

Kinematics: (Branch of mechanics)

CLASSMATE

Date _____

Page _____

Kinematics.

It deals with the study of motion of an object without considering its cause.

=) Types of motion

- One dimensional motion :- The motion of an object along straight line is called One dimensional motion.
In such a motion only one coordinate changes with time. for eg: A car moving in straight path.
- Two dimensional motion :- If the motion of an object can be represented by using two axis, then motion of an object is called two dimensional motion.

In such a motion two coordinates are required to specify the position of an object.

for eg: motion of projectile, car moving in Zigzag path on a ~~level~~ road.

- Three dimensional :- If the motion of object can be represented by using three axis then motion of an object is called three dimensional motion.

In such a motion three coordinates are required to specify the position of an object
for eg: Random motion of molecules of gas, flying bird.

Distance :- The actual path covered by a body from initial position to final position gives the distance.

Distance = actual path length.

It is scalar quantity. It can never be zero, if the body is moving. It is path dependent. SI unit : m
dimensional formula of distance is [L]

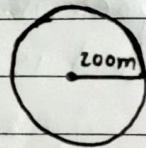
Displacement:- The shortest path between initial and final position gives the displacement.

The displacement = Shortest path length from initial position to final position. It is vector quantity. The direction of displacement is from initial to final position. Displacement of the body can be negative, positive or zero. S.I unit of displacement is m. dimension: [L]

The magnitude of displacement less than or equal to distance, $\text{displacement} \leq \text{distance}$

Question: A body moves 2.5 times around the circular path of radius 200m. Find the distance & displacement.

$$\begin{aligned}\text{Distance} &= 2.5 \times 2\pi r \\ &= 2.5 \times 2 \times \frac{22}{7} \times 200 \\ &= 3141.85 \text{ m.}\end{aligned}$$

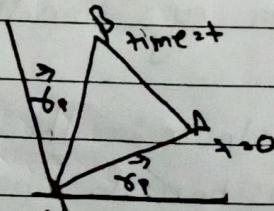


$$\begin{aligned}\text{Displacement} &= 2 \times r \\ &= 2 \times 200 \\ &= 400 \text{ m.}\end{aligned}$$

Let,

\vec{r}_i be the initial position vector of the body and \vec{r}_f be the final position vector of the body, then displacement of the body is given by.

$$\text{displacement } \vec{d} = \vec{r}_f - \vec{r}_i$$



Question: A body is moving such that the position vector of the body is defined by $\vec{r} = 4t^2 \hat{i} + 5t \hat{j} + 6 \hat{k}$ m where t is in second. Find the displacement the body in the first five second.

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) \text{ m}$$

$$\vec{r}_1 = (\vec{r})_{x=0} = (4\cdot 0^2 + 5 \cdot 0 + 6) \text{ m} \\ = 6\hat{k} \text{ m}$$

$$\vec{r}_2 = (\vec{r})_{z=5} = (4 \times 5^2 \hat{i} + 5 \times 5 \hat{j} + 6\hat{k}) \text{ m} \\ = (100\hat{i} + 25\hat{j} + 6\hat{k}) \text{ m}$$

Final displacement (Δr): $\vec{r}_2 - \vec{r}_1$

$$= (100\hat{i} + 25\hat{j} + 6\hat{k}) - 6\hat{k} \\ = (100\hat{i} + 25\hat{j}),$$

Question:

A car driver drives a car 4km east, 5km north and 6km 30° N of east. Find the distance & displacement of the car.

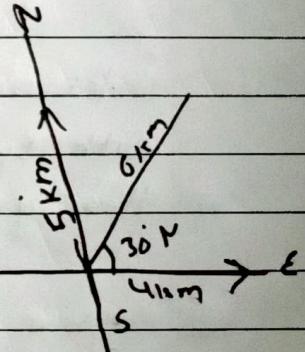
Distance: $4\text{ km} + 5\text{ km} + 6\text{ km}$
 $= 15\text{ km}$

For displacement:

Let, $\vec{A} = 4\text{ km east}$

$\vec{B} = 5\text{ km north}$

$\vec{C} = 6\text{ km } 30^\circ \text{ N of east}$



Vector	along/e with (true) axis	x-component	y-component
\vec{A}	$\theta = 0^\circ$	$A_x(\cos\theta) = 4 = A_x$	$A_x \sin\theta = 0$
\vec{B}	$\theta = 90^\circ$	$B_x(\cos\theta) = 0 = B_x$	$B_x \sin\theta = 5$
\vec{C}	$\theta = 30^\circ$	$C_x(\cos\theta) = 3\sqrt{3} = C_x$	$C_x \sin\theta = 3$

Let \vec{r} be the displacement, having Δx & Δy component along x-axis & y-axis respectively.

Sum of Δx -component $\Delta x = A_x + B_x + C_x = 4 + 0 + 3\sqrt{3} = 4 + 3\sqrt{3}$

Sum of Δy -component $\Delta y = A_y + B_y + C_y = 0 + 5 + 3 = 8\text{ m}$

Resultant vector $|\vec{r}| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$$= \sqrt{(4+3\sqrt{3})^2 + (8)^2}$$

$$= 12.148\text{ m}$$

For direction $\tan\alpha = \frac{\Delta y}{\Delta x} = \frac{8}{4+3\sqrt{3}} = 41.020,$

Speed : Speed is defined as distance covered per unit time.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

- Speed of a body can be zero, positive but not negative.
- Speed is scalar quantity.
- S.I unit of speed is m/s
- Dimensional formula of speed is $[LT^{-1}]$

Types of speed

① Average speed.

Average speed is defined as total distance to total time.

$$\text{Average speed (V}_{av}\text{)} = \frac{\text{(Total distance) } ds}{\text{(Total time) } dt}$$

Let us suppose a body covers a distance $s_1, s_2, s_3, \dots, s_n$ with speed $v_1, v_2, v_3, \dots, v_n$ respectively.

then average speed of a body is given by

$$V_{av} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{t_1 + t_2 + t_3 + \dots + t_n}$$

Special case if $s_1 = s_2 = s_3 = \dots = s_n = s$ (let)

$$\text{Then, } V_{av} = \frac{s + s + s + \dots + s}{t_1 + t_2 + t_3 + \dots + t_n}$$

$$= \frac{ns}{s/v_1 + s/v_2 + s/v_3 + \dots + s/v_n}$$

$$= \frac{ns}{v_1 + v_2 + v_3 + \dots + v_n}$$

For two equal distance

$$V_{av} = \frac{2}{v_1 + v_2} \quad \text{or, } V_{av} = \frac{2v_1 v_2}{v_1 + v_2}$$

② If, $t_1 = t_2 = t_3 = \dots = t_n = t$ (let), then,

$$V_{av} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{t + t + t + \dots + t}$$

$$= \frac{s/v_1 + s/v_2 + s/v_3 + \dots + s/v_n}{n/t}$$

$$V_{av} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n}$$

$$\frac{v_1 + v_2 + \dots + v_n}{n}$$

$$v_1 + v_2 + \dots + v_n$$

for two equal distance time interval.

$$V_{av} = \frac{v_1 + v_2}{2}$$

① Instantaneous speed.

If average Speed is calculated for very short interval of time then it is called as instantaneous speed.

$$V_{in} = \lim_{\Delta t \rightarrow 0} V_{av} // = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t}$$

$= \frac{ds}{dt}$ [The first order derivative of distance gives instantaneous speed.]

In case of uniform motion average speed = instantaneous speed.

Question A body is moving such that $(s = Mt^2 + nt)$ where t is in second find.

① Initial speed of a body at $t = 0$ s.

① Uniform Speed :- Speed of a body is constant with equal interval of time.

Velocity

Velocity is defined as displacement per unit time

$$\text{Velocity } (V) = \frac{S \text{ (displacement)}}{t \text{ (time)}}$$

- Velocity is vector quantity

Velocity of a body tells about how fast and in which direction a body is moving

- Velocity of a body can be positive, negative or zero.

If a body is moving in circular path or curved path the tangent at a point determine the direction of velocity of a body at that point.

Types of Velocity

② Average Velocity : Average Velocity is defined as ratio of total displacement to total time.

$$\text{Avg } V) \vec{V}_{av} = \frac{\text{Total displacement } \vec{AS}}{\text{Total time } \Delta t}$$

\Rightarrow If a body moving in a straight path in particular direction then magnitude of avg. Velocity is equal to avg. speed.

③ Instantaneous Velocity : if average velocity is calculated for very short interval of time then, it is called instantaneous velocity.

$$\text{Instantaneous Velocity } (\vec{V}_{in}) = \lim_{\Delta t \rightarrow 0} \vec{V}_{av}$$

$$= \underset{\Delta t \rightarrow 0}{\rightarrow} \frac{\vec{AS}}{\Delta t}$$

$$= \frac{d\vec{s}}{dt}$$

① Uniform Velocity : If the velocity of a body is constant with time than the body is said to have uniform velocity.

② Non-Uniform Velocity : If the velocity of a body is not constant with time than the body is said to have non-uniform velocity.

Note:

$$\frac{\text{Distance}}{|\text{Displacement}|} \geq 1 \quad \frac{\text{Distance}}{\text{time}} \geq 1 \quad \frac{|\text{Displacement}|}{\text{time}} \geq 1 \quad \frac{\text{Avg Speed}}{(\text{Avg Velocity})} \geq 1$$

Question. A taxi driver drives a taxi from A to B with 60 km/hr and return back to A through same path with 40 km/hr. Find Avg speed and Magnitude of avg velocity of the taxi.

Sol

$$\text{Avg Speed} = \frac{2v_1 v_2}{v_1 + v_2} = \frac{2 \times 60 \times 40}{60 + 40} \\ = 48 \text{ km/hr}$$

$$\text{Magnitude of avg velocity} : - \frac{|\text{Displacement}|}{\text{time}}$$

$$= \frac{0}{t} = 0 \text{ km/hr}$$

Since, Initial and final position of taxi is same point displacement is zero km.

Question. A truck drives a truck for first half an hour with 60 km/hr and another half an hour with 30 m/s in particular direction. Find the average speed & magnitude of avg velocity of the truck.

Here 1st case

2nd case

$$t_1 = 0.5 \text{ hr}$$

$$t_2 = 0.5 \text{ hr}$$

$$v_1 = 60 \text{ km/hr.}$$

$$v_2 = 30 \text{ m/s} = 108 \text{ km/hr}$$

of avg velocity Here, avg speed = $\frac{60 + 108}{2}$

= avg
Speed /

$$= 84 \text{ km/hr} \quad \frac{108}{2} \text{ km/hr}$$

Acceleration :-

The time rate of change of velocity is called acceleration. It is called vector quantity. The direction of acceleration is determined by the change in velocity.

\Rightarrow Acceleration can be +ve, -ve or zero & its SI unit is m/s^2 having dimension $[LT^{-2}]$

If a body is moving in a particular direction in a straight with increasing velocity then acceleration positive & the direction is along the direction of motion of the body & if the body is moving with decreasing velocity then acceleration is negative & the direction is along the opposite to the motion of the body.

Types of Acceleration

① Average Acceleration :- Avg acceleration is defined as change in velocity per unit time

$$\text{Average acceleration } (\vec{a}_\text{av}) = \frac{\text{change in velocity } (\Delta \vec{v})}{\text{time } \Delta t}$$

$$= \vec{v}_f - \vec{v}_i / \Delta t$$

where, v_i is initial velocity & v_f is final velocity of the body.

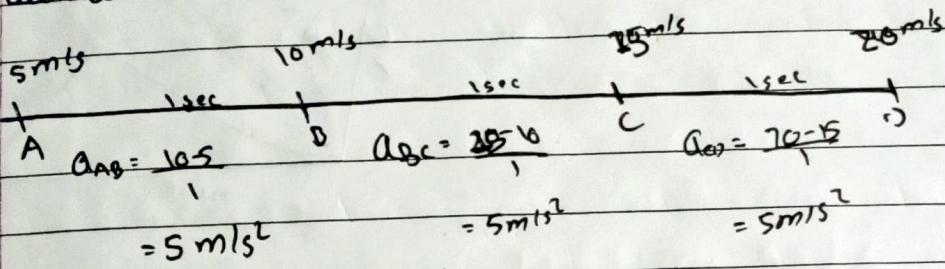
② Instantaneous acceleration :- If average acceleration is calculated for very short period of time then it is called instantaneous acceleration.

$$\text{Instantaneous acceleration } (\vec{a}_\text{in}) = \lim_{\Delta t \rightarrow 0} \vec{a}_\text{av}$$

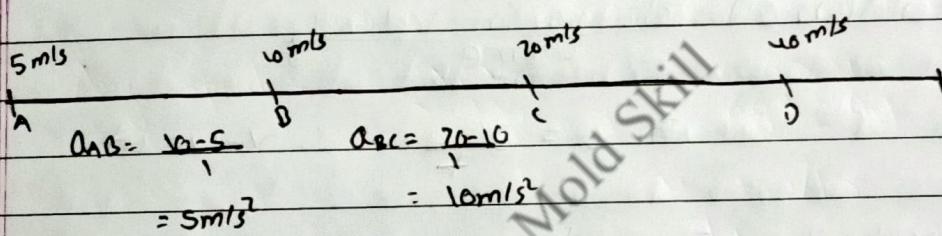
$$= \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d(\vec{v})}{dt}$$

$$\therefore \frac{d(\vec{v})}{dt} = \frac{d^2 \vec{s}}{dt^2}$$

III Uniform acceleration :- If change in velocity is same in equally interval of time, then a body is said to have uniform acceleration.



IV non-uniform acceleration :- If change in velocity is different in equally interval of time, then a body is said to have non-uniform acceleration



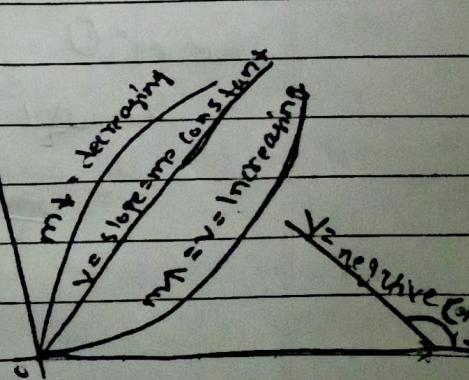
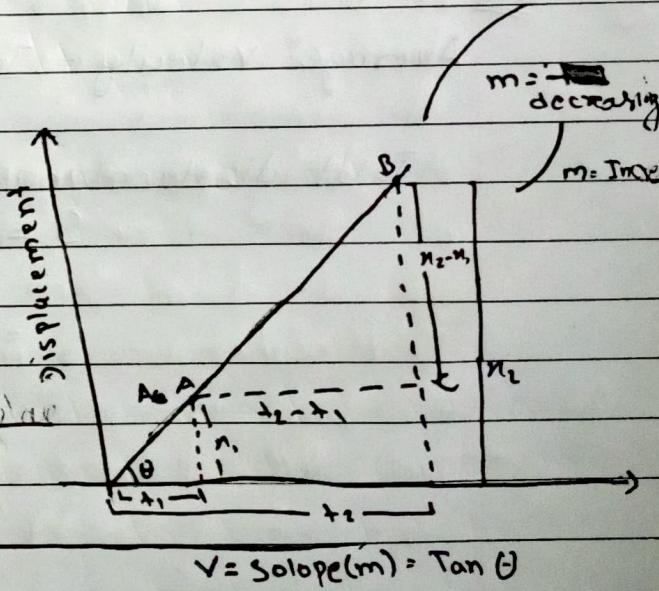
Displacement - time graph

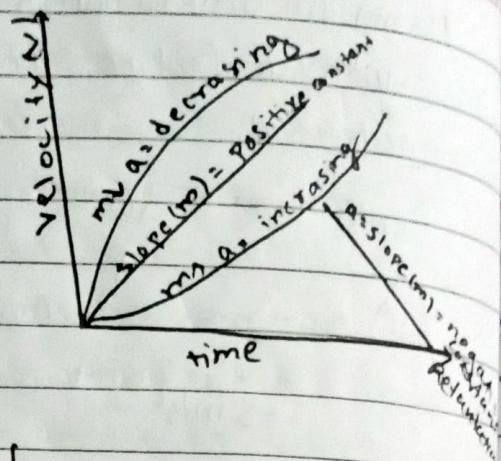
In ΔABC

$$\text{Slope } m = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$$

= change in displacement / time

= Velocity





Equation of uniformly accelerated motion

let us consider, a body moving uniform acceleration. Let u be the initial velocity & v be the velocity of body at time t then

- Acceleration (a) = $\frac{\text{change in velocity}}{\text{time}}$

$$\text{a. } a = \frac{v-u}{t} \quad \therefore v-u = at$$

Also

Average Velocity = Total displacement / time

Total displacement \equiv Average velocity \times time.

$$S = \frac{u+v}{2} \times t \quad \text{--- (1)}$$

$$= u + ut + \frac{at^2}{2} \times t$$

$$S = u + \frac{2ut + at^2}{2}$$

$$S = u + \frac{1}{2}at^2$$

From eqn (1)

$$S = \frac{u+v}{2} \times t$$

$$= \frac{u+v}{2} \times \frac{v-u}{a} = \frac{v^2 - u^2}{2a}$$

$$a = \frac{v-u}{t}$$

$$v^2 = u^2 + 2at$$

equation of uniformly accelerated motion - from velocity-time graph

let us consider a body is moving with uniform acceleration (a). let v be the initial velocity & V be the final velocity of the body at time t

From figure.

$$OA = \text{Initial velocity} = v$$

$$OB = \text{Final velocity} = V$$

$$OD = AC = \text{time interval (t)}$$

$$\textcircled{1} \bullet V = v + at \quad \text{In } \triangle ABC$$

$$\text{Slope } m = \frac{BC}{AC} = \frac{V-v}{t}$$

we know, Velocity time graph gives acceleration.

$$\text{acceleration} = \frac{BC}{AC} = \frac{V-v}{t}$$

$$\textcircled{2} \bullet a = \frac{V-v}{t}$$

$$\textcircled{3} \bullet S = vt + \frac{1}{2}at^2$$

we know velocity ~~vs~~ time graph gives acceleration (slope)

$$a = \frac{BC}{AC} = \frac{V-v}{t} = BC \text{ --- } \textcircled{1}$$

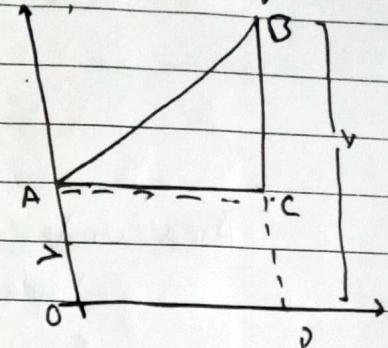
Area under velocity-time graph gives the displacement
Displacement = Area of triangle trapezium (GAB)

$$S = \text{Area of rectangle } OAC + \triangle ABC$$

$$= (OA) \times (OD) + \frac{1}{2} (AC) \times (BC)$$

$$= vt + \frac{1}{2} t \times at$$

$$= vt + \frac{1}{2} at^2$$



$$\Downarrow v^2 = u^2 + 2as$$

we know, velocity time graph gives acceleration,
acceleration = slope (m/s)

$$a = \frac{BC}{AC} \quad \text{or } AC = \frac{BC}{a} \quad \text{---(1)}$$

Area under velocity time graph gives displacement.

Displacement = Area of trapezium OADD + ~~Area of triangle~~

$$S = \frac{1}{2}(OA + BD) \times AC$$

$$S = \frac{1}{2}(u+v) \times BA$$

$$S = \frac{v+u}{2} \times \frac{v-u}{a} \Rightarrow \frac{v^2 - u^2}{2a}$$

question: A body starts from a point and moves with constant acceleration 2 m/s^2 . Find distance travelled by body in 6th second.

Ques: A body is dropped from the top of tower. If it covers half of the total height in last second. Then find the height of the tower.

Ques: A body is dropped from the top of tower of $h=60\text{m}$ at the same time another object B projected vertically upward from the foot of the tower with 20 m/s find where & when they meet each other.

Ques: A body is projected vertically upward with 30 m/s . Find the distance covered by the object in last one s.

Distance travelled by body (A) in particular second.
(i.e t^{th} second)

Let us consider a truck, having mass (m), Initial velocity (v) and uniform acceleration (a)

Let, s_t , s_{t-1} and s_{th} be the distance travelled by body in t sec, $(t-1)$ sec & t^{th} sec respectively.

$$s_t = v_t + \frac{1}{2} a t^2$$

$$s_{t-1} = v_{t-1} + \frac{1}{2} a (t-1)^2$$

From figure,

$$s_{th} = s_t - s_{t-1}$$

$$= v_t + \frac{1}{2} a t^2 - \{ v_{t-1} + \frac{1}{2} a (t-1)^2 \}$$

$$= v_t + \frac{a t^2}{2} - \{ v_t - v + \frac{a}{2} (t^2 - t^2 + 2t - 1) \}$$

$$= v_t + \frac{a t^2}{2} - \cancel{v_t} - v + \frac{a t^2}{2} - \frac{a t}{2} + \frac{a}{2}$$

$$= \cancel{v_t} + \frac{a t^2}{2} - \cancel{v_t} + v - \frac{a t^2}{2} + 2 \cancel{\frac{a t}{2}} - \frac{a}{2}$$

$$s_{th} = v_t + \frac{a t^2}{2}$$

$$s_{th} = v t + \frac{a t^2}{2}$$

$$= v t + \frac{a}{2} (t^2 - 1)$$

$$m = \frac{m}{s} \times s + m s^2 (s-s)$$

$$(m) = [m + m s^{-2} \times s^2]$$

$$= [m, m]$$

$$= [m]$$

Horizontal Velocity = constant //

Projectile Motion,

The object which is projected into air and moves under the action of gravity is called projectile and its motion is called motion.

The path of projectile is called trajectory.
for ex: a stone thrown into space. a bullet fired from gun.

Assumption used in projectile motion.

- The air resistance is neglected
- The effect due to rotation of earth and its curvature is negligible
- At all points of trajectory, the acceleration due to gravity is constant both in magnitude & direction.

Projectile fired at an angle (θ) from a ground

let, us suppose a projectile is fired initial velocity (v) making an angle (θ) with horizontal where

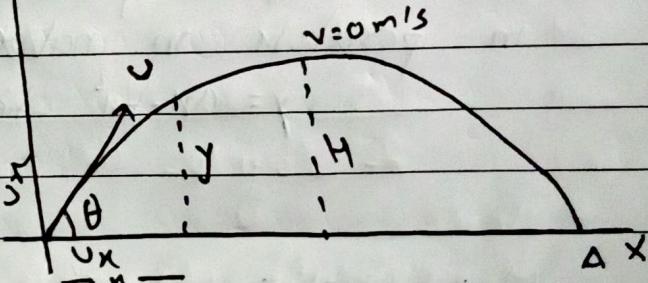
$$v_x = v \cos \theta \quad \& \quad v_y = v \sin \theta$$

between horizontal & vertical velocity

Horizontal velocity remains constant throughout the motion of projectile

Since, horizontal acceleration ($a_x = 0$) but vertical velocity does not remain constant.

Since, vertical acceleration ($a_y = \text{acceleration due to } g$)



Equation of the trajectory.

Let us suppose, a projectile is at point (x) after time (t). If x & y be the horizontal & vertical distance after time (t).

$$\text{Horizontal distance } (S_x) = v \cos \theta t + \frac{1}{2} a x t^2$$

$$x = v \cos \theta t + \frac{1}{2} \times 0 \times t^2,$$

$$\text{or } x = v \cos \theta t \quad : \quad t = \frac{x}{v \cos \theta} \quad \text{--- (1)}$$

$$\text{Vertical distance } (S_y) = v \sin \theta t + \frac{1}{2} a y t^2$$

$$y = v \sin \theta t - \frac{1}{2} g t^2 \quad \text{--- (2)}$$

$$\text{From (1) & (2)} \quad y = v \sin \theta \cdot \frac{x}{v \cos \theta} - \frac{1}{2} g \left(\frac{x}{v \cos \theta} \right)^2$$

$$y = \tan \theta x - \frac{g}{2 v^2 \cos^2 \theta} x^2$$

which is eqn of parabolic, hence the path of projectile is parabolic in nature.

$$y = ax - bx^2 \text{ where } a = \tan \theta, b = \frac{g}{2 v^2 \cos^2 \theta}$$

Max height =

The maximum vertical height attained by the projectile is maximum height. In fig H, maximum height.

$$Y_y = 0 \text{ m/s}$$

$$Y_y^2 = U_y^2 + 2ay S_y$$

$$0 = U_y^2 - 2g H$$

$$\therefore U_y^2 = 2g H$$

$$\therefore H = \frac{U_y^2}{2g} = \frac{U^2 \sin^2 \theta}{2g}$$

Maximum height attained by the projectile depends on initial vertical velocity.

Time to reach max height (known as Ascent)

$t = \text{max time to reach max height}$

$$Y = 0 \text{ m/s}$$

$$V_y = V_y + gt$$

$$0 = V \sin \theta - gt$$

$$\therefore V \sin \theta = gt \quad (\text{Ascent})$$

Time to reach maximum height depends upon initial vertical velocity

- Total time of flight (T)

Time for which projectile remain in air is total time of flight (T)

time taken by the projectile to reach point A from O is (total time of flight) the value of y is 0 at A.

we have,

$$y = V_y t - \frac{1}{2} g t^2$$

$$\text{or } 0 = V_y t - \frac{1}{2} g t^2, \quad \text{or. } V_y t = \frac{1}{2} g t^2$$

$$V \sin \theta = \frac{gt}{2} \quad \therefore t = \frac{2V \sin \theta}{g}$$

The horizontal distance covered by a projectile during its time of flight. (R)

$y \uparrow$

From figure:

OA = Horizontal range

(in meter)

$$a) S_n = V_n t + \frac{1}{2} a_n t^2$$

$$a) R = V \cos \theta t + \frac{1}{2} \times 0 \times t^2$$

$$R = V \cos \theta \times \frac{2V \sin \theta}{g}$$

$$\therefore R = \frac{2V^2 \sin \theta \cos \theta}{g} = \frac{V^2 \sin 2\theta}{g}$$

① Special case

for maximum horizontal range for given velocity of projection value of $\sin 2\theta$ should be maximum

$$R = \frac{v^2 \sin 2\theta}{g}$$

$$\sin 2\theta = 1$$

$$\sin 2\theta = \sin 90^\circ$$

$$\theta = 45^\circ$$

Thus, angle of projection for maximum horizontal range is 45° .

$$\text{Maximum horizontal range } R_{\max} = \frac{v^2}{g},$$

For minimum horizontal range for given velocity of projection value of $\sin 2\theta$ must be, -1,

$$\text{we have } R = \frac{v^2 \sin 2\theta}{g}$$

$$\sin 2\theta = -1$$

$$2\theta = 180^\circ$$

$$\therefore \theta = 90^\circ$$

$$R = \frac{v^2 \sin 2\theta}{g}$$

$$R = v^2 \min (\sin 2\theta)_{\max}$$

$$R = \frac{v^2 \min}{g}$$

$$v_{\min} = \sqrt{Rg}$$

| This is required expression to express that, minimum velocity

Another angle of projection for same horizontal range
 i.e. another angle of projection (θ_1) = $90 - \theta$
 Horizontal range (R_1) = $\frac{v^2 \sin 2\theta}{g}$

$$= \frac{v^2 \sin(90 - \theta)}{g}$$

$$= \frac{v^2 \sin(180 - 2\theta)}{g}$$

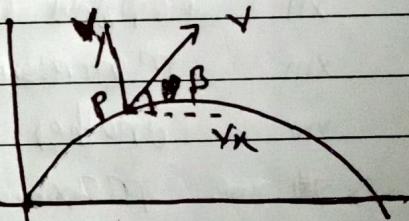
$$= \frac{v^2 \sin 2\theta}{g}$$

Thus another angle of projection
 for same horizontal range is $90 - \theta$

- Velocity of the projectile at any instant.

i.e., V be the velocity of projectile after time t . V_x & V_y be the horizontal and vertical velocity of the projectile respectively after time t .

$$\begin{aligned} \text{we have } V_x &= V_0 \cos \theta + a_x t \\ &= V_0 \cos \theta + 0 \times t \\ &= V_0 \cos \theta - 0 \end{aligned}$$



$$\begin{aligned} \text{Also, } V_y &= V_0 \sin \theta + a_y t \\ &= V_0 \sin \theta - g t - 0 \quad \text{--- (ii)} \end{aligned}$$

Magnitude of the resultant velocity is

$$\begin{aligned} V &= \sqrt{(V_x)^2 + (V_y)^2} \\ &= \sqrt{(V_0 \cos \theta)^2 + (V_0 \sin \theta - gt)^2} \end{aligned}$$

Let β be the angle between resultant velocity and horizontal then $\tan \beta = \frac{V_y}{V_x} = \frac{V_0 \sin \theta - gt}{V_0 \cos \theta}$

$$\beta = \tan^{-1} \left(\frac{V_0 \sin \theta - gt}{V_0 \cos \theta} \right)$$

projectile fired from a top of tower (Horizontally)

Let us suppose a projectile is projected horizontally with a velocity (v).

$v_h = v$ = horizontal velocity of projectile which remains constant throughout the motion. Since horizontal acceleration (a_h) = 0.

$v_y = \cos 90^\circ = 0$ is initial vertical velocity which increases with time, since acceleration vertically acceleration (a_y) = (g) .

equation of trajectory.

Let us suppose a projectile is at point P after time t . x & y give the horizontal & vertical distance of projectile respectively after time t

$$\text{Horizontal distance } s_h = v_h t + \frac{1}{2} a_h t^2$$

$$x = v t + \frac{1}{2} \times 0 \times t^2$$

$$t = x - 0$$

Vertical distance

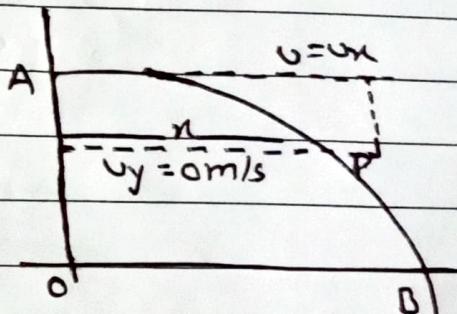
$$y = v_y t + \frac{1}{2} a t^2$$

$$y = v t + \frac{1}{2} (g) (\frac{x}{v})^2 \quad \text{from eq ①}$$

$$y = \frac{1}{2} g \frac{x^2}{v^2} \quad || \quad y = \frac{g}{2v^2} \times x^2$$

$y = ax^2$
 $a = \frac{g}{2v^2}$

which is equation of parabola. hence path of projectile is parabolic in nature.



- Time of flight

Time for which projectile remain in air is time of flight. (time of descent)

From figure time take by the projectile to reach point B from point A, is time of flight.
we have

$$S_y = V_y t + \frac{1}{2} a_y t^2$$

$$h = 0x + \frac{1}{2} (g) t^2$$

$$\frac{g t^2}{2} = h \\ u \quad t = \sqrt{\frac{2h}{g}}$$

- Horizontal range

The horizontal distance covered by the projectile during its time of flight is (Horizontal Flight) (R)

From figure OB is horizontal range.

we have

$$S_x = V_x t + \frac{1}{2} (a_x) t^2$$

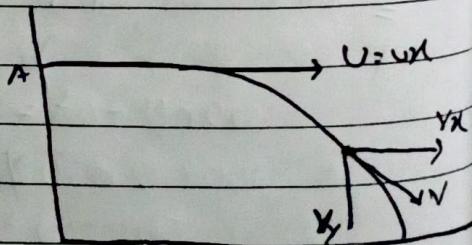
$$R = V_x t$$

$$R = V \cos \theta \times \{ \text{is required eqn} \}$$

$$R = V \cos \theta \times \sqrt{\frac{2h}{g}}$$

- Velocity of the projectile at any instant

let V be the velocity of the projectile after time (t). V_x & V_y be the horizontal & vertical velocity of the projectile after time (t)
we have



$$V_n = V_x + a_{xt}$$

$$= V + a_{xt}$$

$$\therefore V_n = V - \text{---} \textcircled{1}$$

Also, $V_y = V_y + a_{yt}$

$$V_y = 0 + gt \text{ --- } \textcircled{2}$$

magnitude of resultant velocity is

$$V = \sqrt{V_n^2 + V_y^2}$$

$$= \sqrt{V^2 + (gt)^2}$$

$$\therefore V = \sqrt{V^2 + g^2 t^2}$$

Let θ be the angle made by the resultant velocity with horizontal, then, $\tan \theta = \frac{V_y}{V_n}$

$$\theta = \tan^{-1} \left(\frac{gt}{V} \right)$$

Ques A projectile is projected horizontally from the top of tower of height 45m with 10 m/s. calculate

i) time of descent

$$T = \sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2 \times 45}{10}} = 3 \text{ sec}$$

ii) horizontal & vertical distance after 1 sec.

iii) horizontal range

iv) Velocity of the projectile after 1 sec.

v) Velocity of the projectile with which it strike the ground

for vi) horizontal distance

$$V_x = V \cos \theta + + \frac{1}{2} (a) t^2$$

$$V_x = 10 \text{ m/s}$$

Vertical distance,

$$V_y = V \sin \theta + + \frac{1}{2} g t^2$$

$$= \frac{1}{2} \times 10 (3)^2$$

$$= 5 \text{ m}$$

(W)

Relative Velocity

Relative velocity of an object A with respect to the object B is rate of change of position with respect to B.

If \vec{v}_A be the velocity of A with respect to the ground & \vec{v}_B be the velocity of an object with respect to the ground. the relative velocity of an object A with respect to object B is :-

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \\ = \vec{v}_A + (-\vec{v}_B)$$

Relative velocity of an object B is with respect to an object A is :

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A \\ = \vec{v}_B + (-\vec{v}_A)$$

$$\vec{v}_{BA} = |-\vec{v}_{AB}|$$

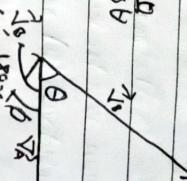
Relative velocity of A with respect to B.

$$\begin{aligned}\vec{v}_{AB} &= \vec{v}_A - \vec{v}_B \\ &= \vec{v}_A + (-\vec{v}_B)\end{aligned}$$

observer,

Magnitude of \vec{v}_{AB} is,

$$|v_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_A \cdot v_B \cos \theta}$$



Let ϕ be the angle made by

v_{AB} vector is v_A vector then

$$\phi = \tan^{-1} \left(\frac{v_B \sin \theta}{v_A - v_B \cos \theta} \right)$$

Special case if $\theta = 0$.

A & B are moving along same direction

$$\begin{aligned}\vec{v}_{AB} &= \sqrt{v_A^2 + v_B^2 - 2v_A \cdot v_B \cos 0} \\ &= \sqrt{v_A^2 + v_B^2 - 2v_A v_B} \rightarrow v_B \\ &= \sqrt{v_A - v_B},\end{aligned}$$

$\theta = 180$

A & B are moving along opposite direction.

$$\vec{v}_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos 180^\circ}$$

$\Rightarrow v_A + v_B$, Maximum

$\theta = 90$

A & B are moving perpendicularly

$$\sqrt{v_A^2 + v_B^2} = \sqrt{v_A^2 + v_B^2 - 2v_A \cdot v_B \cos 90^\circ}$$

$$= \sqrt{v_A^2 + v_B^2}$$

$$\tan \phi = \frac{v_B \sin 90^\circ}{v_A - v_B \cos 90^\circ} = \frac{v_B}{v_A},$$

$$\therefore \phi = \tan^{-1} \left(\frac{v_B}{v_A} \right)$$

He should hold his umbrella at an angle
 # River boat problem:- (पानी की वेग समस्या)

let \vec{V}_b = Velocity of boat in still water.

\vec{V}_r = Velocity of water

\vec{V}_{br} = Resultant velocity of boat in river.

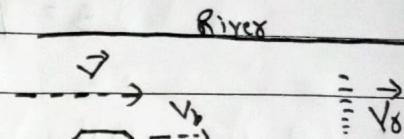
$$V = V_b + V_r$$

Distance travelled by boat in t time

$$\text{Def } V = \frac{S}{t}$$

$$\therefore S = Vt$$

$$= (V_b + V_r) t$$



Upstream: \vec{V}_b is antiparallel to \vec{V}_r

Resultant velocity of boat: $V = V_b - V_r$

Distance travelled by boat in time t time

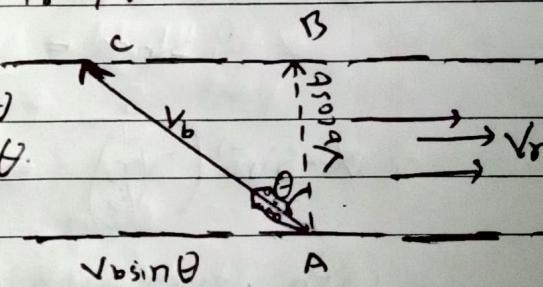
$$S = Vt$$

$$= (V_b - V_r) t$$

$$V = \frac{S}{t}$$

Time to cross the river

The component of V_b (i.e. $V_b \sin \theta$) is cancelled by V_r & $V_b \cos \theta$ responsible for boat to reach the opposite bank.



Time to cross :-

$$t = \frac{AB}{V_b \cos \theta}$$

Minimum time to cross the

For minimum time to cross the river, the value of $\cos \theta$ should be maximum that is

$$\cos \theta = 1$$

$\theta = 0^\circ$ with vertical or 90° with bank bank or current of water.

