Electric potential

Electric potential :- It is defined as amount of work done in bringing a unit positive test charge from infinity to a point in the electric field

i.e $V_A = \frac{W_{\infty A}}{q_0}$ It S.I unit is Joule/q

It's dimension is $\frac{[MLT^{-2}][L]}{[AT]}$

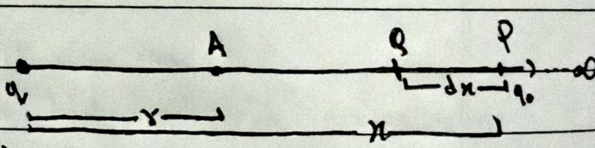
$= [ML^2T^{-3}A^{-1}]$

Electric potential due to a point charge

Coinsder a sourch charge q at a point 'O'. to find Electric potential at point 'a' which is at distance of r from the source charge in region of electric field,

let,

An instant of time 't', the charge is at q_0 poin P at distance x from the source charge as shown in figure.



If V be electric potential at point A then it determined as $V = \frac{W_{\infty A}}{q_0}$ —— ①

If the test charge q_0 is moved from P to Q with a small displacement dx then work done is calculated as

$dw = -F\,dx$ -- ② (negative Sign indicates that work done is agains electric field)

The electrostatic force of repulsion between Source Charge & test charge is given by :-

$$F = \frac{1}{4\pi\epsilon_0} \times \frac{qq_0}{r^2}$$

(i) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

(ii) $\int_a^b x^n dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$

Then eq^n (i) becomes

$$dw = \frac{-1}{4\pi\epsilon_0} \times \frac{qq_0}{r^2} \text{ ---- (iii)}$$

The total work done is given by integrating above eq^n (iii)

$$\int_0^w dw = \int_{\infty}^{r} -F dx$$

$$w = \int_{\infty}^{r} -\frac{1}{4\pi\epsilon_0} \times \frac{qq_0}{x^2} dx$$

$$w = -\frac{1}{4\pi\epsilon_0} qq_0 \int_{\infty}^{r} \frac{1}{x^2} dx$$

$$w = -\frac{1}{4\pi\epsilon_0} qq_0 \int_{\infty}^{r} x^{-2} dx$$

$$w = \frac{-qq_0}{4\pi\epsilon_0}\left[\frac{x^{-2+1}}{-2+1}\right]_{\infty}^{r} = \frac{-qq_0}{4\pi\epsilon_0}\left[\frac{x^{-1}}{-1}\right]_{\infty}^{r}$$

$$w = \frac{qq_0}{4\pi\epsilon_0}\left[\frac{1}{x}\right]_{\infty}^{r}$$

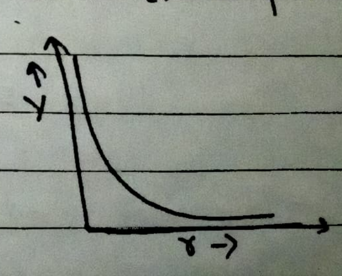
$$w = \frac{qq_0}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$w = \frac{qq_0}{4\pi\epsilon_0 r} \quad //$$

then, eq^n (i) becomes, $V = \frac{q_0 q}{4\pi\epsilon_0 r} \times \frac{1}{q_0}$

$$= \frac{q}{4\pi\epsilon_0 r} \quad //$$

This is required eq^n for electric potential due a point charge at point A.



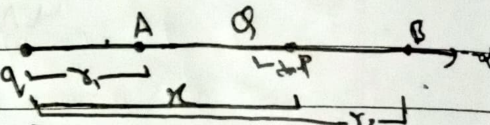
Relation bet^n V & r

The potential difference b/w two point is defined as amount of work done in brigining a unit positive test charge from one point to another

=> Potential difference between two point.

If V_{BA} be the potential difference between two points then it is written as $V_{BA} = V_A - V_B = \frac{W_{BA}}{q_0}$ ---- (i)

At any instant of time, let the test charge is at point P which is at distance of 'x' from the Source charge. as shown in figure :-



The force of repulsion between point charge and source charge is given by $F = \frac{q \cdot q_0}{4\pi\varepsilon_0 x^2}$ ---- (ii)

If the test charge q_0 is displaced to Q with small displacement of dx then the small work done is given by

$$dw = -F dx$$
$$= -\frac{q \cdot q_0}{4\pi\varepsilon_0 x^2} \times dx \text{ ---- (iii)}$$

The total work done is given by integrating the eqn (iii) as.

$$\int_0^{W_{BA}} dw = \int_B^A -\frac{q \cdot q_0}{4\pi\varepsilon_0 x^2} \times dx$$

$$W_{BA} = -\frac{q \cdot q_0}{4\pi\varepsilon_0} \int_{r_2}^{r_1} x^{-2} \times dx \qquad \frac{x^{-1}}{-1}$$

$$= \frac{-q \cdot q_0}{4\pi\varepsilon_0} \left[\frac{x^{-2+1}}{-2+1} \right]_{r_2}^{r_1}$$

$$= \frac{q \cdot q_0}{4\pi\varepsilon_0} \times \left[\frac{1}{x} \right]_{r_2}^{r_1}$$

$$= \frac{q.q_0}{4\pi\varepsilon_0} \times \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$W_{BA} = \frac{q.q_0}{4\pi\varepsilon_0} \times \frac{r_1 - r_2}{r_1 r_2} \text{ --- (iv)}$$

now value of eq^n (i) becomes using (iv)

$$V_{AB} = \frac{q.q_0}{4\pi\varepsilon_0} \times \frac{r_1 - r_2}{r_1 r_2} \times \frac{1}{q_0}$$

$$= \frac{q(r_1 - r_2)}{4\pi\varepsilon_0 (r_1 r_2)}$$

which is requried eq^n for potential difference bet^n two points

Electrical potential energy.

Electrical potential energy is the amount of workdone in briging a charge from infinity to a point.

Suppose a charge q' is braught from infinity to a point then electric potential is calculated as

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

now,

The work done in bringing a charge q' from infinity to point is given by

$$W = q'V$$

$$= q' \times \frac{q}{4\pi\varepsilon_0 r}$$

$$= \frac{qq_1}{4\pi\varepsilon_0 r}$$

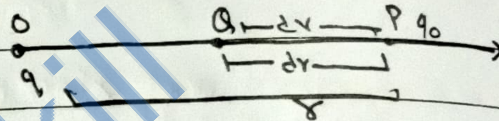
now, The work done is stored in the form of electric potential energy. So $U = \frac{qq'}{4\pi\varepsilon_0 r}$ //

Equipotential surface:-

It is surface that has equal potential energy all over the surface, same at every point on the surface.

⇒ work done Equipotential surface is always zero.

Relationship betn : Potential Gradient



workdone

$P \longrightarrow Q$

$w = -f \times dr$

let us consider a charge q at point O & at any instant of time the test charge q_0 is at P. when the test charge is displace with small displacement dr then workdone is given by

$$W_{PQ} = -f \times dr \quad \text{--- (i)}$$

Again, we know that the force expirened by a test charge is given by $f = q_0 E$ the eqn (i) becomes,

$$W_{PQ} = -q_0 E \times dr \quad \text{--- (ii)}$$

Again, The electric potential difference between P & Q is given by

$$V_P - V_Q = dv = \frac{W_{PQ}}{q_0}$$

$$\therefore W_{PQ} = dv\, q_0 \quad \text{--- (iii)}$$

from (ii) & (iii)

$$-q_0 E \times dr = dv \times q_0$$

or $$E = \frac{dv}{dr} \times \frac{q_0}{-q_0}$$

$$\therefore E = -\frac{dv}{dr}$$

Electron Volt : The electron volt is the amount of work done or kinetic energy gaind by the electron which has been accelerated through potential differene of 1 Volt