HISSAN CENTRAL EXAMINATION - 2080 (2023)

Grade: XI **F.M.: 75**

Time : 3 hrs

MATHEMATICS (0071 D2)

Candidates are required to give their answers in their own words as far as practicable.

GROUP A

Attempt ALL Questions. **[1 × 11 = 11]**

Write the correct option in your answer sheet.

1. If p and q are two statements then the conditional $\sim p \Rightarrow \sim q$ is said to be the ..... of the conditional $p \Rightarrow q$.
   a. converse b. tautology c. inverse d. contra positive

2. What is the value of x if $x + iy = (2 - 3i)(2 + 3i)$?
   a. 0 b. 2 c. 3 d. 13

3. The principal value of $\tan^{-1}(\tan\frac{3\pi}{4})$ is
   a. $\frac{3\pi}{4}$ b. $-\frac{3\pi}{4}$ c. $\frac{\pi}{4}$ d. $-\frac{\pi}{4}$

4. The length of the perpendicular drawn from the point (b, a) on the line $\frac{x}{a}+\frac{y}{b}=0$ is
   a. $\sqrt{a^2+b^2}$ b. $-\sqrt{a^2+b^2}$ c. $\pm\sqrt{a^2+b^2}$ d. 1

5. Three vectors $\vec{i}+\vec{k}$, $\vec{i}+\vec{j}$ and $-\vec{i}-\vec{k}$ are
   a. Linearly independent b. Linearly dependent
   c. coplanar d. perpendicular

6. If $n = 10$, $\sum x^2 = 170$, and $\sum x = 10$ then the standard deviation is
   a. $\sqrt{17}$ b. 4 c. $\sqrt{13}$ d. 3

7. Which one of the following is the value of $\lim_{x\to 2}\frac{x^5-32}{x-2}$?
   a. $\frac{0}{0}$ b. 16 c. 32 d. 80

8. Which one of the following is the derivative of $\sec^2 x$ with respect to tanx ?
   a. tanx b. 2 tan x c. secx. tanx d. secx

9. Which one of the following is the integral of $\int \frac{dA}{1-\sin A}$?

a. $\tan A - \sec A + C$ b. $\sec^2 A + C$

c. $\tan A + \sec A + C$ d. $\sec^2 A + C$

10. Which one of the following is the area bounded by curve $y = 3x$, the x axis and the ordinates $x = 0$ and $x = 4$?

a. 6 $sq.unit$ b. 12 $sq.unit$ c. 24 $sq.unit$ d. 48 $sq.unit$

11. For a continuous function $f(x)$, the equation $f(x) = 0$ has at least one root in the interval (a, b) if

a. $f(a).f(b) = 0$ b. $f(a).f(b) \leq 0$

c. $f(a).f(b) > 0$ d. $f(a) - f(b) = 0$

OR

A bicycle is travelling at the rate of 16m/s. Its velocity slows down at the rate of $4m/s^2$, the time taken by the bicycle to come to rest is

a. 8 sec b. 6 sec c. 4 sec d. 2 sec

GROUP B

[8 × 5 =40]

12. a) Define odd function with an example. [1]

b) Write any one relation that satisfy by A.M, G.M and H.M between any two unequal positive numbers. [1]

c) Write the formula to find the sum of infinite geometric series if the common ratio is less than 1. [1]

d) What is the absolute value of a complex number x + iy? [1]

e) What types of function exists its inverse? Write it. [1]

13. Prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2+b^2+c^2)(a+b+c)(a-b)(b-c)(c-a).$$ [5]

14. a) Solve: $\sec y . \tan y = \sqrt{2}$. [3]

b) If $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$, prove that $x^2+y^2=1$. [2]

15. a) Express $\vec{r} = (8, -5)$ as the linear combination of $\vec{a} = (2, -3)$ and $\vec{b} = (-1, -2)$. [2]

b) Show that the vectors $\vec{x} - 2\vec{y} + 3\vec{z}$, $-2\vec{x} + 3\vec{y} - 4\vec{z}$ and $-\vec{y} + 2\vec{z}$ are Coplanar, where $\vec{x}, \vec{y}, \vec{z}$ are any three vectors. [3]

16. a) Calculate the coefficient of skewness based on mean, mode and standard deviation from the following data:

wages(in Rs)	400	410	420	430	440
No. of persons	2	6	4	8	4

[3]

b) What is the probability that a leap-year selected at random contains 53 Fridays? [2]

17. For any function $f(x) = 2x^3 - 6x^2 + 5$
   a) Find the values of x for $f'(x) = 0$. [1]
   b) Find the sign of $f''(x)$ at the values of x [1]
   c) Find where the graph is concave upward and concave downward [1+1]
   d) If F(x) is the anti-derivative of f(x) then what is the value of $\int_a^b f(x)dx$? [1]

18. a) Evaluate : $\lim_{x \to \theta} \frac{x \sin\theta - \theta \sin x}{x - \theta}$ [3]

   b) Evaluate: $\int e^{\sin^2 x} \sin 2x \, dx$ [2]

19. Find a root of the equation $x^3 - x - 4 = 0$ between 1 and 2 to three place of decimal by Newton-Raphson method. [5]

OR

Forces (A- B), A and (A+ B) act at a point in the direction parallel to the sides of an equilateral triangle taken in order. Find the magnitude and direction of the resultant. [5]

GROUP C

[3 × 8 = 24]

20. a) Rewrite $-1 < x < -\frac{1}{3}$ by using absolute value sign. [2]

   b) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, prove that $x^x \cdot y^y \cdot z^z = 1$. [3]

   c) If α and β are the roots of the equation $x^2 - px + q = 0$, find the equation whose roots are $\alpha^2 \beta^{-1}$ and $\beta^2 \alpha^{-1}$. [3]

21. a) Find the equation of the bisector of the angle between the lines $3x - 2y + 1 = 0$ and $18x + y - 5 = 0$ which contain the origin. [2]

b) Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and curve $x^2 + y^2 = c^2$ are right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$. [3]

c) Find the direction cosines l, m, n of two lines which satisfy the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$. [3]

22. a) Find the derivative of $\frac{1}{\sqrt{5x-2}}$ from first principle. [4]

b) Find the area of the region between the curves $y^2 = 4ax$ and $x^2 = 4ay$. [4]

THE END