TIME SERIES ANALYSIS WITH PYTHON

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July, 13, 2016

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INSTALLATION INSTRUCTIONS

- Please install Conda per 'quick install' instructions: http://conda.pydata.org/docs/install/quick.html
- Make sure you have the following packages installed:
  - pandas
  - numpy
  - Statsmodels
  - scikit-learn
  - scipy
- These would be good to have but are not essential:
  - pytz
  - hmmlearn

OUTLINE

- Why time series?
- Quick Pandas intro
- Dealing with dates in Pandas
- Reading + manipulating time-stamped data
- Common time series analytical tools
- Prediction
- Classification

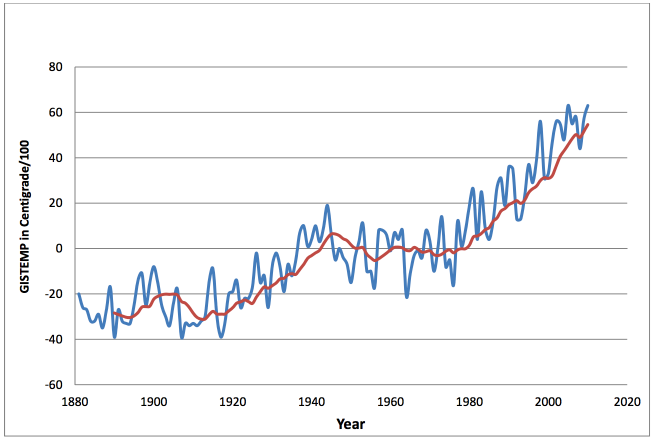
CAVEATS

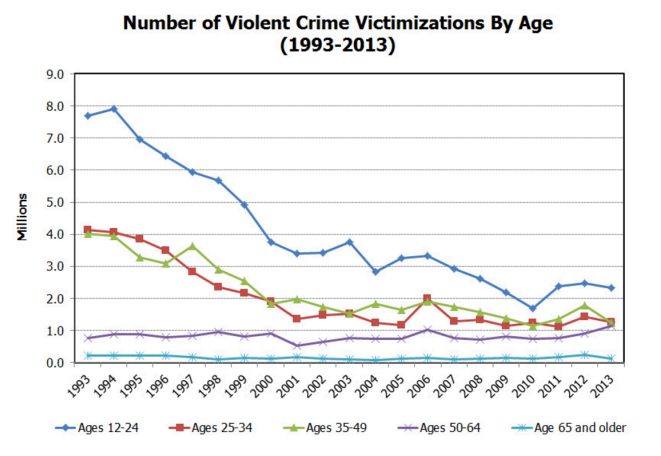
- Time series analysis is a particularly tricky & **controversial** field
- I'll give some background as we move ahead, but **you need to read more** when you want to do a real analysis
- Tests for goodness of fit, etc, are particularly error prone in time series analysis
- Whenever I don't specify, but should, assume it's **iid normally distributed** ('error' terms)

WHAT'S SPECIAL ABOUT TIME SERIES?

WHERE DO TIME SERIES POP UP?

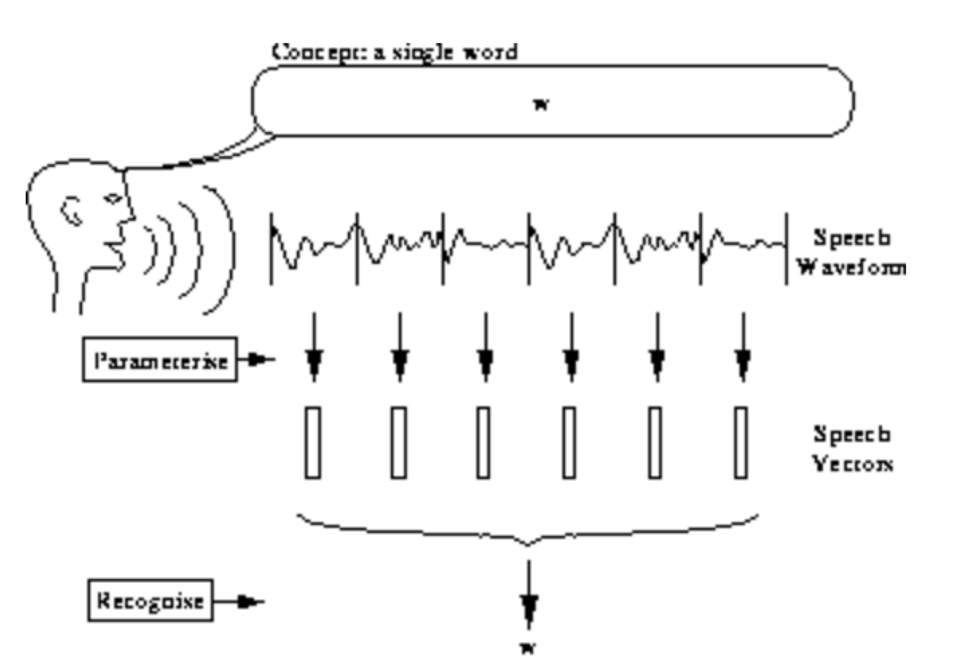
- Many of the most controversial questions arise from time series analysis
- Whenever we want to know the future, we're pretty much stuck with time series analysis
- Ditto for thinking about causality in 'natural experiments'





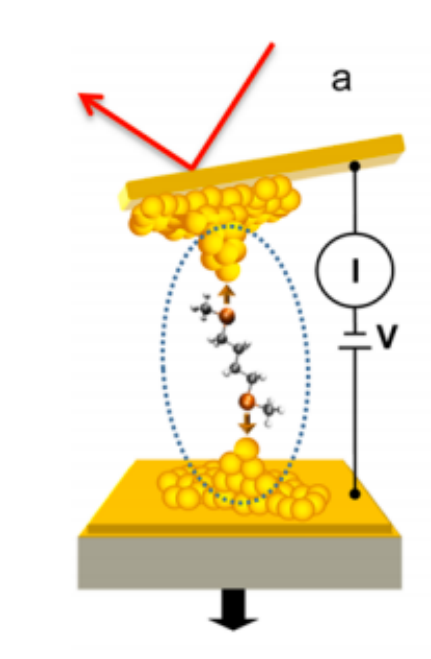
http://www.amstat.org/publications/jse/v21n1/witt.pdf
http://www.forbes.com/sites/neilhowe/2015/05/28/whats-behind-the-decline-in-crime/#3e1e6be07733

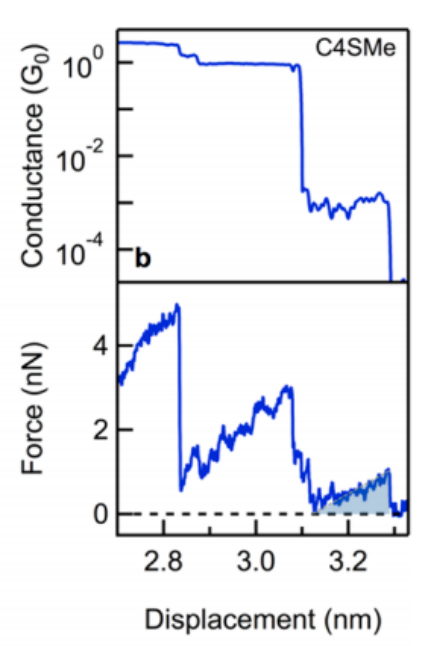
SPEECH RECOGNITION



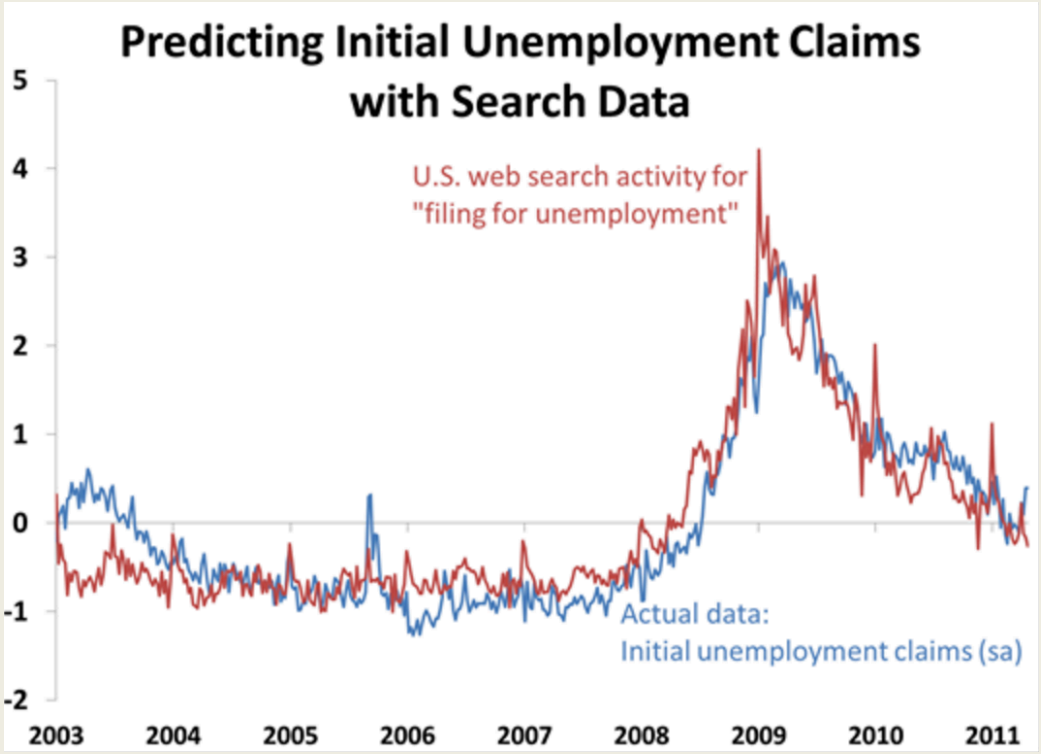
http://www.amstat.org/publications/jse/v21n1/witt.pdf
http://www.forbes.com/sites/neilhowe/2015/05/28/whats-behind-the-decline-in-crime/#3e1e6be07733

PHYSICS EXPERIMENTS



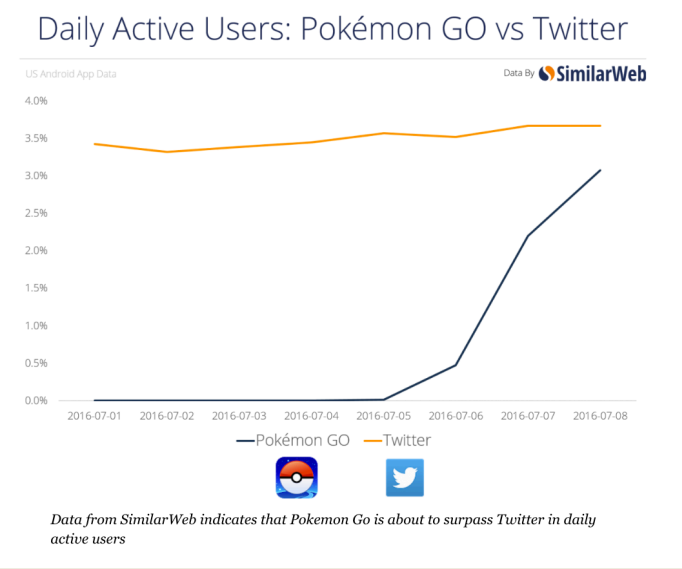


ECONOMICS, GOVERNMENT, POLICY

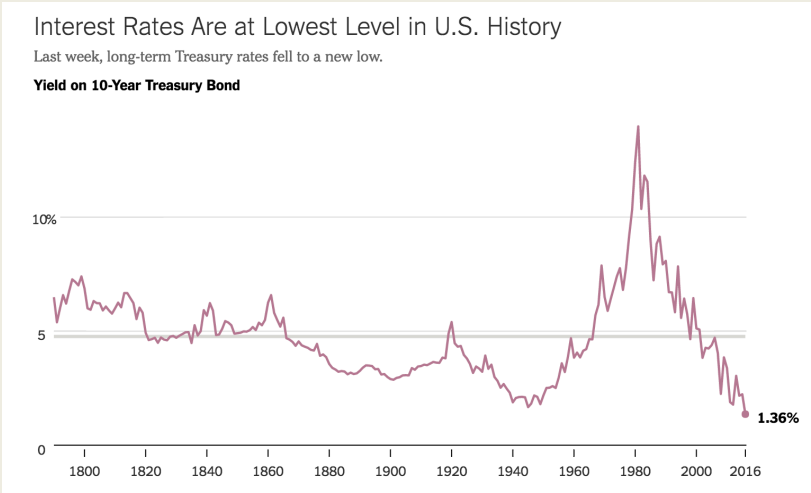


http://freakonomics.com/2011/05/25/mining-for-correlations-it-works/

IN THE NEWS



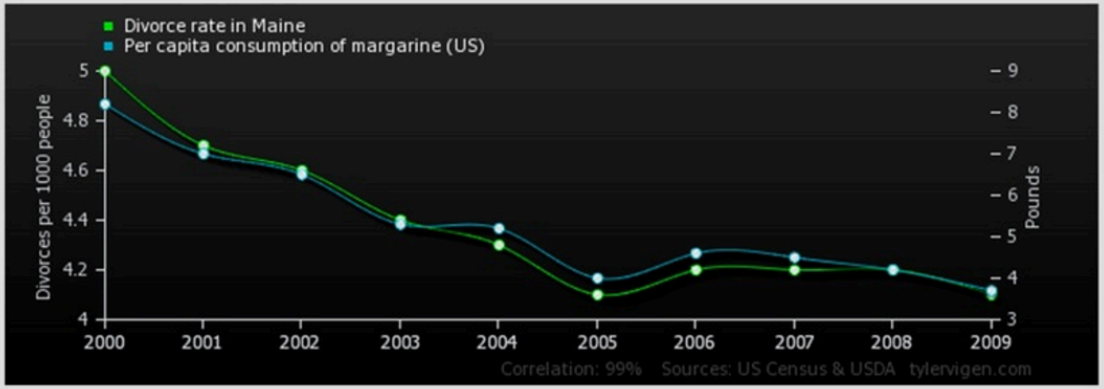
Data from SimilarWeb indicates that Pokemon Go is about to surpass Twitter in daily active users



http://www.forbes.com/sites/jasonevangelho/2016/07/10/pokemon-go-about-to-surpass-twitter-in-daily-active-users/#5c27d4825174

BE CAREFUL! IT'S ESPECIALLY TRUE FOR TIME SERIES THAT YOU NEED TO KNOW SOMETHING ABOUT YOUR DATA

Divorce rate in Maine
correlates with
Per capita consumption of margarine (US)



	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Divorce rate in Maine *Divorces per 1000 people (US Census)*	5	4.7	4.6	4.4	4.3	4.1	4.2	4.2	4.2	4.1
Per capita consumption of margarine (US) *Pounds (USDA)*	8.2	7	6.5	5.3	5.2	4	4.6	4.5	4.2	3.7

Correlation: 0.992558

http://twentytwowords.com/funny-graphs-show-correlation-between-completely-unrelated-stats-9-pictures/

10 MINUTES TO PANDAS

PANDAS USES 'DATA FRAMES' TO PUT DATA IN AN EASY-TO-USE FORMAT

- All the convenience of a SQL-like API, but better
- Fast (in-place) if you know what you're doing
- Fast read/write to standard storage formats like csv and databases
- Let's look at a quick notebook of examples

PANDAS DATA FRAMES ARE BUILT FOR WHAT YOU WANT TO DO ANYWAY

If you're using IPython, tab completion for column names (as well as public attributes) is automatically enabled. Here's a subset of the attributes that will be completed:

```
In [13]: df2.<TAB>
df2.A                  df2.boxplot
df2.abs                df2.C
df2.add                df2.clip
df2.add_prefix         df2.clip_lower
df2.add_suffix         df2.clip_upper
df2.align              df2.columns
df2.all                df2.combine
df2.any                df2.combineAdd
df2.append             df2.combine_first
df2.apply              df2.combineMult
df2.applymap           df2.compound
df2.as_blocks          df2.consolidate
df2.asfreq             df2.convert_objects
df2.as_matrix          df2.copy
df2.astype             df2.corr
df2.at                 df2.corrwith
df2.at_time            df2.count
df2.axes               df2.cov
df2.B                  df2.cummax
df2.between_time       df2.cummin
df2.bfill              df2.cumprod
df2.blocks             df2.cumsum
df2.bool               df2.D
```

http://pandas.pydata.org/pandas-docs/stable/10min.html

DEALING WITH TIME

PANDAS FUNCTIONALITY

- Generate sequences of fixed-frequency dates and time spans
- Conform or convert time series to a particular frequency
- Compute 'relative' dates based on various non-standard time increments (e.g. 5 business days before the last day of the year) or 'roll' dates backward and forward

http://pandas.pydata.org/pandas-docs/stable/timeseries.html

PANDAS TIME-RELATED DATA TYPES

Overview

Following table shows the type of time-related classes pandas can handle and how to create them.

Class	Remarks	How to create
`Timestamp`	Represents a single time stamp	`to_datetime`, `Timestamp`
`DatetimeIndex`	Index of `Timestamp`	`to_datetime`, `date_range`, `DatetimeIndex`
`Period`	Represents a single time span	`Period`
`PeriodIndex`	Index of `Period`	`period_range`, `PeriodIndex`

http://pandas.pydata.org/pandas-docs/stable/timeseries.html

DatetimeIndex

One of the main uses for `DatetimeIndex` is as an index for pandas objects. The `DatetimeIndex` class contains many timeseries related optimizations:

- A large range of dates for various offsets are pre-computed and cached under the hood in order to make generating subsequent date ranges very fast (just have to grab a slice)
- Fast shifting using the `shift` and `tshift` method on pandas objects
- Unioning of overlapping DatetimeIndex objects with the same frequency is very fast (important for fast data alignment)
- Quick access to date fields via properties such as `year`, `month`, etc.
- Regularization functions like `snap` and very fast `asof` logic

http://pandas.pydata.org/pandas-docs/stable/timeseries.html

Time/Date Components

There are several time/date properties that one can access from `Timestamp` or a collection of timestamps like a `DateTimeIndex`.

Property	Description
year	The year of the datetime
month	The month of the datetime
day	The days of the datetime
hour	The hour of the datetime
minute	The minutes of the datetime
second	The seconds of the datetime
microsecond	The microseconds of the datetime
nanosecond	The nanoseconds of the datetime
date	Returns datetime.date
time	Returns datetime.time
dayofyear	The ordinal day of year
weekofyear	The week ordinal of the year
week	The week ordinal of the year
dayofweek	The numer of the day of the week with Monday=0, Sunday=6
weekday	The number of the day of the week with Monday=0, Sunday=6
weekday_name	The name of the day in a week (ex: Friday)
quarter	Quarter of the date: Jan=Mar = 1, Apr-Jun = 2, etc.
days_in_month	The number of days in the month of the datetime
is_month_start	Logical indicating if first day of month (defined by frequency)
is_month_end	Logical indicating if last day of month (defined by frequency)
is_quarter_start	Logical indicating if first day of quarter (defined by frequency)
is_quarter_end	Logical indicating if last day of quarter (defined by frequency)
is_year_start	Logical indicating if first day of year (defined by frequency)
is_year_end	Logical indicating if last day of year (defined by frequency)

http://pandas.pydata.org/pandas-docs/stable/timeseries.html

Offset Aliases

A number of string aliases are given to useful common time series frequencies. We will refer to these aliases as *offset aliases* (referred to as *time rules* prior to v0.8.0).

Alias	Description
B	business day frequency
C	custom business day frequency (experimental)
D	calendar day frequency
W	weekly frequency
M	month end frequency
BM	business month end frequency
CBM	custom business month end frequency
MS	month start frequency
BMS	business month start frequency
CBMS	custom business month start frequency
Q	quarter end frequency
BQ	business quarter endfrequency
QS	quarter start frequency
BQS	business quarter start frequency
A	year end frequency
BA	business year end frequency
AS	year start frequency
BAS	business year start frequency
BH	business hour frequency
H	hourly frequency
T, min	minutely frequency
S	secondly frequency
L, ms	milliseconds
U, us	microseconds
N	nanoseconds

http://pandas.pydata.org/pandas-docs/stable/timeseries.html

DateOffset objects

In the preceding examples, we created DatetimeIndex objects at various frequencies by passing in frequency strings like 'M', 'W', and 'BM to the `freq` keyword. Under the hood, these frequency strings are being translated into an instance of pandas `DateOffset`, which represents a regular frequency increment. Specific offset logic like "month", "business day", or "one hour" is represented in its various subclasses.

Class name	Description
DateOffset	Generic offset class, defaults to 1 calendar day
BDay	business day (weekday)
CDay	custom business day (experimental)
Week	one week, optionally anchored on a day of the week
WeekOfMonth	the x-th day of the y-th week of each month
LastWeekOfMonth	the x-th day of the last week of each month
MonthEnd	calendar month end
MonthBegin	calendar month begin
BMonthEnd	business month end
BMonthBegin	business month begin
CBMonthEnd	custom business month end
CBMonthBegin	custom business month begin
QuarterEnd	calendar quarter end
QuarterBegin	calendar quarter begin
BQuarterEnd	business quarter end
BQuarterBegin	business quarter begin
FY5253Quarter	retail (aka 52-53 week) quarter
YearEnd	calendar year end
YearBegin	calendar year begin
BYearEnd	business year end
BYearBegin	business year begin
FY5253	retail (aka 52-53 week) year
BusinessHour	business hour
CustomBusinessHour	custom business hour
Hour	one hour
Minute	one minute
Second	one second
Milli	one millisecond
Micro	one microsecond
Nano	one nanosecond

http://pandas.pydata.org/pandas-docs/stable/timeseries.html

Anchored Offsets

For some frequencies you can specify an anchoring suffix:

Alias	Description
W-SUN	weekly frequency (sundays). Same as 'W'
W-MON	weekly frequency (mondays)
W-TUE	weekly frequency (tuesdays)
W-WED	weekly frequency (wednesdays)
W-THU	weekly frequency (thursdays)
W-FRI	weekly frequency (fridays)
W-SAT	weekly frequency (saturdays)
(B)Q(S)-DEC	quarterly frequency, year ends in December. Same as 'Q'
(B)Q(S)-JAN	quarterly frequency, year ends in January
(B)Q(S)-FEB	quarterly frequency, year ends in February
(B)Q(S)-MAR	quarterly frequency, year ends in March

http://pandas.pydata.org/pandas-docs/stable/timeseries.html

Holidays / Holiday Calendars

Holidays and calendars provide a simple way to define holiday rules to be used with `CustomBusinessDay` or in other analysis that requires a predefined set of holidays. The `AbstractHolidayCalendar` class provides all the necessary methods to return a list of holidays and only `rules` need to be defined in a specific holiday calendar class. Further, `start_date` and `end_date` class attributes determine over what date range holidays are generated. These should be overwritten on the `AbstractHolidayCalendar` class to have the range apply to all calendar subclasses. `USFederalHolidayCalendar` is the only calendar that exists and primarily serves as an example for developing other calendars.

For holidays that occur on fixed dates (e.g., US Memorial Day or July 4th) an observance rule determines when that holiday is observed if it falls on a weekend or some other non-observed day. Defined observance rules are:

Rule	Description
nearest_workday	move Saturday to Friday and Sunday to Monday
sunday_to_monday	move Sunday to following Monday
next_monday_or_tuesday	move Saturday to Monday and Sunday/Monday to Tuesday
previous_friday	move Saturday and Sunday to previous Friday"
next_monday	move Saturday and Sunday to following Monday

An example of how holidays and holiday calendars are defined:

http://pandas.pydata.org/pandas-docs/stable/timeseries.html

pandas.DataFrame.asfreq

`DataFrame.asfreq`(*freq*, *method=None*, *how=None*, *normalize=False*)

Convert all TimeSeries inside to specified frequency using DateOffset objects. Optionally provide fill method to pad/backfill missing values.

Parameters:

freq *: DateOffset object, or string*

method *: {'backfill', 'bfill', 'pad', 'ffill', None}*

Method to use for filling holes in reindexed Series pad / ffill: propagate last valid observation forward to next valid backfill / bfill: use NEXT valid observation to fill method

how *: {'start', 'end'}, default end*

For PeriodIndex only, see PeriodIndex.asfreq

normalize *: bool, default False*

Whether to reset output index to midnight

Returns:

converted *: type of caller*

http://pandas.pydata.org/pandas-docs/stable/timeseries.html

pandas.DataFrame.resample

`DataFrame.`**`resample`**(*rule*, *how=None*, *axis=0*, *fill_method=None*, *closed=None*, *label=None*, *convention='start'*, *kind=None*, *loffset=None*, *limit=None*, *base=0*)

Convenience method for frequency conversion and resampling of regular time-series data.

Parameters:

rule *: string*
the offset string or object representing target conversion

axis *: int, optional, default 0*

closed *: {'right', 'left'}*
Which side of bin interval is closed

label *: {'right', 'left'}*
Which bin edge label to label bucket with

convention *: {'start', 'end', 's', 'e'}*

loffset *: timedelta*
Adjust the resampled time labels

base *: int, default 0*
For frequencies that evenly subdivide 1 day, the "origin" of the aggregated intervals. For example, for '5min' frequency, base could range from 0 through 4. Defaults to 0

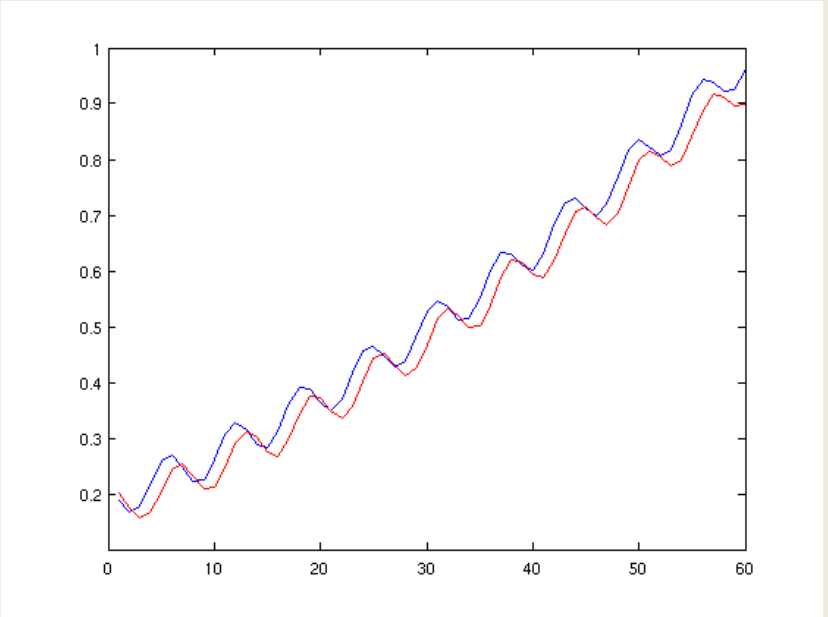
http://pandas.pydata.org/pandas-docs/stable/generated/pandas.DataFrame.resample.html

LAG FUNCTIONS

File Edit View Data Transform Analyze Direct Marketing Graphs Ut

compute previous_id = lag(id).

	id	previous_id	var	var
1	1.00	.		
2	2.00	1.00		
3	2.00	2.00		
4	3.00	2.00		
5	3.00	3.00		
6	3.00	3.00		
7	4.00	3.00		
8	4.00	4.00		
9	4.00	4.00		
10	4.00	4.00		

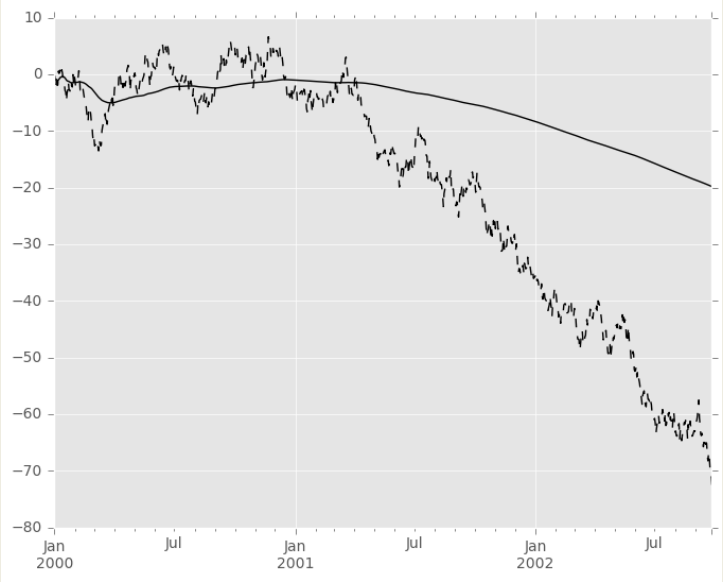


http://stackoverflow.com/questions/18300270/lag-in-time-series-regression-using-libsvm
http://www.spss-tutorials.com/spss-lag-function/

WINDOW FUNCTIONS

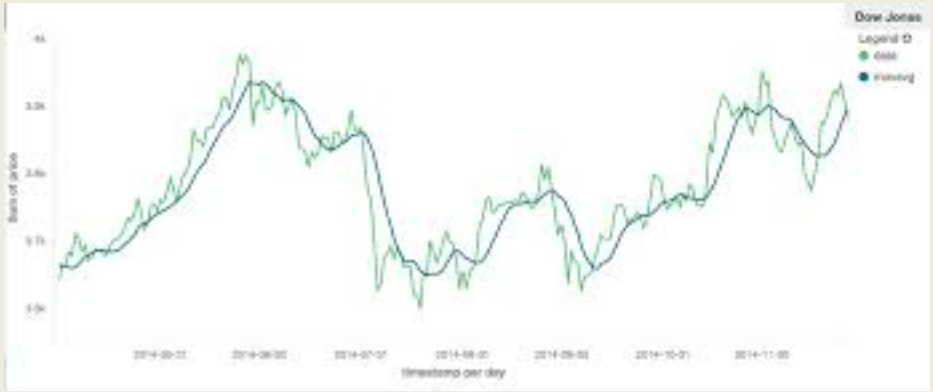
Rolling window

Expanding window



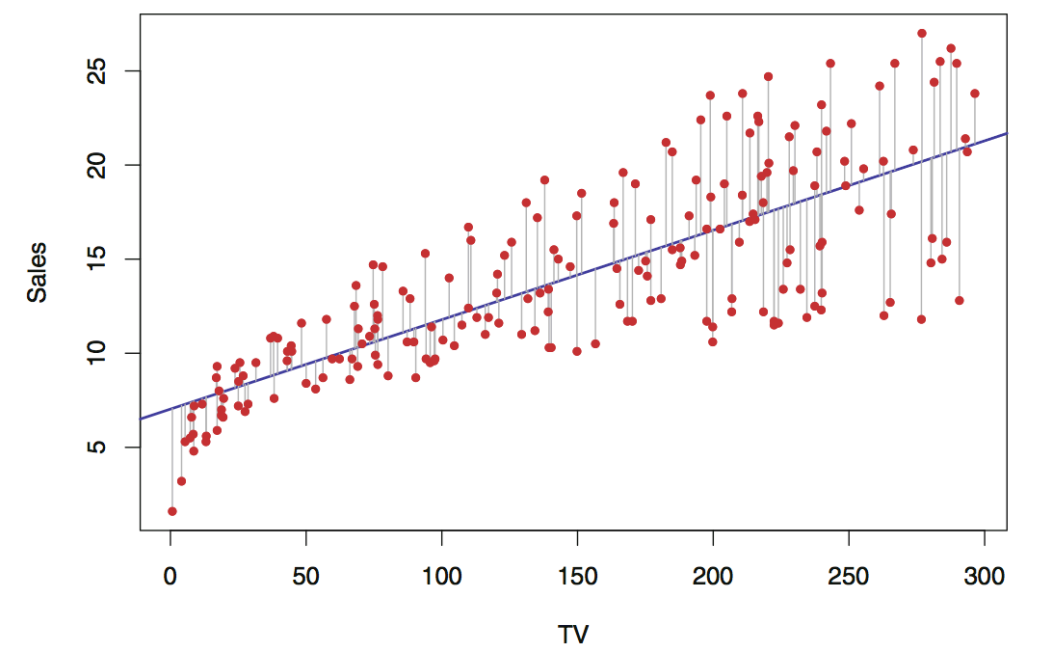
Why use a rolling window function?

What's a little funky here?



LINEAR REGRESSION

LINEAR REGRESSION INTUITION

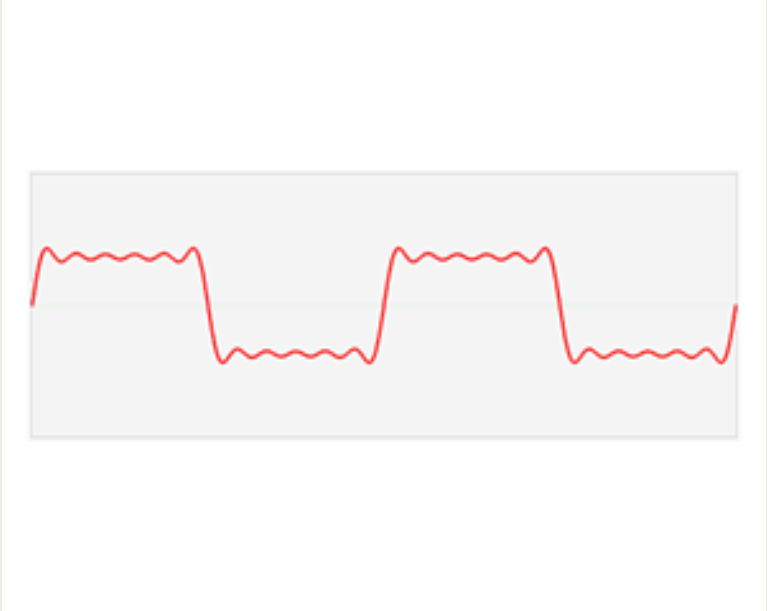


http://datascience.stackexchange.com/questions/6787/is-decision-tree-algorithm-a-linear-or-nonlinear-algorithm

SPECTRAL ANALYSIS

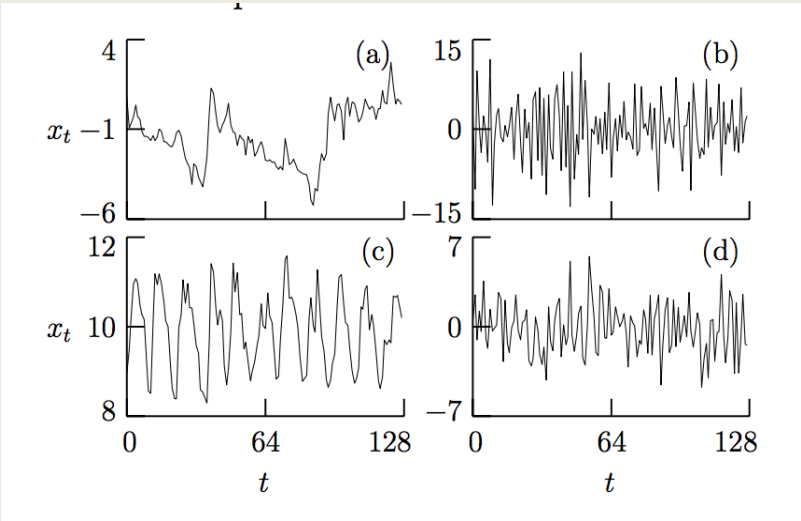
INTUITION

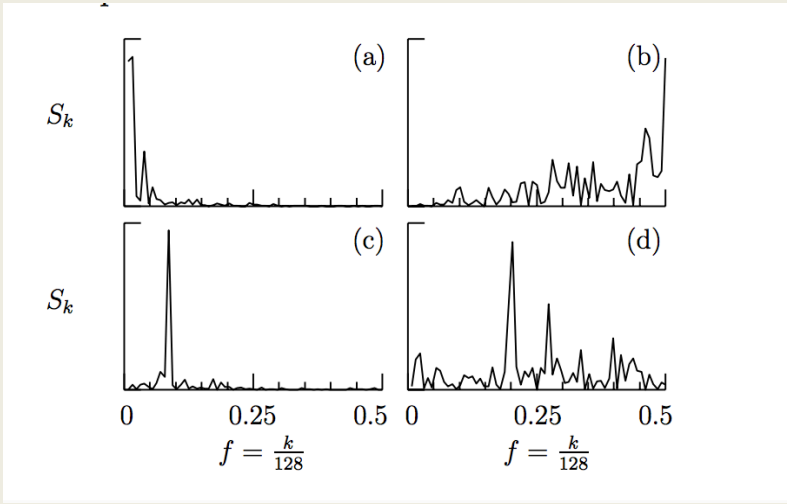
- Decompose a time series into a sum of many many sine or cosine functions
- The coefficients of these functions should have uncorrelated values
- Regression on sinusoids



https://en.wikipedia.org/wiki/Fourier_transform

EXAMPLES

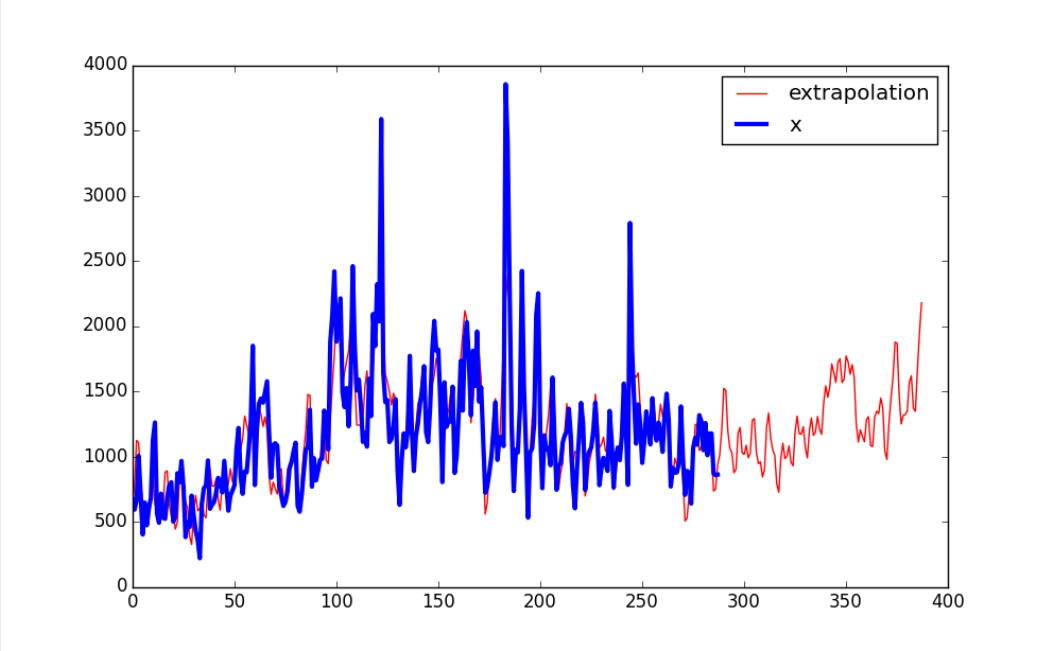




1. What are the advantages?
2. When would this provide useful information?
3. When would this *not* provide useful information?

https://faculty.washington.edu/dbp/PDFFILES/GHS-AP-Stat-talk.pdf

SPECTRA-BASED FIT CAN BE SURPRISINGLY GOOD



SPECTRAL ANALYSIS TURNS UP EVERYWHERE

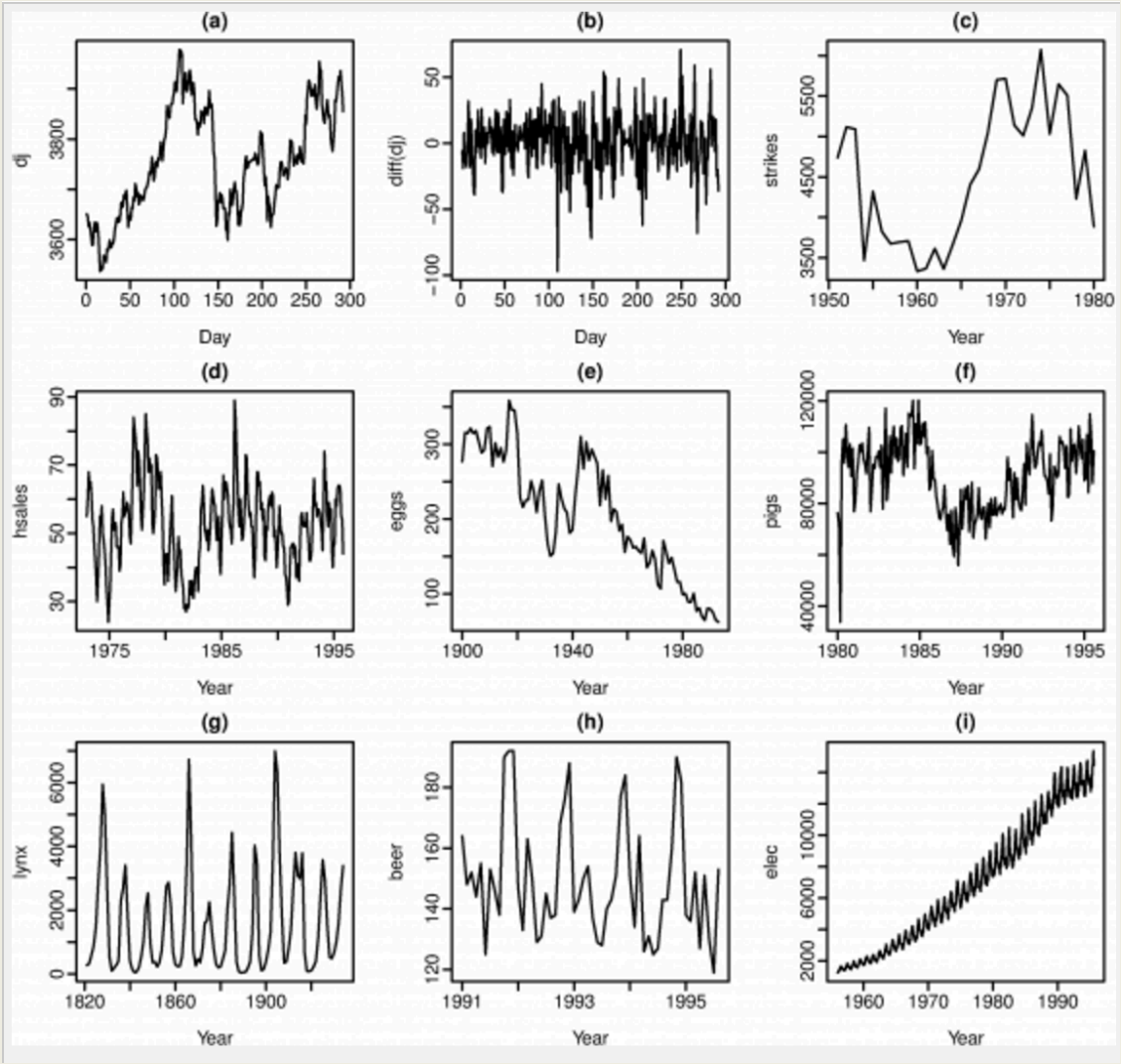
Astronomical data

Paleo-climate proxy data

Biology experiments

PRE-PREDICTION MUNGING & STATIONARITY

STATIONARY TIME SERIES …NOT ALWAYS OBVIOUS

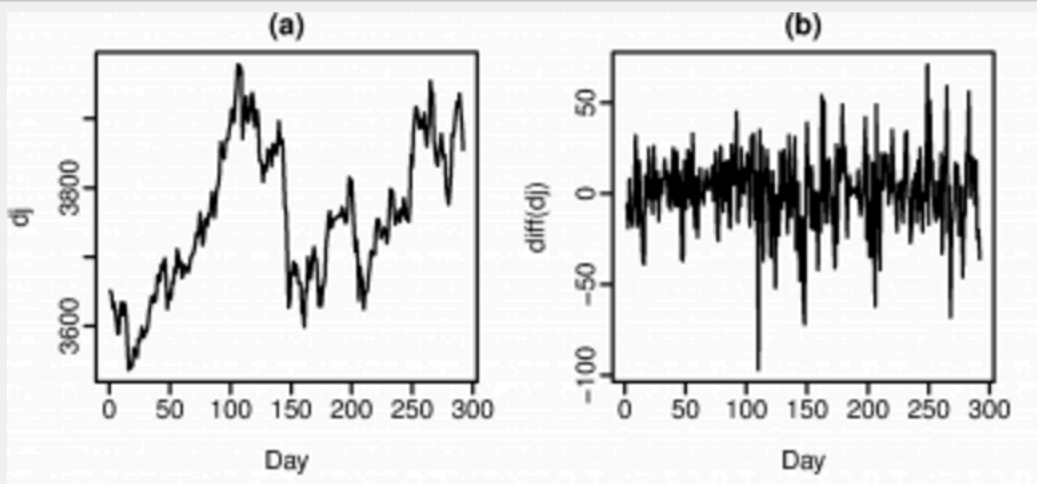


(a) *Dow Jones index on 292 consecutive days;* ***(b)*** *Daily change in Dow Jones index on 292 consecutive days;* ***(c)*** *Annual number of strikes in the US;* ***(d)*** *Monthly sales of new one-family houses sold in the US;* ***(e)*** *Price of a dozen eggs in the US (constant dollars);* ***(f)*** *Monthly total of pigs slaughtered in Victoria, Australia;* ***(g)*** *Annual total of lynx trapped in the McKenzie River district of north-west Canada;* ***(h)*** *Monthly Australian beer production;* ***(i)*** *Monthly Australian electricity production*

Stationary: (b) & (g)

https://www.otexts.org/fpp/8/1

DIFFERENCING TO CREATE STATIONARY TIME SERIES



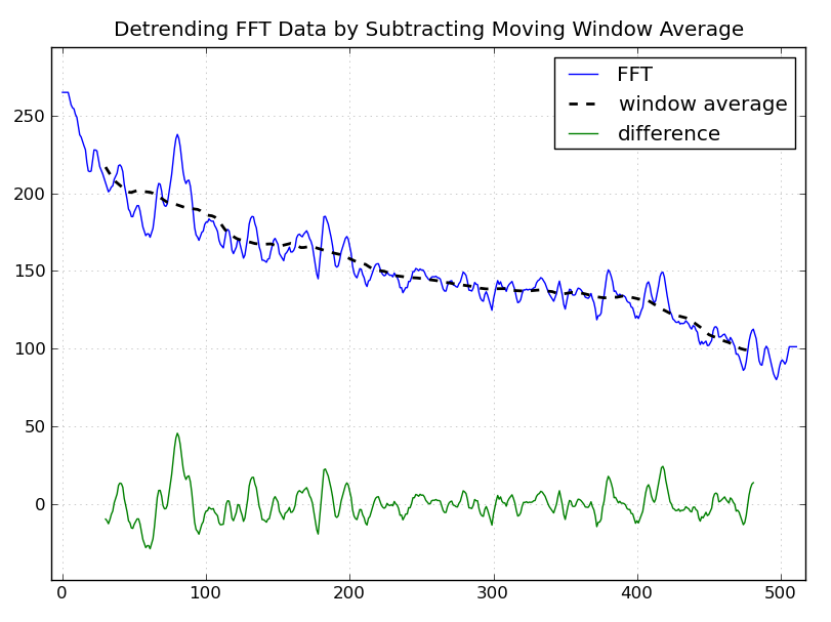
https://www.otexts.org/fpp/8/1

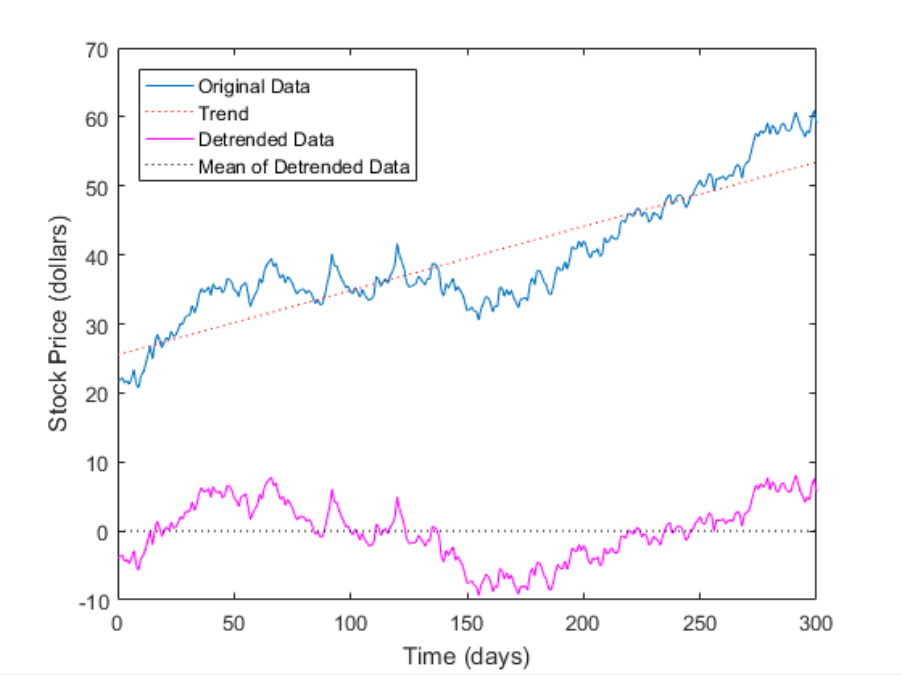
YOU NEED TO REMOVE THE TREND AND SEASONAL ELEMENTS BEFORE FORECASTING

- Most (interesting) data in the real world will show
  - Trends
  - Seasonality
- Most models require data that shows neither of these properties to say something interesting
- In particular, we need a **stationary** time series

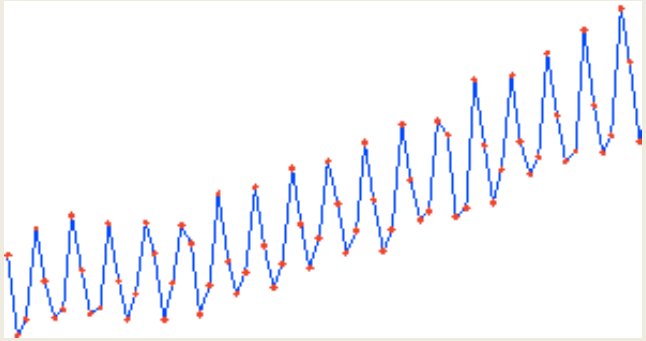
DE-TREND YOUR DATA

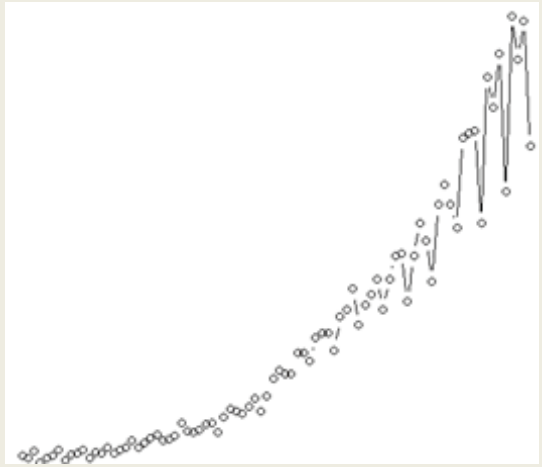
Use local smoothing or a linear regression





SEASONALITY





https://onlinecourses.science.psu.edu/stat510/node/57

REMOVE SEASONALITY

- Simplest: average de-trended values for specific season
- More common: use 'loess' method ('locally weighted scatterplot smoothing')
  - Window of specified width is placed over the data
  - A weighted regression line or curve is fitted to the data, with points closest to center of curve having greatest weight
  - Weighting is reduced on points farthest from regression line/curve and calculation is rerun several times.
  - This yields one point on loess curve
  - Helps reduce impact of outlier points
  - Computationally taxing

DICKEY-FULLER TEST

- Tests the null hypothesis of whether a unit root is present in an autoregressive model
- In plain English, tests whether $\rho = 1$ in

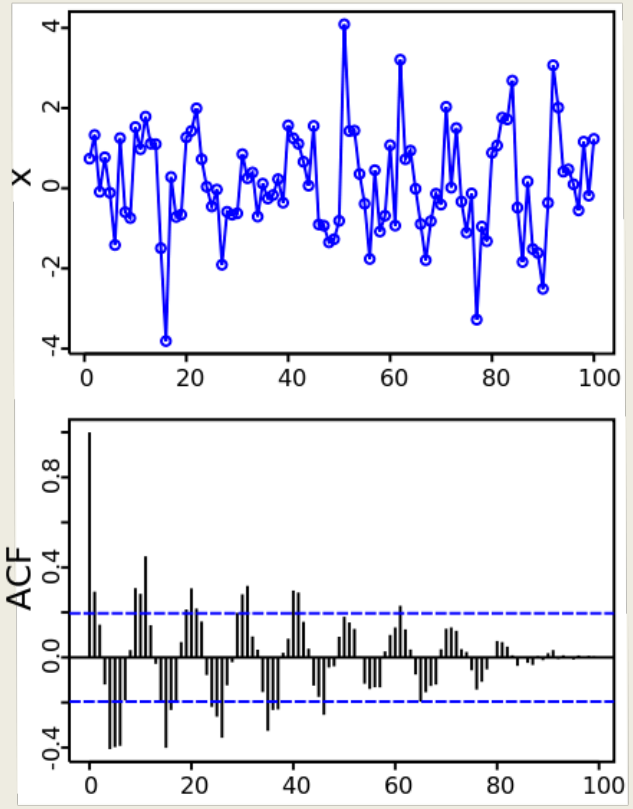
$$y_t = \rho y_{t-1} + u_t$$

- The test gives back several values to help you assess significance with standard p-value reasoning.
- Basic intuition: ρ should not have unit value

SELF-CORRELATION, SELF-EXPLANATION, AND SELF-PREDICTION

AUTOCORRELATION FUNCTION

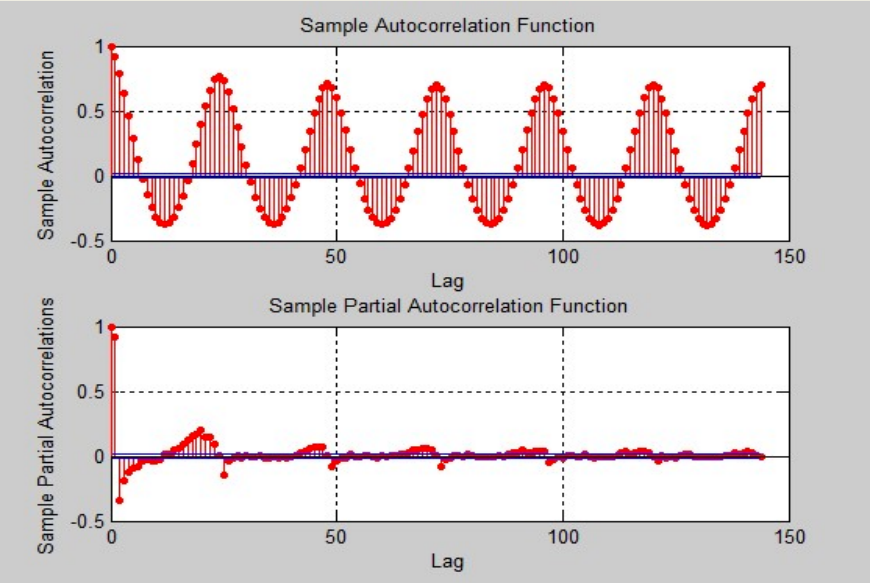
- Used to help identify possible structures of time series data
- Gives a sense of how different points in time relate to each other in a way explained by temporal distance



https://en.wikipedia.org/wiki/Autocorrelation

PARTIAL AUTOCORRELATION FUNCTION

- "gives the partial correlation of a time series with its own lagged values, controlling for the values of time series at all shorter lags"
- Why would this be useful?



https://en.wikipedia.org/wiki/Partial_autocorrelation_function &
https://www.mathworks.com/matlabcentral/answers/160904-interpretation-of-autocorrelation-function-to-determine-number-of-lags-in-ar-model?requestedDomain=www.mathworks.com

FORECASTING

MOVING AVERAGE PROCESS (MA)

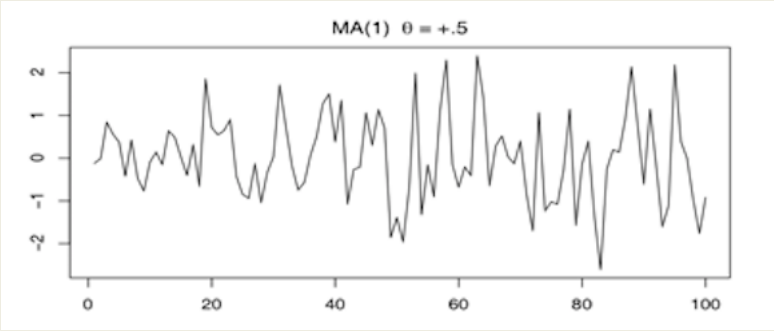
- Defined as having the form:

$$X_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q}$$

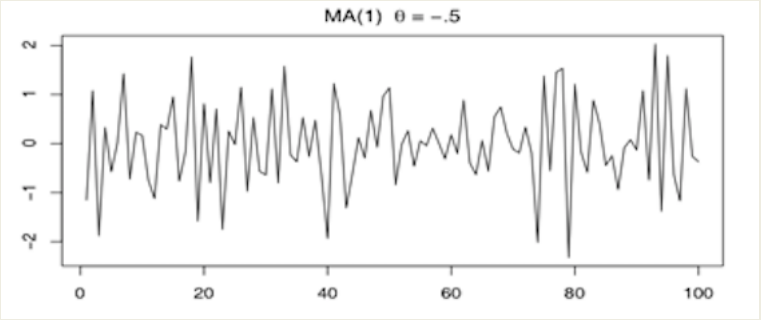
- μ is the mean of the series, θ are parameters, θ_q not 0
- This is a stationary process regardless of values of θ
- Consider an MA(1) process (centered at 0):

$$X_t = \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

$\theta_1 = +.5$



$\theta_1 = -.5$



Time Series Analysis and Applications, Robert H. Shumway and David S. Stoffer
https://en.wikipedia.org/wiki/Moving-average_model

AUTOREGRESSIVE PROCESS (AR)

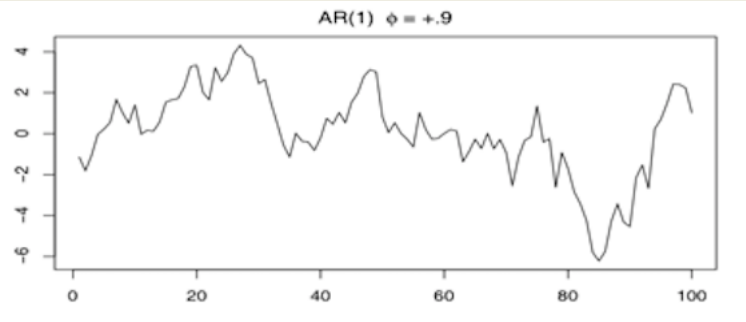
- Defined as having the form:

$$X_t = \phi_1 X_{t-1} + \ldots + \phi_p X_{t-p} + \varepsilon_t$$

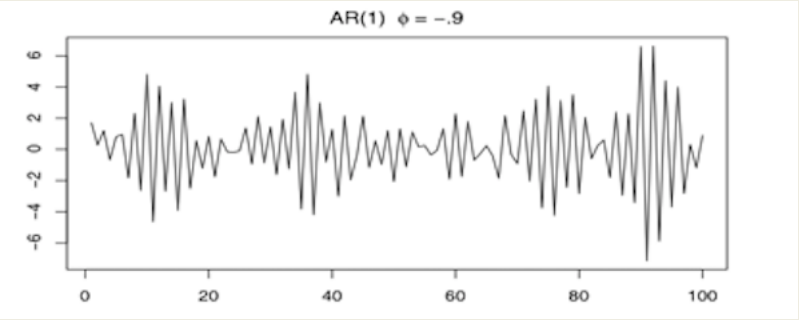
- This is a stationary process if abs(ϕ) < 1
- Consider an AR(1) process:

$$X_t = \phi_1 X_{t-1} + \varepsilon_t$$

$\phi_1 = +.9$



$\phi_1 = -.9$



Time Series Analysis and Applications, Robert H. Shumway and David S. Stoffer
https://en.wikipedia.org/wiki/Moving-average_model

ARIMA MODEL (A.K.A. BOX-JENKINS)

- AR = autoregressive terms
- I = differencing
- MA = moving average
- Hence specified as (autoregressive terms, differencing terms, moving average terms)

ARIMA MODE: 'THE MOST GENERAL CLASS OF MODELS FOR FORECASTING A TIME SERIES WHICH CAN BE **MADE TO BE STATIONARY**

- Statistical properties (mean, variance) constant overt time
- 'its short-term random time patterns always look the same in a statistical sense'
- Autocorrelation function & power spectrum remain constant over time
- Ok to do non-linear transformations to get there
- ARIMA model can be viewed as a combination of signal ad noise
- Extrapolate the signal to obtain forecasts

http://people.duke.edu/~rnau/411arim.htm

APPLYING THE APPROPRIATE ARIMA MODEL

- Need to determine what ARIMA model to use
- Use plot of the data, the ACF, and the PACF
- With the plot of the data: look for trend (linear or otherwise) & determine whether to transform data
- Most software will use a maximum likelihood estimation to determine appropriate ARIMA parameters

ARIMA ANALYSIS TURNS UP EVERYWHERE

Time Series Analysis and Forecasting of Temperatures in the Sylhet Division of Bangladesh

A.H. Nury[1*], M. Koch[2] and M.J.B. Alam[3]

[1] *Department of Civil Engineering, Leading University, Sylhet-3100, Bangladesh*
[2] *Department of Geohydraulics and Engineering Hydrology, Kassel University, Kassel, Germany*
[3] *Department of Civil and Environmental Engineering, Shahjalal University, Sylhet, Bangladesh*
*e-mail: hasancee@yahoo.com

Application of an Autoregressive Integrated Moving Average Model for Predicting the Incidence of Hemorrhagic Fever with Renal Syndrome

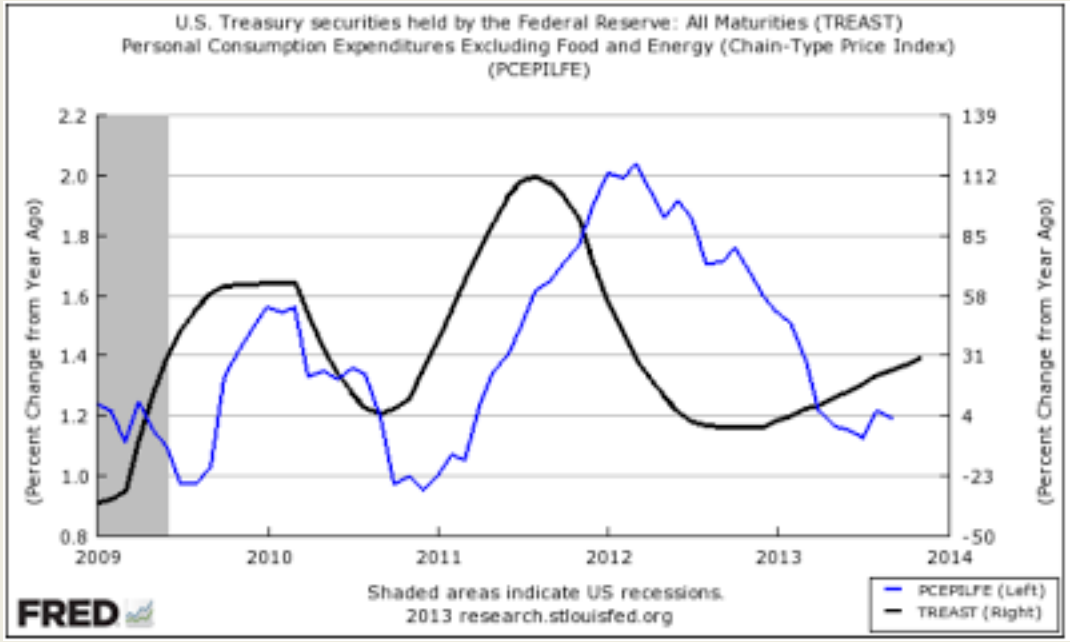
Qi Li,* Na-Na Guo, Zhan-Ying Han, Yan-Bo Zhang, Shun-Xiang Qi, Yong-Gang Xu, Ya-Mei Wei, Xu Han, and Ying-Ying Liu

Forecasting daily maximum surface ozone concentrations in Brunei Darussalam--an ARIMA modeling approach.

Kumar K[1], Yadav AK, Singh MP, Hassan H, Jain VK.

LEARN MORE…

Vector auto-regression works similarly for cases of multivariate time series



$$x_{t,1} = \alpha_1 + \phi_{11} x_{t-1,1} + \phi_{12} x_{t-1,2} + \phi_{13} x_{t-1,3} + w_{t,1}$$

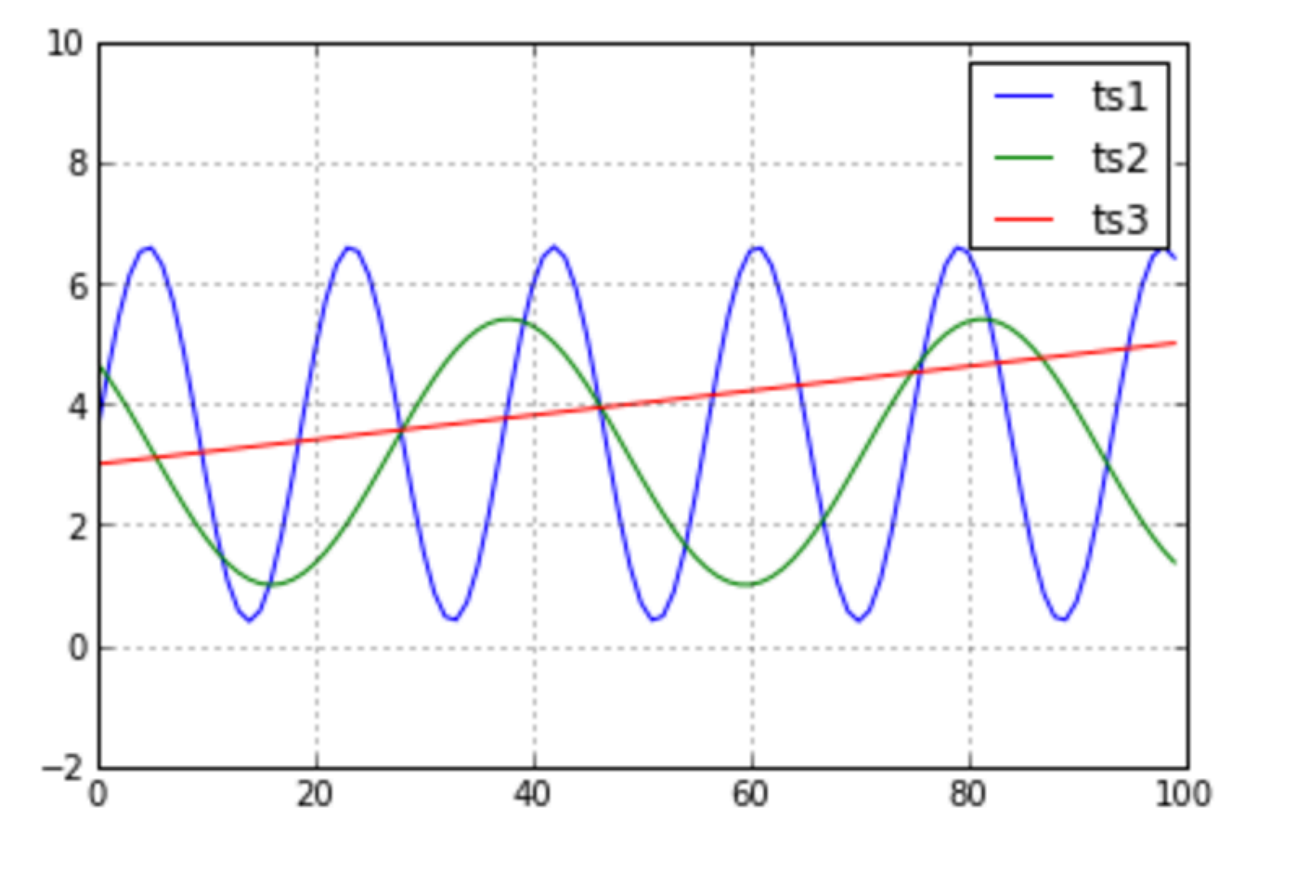
$$x_{t,2} = \alpha_2 + \phi_{21} x_{t-1,1} + \phi_{22} x_{t-1,2} + \phi_{23} x_{t-1,3} + w_{t,2}$$

$$x_{t,3} = \alpha_3 + \phi_{31} x_{t-1,1} + \phi_{32} x_{t-1,2} + \phi_{33} x_{t-1,3} + w_{t,3}$$

http://macromarketmusings.blogspot.com/2013/12/taking-model-to-data.html

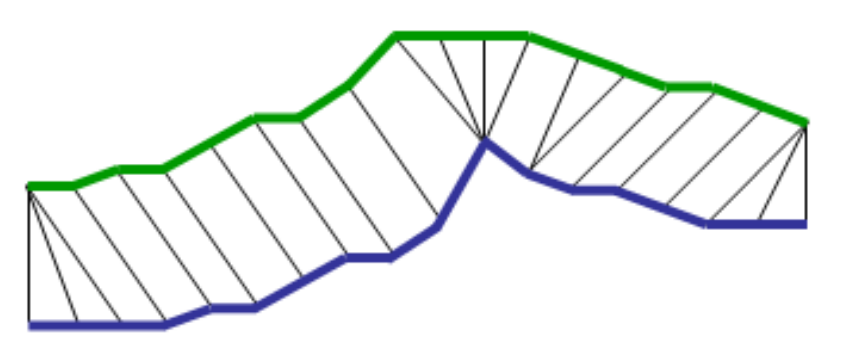
CLUSTERING & CLASSIFICATION (YET ANOTHER ROUTE TO PREDICTION)

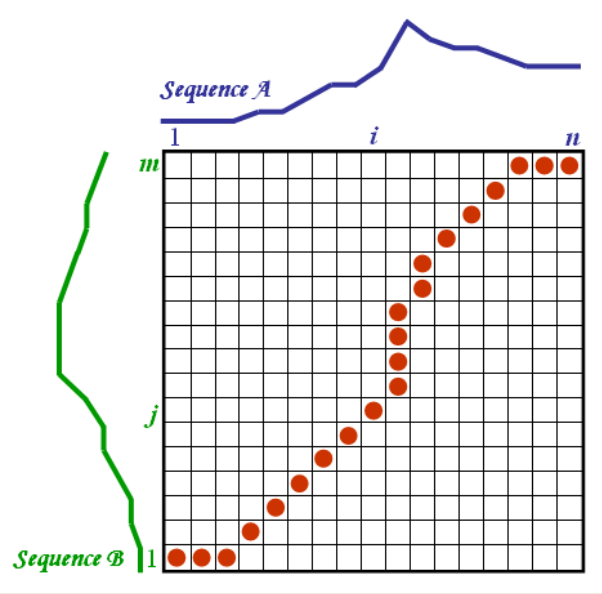
NEED TO THINK CAREFULLY ABOUT DISTANCE METRIC



http://www.psb.ugent.be/cbd/papers/gentxwarper/DTWalgorithm.htm &
http://alexminnaar.com/time-series-classification-and-clustering-with-python.html

DYNAMIC TIME WARPING





http://www.psb.ugent.be/cbd/papers/gentxwarper/DTWalgorithm.htm &
http://alexminnaar.com/time-series-classification-and-clustering-with-python.html

APPLICATIONS

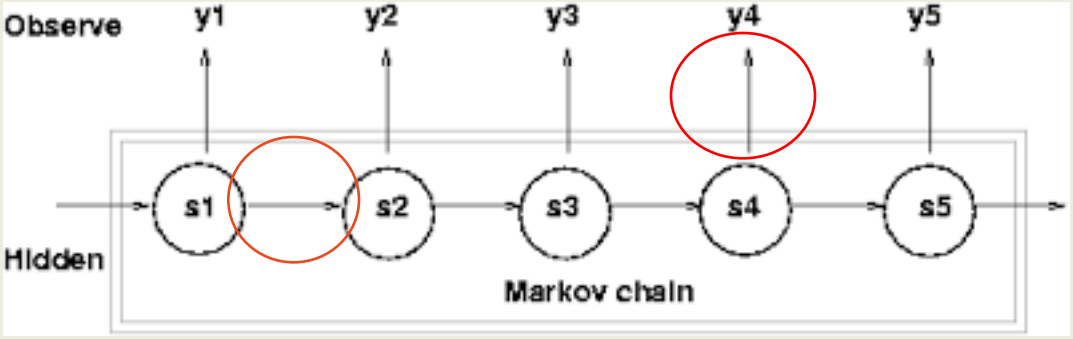
DTW-based clustering

DTW-based nearest neighbor classifcation

HIDDEN MARKOV MODELS

MULTI-STATE TIME-BASED SYSTEMS

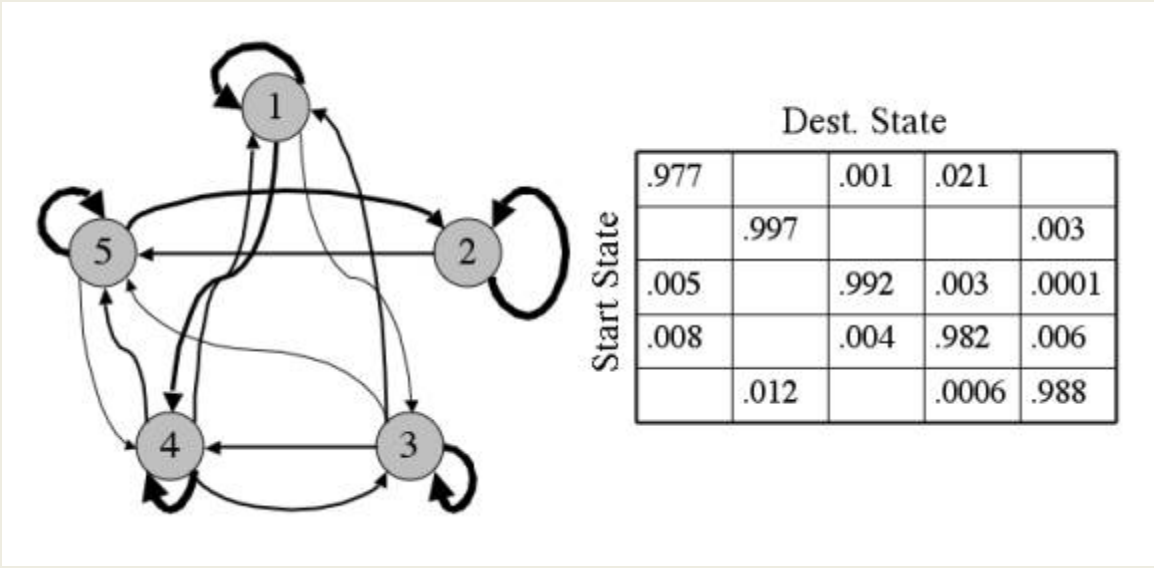
Hidden Markov Models are another way of thinking about time series classification.



https://en.wikipedia.org/wiki/Hidden_Markov_model

HOW IT WORKS

A transition matrix specifies the likelihood of transitioning between states.



Start State \ Dest. State					
	.977		.001	.021	
		.997			.003
	.005		.992	.003	.0001
	.008		.004	.982	.006
		.012		.0006	.988

http://web.mit.edu/9.29/www/neville_jen/hmm/

GET MORE PRACTICE