Thermal Expansion.

Expansion in Solid.

- Most solid expands when they are heated & contract when they are cooled. Expansion occur in substance due to change in temperature is called thermal expansion. this phenomena occurs in all state of matter.

Molecular are held together by strong attractive force called intermolecular ~~solid~~ forces. when solid is heated, vibrational K.E of it's molecules increases & they begins to vibrate is larger amplitude, so they require larger space. As a result, the equilibrium distance betn molecules increases & hence solid expands.

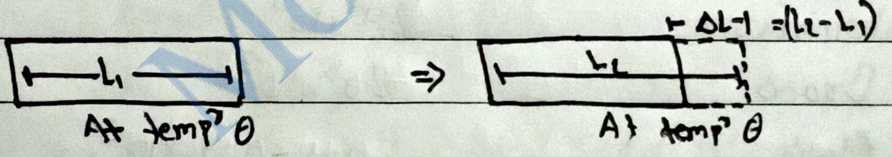
increase in length :- linear expansion in 1 dimension.

Increase in length :- Area expansion in 2 dimension

Cubical expansion :- Volume expansion in 3 dimension

(i) Linear expansion (α alpha)

The increase in length of a solid body due to incre in tempr is called linear expansion.



Consider a metal rod having length L_1 at temperature θ_1. when it is heated, let it's tempr be increase to θ_2 & lenth l_2

So change in temperature, $\Delta\theta = \theta_2 - \theta_1$ & changes in length $\Delta l = l_2 - l_1$

Experimentally, it is found that the increase in the length of body is driectly proportional to. (i) to orginal length(L_1) _ (i)

(ii) to change in tempr, $\Delta\theta$ _ (ii)

Combing eqn (i) & (ii) we get

$$\Delta l \propto l_1 \Delta\theta$$

or $\Delta l = \alpha\, l_1 \Delta\theta_1$ [where. α is the propotionality constant anlso called of linear expansion or linear expansivity of the material of rod] --- (iii)

From eqⁿ (iii)

$$l_2 - l_1 = \alpha l_1 \Delta\theta$$

$$\therefore l_2 = l_1 + \alpha l_1 \Delta\theta$$

$$\therefore l_2 = l_1 (1 + \alpha \Delta\theta) \quad \text{---(iv)}$$

This eqⁿ gives the final length of the road at temp^r θ_2.

From eqⁿ (iii)

$$\alpha = \frac{\Delta l}{l_1 \Delta\theta}$$ let us put $l_1 = 1$ unit of length, $\Delta\theta = 1°$ = unit of temperature.

$$\therefore \alpha = \frac{\Delta l}{1 \cdot 1}$$

linear expensivity of rod is defined as the chang in ~~temperature~~ length per unit original length per unit rise in temperature.

Unit: SI unit : Per kelvin (K^{-1})

C.G.S unit: Per degree celsius ($°C^{-1}$)

Some examples of l.

Material	α ($°C^{-1}$ or K^{-1})
Steel	1.2×10^{-5}
Aluminium	2.4×10^{-5}
Brass	2.0×10^{-5}
Glass	$0.4 - 0.9 \times 10^{-5}$
Copper	1.7×10^{-5}
Gold	1.4×10^{-5}

linear expansity of steel is $1.2 \times 10^{-5}\ K^{-1}$ what does it means?

Ans:

$$\alpha = \frac{\Delta l}{l \Delta\theta}$$, we have:-

let us assume $l = 1$ m, original length, change in temperature, change in temp^r $\Delta\theta = 1K$,

then $\alpha = 1.2 \times 10^{-5}\ K^{-1}$ means, for every 1K chang in temp^r, the 1m original length of steel rod exranded through 1.2×10^{-5} m length.

Superficial Expansion

The expansion in area (i.e in 2 dimension) when temp^r rises is called superficial expansion

Consider the plane sheet of metal having Area A_1 at initial temp^r θ_1 when it is heated to higher temp^r θ_2 ~~its~~ are increases to A_2

So the change in area ΔA is directly proportional to

(i) the original area of plane sheet $\Delta A \propto A_1$ ---(i)

(ii) the change (or rise) in temp^r $\Delta A \propto \Delta\theta$ ---(ii)

Combining eq^n (i) & (ii), we get,

$$\Delta A \propto A_1 \Delta\theta$$

or $\Delta A = \beta A_1 \Delta\theta$ ---(iii) where β is proportional constant known as coefficient of superficial expansivity of material of plane sheet. It unit is K^{-1} in SI unit & $°C^{-1}$ in CGS unit.

for eq^n (iii)

$$A_2 - A_1 = \beta A_1 \Delta\theta$$

$$A_2 = \beta A_1 \Delta\theta + A_1$$

$$A_2 = A_1(1 + \beta\Delta\theta)$$ This eq^n gives final area at temp^r θ_2 Again from eq^n (iv)

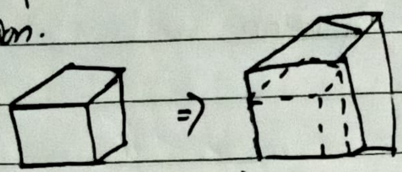
$$\beta = \frac{\Delta A}{A_1 \Delta\theta}$$

So, superficial expansivity is defined as, change in area per unit area per unit rise in temp^r.

$\frac{\alpha}{1} = \beta/2 = \frac{\gamma}{3}$

Cubical Expansion:

Expansion takes places in volume, when temperature rises is called cubical expansion.



let us consider, the solid block of metal having volume V_1 at in initial temperature θ_1 when it is heated, temperature rises & volume increases to V_2

Experimentally.

The change in temperature (volume) of solid block, is directly proportional to the it's original volume.

(i) its original volume $\Delta V \propto V_1$ ---(i)

(ii) change (or rise in temp) $\Delta V \propto \Delta\theta$ --(ii)

Combining eqn (i) & (ii)

or $\Delta V = \lambda V_1 \Delta\theta$ where λ is proportional contant known as coefficient of cubical expansion of a material) --(iii)

from (iii)

$\Delta V = \lambda V_1 \Delta\theta$

$V_2 - V_1 = \lambda V_1 \Delta\theta$

$V_2 = V_1 (1 + \lambda\Delta\theta)$ This gives of the expansion of linear volume at temp θ_2

~~$\therefore V_2 \alpha$~~ $\lambda = \frac{\Delta V}{V_1 \Delta\theta}$.

So, cubical expansion is defined as the chang in volume per unit volume per unit temperature.

$\frac{\alpha}{1} = \beta/2 = \frac{\gamma}{3}$

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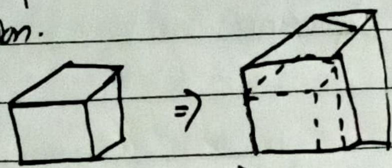

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&

$\lambda = \frac{\Delta V}{V_1 \Delta\theta}$.

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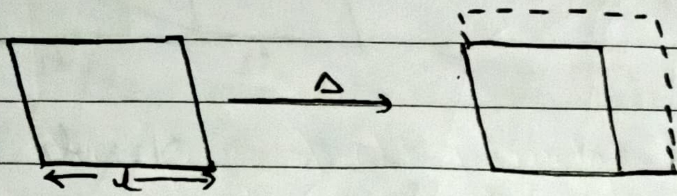
Relation betn linear expansitivity, Superficial expansivity and cubical expansivity.

① Relation betn α & β



Let us consider the square plane sheet of ~~Hepat~~ Metal having length l_1 at initial temp θ_1

$$\therefore \text{Area } A_1 = l_1^2$$

It is heated to tempr θ_2 So, that its length increases to l_2 & Area increases,

$$\text{i.e } A_2 = l_2^2$$

So linear expansion,

$$\text{final length } l_2 = l_1(1+\alpha\Delta\theta) \text{ ---- (i)}$$

where α = linear expansisivity of the material of plane sheet.

So, final Area

$$A_2 = l_2^2 = l_1^2(1+\alpha\Delta\theta)^2$$

$$A_2 = A_1(1+2\alpha\Delta\theta+\alpha^2\Delta\theta^2) \text{ ---(ii)}$$

As the value of α is very small, its higher power values are also very ~~low~~ small value. So, it can be negligible ---(iii)

$\therefore$ eqn (ii) become,

$$A_2 = A_1(1+2\alpha\Delta\theta) \text{ -- (iii)}$$

Again from Superficial expansion,

$$A_2 = A_1(1+\beta\Delta\theta) \text{ --- (iv)}$$

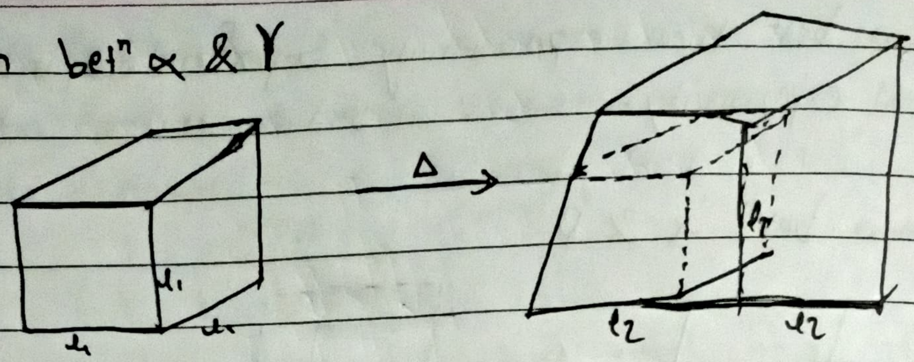
where β = superficial expansivity of the materiel of plane sheet.

Comparing eqn (iii) & iv we get

$$\beta = 2\alpha$$

ie superficial expensivity of substances is two times of its linear expansivity.

(ii) Relation betⁿ α & γ



$V_1 = l_1^3$ $\qquad\qquad$ $V_2 = l_2^3$

let us consider, the cube of substances having length L_1 at initial temperature θ_1

$\therefore$ initial volume, $V_1 = l_1^3$

It is heated to higher temp θ_2 so its length increases to l_2 & volume increases to V_2

$$V_2 = l_2^3$$

$\therefore$ final length,

$$l_2 = l_1(1+\alpha\Delta\theta) \text{ ....(i)}$$

final volume

$$V_2 = (l_2)^3 = l_1^3(1+\alpha\Delta\theta)^3$$

$$\text{or } V_2 = V_1(1+3\alpha\Delta\theta+\underbrace{3\alpha^2\Delta\theta^2+\alpha^3\Delta\theta^3}) \text{ ....(iii)}$$

As α is very small, so, its higher power is also very small in volume, so, it can be neglected,

$$V_2 = V_1(1+3\alpha\Delta\theta)$$

we have from cubical expansion,

$$V_2 = V_1(1+\gamma\Delta\theta) \text{ ...iv,}$$

where γ = cubical expansivity of material of substance.

$\therefore$ Comparing (iii) & (iv)

$$\therefore \gamma = 3\alpha \text{ ...(v)}$$

cubical expansivity of body is three of its linear body.

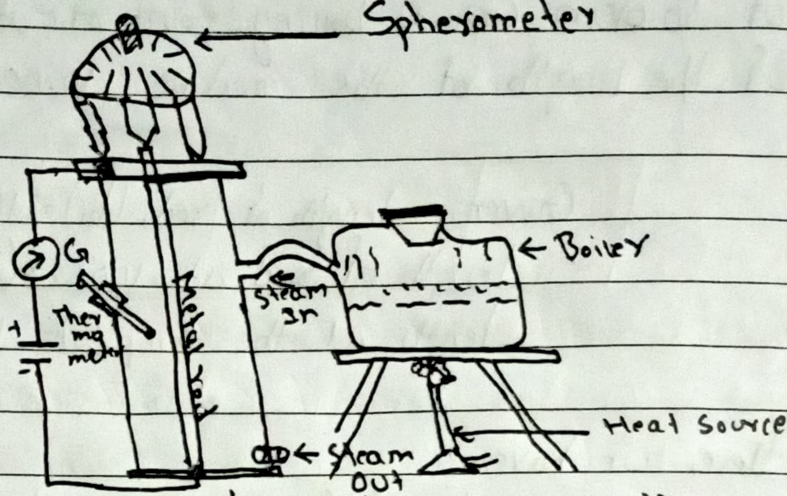
Here, As $\beta = 2\alpha$

so, $\alpha = \beta/2$ $\qquad$ & $\gamma = 3\alpha$

$\alpha = \dfrac{\gamma}{3}$,

$$\therefore \alpha = \frac{\beta}{2} = \frac{\gamma}{3} \text{ ///}$$

Pullinger's Apparatus for determination of coeffient of linear expansion of given soid.



pullinger's appartus is used to determine coeificient of linear expansion of given soid.

Constructi: It consits of the hollow cylinder, in which the given metallic rod is placed. Its one end is fixed at bottom and its upper end is free to expand.

The thermometer is fixed at the middle of the tub to measure temperature. Steam is paused from the boiler inside the cylinder to heat the rod & it is passed out from another opening. At the top of the instrument, spherometer is placed to measure increase in length. It is also connected with the circuit joining the spherometer and the bottom end of the rod, to check wheather the leg of spherometer tuches the rod or not.

Working:- A metalic rod is taken & it's initial length is measured

Original length $= l_1$ ---- (i)

It is placed under (i.e inside) the cylinder & initial tempr is noted Initial tempr $= \theta_1$ --- (ii)

The initial reading of spherometer is noted & it is also rotated upword to some rotation in order to give space for expansion

Now, steam is passed & rod is now expanded and After some time it will attain the final temperature θ_2

final temp^r = θ_2. finally the spherometer is rotated downward in order to touch the rod expanded & final observation are noted.

~~no. of CS~~ Initial Circular Scale division (CSD) = x

no. of rotation raised = n_1.

After expansion,

No. of rotation lowered = n_2 & final CSD = y.

$\therefore$ Increase In length

Δl = no. of rotation raised - [No. of rotation lowered + (Initial CSD - final CSD) $\times$ LC)

$= n_1 - [n_2 + (x-y) \times L.c]$

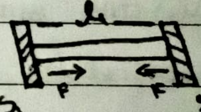
$\therefore$ Coefficient of linear expansion is given by.

$$\alpha = \frac{\Delta l}{l_1 \times (\theta_2 - \theta_1)}$$

By putting the respective Values from observation in above eq^n the Value of α is determined.

Force setup due to expansion or contraction.

Let us, Suppose that a metal rod is fixed at two rigid ends S_1 & S_2. Suppose Initially termp^r θ_1



It is heated to final temp^r θ_2. The rod tries to expand but will not able to do so, due to expand blockage (fixed Support). As a result a force of tension is developed which Compress the rod.

from the linear expansion,

$$l_2 = l_1 (1 + \alpha \Delta\theta)$$

$$\therefore \Delta l = l_2 - l_1 = l_1 \alpha \Delta\theta \text{ ---- (1)}$$

from the defination of young's modulus of elasticity.

$$Y = \frac{\text{Normal stress}}{\text{longitudinal Strain}}.$$

$$y = \frac{F/A}{\Delta l / l_1} = \frac{F l_1}{A \Delta l}$$

$$\text{or} \quad F = \frac{Y A \Delta l}{l_1} \quad \dots (ii)$$

From eqⁿ (i) and (ii)

$$F = \frac{Y A l_1 \alpha \Delta\theta}{l_1}$$

$$\therefore F = T = Y \alpha A \Delta\theta.$$

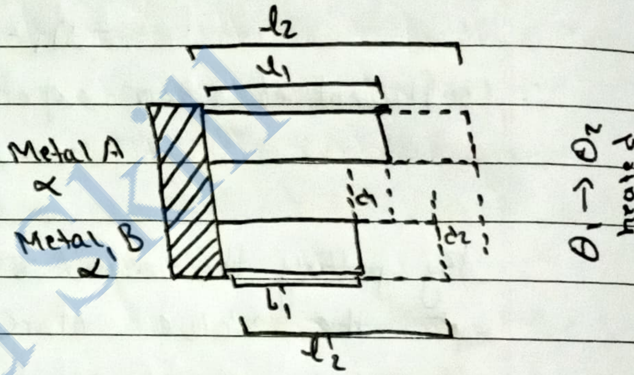
Diffrential expansion

$\Delta l - \Delta l' =$ differential expansion

$= (l_2 - l_1) - (l_2' - l_1')$

$= (l_2' - l_2) - l_1 - l_1'$

$= d_2 - d_1$



Diffrent solids have diffrent values of expansivities & expand by diffrent amount when they are heated through same range of temprature,

Let us consider two metallic rods A & B made of diffrent materials having intial length l_1 & l_1' and linear expansivities α & α' at temp θ_1°c respectively. Let their one end be fixed to rigid support & another end is free to expand. when they are heated to high tempʳ θ_2, their length becomes l_2 & l_2' respectively Here we can write,

For Metal A $\quad l_2 = l_1(1 + \alpha\Delta\theta)$

For Metal B $\quad l_2' = l_1'(1 + \alpha'\Delta\theta)$

let d_1 be diffrenre in length betn metal A & B at initial tempr θ_1 & d_2 be their diffrence in length at tempr θ_2

$$\therefore d_1 = l_1 - l_1' \quad \text{---(iii)}$$

$$\therefore d_2 = l_2 - l_2' \quad \text{---(iv)}$$

now, $d_2 = l_2' - l_2$

$$d_2 = l_1'(1+\alpha'\Delta\theta) - l_1(1+\alpha\Delta\theta)$$

$$d_2 = l_1' + l_1'\alpha'\Delta\theta - l_1 + l_1\alpha\Delta\theta$$

$$d_2 = l_1' - l_1 + (l_1'\alpha' - l_1\alpha)\Delta\theta.$$

$$\text{or } d_2 - d_1 = (l_1'\alpha' - l_1\alpha)\Delta\theta \quad \text{---(v)}$$

This is the expansion for diffrenciential expansion for two rods at two diffrent temperatures.

Condition for zero diffrential expansion.

when the diffrence in length of 2 diffrent rods are same at all tempr i.e $d_1 = d_2$

from eqn (v)

$$d_2 - d_1 = (l_1'\alpha' - l_1\alpha)\Delta\theta$$

$$\therefore (l_1'\alpha' - l_1\alpha)\Delta\theta = 0$$

Here, $\Delta\theta \neq 0$

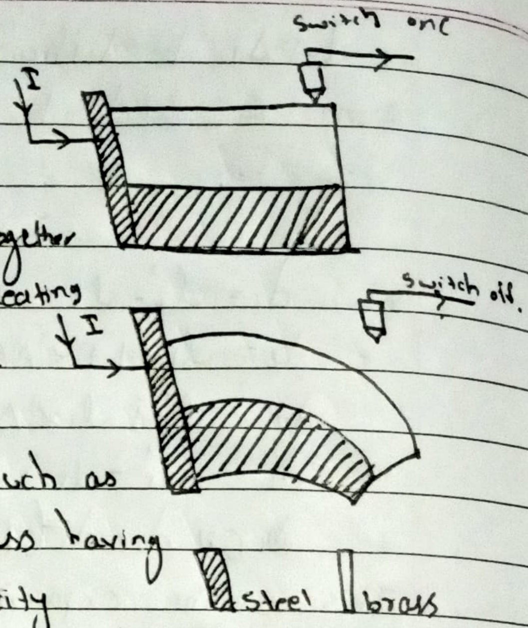
$$\therefore l_1'\alpha' - l_1\alpha = 0$$

$$\text{or } l_1'\alpha' = l_1\alpha$$

$$\text{or } \frac{l_1'}{l_1} = \frac{\alpha}{\alpha'} \quad \text{--- (vi)}$$

Thus, if initial length of materials of rod are so chosen that, the ratio of their initial length is ~~called~~ equivalent to ratio of reciprocal of their ~~length~~ linear expansivities, then differential expansion will be zero.

Bimetallic Thermostat.



- It Consits of two thin metal strips of diffrent expansivities welded together along their length, specially used in heating devices to control over heating of the devices.

when the bimetallic strip such as brass-steel strip is heated, as brass having the higher value of linear expansivity them that of steel, brass expands more than steel. so the strip bends together into an arc with longer brass piece lying on convex surface. As a result the electrical circuit is broken.

It is used for controlling the temp^r of laundry irons, hot water storage tanks, aqranium for tropical fish.

Variation of density with temperature.

we know, mass = volume x density.

$$\therefore \rho = \frac{m}{V} \quad \text{or} \quad \therefore \rho \propto \frac{1}{V}$$

let, us consider, the body of mass 'm' have intial volume V_1 at intial temperature θ_1. It's initial density be ρ_1

$$\rho_1 = \frac{m}{V_1} \text{ --- (i)}$$

when the object is heated to final temp^r θ_2. volume is increased to V_2 & since $\rho = \frac{1}{V}$ density is decreased to ρ_2 given by, $\rho_2 = \frac{m}{V_2}$ --- (ii) let, γ be cubical expansivity of the body ----(iii)

$\therefore V_2 = V_1(1+\gamma\Delta\theta)$ ---(iii)

Substituting eq^n (iii) in eq^n (ii)

$$\rho_2 = \frac{m}{V_1(1+\gamma\Delta\theta)}$$

$$\therefore \rho_2 = \frac{\rho_1}{(1+\gamma\Delta\theta}$$

$$\therefore \rho_1 = \rho_2(1+\gamma\Delta\theta) \text{ ----(iv)}$$

Numerical:-

The density of silver at 0°c is 10310 kg/m³ and the linear expansivity is 0.000019 /°c calculate its density at 100°c

Here, $\theta_1 = 0°c$ & $\theta_2 = 100$

$\Delta\theta = 100°c$

$\alpha = 0.000019$ or, $\gamma = 5.7\times10^{-5}$

$\rho_1 = 10310$ & $\rho_2 = ?$

$\rho_1 = \rho_2(1+\gamma\Delta\theta)$

$10310 = \rho_2(1+5.7\times10^{-5}\times100)$

$10310 = \rho_2(1.0057)$

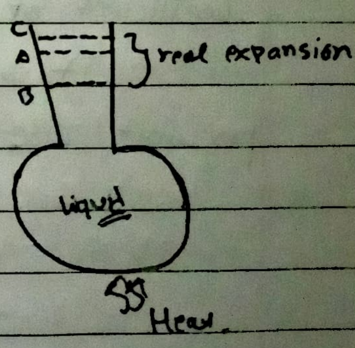
$\rho_2 = 10251.57 \ kgm^{-3}$

Expansion of liquid

AB = expansion of solid container.

CA = Apparent expansion of liquid.

BC = Expansion of solid expansion.



from fig.

CB = AB + CA

Real expansion = Apparent expansion + vol^m expansion of solid vessel

$\therefore \gamma_r V_1\Delta\theta = \gamma_a V_1\Delta\theta + \gamma_g V_1\Delta\theta$

$\Delta V_r = \Delta V_a + \Delta V_g$

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Remining are noted beside this sheet:-

Expansion in liquids

Liquid has only size, but it doesnot have any definite shape. liquid can't be heated directly. So, liquid is heated by putting it inside the vessel.

Suppose the glass vessels contains the liquid of certain Volume upto mark A. when the vessels is heated, at first the vessels expands, so due to expansion, liquid level is decreased to another mark B.

As the heat continue, the liquid also get heated & begins to expand. As the volume expansivity of the liquid is greater than that of soild, liquid expands more than soild. as a result it's level rises upto mark C crossing mark. A. from figure,

CB, ~~insdes~~ + indicates the real expansion of liquid but for the observer, the liquid expanded from intial mark A to final mark C. So CA insidicates the apparent expansion of liquid. & AB indicates the expansion of solid gas vessels.

$$CB = CA + AB$$

real exp^n of liquid = Apparent expansion of liquid. + expansion of vessel.

i.e $\Delta V_r = \Delta V_a + \Delta V_g$ ---- (i)

Here,

Coefficient of real expansion of liquid

It is defined as real increase in volume of liquid per unit rise in temperature.

$$\gamma_r = \frac{\Delta V_r}{V_1 \times \Delta\theta}$$

or, $\Delta V_r = \gamma_r V_1 \Delta\theta$ --- (ii)

Coefficient of apparent expansion of liquid

It is defined as a apparent increase in volume of liquid per unit original volume per unit rise in temperature.

A steel wire 8 m long & 4mm diameter is fixed to two rigid supports. calculate the increase in tension, when the tempr falls by 10°C

Given, $Y = 2 \times 10^{11}$ N/m² and $\alpha = 12 \times 10^{-6}$ °c⁻¹.

difference in tempr $\Delta\theta = 10°c$

~~Given~~ Increase in tension

$$T = Y A \alpha \Delta\theta$$

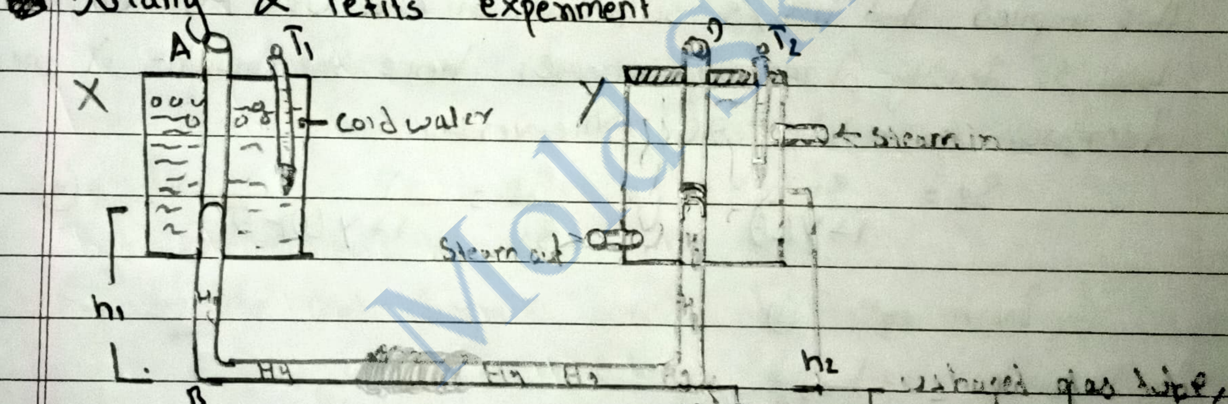
$$T = Y \times \pi r^2 \times \alpha \times \Delta\theta,$$

$$T = 2\times10^{11} \times \frac{22}{7} \times \frac{(4)^2}{4} \times 12\times10^{-6} \times 10$$

$$= 2\times10^{11} \times \frac{11}{875000} \times 1.2\times10^{-4}$$

$$= 301.71 \text{ N}$$

Dulong & Petit's experiment



=) Experiment arrangement for Dulong & petit's experiment.

This experiment is used to determine the real expansivity of liquid It is based on the principle of hydrostatics. It consist of U-shaped glass tube ABCD which contains mercury whose real expansivity is to be found.

Limb AB of the tube is placed inside chamber X, which is maintained at cold temperature using icewater. Limb DC of the tube is placed inside chamber Y which is regulated in hot temperature by providing steam in it.

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Thermometer T_1 & T_2 are fixed at two chambers X & Y respectively to measure their respective temperature. BC tube is covered with wet cloth to prevent unnecessary heat flow from hot tube to cold tube.

After some time, the steady state occurs, so that the hydrostatics pressure of A, B tube is equal to the x.

let In tube AB

h_1 = height of mercury

ρ_1 = density of mercury

θ_1 = temperature of AD

let In tube CD

h_2 = height of mercury

ρ_2 = density of mercury

θ_2 = temperature of CD

$\therefore$ Hydrostatic force of AB = Hydrostatic force of CD,

$$h_1 \rho_1 g = h_2 \rho_2 g$$

$$\frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} \quad \text{--- (I)}$$

This implies that $h \propto 1/\rho$ such that, In cold tube AB tempr is low, so density of mercury increases, hence the height of mercury decreases & In tube DC vice-versa,

$$\rho_2 = \frac{\rho_1}{1+\gamma\Delta\theta} \quad \text{or} \quad \frac{\rho_2}{\rho_1} = \frac{1}{1+\gamma(\theta_2-\theta_1)} \quad \text{--- (II)}$$

from eqn (I)

$$\frac{h_1}{h_2} = \frac{1}{1+\gamma(\theta_2-\theta_1)}$$

$$h_1(1+\gamma(\theta_2-\theta_1)) = h_2$$

$$\text{or } h_1 + h_1\gamma(\theta_2-\theta_1) = h_2$$

$$\text{or } h_1\gamma(\theta_2-\theta_1) = h_2-h_1$$

$$\text{or } \gamma(\theta_2-\theta_1) = \frac{h_2-h_1}{h_1}$$

$$\therefore \gamma(\theta_2-\theta_1) = \frac{\Delta h}{h_1}$$

By substituting $h_2, h_1, \theta_2, \theta_1$ on above eqn γ is determined.

Source of error.

(i) As the distance bet two limbs are little for the difference in height, h_2-h_1 can't be measured accurately

(ii) Temperature of the hot tube & cold tube must be measured accurately

(iii) The heat mustn't be transferred from hot tube to cold tube to prevent this the tube must be taken very narrow & it should be covered with wet cloth.

Anamolous expansion of water.

Generally, matter expands when heated but, water have some exception. water shows anamolous behaviours between 0°C to 4°C such that water contracts on increasing its temperature from 0°C to 4°C instead of expansion. So that 4°C, water attains minimum Volume and max density (i.e $\rho_{max} = 1\ gm/cm^3$).

This kind of unusual expansion of water is called anamolous expansion of water. This property have an importance significance in the nature.

- Significance in the nature.

(i) Aquatic life can survive inside the freezen pound in cold region

During the normal temperature, water in pond or lake is in thermal equilibrium with the surrounding. As the temperature of surrounding falls down & reaches to 4°C, intially the water at surface also becomes at temperature 4°C. we know that at 4°C, water attains maximum density so, being heavy water at 4°C will descend to bottom to displace it with light water which moves upward. when all the water becomes 4°C, the circulation stops. Now the surrounding temp^n is still falling so that water at surface below 4°C being lighter that 4°C water, it always remains at surface & when temperature falls to 0°C, it freezes