Midterm delivery Morten Svendgard

Github link: https://github.com/MSNetrom/COMPSCI294ExperimentalMM_Midterm

6.1

First I want to mention that I have used a very long time on this task, as I feel like I understood the concept, but had a hard time showing it using nearest neighbour. However I think that I have been able to make an algorithm which shows the point we want to show. What we really want to show is that as the dimensionality of our data increases, we have more coordinates and therefore more freedom of how we split up our data. This should give the result that as the dimensionality increases, we get closer and closer to only having to memorize half of our training rows, to correctly classify all the points in the binary classification dataset.

For this task we will do the following:

1. Loop datapoints.
   - If the datapoint can be deleted, and still classified correctly, delete it.
2. Stop when no more deletions are possible.
3. Use remaining datapoints as reference points in the NN algorithm.

This algorithm is does not in general find an optimal solution, but it seems to work well enough for demonstrating the fact that one can (almost) memorize almost all the training data by only using half the data (in theory) when the dimensions increases.

First I have some code for generating random functions:

In [ ]:
```
from sklearn.neighbors import NearestNeighbors
from itertools import product
from collections import namedtuple
import numpy as np

def all_combinations(base_range, num_variables):
    return np.array([i for i in product(range(base_range), repeat=num_variables)])

FunctionConfiguration = namedtuple('FunctionConfiguration', ['num_functions', 'num_variables', 'num_classes'])

class FunctionGenerator:

    def __init__(self, num_functions, num_variables, num_classes):
        self.num_functions = num_functions

        self.rows = all_combinations(2, num_variables).astype(float) #+ np.random.normal(0, noisy_points, (2**num_variabl
        self.labels = None

        if num_variables == 2 and num_functions == 16 and num_classes == 2:
            self.labels_2 = all_combinations(2, len(self.rows))
            self.labels = lambda i: self.labels_2[i]
        else:
            self.labels = lambda i: np.random.randint(num_classes, size=(self.rows.shape[0]))

        self.index = -1

    def __iter__(self):
        return self

    def __next__(self):
        if self.index < self.num_functions - 1:
            self.index += 1
            return self.rows, self.labels(self.index)

        raise StopIteration
```

Then some code for finding a got set of reference points for the nearest neighbour algorithm:

In [ ]:
```
def test_function(data, labels):

    if np.all(labels[0] == labels):
        return 0

    reference_points = data
    reference_labels = labels

    i = 0
    solution_found = False
    while True:

        # Try without reference i
        temp_references = np.vstack((reference_points[:i], reference_points[i+1:]))
        temp_labels = np.hstack((reference_labels[:i], reference_labels[i+1:]))
```

```
        # Can we predict all labels with the remaining references?
        predicted_indices = NearestNeighbors(n_neighbors=1).fit(temp_references).kneighbors(data, return_distance=False).
        predicted_labels = temp_labels[predicted_indices]

        if np.all(predicted_labels == labels):
            reference_points = temp_references
            reference_labels = temp_labels
            solution_found = True
            i %= len(reference_points)

        else:

            i += 1

            if i == len(reference_points):

                if not solution_found:
                    break

                solution_found = False
                i = 0

    return len(reference_points)

def evaluate_functions(f_gen):

    num_references_total = 0

    for data, labels in f_gen:
        num_references_total += test_function(data, labels)

    return num_references_total / f_gen.num_functions
```

Part a.) Now we will actually test how many points we need to remember the table

In [ ]:
```
function_configurations = [
    FunctionConfiguration(num_functions=16, num_variables=2, num_classes=2),
    FunctionConfiguration(num_functions=32, num_variables=3, num_classes=2),
    FunctionConfiguration(num_functions=64, num_variables=4, num_classes=2),
    FunctionConfiguration(num_functions=128, num_variables=5, num_classes=2),
]

for f_conf in function_configurations:

    f_gen = FunctionGenerator(*f_conf)
    avg_references = evaluate_functions(f_gen)

    print(f"functions tested: {f_conf.num_functions}, n_full: {len(f_gen.rows)}, dimensions: {f_conf.num_variables}")
    print(f"avg_points: {avg_references}, n_full / avg_refs {len(f_gen.rows) / avg_references} \n")
```

```
functions tested: 16, n_full: 4, dimensions: 2
avg_points: 2.375, n_full / avg_refs 1.6842105263157894

functions tested: 32, n_full: 8, dimensions: 3
avg_points: 4.625, n_full / avg_refs 1.7297297297297298

functions tested: 64, n_full: 16, dimensions: 4
avg_points: 8.734375, n_full / avg_refs 1.8318425760286225

functions tested: 128, n_full: 32, dimensions: 5
avg_points: 17.109375, n_full / avg_refs 1.8703196347031963
```

We see that as the number of dimensions increaes from 2 to 5 we are able to delete relativly more and more points, and the ratio of the total amount of points to the amount of points we need to remember comes closer to 2.

Part b.) Now we will try to do it for more classes:

For more classes we have the empirical result:

$$bits/parameter = \frac{c}{c-1}$$

In [ ]:
```
function_configurations = [
    FunctionConfiguration(num_functions=16, num_variables=2, num_classes=3),
    FunctionConfiguration(num_functions=32, num_variables=3, num_classes=3),
    FunctionConfiguration(num_functions=64, num_variables=4, num_classes=3),
    FunctionConfiguration(num_functions=128, num_variables=5, num_classes=3),

    FunctionConfiguration(num_functions=32, num_variables=3, num_classes=20),
    FunctionConfiguration(num_functions=64, num_variables=4, num_classes=20),
    FunctionConfiguration(num_functions=128, num_variables=5, num_classes=20),
]

for f_conf in function_configurations:

    f_gen = FunctionGenerator(*f_conf)
```

```
    avg_references = evaluate_functions(f_gen)

    print(f"functions tested: {f_conf.num_functions}, n_full: {len(f_gen.rows)}, dimensions: {f_conf.num_variables}, clas
    print(f"avg_points: {avg_references}, n_full / avg_refs {len(f_gen.rows) / avg_references}")
    print(f"Max expected ratio for {f_conf.num_classes} classes (n_full / avg_refs): {f_conf.num_classes / (f_conf.num_cl
```

```
functions tested: 16, n_full: 4, dimensions: 2, classes: 3
avg_points: 2.9375, n_full / avg_refs 1.3617021276595744
Max expected ratio for 3 classes (n_full / avg_refs): 1.5

functions tested: 32, n_full: 8, dimensions: 3, classes: 3
avg_points: 5.78125, n_full / avg_refs 1.3837837837837839
Max expected ratio for 3 classes (n_full / avg_refs): 1.5

functions tested: 64, n_full: 16, dimensions: 4, classes: 3
avg_points: 11.125, n_full / avg_refs 1.4382022471910112
Max expected ratio for 3 classes (n_full / avg_refs): 1.5

functions tested: 128, n_full: 32, dimensions: 5, classes: 3
avg_points: 22.0078125, n_full / avg_refs 1.4540291089811856
Max expected ratio for 3 classes (n_full / avg_refs): 1.5

functions tested: 32, n_full: 8, dimensions: 3, classes: 20
avg_points: 7.5, n_full / avg_refs 1.0666666666666667
Max expected ratio for 20 classes (n_full / avg_refs): 1.0526315789473684

functions tested: 64, n_full: 16, dimensions: 4, classes: 20
avg_points: 15.25, n_full / avg_refs 1.0491803278688525
Max expected ratio for 20 classes (n_full / avg_refs): 1.0526315789473684

functions tested: 128, n_full: 32, dimensions: 5, classes: 20
avg_points: 30.6015625, n_full / avg_refs 1.0456982384477918
Max expected ratio for 20 classes (n_full / avg_refs): 1.0526315789473684
```

Here we test 2 different number of classes. We test 3 classes, where we see that the expected ratio of total points to reference points comes closer to the expected 1.5 as the dimensions increases (as given by the formula above). We also test with 20 classes, just to demonstrate that now we have to remember close to all points.

6.2

a.) I was a bit unsure if this problem was about a dataset where all elements are binary digits, or just a dataset where we are doing binary classification. I targeted some fully binary datasets, although my algorithms would need small modifications to work for other datasets. After all, all data on a computer is in fact binary anyway. The algorithms work for both binary classification and multi-class classification.

I will investigate some partly self-invented methods, since this seems to be what the task is asking about. Also since this is a question regarding machine learning, we will accept more than just boolean operations. We will use summation and other operations, like you would do in a neural network.

Algorithm 1 (alg_8): This algorithm is inspired by how algorithm 8 in the book tries to find the boundaries needed to separate the data in the table. For our purpose, algorithm 8 has the problem that the same sum of binary digits can correspond to multiple classes. We fix this by using the actual integer value of each row in the table for making each sum unique. We are then able to use if statements to split all classes.

In [ ]:
```
def base2_val(data):
    return sum([d*2**p for p, d in enumerate(data)])

def alg8_model(data, labels):
    table = []

    # Get value
    for i in range(len(data)):
        table.append([base2_val(data[i]), labels[i]])

    cur_class = 0
    sorted_table = np.array(sorted(table, key=lambda x: x[0]))
    if_funcs = []
    text_descriptor = ""
    # Find boundaries for splitted data
    for i in range(len(sorted_table)):
        if sorted_table[i][1] != cur_class:
            if_funcs.append((lambda x, bound=sorted_table[i][0]: base2_val(x) < bound, cur_class))
            text_descriptor += f'if x_int < {sorted_table[i][0]} then {cur_class}\n'
            cur_class = sorted_table[i][1]

    if_funcs.append((lambda basx: True, cur_class))
    text_descriptor += f'else {cur_class}'

    return if_funcs, text_descriptor
```

Algorithm 2 (neighbour): This algorithm is a nearest neighbour-like algorithm. It starts by choosing 1 data point to be used as a reference for others of the same class. Then it investigates how big ball it can make around this data-point, before other classes are included in the ball. Then it repeats this until all data points are included in a ball with the correct label.

In [ ]:
```python
def neighbour_model(data, labels):
    current_data_points = data.copy()
    current_labels = labels.copy()

    # Loop datapoints
    if_statements = []
    text_description = ""
    j = 0
    while len(current_data_points) > 0:

        # Find point furthest away from all other points
        # ...

        # Get a unused points
        current_data_point = current_data_points[0]
        current_label = current_labels[0]

        distance_to_points = np.linalg.norm(current_data_points - current_data_point, axis=1)

        sorted_indices = np.argsort(distance_to_points)
        sorted_neighbours = current_data_points[sorted_indices]
        sorted_labels = current_labels[sorted_indices]

        # Find neoghbours with same class
        same_class_index = 0
        while same_class_index < len(sorted_indices) and sorted_labels[same_class_index] == current_label:
            same_class_index += 1

        same_class_index -= 1 # For 0 entry which is ourself

        # Find where distance changes
        last_distance = 0
        last_dist_change_index = 1
        i = 0
        while i < len(sorted_neighbours):
            dis = np.linalg.norm(sorted_neighbours[i] - current_data_point)

            if dis != last_distance:
                if i >= same_class_index:
                    break

                last_distance = dis
                last_dist_change_index = i

            i += 1

        #print(i, last_dist_change_index)

        # Fin treshold and add if statement
        treshold_distance = np.linalg.norm(sorted_neighbours[last_dist_change_index-1] - current_data_point)
        if_statements.append((lambda x, t=treshold_distance, c=current_data_point: np.linalg.norm(x - c) <= t, current_la
        text_description += f'if ||x - x_{j}|| <= {treshold_distance} then {current_label}\n'

        mask = np.ones(len(current_data_points), dtype=bool)
        mask[sorted_indices[:last_dist_change_index]] = False

        current_data_points = current_data_points[mask]
        current_labels = current_labels[mask]

        j += 1

    return if_statements, text_description
```

We also have this extra code for evaluating the if-statements:

In [ ]:
```python
def ifmodel_eval(data, model):
    for tresh, label in model:
        if tresh(data):
            return label


def evaluator(data, labels, model):
    correct = 0
    for i in range(len(data)):
        if ifmodel_eval(data[i], model) == labels[i]:
            correct += 1

    return correct

def print_evaluation(data, correct, model, data_set_name, model_name):
    print(f'{model_name} on {data_set_name}:')
    print(f'Accuracy: {correct/len(data)}')
    print(f'Dataset length: {len(data)}, if statements: {len(model)}')
    print(f'Dataset dimensions: rows: {len(data)}, dimesions / columns: {len(data[0])}')
    print(f"Total number of points / number of parameters (if statements): {len(data) / len(model)}")
```

b.) We will test our algorithms on some different datasets.

Dataset 1: Full adder. We will see this as a binary classification problem, and use the sum as an input row as well, and use the carry as an output.

Inputs			Outputs	
A	B	C_{in}	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

In [ ]:
```
# Define the inputs and labels
full_adder_inputs = np.array([[0, 0, 0, 0],
                    [0, 0, 1, 1],
                    [0, 1, 0, 1],
                    [0, 1, 1, 0],
                    [1, 0, 0, 1],
                    [1, 0, 1, 0],
                    [1, 1, 0, 0],
                    [1, 1, 1, 1]])

# We define outputs
full_adder_labels  = np.array([0,
                     0,
                     0,
                     1,
                     0,
                     1,
                     1,
                     1])
```

Dataset 2: Truth table for if the sum of binary digits is even for 17 inputs (multivariate XOR)

In [ ]:
```
from itertools import product

def generate_even_table(n):
    inputs = np.array([i for i in product(range(2), repeat=n)])
    outputs = np.array([np.sum(row) % 2 for row in inputs])

    return inputs, outputs

inputs_sum_even, outputs_sum_even = generate_even_table(10)
```

Lets evaluate the algorithms on these datasets.

In [ ]:
```
m_alg8, _ = alg8_model(full_adder_inputs, full_adder_labels)
correct = evaluator(full_adder_inputs, full_adder_labels, m_alg8)
print_evaluation(full_adder_inputs, correct, m_alg8, 'full_adder', 'alg8')
```

```
alg8 on full_adder:
Accuracy: 1.0
Dataset length: 8, if statements: 4
Dataset dimensions: rows: 8, dimesions / columns: 4
Total number of points / number of parameters (if statements): 2.0
```

In [ ]:
```
n_model, _ = neighbour_model(full_adder_inputs, full_adder_labels)
correct = evaluator(full_adder_inputs, full_adder_labels, n_model)
print_evaluation(full_adder_inputs, correct, n_model, 'full_adder', 'neighbour')
```

```
neighbour on full_adder:
Accuracy: 1.0
Dataset length: 8, if statements: 8
Dataset dimensions: rows: 8, dimesions / columns: 4
Total number of points / number of parameters (if statements): 1.0
```

In [ ]:
```
m_alg8, _ = alg8_model(inputs_sum_even, outputs_sum_even)
correct = evaluator(inputs_sum_even, outputs_sum_even, m_alg8)
print_evaluation(inputs_sum_even, correct, m_alg8, 'inputs_sum_even', 'alg8')
```

```
alg8 on inputs_sum_even:
Accuracy: 1.0
Dataset length: 1024, if statements: 683
Dataset dimensions: rows: 1024, dimesions / columns: 10
Total number of points / number of parameters (if statements): 1.499267935578331
```

In [ ]:
```
n_model, _ = neighbour_model(inputs_sum_even, outputs_sum_even)
correct = evaluator(inputs_sum_even, outputs_sum_even, n_model)
print_evaluation(inputs_sum_even, correct, n_model, 'inputs_sum_even', 'neighbour')
```

```
neighbour on inputs_sum_even:
Accuracy: 1.0
Dataset length: 1024, if statements: 1024
Dataset dimensions: rows: 1024, dimesions / columns: 10
Total number of points / number of parameters (if statements): 1.0
```

We see that the alg_8 implementation gets a ratio of rows to if-statements of 2 on dataset 1 and 1.49 on dataset 2. A ratio of 2 is very good, and is the expected possible ratio of rows per parameter. This is quite good regarding the simplicity of the algorithm, and the low dimensional dataset.

The neighbour algorithm uses the same amount of if-statements as the number of rows in the dataset for both dataset. It is therefore not able to generalize anything at these datasets. This was suprisingly bad, but the dimensionality is quite low, and the algorithm is not very sophisticated. It shows slightly better results on the random datasets below.

c.) We will also evaluate the algorithms on random datasets, with different number of rows and dimensions.

We tested the algorithms on the following random data:

- 25 rows, 25 dimensions
- 25 rows, 50 dimensions
- 25 rows, 100 dimensions
- 50 rows, 100 dimensions

We repeat these tests 100 times for each case.

In [ ]:
```python
def generate_unique_vectors(rows, sampling_function):

    seen = set()

    while len(seen) < rows:
        vector = sampling_function()
        seen.add(tuple(vector))

    return np.array(list(seen))
```

In [ ]:
```python
from collections import defaultdict

def random_dataset(rows, dimensions, classes):
    data_input = np.random.randint(0, classes, (rows, dimensions))
    data_output = np.random.randint(0, classes, rows)
    return data_input, data_output

def random_dataset_evaluation(test_scenarios, repeats=10):

    for scenario in test_scenarios:
        print("-----------------------------------")
        correct_counter = defaultdict(lambda: 0)
        if_statement_counter = defaultdict(lambda: 0)

        for _ in range(repeats):
            rows, dimensions, classes = scenario[0]
            data_input, data_output = random_dataset(rows, dimensions, classes)

            for model, model_name in scenario[1]:
                my_model, _ = model(data_input, data_output)

                correct = evaluator(data_input, data_output, my_model)
                correct_counter[model_name] += correct
                if_statement_counter[model_name] += len(my_model)

        for model_name in correct_counter:
            print(f'{model_name} on random dataset {rows}x{dimensions}x{classes}:')
            print(f'Average accuracy: {correct_counter[model_name]/(repeats*rows)}')
            print(f'Average if statements: {if_statement_counter[model_name]/repeats}')
            print(f'Average ratio: {repeats*rows/if_statement_counter[model_name]}')
            print()


# (rows, dimensions, classes), model_creator,
test_scenarios = [((25, 25, 2), [(alg8_model, 'alg8_model'), (neighbour_model, 'neighbour_model')]),
                  ((25, 50, 2), [(alg8_model, 'alg8_model'), (neighbour_model, 'neighbour_model')]),
                  ((25, 100, 2), [(alg8_model, 'alg8_model'), (neighbour_model, 'neighbour_model')]),
                  ((50, 100, 2), [(alg8_model, 'alg8_model'), (neighbour_model, 'neighbour_model')])]

random_dataset_evaluation(test_scenarios, repeats=100)
```

```
----------------------------------
alg8_model on random dataset 25x25x2:
Average accuracy: 1.0
Average if statements: 13.52
Average ratio: 1.849112426035503

neighbour_model on random dataset 25x25x2:
Average accuracy: 1.0
Average if statements: 22.66
Average ratio: 1.1032656663724625

----------------------------------
alg8_model on random dataset 25x50x2:
Average accuracy: 1.0
Average if statements: 13.1
Average ratio: 1.9083969465648856

neighbour_model on random dataset 25x50x2:
Average accuracy: 1.0
Average if statements: 22.05
Average ratio: 1.1337868480725624

----------------------------------
alg8_model on random dataset 25x100x2:
Average accuracy: 1.0
Average if statements: 12.98
Average ratio: 1.926040061633282

neighbour_model on random dataset 25x100x2:
Average accuracy: 1.0
Average if statements: 22.28
Average ratio: 1.1220825852782765

----------------------------------
alg8_model on random dataset 50x100x2:
Average accuracy: 1.0
Average if statements: 25.82
Average ratio: 1.9364833462432223

neighbour_model on random dataset 50x100x2:
Average accuracy: 1.0
Average if statements: 43.13
Average ratio: 1.1592858798979828
```

NB!: *The exact values discussed here changes a bit between every run*

We see that alg_8 is able to generalize quite well, and the ratio of rows to if-statements goes from around 1.8 to over 1.9 as the dimensionality increases from 25 to 100. Also the ratio seems to increase as we do 50 rows instead of 25. This shows that this algorithm seems to converge to the 2 bits of information per paramter ration, which is the best we can expect in general.

For the neighbour algorithm, we also see that the ratio of rows to if-statements increases as the dimension increases, and also when we increase the number of rows in the last case. The best ratio is still 1.16, which is quite bad.

6.3

a.)

We will have a look at huffman coding for this compression. Mark that on uniform random data with high information entropy we can't expect a high compression rate.

We assume our characters are ascii characters of 1 byte each.

We will try some different strings:

- one with reasonable words from some alice in wonderland text (I think), from online.
- one with just random alphabetic charecters, that one would use to form words.
- one with uniformly chosen random charachters from all 256 ascii characters.

In [ ]:
```python
from dahuffman import HuffmanCodec
import requests
import random
import string


def huffman_compress_test(text):
    # Create a HuffmanCodec object
    codec = HuffmanCodec.from_data(text)

    # Encode the text
    compressed_data = codec.encode(text)

    # Decode the encoded data
    decoded_data = codec.decode(compressed_data)
```

```
    print("Original text length:", len(text), "bytes")
    print("Compressed data length:", len(compressed_data), "bytes")
    print("Decoded text:", len(decoded_data))
    equality = text == decoded_data
    print("Reconstruction successful:", equality)
    compression_rate = (len(text)-len(compressed_data))/len(text)
    print("Compression rate", compression_rate)

# URL of the text file from Project Gutenberg
url = "https://www.gutenberg.org/files/11/11-0.txt"
# Download the text file
response = requests.get(url)
normal_text = response.text
print("Normal text compression evaluation:")
huffman_compress_test(normal_text)
print()

# Try random text
# Define the characters to choose from
text_characters = string.ascii_letters + string.digits + string.punctuation + ' '
random_text = random_text = ''.join(random.choice(text_characters) for _ in range(int(1e5)))
print("Random textual-like characters compression evaluation:")
huffman_compress_test(random_text)
print()

# Try all random ascii characters
random_characters = ''.join([chr(i) for i in range(256)])
random_ascii_text = ''.join(random.choice(random_characters) for _ in range(int(1e5)))
print("Random ascii characters compression evaluation:")
huffman_compress_test(random_ascii_text)
print()
```

```
Normal text compression evaluation:
Original text length: 154638 bytes
Compressed data length: 92073 bytes
Decoded text: 154638
Reconstruction successful: True
Compression rate 0.4045900748845691

Random textual-like characters compression evaluation:
Original text length: 100000 bytes
Compressed data length: 83142 bytes
Decoded text: 100000
Reconstruction successful: True
Compression rate 0.16858

Random ascii characters compression evaluation:
Original text length: 100000 bytes
Compressed data length: 100042 bytes
Decoded text: 100000
Reconstruction successful: True
Compression rate -0.00042
```

As expected, a text written by a human has some probabilities that lets us compress it with 40%.

Random characters that we usually see in text can also be compresses by 16%, as these characters are only a subset of the 256 characters representable by 8 bytes, so there are some characters not used in normal text, that have very low probabilities of appearing (some have 0).

The task seems to ask about a string with all characters randomly chosen from all possible characters. This string can't be compressed at all, as the entropy is 8 bits per character, since they are all just as likely to appear. We see this from the last example above.

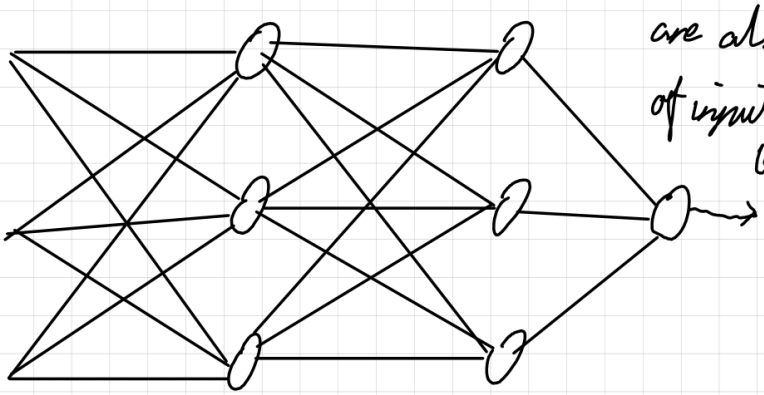
b.) For random characters we can't expect any compression, as the entropy is 8 bits per character. As we see above, what distribution the characters is chosen from affects the compression rate.

8.1

We want to find the memory equivalent capacity (MEC) of the following models:

8.1.)

The memory equivalent capacity of a is:



For this node there are also just 3 bits of input, although 4 bits of capacity

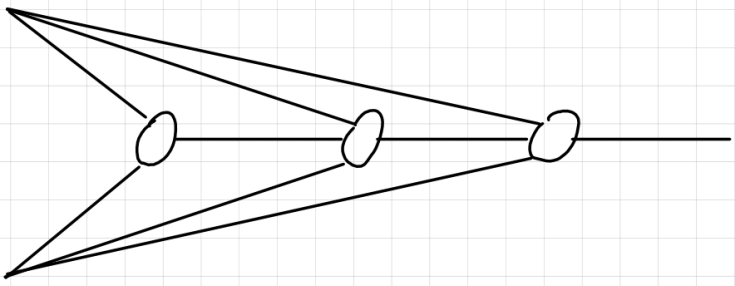
Each of these nodes can save 3+1 bits, that is 3·4 bits

Each node in this part can also save 3·4, but there are really just 3 distinct outputs with 3 distinct bits

Total capacity is therefore:

MEC = 12 + 3 + 3 = 18

b) Then we have this network:



- None of these nodes have their output restricted by the

previous layer. We therefore got

$MEC = 3 + 2(3+1) = 3 + 8 = 11$ bits

c) The maximum amount of rows any network can "memorize" can be said to be infinity if f.ex. all labels are 0, depending on the defenition of "memorize".

However the maximum number of rows we can guarantee to memorize for binary classification are 18 for network a., and 11 for network b.

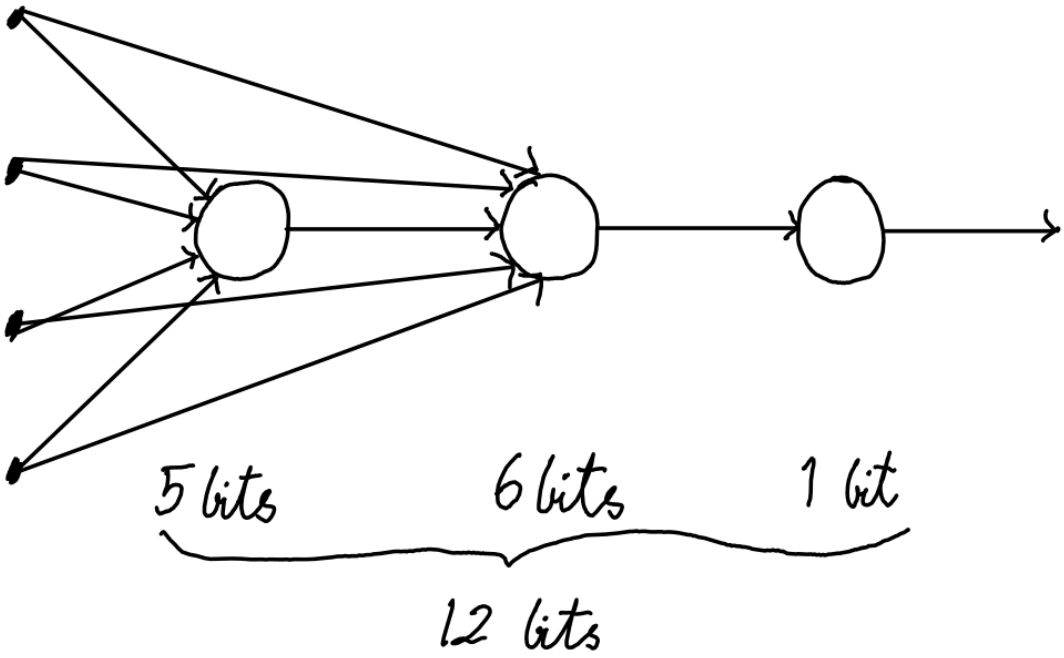
d) MEC is the number of bits needed to make 1 binary desicision per row. For 4 classes, $\log_2(4) = 2$ binary decisions have to be made per row instead, such that one could now guarantee to memorize half the amount of rows compared to binary classification.

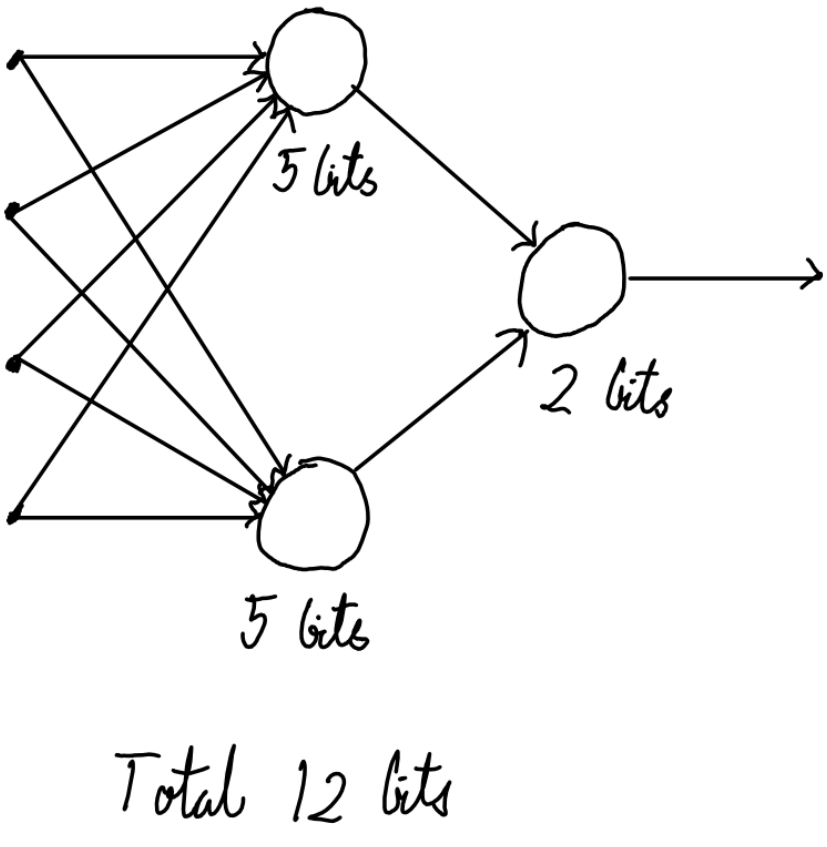
That is:

a.) $\left\lfloor \frac{18}{2} \right\rfloor = 9$ rows

b) $\left\lfloor \frac{11}{2} \right\rfloor = 5$ rows

The definition of MEC says: "A model's intellectual capacity is memory equivalent to n bits when the model is able to represent all 2^n binary labeling functions of n points.". In this problem there are 12 points, and we therefore need a MEC of 12 bits to guarantee memorization. Here are two examples of this:





8.4

a.)

First I want to state that the interpration of this task could be a bit unclear. For example there is a difference in the amount of information that our sensors "experience" given no prior estimation of the likelihood of different events vs. the amount of information we actually pay attention to as a human that can recognize normal or unusual events. We can't turn of our ears, however we subconsciously ignore a lot of the sounds we hear. The

same goes for our eyes. We can't turn of our eyes, but we can ignore a lot of the things we see. After all this is might just a reflection of the amount of the amount of uncertainty we can reduce by focusing on the right sensory information. If there is happening something very visually unusual in my room, I will probably pay more attention. Like if there is a fire. This usually never happens, so paying attention reduces my uncertainty.

The first part of this task seems to be about the amount data that our sensors would take in during a lifetime, and not actually what we pay attention to. The second part about memorization seems to be more about the amount of information we actually pay attention to.

What sensory information can a human take in?

- Vision
- Audio
- Physical touch
- Smell
- Taste
- (Temperature)
- (Balance)

First let's get some more info about each sensor:

- Vision:
  - This article suggests that the retina feeds the brain with 10 million bits per second: https://www.edwardtufte.com/bboard/q-and-a-fetch-msg?msg_id=0002NC#:~:text=The%20research%20suggests%20that%20the,close%20to%20an%20Ethernet%20connection!. Another article claimed 576MP at 30 fps, altough this seems unreasonable, so I will use the first article as my basis.
    - This does not take into account the actual entropy of the data. Let's say I watch a youtube video. The youtube video is compressed, and I will not take in more info than the optimal compression of the video. I will therefore not experience more than 10 million bits per second, unless the compressed video surpasses this amount of information rate. Although for the first part of the task I will focus more on the amount of information.
- Audiotory:
  - A normal healthy human can hear from 20 Hz to 20 kHz.
  - A professional audio setup would use something lik 24 bit at 44.1 kHz (because we can hear up to 20 kHz, and need higher sampling rate). This would be $24 * 44.1 * 1000 bits = 1.0584$ million bits per second.
  - Let's say this is the amount of data we take in every second of listening, since that is what is needed for profeesionals to not notice any loss in quality.
- Other:
  - It is very difficult to measure the amount of information taken in by other sensors, as it would be just guessing. Anyways, some articles suggests that up to 90% of the information we take in comes from the eyes. Let's say the eyes gives 10 million bits per second, the ears 1 million bits per second, and the rest of the sensors 1 million bits per second. This would make the total amount of information taken in about **12 million bits per second**.
  - Let's also assume that we sleep for 8 hours every night, and that our sensors are close to being turned off during this time.

At the age of 23, this would give a total of:

$$12 \cdot 10^6 bits/s \cdot 60 s/min \cdot 60 min/h \cdot (24-8) h/day \cdot 365 days/year \cdot 23 years = 5.8026 \cdot 10^{15} bits$$

How much have me memorized?

- This is a difficult question to answer. First we have to think about how we process data. Even though I see a very detailed view of the world, I don't save these details to memory. If I see a new car, the only really new data I take in is how different concepts that I already now, connects together. This would be concepts like shape and color, that I have already memorized.
- I will choose to focus on the vision part of information, as this is reported to be up to 90% of the information we take in. It would therefore be a good estimate for the total memorization.
- The brain seems to memorize shapes and colors, and then connects these to concepts. These concepts are also memorized.
- Lets look at an image I found online: https://vecta.io/blog/comparing-svg-and-png-file-sizes/penrose-triangle.d6e2a09434.svg:

By using a zipped vectorized format (svgz), they are able to save this image in 621 Bytes, while when using reasonable PNG resolutions they use several kilobytes for 1 image. Let's assume that one scene of my life consists of 1000 shapes of this complexity worth memorizing. This would be

621 kilobytes of information for 1 scene. Usually half of my day is spent between 4-6 different scenes at home, and the other half is spent at the university changing between maybe 10 scenes. With no prior knowledge of the world, I would have to memorize about $621kilobyte/scene \cdot 15scenes = 9315kilobytes = 7.45 \cdot 10^7 bits$ of information. However each day I would only have to memorize the new information I take in. Let's say I have to connect these shapes to new memories at a rate of 10% of the above every day, altough this rate would get smaller the more I experience (we ignore this for simplicity). Let's also assume that we start very fresh, and have to start reconnecting these shapes every 2 years, for reasons like moving to new places, new workplace and similar stuff. This happens 10 times in 23 years. Altough this does not require to relearn everything, let's just assume that a lot of new connections have to be made (at an equivalent size of this). This would give a total of:

$$10 * (7.45 \cdot 10^7 bits + (7.45 \cdot 10^7 bits/day * 0.1 \cdot 365 days/year * 5 years)) = 136707500000 bits = 1.367 \cdot 10^{11} bits$$

I will note that it is very difficult to estimate the amount of information we memorize. This is a very guessy estimate based on the vision part of our mind. But hopefully it is within maybe 4 magnitudes of order of the real amount of information we memorize.

Works of shakespare:

- According to this article, shakespare wrote 38 plays, 2 narrative poems, and 154 sonnets. 884,647 words: https://www.shakespeare-online.com/biography/wordsinvented.html
- According to this website, shakespare wrote about 835000 words: https://www.opensourceshakespeare.org/views/plays/plays_numwords.php.
- According to this wikipedia website, the english language has about 0.6 to 1.3 bits of information per letter: https://en.wikipedia.org/wiki/Entropy_(information_theory)#:~:text=English%20text%20has%20between%200.6,per%20character%20of%20the%
- Combining this information we get a total of:

$$835000 words \cdot 1 bit/word = 835000 bits = 8.35 \cdot 10^5 bits$$

Total information capacity of the brain:

Since image and accoustic data is assumed to be fairly high dimensional, we can assume two bits of information per connection in the brain. Each connection involves two neurons, such that for 10^{11} neurons with 1000 connections each, we get a total of $10^{11} \cdot \frac{1000}{2}$ total connections, that would be the equivalent of parameters for a neural network. This gives the following information capacity for the brain:

$$Information - capcity = 10^{11} \cdot \frac{1000}{2} \cdot 2 = 10^{14} bits$$

This is 9 magnitudes of order more than needed to memorize all of shakespare's plays. It is also 3 orders of magnitude higher than the estimated amount of information we memorize in a lifetime. The memorization estimation was very guessy, but it this might suggest that the brain could memorize much more than I expected, and according to my calculations it might not be full yet.

b.) From what we see in about algorithm 8, we find a certain amount of tresheld needed to classify a table. If we are handling more classes the treshold calculations will still work the same way, although the MEC symbolizes the capacity given that we are decisidng between two classes (binary). If we are instead desiding between 4 classes, it would be the equivalent of doing 2 binary classifications, requiring twice the amount of MEC. We can formulize this as follows:

$$MEC_c = MEC_2 \cdot log_2(c)$$

This would be the extra line of code we would at to the end of algorithm 8.

c.) For regression, One way of looking at it is to say that each row has it's own class. That would lead to to the need for N classes. One class for every row. This would also mean that we need $tresholds = N$ and $classes = N$. We would not need the algorithm for this. The whole calculation can be done in a mathematical expression:

$$MEC_{regression} = log_2(tresholds + 1) \cdot log_2(N) = log_2(N + 1) \cdot log_2(N)$$

It would be easier to give a good approximation of the MEC of a already existing regression network, but estimating the needed MEC for a regression network is a bit more difficult.

9.1

For solving this task we will do the following:

- We will look at the mutual information in the training data to find the MEC needed after compression.
- We will design a CNN that compresses the data reasonably to fit in a fully connected network designed according to the MEC found above.
- We will make the fully connected network after the CNN have maximally the MEC found above. It should probably be lower.

Mutual information in the training data:

From the book we have that the maximal mutual information is given by the following formula:

$$I(X;Y)_{\max} = \min(H(X), H(Y))$$

Therefore we would only need to calculate the entropy of the labels to find an upper bound for the MEC.

The MNIST database contains 60,000 training images and 10,000 testing images. The images are 28x28 pixels grayscale, and each pixel is represented by a value between 0 and 255. This could correspond to a table of **60,000 rows** and **784 columns**, with **10 classes**. This suggests that we would need the following MEC to remember the whole dataset given that the labels are equally distributed:

$$MEC = \lceil 60000 \cdot log_2(10) \rceil = 199316bits$$

Let's calculate this by using the actual MNIST dataset as well:

In [ ]:
```
import torch
from torchvision import datasets, transforms

mnist_dataset = datasets.MNIST(root='./data', train=True, download=True, transform=transforms.PILToTensor())
mnist_labels = mnist_dataset.targets

label_counts = torch.unique(mnist_labels, return_counts=True)[1]
probabilities = label_counts / mnist_labels.shape[0]
entropy = -torch.sum(probabilities * torch.log2(probabilities)).item()

eq_memory_cap = mnist_dataset.data.shape[0]*mnist_dataset.data.shape[1]*mnist_dataset.data.shape[2]*8
print("Entropy per row:", entropy, "Total entropy:", entropy*mnist_labels.shape[0])
print("Memory capacity of a corresponding table:", eq_memory_cap, "bits")
print("Suggested maximum compression rate:", eq_memory_cap/(entropy*mnist_labels.shape[0]))
```

```
Entropy per row: 3.319870948791504 Total entropy: 199192.25692749023
Memory capacity of a corresponding table: 376320000 bits
Suggested maximum compression rate: 1889.2300624767138
```

Maximum compression

We see that both our simple theory and the actual data gives us a MEC of about $199000bits$ or $log_2(10) = 3.3219bits$ per row.

Our input data is $28 * 28 * 8bits = 6272bits$ per row. Our data should maximally be compressed down to $3.3219bits$ (corresponding to 10 classes). This is a compression rate of $6272/3.3219 = 1889$. Since the labels are equally distributed we could simply calculate this by looking at the 10 classes. However a corresping calculation using the whole table is done in the code above.

Thoughts about Fully connected network:

If we were to design the last fully connected part of the model as 1 hidden layer having the same amounts of inputs and neurons, and 1 output layer (10 nodes), we would approximatly need $\sqrt{(199000)} = 446$ neurons. This would give a MEC of:

$$MEC = 446 * (446 + 1) = 199362bits$$

Let's say we need only $199000/4$ bits of memory, since many of the data points are very similar for the different digits in MNIST. Then we could have 1 hidden layer with $\sqrt{(199000/6)} = 223$, giving:

$$MEC = 223 * (223 + 1) = 49952bits$$

We could also add a second hidden layer of a chosen size. It will not add much capacity as the output of the 223 neurons are 223 bits. We could for example add a second layer of 50 neurons, giving 223 extra bits of capacity. Then we add the last layer of 1o neurons, adding another 10 bits. The main benefit of the 3 layers is the theoretical foundation in th book that shows a 3 layer network can represent any functions.

Designing the CNN:

- We will use 3x3 kernels as this seems to be the best choise according to articles. We will also use RELU activation functions.
- Let's first add 2 layers with output_channels = 2*input_channels, stride 1, and then max pooling with stride 2. This does a total compression with ratio 4
- Let's then do 1 layers with output_channel = 3*input_channels, stride 1 and max pooling with stride 2. This does a total compression of $\frac{4}{3}$
- Now our total compression is $\frac{16}{3} = 5.33...$. After some padding and other stuff we end up with 192 outputs.
  - This differs a little bit from what we found above, but this would give approximatly a MEC of $192 * (192 + 1) = 37056bits$, which could work.

Since we are just trying to observe how different models behave, I evaluate accuracies on the whole train and test set every epoch. The **UnOptimalCNN** was designed without having read or considered the material in the book.

In [ ]:
```
class UnOptimalCNN(torch.nn.Module):
    def __init__(self):
        super().__init__()
        self.model = torch.nn.Sequential(
            torch.nn.Conv2d(in_channels=1, out_channels=32, kernel_size=3, stride=1, padding=1),
            torch.nn.ReLU(),
            torch.nn.Conv2d(in_channels=32, out_channels=64, kernel_size=3, stride=1, padding=1),
            torch.nn.ReLU(),
            torch.nn.Conv2d(in_channels=64, out_channels=32, kernel_size=3, stride=2, padding=1),
            torch.nn.ReLU(),
            torch.nn.Conv2d(in_channels=32, out_channels=16, kernel_size=3, stride=2, padding=1),
            torch.nn.ReLU(),
            torch.nn.Conv2d(in_channels=16, out_channels=4, kernel_size=4, stride=3, padding=1),
            torch.nn.ReLU()
        )
        self.linear = torch.nn.Linear(16, 10)

    def forward(self, x):
        x = self.model(x)
        x = self.linear(x.flatten(start_dim=1))
        return x
```

```
    def get_param_count(self):
        return sum(p.numel() for p in self.parameters() if p.requires_grad)


class OptimalCNN(torch.nn.Module):

    def __init__(self):
        super().__init__()

        self.conv_sequence = torch.nn.Sequential(torch.nn.Conv2d(in_channels=1, out_channels=2, kernel_size=3, stride=1,
                    torch.nn.ReLU(),
                    torch.nn.MaxPool2d(kernel_size=3, stride=2),
                    torch.nn.Conv2d(in_channels=2, out_channels=4, kernel_size=3, stride=1, padding=2),
                    torch.nn.ReLU(),
                    torch.nn.MaxPool2d(kernel_size=3, stride=2),
                    torch.nn.Conv2d(in_channels=4, out_channels=12, kernel_size=3, stride=1, padding=2),
                    torch.nn.ReLU(),
                    torch.nn.MaxPool2d(kernel_size=3, stride=2))

        self.full_connected_sequence = torch.nn.Sequential(torch.nn.Linear(12*4*4, 50),
                                                           torch.nn.ReLU(),
                                                           torch.nn.Linear(50, 10))


    def forward(self, x: torch.Tensor) -> torch.Tensor:

        x = self.conv_sequence(x)
        x = self.full_connected_sequence(x.flatten(start_dim=1))

        return x

    def get_param_count(self):
        return sum(p.numel() for p in self.parameters() if p.requires_grad)


def eval_on_set(data_loader, model, normalizer):

    acc = []
    with torch.no_grad():

        for inputs, labels in data_loader:

            outputs = model(normalizer(inputs.float()))

            _, preds = torch.max(outputs, dim=-1)
            acc.append(torch.sum(preds == labels).item() / len(preds))

    return sum(acc) / len(acc)

def train_model(model, data_loader, test_data_loader, criteria, optimizer, normalizer, num_epochs=15):

    train_losses = []
    train_accs = []
    test_accs = []

    for _ in range(num_epochs):

        epoch_loss = []
        for inputs, labels in data_loader:

            # Forward pass
            outputs = model(normalizer(inputs.float()))

            # Calculate the loss, the input is our target
            loss = criteria(outputs, labels)
            epoch_loss.append(loss.item())

            # Backward pass
            optimizer.zero_grad()
            loss.backward()
            optimizer.step()

        model.eval()

        train_accs.append(eval_on_set(data_loader, model, normalizer))
        test_accs.append(eval_on_set(test_data_loader, model, normalizer))

        model.train()

        train_losses.append(sum(epoch_loss) / len(epoch_loss))

        print("Loss:", train_losses[-1], "Train Accuracy:", train_accs[-1], "Test Accuracy:", test_accs[-1])

    return train_losses, train_accs, test_accs

mnist_test_dataset = datasets.MNIST(root='./data', train=False, download=True, transform=transforms.PILToTensor())
mnist_test_data_loader = torch.utils.data.DataLoader(mnist_test_dataset, batch_size=50, shuffle=True)

mnist_data_loader = torch.utils.data.DataLoader(mnist_dataset, batch_size=50, shuffle=True)
```

```
d_mean = mnist_data_loader.dataset.data.float().mean(axis=(0,1,2))
d_std = mnist_data_loader.dataset.data.float().std(axis=(0,1,2))
normalizer = transforms.Normalize(mean=d_mean, std=d_std)


criteria = torch.nn.CrossEntropyLoss()
```

We will first test the CNN that I randomly designed, without reading the book. I have seen that adding a lot of channels is done sometimes, so I tried doing the same. Then adding a fully connected network at the end, that just fits the output dimensions.

In [ ]:
```
my_model = UnOptimalCNN()
optimizer = torch.optim.Adam(my_model.parameters(), lr=0.001)

print("Params:", my_model.get_param_count())
unopt_losses, unopt_train_accs, unopt_test_accs = train_model(my_model, mnist_data_loader, mnist_test_data_loader, criter
```

```
Params: 43102
Loss: 0.3646050670191956 Train Accuracy: 0.9607833333333472 Test Accuracy: 0.9603999999999994
Loss: 0.11629135861177929 Train Accuracy: 0.9683166666666783 Test Accuracy: 0.9665999999999987
Loss: 0.08642246448279668 Train Accuracy: 0.9805833333333427 Test Accuracy: 0.977099999999999
Loss: 0.07034708616614807 Train Accuracy: 0.9806500000000097 Test Accuracy: 0.977699999999999
Loss: 0.059910181468973554 Train Accuracy: 0.9838333333333423 Test Accuracy: 0.9811999999999987
Loss: 0.05291071052304081 Train Accuracy: 0.985483333333341 Test Accuracy: 0.9825999999999993
Loss: 0.04747021285467781 Train Accuracy: 0.9861333333333404 Test Accuracy: 0.9801999999999995
Loss: 0.04404787604097995 Train Accuracy: 0.9900500000000053 Test Accuracy: 0.9852999999999993
Loss: 0.0387213781481114 Train Accuracy: 0.9859000000000079 Test Accuracy: 0.9815999999999991
Loss: 0.03576856106762231 Train Accuracy: 0.9913000000000053 Test Accuracy: 0.9853999999999989
Loss: 0.031172180719737905 Train Accuracy: 0.9901333333333385 Test Accuracy: 0.9844999999999993
Loss: 0.028316093427104838 Train Accuracy: 0.9917166666666714 Test Accuracy: 0.9819999999999993
Loss: 0.026588587773500573 Train Accuracy: 0.9866500000000068 Test Accuracy: 0.9804999999999994
Loss: 0.02302128074490914 Train Accuracy: 0.9949500000000031 Test Accuracy: 0.9867999999999989
Loss: 0.023173135660302553 Train Accuracy: 0.9959000000000028 Test Accuracy: 0.9874999999999988
Loss: 0.020447573606415973 Train Accuracy: 0.9948166666666699 Test Accuracy: 0.9865999999999989
Loss: 0.018069672638042298 Train Accuracy: 0.9959833333333359 Test Accuracy: 0.986499999999999
Loss: 0.01575924227933683 Train Accuracy: 0.9951833333333362 Test Accuracy: 0.985999999999999
Loss: 0.017037724168577975 Train Accuracy: 0.9968833333333355 Test Accuracy: 0.9860999999999995
Loss: 0.014338572864735397 Train Accuracy: 0.9965000000000023 Test Accuracy: 0.9870999999999992
Loss: 0.014156534438284326 Train Accuracy: 0.9962833333333357 Test Accuracy: 0.9845999999999995
Loss: 0.013548931311434748 Train Accuracy: 0.997533333333335 Test Accuracy: 0.9864999999999989
Loss: 0.011871106880838719 Train Accuracy: 0.9972833333333351 Test Accuracy: 0.9855999999999989
Loss: 0.0130908602028173 Train Accuracy: 0.9970833333333352 Test Accuracy: 0.9865999999999989
Loss: 0.011054596326195376 Train Accuracy: 0.996800000000002 Test Accuracy: 0.9852999999999992
Loss: 0.010959177335344066 Train Accuracy: 0.9978166666666682 Test Accuracy: 0.9875999999999995
Loss: 0.011228150185094989 Train Accuracy: 0.9966333333333353 Test Accuracy: 0.9845999999999989
Loss: 0.010453214257311933 Train Accuracy: 0.9961000000000026 Test Accuracy: 0.9852999999999987
Loss: 0.011212065552599123 Train Accuracy: 0.9977000000000015 Test Accuracy: 0.9862999999999984
Loss: 0.009887067923391063 Train Accuracy: 0.9983166666666679 Test Accuracy: 0.9889999999999997
```

In [ ]:
```
my_model = OptimalCNN()
optimizer = torch.optim.Adam(my_model.parameters(), lr=0.001)

print("Params:", my_model.get_param_count())
opt_losses, opt_train_accs, opt_test_accs = train_model(my_model, mnist_data_loader, mnist_test_data_loader, criteria, op
```

```
Params: 10700
Loss: 0.5115602343001714 Train Accuracy: 0.932700000000009 Test Accuracy: 0.9361999999999996
Loss: 0.17103676894214004 Train Accuracy: 0.9561500000000123 Test Accuracy: 0.9580999999999991
Loss: 0.1363471255513529 Train Accuracy: 0.9657833333333453 Test Accuracy: 0.9642999999999989
Loss: 0.11614437231599974 Train Accuracy: 0.9659833333333463 Test Accuracy: 0.9661999999999992
Loss: 0.1052581303017602 Train Accuracy: 0.9628666666666786 Test Accuracy: 0.9641999999999991
Loss: 0.09843741205986589 Train Accuracy: 0.9728666666666781 Test Accuracy: 0.9698999999999994
Loss: 0.09003519797930494 Train Accuracy: 0.9708000000000114 Test Accuracy: 0.9696999999999987
Loss: 0.08462352342049902 Train Accuracy: 0.971000000000012 Test Accuracy: 0.9702999999999995
Loss: 0.08031322523602284 Train Accuracy: 0.9737166666666782 Test Accuracy: 0.9734999999999989
Loss: 0.07564839049368553 Train Accuracy: 0.9785833333333437 Test Accuracy: 0.9758999999999994
Loss: 0.07264277468299649 Train Accuracy: 0.9792833333333436 Test Accuracy: 0.9759999999999988
Loss: 0.06832725098967785 Train Accuracy: 0.9820000000000093 Test Accuracy: 0.9784999999999989
Loss: 0.064458697080845 Train Accuracy: 0.9769333333333439 Test Accuracy: 0.9727999999999993
Loss: 0.06260119709550054 Train Accuracy: 0.984883333333341 Test Accuracy: 0.9809999999999992
Loss: 0.06158828498359071 Train Accuracy: 0.984283333333342 Test Accuracy: 0.9793999999999989
Loss: 0.05895051023796744 Train Accuracy: 0.9815666666666762 Test Accuracy: 0.9789999999999991
Loss: 0.05701038215401544 Train Accuracy: 0.982550000000009 Test Accuracy: 0.9776999999999993
Loss: 0.05420369987080145 Train Accuracy: 0.9840166666666751 Test Accuracy: 0.9787999999999993
Loss: 0.05429654554774364 Train Accuracy: 0.9823833333333418 Test Accuracy: 0.9755999999999999
Loss: 0.05373473305361889 Train Accuracy: 0.9853333333333404 Test Accuracy: 0.978699999999999
Loss: 0.05026453875390871 Train Accuracy: 0.982733333333343 Test Accuracy: 0.9767999999999992
Loss: 0.04997128472479138 Train Accuracy: 0.9845333333333417 Test Accuracy: 0.9787999999999991
Loss: 0.04969971341177976 Train Accuracy: 0.981500000000009 Test Accuracy: 0.9754999999999987
Loss: 0.04586526393992244 Train Accuracy: 0.9866666666666735 Test Accuracy: 0.9789999999999985
Loss: 0.04776960421040712 Train Accuracy: 0.9848833333333412 Test Accuracy: 0.9774999999999996
Loss: 0.04484257025285236 Train Accuracy: 0.9876333333333404 Test Accuracy: 0.9795999999999988
Loss: 0.0442891184570423 Train Accuracy: 0.9805333333333424 Test Accuracy: 0.9718999999999989
Loss: 0.046108622735143094 Train Accuracy: 0.9872166666666734 Test Accuracy: 0.9806999999999987
Loss: 0.04272894681991602 Train Accuracy: 0.98863333333334 Test Accuracy: 0.9799999999999988
Loss: 0.042899339969881115 Train Accuracy: 0.9877000000000072 Test Accuracy: 0.9798999999999991
```

In [ ]:
```
import matplotlib.pyplot as plt
```

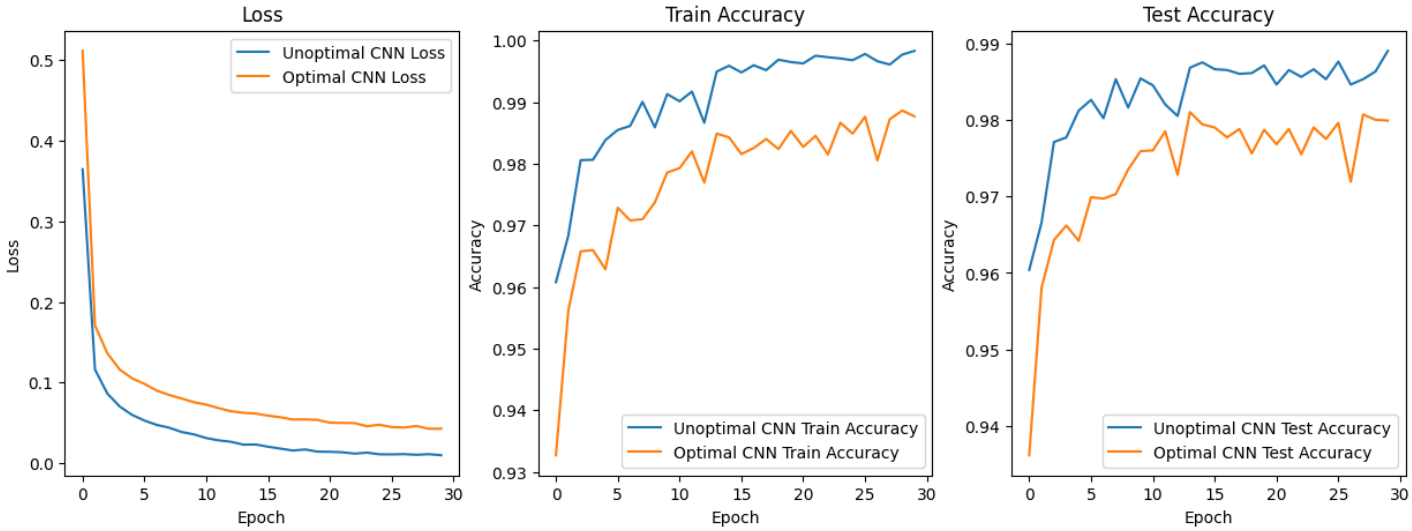
```
fig, ax = plt.subplots(1, 3, figsize=(15, 5))

ax[0].plot(unopt_losses, label='Unoptimal CNN Loss')
ax[0].plot(opt_losses, label='Optimal CNN Loss')
ax[0].legend()
ax[0].set_ylabel('Loss')
ax[0].set_xlabel('Epoch')
ax[0].set_title('Loss')

ax[1].plot(unopt_train_accs, label='Unoptimal CNN Train Accuracy')
ax[1].plot(opt_train_accs, label='Optimal CNN Train Accuracy')
ax[1].legend()
ax[1].set_ylabel('Accuracy')
ax[1].set_xlabel('Epoch')
ax[1].set_title('Train Accuracy')

ax[2].plot(unopt_test_accs, label='Unoptimal CNN Test Accuracy')
ax[2].plot(opt_test_accs, label='Optimal CNN Test Accuracy')
ax[2].legend()
ax[2].set_ylabel('Accuracy')
ax[2].set_xlabel('Epoch')
ax[2].set_title('Test Accuracy')

plt.show()
```



We notice that the **UnOptimalCNN** actually performs quite well. However, it does have a big downside. It uses much more time on training. Additionaly it also has 43102 learnaable parameters, compared to 10300 for the **OptminalCNN**. After all it actually seems like the **OptimalCNN** is the more "optimal", if one are considering run time and number of parameters.

However it is still interesting to see that the **UnOptimalCNN** actually performs quite well, even though it only has 16 neurons in the linear layer, compared to the 192 in the **OptimalCNN**. This might suggest that the CNN is able to compress the data quite well, which might again suggest that the mutual information in the training data is much lower than approximated. After all, the classes are quite uniformally distributed, while the data points belonging to the same class are probably far from uniform. This might lead to a big overestimation of the MEC needed to memorize the data. How big of a fully connected network needed almost becomes like a measure of the mutual information in the data.

I think a question to ask is if we need mostly compression for the MNIST dataset. By looking at CNNs as compression, we could imagine that if every class had very similar data points, we could look at the image as a very inefficient way of representing the digits, and that the CNN would be able to compress the data all the way down to it's minimum representations in 4 bits.