

Quiz for RDC math club entrance

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September, 2020

25' for each question. 100' for full mark.
150 minutes.

Problem 1 Find $a \in \mathbb{R}$ such that

$$\text{sum} \{x \in \mathbb{R} \mid \lfloor x \rfloor (x - \lfloor x \rfloor) = ax^2\} = 420.$$

Problem 2 Calculate

$$\sum_{n=0}^{\infty} \frac{\Re((2+i)^n) \Im((2+i)^n)}{7^n}.$$

($\Re z$ denotes the real part of complex number z , and $\Im z$ denotes the imaginary part of complex number z .)

Problem 3 Let $\{a_j\}_{j \in \mathbb{N}}$ be a strictly increasing sequence such that

$$\{a_j \mid j \in \mathbb{N}\} = \{n^k \mid n, k \in \mathbb{Z}^+, k \geq 2\}.$$

Show that there are infinite many j such that

$$\frac{a_{j+1} - a_j}{9999} \in \mathbb{Z}.$$

Problem 4 Function $f : x \mapsto ax^2 + (2b+1)x - a - 2$, where $a, b \in \mathbb{R}$ and $a \neq 0$, has only one zero on $[3, 4]$. Find $(a^2 + b^2)_{\min}$.

Problem 5 In $\triangle ABC$, $b^2 = ac$, and $2 \sin A = \sin(B - A) + \sin C$. Find $\cos B$.