

1. Define Quadratic equation. Prove that the quadratic equation cannot have more than two roots. Discuss the nature of the roots of a quadratic equation.
2. Show that the roots of the equation  $x^2 - 4lmx + (l^2 + 2m^2)^2 = 0$  are imaginary.
3. For what values of  $m$  will the equation  $x^2 - 2(5 + 2m)x + 3(7 + 10m) = 0$  have equal roots?
4. Prove that the roots of the equation  $x^2 + (2k - 1)x + k^2 = 0$  are real if  $k \leq \frac{1}{4}$ .
5. Discuss the relation between roots and coefficients. Find a quadratic equation whose roots are the square squares of the roots  $3x^2 - 5x + 4 = 0$
6. For what value of  $k$  will the equation  $(3k + 1)x^2 - 5x + 4 = 0$  may have
  - a) Roots equal in magnitude and opposite in sign
  - b) One root zero.
  - c) Reciprocal roots.  $\sim$
  - d) Sum is equal to 4.  $\sim$
7. Find the value of  $m$  for which one root of the equation  $x^2 + mx + 1 = 0$  is the square of other.
8. Find the value of  $P$  in  $2x^2 - (p + 1)x + (p - 1) = 0$  if  $\alpha - \beta = \alpha\beta$ .
9. If the roots of the equation  $rx^2 + sx + t = 0$  be in the ratio of  $m : n$ , prove that

$$\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} + \sqrt{\frac{t}{r}} = 0$$

10. If the roots of the equation  $12x^2 - mx + 5 = 0$  are in the ratio of 2:3. find the value of  $m$ .
11. Determine the condition for a quadratic equation may have i) one root common ii) both root common. Also, prove that  $a^3 + b^3 + c^3 = 3abc$  if the equations  $ax^2 + bx + c = 0$  and  $bx^2 + cx + a = 0$  have a common root.
12. If  $P$  be the length of the perpendicular dropped from the origin on the line  $\frac{x}{a} + \frac{y}{b} = 1$ , prove that

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$$

13. The length of the perpendicular drawn from the point  $(a, 3)$  on the line  $3x + 4y + 5 = 0$  is 4. Find the value of  $a$ .
14. Find the equation of the two straight lines each of which is parallel to and at a distance of  $\sqrt{5}$  from the line  $x + 2y - 7 = 0$
15. Find the equation of the line which is at right angles to  $3x + 4y = 12$ , such that its perpendicular distance from the origin is equal to the length of the perpendicular from  $(3, 2)$  On the given line.
16. If  $p$  and  $p'$  be the length of the perpendiculars from the origin upon the straight line whose equations are  $x \sec \theta + y \csc \theta = a$  and  $x \cos \theta - y \sin \theta = a \cos 2\theta$ , prove that  $4p^2 + p'^2 = a^2$



17. The origin is a corner of a square and two of its sides are  $y + 2x = 0$  and  $y + 2x = 3$ . Find the equation of the other two sides.
18. Find the equation of the two lines represented by  $x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$ . Prove that the two lines are parallel. Also, find the distance between them.
19. If the line pairs  $ax^2 + 2hxy + by^2 = 0$  and  $a'x^2 + 2h'xy + b'y^2 = 0$  have the same bisectors, prove that  $h(a' - b') = h'(a - b)$ .
20. Find the value of  $k$  so that the lines which join the origin to the point of intersection of the lines  $y - x = k$  and the curve  $x^2 + y^2 + 4x - 6y - 36 = 0$  may be at right angles.
21. Define Conjunction and Disjunction.
22. Define Conditional and Bi-conditional with example.
23. Let  $p, q, r$  and  $s$  be four simple statements. If  $p$  is true,  $q$  is false,  $r$  is true and  $s$  is false, find the truth values of the following compound statements.  
 a)  $p \wedge q$       b)  $(p \vee q) \wedge (r \vee s)$       c)  $[(p \wedge q) \Rightarrow p] \Rightarrow (q \wedge \sim q)$  is a contradiction
24. Let  $A, B$  and  $C$  be the subsets of a universal set  $U$ . Then prove that  
 i)  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$   
 ii)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
25. State and prove De-Morgan's Law.
26. For any two real numbers  $x$  and  $y$ . Prove that  
 a)  $|x + y| \leq |x| + |y|$   
 b)  $|x - y| \geq |x| - |y|$
27. Prove that if  $x \in \mathbb{R}$  and  $a$  be any positive real number then  $|x| < a \Rightarrow -a < x < a$  and conversely.
28. Solve the following inequalities  
 a)  $6 + 5x - x^2 \geq 0$   
 b)  $\frac{x(x+2)}{x-1} \leq 0$
29. Solve the inequalities of  $|2x + 1| \geq 3$
30. Compute the following limit  
 a)  $\lim_{x \rightarrow 0} \frac{4x^3 - x^2 + 2x}{3x^2 + 4x}$       b)  $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^2 - 7x + 10}$       c)  $\lim_{x \rightarrow a} \frac{\sqrt{3a-x} - \sqrt{x+a}}{4(x-a)}$
31. Calculate the following limits  
 a)  $\lim_{x \rightarrow \infty} (\sqrt{x} - \sqrt{x-3})$       b)  $\lim_{x \rightarrow \infty} \sqrt{x}(\sqrt{x} - \sqrt{x-a})$
32. Evaluate the following  
 a)  $\lim_{x \rightarrow a} \frac{\sin(x-a)}{x^2 - a^2}$       b)  $\lim_{x \rightarrow 0} \frac{1 - \cos 6x}{x^2}$       c)  $\lim_{x \rightarrow \theta} \frac{x \cot \theta - \theta \cot x}{x - \theta}$