MECH650-Clearpath Husky Robot: Forward and Inverse Kinematics Mathematical Analysis

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1 Introduction and Robot Specifications

The Clearpath Husky is an unmanned ground vehicle (UGV) designed for outdoor robotics research. It employs a skid-steer configuration with four wheels, where the left and right sides are driven independently.

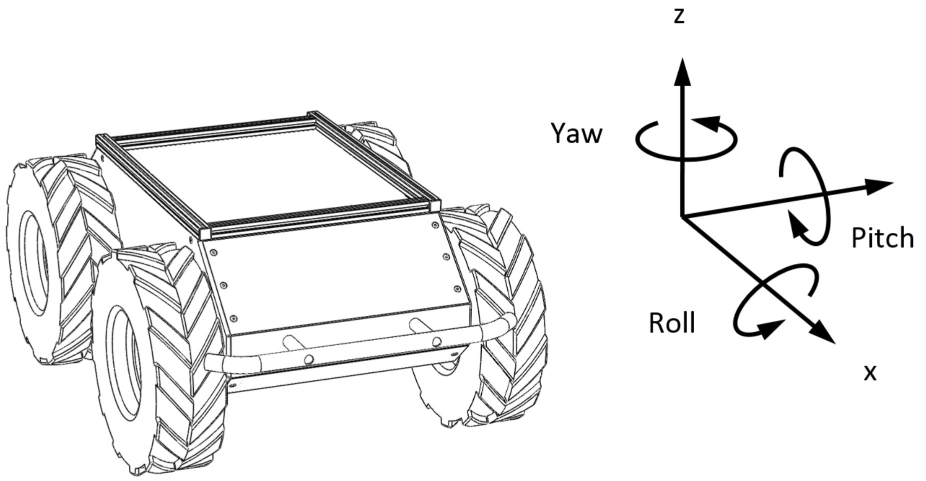

Figure 1: Husky's front is shown. When commanded with a positive translational velocity (forward), wheels travel in the positive X-direction.

2 Coordinate Frame Conventions

Following ISO 8855 and ROS REP-103 standards:

- **x-axis**: Points forward (direction of positive linear velocity)
- **y-axis**: Points left
- **z-axis**: Points upward
- **Origin**: Located at the center point between the wheels (for 2-wheel model) or geometric center (for 4-wheel model)

2.1 Key Specifications:

- **Track Width (l)**: 555 mm (0.555 m) - distance between left and right wheel centers
- **Wheelbase**: 512 mm (0.512 m) - distance between front and rear axles
- **Wheel Radius (r)**: Approximately 165 mm (0.165 m) based on 13" diameter wheels
- **Maximum Speed**: 1.0 m/s (A200 series)
- **Drive Configuration**: 4-wheel differential drive (skid-steer)

2.2 State Vector

The robot's pose in the global frame is represented as:

$$\mathbf{q} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{1}$$

where:

- (x, y) = position in the global coordinate frame
- θ = orientation (yaw angle) with respect to the global x-axis

3 Kinematic Model Development

3.1 $\mathcal{T}$ Task 2: Define parameters and variables

Robot parameters (with symbols and values):

- ____________________ = __________
- ____________________ = __________

Motion variables:

- ____________________ = __________
- ____________________ = __________

3.2 $\mathcal{T}$ Task 3: Derive the inverse kinematics

For the kinematic model you identified:

1. **Write the relationship between robot motion and wheel velocities:**
   Left side velocity: $v_L =$ ____________________

   Right side velocity: $v_R =$ ____________________
2. **Convert to wheel angular velocities:**
   $\varphi_1 =$ ____________________

   $\varphi_2 =$ ____________________
3. **Express in matrix form:**

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{bmatrix} \underline{\hspace{1cm}} \\ \underline{\hspace{1cm}} \end{bmatrix} \tag{2}$$

3.3 $\mathcal{T}$ Task 4: Derive the forward kinematics

Now derive the forward kinematics - the relationship between wheel motions and robot motion:

1. **Express the robot's linear and angular velocities in terms of wheel velocities:**

   Linear velocity: $v =$ ____________________

   Angular velocity: $\omega =$ ____________________

2. **Write the transformation from robot body frame to global frame:**

$$\dot{x} = \underline{\hspace{3cm}} \tag{3}$$
$$\dot{y} = \underline{\hspace{3cm}} \tag{4}$$
$$\dot{\theta} = \underline{\hspace{3cm}} \tag{5}$$

3. **Express in matrix form relating wheel velocities to robot pose derivatives:**

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \tag{6}$$

   Hint: This should involve both the wheel radius r, track width l, and trigonometric functions of θ.

Hints

- Think about how many degrees of freedom the robot has
- Consider what happens when both left wheels rotate at the same speed
- For skid-steer, front and rear wheels on each side typically rotate together

4 Part 4 - Numerical Verification

4.1 Inverse Kinematics Calculations

Using your derived inverse kinematics equations and the Husky's actual parameters, calculate wheel angular velocities for:

- **Pure forward motion at 0.5 m/s**
  - $\varphi_1 =$ __________rad/s
  - $\varphi_2 =$ __________rad/s
  - Do these values make sense? Why? ____________________
- **Pure rotation at 1 rad/s** (turning in place)
  - $\varphi_1 =$ __________rad/s
  - $\varphi_2 =$ __________rad/s
  - What do you notice about these values? ____________________

- **Following a circular arc**: v = 0.3 m/s, ω = 0.4 rad/s
  - $\varphi_1 =$ ___________rad/s
  - $\varphi_2 =$ ___________rad/s
  - Which wheel is faster? Why? ______________________

4.2 Forward Kinematics Calculations

Using your derived forward kinematics equations, calculate the robot's motion for these wheel speeds:

- **Both wheels at 2 rad/s**: $\varphi_1 = \varphi_2 = 2$ rad/s
  - Robot linear velocity: $v =$ ___________m/s
  - Robot angular velocity: $\omega =$ ___________rad/s
  - What type of motion is this? ______________________
- **Opposite wheel speeds**: $\varphi_1 = -1.5$ rad/s, $\varphi_2 = +1.5$ rad/s
  - Robot linear velocity: $v =$ ___________m/s
  - Robot angular velocity: $\omega =$ ___________rad/s
  - What type of motion is this? ______________________
- **Unequal wheel speeds**: $\varphi_1 = 1$ rad/s, $\varphi_2 = 3$ rad/s
  - Robot linear velocity: $v =$ ___________m/s
  - Robot angular velocity: $\omega =$ ___________rad/s
  - Is the robot moving forward and turning? ______________________