

# HISSAN CENTRAL EXAMINATION - 2080 (2023)

Grade: XI

F.M.: 75

Time : 3 hrs

## MATHEMATICS (0071 D2)

Candidates are required to give their answers in their own words as far as practicable.

### GROUP A

[1 × 11 = 11]

Attempt ALL Questions.

Write the correct option in your answer sheet.

1. If  $p$  and  $q$  are two statements then the conditional  $\sim p \Rightarrow \sim q$  is said to be the ..... of the conditional  $p \Rightarrow q$ .  
a. converse      b. tautology      c. inverse      d. contra positive
2. What is the value of  $x$  if  $x + iy = (2 - 3i)(2 + 3i)$ ?  
a. 0      b. 2      c. 3      d. 13
3. The principal value of  $\tan^{-1}(\tan \frac{3\pi}{4})$  is  
a.  $\frac{3\pi}{4}$       b.  $-\frac{3\pi}{4}$       c.  $\frac{\pi}{4}$       d.  $-\frac{\pi}{4}$
4. The length of the perpendicular drawn from the point  $(b, a)$  on the line  $\frac{x}{a} + \frac{y}{b} = 0$  is  
a.  $\sqrt{a^2 + b^2}$       b.  $-\sqrt{a^2 + b^2}$       c.  $\pm \sqrt{a^2 + b^2}$       d. 1
5. Three vectors  $\vec{i} + \vec{k}$ ,  $\vec{i} + \vec{j}$  and  $-\vec{i} - \vec{k}$  are  
a. Linearly independent      b. Linearly dependent  
c. coplanar      d. perpendicular
6. If  $n = 10$ ,  $\sum x^2 = 170$ , and  $\sum x = 10$  then the standard deviation is  
a.  $\sqrt{17}$       b. 4      c.  $\sqrt{13}$       d. 3
7. Which one of the following is the value of  $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2}$  ?  
a.  $\frac{0}{0}$       b. 16      c. 32      d. 80
8. Which one of the following is the derivative of  $\sec^2 x$  with respect to  $\tan x$  ?  
a.  $\tan x$       b.  $2 \tan x$       c.  $\sec x \cdot \tan x$       d.  $\sec x$



9. Which one of the following is the integral of  $\int \frac{dA}{1-\sin A}$  ?

a.  $\tan A - \sec A + C$

b.  $\sec^2 A + C$

c.  $\tan A + \sec A + C$

d.  $\sec^2 A + C$

10. Which one of the following is the area bounded by curve  $y = 3x$ , the x axis and the ordinates  $x=0$  and  $x=4$  ?

a. 6 sq.unit    b. 12 sq.unit    c. 24 sq.unit    d. 48 sq.unit

11. For a continuous function  $f(x)$ , the equation  $f(x) = 0$  has at least one root in the interval  $(a, b)$  if

a.  $f(a) \cdot f(b) = 0$

b.  $f(a) \cdot f(b) < 0$

c.  $f(a) \cdot f(b) > 0$

d.  $f(a) - f(b) = 0$

OR

A bicycle is travelling at the rate of 16m/s. Its velocity slows down at the rate of  $4\text{m/s}^2$ , the time taken by the bicycle to come to rest is

a. 8 sec

b. 6 sec

c. 4 sec

d. 2 sec

### GROUP B

[8 × 5 = 40]

12. a) Define odd function with an example. [1]

b) Write any one relation that satisfy by A.M, G.M and H.M between any two unequal positive numbers. [1]

c) Write the formula to find the sum of infinite geometric series if the common ratio is less than 1. [1]

(d) What is the absolute value of a complex number  $x + iy$ ? [1]

e) What types of function exists its inverse? Write it. [1]

13. Prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2 + b^2 + c^2)(a+b+c)(a-b)(b-c)(c-a). \quad [5]$$

14. (a) Solve:  $\sec y \cdot \tan y = \sqrt{2}$ . [3]

b) If  $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$ , prove that  $x^2 + y^2 = 1$ . [2]

15. a) Express  $\vec{r} = (8, -5)$  as the linear combination of  $\vec{a} = (2, -3)$  and  $\vec{b} = (-1, -2)$ . [2]

b) Show that the vectors  $\vec{x} - 2\vec{y} + 3\vec{z}$ ,  $-2\vec{x} + 3\vec{y} - 4\vec{z}$  and  $-\vec{y} + 2\vec{z}$  are Coplanar, where  $\vec{x}$ ,  $\vec{y}$ ,  $\vec{z}$  are any three vectors. [3]



16. a) Calculate the coefficient of skewness based on mean, mode and standard deviation from the following data:

wages(in Rs)	400	410	420	430	440
No. of persons	2	6	4	8	4

- b) What is the probability that a leap-year selected at random contains 53 Fridays? [3]
17. For any function  $f(x) = 2x^3 - 6x^2 + 5$
- a) Find the values of  $x$  for  $f'(x) = 0$ . [1]
- b) Find the sign of  $f''(x)$  at the values of  $x$  [1]
- c) Find where the graph is concave upward and concave downward [1+1]
- d) If  $F(x)$  is the anti-derivative of  $f(x)$  then what is the value of  $\int_a^b f(x)dx$ ? [1]

18. a) Evaluate :  $\lim_{x \rightarrow \theta} \frac{x \sin \theta - \theta \sin x}{x - \theta}$  [3]

- b) Evaluate:  $\int e^{\sin^2 x} \sin 2x dx$  [2]

19. Find a root of the equation  $x^3 - x - 4 = 0$  between 1 and 2 to three place of decimal by Newton-Raphson method. [5]

OR

Forces (A - B), A and (A + B) act at a point in the direction parallel to the sides of an equilateral triangle taken in order. Find the magnitude and direction of the resultant. [5]

GROUP C

[3 × 8 = 24]

20. a) Rewrite  $-1 < x < -\frac{1}{3}$  by using absolute value sign. [2]
- b) If  $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ , prove that  $x^x \cdot y^y \cdot z^z = 1$ . [3]
- c) If  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 - px + q = 0$ , find the equation whose roots are  $\alpha^2 \beta^{-1}$  and  $\beta^2 \alpha^{-1}$ . [3]



21. a) Find the equation of the bisector of the angle between the lines  $3x - 2y + 1 = 0$  and  $18x + y - 5 = 0$  which contain the origin. [2]
- b) Prove that the straight lines joining the origin to the point of intersection of the line  $\frac{x}{a} + \frac{y}{b} = 1$  and curve  $x^2 + y^2 = c^2$  are right angles if  $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$ . [3]
- c) Find the direction cosines  $l, m, n$  of two lines which satisfy the equations  $l + m + n = 0$  and  $l^2 + m^2 - n^2 = 0$ . [3]
22. a) Find the derivative of  $\frac{1}{\sqrt{5x-2}}$  from first principle. [4]
- b) Find the area of the region between the curves  $y^2 = 4ax$  and  $x^2 = 4ay$ . [4]

THE END

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