

Vector

classmate

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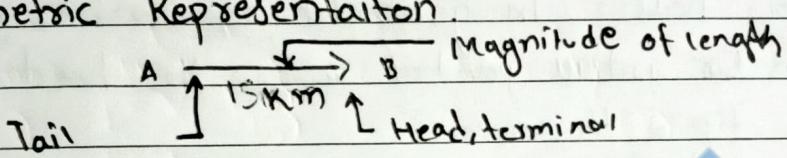
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Representation of Vector :-

\Rightarrow Algebraic Representation direction
- acceleration = \vec{a} ← Magnitude

$$\text{Magnitude of } \vec{a} = |\vec{a}| = a,$$

\Rightarrow Geometric Representation.



Scalar quantity: The physical quantity which have only magnitude but no direction is called Scalar quantity

The Scalar quantity is represented by Simple alphabetical letter. for ex Mass can be represented by m'

Scalar quantity obey simple algebraic rule for ex mass, length, time, temperature, Speed, current, energy.

Area, Volume, density, Pressure, angle, electric flux, magnetic flux volume flux, liquid flux, etc.

Vector quantity : This physical quantity which have both magnitude & direction are called Vector quantity. Necessary Condition of vector is direction. Sufficient Condition of Vector is Vector rule.

Example of Vector quantity are :- Displacement, Velocity, acceleration, momentum, impulse, force, angular displacement, angular, velocity, angular acceleration, angular momentum, angular torque, current density, current element, electric field density, gravitational intensity, magnetic field intensity. Electric flux density, magnetic flux density, magnetic vector potential... alimetry length, alimetry area, alimetry angle etc. (gradient of scalar are always vector)

Representation of Vector

Algebraic representation :- Vectors are represented by simple alphabets with arrow over it. for acceleration = \vec{a}

Geometric representation: Vectors are represented by a straight line with arrow head. In geometric representation tail 'o' is called origin or initial point of vector. the head 'p' is called terminal or tip of the vector.

the length of vectors gives magnitude whereas arrow defines the direction of the vector.

Types of Vectors

① Null / Vectors :-

\Rightarrow Magnitude = 0

$$\Rightarrow |\vec{a}| = a = 0$$

\Rightarrow It has arbitrary direction.

② Unit / vectors

\Rightarrow It has unit magnitude

\Rightarrow Direction Along the direction of vec.

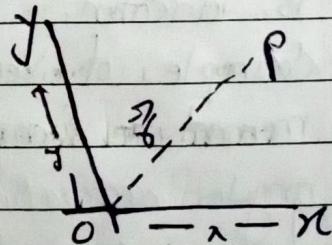
\Rightarrow Unit vectors are unitless & dimensionless

Position vectors:-

\Rightarrow It is used to define position of a particle from certain reference point (i.e. origin)

$$\vec{OP} = \vec{a} = x\hat{i} + y\hat{j}$$

$$OP = r = \sqrt{x^2 + y^2}$$

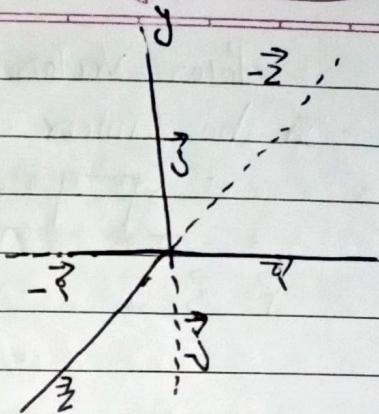


Equal Vectors:-

Magnitude & direction are same

$$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$OP = \sqrt{x^2 + y^2 + z^2}$$



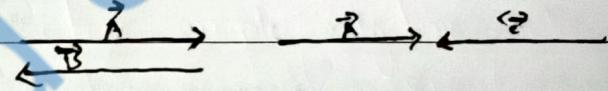
Parallel Vectors :-

- Direction = Same.
- whatever be the magnitude
- All equal vectors are parallel.



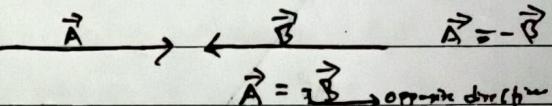
Anti parallel Vector :-

- Direction = Opposite
- whatever be the magnitude.



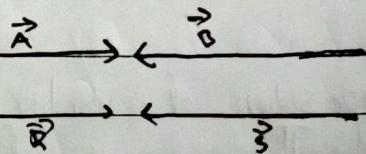
Negative vector :-

- Magnitude same but direction opposite.



Collinear vector :-

- lie along the same line
- All parallel vectors are collinear.
- All anti-parallel vectors are collinear

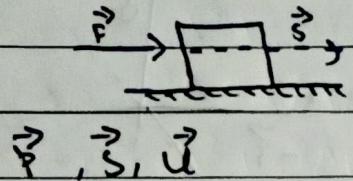


Coplanar vector :-

- lie in the same plane but it must not be parallel or anti-parallel

Polar Vectors

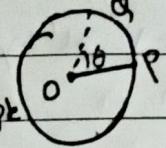
- It has linear effect.



$$\vec{F}, \vec{S}, \vec{R}$$

Axial Vector

- It has rotational effect



e.g. Angular disp.

" velocity

" momentum

" acceleration

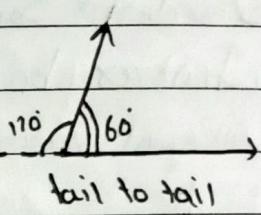
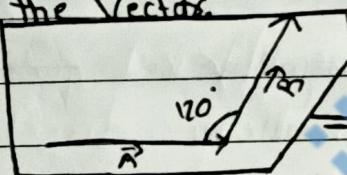
Torque ($\vec{\tau}$) = ... etc.

Comp Angle between the Vectors

↳ tail to tail or

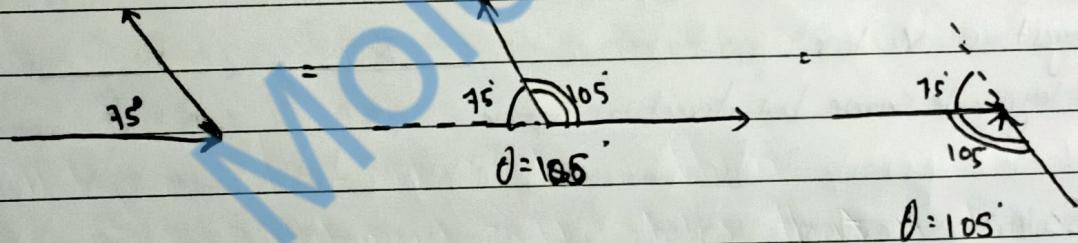
↳ Head to Head

Range 0 to 180°



For example:-

Head to Head.



$\theta = 180^\circ$

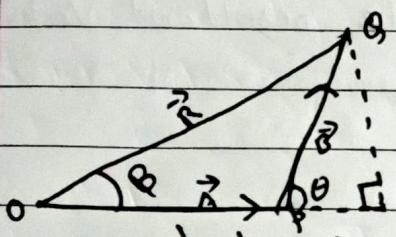
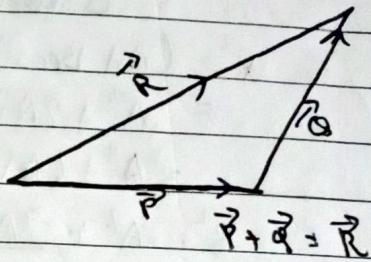
Composition of Vector (Addition of Vectors)

The process of obtaining Single Vector from two or more Vector is called Composition of Vector. The single Vector obtained as a result is called resultant Vector.

- Triangle law of Vector addition:- It states that "if two sides of a triangle represent two vector taken in the same order then third side taken in opposite order represent the resultant vector."

- tail + head or] same order
 ↳ Head + tail (Add)

- Head + Head or] opposite order
 tail + tail



$$\vec{P} + \vec{Q} = -\vec{R}, \vec{P} + \vec{Q} + \vec{R} = 0,$$

Let \vec{P} & \vec{Q} be the two vectors represented by two sides \vec{OP} & \vec{OQ} of the triangle OQP respectively taken in the same order.

According to triangle law of vector addition, the third side \vec{OQ} vector ~~is taken~~ in opposite order represent the resultant vector (\vec{R}).

Let us produce \vec{OP} to M and draw a perpendicular QM as shown in the figure

$\angle QPM = \theta$ is angle between \vec{OQ} & \vec{P} and
 $\angle QOM = \beta$ is angle between the resultant vector (\vec{R}) with \vec{A}

for magnitude of \vec{R}

In $\triangle OQM$

$$OQ^2 = OM^2 + QM^2$$

$$R^2 = (OP + PM)^2 + QM^2 \quad \text{--- (1)} \quad R^2 = (OA + AM)^2 + QM^2 \quad \text{--- (2)}$$

In $\triangle PQM$

$$\cos \theta = \frac{PM}{PQ}$$

$$\sin \theta = \frac{QM}{PQ}$$

$$\begin{aligned} PM &= \cos \theta PQ \\ &= \cos \theta B \end{aligned}$$

$$\begin{aligned} QM &= PQ \sin \theta \\ QM &= \sin \theta B, \end{aligned}$$

Substituting the value of PM & OM in eqn ①

$$\begin{aligned} R^2 &= (A + B \cos \theta)^2 + (B \sin \theta)^2 \\ &= A^2 + 2AB \cos \theta + B^2 \cos^2 \theta + B^2 \sin^2 \theta \\ &= A^2 + 2AB \cos \theta + B^2 (\cos^2 \theta + \sin^2 \theta) \end{aligned}$$

$$R^2 = A^2 + 2AB \cos \theta + B^2$$

$$\therefore R = \sqrt{A^2 + B^2 + 2AB \cos \theta} \quad \text{--- (1)}$$

eqn ① gives the magnitude of the resultant vector. (R)

For dissection of \vec{R}

In $\triangle OQM$

$$\tan \beta = \frac{OM}{PM}$$

$$= \frac{OM}{OP + PM} = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\beta = \tan^{-1} \left(\frac{B \sin \theta}{A + B \cos \theta} \right)$$

The angle β defines the direction of the resultant vector with A vector.

Parallelogram law of Vector addition

Statement :- It states that if two vectors acting simultaneously at the point are represented by two adjacent sides of a parallelogram then diagonal passing through that point represent the resultant vector.

Let \vec{A} & \vec{B} be the two vectors represented by two sides OA & OB of the parallelogram respectively according to parallelogram law of vector addition. the diagonal OG drawn from the join of \vec{A} & \vec{B} represent the resultant vector.

Let us produce OP to M & draw a \perp perpendicular GM as shown in the figure. $\angle NO M = \angle OPM = \theta$ is the angle between \vec{A} & \vec{B} .

$\angle QOM = \beta$ is the angle between (\vec{R}) & (\vec{A})
for magnitude of \vec{R}

In ΔOQM

$$OQ^2 = OM^2 + QM^2$$

$$R^2 = (OP + PM)^2 + QM^2$$

$$R^2 = (A + PM)^2 + QM^2 - - (i)$$

In ΔPGM

$$\cos \theta = \frac{PM}{PQ}$$

$$PM = PQ \cos \theta$$

$$PM = A \cos \theta$$

$$\sin \theta = \frac{QM}{PQ}$$

$$QM = PQ \sin \theta$$

$$QM = B \sin \theta$$

Substituting the value of PM & QM .

$$R^2 = (A + B \cos \theta)^2 + (B \sin \theta)^2$$

$$R^2 = A^2 + 2AB \cos \theta + B^2 \cos^2 \theta + B^2 \sin^2 \theta$$

$$R^2 = A^2 + 2AB \cos \theta + B^2 (\sin^2 \theta + \cos^2 \theta)$$

$$R = \sqrt{A^2 + 2AB \cos \theta + B^2}$$

Special Cases:

i) If two vectors are in same direction i.e. $\theta = 0^\circ$

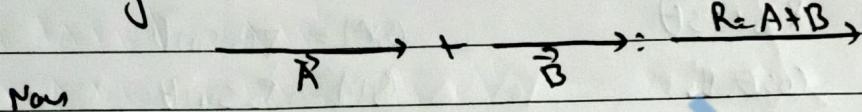
$$R^2 = \sqrt{A^2 + 2AB \cos 0 + B^2}$$

$$R^2 = \sqrt{A^2 + 2AB + B^2}$$

$$R^2 = (A+B)^2$$

$$R = A+B \quad (\text{maximum})$$

The magnitude of resultant vector is maximum in this case



Now

$$\text{For direction, } \tan \beta = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\beta = \tan^{-1}(\theta)$$

$$\beta = 0^\circ$$

The direction of resultant vector is along the direction either vector.

ii) If two vectors are in opposite direction i.e. 180°

$$R^2 = \sqrt{A^2 + 2AB \cos 180^\circ + B^2}$$

$$= \sqrt{A^2 - 2AB + B^2}$$

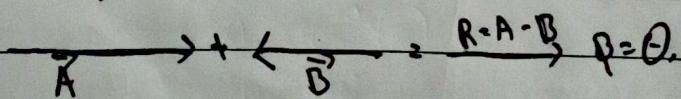
$$= \sqrt{(A-B)^2} \quad \text{or, } \sqrt{(B-A)^2}$$

$$R = A-B \quad \text{if } A > B$$

$$R = B-A \quad \text{if } B > A$$

$$R = |A-B| = \text{minimum}$$

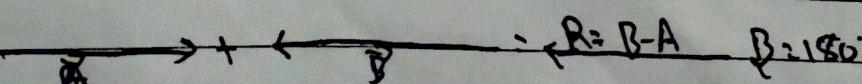
The magnitude of resultant vector is minimum in this case if $A > B$



If $B > A$

$$|\vec{A} + \vec{B}| = |A - B|$$

$$\theta = 180^\circ$$



The direction of the resultant vector is along the vector having high mag.

If two vectors are perpendicular to each other : $\theta = 90^\circ$

$$R = \sqrt{A^2 + 2AB \cos 90^\circ + B^2}$$

$$R = \sqrt{A^2 + B^2}$$

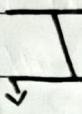
$$\text{now, } \tan \beta = \frac{B \sin 90^\circ}{A + B \cos 90^\circ}$$

$$= \frac{AB}{A+0}$$

$$\beta = \tan^{-1} (B/A)$$

Range of R

$$|A-B| = \text{minimum}$$



$$|A+B| = \text{Maximum}$$



iii) find the resultant vector of two vectors of same magnitude

Here, let two vectors be, \vec{a} & \vec{b}

According to que $|a| = |b|$ then $a = b = x$ (let), now

According to triangle law of vector addition,

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$= \sqrt{x^2 + x^2 \cos^2 \theta + x^2}$$

$$= \sqrt{2x^2 + 2x^2 \cos^2 \theta}$$

$$= \sqrt{2x^2 \cdot (1 + \cos \theta)}$$

$$= \sqrt{2x^2 \cdot 2 \cos^2 \frac{\theta}{2}}$$

$$= \sqrt{(2x \cos \frac{\theta}{2})^2}$$

$$= 2x \cos \frac{\theta}{2}$$

Let, β be the angle between \vec{R} and \vec{a}

$$\tan \beta = \frac{B \sin \theta}{A + B \cos \theta} = \frac{x \sin \theta}{x + x \cos \theta} = \frac{x \sin \theta}{x(1 + \cos \theta)} = \frac{x \sin \theta/2 \cdot \cos \theta/2}{2 \cos^2 \theta/2}$$

$$\tan \beta = \frac{\sin \theta/2}{\cos \theta/2}$$

$$\tan \beta = \tan \theta/2$$

$$\therefore \beta = \theta/2$$

Show that the resultant vector bisect the two vectors having same magnitude.

i) If $\vec{A} + \vec{B} = \vec{C}$ and $A + B = C$ then find the angle between \vec{A} & \vec{B}

Soln: ii) If $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$ then find the angle between \vec{A} & \vec{B}

iii) If $\vec{A} + \vec{B} = \vec{C}$ and $A - B = C$ then find the angle between \vec{A} & \vec{B}

iv) If $\vec{A} + \vec{B} = \vec{C}$ and $B - A = C$

v) If $|\vec{A} + \vec{B}| = A + B$

vi) If $|\vec{A} + \vec{B}| = A - B$

vii) If $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2}$

viii) If $\vec{A} + \vec{B} = \vec{C}$, Range of C ?

for i) let θ be the angle between the \vec{A} & \vec{B}

Given that $\vec{A} + \vec{B} = \vec{C}$

$$c = \sqrt{A^2 + 2AB \cos \theta + B^2}$$

$$(A+B)^2 = A^2 + 2AB \cos \theta + B^2$$

$$A^2 + 2AB + B^2 = A^2 + 2AB \cos \theta + B^2$$

$$2AB = 2AB \cos \theta$$

$$\cos \theta = 1$$

$$\theta = 0^\circ$$

for vi) let θ be the angle between the \vec{A} & \vec{B}

Given that $A^2 + B^2 = C^2$

$$c = \sqrt{A^2 + 2AB \cos \theta + B^2}$$

$$c^2 = A^2 + 2AB \cos \theta + B^2$$

$$A^2 + B^2 = A^2 + 2AB \cos \theta + B^2$$

$$2AB \cos \theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$\theta = 90^\circ$$

Ex III if $\vec{A} + \vec{B} = \vec{C}$ and $A \cdot B = C$ then find the angle

let θ be the angle between the $\vec{A} + \vec{B}$

Given that $A \cdot B = C$ now

According vector law of addition

$$C = \sqrt{A^2 + 2AB \cos \theta + B^2}$$

$$\text{Squaring both side } (A \cdot B)^2 = A^2 + 2AB + B^2$$

$$\text{or } A^2 - 2AB + B^2 = A^2 + 2AB + B^2$$

$$-2AB = 2AB \cos \theta$$

$$\cos \theta = -1$$

$$\theta = \cos^{-1}(-1)$$

$$\theta = 180^\circ$$

Now if $\vec{A} + \vec{B} = \vec{C}$

let θ be the angle between the $\vec{A} \& \vec{B}$

Given that $B \cdot A = C$,

According to Vector law of addition

$$C = \sqrt{A^2 + 2AB \cos \theta + B^2}$$

$$\text{Squaring both } (B \cdot A)^2 = A^2 + 2AB \cos \theta + B^2$$

$$B^2 - 2AB + A^2 = A^2 + 2AB \cos \theta + B^2$$

$$-2AB = 2AB \cos \theta$$

$$\cos \theta = -1$$

$$\therefore \theta = \cos^{-1}(-1)$$

$$\therefore \theta = 180^\circ$$

for v

let θ be the angle between them $\vec{A} + \vec{B}$

According to vector law of Addition,

$$\vec{A} + \vec{B} = \sqrt{A^2 + 2AB \cos\theta + B^2}$$

$$\text{Square both } (\vec{A} + \vec{B})^2 = A^2 + 2AB \cos\theta + B^2$$

$$\text{or } A^2 + 2AB + B^2 = A^2 + 2AB \cos\theta + B^2$$

$$\text{or } 2AB = 2AB \cos\theta$$

$$\text{or } \cos\theta = 1$$

$$\therefore \theta = 0^\circ$$

for vi

let θ be the angle between them $\vec{A} + \vec{B}$

Given that $|\vec{A} + \vec{B}| = A - B$,

According to vector law of addition,

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + 2AB \cos\theta + B^2}$$

$$\text{or } (\vec{A} - \vec{B})^2 = A^2 + 2AB \cos\theta + B^2$$

$$\text{or } A^2 + 2AB + B^2 = A^2 + 2AB \cos\theta + B^2$$

$$\text{or } -2AB = 2AB \cos\theta$$

$$\text{or } \cos\theta = \cos^{-1}(-1)$$

$$\therefore \theta = 180^\circ$$

for vii

let θ be the angle between the \vec{A} & \vec{B}

Given that, $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2}$

According to vector law of addition,

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + 2AB \cos\theta + B^2}$$

$$(\sqrt{A^2 + B^2})^2 = \sqrt{A^2 + 2AB \cos\theta + B^2}$$

$$A^2 + B^2 = A^2 + 2AB \cos\theta + B^2$$

$$\cos\theta = 0$$

$$\theta = 90^\circ$$

viii If $\vec{A} + \vec{B} = \vec{C}$, Range of C

the Range of resultant C should be,

$$\text{Min} = |\vec{A} - \vec{B}|$$

$$\text{Max} = |\vec{A} + \vec{B}|$$

$$\text{Range} : -|\vec{A} - \vec{B}| \leq C \leq |\vec{A} + \vec{B}|$$

$$\text{Range} : |\vec{A} - \vec{B}| \leq |\vec{C}| \leq \vec{A} + \vec{B}$$

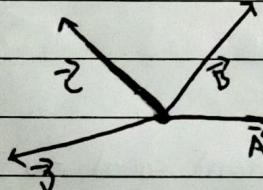
$$|\vec{A} - \vec{B}| \leq R \leq \vec{A} + \vec{B}$$

$$|\vec{A} - \vec{B}| \leq |\vec{A} + \vec{B}| \leq \vec{A} + \vec{B}$$

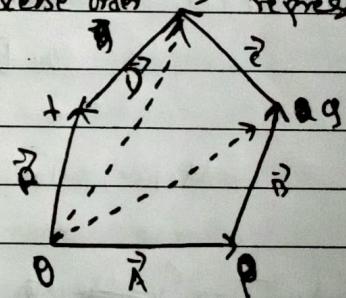
Polygon law of vector addition. (Extension of triangle law)

It state that if a number of vectors be represented both in magnitude and direction by the sides of the polygon taken in the opposite sum order then the resultant is represented completely in magnitude and direction by closing side of polygon taken in opposite order.

Let, In figure $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ are represented by side OP, PQ, QS, ST respectively of the polygon. According to polygon law of vector addition the closing side OT taken in the reverse order represent the resultant vector \vec{R} [$\because \vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{R}$]



It can be drawn as



Construction: O to Q, O to S,

In $\triangle OQS$,

In $\triangle OQS$

By triangle law of vector addition

By triangle law of vector addition,

$$\vec{OQ} + \vec{QS} = \vec{OS}$$

$$\vec{OQ} + \vec{QS} = \vec{OS}$$

$$\therefore \vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{OS} \quad \text{from } \textcircled{1}$$

$$\therefore \vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{OS}$$

In $\triangle OST$

By triangle law of vector addition

$$\vec{OS} + \vec{ST} = \vec{OT}$$

$$\therefore \vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{OT}$$

Resolution of Vector :-

A splitting of a vector into its constituent component is called resolution of vector.

- Rectangular resolution of Vector :-

The splitting of a vector into two mutually perpendicular component in two dimension is called rectangular resolution of vector and the components are called rectangular component.

In figure, \vec{R} is represented by OP vector. R_x & R_y are the component of \vec{R} along x-axis and y-axis respectively.

In $OPMA$

$$\text{or } \cos\theta = \frac{OM}{OP}$$

$$\text{or } \sin\theta = \frac{MP}{OM}$$

$$\text{or } R \cos\theta = OM$$

$$\text{or } R \sin\theta = MP \quad (\because MP = NO)$$

$$\therefore R_x = R \cos\theta \dots (i)$$

$$\therefore R_y = R \sin\theta \dots (ii)$$

Squaring and adding eqn (i) and (ii)

$$R_x^2 + R_y^2 = R \sin^2\theta + R^2 \cos^2\theta \\ = R^2 (\sin^2\theta + \cos^2\theta)$$

$$\text{or } R^2 = R_x^2 + R_y^2$$

$$\therefore R = \sqrt{R_x^2 + R_y^2} \dots (iv)$$

Dividing eqn (ii) by eqn (i)

$$\frac{R_y}{R_x} = \frac{R \sin\theta}{R \cos\theta}$$

$$\therefore \frac{R_y}{R_x} = \tan\theta.$$

In Vector form

$$\vec{OP} = \vec{OM} + \vec{MP}$$

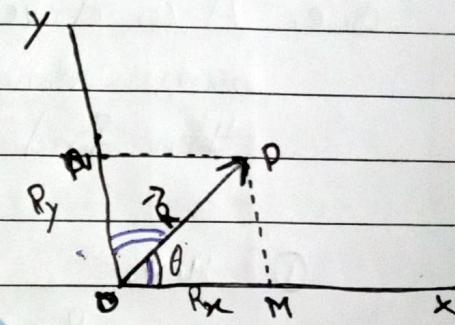
$$\vec{R} = \vec{R}_x + \vec{R}_y$$

$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

$$\vec{R} = R \cos\theta \hat{i} + R \sin\theta \hat{j}$$

If R_x & R_y & R_z be the component of \vec{R} along x-axis, y-axis and z-axis respectively then $\vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z$ Also $= \vec{R} = R_x \hat{i} + R_y \hat{j} + R_z \hat{k}$

$$\therefore R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$



Multiplication of vector:

$$2\vec{A}$$

① By number.

\vec{A} = vector, the $n\vec{A}$ = Vector.

n = number,

e.g. $\vec{A} = 4 \text{ km east}$

$n=2$

$n\vec{A} = 2 \times 4 \text{ km east}$

= 8 km east .

i) $n=-2$

$n\vec{A} = -2 \times 4 \text{ km east}$

= -8 km east

= 8 km west .

② By a scalar.

\vec{A} = Vector

e.g. \vec{a} = acceleration (vector)

λ = Scalar

m = mass (scalar)

$\lambda\vec{A}$ = Vector,

$m\vec{a} = \vec{F}$ = Force (vector)

e.g. \vec{m} = mass (scalar)

\vec{v} = Velocity (vector)

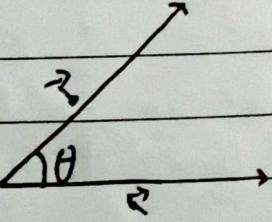
$m\vec{v}$ = momentum (P) Vector....

③ By a vector.

i) scalar product (Dot product)

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos\theta$$

$$= ab \cos\theta \text{ (scalar)}$$



e.g:

$$\text{i) work (W)} = \vec{F} \cdot \vec{s} = Fs \cos\theta \text{ (scalar)}$$

$$\text{ii) Power (P)} = \vec{F} \cdot \vec{v} = Fv \cos\theta \text{ (scalar)}$$

→ velocity.

Special cases

$$\text{i) } \theta = 0^\circ, \vec{a} \cdot \vec{b} = ab \cos 0^\circ = ab$$

$$\text{ii) } \theta = 90^\circ, \vec{a} \cdot \vec{b} = ab \cos 90^\circ = 0$$

$$\text{iii) } \theta = 180^\circ, \vec{a} \cdot \vec{b} = ab \cos 180^\circ = -ab$$

Dot product of unit vector.

$$|\hat{i}| = |\hat{j}| = |\hat{k}| \approx 1$$

$$\hat{i} \cdot \hat{i} = |\hat{i}| |\hat{i}| \cos 0^\circ = 1 \times 1 = 1$$

$$\hat{j} \cdot \hat{j} = |\hat{j}| |\hat{j}| \cos 0^\circ = 1 \times 1 = 1$$

$$\hat{k} \cdot \hat{k} = |\hat{k}| |\hat{k}| \cos 0^\circ = 1$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1,$$

properties

$$\textcircled{1} \vec{A} \cdot \vec{A} = |\vec{A}| |\vec{A}| \cos 0^\circ$$

$$= A \times A \times 1$$

$$= A^2$$

$$\textcircled{2} \vec{A} \cdot \vec{B} = AB \cos \theta$$

$$= BA \cos \theta$$

$$= |\vec{A}| |\vec{B}|$$

Now,

$$\hat{i} \cdot \hat{j} = |\hat{i}| |\hat{j}| \cos 90^\circ = 0$$

$$\hat{j} \cdot \hat{k} = |\hat{j}| |\hat{k}| \cos 90^\circ = 0$$

$$\hat{k} \cdot \hat{i} = |\hat{k}| |\hat{i}| \cos 90^\circ = 0$$

$$\textcircled{3} \vec{A} \cdot (\vec{B} + \vec{C}) \Rightarrow \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 0,$$

~~Properties~~

$$\textcircled{4} \text{ If } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \quad \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\begin{aligned} \vec{A} \cdot \vec{B} &= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ &= A_x B_x \hat{i} \cdot \hat{i} + A_y B_y \hat{j} \cdot \hat{j} + A_z B_z \hat{k} \cdot \hat{k} \\ &= A_x B_x + A_y B_y + A_z B_z. \end{aligned}$$

Vector product. (cross product)

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$$

$$= ab \sin \theta \hat{n} \text{ (vector)}$$

\hat{n} magnitude unit vector

\hat{n} represents the direction of $\vec{a} \times \vec{b}$ which is perpendicular to the plane formed by \vec{a} & \vec{b} .

Ex: e.g.

$$\textcircled{1} \text{ Velocity } (\vec{v}) = \vec{\omega} \times \vec{R} = \text{Vector}$$

$$\text{Torque } (\vec{T}) = \vec{\theta} \times \vec{F} = \text{Vector.}$$

For direction,

$$\vec{R} \times \vec{F} = \text{outward}, \quad \vec{\theta} \times \vec{F} = \text{Inward.}$$

$$(\vec{a} \times \vec{b}) = \perp \vec{a}, \vec{b}, \vec{a} \pm \vec{b}$$

$$(\vec{b} \times \vec{a}) = \perp \vec{a}, \vec{b} (a \pm b)$$

$$(b \pm a)$$

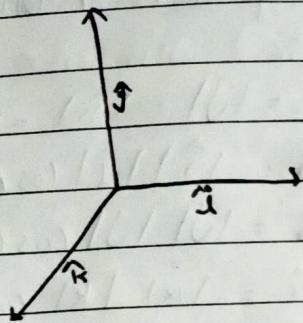
Cross product vector of unit vector.

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

$$\hat{i} \times \hat{i} = \hat{i} \cdot \hat{i} \sin 0^\circ = \vec{0}$$

$$\hat{j} \times \hat{j} = \hat{j} \cdot \hat{j} \sin 0^\circ = \vec{0}$$

$$\hat{k} \times \hat{k} = \hat{k} \cdot \hat{k} \sin 0^\circ = \vec{0}$$

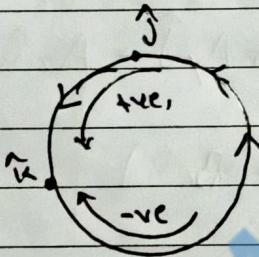


$$\hat{i} \times \hat{j} = |\hat{i}| |\hat{j}| \sin 90^\circ \hat{k} = \hat{k}$$

$$\hat{j} \times \hat{k} = |\hat{j}| |\hat{k}| \sin 90^\circ \hat{i} = \hat{i}$$

$$\text{So, } \vec{B} \times \vec{C} = -(\vec{C} \times \vec{B})$$

$$\hat{j} \times \hat{i} = -(\hat{i} \times \hat{j})$$



$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = +\hat{j}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

$$(ii) \vec{A} \times \vec{A} = |\vec{A}| |\vec{A}| \sin 0^\circ$$

$$= \vec{0}$$

$$|\vec{A} \times \vec{A}| = 0$$

$$(iii) \text{ If } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Geometrical meaninging of cross-product.

$$\sin \theta = \frac{CD}{OC}$$

$$CD = OC \sin \theta$$

$$\therefore CD = OB \sin \theta$$

$$|\vec{OA} \times \vec{OB}| = ab \sin \theta.$$

$$= (OA) \times (CD)$$

= Base x height

= Area of parallelogram.

