Vector

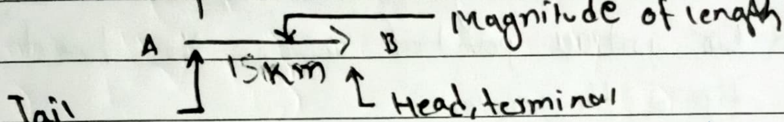
Representation of Vector :-

⇒ Algebric Representation Direction

- accleration = $\vec{a}$ ← Magnitude

Mgnitude of $\vec{a}$ = $|\vec{a}|$ = a,

⇒ Geometric Representaiton.



Scalr quantity :

The physical quantity which have only magnitude but no direction is called Scalar quantity. The Scalr quantity is represented by Simple alphabetical letter. for ex Mass can be represented by 'm'

Scalar quantity obey simple algebric rule for ex mass, length, time, temperature, Speed, Current, energy. Area, Volume, density, Pressure, angle, electric flux, magnetic flux Volume flux, liquid flux, etc.

Vector quantity :-

This physical quantity which have both magnitude & direction are called Vector quantity. Necessary Condition of vector is direction. Sufficient Condition of vector is Vector rule. Example of Vector quantity are :- Displacement, Velocity, accelerati momentum, impulse, force, angular displacement, angular velocity, angular acceleration, angular momentum, angular torque, current density, current element, electricfield density, gravitional intensity. magnetic field intensity. electric flux density, magnetic flux density. magnetic vector potential...alimentory length, alimentry area, alimentory angle etc. (gradient of scalar are always vector)

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Representation of Vector,

Algebric representation :- Vector are represented by simple alphabet with arrow over it. for acceleration : $\vec{a}$

Geometric representation:- Vector are represented by a straight line with arrow head. In geometric representation tail 'o' is called origin or intial point of vector. the head 'p' is called terminal or tip of the vector.

the length of vectors gives magnitude whereas arrow defines the direction of the vector.

Types of Vector.

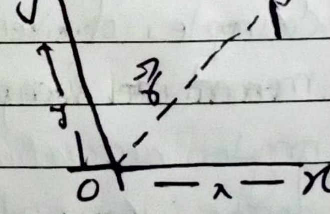
① Null/Vector:-
⇒ Magnitude = 0
⇒ $|\vec{a}| = a = 0$
⇒ It has arbitary direction.

⑪ Unit /Vector
⇒ It has unit magnitude
⇒ Direction Along the direction of vector
⇒ unit Vector are unitless & dimensionless

Position vector:-
⇒ It is use to define position of a particle from certain refrence point (i.e. origin)

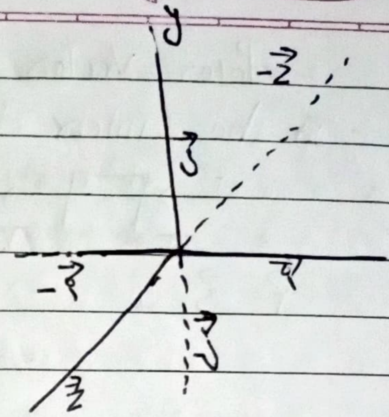
$\vec{OP} = \vec{r} = x\hat{i} + y\hat{j}$

$OP = r = \sqrt{x^2+y^2}$



Equal Vector:-
Magnitude & Direction are same.

$$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$OP = \sqrt{x^2+y^2+z^2}$$

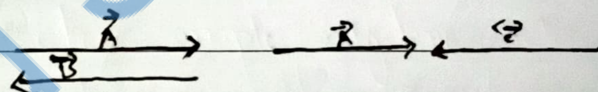


Parallel Vector:-

→ Direction = Same.
→ whatever be the magnitude
→ All equal vector are parallel.

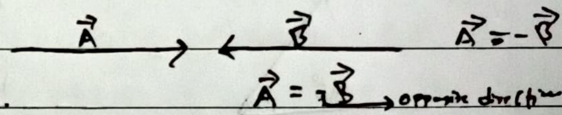
Anti parallel Vector:-

- Direction = Opposite
- whatever be the magnitude.



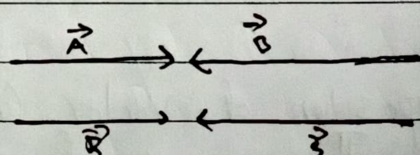
Negative vector

- Magnitude same but direction opposite.



Collinear vector

- lie along the same line
- All parallel vector are collinear.
- All anti parallel vector are collinear

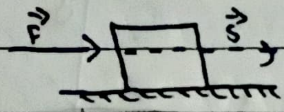


Coplanar vector:-

→ lie in the same plane but it must not be parallel or antiparallel

Polar Vector

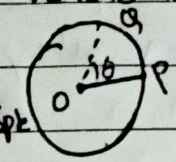
- It has linear effect.



$\vec{P}$, $\vec{S}$, $\vec{u}$

Axial Vector

It has rotaional effect



eg:- Angular ~~disp~~

" velcity

" momentum

" acleration

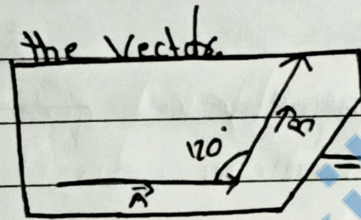
Torqu ($\vec{\tau}$) = ... etc.

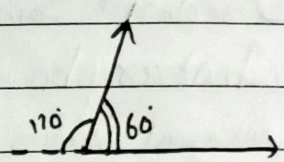
~~Comp~~ Angle between the Vectors.

↳ tail to tair ~~or~~

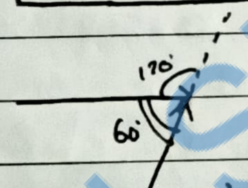
↳ Head to Head

Range 0° to 180°



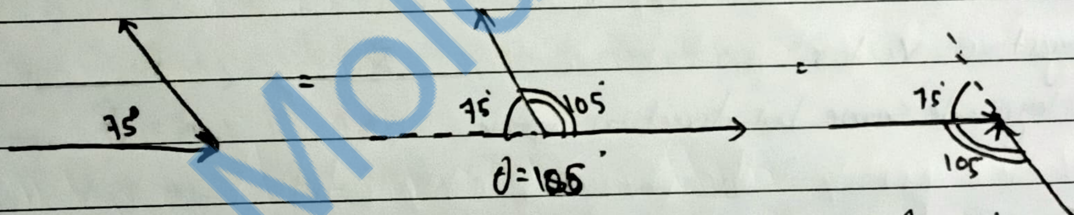


tail to tail



Head to Head.

for example:-



θ = 105°

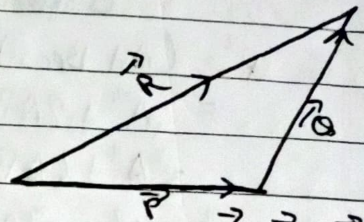
Composition of Vector (Addition of Vector)

The process of obtaining single vector from two or more vector is called Composition of Vector. The single vector obtained as a result is called resultant vector.

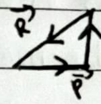
- Triangle law of Vector addition:- It states that "If two sides of a traingle represent two vector taken in the same order then third side taken in opposite order represent the resultant vect

↳ tail + head or
↳ Head + tail] — same order (Add)

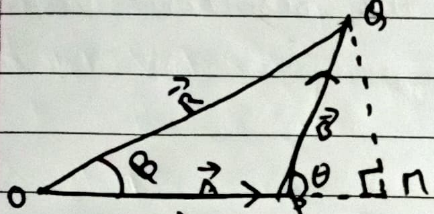
↳ Head + Head or
tail + tail] opposite order



$\vec{P} + \vec{Q} = \vec{R}$



$= \vec{P} + \vec{Q} = -\vec{R}$, $\vec{P} + \vec{Q} + \vec{R} = 0$,



let $\vec{A}$ & $\vec{B}$ be the two vector represented by two sides $\vec{OP}$ & $\vec{PQ}$ of the traingle OPQ respectively taken in the same order.

According to traingle law of vector addition, the third side $\vec{OQ}$ vector ~~rep~~ take in opposite order represent the resultant vector $(\vec{R})$.

Let us produce $\vec{OP}$ to M and draw a perpendicular QM as shown in the figure

$\angle QPM = \theta$ is angle between $\vec{R}$ & $\vec{B}$ and
$\angle QOM = \beta$ is angle between the resultant vector $(\vec{R})$ with $\vec{A}$

for magnitude of $\vec{R}$

In ΔOQM

$$OQ^2 = OM^2 + QM^2$$

$$R^2 = (OP + PM)^2 + QM^2 \quad \text{---- (1)} \quad \Big/ \quad R^2 = (A + PM)^2 + QM^2 \text{ ---} \textcircled{1}$$

In ΔPQM

$\cos\theta = PM/PQ$	$\sin\theta = \frac{QM}{PQ}$
$PM = \cos\theta \, PQ$	$QM = PQ \sin\theta$
$= \cos\theta \, B$	$QM = \sin\theta \, B$,

Substituting the value of PM & QM in eqⁿ (I)

$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$
$$= A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta$$
$$= A^2 + 2AB\cos\theta + B^2(\cos^2\theta + \sin^2\theta)$$
$$R^2 = A^2 + 2AB\cos\theta + B^2$$
$$\therefore R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \text{---- (II)}$$

eqⁿ (II) Gives the magnitude of the resultant Vector. $(\vec{R})$

For direction of $\vec{R}$

In Δ OQM

$$\tan\beta = \frac{QM}{OM}$$
$$= \frac{QM}{OP + PM} = \frac{B\sin\theta}{A + B\cos\theta}$$

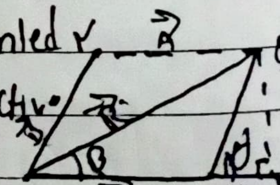
$$\beta = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

The angle β defines the direction of the resultant vector with A vector.

Parallelogram Law of Vector addition

Statement :- It states that if two vectors acting simultenously at the point are represented by two coduratent side of a parallelogram then diagnol passing through that point represnst the resultant Vector.

let $\vec{A}$ vector & $\vec{B}$ be the two vector represented by two sides $\vec{OP}$ & $\vec{ON}$ of the parallelogram respectively according to parallelogram law of Vector addition the diagonal OQ vector drawn from the tain of $\vec{A}$ & $\vec{B}$ represent the resultant Vector.



Let us produce OP to M & draw a ⊥ perpendicul QM as shown in the figure. $\angle NOM = \angle QPM = \theta$ is a the agle betⁿ $\vec{A}$ & $\vec{B}$ and.

$QOM = \beta$ is the angle between $|\vec{R}|$ & $\vec{A}$

for magnitude of $\vec{R}$

In ΔOQM

$$OQ^2 = OM^2 + QM^2$$

$$R^2 = (OP + PM)^2 + QM^2$$

$$R^2 = (A + PM)^2 + QM^2 \quad \text{--- (1)}$$

In ΔPQM

$\cos\theta = PM/PQ$ $\qquad$ $\sin\theta = QM/PQ$

$PM = PQ\cos\theta$ $\qquad$ $QM = PQ\sin\theta$

$PM = \cos\theta B,$ $\qquad$ $QM = B\sin\theta$

Substituting the value of PM & QM.

$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$R^2 = A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta$$

$$R^2 = A^2 + 2AB\cos\theta + B^2(\sin^2\theta + \cos^2\theta)$$

$$R = \sqrt{A^2 + 2AB\cos\theta + B^2}$$

Special cases:

i) If two vectors are in same direction. i.e $\theta = 0°$

$$R = \sqrt{A^2 + 2AB\cos\theta + B^2}$$

$$R = \sqrt{A^2 + 2AB + B^2}$$

$$R = \sqrt{(A+B)^2}$$

$$R = A+B \quad \text{(maximum)}$$

The magnitude of resultant vector is maximum in this case

$$\vec{A} + \vec{B} = \xrightarrow{R = A+B}$$

Now

for direction, $\tan\beta = \dfrac{B\sin\theta}{A + B\cos\theta}$

$$\beta = \tan^{-1}(0)$$

$$\beta = 0$$

The direction of resultant vector is along the direction either vector.

(ii) If two vectors are in opposite direction. i.e $= 180°$

$$R = \sqrt{A^2 + 2AB\cos 180 + B^2}$$

$$= \sqrt{A^2 - 2AB + B^2}$$

$$= \sqrt{(A-B)^2} \quad \text{or,} \quad \sqrt{(B-A)^2}$$

$R = A - B$ if $A > B$

$R = B - A$ if $B > A$

$R = |A - B| =$ minimum,

The magnitude of resultant vector is minimum in this case

if $A > B$

$$\vec{A} + \overleftarrow{B} = \xrightarrow{R = A-B} \quad \beta = 0$$

$$|\vec{A} + \vec{B}| = |A - B|$$

$$\theta = 180°$$

if $B > A$

$$\vec{A} + \overleftarrow{B} = \xleftarrow{R = B-A} \quad \beta = 180°$$

The direction of the resultant vector is along the vector having high mag if two vector are perpendicular to each other : $\theta = 90^\circ$

$$R = \sqrt{A^2 + 2AB\cos 90 + B^2}$$

$$R = \sqrt{A^2 + B^2}$$

now, $$\tan\beta = \frac{B\sin 90^\circ}{A + B\cos 90^\circ}$$

$$= \frac{B}{A+0}$$

$$\beta = \tan^{-1}(B/A)$$

Range of R

$|A-B|$ = minimum $\qquad$ $|A+B|$ = Maximum

$\boxed{iii}$ find the resultant vector of two vector of same magnitude

Here, let two vectors be, $\vec{a}$ & $\vec{b}$

Acording to que $|\vec{a}| = |\vec{b}|$ then $a = b = x$ (let), now

According to traingle low of vector addition,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{x^2 + 2x^2\cos\theta + x^2}$$

$$= \sqrt{2x^2 + 2x^2\cos\theta}$$

$$= \sqrt{2x^2 \cdot (1 + \cos\theta)}$$

$$= \sqrt{2x^2 \cdot 2\cos^2\frac{\theta}{2}}$$

$$= \sqrt{\left(2x\cos\frac{\theta}{2}\right)^2}$$

$$= 2x\cos\frac{\theta}{2}$$

let, β be the angle between $\vec{R}$ and $\vec{A}$

$$\tan\beta = \frac{B\sin\theta}{A + B\cos\theta} = \frac{x\sin\theta}{x + x\cos\theta} = \frac{x\sin\theta}{x(1+\cos\theta)} = \frac{\cancel{2}\sin\theta/2 \cdot \cancel{\cos\theta/2}}{\cancel{2}\cos^{\cancel{2}}\theta/2}$$

$$\tan\beta = \frac{\sin\theta/2}{\cos\theta/2} \qquad \tan\beta = \tan\theta/2$$

$$\therefore \beta = \theta/2$$

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Show that the resultant vector bisect the two vector having same magnitude.

i) if $\vec{A}+\vec{B}=\vec{C}$ and $A+B=C$ then find the the angle between $\vec{A}$ & $\vec{B}$

Solution ii) if $\vec{A}+\vec{B}=\vec{C}$ and $A^2+B^2=C^2$ then find the angle between $\vec{A}$ & $\vec{B}$

iii if $\vec{A}+\vec{B}=\vec{C}$ and $A-B=C$ then find the angle between $\vec{A}$ & $\vec{B}$

iv if $\vec{A}+\vec{B}=\vec{C}$ and $B-A=C$

v if $|\vec{A}+\vec{B}|=A+B$

vi if $|\vec{A}+\vec{B}|=A-B$

vii if $|\vec{A}+\vec{B}|=\sqrt{A^2+B^2}$

viii if $\vec{A}+\vec{B}=\vec{C}$, Range of C?

for i) let θ be the angle between the $\vec{A}$ & $\vec{B}$

Given that $\vec{A}+\vec{B}=\vec{C}$

$$C=\sqrt{A^2+2AB\cos\theta+B^2}$$

$$(A+B)^2=A^2+2AB\cos\theta+B^2$$

$$A^2+2AB+B^2=A^2+2AB\cos\theta+B^2$$

$$2AB=2AB\cos\theta$$

$$\cos\theta=1$$

$$\theta=0^\circ$$

for ii) let θ be the angle between the $\vec{A}$ & $\vec{B}$

Given that $A^2+B^2=C^2$

$$C=\sqrt{A^2+2AB\cos\theta+B^2}$$

Sq.. $$C^2=A^2+2AB\cos\theta+B^2$$

$$A^2+B^2=A^2+2AB\cos\theta+B^2$$

$$2AB\cos\theta=0$$

$$\theta=\cos^{-1}(0)$$

$$\theta=90^\circ$$

for III if $\vec{A} + \vec{B} = \vec{C}$ and $A - B = C$ then find the angle

let θ be the angle between the $\vec{A} + \vec{B}$

Given that $A - B = C$ now

According vector law of addition

$$C = \sqrt{A^2 + 2AB\cos\theta + B^2}$$

Squaring both side $(A-B)^2 = A^2 + 2AB + B^2$

$$\text{or } A^2 - 2AB + B^2 = A^2 + 2AB\cos\theta + B^2$$

$$-2A\cancel{B} = 2A\cancel{B}\cos\theta$$

$$\cos\theta = -1$$

$$\theta = \cos^{-1}(-1)$$

$$\theta = 180$$

for IV if $\vec{A} + \vec{B} = \vec{C}$

let θ be the angle between the $\vec{A}$ & B^2

Given that $B - A = C$,

According to vector law of addition

$$C = \sqrt{A^2 + 2AB\cos\theta + B^2}$$

Squaring both $(B-A)^2 = A^2 + 2AB\cos\theta + B^2$

$$B^2 - 2AB + A^2 = A^2 + 2AB\cos\theta + B^2$$

$$-2AB = 2AB\cos\theta$$

$$\cos\theta = -1$$

$$\therefore \theta = \cos^{-1}(-1)$$

$$\therefore \theta = 180^\circ$$

for v

let θ be the angle between the $\vec{A} + \vec{B}$

According to vector law of Addition,

$$A+B = \sqrt{A^2 + 2AB\cos\theta + B^2}$$

Square both $(A+B)^2 = A^2 + 2AB\cos\theta + B^2$

or $A^2 + 2AB + B^2 = A^2 + 2AB\cos\theta + B^2$

or $2AB = 2AB\cos\theta$

or $\cos\theta = 1$

$\therefore \theta = 0^\circ$

for vi

let θ be the angle between the $\vec{A} + \vec{B}$

Given that $|\vec{A} + \vec{B}| = A - B$.

According to vector law of addition,

$$|\vec{A}+\vec{B}| = \sqrt{A^2 + 2AB\cos\theta + B^2}$$

or $(A-B)^2 = A^2 + 2AB\cos\theta + B^2$

or $A^2 - 2AB + B^2 = A^2 + 2AB\cos\theta + B^2$

or $-2\cancel{AB} = 2\cancel{AB}\cos\theta$

or $\theta = \cos^{-1} = -1$

$\therefore \theta = 180^\circ$

for vii

let θ be the angle between the $\vec{A}$ & $\vec{B}$

Given that, $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2}$

According to vector law of addition

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + 2AB\cos\theta + B^2}$$

$$\left(\sqrt{A^2+B^2}\right)^2 = \sqrt{A^2 + 2AB\cos\theta + B^2}$$

$$A^2 + B^2 = A^2 + 2AB\cos\theta + B^2$$

$$\cos\theta = 0$$

$$\theta = 90^\circ$$

viii If $\vec{A} + \vec{B} = \vec{C}$, Range of $\vec{C}$

the Range of resultant C should be,

Min $= |\vec{A} - \vec{B}|$

Max $= |\vec{A} + \vec{B}|$

~~Range :- $|\vec{A} - \vec{B}| \le C \le |\vec{A} + \vec{B}|$~~

Range :- $|A - B| \le |\vec{R}| \le A + B$

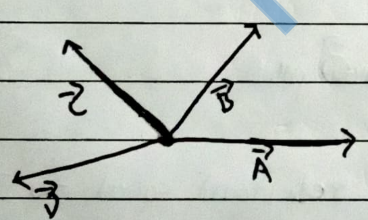
$|A - B| \le R \le A + B$

$|A - B| \le |\vec{A} + \vec{B}| \le A + B$

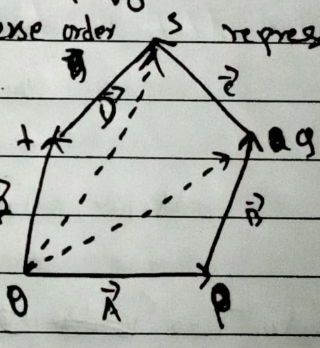
Polygon law of Vector addition. (extension of triangle law)

It State that if a number of vectors be represented both in magnitude and direction by the sides of the polygon taken in the ~~opposite~~ Same order then the resultant is represented completely in magnitude and direction by closing side of polygon taken in opposite order.

let, In figure $\vec{A}$, $\vec{B}$, $\vec{C}$ & $\vec{D}$ are represented by side $\vec{OP}$, $\vec{PQ}$, $\vec{QS}$, $\vec{ST}$ respectively of the polygon. According to polygon law of vector addition the closing side $\vec{OT}$ taken in the reverse order represent the resultant vector $\vec{R}$ [$\therefore \vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{R}$]



It can be drawn ~~writte~~ as



Construction: O to Q, O to S,

In ΔOPQ,

By triangle law of vector addition

$\vec{OP} + \vec{PQ} = \vec{OQ}$

$\therefore \vec{A} + \vec{B} = \vec{OQ}$ ... ①

In ΔOQS

By Δ law of Vector addition,

$\vec{OQ} + \vec{QS} = \vec{OS}$

$\therefore \vec{A} + \vec{B} + \vec{C} = \vec{OS}$ from ①

In ΔOST

By triangle law of vector addition

$\vec{OS} + \vec{ST} = \vec{OT}$

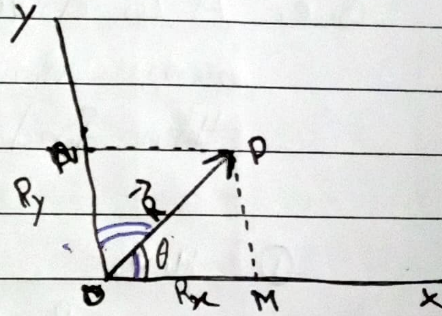
$\therefore \vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{OT}$

Resolution of Vector :-

A splitting of a vector into its constituent component is called resolution of vector.

- Rectangular resolution of vector :-

The splitting of a vector into two mutually perpendicular component in two dimension is called rectangular resolution of vector. and the components are called rectangular component



In figure, $\vec{R}$ is represented by OP vector. R_x & R_y are the component of $\vec{R}$ along x-axis and y-axis respectively

In OPMΔ

or $\cos\theta = \frac{OM}{OP}$ or $\sin\theta = \frac{MP}{OM}$

or, $R\cos\theta = OM$ or $R\sin\theta = MP$ $(\because MP = NO)$

$\therefore R_x = R\cos\theta$ ---(i) $\therefore R_y = R\sin\theta$ ---(ii)

Squaring and adding eq^n (i) and (ii)

$R_x^2 + R_y^2 = R^2\sin^2\theta + R^2\cos^2\theta$

$= R^2(\sin^2\theta + \cos^2\theta)$

or $R^2 = R_x^2 + R_y^2$

$\therefore R = \sqrt{R_x^2 + R_y^2}$ ---- (x)

Dividing eq^n (ii) by eq^n (i)

$\frac{R_y}{R_x} = \frac{R\sin\theta}{R\cos\theta}$

$\therefore \frac{R_y}{R_x} = \tan\theta.$

In Vector form

$\vec{OP} = \vec{OM} + \vec{MP}$

$\vec{R} = \vec{R_x} + \vec{R_y}$

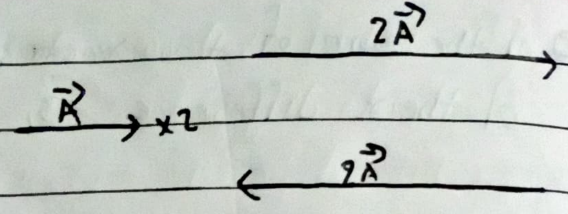
$\vec{R} = R_x\hat{i} + R_y\hat{j}$

$\vec{R} = R\cos\theta\hat{i} + R\sin\theta\hat{j}$.

If R_x & R_y & R_z be the component of $\vec{R}$ along x-axis, y-axis and z-axis respectively then $\vec{R} = \vec{R_x} + \vec{R_y} + \vec{R_z}$ Also $\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$

$\therefore R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

Multiplication of Vector:



(a) By number.

$\vec{A}$ = Vector. the $n\vec{A}$ = Vector.

n = number,

eg $\vec{A}$ = 4 km east

n = 2

$n\vec{A}$ = 2×4 km east

= 8 km east.

i) n = −2

$n\vec{A}$ = −2×4 km east

= −8 km east

= 8 km west.

(ii) By a Scalar.

$\vec{A}$ = Vector

λ = Scalar

$\lambda\vec{A}$ = Vector,

eg: $\vec{a}$ = acceleration (vector)

m = mass (scalar)

$m\vec{a} = \vec{F}$ = force (vector)

eg: m = mass (scalar)

$\vec{v}$ = Velocity (Vector)

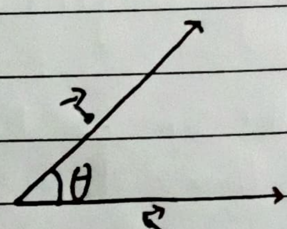
$m \times \vec{v}$ = momentum (P) Vector.....

(iii) By a vector.

(i) Scalar product (Dot product)

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$= ab\cos\theta$ (Scalar)



eg:

(i) work (W) = $\vec{F} \cdot \vec{S}$ = $Fs\cos\theta$ (scalar)

(ii) Power (P) = $\vec{F} \cdot \vec{v}$ = $Fv\cos\theta$ (Scalar) $\vec{v}$ velocity.

Special cases

i) $\theta = 0$, $\vec{a}\cdot\vec{b} = ab\cos\theta = ab$,,

$\theta = 90$, $\vec{a}\cdot\vec{b} = ab\cos 90 = 0$,

$\theta = 180$ $\vec{a}\cdot\vec{b} = ab\cos 180 = -ab$,,

Dot product of unit vector.

$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

$\hat{i}\cdot\hat{i} = |\hat{i}||\hat{i}|\cos 0^\circ = 1\times1 = 1$

$\hat{j}\cdot\hat{j} = |\hat{j}||\hat{j}|\cos 0^\circ = 1\times1 = 1$

$\hat{k}\cdot\hat{k} = |\hat{k}||\hat{k}|\cos 0 = 1$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1,$

Now,

$\hat{i}\cdot\hat{j} = |\hat{i}||\hat{j}|\cos 90 = 0$

$\hat{j}\cdot\hat{k} = |\hat{j}||\hat{k}|\cos 90 = 0$

$\hat{k}\cdot\hat{i} = |\hat{k}|\cdot|\hat{i}|\cos 90 = 0$

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = \hat{j}\cdot\hat{i} = \hat{k}\cdot\hat{j} = \hat{i}\cdot\hat{k} = 0,$

~~properties~~.

propirties

(i) $\vec{A}\cdot\vec{A} = |\vec{A}||\vec{A}|\cos 0$

$= A\times A\times1$

$= A^2$

(ii) $\vec{A}\cdot\vec{B} = AB\cos\theta$

$= BA\cos\theta$

$= |\vec{B}|\cdot|\vec{A}|$

(iii) $\vec{A}\cdot(\vec{B}+\vec{C}) \Rightarrow \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

(iv) (i) $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ $\quad \vec{B} = B_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_xB_x\,\hat{i}\cdot\hat{i} + A_yB_y\,\hat{j}\cdot\hat{j} + A_zB_z\,\hat{k}\cdot\hat{k}$

$= A_xB_x + A_yB_y + A_zB_z.$

Vector product. (cross product)

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$

$= \underbrace{ab\sin\theta}_{\text{magnitude}}\ \underbrace{\hat{n}}_{\text{unit vector}}$ (vector)

$\hat{n}$ represents the direction of $\vec{a}\times\vec{b}$ which is perpendicular to the plane formed by $\vec{a}$ & $\vec{b}$.

for eg

(i) Velocity $(\vec{v}) = \vec{\omega}\times\vec{R}$ = vector

Torque $(\vec{\tau}) = \vec{r}\times\vec{F}$ = vector.

$(\vec{a}\times\vec{b}) = \perp\ \vec{a},\ \vec{b},\ \vec{a}\ne\vec{b}$

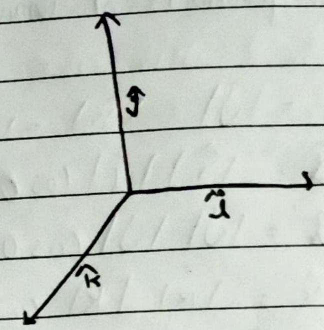
$(\vec{b}\times\vec{a}) = \perp\ \vec{a},\ \vec{b}\ (\vec{a}\ne\vec{b})$

$(\vec{b}\ne\vec{a})$

For direction,

$\vec{a}\times\vec{b}$ = outward, $\vec{b}\times\vec{a}$ = Inward.

Cross product ~~vector~~ of unit vector.



$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

$\hat{i} \times \hat{i} = |\hat{i}||\hat{i}| \sin 0^\circ \, \hat{n}$
$= \vec{0}$

$\hat{j} \times \hat{j} = |\hat{j}||\hat{j}| \sin 0 \, \hat{n}$
$= \vec{0}$

$\hat{k} \times \hat{k} = |\hat{k}|.|\hat{k}| \sin 0 \, \hat{k}$
$= \vec{0}$

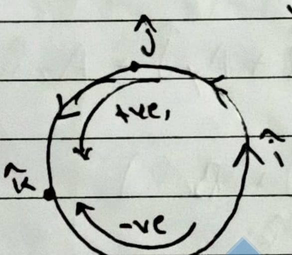
* $\hat{i} \times \hat{j} = |\hat{i}||\hat{j}| \sin 90 \, \hat{k} = \hat{k}$

* $\hat{k} \times \hat{i} = |\hat{k}||\hat{i}| \sin 90 \, \hat{j} = \hat{j}$

* $\hat{j} \times \hat{k} = |\hat{j}||\hat{k}| \sin 90 \, \hat{i} = \hat{i}$

So, $\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$

$\hat{j} \times \hat{i} = -(\hat{i} \times \hat{j})$



$\hat{i} \times \hat{j} = \hat{k}$
$\hat{j} \times \hat{k} = \hat{i}$
$\hat{k} \times \hat{i} = +\hat{j}$

$\hat{k} \times \hat{j} = -\hat{i}$
$\hat{j} \times \hat{i} = -\hat{k}$
$\hat{i} \times \hat{k} = -\hat{j}$

(ii) $\vec{A} \times \vec{A} = |\vec{A}||\vec{A}| \sin 0$
$= \vec{0}$

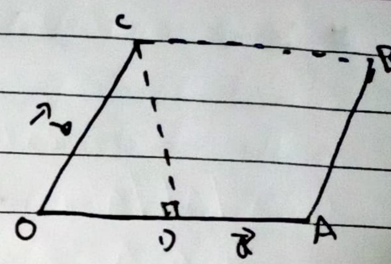
$|\vec{A} \times \vec{A}| = |0|$

(iii) i) $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{j}$

$$\vec{A} \times \vec{B} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Geometrical meaning of cross-product.



$\sin\theta = \frac{CD}{OC}$

$CD = OC \sin\theta$

$\therefore CD = \vec{b} \sin\theta$

$|\vec{a} \times \vec{b}| = ab \sin\theta.$

$= |OA| \times (CD)$

= Base × height

= Area of parllelogram.