MATLAB Implementation of Computed Tomography Using Fan Beam Projections

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***Abstract*—Computed Tomography(CT) is a widely used medical imaging modality. It utilizes ionizing radiation to display human anatomy and abnormalities. The images may be generated using either parallel or fan beam projections. In this documentation a MATLAB implementation of computed tomography projection, back projection, and filtered back projection algorithm is demonstrated. For the forward problem fan beam projections are utilized. Motivation of this work is to provide simulation tools for the analysis of CT and its characteristics. In this way, experimental work may be avoided to some extent. In addition to the developed algorithms, further reconstruction algorithms are discussed in detail.**

Keywords—computed tomography, projection, back projection, filtered back projection, MATLAB, imaging, fan beam, ART, SIRT, SART

I. Introduction

In December 22, 1895 Wilhelm K. Rontgen was probably not aware of the fact that he invented a groundbreaking medical imaging modality. However, he was aware that the X-rays had some interesting properties. Lighting the first fire, Roentgen allowed other researchers to develop X-ray imaging modalities by publishing "A New Kind of Rays". After this publication, handful of researchers started to develop X-rays both for medical and commercial use. Dr Edwin Frost was the first one using X-ray radiography for diagnostic purposes. E. Haschek and O. Lindenthal developed further diagnostic approaches such as contrast enhanced vein imaging using X-rays. On the other hand, researchers like Mihran Kassabian were observing the effects of X-rays on human tissues.

World War II caused a significant acceleration in both scientific and experimental works. With the help of mathematical background provided by Johann Radon, in 1967 Godfrey Hounsfield invented the first commercially viable Computed Tomography [1]. After this invention medical imaging reseacrhes have gained further acceleration and resulted in invention of modalities such as SPECT, PET and so on.

In this project, the aim was to implement a software-based simulation of CT that utilizes fan beams. Motivation behind this aim was to allow a researcher to make development on CT without harming any living organism. Furthermore, characteristics and limitations of CT can be examined through a software-based simulation. With the help of MATLAB, projection, back projection, and filtered back projection algorithms are implemented and resulting projection data and back projected images are discussed in detail. Changing crucial variables such as beam number is experimented and results are noted.

II. Theory and Algorithm

Here implemented algorithm will be explained using pseudocode and related background will be presented. Projection and back projection algorithms are explained separately although they are very similar and closely related because projection and back projection can be run independently. Indeed, back projection algorithm should obtain only projection matrix where projection algorithm takes only image data.

A. Projection

Projection Program Starts

Load the image

%Here image that is to be projected is loaded and transformed to a matrix.

Obtain the size

Get number of projection angle steps, number of sampling points, distance, and projection angle range

Create x, and y coordinate arrays

%Mathematically taking a projection means calculating a line integral. In CT, this projection naturally occurs through X-ray beams. In the case of this project mathematical calculations should be done through (1). However, conversion to cartesian coordinates yields (2). Through (2) projection is calculated for each t and theta value. However, since the fan beam projections are used t and theta values are composed of distance, gamma, and beta values. The relations can be seen in Fig.1 and equations (3) and (4).

$$p_\theta(t) = \int_{(\theta,t)line} f(x,y)\,ds \tag{1}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - t)\,dx\,dy \tag{2}$$

$$\beta + \gamma = \theta \tag{3}$$

$$t = D\sin\gamma \tag{4}$$

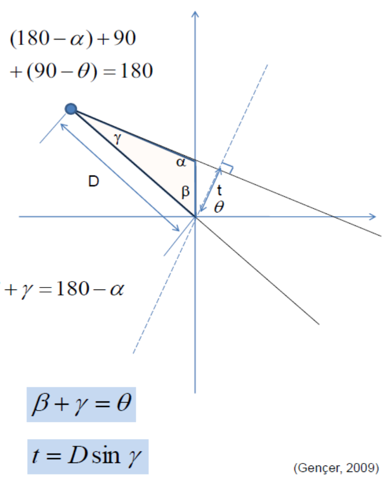

Figure 1: Angle relations

Open two for loops to calculate projection for each beta and gamma pair

Clear dynamic memory

%Intersection points of rays with image should be found to calculate total attenuation for that beam.

Find intersection points using (3) and (4)

$$y = (D * \sin(\gamma)/sin(\beta + \gamma) - (x * cot(\beta + \gamma)) \tag{5}$$

$$x = (D * \sin(\gamma)/cos(\beta + \gamma) - (x * tan(\beta + \gamma)) \tag{6}$$

%Here it is important to note that some irrelevant x and y coordinates may be present since calculation of above equation is done for all space. Also, there may be duplicate x and y pairs since to calculation from (5) and (6) may yield same point twice.

Determine unique and relevant x and y pairs

Sort x and y pairs

If there is not a relevant point equate total attenuation for that beam to zero

%Above means beam does not go through image

If there is only one relevant point equate total attenuation for that beam to zero

%Above means beam is tangential to image

Calculate distance and middle points through (7) and (8)

$$dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{7}$$

$$mid = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \tag{8}$$

%Distance calculation is needed for attenuation calculation. On the other hand, middle point calculations provide address of image pixel.

Correct address information using middle point data

Calculate total attenuation for a beam by summing multiplication of image attenuation data and distance passed through each pixel

Assign calculated attenuation for specific beam and step number in projection matrix

Projection Program Ends

B. Back Projection

Back Projection Program Starts

Load projection matrix, size of image, and distance

%From projection matrix number of projection angle steps and number of sampling points can be found.

Filtering Program Starts

%Back projection is calculated according to (9) where variables are defined in equations (10) and (11).

$$f(r,\phi) = \int_0^{2\pi} \underbrace{\frac{1}{L^2}}_{\text{Distance weight}} Q_\beta(\gamma')d\beta \tag{9}$$

$$Q_\beta(\gamma) = R'_\beta(\gamma) * g(\gamma) \tag{10}$$

$$R'_\beta(\gamma) = R_\beta(\gamma) \underbrace{D\cos\gamma}_{\text{Angular weight}} \tag{11}$$

In mathematical case, back projection corresponds to summing all projection values for each pixel of image. However, the issue with back projection is obvious when impulse response of back projection algorithm is calculated [2]. To correct this issue back projection should be calculated by filtering the projections with (12). For practical purposes, a triangular filter is satisfactory for filtering.

$$g(\gamma) = \frac{1}{2}\left(\frac{\gamma}{\sin\gamma}\right)^2 h(\gamma) \tag{12}$$

Design desired filter in frequency domain

For each theta angle projection multiply filter with projection and then take inverse Fourier Transform

Filtering Program Ends

Create x, and y coordinate arrays

Clear dynamic memory

Find intersection points using (5) and (6)

Determine unique and relevant x and y pairs

Sort x and y pairs

If there is not a relevant point do nothing

%Above means beam does not go through image

If there is only one relevant point do nothing

%Above means beam is tangential to image

Calculate distance and middle points through (5) and (6)

Correct address information using middle point data

Assign projection data for each beam according to distance and address values of each pixel.

Calculate summation of assigned pixel values

Normalize the image matrix

Display the filtered back projected image

Load original image for quantitative measurement

Calculate error by summing absolute differences for each pixel between original and back projected image

Normalize the error to avoid size dependent error calculation

III. Results

In this subsection results will be presented. Raeder should realize that for image figures horizontal axis is x and vertical axis is y coordinate respectively.

A. Projection

Projection algorithm provides the base of this project. Even though it may not seem important as it is, remembering only roentgen images should remind us the importance of a single projection. Simple chest X-ray image might and had saved lots of lives. Having this understanding sets a serious motivation for observation of projection data.

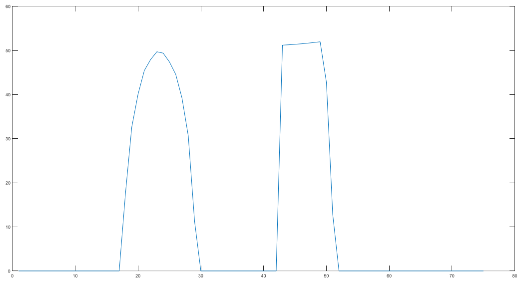

Figure 2: Projection of square_circle using fanbeam function

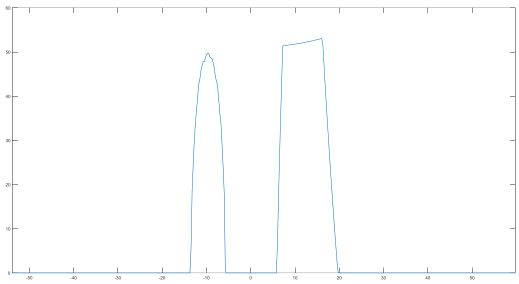

Figure 3: square_circle using developed algorithm

MATLAB provides a built-in function named fanbeam and by comparing Fig. 2 and Fig. 3 reader may be convinced that the developed algorithm is working properly. Furthermore, the effect of changing distance can be interpreted from GUI screens presented in Fig. 4 and Fig. 5.

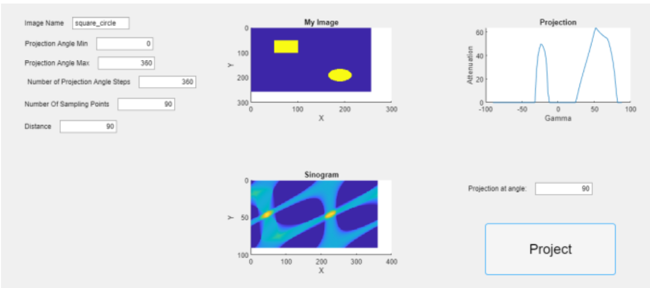

Figure 4: square_circle GUI Screen with distance=90

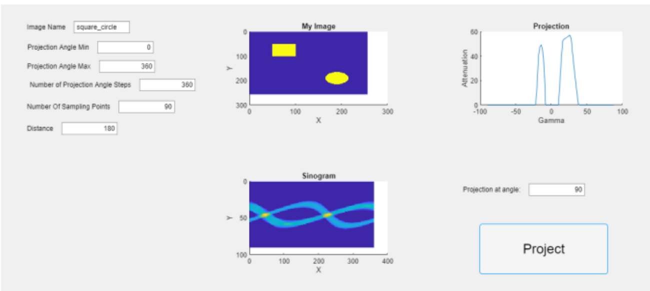

Figure 5: square_circle GUI Screen with distance=180

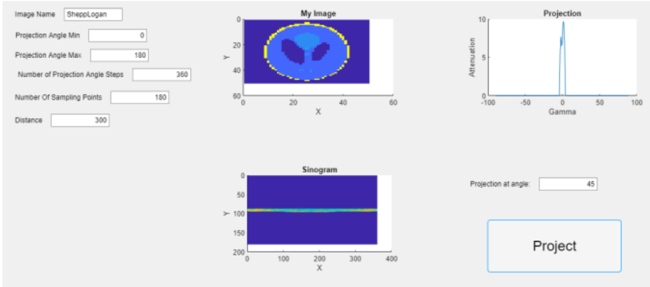

Figure 6: SheppLogan GUI Screen with angle=45

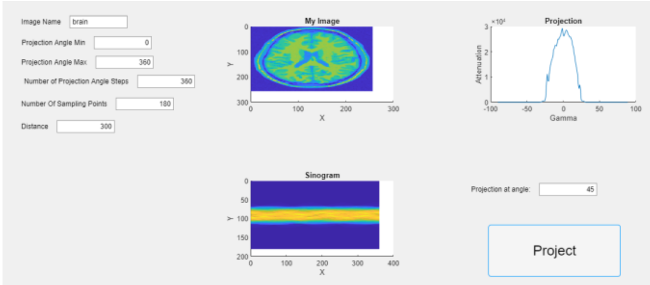

Figure 7: brain GUI Screen with angle=45

The projection taken at 45 degrees angle of SheppLogan and brain are shown in Fig. 6 and 7. Although for untrained eyes projections may seem meaningless, trained physicians may infer information from even from single fan beam projections. Please realize sinograms are also included to provide references.

B. *Filtered BackProjection*

Backprojected images with different number of projection angle steps and number of sampling points are presented in Fig. 8 to 10.

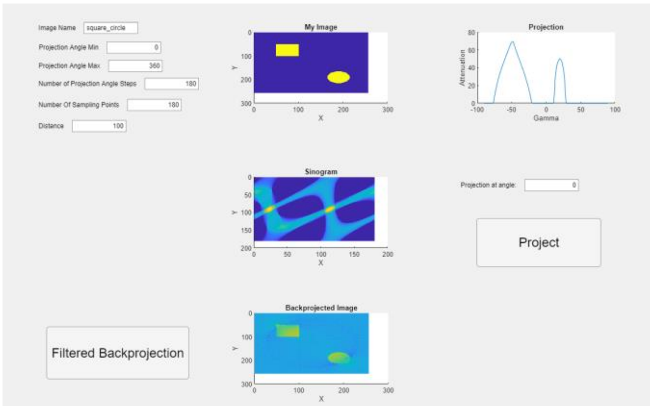

Figure 8: square_circle GUI Screen for Back Projection

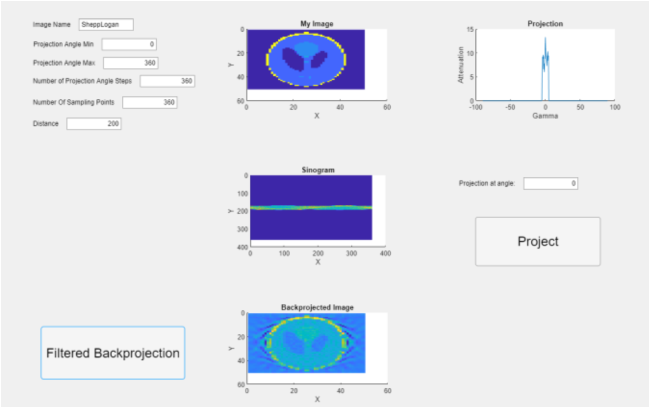

Figure 9: SheppLogan GUI Screen for Back Projection

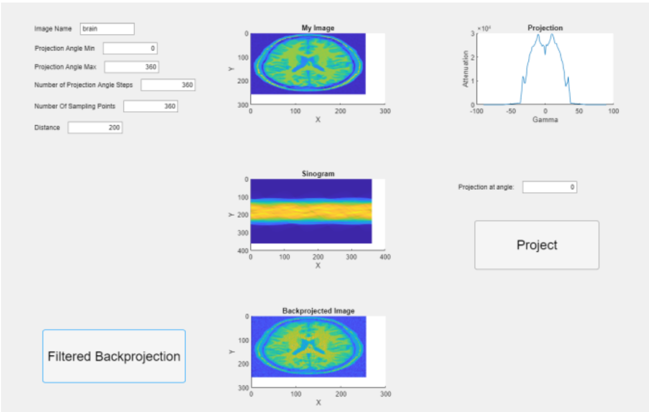

Figure 10: brain GUI Screen for Back Projection

In addition to GUI screens to observe the effect of filtering Fig. 11 and Fig. 12 is displayed. As one can see, in Fig. 11 only a silhouette can be differentiated whereas in Fig. 12 reconstruction is almost clear.

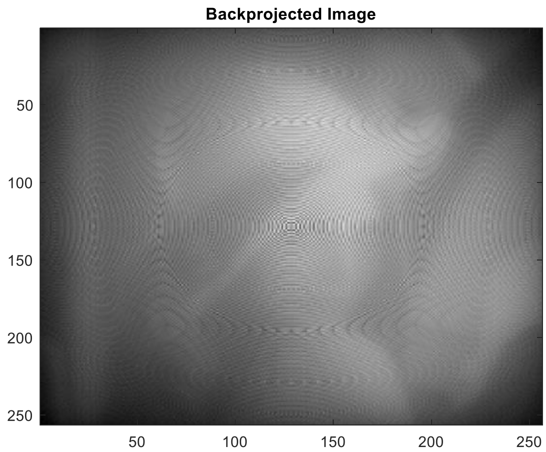

Figure 11: lena.mat only back projected

Figure 12: lena.mat only back projected

IV. Discussion and Conclusion

It is a very well-known fact that number of projection angle steps, number of sampling points, distance, projection angle range, filtering, image size, distance, and original image characteristics have significant importance on back projected image quality. With this project all these criteria and their effects can be examined. Indeed, the aim of the project was to detect and adjust parameters effecting back projected image quality.

First focusing on number of projection angle steps and number of sampling points, it is known that increasing them provides more detailed projection. In the GUI, user can test increasing these numbers and see the effect on the image. During the test user will also realize increased duration of processing times meaning heavier computational load. Moreover, at this point it is extremely important to note increasing beam and step numbers means exposing patient to more ionizing radiation. Therefore, optimization of image quality versus ionizing radiation should be done by qualified experts.

Secondly, the effect of filtering can be understood by comparing results. Despite the longer run time caused, it is definitely worth and necessary to implement an appropriate filtering scheme. As stated earlier, a practical filter was used in this algorithm, but more sophisticated filter would yield more accurate results.

Having mentioned further solutions, algebraic image reconstruction algorithms like ART, SART, and SIRT can also be developed for small images. These methods are developed by first initializing a guess. Then projecting initial guess to fist line of projection. Later, reproject the resulting point in the second line and finally iterating these steps. Specifically, where ART updates image matrix after each projection, SIRT updates it at the end of each iteration and provides better convergence. Because of limited time and computational power, development of these algorithms are hold out of the scope of this project[3].

To sum it all up, in this project a CT algorithm and effects of parameters used during algorithm implementation are discussed. A researcher should utilize such algorithms to observe and improve CT. Simulation results are presented to make a solid understanding of effects of number of projection angle steps, number of sampling points, distance, projection angle range, filtering, image size, distance, and original image characteristics on back projected image quality.

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