

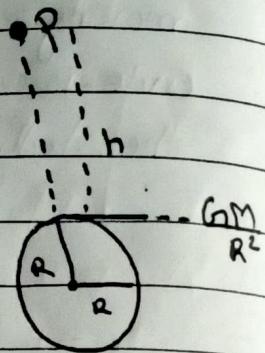
# GRAVITATION

@ Variation with altitude/height.

Let us suppose a spherical shape earth of Radius 'R' and mass 'M'.

The accl. due to gravity at the surface of the earth is given by:-

$$g = \frac{GM}{(R)^2} \quad \text{--- (1)}$$



At point 'P' let us suppose a small body of (m) at point 'P' which is at a distance of  $(R+h)$  from centre of the earth. The acceleration due to gravity at point P is given by

$$g' = \frac{GM}{(R+h)^2} \quad \text{--- (2)}$$

Dividing eqn (2) by eqn (1)

$$\frac{g'}{g} = \frac{GM}{(R+h)^2} \times \frac{(R)^2}{GM}$$

$$\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2$$

$$\frac{g'}{g} = \frac{1}{\left(\frac{R+h}{R}\right)^2}$$

$$\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2$$

$$\therefore \frac{g'}{g} = \left(1 + \frac{h}{R}\right)^{-2}$$

$$\text{or } \frac{g'}{g} = 1 - \frac{2h}{R} \quad (\text{using binomial expansion})$$

$$g' = g \left(1 - \frac{2h}{R}\right) \quad \text{--- req'd eqn.}$$

This required exp<sup>n</sup> of acceleration due to gravity with height alteration due to gravity decrease when height increase from small

If the acceleration due to gravity on the surface of earth is  $9.8 \text{ m/s}^2$ , what will acceleration due to gravity on the surface of planet whose mass and radius both are 2 times corresponding quantities for the earth,

$$\text{For earth } g = \frac{GM}{r^2}$$

For unknown planet

mass =  $2M$  and radius =  $2r$  then

$$g' = \frac{G \cdot 2M}{(2r)^2}$$

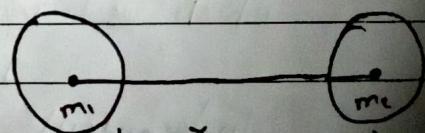
$$g' = \frac{2}{4} \frac{GM}{r^2}$$

$$= \frac{1}{2} \times 9.8$$

$$= 4.9$$

## # Newton's law of gravitation.

It states "The for every particle in the universe experiences a force of attraction that is directly proportional to product of their masses and inversely proportional to the square of distance between their masses."



Let us consider two bodies having masses  $m_1$  &  $m_2$  respectively separated by the distance ' $x$ ', let us suppose ( $F$ ) be the force between them.

By statement,

$$F \propto m_1 m_2 \dots \textcircled{1}$$

$$F \propto \frac{1}{x^2} \dots \textcircled{2}$$

Combining eqn 1 & 2

$$F \propto \frac{m_1 m_2}{x^2}$$

$$\therefore F = \frac{GMm}{r^2} \quad [\text{req eqn}]$$

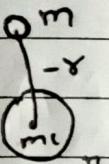
where,  $G$  is the proportionality constant known as gravitational constant.

# For  $g = \frac{GM}{r^2}$

Let us consider a body having  $M$ , & small body having  $m$ .

By the Newton's law of gravitation, force bet. body A & B is given by.

$$F = \frac{GMm}{r^2} \quad \text{--- (1)}$$



By the Newton's 2nd law of motion.

$$F = mxg \quad \text{--- (2)}$$

From (1) & (2)

$$mxg = \frac{GMm}{r^2}$$

$$g = \frac{GM}{r^2}$$

which is required eqn of gravity due to acceleration,

It also state that acceleration due to gravity independent of falling object.

$$V \text{ of sphere} = \frac{4}{3} \pi R^3$$

# Variation with depth.

Let us suppose a spherical shaped earth of radius 'R', mass 'M' and density  $\rho$ . The acceleration due to gravity on surface of earth is

$$g = \frac{GM}{R^2} \quad [M = \rho \times V]$$

$$g = \frac{G\rho}{R^2} \times \frac{4}{3} \pi R^3$$

$$g = \frac{4}{3} G \rho \pi \dots \textcircled{1}$$

Let us suppose a small body of mass 'm' is at dept 'd' from the surface of the earth where equivalent mass of earth is ' $M'$ '

$$g' = \frac{GM'}{r^2} \quad [\text{where, } (R-d)^2 = r^2] \text{ from figure.}$$

$$g' = \frac{G\rho}{(R-d)^2} \times \frac{4}{3} \pi (R-d)^3$$

$$g' = \frac{4}{3} G \rho \pi (R-d) \dots \textcircled{2}$$

Dividing relation  $\textcircled{2}$  by  $\textcircled{1}$

$$\frac{g'}{g} = \frac{\frac{4}{3} G \rho \pi (R-d)}{\frac{4}{3} G \rho \pi R}$$

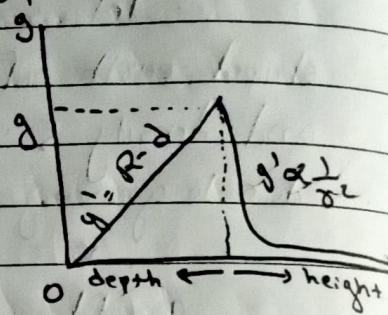
$$\therefore \frac{g'}{g} = \frac{R-d}{R}$$

$$g' = g \left( \frac{R-d}{R} \right) \quad g' = g \left( 1 - \frac{d}{R} \right)$$

This is the required expression of variation of acc. due to depth. It is found that acc due to g decreases on depth from its surface,



Variation of  $g$  with depth & height



### # Gravitational Potential

The amount of work done required to bring a unit mass from infinity to a point is called

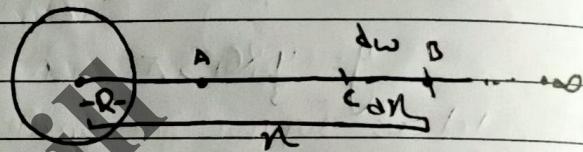
gravitational potential of a

body at that point it is also

defined as potential energy required to bring

from infinity to that point is ~~unit~~ gravitational potential

energy per unit mass



Let us suppose spherical shaped earth of mass  $m$  and radius  $R$ , let us suppose a unit mass is at distance  $r$  from center of the earth.

The gravitational force between earth at that unit body is given by

$$F = \frac{GMm}{r^2} : m=1 \text{ and } R=r..$$

$$= \frac{GM}{r^2}$$

The small amount of work done by bringing a unit from  $b$  to  $c$  with displacement  $dr$  is given,

$$dw = F \cdot d$$

$$= \frac{GM}{r^2} \times dr$$

Integrating on both sides

$$\int_{\infty}^R dw = \int_{\infty}^R GM \frac{dx}{x^2} \times dx,$$

$$dw = GM \int_{\infty}^R \frac{dx}{x^2}$$

$$= GM \int_{\infty}^R x^{-2} dx$$

$$= GM \left[ \frac{x^{-2+1}}{-2+1} \right]_{\infty}^R$$

$$= GM \left[ -\frac{1}{x} \right]_{\infty}^R = GM \left[ -\frac{1}{R} - \frac{1}{\infty} \right]$$

$$dw = \frac{GM}{R}$$

### \* Escape Velocity

The minimum velocity with which a body must be projected upward so that it can escape into space by overcoming gravitational force or field.

Let us suppose a spherical earth of radius ( $R$ ) & mass ( $M$ ) let us suppose a small body of mass ( $m$ ) is at a distance of  $x$  from centre of the earth.

Let  $dw$  be small work done due to displacement  $dx$ . the gravitation force at point A. be  $F$

$$F = \frac{GM}{x^2} \dots \frac{GM}{x^2} - 0$$

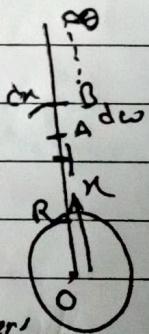
The small work done is given by

$$v = F \times d$$

$$\int_R^{\infty} v = \int_R^{\infty} \frac{GMm}{x^2} dx \quad \text{Integrating on both m}$$

$$v = \int_R^{\infty} \frac{GMm}{x^2} dx = v = GMm \left[ \frac{x^{-2+1}}{-2+1} \right]_R^{\infty}$$

$$\therefore v = GMm \left[ -\frac{1}{x} \right]_R^{\infty}$$



$$U_{\infty} = \frac{1}{R}$$

$$U = -\frac{GMm}{R}$$

This is the required eqn of potential.

# escape velocity

Let  $v_e$  be the velocity of body. The kinetic energy of body is given by  $\frac{1}{2}mv^2$ ,

By conservation of energy,

$$\frac{1}{2}mv_e^2 = \frac{1}{2}mv^2$$

$$\text{or } \frac{GMm}{R} = v_e^2$$

$$\therefore v_e = \sqrt{\frac{GM}{R}} \quad \text{--- (5)}$$

$$\text{Also we have } g = \frac{GM}{R^2} \quad \therefore GM = gR^2 \quad \text{--- (6)}$$

Substituting the value of  $gR^2$  in eqn (5)

$$v_e = \sqrt{\frac{gR^2}{R}}$$

$$\therefore v_e = \sqrt{gR} \quad \text{--- (7)}$$

This the required expression of escape velocity.  
It doesn't depend on angle of projection.

For earth

$$g = 9.8 \quad R = 6400 \text{ km}$$

$$v_e = \sqrt{2 \times 9.8 \times 6400 \times 10^3}$$

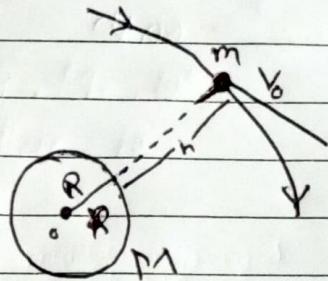
$$\approx 11.2 \text{ km/s}$$

$$V_o = \sqrt{2} V_0$$

## # Orbital Velocity

The orbital velocity is defined as the minimum velocity required to maintain a satellite into a stable orbit.

A satellite of mass ( $m$ ) is moving in a circular orbit with radius  $r$  ( $R+h$ ) with orbital velocity ( $V_o$ ). Let us suppose  $M$  be mass of earth ( $R$ )



The gravitational force provides necessary centripetal force to keep it in a circular path

$$\text{ie } F_g = F_c$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GM}{r+h} = \frac{v^2}{r}$$

$$\therefore V_o = \sqrt{\frac{GM}{r+h}} \quad \text{when } h = \text{height of satellite.}$$

This is required eqn of orbital velocity.