MECH650-Clearpath Husky Robot: Forward and Inverse Kinematics Mathematical Analysis

SOLUTION MANUAL

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1 Introduction and Robot Specifications

The Clearpath Husky is an unmanned ground vehicle (UGV) designed for outdoor robotics research. It employs a skid-steer configuration with four wheels, where the left and right sides are driven independently.

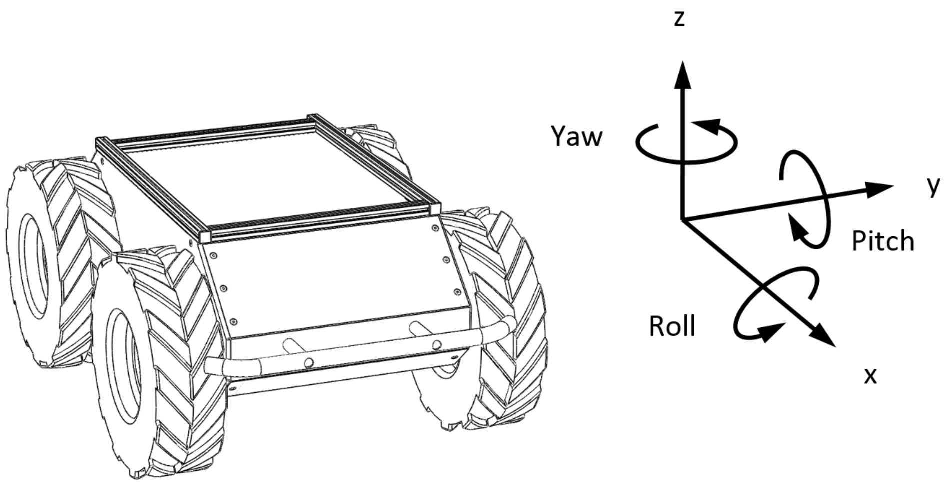

Figure 1: Husky's front is shown. When commanded with a positive translational velocity (forward), wheels travel in the positive X-direction.

2 Coordinate Frame Conventions

Following ISO 8855 and ROS REP-103 standards:

- **x-axis**: Points forward (direction of positive linear velocity)
- **y-axis**: Points left
- **z-axis**: Points upward
- **Origin**: Located at the center point between the wheels (for 2-wheel model) or geometric center (for 4-wheel model)

2.1 Key Specifications:

- **Track Width (l)**: 555 mm (0.555 m) - distance between left and right wheel centers
- **Wheelbase**: 512 mm (0.512 m) - distance between front and rear axles
- **Wheel Radius (r)**: Approximately 165 mm (0.165 m) based on 13" diameter wheels
- **Maximum Speed**: 1.0 m/s (A200 series)
- **Drive Configuration**: 4-wheel differential drive (skid-steer)

2.2 State Vector

The robot's pose in the global frame is represented as:

$$\mathbf{q} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{1}$$

where:

- (x, y) = position in the global coordinate frame
- θ = orientation (yaw angle) with respect to the global x-axis

3 Kinematic Model Development

3.1 $\mathcal{T}$ Task 2: Define parameters and variables

Robot parameters (with symbols and values):

- **Track width: l = 0.555 m**
- **Wheel radius: r = 0.165 m**

Motion variables:

- **Linear velocity: v = m/s**
- **Angular velocity: ω = rad/s**

3.2 $\mathcal{T}$ Task 3: Derive the inverse kinematics

For the kinematic model you identified:

1. **Write the relationship between robot motion and wheel velocities:**

   Left side velocity: $v_L = v - \frac{l\omega}{2}$

   Right side velocity: $v_R = v + \frac{l\omega}{2}$

2. **Convert to wheel angular velocities:**

   $\varphi_1 = \frac{v_L}{r} = \frac{v - \frac{l\omega}{2}}{r}$

   $\varphi_2 = \frac{v_R}{r} = \frac{v + \frac{l\omega}{2}}{r}$

3. **Express in matrix form:**

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & -\frac{l}{2r} \\ \frac{1}{r} & \frac{l}{2r} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{2}$$

3.3 $\mathcal{T}$ Task 4: Derive the forward kinematics

Now derive the forward kinematics - the relationship between wheel motions and robot motion:

1. **Express the robot's linear and angular velocities in terms of wheel velocities:**

   Linear velocity: $v = \frac{r(\varphi_1+\varphi_2)}{2}$

   Angular velocity: $\omega = \frac{r(\varphi_2-\varphi_1)}{l}$

2. **Write the transformation from robot body frame to global frame:**

$$\dot{x} = v\cos\theta \tag{3}$$
$$\dot{y} = v\sin\theta \tag{4}$$
$$\dot{\theta} = \omega \tag{5}$$

3. **Express in matrix form relating wheel velocities to robot pose derivatives:**

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r\cos\theta}{2} & \frac{r\cos\theta}{2} \\ \frac{r\sin\theta}{2} & \frac{r\sin\theta}{2} \\ -\frac{r}{l} & \frac{r}{l} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \tag{6}$$

   Hint: This should involve both the wheel radius r, track width l, and trigonometric functions of θ.

Hints

- Think about how many degrees of freedom the robot has
- Consider what happens when both left wheels rotate at the same speed
- For skid-steer, front and rear wheels on each side typically rotate together

4 Part 4 - Numerical Verification

Given parameters: $l = 0.555$ m, $r = 0.165$ m

4.1 Inverse Kinematics Calculations

Using your derived inverse kinematics equations and the Husky's actual parameters, calculate wheel angular velocities for:

- **Pure forward motion at 0.5 m/s**

  Solution: For pure forward motion: $v = 0.5$ m/s, $\omega = 0$ rad/s

  - $\varphi_1 = \frac{0.5-0}{0.165} = 3.03$ rad/s
  - $\varphi_2 = \frac{0.5+0}{0.165} = 3.03$ rad/s
  - Do these values make sense? Why? **Yes, both wheels rotate at the same speed for straight motion**

- **Pure rotation at 1 rad/s** (turning in place)

  Solution: For pure rotation: $v = 0$ m/s, $\omega = 1$ rad/s

- $\varphi_1 = \frac{0 - \frac{0.555 \times 1}{2}}{0.165} = -1.68$ rad/s
- $\varphi_2 = \frac{0 + \frac{0.555 \times 1}{2}}{0.165} = +1.68$ rad/s
- What do you notice about these values? **They are equal in magnitude but opposite in sign**

- **Following a circular arc**: v = 0.3 m/s, $\omega = 0.4$ rad/s

  Solution: For circular arc motion: $v = 0.3$ m/s, $\omega = 0.4$ rad/s

  - $\varphi_1 = \frac{0.3 - \frac{0.555 \times 0.4}{2}}{0.165} = 1.15$ rad/s
  - $\varphi_2 = \frac{0.3 + \frac{0.555 \times 0.4}{2}}{0.165} = 2.49$ rad/s
  - Which wheel is faster? Why? **Right wheel is faster because the robot is turning left**

4.2 Forward Kinematics Calculations

Using your derived forward kinematics equations, calculate the robot's motion for these wheel speeds:

- **Both wheels at 2 rad/s**: $\varphi_1 = \varphi_2 = 2$ rad/s

  Solution:

  - Robot linear velocity: $v = \frac{0.165(2+2)}{2} = 0.33$ m/s
  - Robot angular velocity: $\omega = \frac{0.165(2-2)}{0.555} = 0$ rad/s
  - What type of motion is this? **Pure translation (straight line motion)**

- **Opposite wheel speeds**: $\varphi_1 = -1.5$ rad/s, $\varphi_2 = +1.5$ rad/s

  Solution:

  - Robot linear velocity: $v = \frac{0.165(-1.5+1.5)}{2} = 0$ m/s
  - Robot angular velocity: $\omega = \frac{0.165(1.5-(-1.5))}{0.555} = 0.89$ rad/s
  - What type of motion is this? **Pure rotation (turning in place)**

- **Unequal wheel speeds**: $\varphi_1 = 1$ rad/s, $\varphi_2 = 3$ rad/s

  Solution:

  - Robot linear velocity: $v = \frac{0.165(1+3)}{2} = 0.33$ m/s
  - Robot angular velocity: $\omega = \frac{0.165(3-1)}{0.555} = 0.59$ rad/s
  - Is the robot moving forward and turning? **Yes, it's moving forward while turning right**