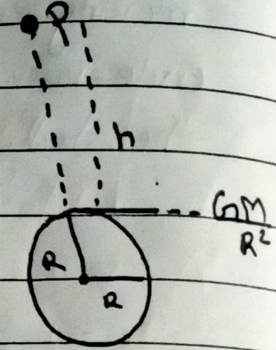
Gravitation

ⓐ Variation with altitude/height.

let us suppose a spherical shape earth of Radius 'R' and mass (M). the accl. due to gravity at the surface of the earth is given by:-



$$g = \frac{GM}{(R)^2} \quad \text{---(I)}$$

At poin 'p' let us suppose a small body of (m) at poin 'p' which is at a distance of (R+h) from centre of the earth. The accleration due to gravity at point P is given by

$$g' = \frac{GM}{(R+h)^2} \quad \text{---(II)}$$

dividing eq^n (II) by eq^n (I)

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \times \frac{(R)^2}{GM}$$

$$\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2$$

$$\frac{g}{g'} = \left(\frac{R}{R} + \frac{R}{h}\right)^2$$

$$\frac{g}{g'} = (1 + R/h)^{-2}$$

$$\frac{g'}{g} = \frac{1}{\left(\frac{R+h}{R}\right)^2}$$

$$\frac{g'}{g} = \left(\frac{R}{R} + \frac{h}{R}\right)^{-2}$$

Here, $h << R$

$$\therefore \frac{g'}{g} = \left(1 + \frac{h}{R}\right)^{-2}$$

or $\frac{g'}{g} = 1 - \frac{2h}{R}$ (using binomial expansion)

$$g' = g\left(1 - \frac{2h}{R}\right) \quad \text{... req eq}^n$$

This required exp^n of acceleration due to gravity with height acceleration due to gravity decrease when height increase from surface

If the accleration due to gravity on the surface of earth is 9.8 m/s^2, what will accleration due to gravity on the surface of planet whose mass and radius both are 2 times ~~are~~ corresponding quanties for the erth,

For earth $g = \frac{GM}{r^2}$

For unknow planet
mass $= 2M$ and radius $= 2r$ then

$$g' = \frac{G\,2M}{(2r)^2}$$

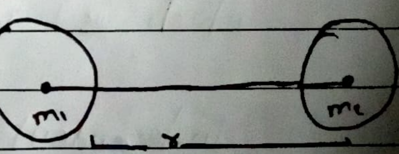
$$g' = \frac{2}{4}\,\frac{GM}{r^2}$$

$$= \frac{2}{4} \times 9.8$$

$$= 4.9$$

Newton's law of gravitation.

It state ~~that~~ the ~~for~~ every particle in the universe expirences a force of attraction that is directly proportiona to product of their masses and inversely proportion to the square of distance between their masses.



let us consider ~~two~~ body having masses m_1 & m_2 respectively separated by the distance 'r', let us suppose (F) be the force between them.

By statement,

$$F \propto m_1 m_2 \quad \text{---} (i)$$

$$F \propto 1/r^2 \quad \text{---} (ii)$$

Combining eq^n (i) & (ii)

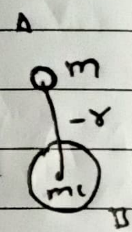
$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = \frac{GMM_1}{r^2} \quad [\text{req eq}^n]$$

wher G is the proportanality constant know as gravitation constant.

for $g = \frac{GM_1}{r^2}$

let us consider a body having M_1 & small body having (m).



By the newtons law of gravitation, force bet^n body A & B is given by

$$F = \frac{GM_1 m}{r^2} \quad \text{—(I)}$$

By the newton's 2nd law of motion.

$$F = m \times g \quad \text{—(II)}$$

from (I) & (II)

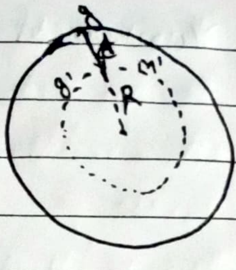
$$m \times g = \frac{GM_1 m}{r^2}$$

$$g = \frac{GM_1}{r^2}$$

which is required eq^n of gravity due to accleration,
It also state that acceleration due to gravity independent of falling object.

V of sphere $= \frac{4}{3}\pi R^3$

Variation with depth.



let us suppose a spherical shaped earth of radius 'R', mass 'M' and density ρ. the accleration due to gravity on surface of earth is

$$g = \frac{GM}{R^2} \qquad [M = \rho \times V]$$

$$g = \frac{G\rho}{R^2} \times \frac{4}{3}\pi R^3$$

$$g = \frac{4}{3} RG\rho\pi \quad \text{--- (i)}$$

let us suppose a small body of mass 'm' is at dept d from the surface of the earth where eqivalent mass of earth is (M')

$$g' = \frac{GM}{r^2} \quad [\text{where, } (R-d)^2 = r^2] \text{ from figure.}$$

$$g' = \frac{G\rho}{(R-d)^2} \times \frac{4}{3}\pi (R-d)^3$$

$$g' = \frac{4}{3} G\rho\pi (R-d) \quad \text{--- (ii)}$$

Dividing relation (ii) by (i)

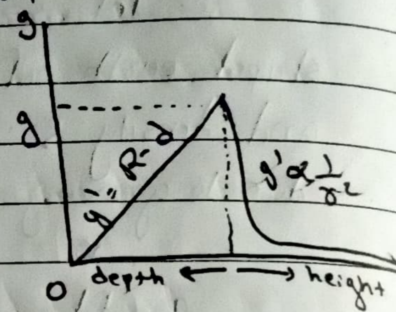
$$\frac{g'}{g} = \frac{4}{3} G\rho\pi (R-d) \times \frac{3}{4 G\rho\pi R}$$

$$\frac{g'}{g} = \frac{R-d}{R}$$

$$g' = g\left(\frac{R-d}{R}\right) \quad g' = g\left(1 - \frac{d}{R}\right)$$

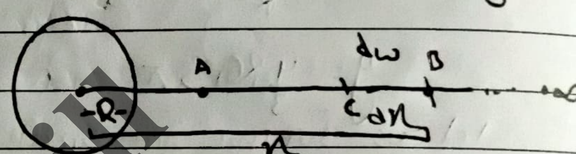
This the required expression of variation of acc. due to earth depth. It is found that acc due g decreases on depth from its surface,

Varition of g with depth & height.



Gravitational Potential

The abamaint of work done required to bring a a unit mass from infinity to a point is called gravitational potential of a body at that point it is also defined as potential energy required to bring from infinity to that point [redacted] ~~gravit~~ potential energy per unit mass



let us suppose spherical shaped earth of mass m and radius (r), let us suppose a unit mass is at distance r from center of the earth.

The gravitational force between earth at that unit body is given by

$$F = \frac{GMm}{(R)^2} \quad ; m=1 \text{ and } R=r..$$

$$= \frac{GM}{r^2}$$

The Small amount of work done to bring a unit from b to c with displacement dr is given,

$$dw = F \times d$$

$$= \frac{GM}{r^2} \times dr,$$

Intregating on both side $\int_{\infty}^{R}$

$$\int_{\infty}^{R} dw = \int_{\infty}^{R} \frac{GM}{x^2} \times dx,$$

$$dw = GM \int_{\infty}^{R} \frac{dx}{x^2}$$

$$= GM \int_{\infty}^{R} x^{-2} dx$$

$$= GM \left[\frac{x^{-2+1}}{-2+1} \right]_{\infty}^{R}$$

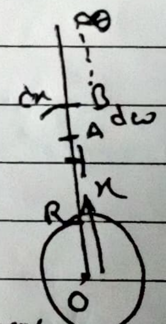
$$= GM \left[-\frac{1}{R} \right]_{\infty}^{R} = GM \left[-\frac{1}{R} - \frac{1}{\infty} \right]$$

$$dw = \frac{GM}{R}$$

\# Escape Velocity

The minimum velocity with which a body most be projected upward so, that it can escape into space by overcoming gravitational force or field.

let us suppose, a Spheric earth of radius (r) & mass (M) let us suppose a small body of mass (m) is at a distance of x from centre of the earth.



let dw be small work done due to displacement dx. the gravitation force at point A. be

$$F = \frac{GM}{(r)^2} \cdots \frac{GM}{x^2} \quad \text{--(1)}$$

The small work done is given by

$$U = f \times d$$

$$U = f \times dx \quad \text{Intergating on both side}$$

$$\int_{R}^{\infty} U = \int_{R}^{\infty} \frac{GMm}{x^2} \times dx$$

$$U = GMm \int_{R}^{\infty} x^{-2} dx \quad \cdot \quad U = GMm \left[\frac{x^{-2+1}}{2+1} \right]_{R}^{\infty}$$

$$\therefore U = GMm \left[-\frac{1}{x} \right]_{R}^{\infty}$$

$$U = GMm\left[-\frac{1}{\infty} + \frac{1}{R}\right]$$

$$U = \frac{GMm}{R}$$

This is the required eq^n of potential.

~~Escape velocity~~

let V_e be the velocity of body. The kinetic energy of body is given by $\frac{1}{2}mv^2$,

By conservation of energy,

$$\frac{GM\cancel{m}}{R} = \frac{1}{2}\cancel{m}v_e^2$$

or $$\frac{2GM}{R} = v_e^2$$

$$\therefore v_e = \sqrt{\frac{2GM}{R}} \quad \text{---} (5)$$

Also we have $g = \frac{GM}{R^2}$ $\therefore GM = gR^2$ ... (6)

Substituting the value of gR^2 in eq^n (5)

$$v_e = \sqrt{\frac{2gR^2}{R}}$$

$$\therefore v_e = \sqrt{2gR} \quad \text{---} (7)$$

This the required expression of escape velocity.

It doesn't depend on angle of projection.

// for earth

$g = 9.8$ $R = 6400$ km

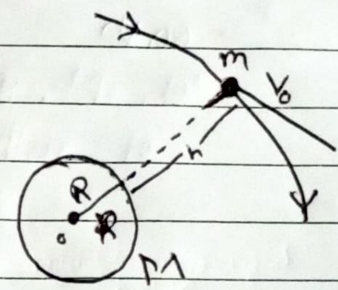
$$v_e = \sqrt{2 \times 9.8 \times 6400 \times 10^{-3}}$$

$$= 11.2 \text{ km/s}$$

$$V_e = \sqrt{2}\, V_o$$

Orbital Velocity

The orbital velocity is defined as the minimum velocity required to maintain a satellite into a stable orbit



A satellite of mass (m) is moving in a circular orbit with radius $r = (R + h)$ with orbital velocity (V_o). let us suppose (M) be mass of earth (R) radius

The gravitational force provides necessary centripetal force to keep in a circular path

ie $F_g = F_c$

$$\frac{GMm}{r^2} = \frac{m v_o^2}{r}$$

$$\frac{GM}{r+h} = (V_o)^2$$

$$\text{or } V_o = \sqrt{\frac{GM}{r+h}} \quad \text{---(1)}$$ when h is height of satellite.

This is required eq^n of orbital velocity.