$s \rightarrow \theta$
$v \rightarrow \omega$
$u \rightarrow \omega_0$
$a \rightarrow \alpha$

Circular Motion

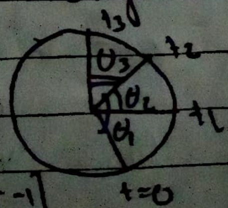
Linear Motion	Angular/circular motion
1. displacement : s	1. Angular displacement : θ
2. Velocity $v_{avg} = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$	2. Angular Velocity $\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$
3. $v_{ins} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$	3. $\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
4. Accleration $a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$	4. Angular Accleration $\alpha_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$
5. $a_{ins} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$	5. $\alpha_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$
6. $a = \frac{v-u}{t}$	6. $\alpha = \frac{\omega - \omega_0}{t}$
7. $s = ut + \frac{1}{2}at^2$ →	7. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
8. $v^2 = u^2 + 2as$ →	$\omega^2 = \omega_0^2 + 2\alpha\theta$
9. $v = u + at$ →	$\omega = \omega_0 + \alpha t$
10. $s = \left(\frac{u+v}{2}\right) \times t$ →	$\alpha = \left(\frac{\omega_0 + \omega}{2}\right) \times t$
11. $v = r\omega$	12. $a = r\alpha$
13. $\omega = 2\pi f = \frac{2\pi}{T}$	14. $T = 1/f$

\# Angular displacement :- The angle made by a rotating body in a circular path is called angular displacement. It is a scalar quantity and expressed in radian S.I.

$$\theta = l / r$$

\# Angular velocity : The time rate of change in angular displacement is angular Velocity.

$$\bar{\omega} = \omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$



It is vector quantity / unit : rad/s / Dim :- $[T^{-1}]$

Direction : Towards the axis of rotation.

Instantenous angular Velocity :- The ~~initial~~ limiting value of the average of angular velocity at any instant of time.

$$\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular Accleration (α) : Rate of change in angular velocity is called angular accleration.

$$\alpha_{avg} = \bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

It is vector quantity (i.e rad/s^2), dim:. $[T^{-1}]$, towards the an

Instantenous angular acceleration :- The limiting value of average of angular accleration is called instantenous accleration.

$$\alpha_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

$$= \frac{d(d\theta/dt)}{dt}$$

$$\therefore \alpha_{ins} = \frac{d^2\theta}{dt^2}$$

$\Rightarrow$ Relation between linear velocity and angular velocity.

let us suppose a body is moving in a circular path with displacement 's' radius 'r' and angular displacement 'θ' we have

$$\theta = \frac{s}{r} \qquad \therefore s = \theta r \quad \text{--(i)}$$

differenciating with respect to t.

$$\frac{ds}{dt} = \frac{d(\theta r)}{dt}$$

$$\frac{ds}{dt} = r \times \frac{d\theta}{dt} + \cancel{\theta \times \frac{dr}{dt}}$$ (0, bcz r is constant)

$$v = r \times \omega \quad \text{---(ii)}$$

$\Rightarrow$ (2) is the relation bet^n linear & angular accleration.

In vector notation $\vec{v} = \vec{\omega} \times \vec{r}$

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differenciating eqⁿ (ii) with respect to time t,

$s = r\theta$
$v = r\omega$
$a = r\alpha$

$$v = r \times \omega$$

$$\Rightarrow \frac{dv}{dt} = \frac{d(r\omega)}{dt}$$

$$\Rightarrow \frac{dv}{dt} = r \times \frac{d\omega}{dt} + \omega \times \cancel{\frac{dr}{dt}}$$ r is constant.

$$\Rightarrow \frac{dv}{dt} = r\alpha + 0.$$

$$\therefore a_T = r\alpha \quad \text{--- (iii)}$$

In vector notation. $\vec{a_t} = \vec{\alpha} \times \vec{r}$

This is the relation for linear & angular acceleration

Tangential acceleration (a_T) :- The rate of change of tangential velocity

$$a_T = \frac{dv}{dt} \quad ; \quad a_T = r\alpha.$$

$\Rightarrow$ $a_T = 0$, for uniform circular motion.

$a_T \neq 0$, for uniform circular motion, It's unit is m/s^2.

Centripetal or radial accleration :- (a_c)

centripe: The accleration produced in a moving body towards its centre due to change in direction of velocity at every point in a circular path is called centrapetal accleration.

$$a_r = a_c = \frac{v^2}{r} = \omega^2 r$$

unit :- m/s^2

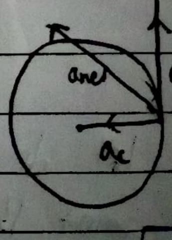
dimension :- $[LT^{-2}]$

$a_c \neq 0$ for uniform motion

$a_c \neq 0$ for non-uniform motion.

$$a_c = \frac{v^2}{r} \quad \begin{matrix} \rightarrow \text{speed} \\ \rightarrow \text{radius} \end{matrix}$$

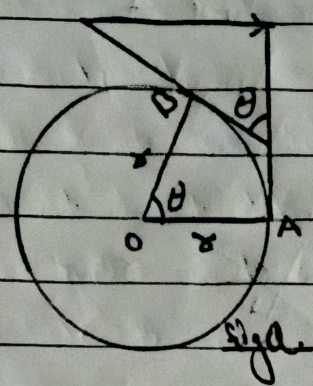
Note $a_c \perp a_T$

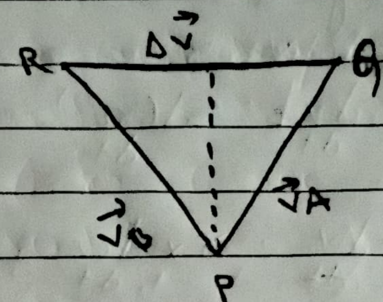


$a_T \perp a_c$

$\cos 90° = 0$

$a_{net} = \sqrt{a_c^2 + a_T^2}$





when a body moves in a circular path with uniform motion then direction of velocity changes at every point in the circular path due to which a centripetal accleration is produced

let us suppose a body is moving in a circlular path A to B with linear Velocity v_A and v_B respectively θ and r represents angular displacement and radius radius respectively.

In fig (b) let $|v_A| = |v_B| = v$

$$\vec{PR} = \vec{PQ} + \vec{QR}$$

$$\vec{v_B} = \vec{v_A} + \vec{\Delta v}$$

$$\therefore \Delta v = \vec{v_B} - \vec{v_A} \quad \text{---} (1)$$

now, $|\Delta\vec{v}| = |\vec{v_B} - \vec{v_A}|$

$$= \sqrt{v_B^2 + \vec{v_A} - 2v_B v_A \cos\theta}$$

$$= \sqrt{v^2 + v^2 - 2v^2\cos\theta}$$

$$= \sqrt{2v^2 - 2v^2\cos\theta}$$

$$= \sqrt{2v^2(1-\cos\theta)}$$

$$= \sqrt{2v^2 \times 2\sin^2\frac{\theta}{2}}$$

$$|\Delta\vec{v}| = 2v\sin\frac{\theta}{2}$$

let dt is a time taken by a moving body to reach at point B from A

Centripetal acceleration $(a_c) = \frac{\Delta v}{\Delta t} = \frac{2v\sin\theta/2}{\frac{r\theta}{v}}$

$$= \frac{2v^2\sin\theta/2}{r\theta}$$

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As, $\Delta t \to 0$ & $\theta/2 \to 0$

$$a_c = \frac{2v^2}{r} \times \frac{\sin\theta/2}{\theta}$$

$$= \lim_{\theta/2 \to 0} \frac{\sin\theta/2}{\theta} \times \frac{2v^2}{r}$$

$$= \lim_{\theta/2 \to 0} \frac{\sin\frac{\theta}{2}}{\theta/2} \times \frac{1}{2} \times \frac{2v^2}{r}$$

$$= \lim_{\theta/2 \to 0} 1 \times \frac{v^2}{r}$$

$$\therefore \frac{v^2}{r}$$

This is the required expression of centreptal accleration, it is always directed towards centre along radius (also known as radical ac

The force which is required to keep a body in a circular path is called centripetal force. It is given by.

$$F = ma_c$$

$$F = \frac{m \times v^2}{r} \quad \text{also, } F = m\omega^2 r$$

A body moves in a circular path, its angular displacement is given by. $\theta = 2t^3 + 5t + 8$

i) average angular velocity for $t = 0$ to 5 sec.

ii) Instantaneous angular velocity for $t = 2$ sec

iii) instantaneous angular accleration for $t = 2$ sec.

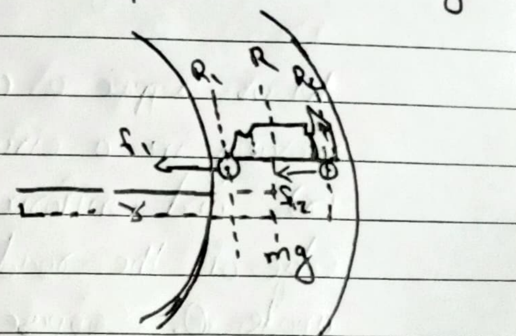
iv) tangential accleration for $t = 2$ sec

vi) Resultant accleration for $t = 2$ sec. $(r = 2m)$

v) centripetal accleration for $t = 2$ sec

Motion of a vechile in a leveled road.

when a truck moves in a curved path (laveled road) the frictional force between tyre & road provides necessary centripetal force



let us suppose, a vechile of mass m, is moving in a curved laveled road with radius (r) & speed (v). let, r_1 & r_2 be the normal reaction of tyres. also, f_1 & f_2 are ~~centripetal~~ frictional force between tyre & road which balances centripetal force

we have, $\left.\begin{matrix} f_1 = \mu R_1 \\ f_2 = \mu R_2 \end{matrix}\right\}$ --- eq^n (i) where μ = coeff of friction))

The vertically downward forces (weight) is balanced by normal reaction $R = mg$ ----- eq^n (ii)

The frictional force provides necessary centripetal force i.e =

$$f_1 + f_2 = \frac{mv^2}{r} \text{ --- (iii)}$$

from (i) & (iii)

$$\mu R_1 + \mu R_2 = \frac{mv^2}{r}$$

$$\mu (R_1 + R_2) = mv^2/r$$

$$\mu R = \frac{mv^2}{r} \text{ --- (iv)}$$

dividing eq^n (iv) by eq^n (ii)

$$\frac{\mu R}{R} = \frac{\frac{mv^2}{r}}{mg}$$

$$\mu = \frac{mv^2}{mgr} \quad = \quad gr\mu = v^2$$

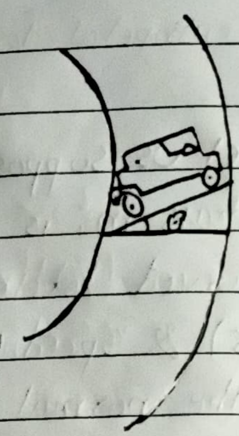
$$\therefore v = \sqrt{\mu r g} \text{ --- (v)}$$

This is the required velocity of a vechile to take a safe turn. The velocity should be less than that of eq^n (v)

Banking of road

The phenomenon of raising outer edge of road over ~~its~~ inner edge of road through certain angle is called banking

let us suppose a car mass having (m) is moving in a circular path of radius (r) and uniform velocity (v). outer edge of the road is banked by angle θ. Suppose R is the normal reaction



(i) R $\cos\theta$ balances ~~the~~ weight of a car.

$$\therefore R\cos\theta = mg$$

(ii) R $\sin\theta$ provides necessary centripetal force.

$$R\sin\theta = \frac{mv^2}{r} \text{ — (ii)}$$

Dividing eqⁿ (ii) by (i)

$$\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{r} \times \frac{1}{mg}$$

$$\tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$\therefore v = \sqrt{rg\tan\theta}$$ This is the required expression of banking of road

Question At what angle should a car be banked, ~~[illegible]~~ so that car running at 60 km/hr be saved to go around the circular turn of 1200 m r. (9.4 m)

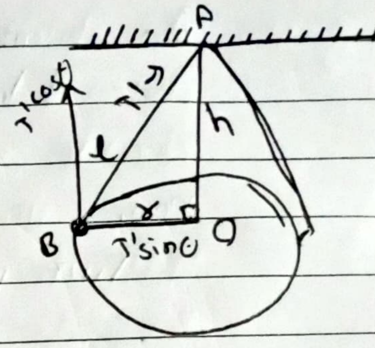
From banking of road $\theta = \tan^{-1}\left(\cancel{\frac{(60)^2}{9.8 \times 0.4}}\right)$

$$= \tan^{-1}\left(\frac{(50/3)^2}{9.8 \times 1206}\right)$$

$$= \tan^{-1}(0.5694)$$

$$= 7.907$$

Conical Pendulum: (Horizontal Motion)



let us suppose a metal bob by a string of length l and radius r of circ. path movies in horizontal circle. let θ be the angle made by thread with vertical height (h). let us suppose

T' : Tension in the string.

(I) $T'\cos\theta$ = balances weight (i.e mg) $= mg$

(II) $T'\sin\theta$ = balances centripetal force (only if necessary) $= \frac{mv^2}{r}$

Dividing eq^n (II) by (I)

$$\frac{T'\sin\theta}{T'\cos\theta} = \frac{mv^2}{r} \times \frac{1}{mg}$$

$$\tan\theta = v^2/rg \quad \therefore v^2 = \sqrt{\tan\theta \times rg} \quad \text{---- (III)}$$

In Δ AOB

$$\sin\theta = \frac{r}{l} \quad \therefore r = l\sin\theta \quad \text{-- (IV)}$$

Also, $\cos\theta = \frac{OP}{AB} = \frac{h}{l} \quad \therefore h = l\cos\theta$ --- (V)

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{rg\tan\theta}}$$

$$= \frac{2\pi \sqrt{r}.\sqrt{r}}{\cancel{\sqrt{r}}\sqrt{g}\tan\theta} = \frac{2\pi\sqrt{r}}{\sqrt{g}\tan\theta} = 2\pi\sqrt{\frac{l\sin\theta}{g\tan\theta}}$$

$$\therefore T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

Also from, eq^n (V)

$$T = 2\pi\sqrt{h/g}$$

Question: A bob of mass (m) 1kg is attached to a string of 1m long and made to revolve in a horizontal circle (conical pendulum) of radius (0.6 m) find the period of motion and tension in the String

mass of bob (m) = 1kg.
length of string (l) = 1m
m radius of (r) = 0.6 m

now,
Time period is given by
$$T = 2\pi \times \sqrt{\frac{l\cos\theta}{g}} \quad \text{---} (1)$$

from, $r = l \sin\theta$
or $\theta = \sin^{-1}\left(\frac{0.6}{1}\right)$

$\theta = 36.87$

from (1) $T = 2 \times \frac{22}{7} \times \sqrt{\frac{1 \times \cos(36.87)}{10}}$

$= 1.77$

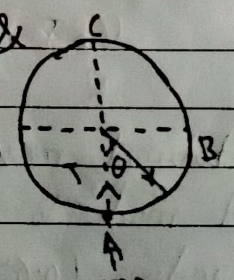
now Tension $T' = mg / \cos\theta$
$= \frac{1 \times 10}{0.799}$

$= 12.56$ N.

Vertical Motion

let us suppose a ~~truck~~ of mass (m) is tied with thread which is allowed to move in ~~circle~~ a vertical circle of radius r in anticlockwise direction.

let θ is the angle between thread & starting point a & (T) is the tension of the string. $T - mg\cos\theta$ provides necessary centripetal force to move in circular path.



$$T - mg\cos\theta = \frac{mv^2}{r}$$

(i) A lowest point A $\theta = 0^\circ$

$$T_A - mg\cos\theta = \frac{mv^2}{r}$$

$$\boxed{\therefore T_A = \frac{mv^2}{r} + mg}$$ Max

(ii) A point B

$$T_B - mg\cos\theta = \frac{mv^2}{r}$$

$$\boxed{\therefore T_B = \frac{mv^2}{r}}$$

(iii) At point C $\theta = 180$

$$T_C - mg\cos 180 = \frac{mv^2}{r}$$

$$\boxed{T_C = \frac{mv^2}{r} - mg}$$ Min

(iv) A point D = $\theta = 270$

$$T_D - mg\cos 270 = \frac{mv^2}{r}$$

$$\boxed{T_D = \frac{mv^2}{r}}$$

At point C

$$T + mg = \frac{mv^2}{r}$$

At highest point $T \to 0$ then, $mg = \frac{mv^2}{r}$

or $v^2 = rg$

$\therefore v = \sqrt{rg}$ --- (i)

~~At point A,~~

$\cancel{T - mg = \frac{mv^2}{r}}$

By using principle of conservation energy at AC,

Total energy at a = total energy at c,

$= \frac{1}{2} \times m \times v^2 + 0 = m \times g \times h$

$= \frac{1}{2} \times m v_a^2 = \frac{1}{2} \times m v_c^2 + m \times g \times 2r$ [r = radius]

$= v_a^2 = v_c^2 + 4gr$ --- (ii)

From (i) & (ii)

$v_a^2 = v_c^2 + 4gr$

$v_a^2 = gr + 4gr$

$v_a = \sqrt{5gr}$, $= \sqrt{5} v_c$ → $v_a = \sqrt{5gr}$

$V_a = \sqrt{5} v_c$,

[illegible] ,,

$\therefore V_c = \sqrt{rg}$ & [illegible] ,,