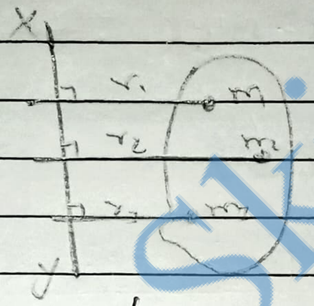
Radius of Gyration :

let us suppose a rigid body containing n particles having masses $m_1, m_2 \dots m_n$ at a distance $r_1, r_2 \dots r_n$ respectively from axis of rotation.

The moment of inertia is given by

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2 \quad \text{--- (i)}$$



let, $m_1 = m_2 = m$ (mass of each particles)

$$I = m(r_1^2 + r_2^2 + \dots r_n^2)$$

Also, $MK^2 = I$ --- (iii) M = total mass of a body.
K = radius of gyration.

from (ii) & (iii)

$$MK^2 = m(r_1^2 + r_2^2 + \dots + r_n^2)$$

$$n\cancel{m}K^2 = \cancel{m}(r_1^2 + r_2^2 + \dots + r_n^2)$$

$$K = \sqrt{\frac{r_1^2 + r_2^2 + \dots + r_n^2}{n}}$$

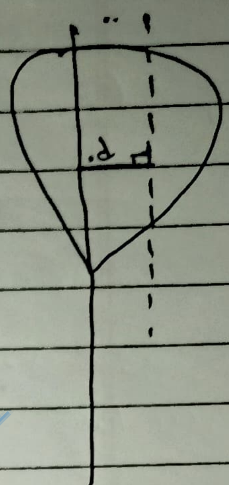
The radius of gyration is also define as square root of average of perpendicular distance of particle from axis of rotation

It is defined as the distance from the axis of rotation to a point where the total mass of the body is supposed to be concentrated.

// Parallel axis theorem of m.I

The moment of inertia of a body about an axis is the sum of moment of inertia about parallel axis through its center of mass & product of mass & square of distance between parallel axis

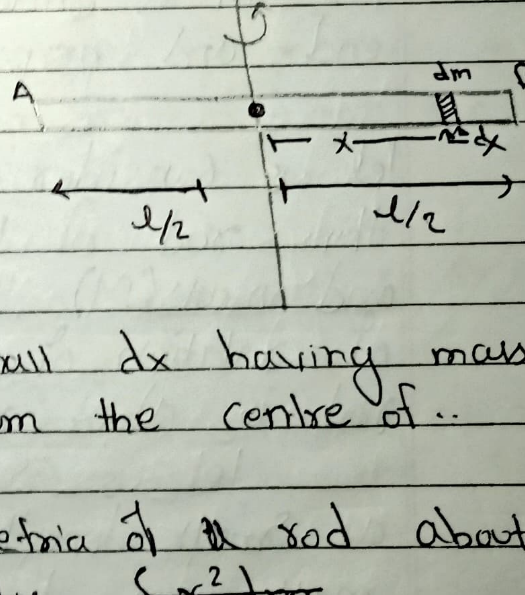
$$I = I_{cm} + Md^2$$



Calculation of Moment of Inertia of Rigid bodies.

- Moment of Inertia a uniform road about an axis through its centre & perpendicular to is length

let us suppose a uniform thin rood of length (l), let us suppose XOY axis of rotation passing through the centre of rod perpendicular to its length,



let us suppose a small dx having mass dm is at a distance x from the centre of.. (axis of rotation)

The moment of inertia of a rod about an axis of rotation is given by ~~$\int x^2 dm$~~

$$I = \int x^2 \, dm$$

$$I = \int_{-l/2}^{l/2} x^2 \, dm \quad --(1)$$

The linear mass density of a uniform rod remains constant at everypoint i.e $\frac{M}{l} = \frac{dm}{dx}$ then $dm = \frac{M}{l} dx$

$$I = \int_{-l/2}^{l/2} \frac{M}{l} x^2 \, dx$$

$$= \frac{M}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{l/2}$$

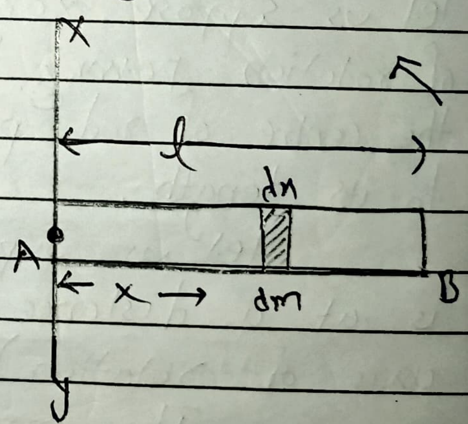
$$= \frac{M}{l}\left\{\frac{l^3}{8\times3} - \left(-\frac{l^3}{8\times3}\right)\right\}$$

$$= \frac{M}{l}\left\{\frac{l^3}{24} + \frac{l^3}{24}\right\}$$

$$= \frac{2 l^3 M}{24 l}$$

$$= \frac{l^2 M}{12}$$

2. M.I of a uniform rod passing through one end and perpendicular to length



let us consider a uniform thin rod of length l and mass(M) the axis of rotation of a uniform rod is about x-y

let us suppose a small length dx of mass dm, located at a distance (x) from axis of rotation, the linear mass density of a uniform rod remains constant at every point

$$\frac{M}{l} = \frac{dm}{dx} \quad \text{or,} \quad dm = \frac{M}{l} \times dx$$

The moment of inertia of rod passing through one fixed end is

$$I = \int_0^{l} x^2 \, dm$$

$$I = \int_0^{l} \frac{M}{l} \times dx \times x^2$$

$$I = \frac{M}{l}\left[\frac{x^3}{3}\right]_0^{l}$$

$$= \frac{M}{l} \frac{l^3}{3}$$

$$\therefore \frac{Ml^2}{3} \quad - (i)$$

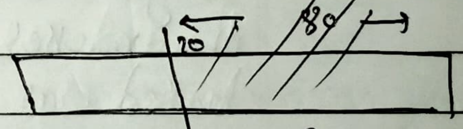
we know,

$I = MK^2 \quad -- (ii)$ from (i) & (ii) $K = \frac{l}{\sqrt{3}}$,

Calculate the rotational inertia of a meter stick, with mass 0.56 kg, about an axis perpendicular to the stick & located at the 70cm mark. (treat the stick as thin rod)

Sol^n

$m = 0.56$ kg

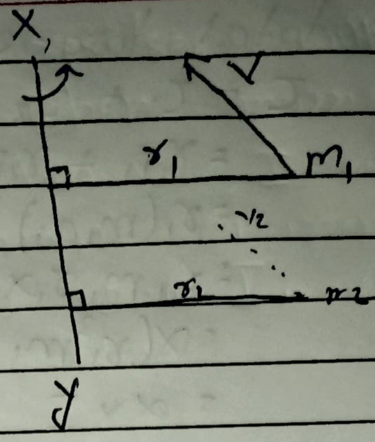


What is moment of inertia of a 1m stick of a mass 300g, about an axis at right angle to the stick and located at 30cm mark.

The moment of inertia of a uniform rod of mass 0.5 kg & length 1 m is 0.10 kg/m^2 about a line perpendicular to rod. find the distance of this line from the middle poin of the rod,

(a)

$L = I\omega$



$$\left.\begin{aligned} v_1 &= r_1\omega \\ v_2 &= r_2\omega \\ &\vdots \\ v_n &= r_n\omega \end{aligned}\right\} - (1)$$

Total angular momentum of a system is given by

$L = L_1 + L_2 + \ldots + L_n$
$= m_1 v_1 r_1 + m_2 v_2 r_2 + \ldots + v_n r_n m_n$
$= m_1 r_1 \omega r_1 + m_2 r_2 \omega r_2 + \ldots + m_n r_n \omega r_n$
$= \omega(m_1 r_1^2 + \ldots m_n r_n^2)$
$L = I\omega$

This is the required exp of angular momentum it is found that ~~a~~ the angular momentum is product of moment of inertia and angular velocity about an axis of ~~a~~ rotation.

Principle of conservation of angular momentum.

In absence of external torque acting on a system, the total angular momentum of a system remains constant

i.e L = constant, $I\omega$ = constant,

let us suppose a rigid body, is rotating with angular velocity ω, moment of inertia about an axis i, and angular momentum l, angular momentum is given by

$l = I\omega$

differenciating with respect to time,

$$L = I\omega \quad -①$$

$$\frac{dL}{dt} = \frac{d I\omega}{dt}$$

$$\frac{dL}{dt} = \cancel{\frac{dI}{dt}} \times \omega + I \times \frac{d\omega}{dt}$$

$$\Rightarrow \frac{dL}{dt} = I\alpha$$

* The rate of change of angular momentum gives torque,

In absence of torque, $\tau = 0$

$$\frac{dL}{dt} = 0$$

$$dL = 0$$

Integrating on both side

$$\int dL = \int 0$$

$L = 0$ (constant)

$I\omega$ = constant

$$I \propto \frac{\text{constant}}{\omega}$$

$$I \propto \frac{T}{2\pi} \qquad [\because I \propto T]$$