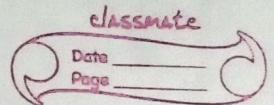


$$\begin{aligned} s &\rightarrow \theta \\ v &\rightarrow \omega \\ u &\rightarrow \omega_0 \\ a &\rightarrow \alpha \end{aligned}$$



Circular Motion,

Linear Motion

1. Displacement: s

2. Velocity

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$$

$$3. V_{avg} = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

4. Acceleration

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$5. a_{avg} = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$6. a = \frac{v - u}{t}$$

$$7. s = u t + \frac{1}{2} a t^2$$

$$8. v^2 = u^2 + 2as$$

$$9. v = u + at$$

$$10. s = \left(\frac{u+v}{2} \right) t$$

$$11. v = u + at$$

$$13. \omega = 2\pi f = \frac{2\pi}{T}$$

Angular/circular motion

i. Angular displacement: θ

2. Angular Velocity

$$\omega_{avg} = \frac{\Delta \theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

$$3. \omega_{avg} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

4. Angular Acceleration

$$\alpha_{avg} = \frac{\Delta \theta}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

$$5. \alpha_{avg} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$

$$6. \alpha = \frac{\omega - \omega_0}{t}$$

$$7. \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$8. \omega^2 = \omega_0^2 + 2\alpha\theta$$

$$9. \omega = \omega_0 + \alpha t$$

$$10. \alpha = (\omega_0 + \omega)t$$

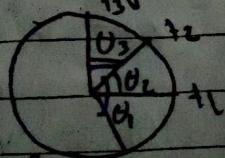
$$11. a = r\alpha$$

$$14. T = \frac{2\pi}{f}$$

Angular displacement:- The angle made by a rotating body in a circular path is called angular displacement. It is a scalar quantity and expressed in radians s.t $\theta = l/r$

Angular Velocity: The time rate of change in angular displacement is angular Velocity.

$$\bar{\omega} = \omega_{avg} = \frac{\Delta \theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$



It is vector quantity / unit: rad/s / dim: [T^-1] * t = 0
Direction: Towards the axis of rotation.

Instantaneous angular velocity :- The initial limiting value of the average angular velocity at any instant of time.

$$\omega_{\text{ins}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular Acceleration (α) : Rate of change in angular Velocity is called angular acceleration.

$$\alpha_{\text{avg}} = \bar{\omega} = \frac{\Delta \theta}{\Delta t}$$

It is vector quantity (i.e rad/s^2), dim: $[T^{-1}]$, towards the axis

Instantaneous angular acceleration :- The limiting value of average angular acceleration is called instantaneous acceleration.

$$\alpha_{\text{ins}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$

$$= \frac{d(\Delta \theta/dt)}{dt}$$

$$\therefore \alpha_{\text{ins}} = \frac{d^2\theta}{dt^2}$$

\Rightarrow Relation between linear velocity and angular Velocity.

let us suppose a body is moving in a circular path with displacement 's' radius 'r' and angular displacement ' θ ' we have

$$\theta = \frac{s}{r} \quad \therefore s = \theta r \quad \text{--- (1)}$$

differentiating with respect to t.

$$\frac{ds}{dt} = \frac{d(\theta r)}{dt}$$

$$\frac{ds}{dt} = r \times \frac{d\theta}{dt} + \theta \times \frac{dr}{dt} \quad (0, \text{ } r \text{ is constant})$$

$$v = r \times \omega \quad \text{--- (2)}$$

\Rightarrow (2) is the relation betⁿ linear & angular acceleration.

In Vector notation $\vec{v} = \vec{\omega} \times \vec{r}$

differentiating eqn ① with respect to time - t,

$$v = \gamma \times \omega$$

$$\Rightarrow \frac{dx}{dt} = \frac{d(\gamma \omega)}{dt}$$

$$s = \gamma \theta$$

$$v = \gamma \omega$$

$$a = \gamma \alpha$$

$$\Rightarrow \frac{dy}{dt} = \gamma \times \frac{d\omega}{dt} + \omega \times \frac{dx}{dt} \quad \gamma \text{ is constant.}$$

$$\Rightarrow \frac{dy}{dt} = \gamma \alpha + 0.$$

$$\therefore \alpha_r = \gamma \alpha \quad \text{--- (ii)}$$

$$\text{In vector notation, } \vec{a}_r = \vec{\omega} \times \vec{\gamma}$$

This is the relation for linear & angular acceleration

Tangential acceleration (a_t) :- The rate of change of tangential velocity.

$$\alpha_t = \frac{dv}{dt} : a_t = \gamma \alpha.$$

$\Rightarrow a_t = 0$, for uniform circular motion.

$a_t \neq 0$, for uniform circular motion, Its unit is m/s^2 .

Centripetal or radial acceleration :- (a_c)

Centripetal: The acceleration produced in a moving body towards its centre due to change in direction of velocity at every point in a circular path is called centripetal acceleration.

$$a_c = a_r = \frac{v^2}{r} = \omega^2 r$$

unit :- m/s^2

dimension:- $[LT^{-2}]$

$a_c \neq 0$ for uniform motion

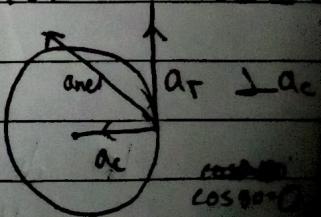
$a_c \neq 0$ for non-uniform motion.

$$a_c = \frac{v^2}{r} \rightarrow \text{Speed}$$

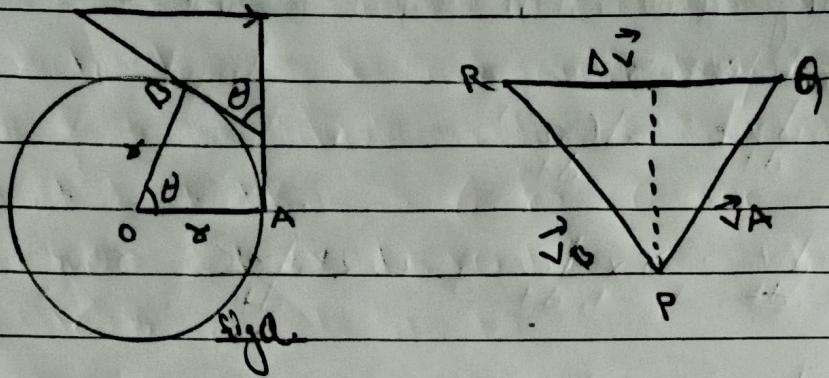
$r \rightarrow \text{radius}$

Note

$a_r \propto a_c$



Ans of Ques 1



when a body moves in a circular path with uniform motion then direction of velocity changes at every point in the circular path due to which a centripetal acceleration is produced

let us suppose a body is moving in a circular path A to B with linear velocity V_A and V_B respectively θ and χ represents angular displacement and radius radius respectively.

In fig ① let $|V_A| = |V_B| = \sqrt{V}$

$$\vec{PR} = \vec{PQ} + \vec{QR}$$

$$\vec{X_B} = \vec{V_A} + \vec{\Delta X}$$

$$\therefore \vec{\Delta X} = \vec{V_B} - \vec{V_A} \quad \dots \text{①}$$

$$\text{now, } |\vec{\Delta X}| = |\vec{V_B} - \vec{V_A}|$$

$$= \sqrt{V_B^2 + V_A^2 - 2V_B V_A \cos \theta}$$

$$= \sqrt{\theta^2 V^2 + V^2 - 2V^2 \cos \theta}$$

$$= \sqrt{2\chi^2 - 2V^2 \cos \theta}$$

$$= \sqrt{2V^2(1 - \cos \theta)}$$

$$= \sqrt{2V^2 \times 2 \sin^2 \frac{\theta}{2}}$$

$$|\vec{\Delta X}| = 2V \sin \frac{\theta}{2}$$

let t is a time taken by a moving body to reach at point B from A

$$\text{Centrifugal acceleration (A_c)} = \frac{\Delta X}{t} = \frac{2V \sin \theta / 2}{t}$$

$$= \frac{2V^2 \sin \theta / 2}{t}$$

As, $\Delta t \rightarrow 0$ & $\theta/2 \rightarrow 0$

$$a_c = \frac{r v^2}{\delta} \times \frac{\sin \theta/2}{\theta}$$

$$= \lim_{\theta/2 \rightarrow 0} \frac{\sin \theta/2}{\theta} \times \frac{r v^2}{\delta}$$

$$= \lim_{\theta/2 \rightarrow 0} \frac{\sin \theta/2}{\theta/2} \times \frac{1}{2} \times \frac{2v^2}{\delta}$$

$$= \lim_{\theta/2 \rightarrow 0} 1 \times \frac{v^2}{\delta}$$

$$\therefore \frac{v^2}{\delta}$$

This is the required expression of centripetal acceleration, it is always directed towards centre along radius (also known as radial acceleration)

The force which is required to keep a body in a circular path is called centripetal force. It is given by.

$$F = m a_c$$

$$F = m \times \frac{v^2}{\delta} \quad \text{also, } F = m \omega^2 r$$

A body moves in a circular path, its angular displacement is given by $\theta = 2t^3 + 5t + 9$

i) average angular velocity for $t = 0$ to 5 sec.

ii) Instantaneous angular velocity for $t = 2$ sec

iii) instantaneous angular acceleration for $t = 2$ sec.

iv) tangential acceleration for $t = 2$ sec

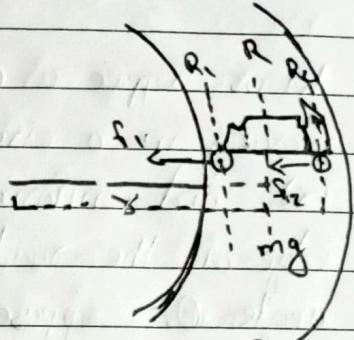
v) Resultant acceleration for $t = 2$ sec. ($r = 2m$)

vi) centripetal acceleration for $t = 2$ sec

Motion of a vehicle in a leveled road.

When a truck moves in a curved path (leveled road) the frictional force between tyre & road provides necessary centripetal force

Let us suppose, a vehicle of mass m , is moving in a curved leveled road with radius R & speed v . Let, f_1 & f_2 be the normal reaction of tyres. Also, f_1 & f_2 are ~~centrifugal~~ frictional force between tyre & road which balances centripetal force



$$\text{we have, } f_1 = \mu R_1 \gamma \quad \text{eqn ①} \quad \text{where } \mu = \text{coeff of friction}$$

$$f_2 = \mu R_2 \gamma$$

The vertically downward forces (weight) is balanced by normal reaction $R = mg$ ~~total~~ eqn ②

The frictional force provides necessary centripetal force i.e.

f_1 + f_2 = \frac{mv^2}{R} \quad \text{③}

From ① & ③

$$\mu R_1 + \mu R_2 = \frac{mv^2}{R}$$

$$\mu(R_1 + R_2) = \frac{mv^2}{R}$$

$$\mu R = \frac{mv^2}{R} \quad \text{④}$$

dividing eqn ③ by eqn ④

$$\mu R = \frac{mv^2}{R}$$

$$R = \frac{mv^2}{\mu g}$$

$$\mu = \frac{mv^2}{mgR} = \frac{v^2}{gR}$$

$$\therefore v = \sqrt{Rg} \quad \text{or}$$

This is the required velocity of a vehicle to take a safe turn. The velocity should be less than that of eqn ④

Banking of road

The phenomenon of raising outer edge of road inner over edge of road through certain angle is called banking

Let us suppose a car mass m is moving in a circular path of radius R with uniform velocity V . Outer edge of the road is banked by angle θ . Suppose R is the normal reaction.

i) $R \cos \theta$ balances the weight of a car.
 $\therefore R \cos \theta = mg$

ii) $R \sin \theta$ provides necessary centripetal force.
 $R \sin \theta = mv^2/R$

$$\frac{R \sin \theta}{R \cos \theta} = \frac{mv^2}{gR}$$

$$\tan \theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$$

$\therefore v = \sqrt{rg \tan \theta}$ This is the required expression of banking of road

Question A) What angle should a car be banked so that car running at 60 km/hr. be saved to go around the circular turn of 1200 m π . (0.4 m)

From banking of road $\theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$

$$= \tan^{-1} \left(\frac{(50/3)^2}{0.4 \times 1200} \right)$$

$$= 22.3 \tan^{-1} (0.5694)$$

$$= 22.3 \times 27.1$$

Conical Pendulum: (Horizontal Motion)

Let us suppose a metal bob by a string of length l . and radius x of circular path moves in horizontal circle. Let θ be the angle made by thread with vertical height (h). Let us suppose

T' = Tension in the string.

$$\textcircled{1} \quad T' \cos \theta = \text{balances weight (i.e. } mg) = mg$$

$$\textcircled{2} \quad T' \sin \theta = \text{balances centripetal force (only if necessary)} = \frac{mv^2}{r}$$

Dividing eqn \textcircled{2} by \textcircled{1}

$$\frac{T' \sin \theta}{T' \cos \theta} = \frac{mg - mv^2/r}{mg} \times \frac{1}{mg}$$

$$\tan \theta = v^2/r g \therefore v^2 = \sqrt{\tan \theta \times r g} \quad \textcircled{3}$$

In $\triangle AOB$

$$\sin \theta = \frac{x}{l} \therefore x = l \sin \theta \quad \textcircled{4}$$

$$\text{Also, } \cos \theta = \frac{OP}{AB} = \frac{l}{r} \therefore h = l \cos \theta \quad \textcircled{5}$$

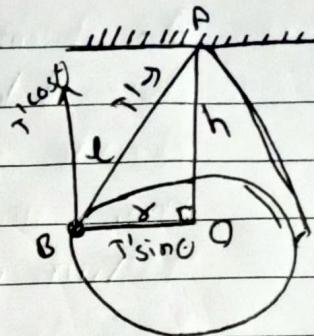
$$T = \frac{2\pi r}{v} = \frac{2\pi x}{rg \tan \theta}$$

$$= \frac{2\pi \cancel{r} \cancel{g} \cdot \cancel{r}}{\cancel{r} \sqrt{g \tan \theta}} = \frac{2\pi \cancel{r}}{\cancel{g} \tan \theta} = 2\pi \sqrt{\frac{1}{g \tan \theta}}$$

$$\therefore T = 2\pi \sqrt{\frac{1}{g \tan \theta}}$$

Also from, eqn \textcircled{3}

$$T = 2\pi \sqrt{\frac{h}{g}}$$



Question: A bob of mass (m) 1kg is attached to a string of 1m long and made to revolve in a horizontal circle (conical pendulum) of radius (0.6 m) find the period of motion and tension of the string

mass of bob (m) = 1 kg.

length of string (l) = 1 m

radius of (r) = 0.6 m

Now,

Time period is given by

$$T = 2\pi \times \sqrt{\frac{l \cos \theta}{g}} \quad \text{--- (1)}$$

$$\text{From, } \theta = l \sin \theta$$

$$\text{or } \theta = \sin^{-1}(0.6)$$

$$\theta = 36.87^\circ$$

$$\text{From (1)} \quad T = 2 \times \frac{22}{7} \times \sqrt{\frac{1 \times \cos(36.87^\circ)}{10}}$$

$$= 1.77$$

$$\text{Now Tension } T' = mg / \cos \theta \\ = \frac{1 \times 10}{0.799}$$

$$= 12.56 \text{ N.}$$

Vertical Motion

Let us suppose a truck of mass (m) is tied with thread which is allowed to move in circle of radius r in anticlockwise direction.

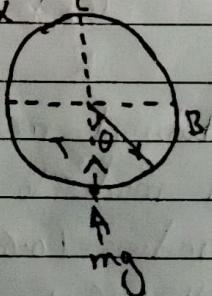
Let θ is the angle between thread &

String point A & (T) is the tension

of the string $T - mg \cos \theta$ provides necessary centripetal force to move

in circular path.

$$T - mg \cos \theta = \frac{mv^2}{r}$$



① At lowest point A $\theta = 0^\circ$

$$T_A - mg \cos(\theta) = \frac{mv^2}{r}$$

$$\therefore T_A = \frac{mv^2}{r} + mg$$

Max

⑩ A point B

$$T_0 - mg \cos \theta = \frac{mv^2}{r}$$

$$\therefore 8 \cdot T_B = \frac{m v^2}{8}$$

At point C $\theta = 180$

$$T_c - mg \cos 180 = \frac{mv^2}{r}$$

$$T_c = \frac{mv^2}{r} - mg$$

13 A point $D = \theta : 770$

$$T_0 - mg \cos \beta = \frac{mv^2}{r}$$

$$T_0 = \frac{m v^2}{\gamma}$$

At point c

$$T + mg = \frac{mv^2}{r}$$

At highest point $T \rightarrow 0$ then, $mg = \frac{mv^2}{r}$

$$\therefore v = \sqrt{8g} \quad \text{--- (Q)}$$

~~A₁ join A₁~~

T-mg-~~mt~~[?]

By using principle of conservation energy at AC,
 Total energy at A = total energy at C.

$$= \frac{1}{2} m x^2 + \frac{1}{2} I \omega^2 = mgh$$

$$= \frac{1}{2} \times m \times v^2 \quad \text{or} \quad m \times g \times h$$

$$= \frac{1}{2} \times m v_a^2 = \frac{1}{2} \times m v_c^2 + m x g \times 2r \quad [r = \text{radius}]$$

$$= Y_a^2 = Y_c^2 + Y_{gr}. \quad \text{--- ②}$$

From 0 & 10

$$Y_a^2 = Y_C^2 + 4\alpha y$$

$$Y_a^2 = g\sigma + 4g\mu$$

$$V_a = \sqrt{50\pi}$$

$$= \sqrt{5} v_c$$

$$V_A = \sqrt{5gY}$$

$$N_a = \sqrt{5} V c^2,$$