

## # Thermal Expansion.

Expansion :- Solid.

- Most solid expands when they are heated & contract when they are cooled. Expansion occurs in substance due to change in temperature is called thermal expansion. This phenomena occurs in all state of matter.

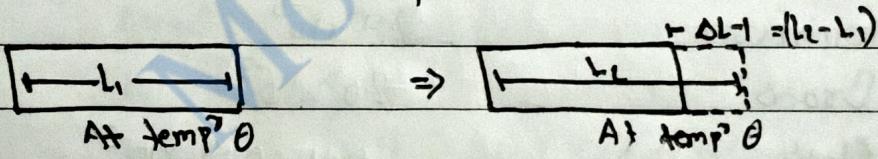
Molecules are held together by strong attractive force called intermolecular solid forces. When solid is heated, vibrational K.E. of its molecules increases & they begin to vibrate with larger amplitude, so they require larger space. As a result, the equilibrium distance b/w molecules increases & hence solid expands.  
Increase in length :- linear expansion in 1 dimension.

Increase in length :- Area expansion in 2 dimension

Cubical expansion :- Volume expansion in 3 dimension

### ① Linear expansion ( $\alpha$ alpha)

The increase in length of a solid body due to increase in temp<sup>r</sup> is called linear expansion.



Consider a metal rod having length  $L_1$  at temperature  $\theta_1$ . When it is heated, let its temp<sup>r</sup> be increased to  $\theta_2$  & length  $\Delta L_2$ .

So change in temperature,  $\Delta \theta = \theta_2 - \theta_1$ , & change in length  $\Delta L = L_2 - L_1$

Experimentally, it is found that the increase in the length of body is directly proportional to. (i) to original length ( $L_1$ ) — ①

(ii) to change in temp<sup>r</sup>,  $\Delta \theta$  — ②

Combining eqn ① & ② we get

$$\Delta L \propto L_1 \Delta \theta$$

or  $\Delta L = \alpha L_1 \Delta \theta$  [where  $\alpha$  is the proportionality constant also called coefficient of linear expansion or linear expansibility of the material of rod] — ③

from eqn (iii)

$$l_2 - l_1 = \alpha l_1 \Delta \theta$$

$$\therefore l_2 = l_1 + \alpha l_1 \Delta \theta$$

$$\therefore l_2 = l_1 (1 + \alpha \Delta \theta) \quad (\text{iv})$$

This eqn gives the final length of the road at temp  $\theta_2$ .

From eqn (iii)

$$\alpha = \frac{\Delta l}{l \cdot \Delta \theta} \quad \text{let us put } l_1 = 1 \text{ unit of length}$$

$\Delta \theta = 1^\circ$  : unit of temperature.

$$\therefore \alpha = \frac{\Delta l}{1 \cdot 1}$$

Linear expansivity of rod is defined as the change in temperature length per unit original length per unit rise in temperature.

Unit SI unit : Per Kelvin ( $K^{-1}$ )

(Gr.S unit) : Per degree celsius ( ${}^{\circ}\text{C}^{-1}$ )

Some examples of  $\alpha$

Material	$\alpha ({}^{\circ}\text{C}^{-1} \text{ or } K^{-1})$
Steel	$1.2 \times 10^{-5}$
Aluminium	$9.4 \times 10^{-5}$
Bronze	$2.0 \times 10^{-5}$
Glass	$0.4 - 0.9 \times 10^{-5}$
Copper	$1.7 \times 10^{-5}$
Gold	$1.4 \times 10^{-5}$

# linear expansivity of steel is  $1.2 \times 10^{-5} K^{-1}$  what does it means?

Ans

$$\alpha = \frac{\Delta l}{l \Delta \theta}, \text{ we have:-}$$

Let us

assume  $l=1 \text{ m}$ , original length, change in temperature, change in temp  $\Delta \theta = 1K$ ,

then  $\alpha = 1.2 \times 10^{-5} K^{-1}$  meas. i.e. every  $1K$  change in temp, the  $1 \text{ m}$  original length of steel ro.1 exanxted through  $1.2 \times 10^{-5} \text{ m}$  length.

## Superficial Expansion

The expansion in area (i.e. in 2 dimension) when temp<sup>r</sup> rises is called Superficial expansion

Consider the plane sheet of metal having Area  $A_1$  at initial temp<sup>r</sup>  $\theta_1$ , when it is heated to higher temp<sup>r</sup>  $\theta_2$  its area increases to  $A_2$

So the change in area  $\Delta A$  is directly proportional to

i) the original area of plane sheet  $\Delta A \propto A_1$  --- (i)

ii) the change (or rise) in temp<sup>r</sup>  $\Delta A \propto \Delta \theta$  --- (ii)

Combining eq<sup>n</sup> (i) & (ii), we get,

$$\Delta A \propto A_1 \Delta \theta$$

or  $\Delta A = \beta A_1 \Delta \theta$  --- (iii) where  $\beta$  is proportional

constant known as coefficient of superficial expansivity of material of plane sheet. If unit is  $K^{-1}$  in SI unit &  $^{\circ}C^{-1}$  in CGS unit.

for eq<sup>n</sup> (iii)

$$A_2 - A_1 = \beta A_1 \Delta \theta$$

$$A_2 = \beta A_1 \Delta \theta + A_1$$

$$A_2 = A_1 (1 + \beta \Delta \theta) \quad \text{The eq<sup>n</sup> gives final area}$$

at temp<sup>r</sup>  $\theta_2$ . Again from eq<sup>n</sup> (iii)

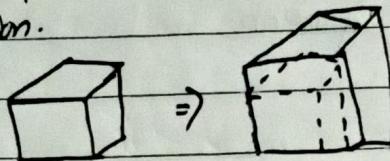
$$\beta = \frac{\Delta A}{A_1 \Delta \theta}$$

So, Superficial expansivity is defined as, change in area per unit area per unit rise in temp<sup>r</sup>.

$$\frac{\alpha}{T} = \frac{\beta}{V_2} = \frac{r}{3}$$

## Cubical Expansion:

Expansion takes place in volume, when temperature rises is called Cubical expansion.



Let us consider, the solid block of metal having volume  $V_1$  at initial temperature  $\theta_1$ . When it is heated, temperature rises & volume increases to  $V_2$  experimentally.

The change in temperature (Volume) of solid block, is directly proportional to the its original Volume.

- (i) its original Volume  $\Delta V \propto V_1$  --- ①
- (ii) change (or rise in temp)  $\Delta V \propto \Delta \theta$  --- ②

Combining eqn ① & ②

$\Delta V = \lambda V_1 \Delta \theta$  where  $\lambda$  is proportional constant known as coefficient of cubical expansion of a material) --- ③

From ③

$$\Delta V \propto V_1 \Delta \theta$$

$$V_2 - V_1 = \lambda V_1 \Delta \theta$$

$V_2 - V_1 = \lambda V_1 \Delta \theta$  (1 +  $\lambda \Delta \theta$ ) This gives of the expansion of linear volume at temp  $\theta_2$

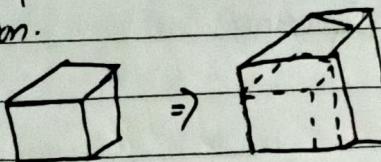
$$\therefore \cancel{V_1 \propto} \quad \lambda = \frac{\Delta V}{V_1 \Delta \theta}$$

So, cubical expansion is defined as the change in volume per unit volume per unit temperature.

$$\frac{\alpha}{T} = \frac{\beta}{2} = \frac{r}{3}$$

### Cubical Expansion:

Expansion takes place in volume, when temperature rises is called Cubical expansion.



Let us consider, the solid block of metal having volume  $V_1$ , at an initial temperature  $\theta_1$ . When it is heated, temperature rises & volume increases to  $V_2$  experimentally.

The change in temperature (volume) of solid block is directly proportional to the its original volume.

$$\text{i} \text{.i} \text{t's original volume } \Delta V \propto V_1, \dots \text{---(i)}$$

$$\text{ii) change (or rise in temp) } \Delta V \propto \Delta \theta \dots \text{---(ii)}$$

Combining eqn (i) & (ii)

$$\text{or } \Delta V = \lambda V_1 \Delta \theta \text{ where } \lambda \text{ is proportional constant known as coefficient of cubical expansion of a material} \dots \text{---(iii)}$$

From (iii)

$$\Delta V = \lambda V_1 \Delta \theta$$

$$V_2 - V_1 = \lambda V_1 \Delta \theta$$

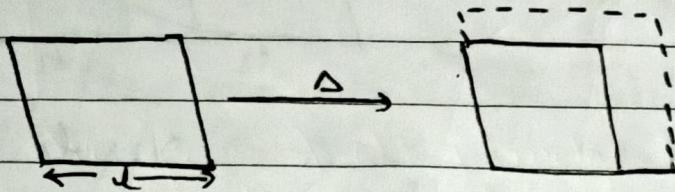
$$V_2 = V_1 (1 + \lambda \Delta \theta) \text{ This gives } \text{ of the expansion of initial volume at temp } \theta_2$$

$$\therefore \cancel{V_1 \Delta \theta} \quad \lambda = \frac{\Delta V}{V_1 \Delta \theta}$$

So, Cubical expansion is defined as the change in volume per unit volume per unit temperature.

Relation bet<sup>n</sup> linear expansivity, Superficial expansivity and cubical expansivity.

### ① Relation bet<sup>n</sup> $\alpha$ & $\beta$



Let us consider the square plane sheet of Metal having length  $l_1$  at initial temp  $\theta_1$ .

$$\therefore \text{Area}, A_1 = l_1^2$$

It is heated to temp  $\theta_2$  so, that its length increases to  $l_2$  & Area increases,

$$\text{i.e } A_2 = l_2^2$$

So linear expansion,

$$\text{Final length, } l_2 = l_1 (1 + \alpha \Delta \theta) \dots \text{(i)}$$

where  $\alpha$  = linear expansivity of the material of plane sheet.

So, Final Area

$$A_2 = l_2^2 = l_1^2 (1 + \alpha \Delta \theta)^2$$

$$A_2 = A_1 (1 + 2\alpha \Delta \theta + \underbrace{\alpha^2 \Delta \theta^2}_{\text{negligible}}) \dots \text{(ii)}$$

As the value of  $\alpha$  is very small, its higher power values are also very small value. So, it can be negligible.  $\dots \text{(iii)}$

$\therefore$  eq<sup>n</sup> (ii) become,

$$A_2 = A_1 (1 + 2\alpha \Delta \theta) \dots \text{(iv)}$$

Again from Superficial expansion.

$$A_2 = A_1 (1 + \beta \Delta \theta) \dots \text{(v)}$$

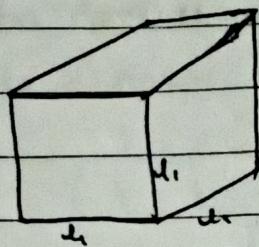
where  $\beta$  = superficial expansivity of the material of plane sheet.

Comparing eq<sup>n</sup> (iv) & (v) we get

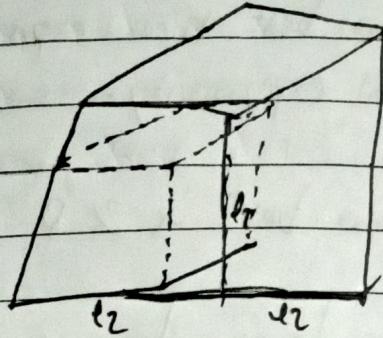
$$\beta = 2\alpha$$

i.e superficial expansivity of substances is two times of its linear expansivity.

(ii) Relation betw  $\alpha$  &  $\gamma$



$\Delta$



$$V_1 = l_1^3$$

$$V_2 = l_2^3$$

let us consider, the cube of substances having length  $l_1$  at initial temperature  $\theta_1$

initial volume,  $V_1 = l_1^3$

It is heated to higher temp  $\theta_2$  so. its length increases to  $l_2$  & volume increases to  $V_2$

$$V_2 = l_2^3$$

$\therefore$  final length,

$$l_2 = l_1 (1 + \alpha \Delta \theta) \dots \text{(i)}$$

final volume

$$V_2 = (l_2)^3 = l_1^3 (1 + \alpha \Delta \theta)^3$$

$$\text{or } V_2 = V_1 (1 + 3\alpha \Delta \theta + 3\alpha^2 \Delta \theta^2 + \alpha^3 \Delta \theta^3) \dots \text{(ii)}$$

As  $\alpha$  is very small, so, its higher power is also very small in Volume. So, it can be neglected,

$$V_2 = V_1 (1 + 3\alpha \Delta \theta)$$

we have from cubical expansion,

$$V_2 = V_1 (1 + \gamma \Delta \theta) \dots \text{iv}$$

where  $\gamma$  = cubical expansivity of material of substance.

$\therefore$  Comparing (ii) & (iv)

$$\therefore \gamma = 3\alpha \dots \text{v}$$

cubical expansivity of body is three of its linear body.

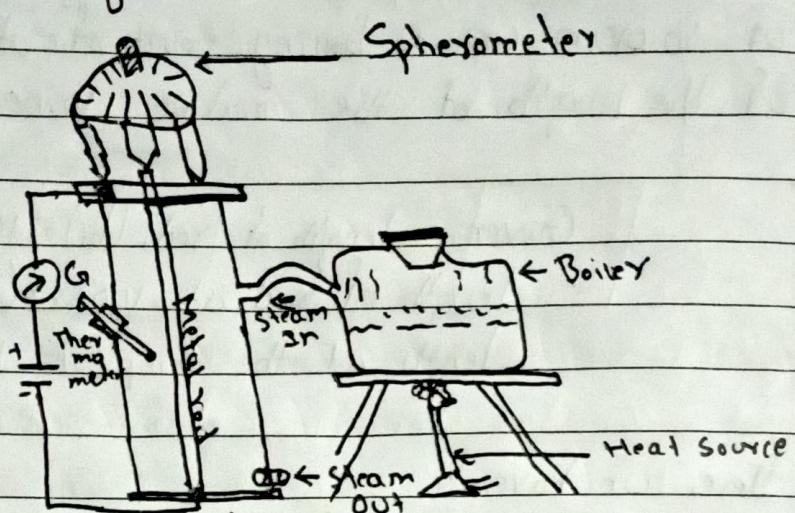
Here, As  $\alpha = \beta / 2$

$$\text{so, } \alpha = \beta / 2 \quad \& \quad \gamma = 3\alpha$$

$$\alpha = \frac{\gamma}{3}$$

$$\therefore \alpha = \frac{\gamma}{2} = \frac{\gamma}{3}$$

# Pullinger's Apparatus for determination of coefficient of linear expansion of given solid.



Pullinger's apparatus is used to determine coefficient of linear expansion of given solid.

Construction:-

It consists of the hollow cylinder, in which the given metallic rod is placed. Its one end is fixed at bottom and its upper end is free to expand.

The thermometer is fixed at the middle of the tube to measure temperature. Steam is passed from the boiler inside the cylinder to heat the rod & it is passed out from another opening. At the top of the instrument, spherometer is placed to measure increase in length. It is also connected with the circuit joining the spherometer and the bottom end of the rod, to check whether the leg of spherometer touches the rod or not.

Working:- A metallic rod is taken & its initial length is measured

$$\text{Original length} = l_1 \dots \text{(i)}$$

It is placed under (i.e. inside) the cylinder & initial temp<sup>r</sup> is noted Initial temp<sup>r</sup> =  $\theta_1 \dots \text{(ii)}$

The initial reading of spherometer is noted & it is also rotated upward to some rotation in order to give space for expansion.

Now, steam is passed & rod is now expanded and after some time it will attain the final temperature  $\theta_2$ .

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final temp<sup>r</sup> =  $\theta_2$ . Finally the spherometer is rotated downward in order to touch the rod expanded & final observation are noted.

no. of cs: Initial Circular Scale division (CSD) =  $x$   
no. of rotation raised =  $n_1$ .

After expansion,

No. of rotation lowered =  $n_2$  & final CSO =  $y$ .

∴ Increase In length

$$\Delta l = \text{no. of rotation raised} - [\text{No. of rotation lowered} + (\text{Initial CSO} - \text{final CSO}) \times L_c] \\ = n_1 - [n_2 + (x-y) \times L_c]$$

∴ Coefficient of linear expansion is given by.

$$\alpha = \frac{\Delta l}{L_c \times (\theta_2 - \theta_1)}$$

By putting the respective values from observation in above eq<sup>n</sup> the value of  $\alpha$  is determined

# Force setup due to expansion or contraction.

Let us, suppose that a metal rod is fixed at two rigid ends  $S_1$  &  $S_2$ . Suppose initial temp<sup>r</sup>  $\theta_1$ . It is heated to final temp<sup>r</sup>  $\theta_2$ . The rod tries to expand but will not able to do so, due to expand blockage (fixed support). As a result a force of tension is developed which compress the rod.

from the linear expansion,

$$l_2 = l_1 (1 + \alpha \Delta \theta)$$

$$\therefore \Delta l = l_2 - l_1 = l_1 \times \alpha \Delta \theta \quad \dots \dots (1)$$

from the definition of young's modulus of elasticity.

$$Y = \frac{\text{Normal Stress}}{\text{Longitudinal Strain}}$$

$$\gamma = \frac{F_A}{A \Delta l_1} = \frac{F_A l_1}{A \Delta l}$$

$$\text{or } F = Y A \frac{\Delta l}{l_1} \dots (i)$$

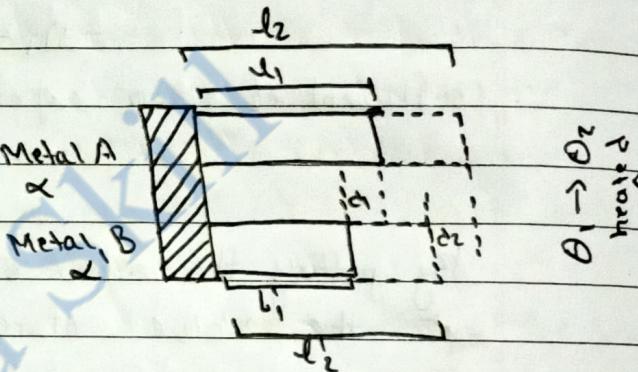
From eqn (i) and (ii)

$$F = \frac{Y A l_1 \alpha \Delta \theta}{l_1}$$

$$\therefore F = T = Y \alpha A \Delta \theta.$$

### # Differential expansion

$$\begin{aligned}\Delta l - \Delta l' &= \text{differential expansion of Metal A} \\ &= (l_2 - l_1) - (l_1' - l_1) \\ &= (l_2 - l_1') - l_1 - l_1' \\ &= d_2 - d_1\end{aligned}$$



Different solids have different values of expansivities & expand by different amount when they are heated through same range of temperature.

Let us consider two metallic rods A & B made of different materials having initial length \$l\_1\$ & \$l\_1'\$ and linear expansivities \$\alpha\$ & \$\alpha'\$ at temp \$\theta\_1\$ c respectively. Let their one end be fixed to rigid support & another end is free to expand when they are heated to high temp \$\theta\_2\$, their length becomes \$l\_2\$ & \$l\_2'\$ respectively. Here we can write:

$$\text{For Metal A } \Rightarrow l_2 = l_1(1 + \alpha \Delta \theta)$$

$$\text{For Metal B } \Rightarrow l_2' = l_1'(1 + \alpha' \Delta \theta)$$

let  $d_1$  be difference in length bet<sup>n</sup> metal A & B at initial temp<sup>r</sup>  $\theta_1$  &  $d_2$  be their difference in length at temp<sup>r</sup>  $\theta_2$   
 $\therefore d_1 = l_1 - l_1' \dots (iii)$   
 $\therefore d_2 = l_2 - l_2' \dots (iv)$

$$\text{now, } d_2 = l_2' - l_2$$

$$d_2 = l_1'(1 + \alpha' \Delta \theta) - l_1(1 + \alpha \Delta \theta)$$

$$d_2 = l_1' + l_1 \alpha' \Delta \theta - l_1 - l_1 \alpha \Delta \theta$$

$$d_2 = l_1' - l_1 + (l_1 \alpha' - l_1 \alpha) \Delta \theta.$$

$$\therefore d_2 - d_1 = (l_1 \alpha' - l_1 \alpha) \Delta \theta \dots (v)$$

This is the expansion for differential expansion for two rods at two different temperatures.

# Condition for zero differential expansion.

when the difference in length of 2 different rods are same at all temp<sup>r</sup> i.e.  $d_1 = d_2$

from eq<sup>n</sup> (v)

$$d_2 - d_1 = (l_1 \alpha' - l_1 \alpha) \Delta \theta$$

$$\therefore (l_1 \alpha' - l_1 \alpha) \Delta \theta = 0$$

Here,  $\Delta \theta \neq 0$

$$\therefore l_1 \alpha' - l_1 \alpha = 0$$

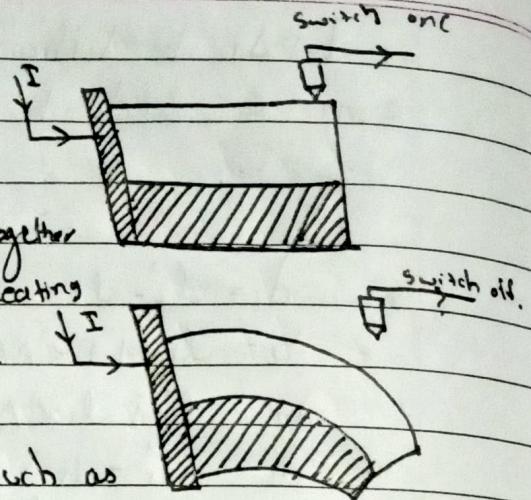
$$\therefore l_1 \alpha' = l_1 \alpha$$

$$\therefore \frac{l_1'}{l_1} = \frac{\alpha}{\alpha'} \dots (vi)$$

Thus, if initial lengths of materials of rod are so chosen that, the ratio of their initial length is called equivalent to ratio of reciprocal of their linear expansivities, then differential expansion will be zero.

## # Bimetallic Thermostat.

- It consists of two thin metal strips of different expansivities welded together along their length, specially used in heating devices to control over heating of the devices.



When the bimetallic strip such as brass-steel strip is heated, as brass having the higher value of linear expansivity than that of steel, brass expands more than steel. So the strip bends together into an arc with longer brass piece lying on convex surface. As a result the electrical circuit is broken.

It is used for controlling the temp of laundry irons, hot water storage tanks, aquarium for tropical fish.

## # Variation of density with temperature.

We know, mass = volume  $\times$  density.

$$\therefore \rho = \frac{m}{V} \quad \text{or} \quad \rho \propto \frac{1}{V}$$

Let us consider, the body of mass 'm' have initial volume  $V_1$  at initial temperature  $\theta_1$ . Its initial density be  $\rho_1$ .

$$\rho_1 = \frac{m}{V_1} \quad \text{--- (i)}$$

When the object is heated to final temp  $\theta_2$ , volume is increased to  $V_2$  & since  $\rho = \frac{m}{V}$  density is decreased to  $\rho_2$

$$\text{given by, } \rho_2 = \frac{m}{V_2} \quad \text{--- (ii)}$$

Let,  $\gamma$  be cubical expansivity of the body --- (iii)

$$\therefore \gamma_2 = V_1(1 + \gamma \Delta \theta) \dots \text{--- (ii)}$$

Substituting eq<sup>n</sup> (ii) in eq<sup>n</sup> (i)

$$S_2 = \frac{m}{V_1(1 + \gamma \Delta \theta)}$$

$$\therefore S_2 = \frac{S_1}{(1 + \gamma \Delta \theta)}$$

$$\therefore S_1 = S_2(1 + \gamma \Delta \theta) \dots \text{--- (iv)} \quad \checkmark$$

Numerical:-

The density of Silver at  $0^\circ\text{C}$  is  $10310 \text{ kg/m}^3$  and the linear expansivity is  $0.000019/\text{C}$  calculate its density at  $100^\circ\text{C}$

$$\text{Here, } \theta_1 = 0^\circ\text{C} \quad \& \quad \theta_2 = 100^\circ\text{C}$$

$$\Delta \theta = 100^\circ\text{C}$$

$$\alpha = 0.000019 \quad \& \quad \gamma = 5.7 \times 10^5$$

$$S_1 = 10310 \quad \& \quad S_2 = ?$$

$$S_1 = S_2(1 + \gamma \Delta \theta)$$

$$10310 = S_2(1 + 5.7 \times 10^5 \times 100)$$

$$10310 = S_2(1.0057)$$

$$S_2 = 10251.57 \text{ kg/m}^3$$

## # Expansion of liquid

AB = expansion of solid container.

CA = Apparent expansion of liquid.

BC = Expansion of solid expansion.

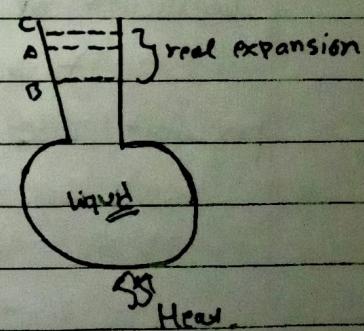
from fig.

$$CB = AB + CA$$

Real expansion = Apparent expansion +  $\frac{\text{volume}}{\text{area}}$  expansion of solid vessel

$$\therefore \gamma V_1 \Delta \theta = \gamma_a V_1 \Delta \theta + \gamma_g V_1 \Delta G$$

$$\Delta V_r = \Delta V_a + \Delta V_g$$



Remaining are noted beside this sheet:-

## # Expansion in liquids

Liquid has only size, but it does not have any definite shape. Liquid can't be heated directly. So, liquid is heated by putting it inside the vessel.

Suppose the glass vessels contains the liquid of certain volume upto mark A. When the vessel is heated, at first the vessel expands, so due to expansion, liquid level is decreased to another mark B.

As the heat continues, the liquid also gets heated & begins to expand. As the volume expansivity of the liquid is greater than that of solid, liquid expands more than solid. As a result its level rises upto mark C crossing mark A. From figure,

(B) indicates the real expansion of liquid but for the observer, the liquid expanded from initial mark A to final mark C. So (A) indicates the apparent expansion of liquid. & AB indicates the expansion of solid gas vessels.

$$\Delta V_r = \Delta V_a + \Delta V_g$$

real exp<sup>n</sup> of liquid = Apparent exp<sup>n</sup> of liquid + expansion of vessel

$$\text{i.e. } \Delta V_r = \Delta V_a + \Delta V_g \dots \text{--- (1)}$$

Here,

Coefficient of real expansion of liquid

$\gamma_r$  is defined as real increase in volume of liquid per unit rise in temperature.

$$\gamma_r = \frac{\Delta V_r}{V_i \times \Delta \theta}$$

$$\text{or, } \Delta V_r = \gamma_r V_i \Delta \theta \dots \text{--- (1)}$$

Coefficient of apparent expansion of liquid

$\gamma_a$  is defined as an apparent increase in volume of liquid per unit original volume per unit rise in temperature.

A steel wire 8 m long & 4mm diameter is fixed to two rigid supports. calculate the increase in tension, when the temp falls by 10°C

Given,  $\gamma = 7 \times 10^7 \text{ N/m}^2$  and  $\alpha = 12 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}$ .

Difference in temps  $\Delta\theta = 10^{\circ}\text{C}$

~~Gross~~ Increase in tension

$$T = \gamma A \alpha \Delta\theta$$

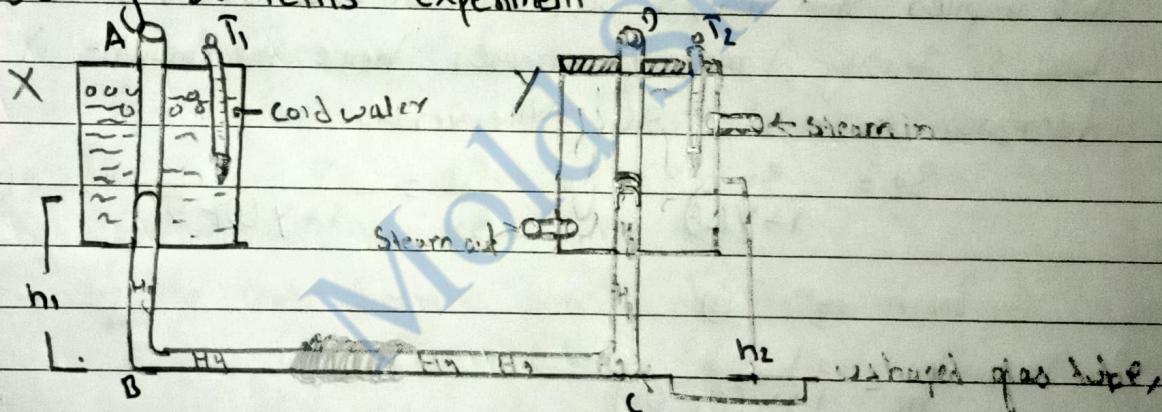
$$T = \gamma \times \cancel{\pi r^2} \times \alpha \times \Delta\theta,$$

$$T = 7 \times 10^7 \times \frac{\pi}{4} \times \frac{4}{4}^2 \times 12 \times 10^{-6} \times 10$$

$$= 7 \times 10^7 \times \frac{\pi}{4} \times 1.2 \times 10^{-7}$$

$$= 301.71 \text{ N.}$$

### • Dulong & Petit's experiment



⇒ Experiment arrangement for Dulong & petit's experiment.

This experiment is used to determine the real expansivity of liquid. It is based on the principle of hydrostatics. It consists of U-shaped glass tube ABCD which contains mercury whose real expansivity is to be found.

Limb AB of the tube is placed inside chamber X, which is maintained at cold temperature using ice-water. Limb CD of the tube is placed inside chamber Y which is regulated in hot temperature by providing steam in it.

Thermometer  $T_1$  &  $T_2$  are fixed at two chambers  $X$  &  $Y$  respectively to measure their respective temperature. BC tube is covered with wet cloth to prevent unnecessary heat flow from hot tube to cold tube.

After some time, the steady state occurs, so that the hydrostatic pressure of A, B tube is equal to tube C.  
let  $h$  in tube AB

$$h_1 = \text{height of mercury}$$

$$\rho_1 = \text{density of mercury}$$

$$\theta_1 = \text{temperature of AB}$$

$$\therefore \text{Hydrostatic force of AB} = \text{Hydrostatic force of CD}$$

$$h_1 \rho_1 g = h_2 \rho_2 g$$

$$\frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} \dots \textcircled{1}$$

$$h_2 = \text{height of mercury}$$

$$\rho_2 = \text{density of mercury}$$

$$\theta_2 = \text{temperature of CD}$$

This implies that  $h_2 \propto \rho_2$  such that, In cold tube AB temp is low, so density of mercury increases, hence the height of mercury decreases & in tube DC vice-versa.

$$\rho_2 = \frac{\rho_1}{1 + \gamma \Delta \theta} \quad \text{or} \quad \frac{\rho_2}{\rho_1} = \frac{1}{1 + \gamma (\theta_2 - \theta_1)} \dots \textcircled{2}$$

From eqn 1

$$\frac{h_1}{h_2} = \frac{1}{1 + \gamma (\theta_2 - \theta_1)}$$

$$h_1 (1 + \gamma (\theta_2 - \theta_1)) = h_2$$

$$\therefore h_1 + h_1 \gamma (\theta_2 - \theta_1) = h_2$$

$$\therefore h_1 \gamma (\theta_2 - \theta_1) = h_2 - h_1$$

$$\therefore \gamma (\theta_2 - \theta_1) = \frac{h_2 - h_1}{h_1}$$

$$\therefore \gamma (\theta_2 - \theta_1) = \frac{h_2 - h_1}{h_1}$$

By substituting  $h_2, h_1, \theta_2, \theta_1$  on above eqn  $\gamma$  is determined.

## # Source of error.

- ① As the distance bet two limbs are little for the difference in height, h<sub>1</sub>-h<sub>2</sub> can't be measured accurately.
- ② Temperature of the hot tube & cold tube must be measured accurately.
- ③ The heat mustn't be transferred from hot tube to cold tube to prevent this the tube must be taken very narrow & it should be covered with wet cloth.

## # Anomalous expansion of water.

Generally, matter expands when heated but, water have some exception. Water shows anomalous behaviour between 0°C to 4°C such that water contracts on increasing its temperature from 0°C to 4°C instead of expansion. So at 4°C, water attains minimum volume and max density (i.e  $S_{\text{max}} = 1 \text{ gm/cm}^3$ ).

This kind of unusual expansion of water is called anomalous expansion of water. This property have an importance significance in the nature.

- Significance in the nature.
- ① Aquatic life can survive inside the frozen pond in cold region.
  - During the normal temperature, water in pond or lake is in thermal equilibrium with the surrounding. As the temperature of surrounding falls down & reaches to 4°C, initially the water at surface also becomes at temperature 4°C. We know that at 4°C, water attains maximum density. So, being heavy water at 4°C will descend to bottom to displace it with light water which moves upward. When all the water becomes 4°C, the circulation stops. Now the surrounding temp is still falling so that water at surface below 4°C being lighter than 4°C water, it always remains at surface & when temperature falls to 0°C, it freezes.