ELEC-E8101 Group project:
Lab A report
Group 14

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Instructions: *For this lab report there is no size limit on your report, but try to be concise.*

Reporting of Task 4.1

We want to express the linearized EOM in the form of a general linear time-invariant (LTI) SS system:

$$\begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + Bu \\ y = C\mathbf{x} + Du \end{cases}$$

The input is the voltage applied to the motors and the measurement is the angular deviation of the balancing robot from the vertical upright position:

$$u = v_m$$
$$y = \theta_b$$

We choose state $\mathbf{x}$ as:

$$\begin{bmatrix} x_w \\ \dot{x}_w \\ \theta_b \\ \dot{\theta}_b \end{bmatrix}$$

The obtained A, B, C and D matrices are here presented in parametric form:

$$A =$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -2\,\frac{\left(l_b\,\left(l_b+l_w\right)m_b+I_b\right)\left(K_e\,K_t+1/2\,b_f\,R_m\right)}{\left(\left(I_b\,{l_w}^2+{l_b}^2\left({l_w}^2m_w+I_w\right)\right)m_b+I_b\,\left({l_w}^2m_w+I_w\right)\right)R_m} & -\frac{{m_b}^2{l_b}^2{l_w}^2g}{\left(\left(m_w+m_b\right){l_w}^2+I_w\right)I_b+{l_b}^2m_b\,\left({l_w}^2m_w+I_w\right)} & 2\,\frac{l_w\,\left(m_b\,{l_b}^2+m_b\,l_b\,l_w+I_b\right)\left(K_e\,K_t+1/2\,b_f\,R_m\right)}{\left(\left(\left({l_b}^2m_w+I_b\right)m_b+I_b\,m_w\right){l_w}^2+I_w\,\left(m_b\,{l_b}^2+I_b\right)\right)R_m} \\ 0 & 0 & 0 & 1 \\ 0 & 2\,\frac{\left(\left(m_w+m_b\right){l_w}^2+m_b\,l_b\,l_w+I_w\right)\left(K_e\,K_t+1/2\,b_f\,R_m\right)}{l_w\,\left(\left(\left({l_b}^2m_w+I_b\right)m_b+I_b\,m_w\right){l_w}^2+I_w\,\left(m_b\,{l_b}^2+I_b\right)\right)R_m} & \frac{m_b\,\left(\left(m_w+m_b\right){l_w}^2+I_w\right)g l_b}{\left(\left({l_b}^2m_w+I_b\right)m_b+I_b\,m_w\right){l_w}^2+I_w\,\left(m_b\,{l_b}^2+I_b\right)} & -2\,\frac{\left(\left(m_w+m_b\right){l_w}^2+m_b\,l_b\,l_w+I_w\right)\left(K_e\,K_t+1/2\,b_f\,R_m\right)}{\left(\left(\left({l_b}^2m_w+I_b\right)m_b+I_b\,m_w\right){l_w}^2+I_w\,\left(m_b\,{l_b}^2+I_b\right)\right)R_m} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 2\,\dfrac{l_w\,K_t\,\left(m_b\,{l_b}^2 + m_b\,l_b\,l_w + I_b\right)}{\left(\left(\left(m_w + m_b\right){l_w}^2 + I_w\right)I_b + {l_b}^2 m_b\,\left({l_w}^2 m_w + I_w\right)\right)R_m} \\ 0 \\ -2\,\dfrac{K_t\,\left(\left(m_w + m_b\right){l_w}^2 + m_b\,l_b\,l_w + I_w\right)}{\left(\left(\left({l_b}^2 m_w + I_b\right)m_b + I_b\,m_w\right){l_w}^2 + I_w\,\left({l_b}^2 m_b + I_b\right)\right)R_m} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

Here we report matrices A, B, C and D in numeric form:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -773.7853734 & -6.573516819 & 16.24949284 \\ 0 & 0 & 0 & 1 \\ 0 & 3313.238430 & 63.07193800 & -69.57800702 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 36.59795686 \\ 0 \\ -156.7072230 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

Reporting of Task 4.2

The transfer function of an LTI SS system is given by the formula:

$$G(s) = C(sI - A)^{-1}B + D$$

The result reported has been obtained by computing an analytical expression of the trasnfer function and then substituting the data.

$$G(s) = 0.001240873320\,\frac{s}{(s + 843.4002)\,(s - 5.6790)\,(s + 5.6422)}$$

At first we used the numeric forms of the matrices A, B, C and D to define the SS system in `MATLAB` and then to compute the transfer function. This however resulted in strange results, giving us additional poles and complex conjugate zeros (with real part $= 0$ and imaginary part $= \pm i$). We then tried to go fully analytical and we got much more reasonable results; in this phase we also compared the results obtained using `Maple` and `MATLAB`, concluding that they are quite different since these softwares run in completely different manners. From this point onwards we only used `MATLAB`.

Reporting of Task 4.3

Reporting of Task 4.4

Reporting of Task 4.5

Reporting of Task 4.6

Reporting of Task 4.7

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