Periodic Motion.

The motion of an object which repeats after certain time interval is known as periodic Motion

for eg:- Motion of pendulum
Motion of object in mass Spring System

Simple Harmonic Motion. (SHM)

(displacement is directly proportional with accleration)

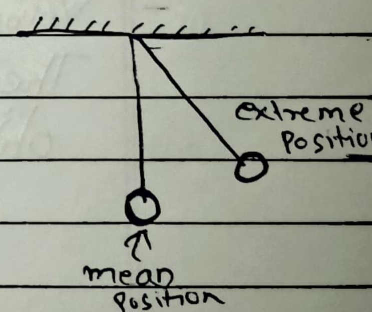
=) The motion of an object in which accleration is directly proportional to the displacement & the motion is always directed towards the mean position then such type of motion is known as SHM

if a be the accleration of object and y be its displacement from mean position then accleration is directly proportional with displacement.

$$a \propto y$$

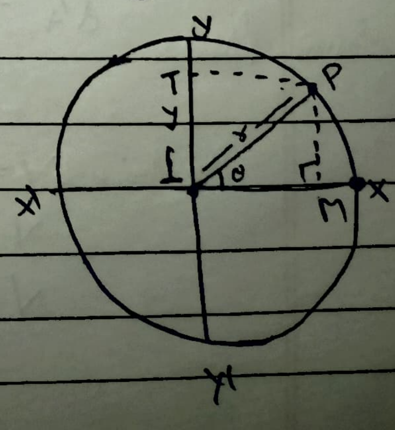
$$a = -ky,$$

Here (-ve) indicates motion is always directed towards the mean position.



Circular motion as Simple harmonic motion.

let us consider an object of mass (m) moving around the circular path of centre 'o' and radius 'r' as shown in figure.



let at any instant of time (t) object raised to the point 'p' making an angle θ with the centre of the circle.

Here,

$$\omega = \text{angular velocity} = \frac{\theta}{t}$$

$$\therefore \theta = \omega \times t \quad \text{--- (i)}$$

In Δ OMP

$$\sin\theta = \frac{PM}{OP}$$

$$\sin\theta = \frac{y}{r}$$

$$\therefore y = r\sin\theta, \text{ from (i)}$$

$$y = r\sin(\omega \times t) \quad \text{(ii)}$$

Eqn (ii) gives the displacement of an object executing SHM.

- Velocity of SHM

The rate of change of displacement of an object showing SHM is known as its velocity.

$$v = \frac{dy}{dt}$$

$$\frac{dy}{dt} = r\sin\omega t$$

$$= r \times \cos(\omega t) \times \omega$$

$$\therefore v = r\omega\cos\omega t$$

$$v = r\omega \times \sqrt{1 - \sin^2\omega t}$$

$$v = r\omega\sqrt{1 - \frac{y^2}{r^2}}$$

$$\frac{\cancel{r}\omega\sqrt{r^2 - y^2}}{\cancel{r}}$$

$$\therefore v = \omega\sqrt{r^2 - y^2} \quad \text{(iii)}$$

∴ At mean position

$y = 0$

$v = \omega\sqrt{r^2 - y^2}$

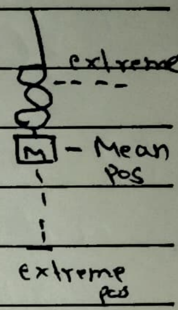
$v = \omega \times r$ // max

∴ At extreme position

$y = r$

$v = \omega\sqrt{r^2 - y^2}$

$= 0$ // min,



- Accleration

The rate of change of velocity of an object showing SHM is known as accleration,

$$a = \frac{dv}{dt}$$

$$a = \frac{d\, r\omega\cos\omega t}{dt}$$

$$= r\omega(-\sin\omega t)\,\omega$$

$$= -r\omega^2\sin\omega t$$

$$a = -\omega^2 y$$

$$a \propto y$$

Hence, the motion of an object revolving around the circular path is SHM

∴ at mean position

$y = 0$

$a = -\omega^2 y$

$a = 0$ min

∴ At extreme position

$y = r$

$a = \omega^2 y$

$a = \omega^2 r$ max

- Time period (T)

The time taken by an object to cover one complete cycle during SHM is known as time period

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{a/y}} \qquad \therefore a = \omega^2 y$$

$$\therefore T = 2\pi\sqrt{y/a}$$

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{accleration}}}$$ //

- Frequency :- The number of cycles per Second made by an object showing SHM is known as frequency

Here,

$$f = \frac{1}{T} = \frac{1}{2\pi\sqrt{y/a}}$$

$$f = \frac{1}{2\pi} \times \sqrt{\frac{a}{y}}$$ //

Question The displacement of the particle executing SHM is represented by an equation

$$y = 20 \sin(12\pi t)$$

Find i) Amplitude (max displacement)

ii) Velocity & Accleration at time $t = 2$ sec

ii) Velocity $v = \frac{dy}{dt}$

$$= \frac{20 \times \sin(12\pi t)}{dt}$$

$$= 20 \times \cos(12\pi t) \times 12\pi$$

$$= 240\pi \cos(12\pi t)$$

At time $t = 2$ sec

$$= 240 \times \pi \times \cos(12\pi \times 2)$$

$$= 240\pi$$

Accleration,

$$\frac{dv}{dt} = \frac{d}{dt} 240\pi \cos(12\pi t)$$

Circular motion periodic motion hunxa
but accleration $\not\propto$ displacement,
तर 1D dimension मा हुँदैन SHM हुन्छ।

$= 240 \times \pi \times -\sin(12\pi t) \times 12 \times \pi$

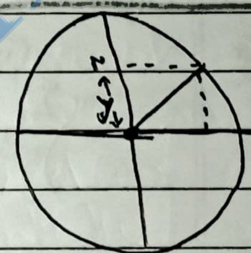
At time $t = 2$ sec

$= -240 \times \pi \times \sin(12 \times \pi \times 2) \times 12 \times \pi$

① for Amplitude,
Comparing with $y = r\sin\theta$ or $y = r\sin(\omega t)$

$r = 20$
$\omega = 12\pi$ } Ans

Energy of Simple Harmonic motion (SHM)



$F = ma$
$= m\omega^2 y$ — ①

$dw = F \times d$
$= F \times dy = m\omega^2 y$

$\int dw = \int_0^y m\omega^2 y \, dy$

$w = m\omega^2 y^2 / 2$

$\therefore PE = \frac{1}{2} \times m\omega^2 y^2$

$\Rightarrow K.E = \frac{1}{2} \times m \times v^2 = \frac{1}{2} \times m (\omega \sqrt{r^2 - y^2})$

$= \frac{1}{2} m\omega^2 (r^2 - y^2)$

The sum of kinetic energy & potential energy of object showing SHM is known as the total energy of SHM

Here,
let us consider an object of mass m executing SHM here y be the displacement of the object from mean position then its acceleration will be,

$$a = -\omega^2 y \quad \text{--- (i)}$$

The restoring force acting within the object

$$F = m \times a$$

$$= m\omega^2 y$$

Here dw be the small work to be done to displace an object through small displacement dy then,

$$dw = F\,dy$$

$$= m\omega^2 y\,dy \quad \text{--- (iii)}$$

To find the work done we have to integrate equation.

(iii) from 0 to y

$$\int_0^y dw = \int_0^y m\omega^2 y\,dy$$

$$w = m\omega^2 \left[\frac{y^2}{2}\right]_0^y$$

$$= m\omega^2 \frac{y^2}{2}$$

$$\therefore w = \frac{1}{2} \times m \times \omega^2 \times y^2, \quad \text{--- (iv)}$$

The amount of workdone is Stored in a form of P.E of SHM

Similarly,

The kinetic energy of a an object Showing SHM is :-

$$K.E = \frac{1}{2} \times m \times v^2$$

$$= \frac{1}{2} \times m \times (\omega\sqrt{r^2 - y^2})^2$$

$$= \frac{1}{2} \times m\omega^2 (r^2 - y^2) \quad \text{--- (vi)}$$

Similarly To total energy of an object showing SHM is:

$$T.E = K.E + P.E$$

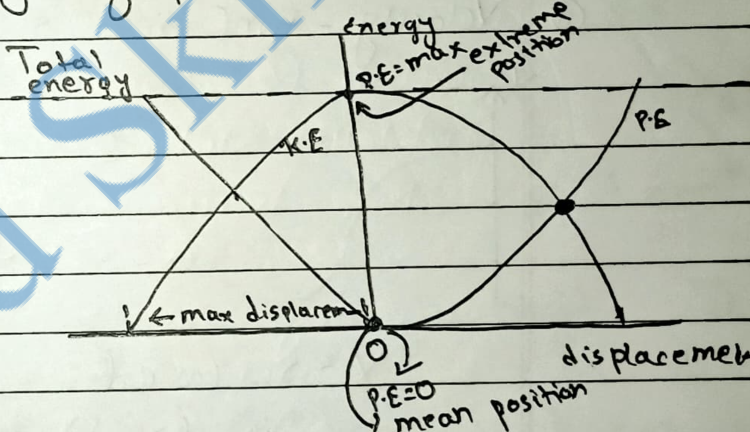
$$= \frac{1}{2} \times m\omega^2(r^2 - y^2) + \frac{1}{2} \times m\omega^2 y^2$$

$$= \frac{1}{2} m\omega^2(r^2 - y^2 - y^2)$$

∴ Total energy of an object is given by

$$= \frac{1}{2} \times m\omega^2(r^2)$$

Thus, total energy of body is always constant and can be represented by graph.



Question find the displacement from the mean position at which the value of K.E & P.E is equal,

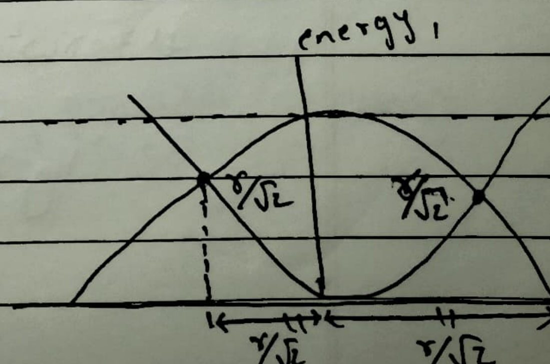
$$P.E = \frac{1}{2} \times m\omega^2 y^2 \quad \& \quad K.E = \frac{1}{2} m\omega^2(r^2 - y^2) \quad \Big| \; y^2 = r^2 - y^2$$

$$\frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2(r^2 - y^2)$$

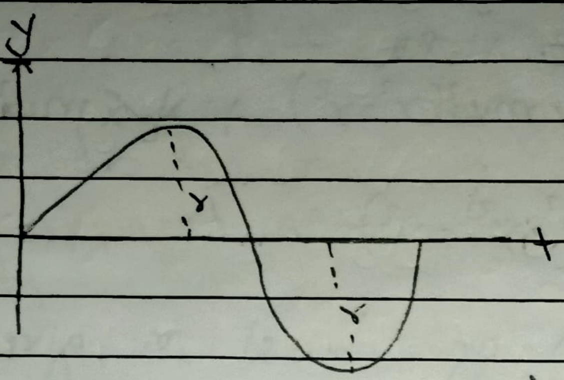
$$y^2 = r^2 - y^2$$

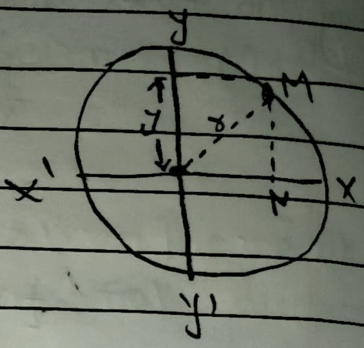
$$2y^2 = r^2$$

$$y = \frac{r}{\sqrt{2}},$$



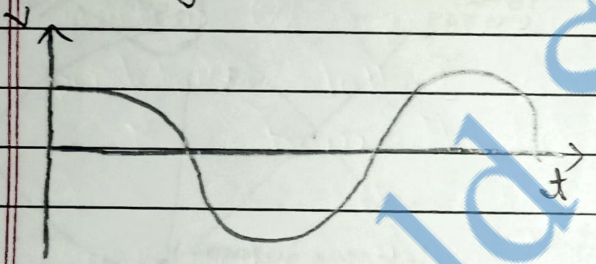
Graphical representation of SHM





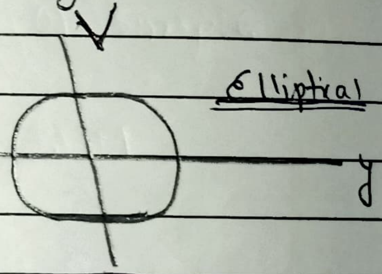
① fig: Displacement - time graph //

② Velocity - time



$v = r\omega\cos\omega t$

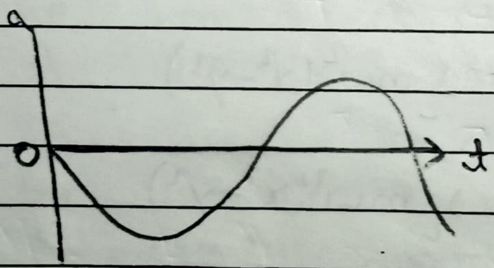
③ Velocity displacement.



$v = \omega\sqrt{r^2-y^2}$

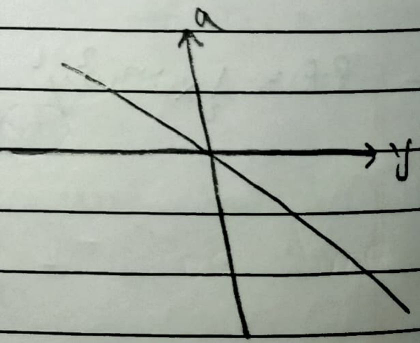
fig: (v-t) & (v-y) graph of SHM.

④ Accleration time



$a = -r\omega^2\sin\omega t$

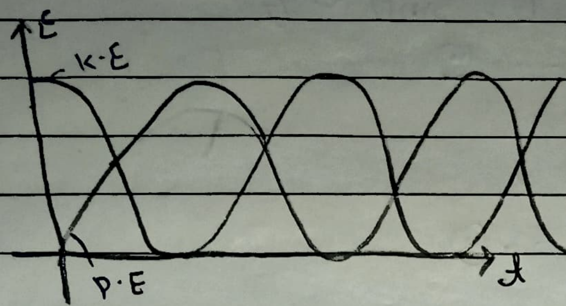
⑤ Accretion - displacement,



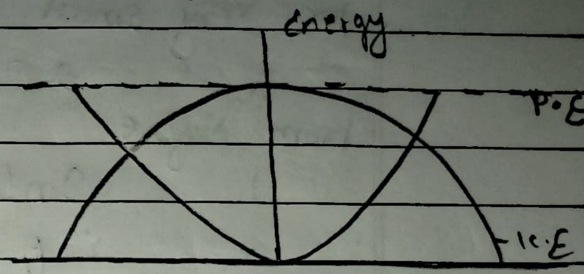
$a = -\omega^2 y$

fig: (a-t) & (a-y) graph of SHM

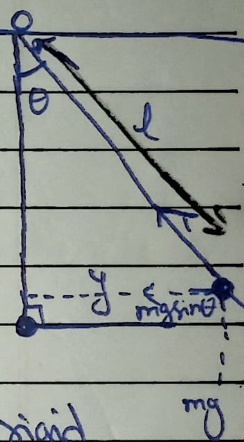
(vi) Energy time



(vii) Energy displacement



Simple - pendulum //



A heavy point object supended through rigid support by weightless, inflexible & inextensible string which can freely shows to and fro motion about the mean position consists of simple pendulum.

let us consider a bod of mass 'm' is suspended by a string through rigid support at point 'O'. The distance between point O to the C.G of the bob is known as effective length of the pendulum.

Here, when the pendulum bob is displaced from mean position P to extreme position (M) it makes an small angle θ with vertical direction. At point P the weight of bob can be resolved into two componets.

The component $mg\cos\theta$ balence the tension in the string. $\therefore mg\cos\theta = T$ ... (1)

Similarly, the component $mg\sin\theta$ provides necessary restoring force to the pendulum bob so,

$$F = -mg\sin\theta$$

$$ma = -mg\sin\theta$$

$$a = -g\sin\theta$$

for very small angle θ, $\sin\theta \approx \theta$

$$a = -g\theta \quad \text{--- (ii)}$$

from figure

$$\sin\theta = y/l$$

$$\theta = y/l \quad \text{--- (iii)}$$

from eq^n (ii) & (iii) we have,

$$a = -g/l \, y \quad \text{--- (iv)}$$

$$a = -ky$$

$$\therefore a \propto y \quad \text{--- (v)}$$

Equation (v) represents that motion of simple pendulum follows SHM

Now, from the characteristics of SHM

$$a = -\omega^2 y \quad \text{--- (vi)}$$

comparing eq^n (iv) & (vi)

$$\omega^2 = g/l$$

$$\left(2\pi/T\right)^2 = g/l$$

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$T = 2\pi\sqrt{l/g} \quad \text{--- (vii)}$$

This is the required expression for the time period of simple pendulum.

$$f = \frac{1}{2\pi}\sqrt{g/l}$$

- Second's pendulum.

The pendulum which time period is 2 second is called second's pendulum.

$$2 = 2\pi\sqrt{l/g} \qquad g = 9.8,$$

$$\left(\frac{2}{2\pi}\right)^2 = \frac{l}{9.8},$$

$$\therefore l = 0.992 \text{ m}, \approx 1\text{m},$$

The time period

Question: what will be the time period of second's pendulum when it is taken to the moon

$$T = 2\pi\sqrt{l/g}$$

$$T' = 2\pi\sqrt{l/g'}$$

$$\frac{T'}{T} = \frac{\cancel{2\pi}}{\cancel{2\pi}}\sqrt{\frac{\cancel{l}}{g'} \times \frac{g}{\cancel{l}}}$$

$$\frac{T'}{2} = \sqrt{\frac{g}{g/6}}$$

$$T' = 2\sqrt{6}$$

Date
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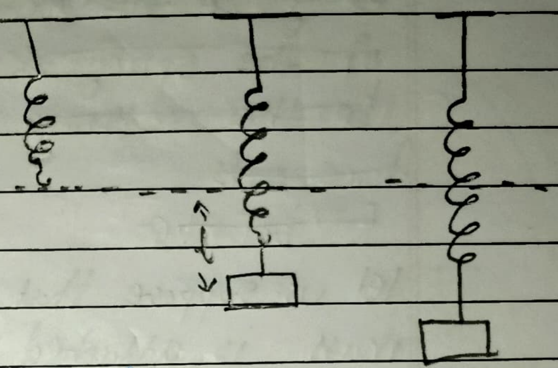
Vertical Oscillation of Spring-mass System



let, us consider a spring constant k suspended through a rigid support as shown in figure.

A small mass m is attached to its end such that is strengthen by a length of l

In this case force of elongation of spring will be

$$F_1 = -kl \quad \text{---} (i)$$

If the mass of is slightly displaced through its mean position & ~~replaced~~ released it will shows to and fro motion.

Here, if y be the displacement of this mass then the total force of elongation of spring is

$$F_2 = -k(l+y) \quad \text{---} (ii)$$

The restoring force acting within the mass m is

$$F = -k(l+y) + kl$$

$$= -ky$$

$$ma = -ky$$

$$a = \frac{-k}{m} y \quad \text{--} (iii) \quad \therefore \; a \propto y \quad \text{--} (iv)$$

From eqn (iv) it is seen that vertical oscillations of spring-mass system shows SHM.

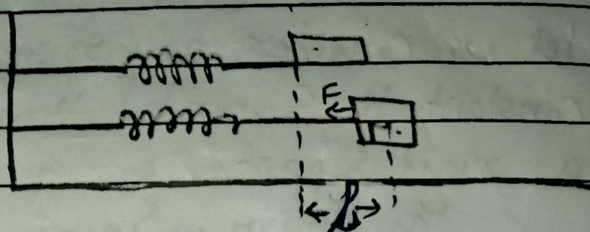
we know from the characteristics of SHM, $a = -\omega^2 y$,

comparing (iii) & (v) we get $\omega^2 = \frac{k}{m}$, $\omega = \sqrt{k/m}$

$$\frac{2\pi}{T} = \sqrt{\frac{k}{M}} \quad \therefore \; T = \sqrt{\frac{m}{k}} \times 2\pi$$

Horizontal Oscillation of Spring-mass System.

~~let y be the extension of the spring & F be the restoring force acts~~



~~F be the~~

let us suppose that one end of string S of negligible mass is attached to the wall and other to an object of mass, m,

let l be the extension of the spring and F be the restoring force set up in the spring. Then from Hooke's law,

$$F \propto l \quad \text{or,} \quad F = -Kl$$

where K is known as force constant of the spring

if a is the accleration produced in the mass, then we have, $F = ma$

$$ma = -kl$$

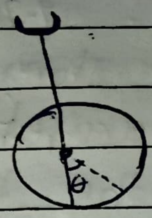
$$a = \frac{-k}{m} l$$

$$a = -\omega^2 l \qquad \frac{K}{m} = \omega^2,$$

for T:

$$T = 2\pi\sqrt{\frac{m}{K}}$$

Angular Simple Harmonic Motion



Let us consider a metal disc of Moment of inertia I is suspended through rigid support by a string of torsional constant k as shown in figure.

Here, the disc is rotated by an angular displacement of θ such that the restoring torque produced within it is $\tau \propto \theta$

$$\tau = -k\theta \quad \text{--- (i)}$$

where k is the proportionality constant which is also known as the torsional constant of the spring.

we know that

$$\tau = I\alpha$$

$$I\alpha = -k\theta$$

$$\alpha = \frac{-k}{I}\theta \quad \text{--- (ii)}$$

$$\alpha \propto \theta \quad \text{--- (iii)}$$

Eqn (iii) represents that the disc shows angular SHM

from eqn (ii)

$$\omega^2 = \frac{k}{I}$$

$$\omega = \sqrt{\frac{k}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{k}} \quad \text{(v)}$$

$y = x\cos\omega t$

Oscillations,

The to & fro motion of an object about mean positions is known as oscillations.

Generally oscillations are of three types

1. free oscillations
2. Damped oscillations
3. Forced Oscillations,

i) The oscillations of an object in absence of external resistive force is known as free oscillations with its natural frequency.

for eg: The oscillations simple pendulum in Vaccum

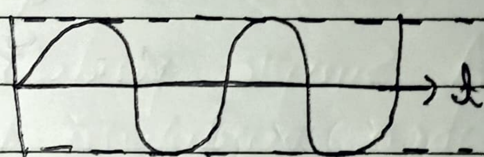


fig: Amplitude time graph of free oscillation.

ii) Damped Oscillation.

The Oscillation of an object in presence of external resistive force is known as damped oscillation. In damped oscillation amplitude gradually decrease with time.

For eg: The oscillation of simple pendulum in lab

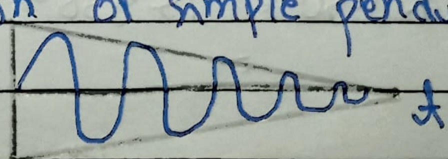


fig:- Amplitude time graph of damped oscillation

(iii) Forced Oscillation:-

If the object oscillate in presence of external force then such type of scillation is Known as forced oscillation. The external force supports the oscillation of an object.

Resonance:

If the natural frequency of vibration of an object matched with external frequency then object Vibrates with maximum amplitude then this is Known as resonance.

For ex: → The Cracking glass in a closed room when loudspeaker is played with high db.

→ The destruction of building during Earthquake