Kinematics: (Branch of mechanics)

Kinematics.

It deals with the study of motion of an object without considering its cause

=) Types of motion

- one dimensional motion :- The motion of an object along Straigh line is called One dimensional motion

In such a motion only one coordinate changes with time. for eg: A car moving in straigh path

- Two dimensional motion :- If the motion of an object can represented by using two axis. then motion of an object is called two dimensional motor.

In such a motion two coordinat are required to specify the position of an object. for eg: motion of projectile, car moving in zigzag path on a ~~labeld~~ level road.

- Three dimensional :- If the motion of object can be represented ~~can~~ using three axis then motion of an object is called three dimensional motion.

In such a motion three coordinates are required to specify the position of an object for eg: Random motion of molecles of gas, flying bird.

Distance :- The actual path covered by a body from initial position to final position gives the distance,

distance = actual path length.

It is scalar quantity. It can never be zero, if the body is moving. It is path dependent. SI unit: m dimensional formula of distance is [L]

Displacement:- The shortest path between initial and final position gives the displacement.

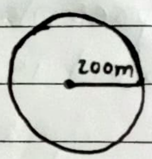
The displacement = shortest path length for initial position to final position. It is vector quantity. The direction of displacement is from initial to final position

displacement of the body can be negative, positive or zero

S.I unit of displacement is m. dimension: [L]

The magnitude of displacement less than or equal to distance, displacement ≥ |displacement|

Question: A body moves 2.5 times around the circular path of radius 200m find the distance & displacem



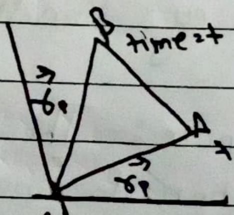
$$\text{distance} = 2.5 \times 2\pi r$$
$$= 2.5 \times 2 \times \frac{22}{7} \times 200$$
$$= 3142.85\,m$$

$$\text{Displacement} = 2 \times r$$
$$= 2 \times 200$$
$$= 400m,$$

let,

$\vec{r}_i$ be the initial position vector of the body and $\vec{r}_f$ be the final position vector of the body, then displacement of the body is given by.

$$\text{displacement } \Delta\vec{r} = \vec{r}_f - \vec{r}_i$$



Question: A body is moving such that the position vector of the body is defined by $\vec{r} = (4t^2\hat{i} + 5t\hat{j} + 6\hat{k})m$ where t is in second. find the displacement the body in the first five second.

$\vec{r} = (4t^2\hat{i} + 5t\hat{j} + 6\hat{k})m$

$\vec{r_i} = (\vec{r})_{t=0} = (4.0^2 + 5\times0 + 6)m$

$= 6\hat{k}\, m$

$r_f = (\vec{r})_{t=5} = (4\times5^2\hat{i} + 5\times5\hat{j} + 6\hat{k})m$

$= (100\hat{i} + 25\hat{j} + 6\hat{k})m$

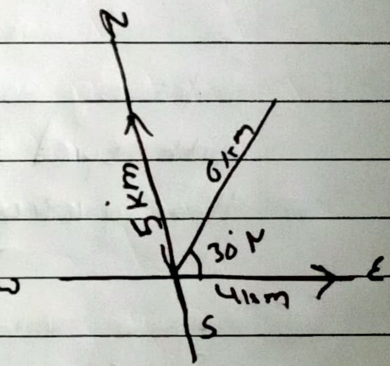
final displacement $(\Delta r) = \vec{r_f} - \vec{r_i}$

$= (100\hat{i} + 25\hat{j} + 6\hat{k}) - 6\hat{k}$

$= (100\hat{i} + 25\hat{j})$

Question:

A car driver drives a car 4km east, 5km north and 6km 30° N of east. find the distance & displacement of the car.



Distance: 4km + 5km + 6km

= 15 km

for displacement:

let, $\vec{A}$ = 4km east

$\vec{B}$ = 5km north

$\vec{C}$ = 6 km 30° N of east

Vector	angle with (+ve) axis	x-component	y component
$\vec{A}$	$\theta = 0°$	$A\times\cos\theta = 4 = A_x$	$A\times\sin\theta = 0$
$\vec{B}$	$\theta = 90°$	$B\times\cos\theta = 0 = B_x$	$B\times\sin\theta = 5$
$\vec{C}$	$\theta = 30°$	$C\times\cos\theta = 3\sqrt{3} = C_x$	$C\times\sin\theta = 3$

let $\vec{d}$ be the displacement, having d_x & d_y component along x-axis & y-axis respectively.

Sum of d_x- component $d_x = A_x + B_x + C_x = 4+0+3\sqrt{3} = 4+3\sqrt{3}$

Sum of d_y-component $d_y = A_y + B_y + C_y = 0+5+3 = 8m$

resultant vector $|\vec{d}| = \sqrt{(d_x)^2 + (d_y)^2}$

$= \sqrt{(4+3\sqrt{3})^2 + (8)^2}$

$= 12.188m$

For direction $\tan\alpha = \frac{d_y}{d_x} = \frac{8}{4+3\sqrt{3}} = 41.020$,

Speed : Speed is defined as distance covered per unit time

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

- speed of a body can be zero, positive but not negative.
- speed is scalar quantity.
- S.I unit of speed is m/s
- dimensional formula of speed is $[LT^{-1}]$

Types of speed

(1) Average speed.

Average speed is defined as total distance to total time.

$$\text{Average Speed } (V_{av}) = \frac{(\text{Total distance})\ \Delta s}{(\text{Total time})\ \Delta t}$$

let us suppose a body covers a distance $s_1, s_2, s_3 \ldots s_n$ with speed $v_1, v_2, v_3 \ldots v_n$ respectively.
then average speed of a body is given by

$$V_{av} = \frac{s_1 + s_2 + s_3 \ldots + s_n}{t_1 + t_2 + t_3 \ldots + t_n}$$

$$\text{time} = \frac{\text{distance } s}{\text{speed } v}$$

Special case (i) if $s_1 = s_2 = s_3 = \ldots s_n = s$ (let)

then, $V_{av} = \dfrac{s + s + s + \ldots + s_n}{t_1 + t_2 + t_3 + \ldots + t_n}$

$$= \frac{ns}{s/v_1 + s/v_2 + s/v_3 + \ldots s/v_n}$$

$$= \frac{n\cancel{s}}{\cancel{s}(1/v_1 + 1/v_2 + 1/v_3 + \ldots + 1/v_n)}$$

for two equal distance

$$V_{av} = \frac{2}{1/v_1 + 1/v_2} \quad \text{or,} \quad V_{av} = \frac{2v_1v_2}{v_1 + v_2}$$

(ii) if, $t_1 = t_2 = t_3 \ldots t_n = t$ (let), then,

$$V_{av} = \frac{s_1 + s_2 + s_3 \ldots + s_n}{t + t + t + \ldots t}$$

$$= \cancel{\frac{s_1}{t} + \frac{s_2}{t} + \frac{s_3}{t}} \ldots \frac{v_1 t + v_2 t + v_3 t \ldots + v_n t}{nt}$$

$$V_{av} = \frac{V_1 + V_2 + V_3 + \dots V_n}{n}$$

$4t^2 + nt$

$8t +$

for two equal ~~distance~~ time interval.

$$V_{av} = \frac{V_1 + V_2}{2}$$

(ii) Instantenous speed.

If average speed is calculated for very short interval of time then it is called as instantenous speed.

$$V_{in} = \lim_{\Delta t \to 0} V_{av} \quad // = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

$$= \frac{ds}{dt}$$ The first order derivative of distance gives instaneous speed.

In case of uniform motion average speed = instantenous speed.

Question A body is moving such that $(S = 4t^2 + nt)$ wher t is in Second find.

(i) Initial speed of a body at $t = 0$ s.

(C) Uniform Speed :- Speed of a body is constant with equal interval of time.

Velocity

Velocity is defined as displacement per unit time

$$\text{Velocity } (V) = \frac{S \text{ (displacement)}}{t \text{ (time)}}$$

- Velocity is vector quantity
  Velocity of a body tells about how fast and in which direction a body is moving
- velocity of a body can be positive, negative or zero.

If a body is moving in circular path or curved path the tangent at a point determine the direction of velocity of a body at that point.

Types of Velocity

(A) Average Velocity : Average Velocity is defined as ratio of total displacement to total time.

$$\text{Avg (V). } \vec{V}_{av} = \frac{\text{Total displacement}}{\text{Total time}} \quad \frac{\Delta \vec{S}}{\Delta t}$$

⇒ If a body moving in a straight path in particular direction then magnitude of avg. Velocity is equal to avg. speed.

(B) Instantaneous Velocity: if average velocity is calculated for very short interval of time then, it is called Instantaneous Velocity.

$$\text{Instantaneous Velocity } (\vec{V}_{in}) = \lim_{\Delta t \to 0} \vec{V}_{av}$$

$$= \lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t}$$

$$= \frac{d\vec{s}}{dt}.$$

① Uniform Velocity : If the velocity of a body is constant with time than the body is said to have uniform velocity.

② Non-Uniform velocity : If the velocity of a body is not constant with time than the body is said to have non-uniform velocit

Note:

$$\frac{\text{Distance}}{|\text{Displacement}|} \geq 1 \qquad \frac{\frac{\text{Distance}}{\text{time}}}{\frac{|\text{Displacement}|}{\text{time}}} \geq 1 \qquad \frac{\text{Avg speed}}{|\text{Avg velocity}|} \geq 1$$

Question A taxi driver drives a taxi from A to B with 60 km/hr and return back to A through same path with 40 km/hr find Avg speed and Magnitude of avg velocity of the taxi.

Solⁿ

$$\text{Avg Speed} = \frac{2v_1 v_2}{v_1+v_2} = \frac{2\times 60\times 40}{60+40}$$

$$= 48 \text{ km/hr}$$

$$\text{Magnitude of avg Velocity} :- \frac{|\text{displacement}|}{\text{time}}$$

$$= \frac{0}{t} = 0 \text{ km/hr}$$

Since, Initial and final position of taxi is same point displacement is Zero. km.

Question A truck driver drives a truck for first ~~hour~~ half an hour with 60 km/hr and another half an hour with 30 m/s in a straight path in a particular direction. find the average speed & magnitude of avg velocity of the truck.

for st. path in particular direction magnitude of avg velocity = avg Speed//

Here 1st case — 2nd case

$t_1 = 0.5$ hr — $t_2 = 0.5$ hr

$v_1 = 60$ km/hr. — $v_2 = 30$ m/s $= 108$ km/hr

Here, $\text{avg speed} = \frac{60+108}{\cancel{}}$

$= \cancel{61.8 \text{ km/hr}}$ $\frac{168}{2}$ km/hr

Accleration :-
The time rate of change of velocity is called accelerate. It is called vector quantity. The direction of acceleration is determined by the change in velocity
=> Accleration can be (+)ve, (-)ve or zero & It's SI unit is m/s^2 having dimension $[LT^{-2}]$
if a body is moving in a particular direction in a straight with increasing velocity then acceleration positive & the direction is along the direction of motion of the body & if the body is moving with decreasing velocity then acceleration is negative & the direction is ~~along~~ the opposite to the motion of the body.

Types of Accleration

① Average Accleration :- Avg acceleration is defined as change in velocity per unit time

$$\text{Average acceleration } (\vec{a}_v) = \frac{\text{change in velocity } (\Delta \vec{v})}{\text{time } \Delta t}$$

$$= \vec{v}_F - \vec{v}_i / \Delta t$$

where, v_i initial velocity & v_F is final velocity of the body.

② Instanteous acceleration :- if average acceleration is calculated for very short period of time then it is called Instantenous acceleration.

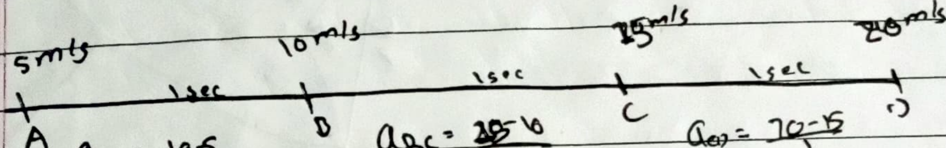
$$\text{Instantaneous accleration } (\vec{a}_{in}) = \lim_{\Delta t \to 0} \vec{a}_v$$

$$= \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d(\vec{v})}{dt}$$

$$= \frac{d}{dt}\left(\frac{d\vec{s}}{dt}\right) = \frac{d^2\vec{s}}{dt^2}$$

Prop 1.2,3.4

III uniform accleration :- if change in velocity is same in equally interval of time, then a body is said to be have uniform acceleration.

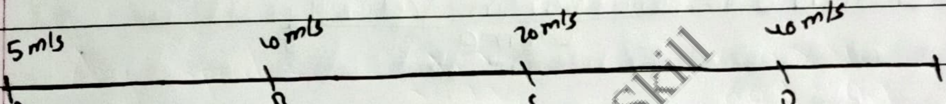


$a_{AB} = \frac{10-5}{1}$ $= 5\ m/s^2$

$a_{BC} = \frac{15-10}{1}$ $= 5\ m/s^2$

$a_{CD} = \frac{20-15}{1}$ $= 5\ m/s^2$

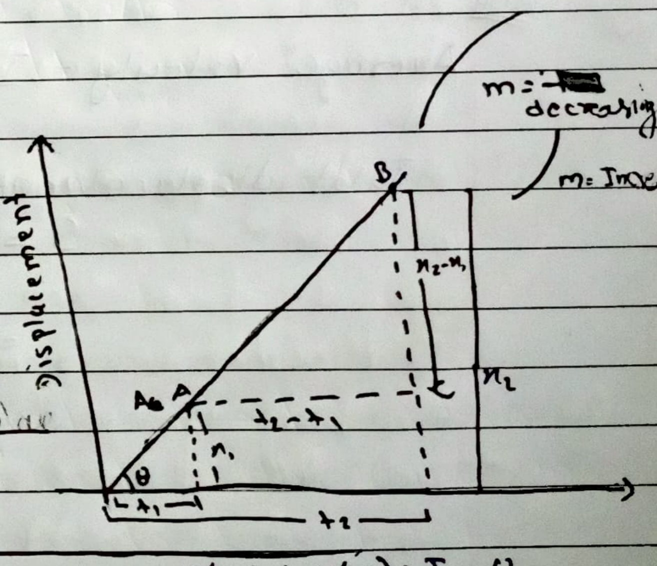
IV non-uniform acceleration :- if change in velocity is different in equally interval of time, then a body is said to be have non-uniform acceleration



$a_{AB} = \frac{10-5}{1}$ $= 5m/s^2$

$a_{BC} = \frac{20-10}{1}$ $= 10m/s^2$

Displacement - time graph



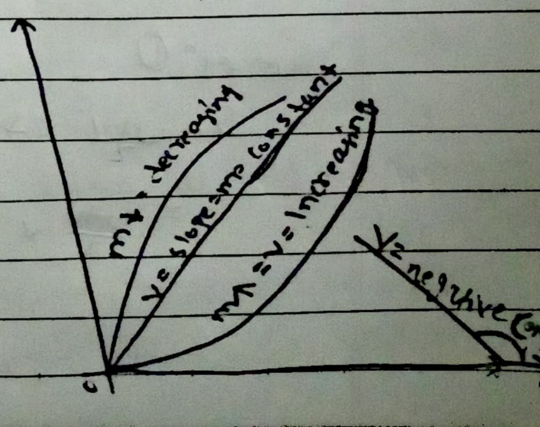
In ΔABC

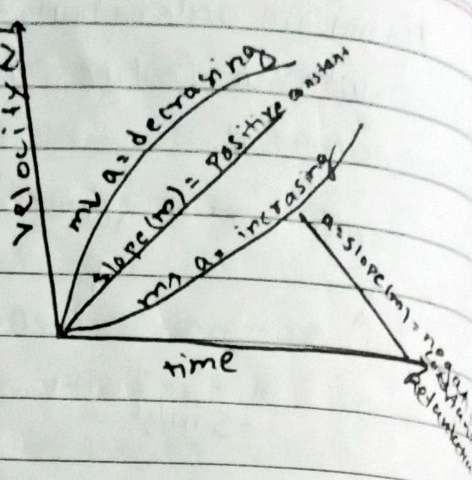
Slope $m = \tan\theta = \frac{x_2 - x_1}{t_2 - t_1}$

$= \frac{\text{change in displac}}{\Delta t}$

= velocity

V = Solope(m) = Tan θ





Equation of uniformly accleraled motion

let us consider, a body moving uniform acceleration. let u be the intial velocity & v be the velocity of body at time t then

- Accleration (a) = $\frac{\text{change in velocity}}{\text{time}}$

$$a = \frac{v-u}{t} \quad \therefore v-u = at$$

Also

Average velocity = $\frac{\text{Total displacement}}{\text{time}}$

Total displacement = Average velocity × time.

$$S = \frac{u+v}{2} \times t \quad \text{--- (1)}$$

$$= \frac{u+u+at}{2} \times t$$

$$S = \frac{2ut + at^2}{2}$$

$$S = ut + \tfrac{1}{2}at^2$$

from eqn (1)

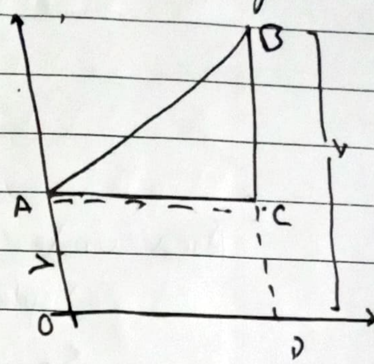
$$S = \frac{u+v}{2} \times t$$

$$= \frac{u+v}{2} \times \frac{v-u}{a} = \frac{v^2-u^2}{2a} \qquad a = \frac{v-u}{t}$$

$$v^2 = u^2 + 2as$$

Equation of uniformly acclerated motion from velocity-time graph

let us consider a body is moving with uniform accleration (a). let u be the Initial velocity & v be the final velocity of the body at time t.



From figure.

OA = Initial velocity = u

BD = final velocity = V

OD = AC = time interval (t)

(i) $v = u + at$ In ΔABC

Slope $m = \tan\theta = \frac{BC}{AC}$

we know, velocity time graph gives accelaration.

$$\text{accleration} = \frac{BC}{AC} = \frac{BD - CD}{AC}$$

$$a = \frac{v - u}{t}$$

(ii) $S = ut + \frac{1}{2}at^2$

we know velocity time graph gives accleration (slope)

$$a = \frac{BC}{AC} = \quad at = BC \text{ --- (1)}$$

Area under velocity-time graph gives the displacement

Displacement = Area of ~~triangle~~ trapezium OABD

S = Area of rectangle OACD + ΔABC

$$= (OA) \times (OD) + \frac{1}{2}(AC) \times (BC)$$

$$= ut + \frac{1}{2}t \times at$$

$$= ut + \frac{1}{2}at^2$$

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III $v^2 = u^2 + 2as$

we know, velocity time graph, slope (m) gives acceleration.

acceleration(a) = slope (m)

$a = \frac{BC}{AC}$ or $AC = \frac{BC}{a}$ --- ①

Area under velocity time graph gives displacement.

Displacemen = Area of trapezium OADD ~~+ Area of Trapezium~~

$S = \frac{1}{2}(OA + BD) \times AC$

$S = \frac{1}{2}(u+v) \times \frac{BC}{a}$

$S = \frac{v+u}{2} \times \frac{v-u}{a} = \frac{v^2 - u^2}{2a}$

Question. A body ~~starts~~ starts from a point and moves with constant acceleration $2\,m/s^2$. find distance ~~d~~ travelled by body in 6^{th} second.

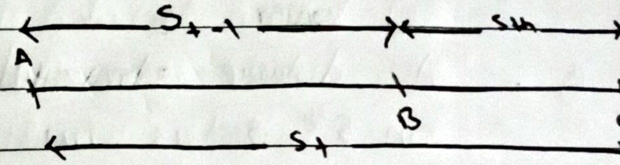
Qu A body is dropped from the top of tower. If it covers half of the total height in last second. then find the heigh of the ~~two~~ tower.

Que A body is dropped from the top of tower of $h = 60m$ at the same another object is projected vertically upward from the ~~vo~~ foot of the tower with $20\,m/s$ find wher & when they meet each other.

Que A body is projected vertically upward with $30\,m/s$. find the distance covered by the object in last one se

Distance travelled by body (A) in particular Second.
(i.e t^{th} second)

let us consider a truck, having mass (m), Initial velocity (u) and uniform accleration (a)



let, S_t, S_{t-1} and S_{th} be the distance travelled by body in t sec, (t-1) sec & t^{th} sec respectively.

$$S_t = ut + \frac{1}{2}at^2$$

$$S_{t-1} = u(t-1) + \frac{1}{2}a(t-1)^2$$

from figure,

$$S_{th} = S_t - S_{t-1}$$

$$= ut + \frac{1}{2}at^2 - \{u(t-1) + \frac{1}{2}a(t-1)^2\}$$

$$= ut + \frac{at^2}{2} - \{ut - u + \frac{a}{2}(t^2 - 2t + 1)\}$$

$$= ut + \frac{at^2}{2} - \{ut - u + \frac{at^2}{2} - \frac{2at}{2} + \frac{a}{2}\}$$

$$= \cancel{ut} + \cancel{\frac{at^2}{2}} - \cancel{ut} + u - \cancel{\frac{at^2}{2}} + \frac{2at}{2} - \frac{a}{2}$$

$S_{th} = u + \frac{a}{2}(2t-1)$

$S_{th} = u \times 1sec + \frac{a}{2}(2t - 1sec)$

$$S_{th} = u + \frac{2at - a}{2}$$

$$S_{th} = u + \frac{2at - a}{2}$$

$$= u + \frac{a}{2}(2t - 1)$$

$m = \frac{m}{s} \times s + ms^{-2}(s-s)s$

$[m] = [m + ms^{-2} \times s^2]$

$= [m + m]$

$= [m]$

Horizontal Velocity = constant //

Projectile Motion,

The object which is projected into air and moves under the action of gravity is called projectile and its motion is called motion.

The path of projectile is called tragectory.

for ex: a stone thrown into space. a bullet fired from gun.

Assumption used in projectile motion.

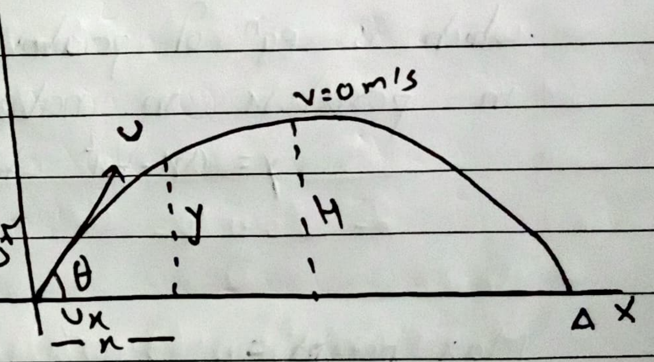
- The air resistance is neglated
- The effect due to rotation of earth and it's Curvature is negligible
- At all points of tragectory, the accleration due to gravity is constant both in magnitude & direction.

Projectile fired at an angle (θ) from a ground

let, us suppose a projectile is fired initial Velocity (U) making an angle (θ) with ~~on~~ Horizontal where

$u_x = u\cos\theta$ & $u_y = u\sin\theta$

be the horizontal & vertical Velocity



Horizontal velocity remains constant throughout the motion of projectile

Since, horizontal accleration ($a_x = 0$) but vertical Velocity donot remains constant.

Since, Vertical accleration (a_y) = accleration due to 'g'

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Equation of the trajectory.

let us suppose, a projectile is act at poin (P) after time (t). x & y be the horizontal & vertical distance after time (t)

$$\text{Horizontal distance } (S_x) = u_x t + \frac{1}{2} a_x t^2$$

$$x = u\cos\theta \, t + \frac{1}{2} \times 0 \times t^2,$$

$$\therefore x = u\cos\theta \, t \quad \therefore t = \frac{x}{u\cos\theta} \text{ --- (1)}$$

$$\text{vertical distance } (S_y) = u_y t + \frac{1}{2} a_y t^2$$

$$y = u\sin\theta \, t - \frac{1}{2} g t^2 \text{ --- (ii)}$$

from (1) & (ii) $y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2$

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta} \times x^2$$

which is eq^n of parabolic, hence the path of projectile in parabolic in nature.

$$y = ax - bx^2 \text{ where } a = \tan\theta, \; b = \frac{g}{2u^2\cos^2\theta}$$

Max height =

The maximum vertical height attained by the projectile is maximum height. In fig H, maximum height.

$$V_y = 0 \text{ m/s}$$

$$V_y^2 = u_y^2 + 2a_y S_y$$

$$0 = u_y^2 - 2gH$$

$$\therefore u_y^2 = 2gH$$

$$\therefore H = \frac{u_y^2}{2g} \quad \therefore H = \frac{u^2\sin^2\theta}{2g}$$

Maximum height attained by the projectile depends on Intial Vertical Velocity.

Time to reach Max height (known as t_{ascent})

t: max time to reach max height (

$v = 0\,m/s$

$v_y = u_y + gt$

$0 = u\sin\theta - gt$

$\frac{u\sin\theta}{g} = t$ (t_{ascent})

Time to reach maximum height depends upon initial vertical velocity.

- Total time of flight (T)

Time for which projectile remain in air is total time of flight (T). In fig time taken by the projectile to reach point A from O is (total time of flight) the value of y is 0 at A.

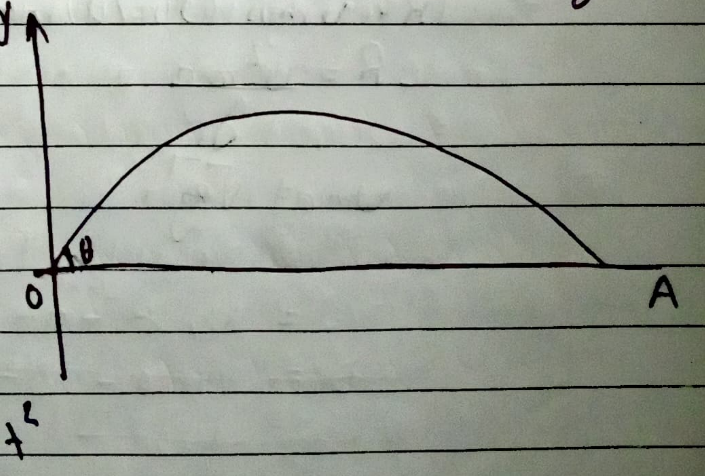
we have,

$y = u_y t - \frac{1}{2} g t^2$

or $0 = u_y t - \frac{1}{2} g t^2$, or $u_y \cancel{t} = \frac{1}{2} g t^{\cancel{2}}$

$u\sin\theta = \frac{gt}{2}$ $\therefore t = \frac{2u\sin\theta}{g}$

The horizontal distance covered by a projectile during its time of flight. (R)



from figure:

OA = horizontal range

we have,

$S_x = u_x t + \frac{1}{2} a_x t^2$

$R = u\cos\theta\, t + \frac{1}{2} \times 0 \times t^2$

$R = u\cos\theta \times \frac{2u\sin\theta}{g}$ $\therefore R = \frac{2u^2 \sin\theta \cdot \cos\theta}{g} = \frac{u^2 \sin 2\theta}{g}$

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① Special case

For maximum horizontal range for given velocity of projection value of $\sin 2\theta$ should be maximum

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\theta = 1$$

$$\sin 2\theta = \sin 90^\circ$$

$$\theta = 45^\circ$$

Thus, angle of projection for maximum horizontal range is 45°.

Maximum horizontal range $R_{max} = \frac{u^2}{g}$,

For minimum horizontal range for given velocity of projection

Value of $\sin 2\theta$ must be, -1,

we have $R = \frac{u^2 \sin 2\theta}{g}$

$$\sin 2\theta = -1$$

$$2\theta = 180$$

$$\therefore \theta = 90^\circ$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R = u^2_{min} (\sin 2\theta)_{max}$$

$$R = \frac{u^2_{min}}{g}$$

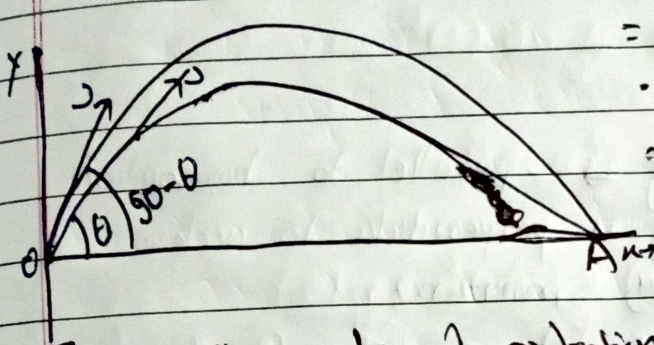
$$u_{min} = \sqrt{Rg}$$

This is required expression to express that, minimum velocity

Another angle of projection for same horizontal range

let another angle of projection $(\theta_1) = 90-\theta$

Horizontal range $(R_1) = \frac{u^2 \sin 2\theta_1}{g}$



$= \frac{u^2 \sin 2(90-\theta)}{g}$

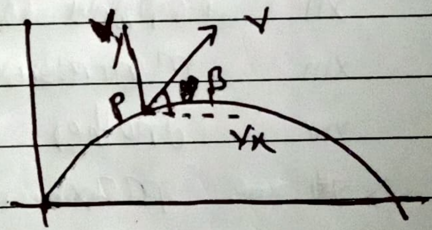
$= \frac{u^2 \sin(180-2\theta)}{g}$

$= \frac{u^2 \sin 2\theta}{g}$

Thus another angle of projection for same horizontal range is $90-\theta$

- Velocity of the projectile an any instant.

let, V be the velocity of projectile after time t. V_x & V_y be the horizontal and vertical velocity of the projectile respectively after time t.



we have $V_x = u_x + a_x t$

$= u\cos\theta + 0 \times t$

$= u\cos\theta$ --- (i)

Also, $V_y = u_y + a_y t$

$= u\sin\theta - gt$ --- (ii)

Magnitude of the resultant velocity is

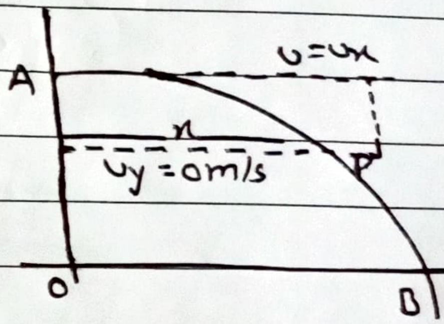
$V = \sqrt{(V_x)^2 + (V_y)^2}$

$= \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

let β be the angle between resultant velocity an horizontal then $\tan\beta = \frac{V_y}{V_x} =$

$\beta = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$

projectile fired from a top of a tower (Horizontally)



let us suppose a projectile is projected horizontally with a velocity (u).
$v_x = u$ = horizontal velocity of projectile which remains constant throughout the motion. Since horizontal accleration $(a_x) = 0$. $v_y = \cos 90 = 0$ is intial vertical velocity which increases with time, sine accleration vertical accleration $(a_y) = (g)$.

equation of trajectory.

Let us suppos a projectile is at point P after time t. x & y give the horizontal & vertical distance of projectile respectively after time (t)

Horizontal distance $S_x = v_x t + \frac{1}{2} a_x (t)^2$

$$x = ut + \frac{1}{2} \times 0 (t)^2$$

$$t = \frac{x}{u} \quad \text{--- (1)}$$

vertical distance

$$S_y = v_y t + \frac{1}{2}(a)(t)^2$$

$$y = 0 \times t + \frac{1}{2}(g)\left(\frac{x}{u}\right)^2 \text{ from eq (1)}$$

$$y = \frac{1}{2} g \frac{x^2}{u^2} \quad \| \quad y = \frac{g}{2u^2} \times x^2$$

$$\boxed{\begin{matrix} y = ax^2 \\ a = g/(2u^2) \end{matrix}}$$

which is equation of parabola. hence path of projectile is parabolic in nature.

- Time of flight

Time for which projectile remain in air is time of flight. (time of descent)

From figure time take by the projectile to reach point B from point A. is time of flight.

we have

$$S_y = u_y t + \frac{1}{2} a_y (t)^2$$

$$h = 0 \times t + \frac{1}{2}(g) T^2$$

$$\frac{gT^2}{2} = h$$

$$T = \sqrt{\frac{2h}{g}}$$

- Horizontal range

The horizontal distance covered by the projectile during its time of flight is (Horizontal flight) (R)

From figure OB is horizontal range.

we have

$$S_x = u_x t + \frac{1}{2}(a_x)(t)^2$$

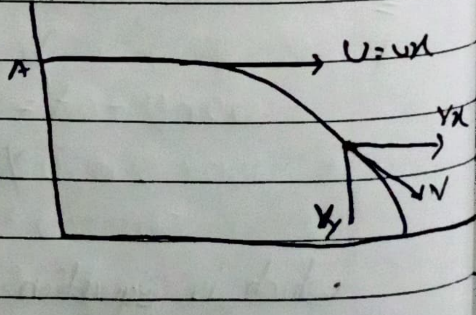
$$R = u_x t$$

$$R = u \cos\theta \times t \quad \text{(is required eq}^n\text{)}$$

$$R = u\cos\theta \times \sqrt{\frac{2h}{g}}$$

- velocity of the projectile at any instant

let v be the velocity of the projectile after time (t). v_x & v_y be the horizontal & vertical velocity of the projectile after time (t)



we have

$V_x = U_x + a_x t$ Also, $V_y = U_y + a_y t$

$= u + 0 \times t$ $V_y = 0 + gt$ ---(1)

$\therefore V_x = u$ ---(1)

magnitude of resultant velocity is

$$V = \sqrt{V_x^2 + V_y^2}$$
$$= \sqrt{u^2 + (gt)^2}$$
$$\therefore V = \sqrt{u^2 + g^2 t^2},$$

let θ be the angle made by the resultant velocity with horizontal, then, $\tan\theta = \frac{V_y}{V_x}$

$$\theta = \tan^{-1}\left(\frac{gt}{u}\right)$$

Quesⁿ A projectile is projected horizontally from the top of tower of height 45m . with 10m/s. Calculate

(i) time of descent

$$T = \sqrt{\frac{2h}{g}}$$
$$= \sqrt{\frac{2 \times 45}{10}} = 3 \text{ sec}$$

45m

(ii) horizontal & vertical distance after (1) sec.
(iii) horizontal range
(iv) Velocity of the projectile after 1 sec.
(v) velocity of the projectile with wich it strike the ground

for (ii) horizontal distance

$$U_x = U\cos\theta t + \frac{1}{2}(a)t^2$$
$$U_x = 10 \text{ m}$$

vertical distance,

$$U_y = U\sin\theta t + \frac{1}{2}gt^2$$
$$= \frac{1}{2} \times 10 (1)^2$$
$$= 5\text{m},$$

Relativ velocity

Relative velocity of an object A with respect to the object B is rate of change of position with respect to B. If $\vec{V}_A$ be the velocity of A with respect to the ground & $\vec{V}_B$ be the velocity of an object with respect to the ground. the relative velocity of an object A with respect to object B is :-

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$
$$= \vec{V}_A + (-\vec{V}_B)$$

Relative velocity of an object B is with respect to an object A is :

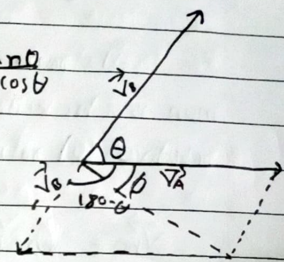
$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$
$$= \vec{V}_B + (-\vec{V}_A)$$

$$\vec{V}_{BA} = |-\vec{V}_{AB}|$$

Relative velocity of A with respect to B.

$$\vec{V_{AB}} = \vec{V_A} - \vec{V_B}$$
$$= \vec{V_A} + (-\vec{V_B})$$

$$\tan\phi = \frac{B\sin\theta}{A \pm B\cos\theta}$$



observer,

Magnitude of $\vec{V_{AB}}$ is,

$$V_{AB} = \sqrt{V_A^2 - 2V_A \cdot V_B \cos\theta + V_B^2}$$

let ϕ be the angle made by V_{AB} vector is V_A vector than

$$\phi = \tan^{-1}\left(\frac{V_B \sin\theta}{V_A - V_B\cos\theta}\right)$$

Special case if $\theta = 0$.

A & B are moving along same direction

$$\vec{V_{AB}} = \sqrt{V_A^2 + V_B^2 - 2V_A \cdot V_B \cdot \cos 0}$$
$$= \sqrt{V_A^2 - 2V_AV_B + V_B^2}$$
$$= V_A - V_B,$$

$\theta = 180$

A & B are moving along opposite directi.

$$\vec{V_{AB}} = \sqrt{V_A^2 + V_B^2 - 2V_AV_B \cdot \cos 180}$$
$$= V_A + V_B \text{ ,, Maximum}$$

$\theta = 90$

A & B are moving perpendicular

$$\vec{V_{AB}} = \sqrt{V_A^2 + V_B^2 - 2V_A \cdot V_B \cdot \cos 90}$$
$$= \sqrt{V_A^2 + V_B^2}$$

$$\tan\phi = \frac{V_B \sin 90}{V_A - V_B\cos 90} = \frac{V_B}{V_A},$$

$$\therefore \phi = \tan^{-1}\left(\frac{V_B}{V_A}\right)$$

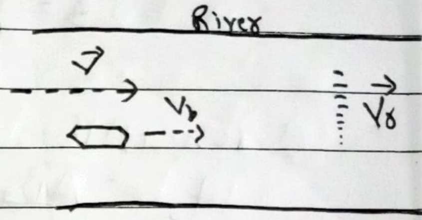
He should hold his umbreall at an angle

River boat problem :- (पानी की सापेक्ष

let $\vec{V_b}$ = Velocity of boat in still water.

$\vec{V_r}$ = Velocity of water

$\vec{V_{br}}$ = Resultant velocity of boat in river.



$V = V_b + V_r$

Distance travelled by boat in t time

$V = \frac{S}{t}$

$\therefore S = Vt$

$= (V_b + V_r)\, t$

Upstream : $\vec{V_b}$ is antiparallel to $\vec{V_r}$

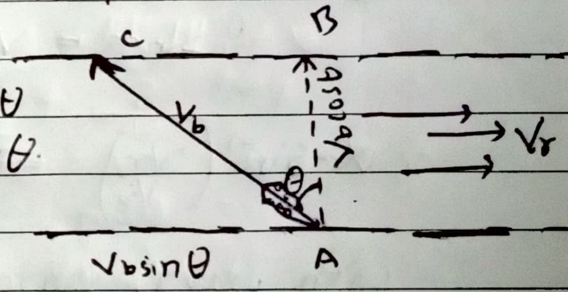
Resultant velocity of boat : $V = V_b - V_r$

Distance travelled by boat in time t time

$S = Vt$ $\quad V = \frac{S}{t}$

$= (V_b - V_r)\, t$

Time to cross the river



The component of V_b (i.e $V_b \sin\theta$ is cancelled by V_r) & $V_b \cos\theta$ responsible for boat to reach the opposite bank.

Time to cross :-

$t = \frac{AB}{V_b \cos\theta}$

Minimum time to cross the

for minimum time to cross the river. the value of $\cos\theta$ should be maximum that 1

$\cos\theta = 1$

$\theta = 0^\circ$ with vertical or. 90° with bank bank or current of water.