

Group 'A'

[11 × 1 = 11]

1. The statement $p \vee \sim p$ is a
 - a) Contradiction
 - b) Tautology
 - c) Contrapositive
 - d) None of these
2. For two non-empty sets A and B, then $A - B =$
 - a) $A \cap \bar{B}$
 - b) $B \cap \bar{A}$
 - c) $B - A$
 - d) $\bar{A} \cap \bar{B}$
3. The function $f: R \rightarrow R$ defined by $f(x) = x^2$ is
 - a) one to one
 - b) onto
 - c) one to one and onto
 - d) neither one to one nor onto
4. If A is a square matrix, then $A - A^T$ is a
 - a) skew-symmetric matrix
 - b) symmetric matrix
 - c) diagonal matrix
 - d) identity matrix
5. If α and β are the roots of the equation $3x^2 - 2x - 1 = 0$ then the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to
 - a) -2
 - b) -2/3
 - c) -1/3
 - d) 1/2
6. If $z = \frac{1}{1+2i}$, then $\bar{z} =$
 - a) $\frac{1}{5} - \frac{2}{5}i$
 - b) $\frac{1}{5} + \frac{2}{5}i$
 - c) $-\frac{1}{5} + \frac{2}{5}i$
 - d) $-\frac{1}{5} - \frac{2}{5}i$
7. The perpendicular distance between the two lines $4x - 3y = 12$ and $4x - 3y = 2$ is
 - a) 12/5
 - b) 2/5
 - c) 2
 - d) 10

8. The obtuse angle between the line pair $2x^2 + 7xy + 3y^2 = 0$ is
 a) 120° b) 135° c) 150° d) none of them
9. If the direction ratios of a line are 1, 2, -2 then its direction cosines are
 a) $1/4, 1/2, -1/2$ b) $1/9, 2/9, -2/9$
 c) $1/5, 2/5, -2/5$ d) $1/3, 2/3, -2/3$
10. The two vectors (3, -6) and (1, 4) are
 a) linearly independent b) linearly dependent
 c) collinear d) none of them
11. If $\lim_{x \rightarrow 3} \frac{x^n - 3^n}{x - 3} = 108$ then the value of $n =$
 a) 6 b) 5 c) 4 d) 3

Group 'B'

[8 × 5 = 40]

12. a) Define conditional of two statements [1]
 b) Compute the truth table of the statement $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ [2]
 c) Prove that $\overline{A \cup B} = \bar{A} \cap \bar{B}$ [2]
13. a) For any two real numbers x and y prove that $|x - y| \geq |x| - |y|$ [2]
 b) Solve the inequality $x^2 - 2x - 3 \geq 0$ [3]
14. a) Show that $\begin{vmatrix} a - b - c & 2a & 2a \\ 2b & b - c - a & 2b \\ 2c & 2c & c - a - b \end{vmatrix} = (a + b + c)^3$ [3]
 b) Given a matrix $\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix}$ find a matrix $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ such that they are inverse of each other. [2]
15. a) Determine the nature of the roots of $x^2 - 6x + 5 = 0$ [1]
 b) If $a + b + c = 0$ solve the equation
 $(b + c)x^2 + (c + a)x + (a + b) = 0$ [2]
 c) Prove that $(1 - i^3)^6 \cdot \left(1 - \frac{1}{i^3}\right)^6 = 64$ [2]

16. a) i) Define absolute value of a complex number. [1]

ii) Write down its any four properties. [1]

b) Find the equation of the two straight line drawn through the point $(0,a)$ on which the perpendicular drawn from the point $(2a,2a)$ are each of length a . [3]

17. a) Find the equation to the straight lines passing through $(1,1)$ and parallel to the lines represented by $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$ [3]

b) Find the point where the line joining the points $(2,-3,1)$ and $(3,-4,-5)$ cuts the plane $2x + y + z = 7$. [2]

18. a) Define collinear of a vector. [1]

b) Show that the $\vec{a} - 2\vec{b} + 3\vec{c}$, $-2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-\vec{b} + 2\vec{c}$ are collinear vector. [2]

c) Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x} - \sqrt{x-3})$ [2]

19. a) Define continuity of a function at a point. [1]

b) Discuss the continuity of function at a given point

$$f(x) = \begin{cases} 2 - x^2 & \text{for } x \leq 2 \\ x - 4 & \text{for } x > 2 \end{cases} \text{ at } x=2$$
 [2]

c) Find the derivative of $\frac{x^2 - a^2}{x^2 + a^2}$ [2]

20. a) If $A = \begin{pmatrix} 2 & 4 & 3 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{pmatrix}$

i) find A^T [1]

ii) Show that the sum of the given matrix and its transpose is a symmetric matrix. [1]

iii) Show that the difference of the given matrix and its transpose is a skew-symmetric matrix. [1]

iv) Express the given matrix A as the sum of the symmetric and skew-symmetric matrix form. [1]

b) If Z and W are two complex numbers, prove that $|Z - W| \geq |Z| - |W|$ [4]

21. a) Prove that the straight lines joining the origin to the point of intersection of line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are right angles if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$ [4]

b) Find the direction cosines of two lines which satisfy the relation:

$l + m + n = 0$ and $2lm - mn + 2nl = 0$ also find the angle between the lines. [4]

22. a) Evaluate $\lim_{x \rightarrow \theta} \frac{x \cos \theta - \theta \cos x}{x - \theta}$ [3]

b) Use the chain rule find $\frac{dy}{dx}$ of $y = \frac{t}{t^2 - 1}$ and $t = 3x^2 + 1$ [2]

c) Find from the first principles, the derivative of $\cos^2 x$ [3]