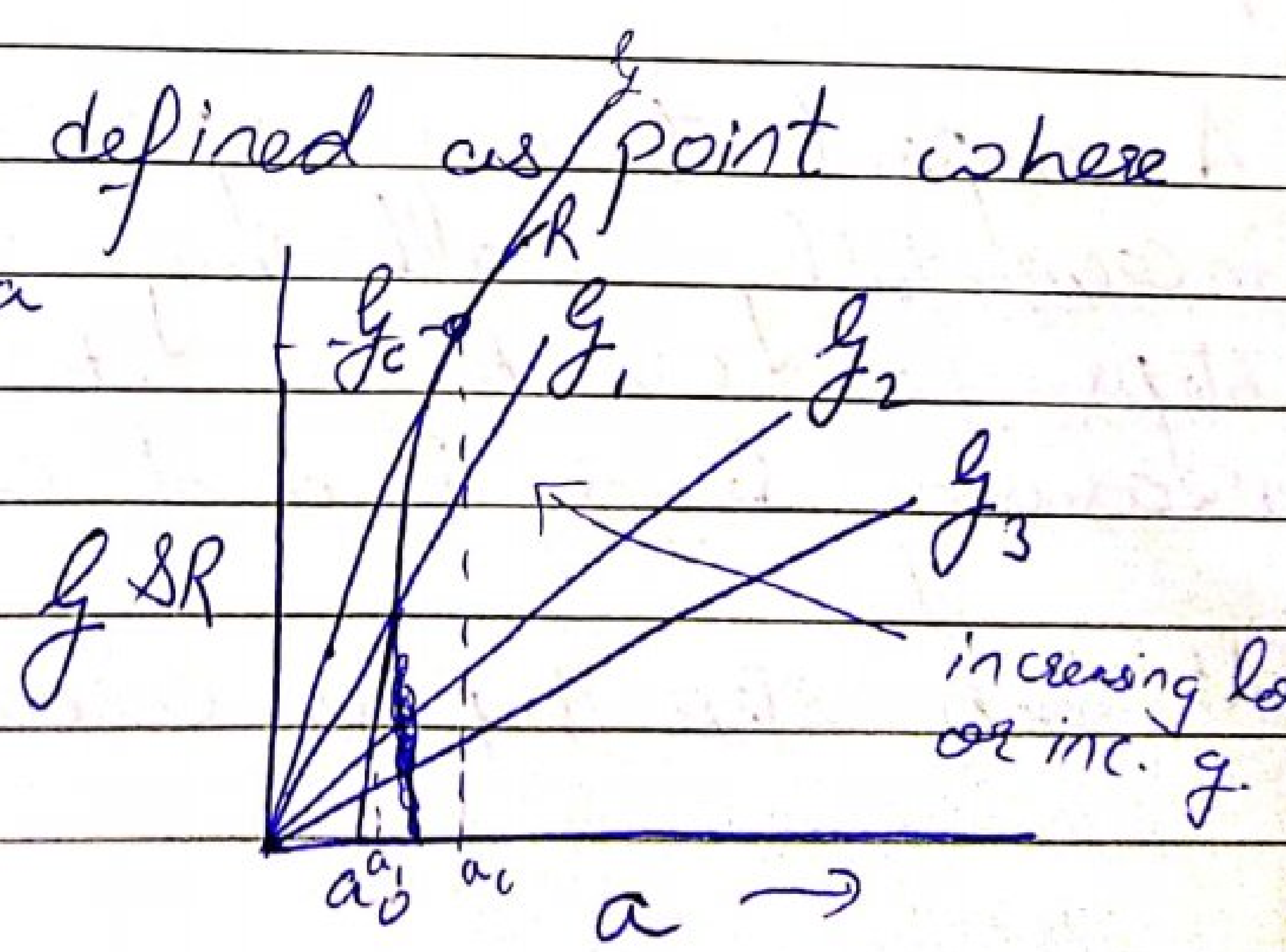
→ It provides a record of toughness of a material as the crack extends under increasing crack extension forces

→ An 'R-curve' is a graphical representation of the resistance to crack propagation R, versus crack length 'a'.

→ It was observed that failure or crack instability will occur when rate of change of strain energy release rate $\partial G/\partial a$ equals the rate of change in resistance to crack growth $\partial R/\partial a$.

→ Thus, G_c is defined as point where $\partial G/\partial a = \partial R/\partial a$

Ductile material →



→ The crack does not extend untill the curve become tangent to the R curve where upon unstable fracture occurs ($G = G_c$)

→ For a more general case of a material with some ductility, crack extension occurs when $G > R$.

→ Consider, the G curve labelled as G_1. At the load or stress corresponding to this value at G_1, the crack will propagate stably from an original length a_0 to a_1 since $G > R$.

→ However, the crack will not extend beyond a_1, because $G < R$. For additional crack extension to occur the value of G must increase untill $G = R$, when unstable fracture occur.

→ ASTM has established a standard procedure for determining "R curve"

→ A compact tension specimen is loaded incrementally, allowing time between steps for crack to stabilize before measuring load 'P' and crack length 'a'

→ At each step K_R is calculated

$$K_R = \frac{P}{B\sqrt{W}} \times f\left(\frac{a}{w}\right)$$

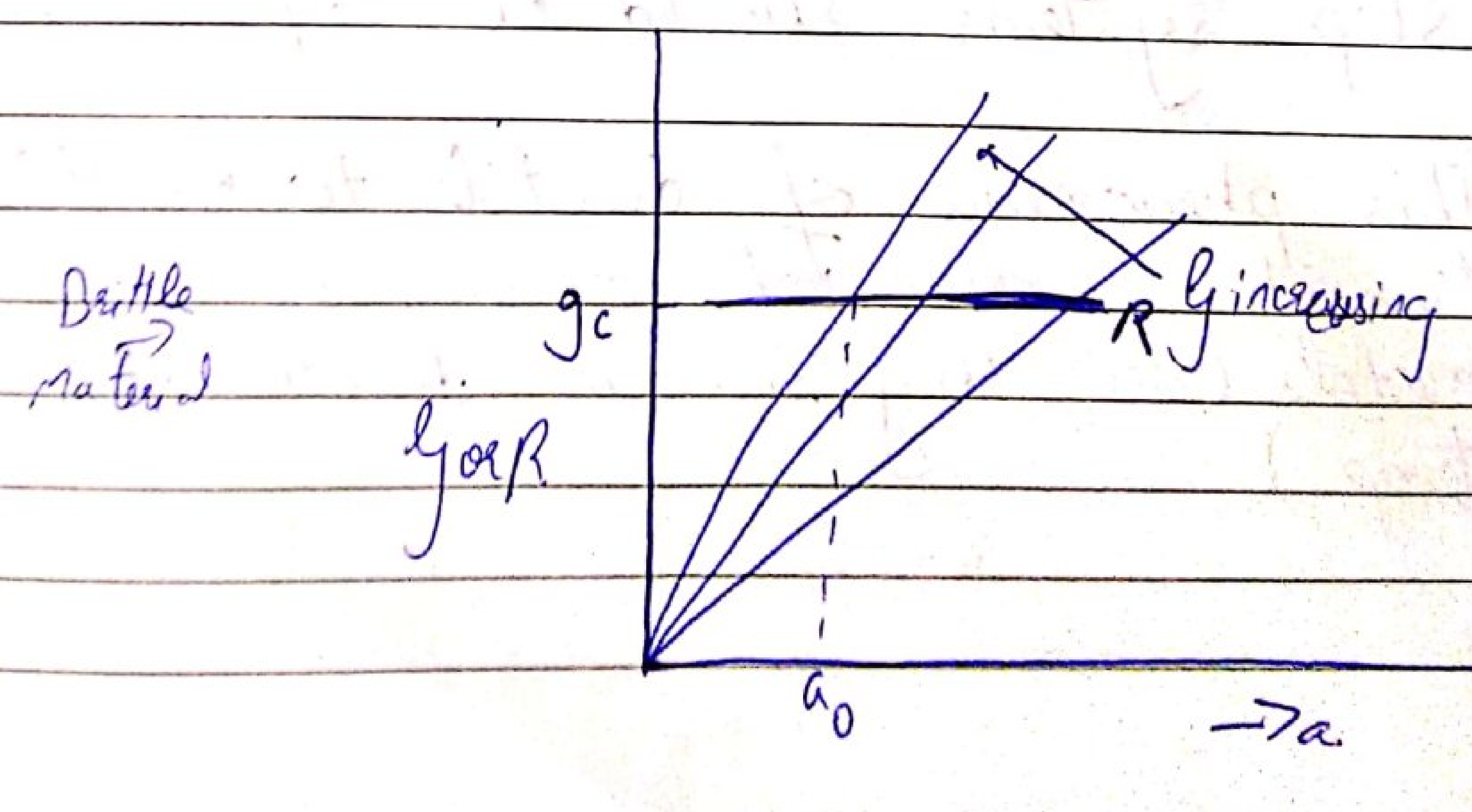
where
$$f\left(\frac{a}{w}\right) = \left[2 + \frac{a}{w} \Big/ \left(1 - \frac{a}{w}\right)^{3/2}\right]\left[0.866 + 4.64\left(\frac{a}{w}\right) - 13.32\left(\frac{a}{w}\right)^2 + 14.72\left(\frac{a}{w}\right)^3 - 5.6\left(\frac{a}{w}\right)^4\right]$$

B = specimen thickness
w = crack width.

→ The crack length 'a' used in eq. is the effective crack length where the physical crack growth is corrected for the plastic zone, r_p

$$\boxed{a_{eff} = a_0 + \Delta a + r_p}$$

→ For low strength, high-toughness material, the correction for the plastic zone should be used.



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DUCTILE TO BRITTLE TRANSITION

→ Certain material which are ductile at given temp. (Eg R.T.) become brittle at lower temp.

→ The temp. at which this happens is termed as ductile to brittle transition temp. (DBTT)

→ DBT can cause problems in Components which operate in ambient and low temp. condition

→ Typically the phenomena is reported in polycrystalline material

→ Deformation should be continuous across grain boundry in polycrystals for them to be ductile.

→ This implies that five independent slip systems should be operative

→ This phenomena of ductile to brittle transition is not observed in FCC metals (i.e. they remain ductile to low temp.)

→ Common BCC metals become brittle at low temp. ~~[illegible]~~

Causes of Ductile to Brittle Transition

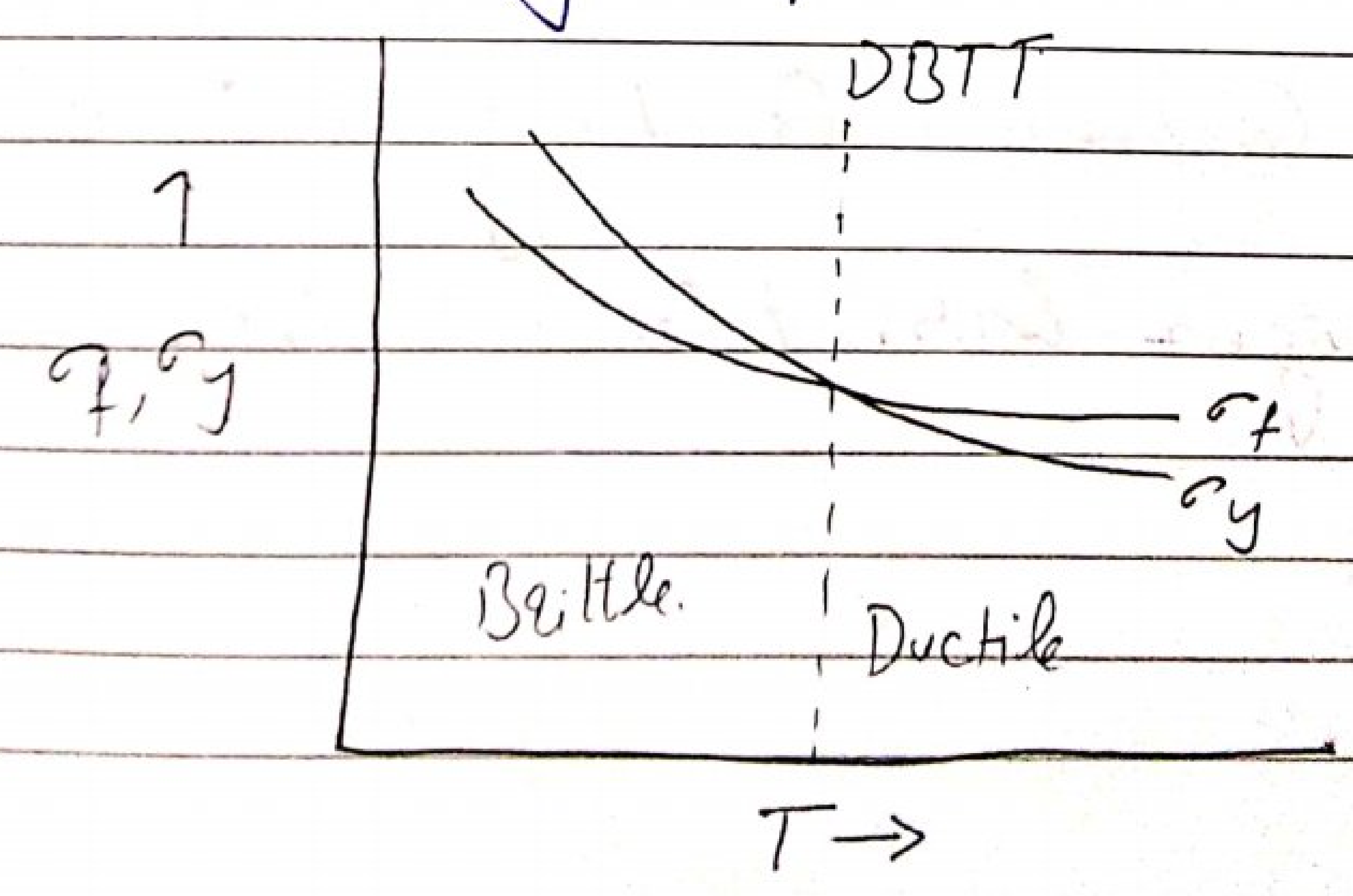
→ Both the fracture stress (σ_f) and yield stress (σ_y) are temp. dependent.

→ However, ~~the~~ as slip is a thermally activated process, the yield stress is a stronger function of temp. as compared to fracture stress.

Ductile fs > ys.
Brittle ys > fs

→ If one looks at the Griffith's criteria of fracture, σ_f has a slight dependence on temp. as $\underset{\text{Modulus}}{E}$ inc. with decreasing the temp.

→ $\overset{\text{yield strength}}{\sigma_y}$ on the other hand has a steeper inc. with decreasing temp.



PROTECTION AGAINST BRITTLE FRACTURE

→ Lower value of surface Energy (γ) implies a lower fracture stress. Increase in fracture stress or surface energy can be done by chemical adsorption of molecule on crack surfaces.

→ Removal of surface crack ⇒ Etching of glass.

→ Use fine grain Ceramic (~0.1 μm) : dec. in grain size dec. in size of crack

→ Introduce residual compressive stresses : Toughened glasses Eg Short peening is done, surface of molten glass is left in atmosphere

ROLE OF ENVIRONMENT IN FRACTURE

① Stress Corrosion Cracking (SCC)

② Hydrogen Embrittlement (HE)

Stress Corrosion Cracking (SCC)

- In SCC presence of a chemical species can enhance crack propagation & reduce fracture stress.
- This phenomena may lead to sudden failure of ductile material especially at high temp.
- The chemical agent causing stress corrosion cracking is one which is normally corrosive to the metal/alloy involved.
- Similar to a criticle value of the stress intensity factor (K_{IC}) in normal fracture mechanics, we can define a criticle stress intensity factor in the presence of a corrosive environment. (K_{ISCC})
- Several accidents like explosion of boilers, rupture of gas pipes have happened due to this process.

Till here in Mid sem

Hydrogen Embrittlement (HE).

-> Another related phenomena, which can be classified under the broad ambit of SCC is hydrogen embrittlement.

-> Hydrogen can be introduced into the material during processing (welding, pickering etc.) or in service (from nuclear reactor, corrosive environment).

-> This [H] diffuses into interstitial site of metals and alloys and causes hinderance to dislocation motion of ductile materials.