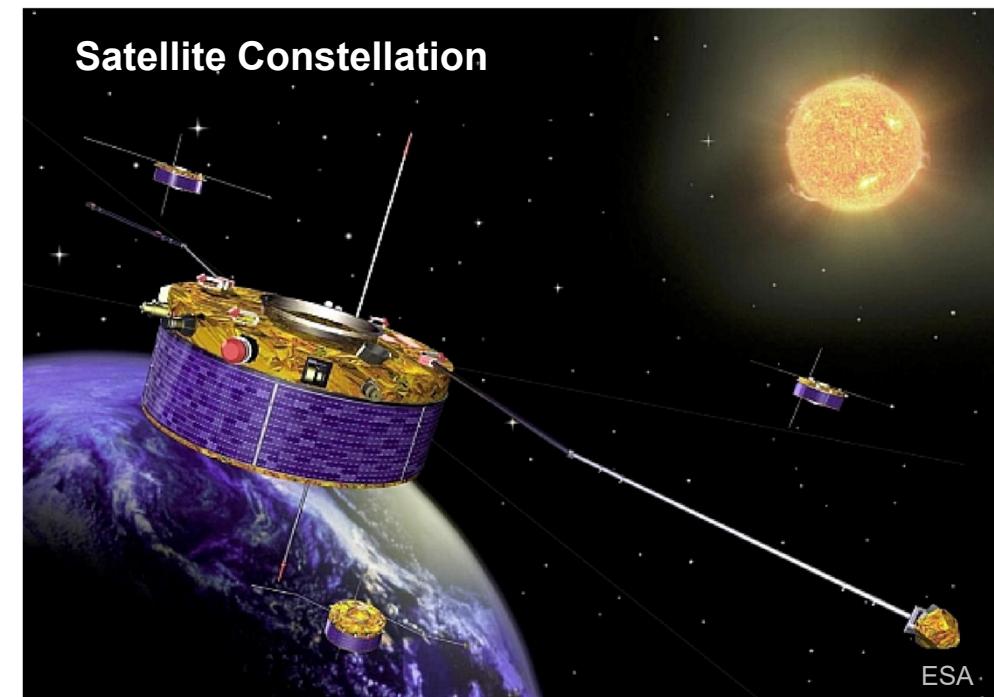
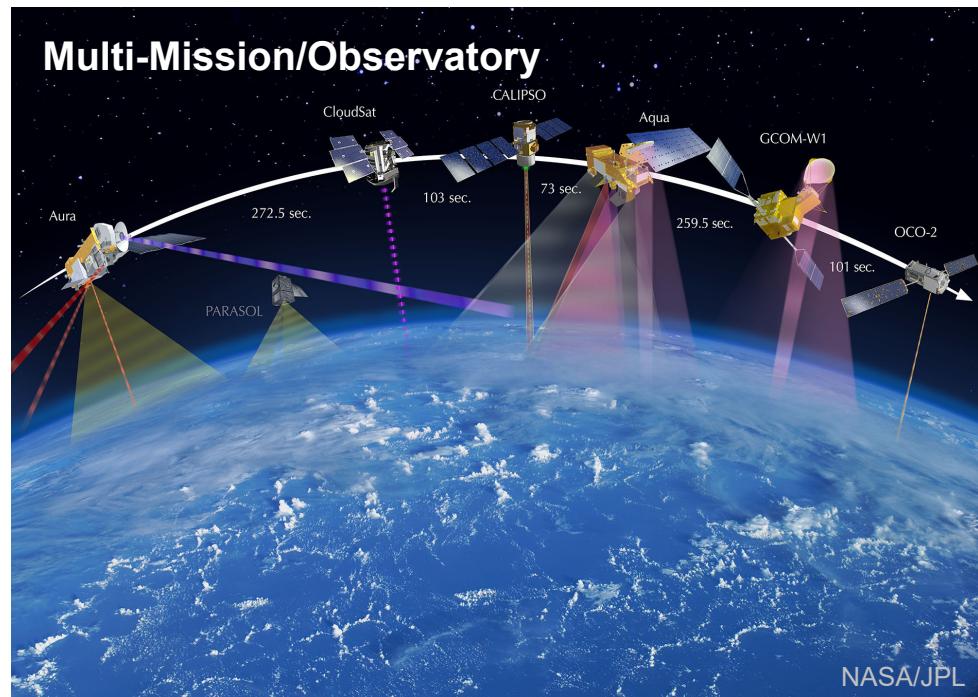


# Multi-Point Observations

Sarah K. Vines

Magnetosphere Online Seminar Series  
November 16, 2020

# Multi-point observations take many forms



With multi-point measurements and techniques, we can explore system-wide dynamics and local variations beyond what can be done with just one observatory.

# **Multi-Point Observations: The Multi-Observatory View**

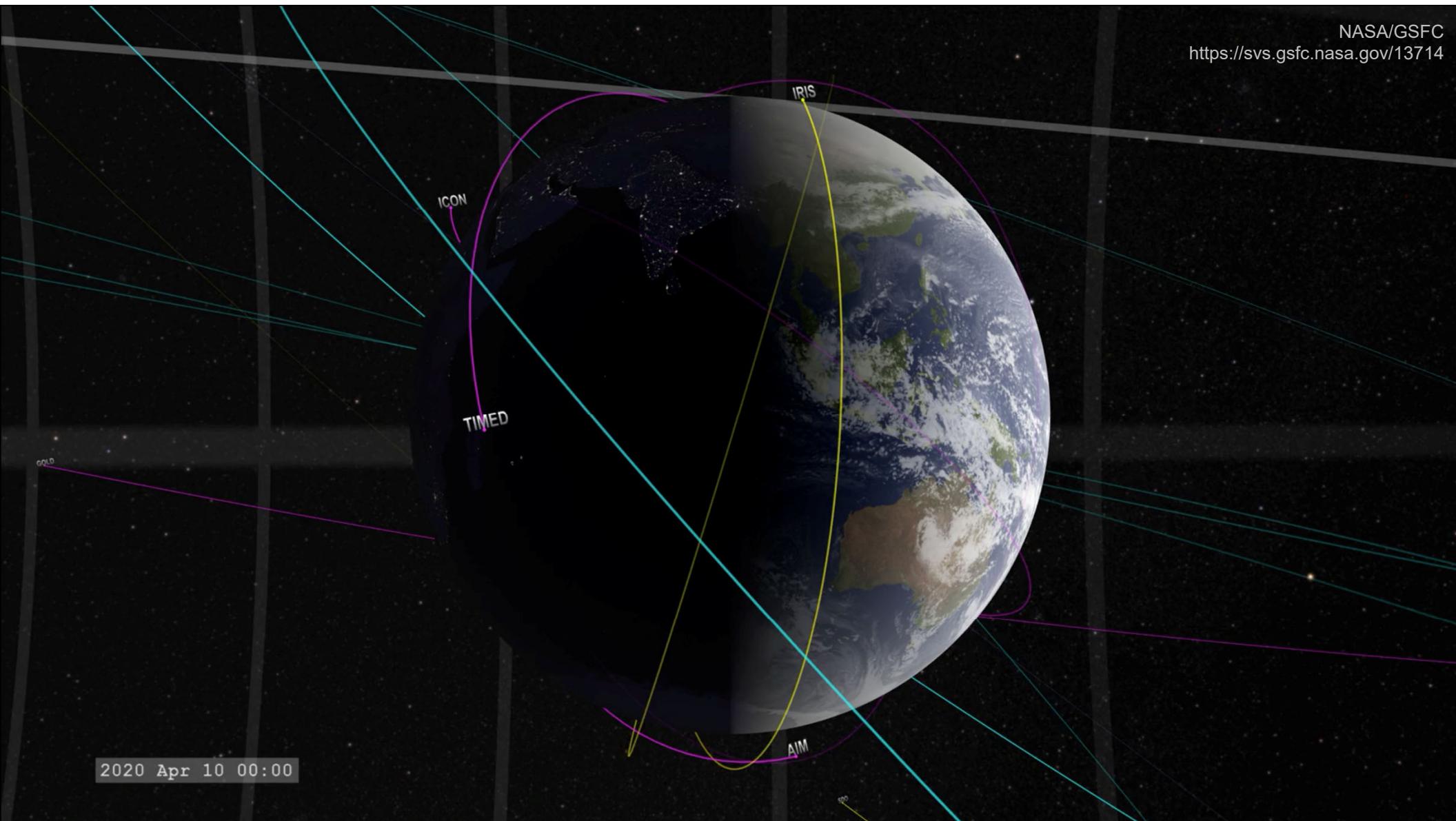
**Timing & Causality**

**Spatial Distribution**

**Measurement Synthesis**



NASA/GSFC  
<https://svs.gsfc.nasa.gov/13714>

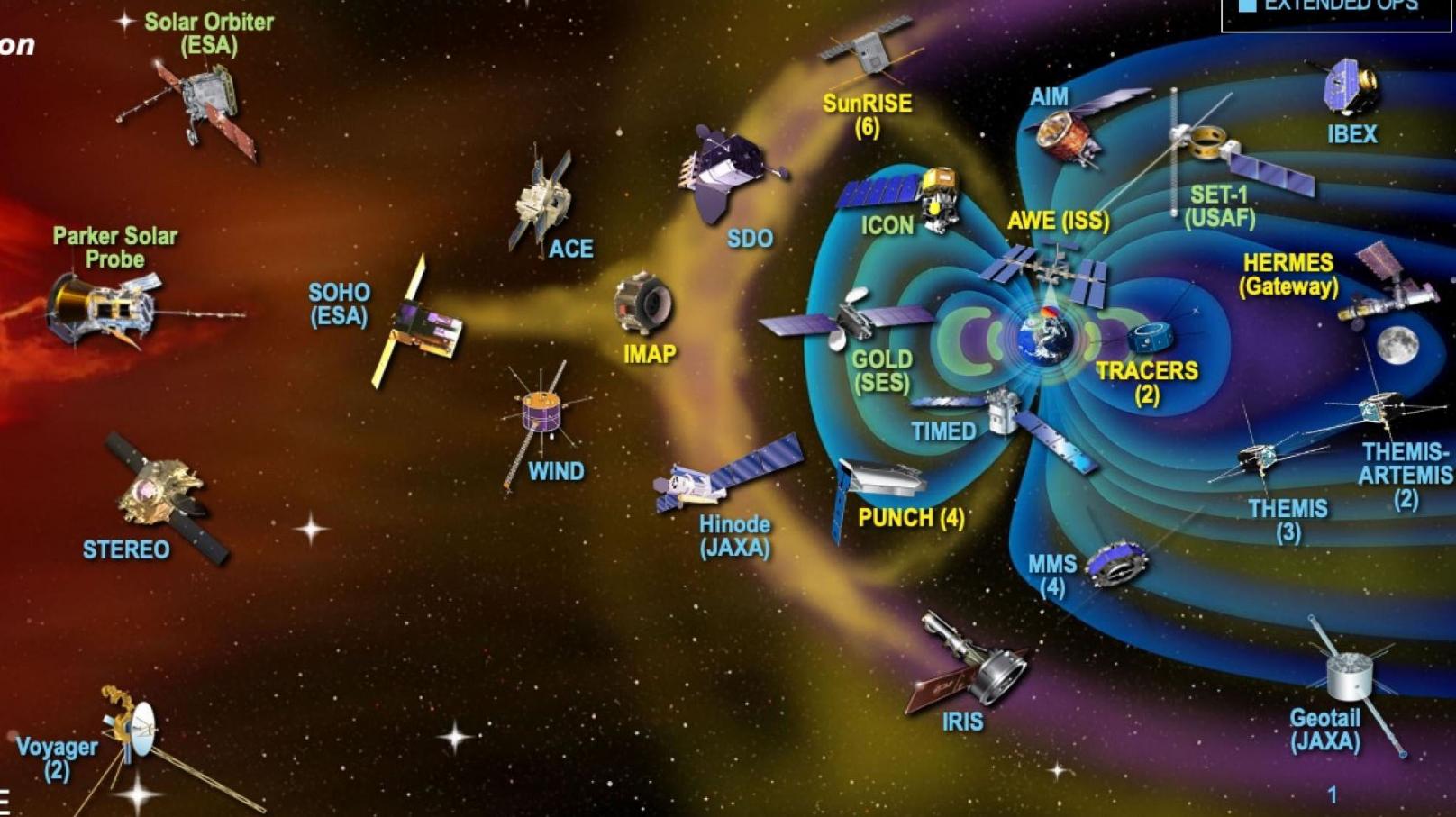


June 2020

# HELIOPHYSICS SYSTEM OBSERVATORY

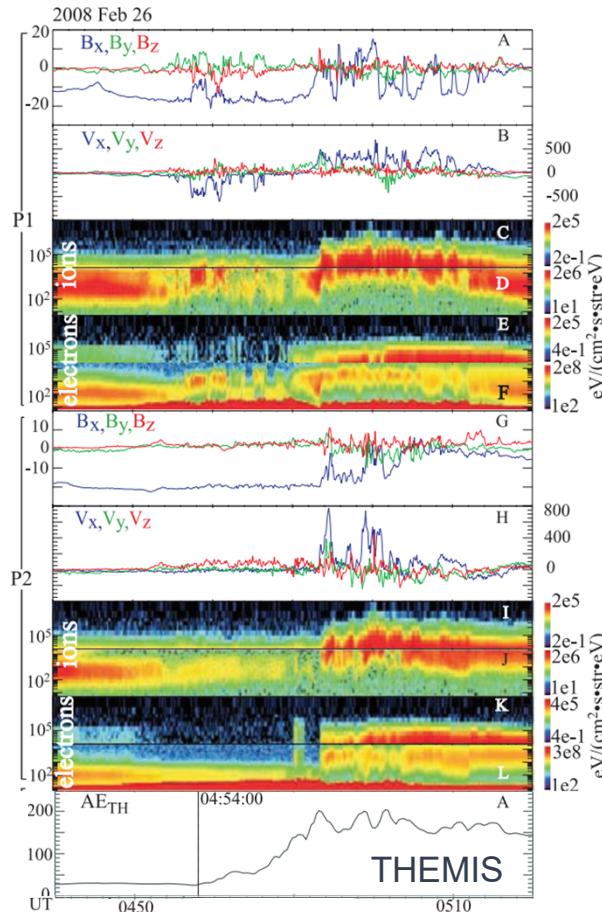
- **20 Operating Missions with 27 Spacecraft**
- **6 Missions in Formulation**

+ other assets in the inner magnetosphere and LEO, on the ground, in the solar wind, at nearby planetary bodies, and previous missions with data to dig into!

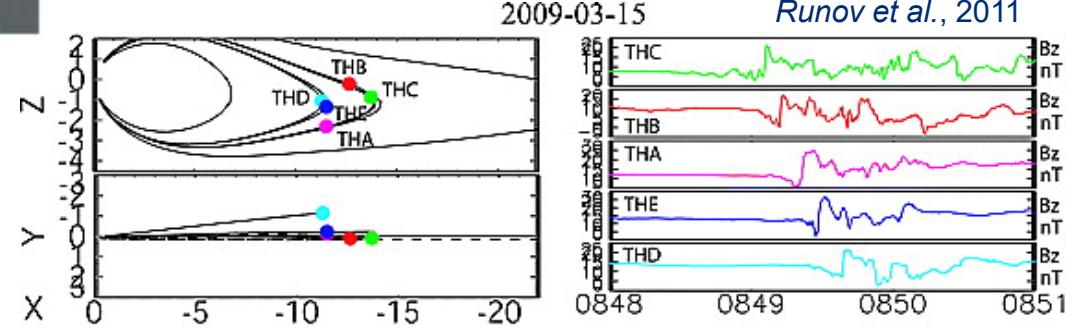
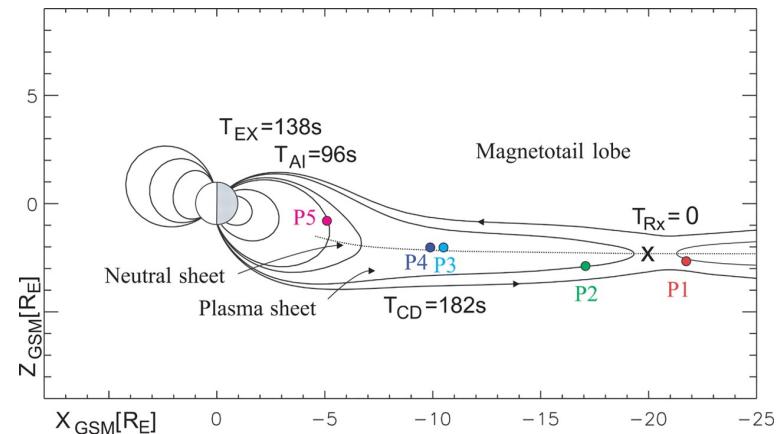
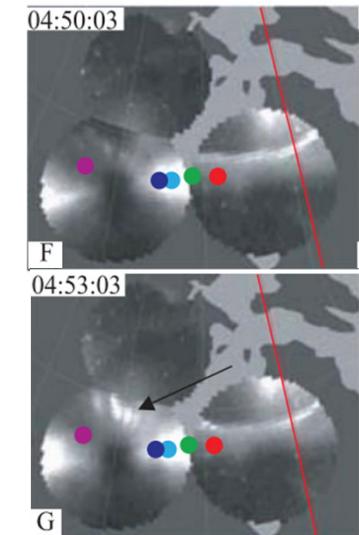


# Examples of the extended HSO at work

Timing, causality, and large-scale spatial distribution: Substorms



*Angelopoulos et al., 2008*



# Examples of the extended HSO at work

Timing, causality, and large-scale spatial distribution: Energetic Particle Injections

Tsyganenko-Sitnov [2005]  
GSM-Coordinates  
07 Apr. 2016 / 01:20 UT

Geotail

MMS  
RBSP-B  
RBSP-A

LANL-02A  
LANL-01A  
THEMIS-D  
THEMIS-E

1994-084  
1991-080

Y

X

N

Z

Turner et al., 2017

a) MMS, LANL-GEO, and Van Allen Probes Electrons

b) SOPA e<sup>-</sup> counts

c) SOPA e<sup>-</sup> counts

d) MagEIS e<sup>-</sup> flux

e) FEEPS e<sup>-</sup> flux

f) MagEIS e<sup>-</sup> flux

g) LANL-GEO, THEMIS, and Geotail Electrons

h) SOPA e<sup>-</sup> counts

i) SST e<sup>-</sup> flux

j) SST e<sup>-</sup> flux

k) SOPA e<sup>-</sup> counts

l) EPI e<sup>-</sup> Int. Flux

hhmm 2016 Apr 07

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Vines | Magnetosphere Online Seminar Series

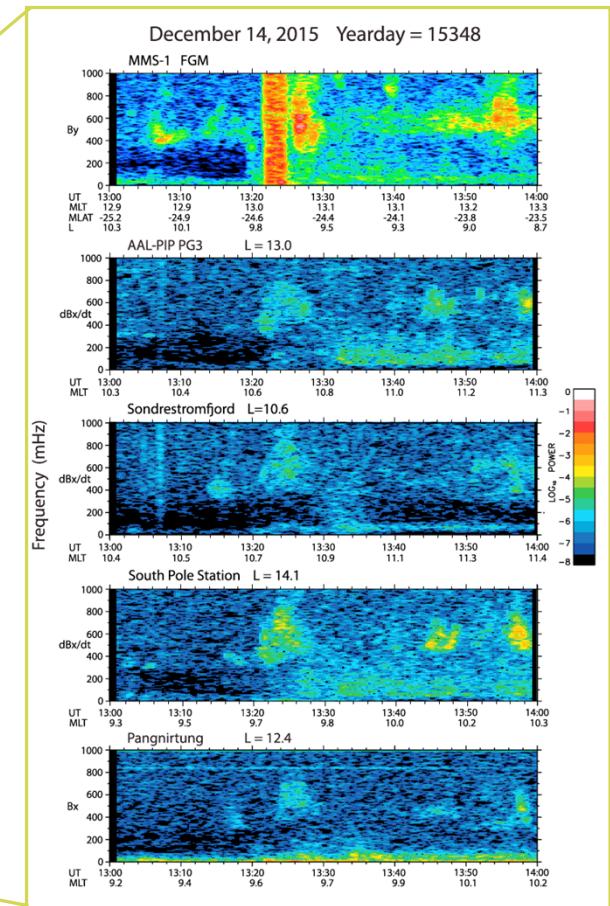
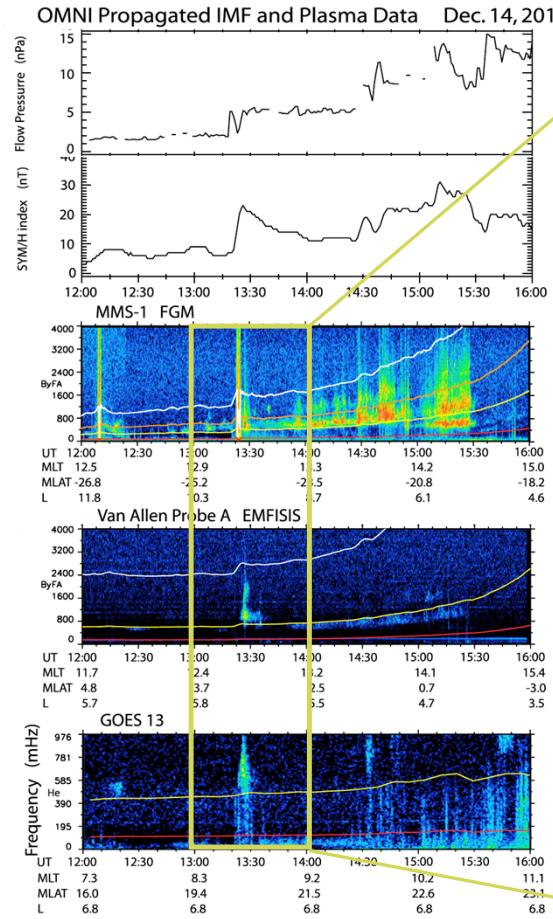
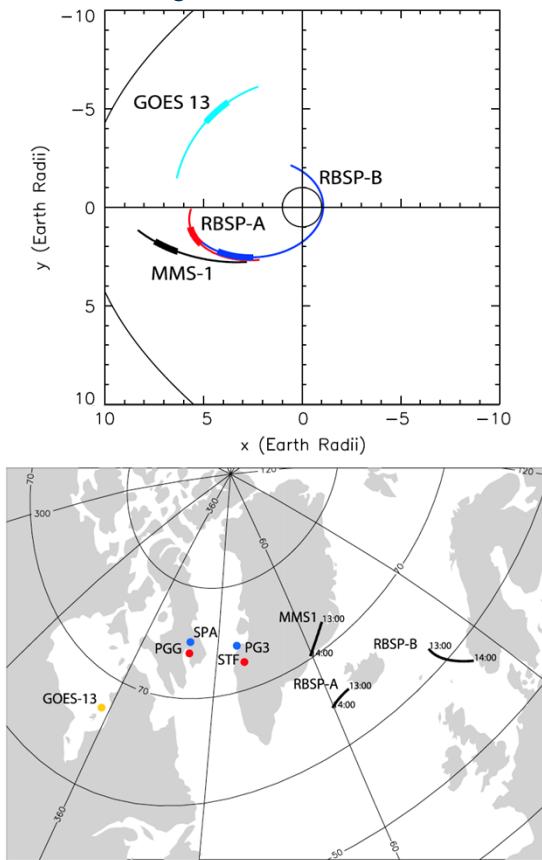
16 November 2020

7

# Examples of the extended HSO at work

Timing, causality, and large-scale spatial distribution: Wave generation

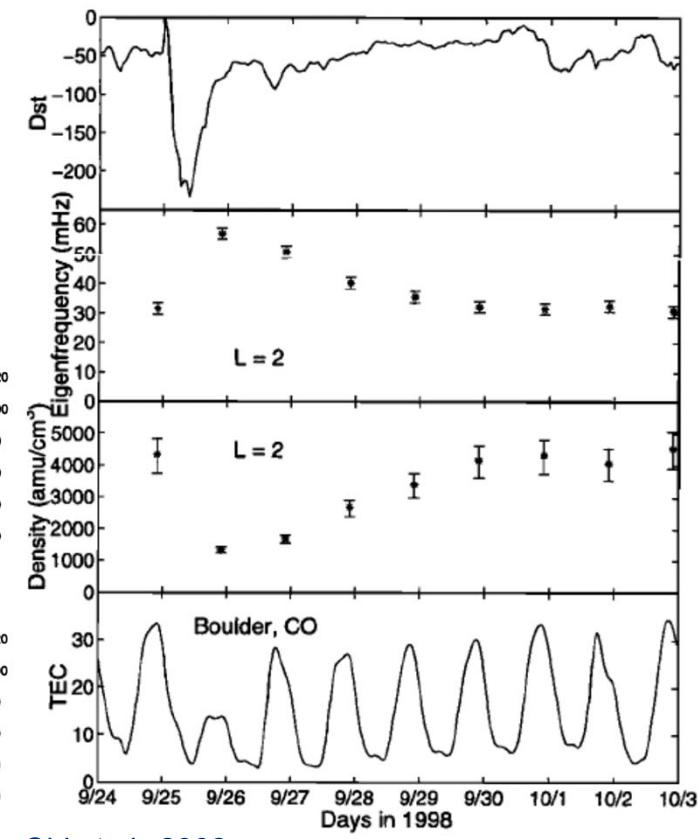
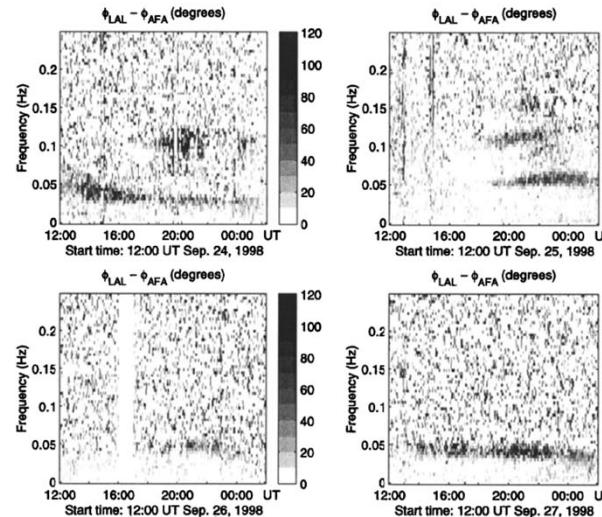
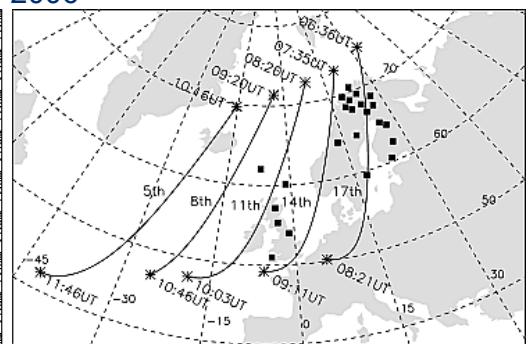
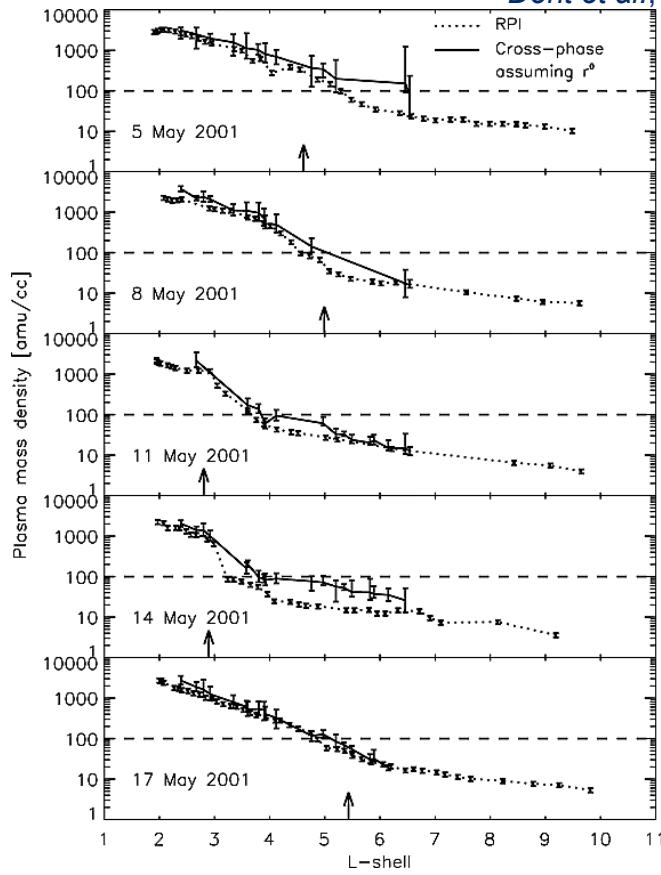
Engebretson et al., 2018



# Examples of the extended HSO at work

Timing, causality, and large-scale spatial distribution: Plasmaspheric mass density and refilling

Dent et al., 2006



Chi et al., 2002



# Things to keep in mind for multi-observatory studies

## *Caveats and uncertainties*

- Understanding timing delays, specifically when comparing upstream solar wind measurements to measurements within the magnetosphere or polar cap region
- Field line and footpoint mapping from spacecraft at large radial distances to the ionosphere and ground, especially for observations in the magnetotail
- Differences in instrument types and capabilities between missions, especially for quantitative comparisons of measurements between different satellite missions

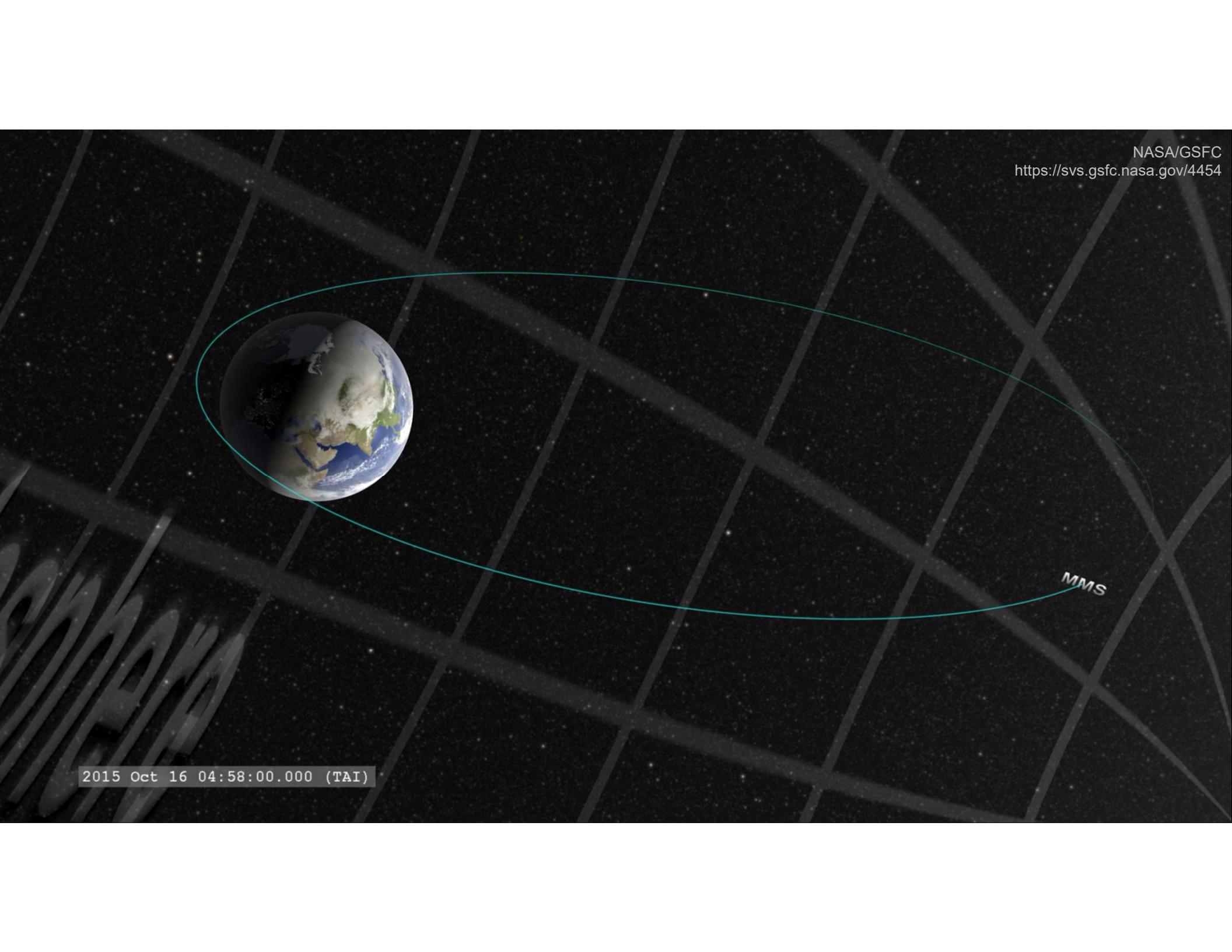
# Multi-Point Observations: The Constellation View

Timing and Phase Correlations

Spatial Gradients

Event Reconstruction





NASA/GSFC  
<https://svs.gsfc.nasa.gov/4454>

2015 Oct 16 04:58:00.000 (TAI)

# Timing Analysis from Constellations

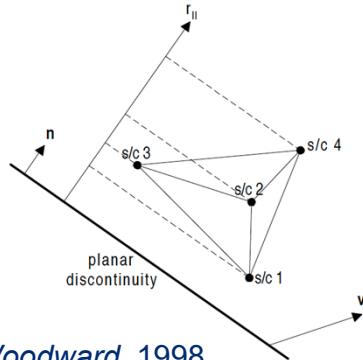
## Boundary/Discontinuity Analysis

- Discontinuity analysis uses the relative location and time difference between multiple spacecraft to determine the orientation and motion of a structure:

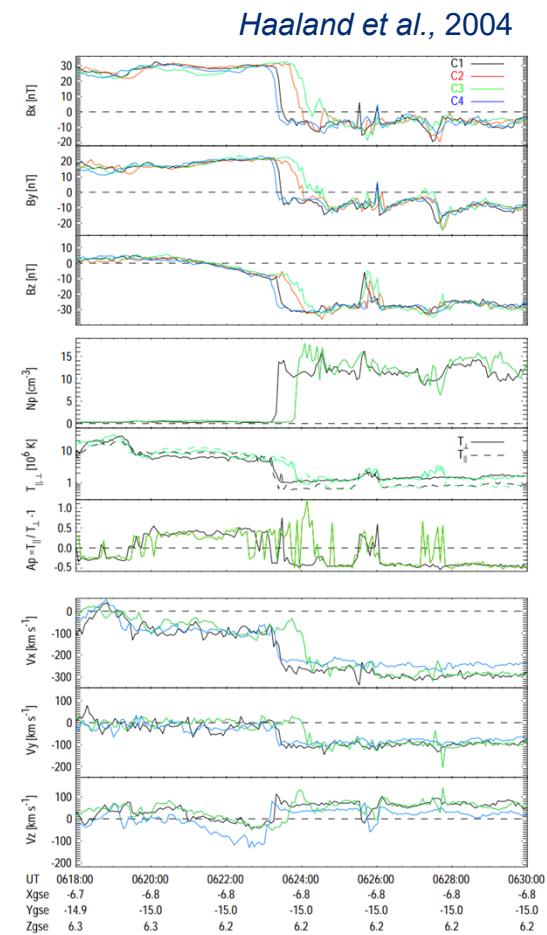
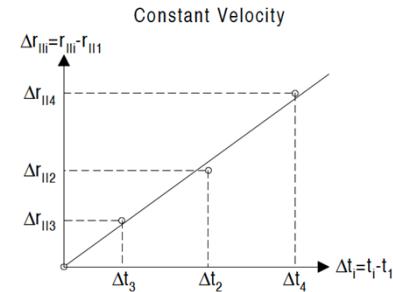
$$(V_{\text{sh}}^{\text{arb}} t_{\alpha\beta}) \cdot \hat{n} = \mathbf{r}_{\alpha\beta} \cdot \hat{n} \rightarrow \begin{pmatrix} \mathbf{r}_{12} \\ \mathbf{r}_{13} \\ \mathbf{r}_{14} \end{pmatrix} \cdot \frac{1}{V_{\text{sh}}^{\text{arb}}} \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \begin{pmatrix} t_{12} \\ t_{13} \\ t_{14} \end{pmatrix}$$

(Schwartz, 1998)

- Generally assumes a planar boundary or structure with a constant velocity, thickness, or acceleration
  - Has been used typically to determine properties of large-scale boundaries like the magnetopause (e.g., Haaland et al., 2004) and the bow shock (e.g., Russell et al., 1983; Schwartz, 1998)



Dunlop & Woodward, 1998



# Timing Analysis from Constellations

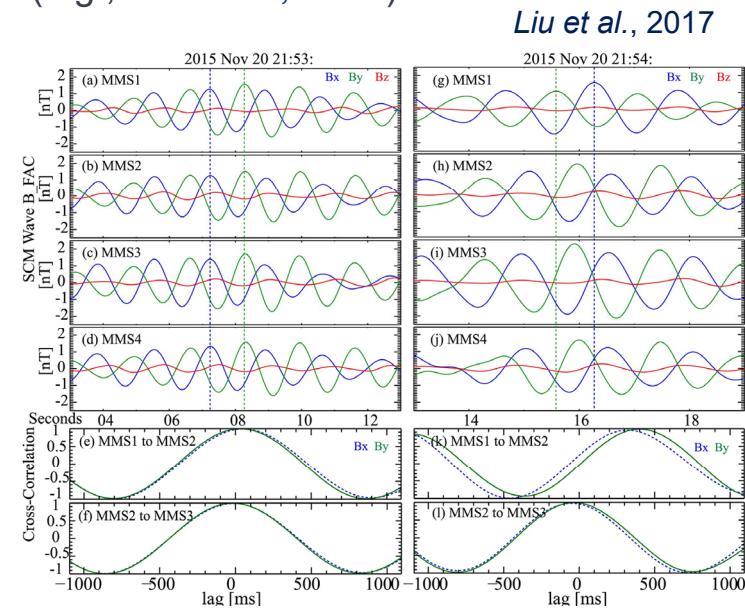
## Phase Differencing

- Using the known separation of individual spacecraft and the assumption that all spacecraft are observing the same waveform, the time difference in the phase of the waveforms between each location can determine the wave vector direction (e.g., *Balikhin et al.*, 1997; *Pinçon & Glassmeier*, 2008)
- Cross-correlation analysis of the waveforms are used to determine a time lag between observing points
- For four spacecraft, there are multiple triplet-pair combinations which can be used to determine an average wave vector ( $\mathbf{k}$ ) and standard deviations in the results (e.g., *Lee et al.*, 2019)

$$\mathbf{A}(\mathbf{r}, t) = \sum_{\omega} \mathbf{A}_{\omega} \exp i (\mathbf{k} \cdot \mathbf{r} - \omega t) + c.c.$$

$$\begin{aligned}\Delta\psi(\omega) &= \psi_{\alpha}(\omega) - \psi_{\beta}(\omega) \\ &= (\mathbf{k} \cdot \mathbf{r}_{\alpha} - \omega t) - (\mathbf{k} \cdot \mathbf{r}_{\beta} - \omega t) \\ &= |\mathbf{k}| |\mathbf{r}_{\alpha\beta}| \cos(\theta_{kr})\end{aligned}$$

(*Pinçon & Glassmeier*, 2008)



# Spatial Analysis from Constellations

## Gradients in the Magnetic Field

- Several methods for estimating spatial variations in the magnetic field, extricating current density, and determining properties of magnetic field structures like flux ropes/FTEs
- Applied point-wise in time to disentangle temporal versus spatial changes
- Some examples utilizing the magnetic field gradient, divergence, and curl:
  - Maximum Directional Derivative and Spatiotemporal Differencing methods (e.g., *Shi et al.*, 2005; 2006)
  - Magnetic rotation analysis and magnetic curvature (e.g., *Shen et al.*, 2003; 2007; *Zhao et al.*, 2016; *Akhavan-Tafti et al.*, 2019)
  - **Curlometer** (e.g., *Dunlop et al.* 1988; 2002; *Dunlop & Harvey*, 2008; *Robert et al.*, 1998)

# Gradients in the Magnetic Field

The Tetrahedron, Reciprocal Vectors, and the Curlometer

- For 3D linear gradients, a minimum number of 4 spacecraft can be used if the constellation has tetrahedral configuration
  - Three distinct surfaces, with a fourth surface of the tetrahedron available for redundancy
- Deriving gradients using a tetrahedral configuration can be described through the volumetric tensor and reciprocal vectors

(Chanteur, 1998; Harvey, 1998; Chanteur & Harvey, 1998; Vogt et al., 2008)

$$\mathbf{R} = \frac{1}{N} \sum_{\alpha=1}^N (\mathbf{r}_\alpha - \mathbf{r}_b) (\mathbf{r}_\alpha - \mathbf{r}_b)^T = \frac{1}{N} \sum_{\alpha=1}^N \mathbf{r}_\alpha \mathbf{r}_\alpha^T - \mathbf{r}_b \mathbf{r}_b^T ; \quad \mathbf{k}_\alpha = \frac{\mathbf{r}_{\beta\gamma} \times \mathbf{r}_{\beta\lambda}}{\mathbf{r}_{\beta\alpha} \cdot (\mathbf{r}_{\beta\gamma} \times \mathbf{r}_{\beta\lambda})}$$

$$\nabla g \simeq \nabla \tilde{g} = \sum_{\alpha=0}^3 \mathbf{k}_\alpha g_\alpha ; \quad \nabla \cdot \mathbf{V} \simeq \nabla \cdot \tilde{\mathbf{V}} = \sum_{\alpha=0}^3 \mathbf{k}_\alpha \cdot \mathbf{V}_\alpha ; \quad \nabla \times \mathbf{V} \simeq \nabla \times \tilde{\mathbf{V}} = \sum_{\alpha=0}^3 \mathbf{k}_\alpha \times \mathbf{V}_\alpha$$

- The assumption of linear gradients and the use of reciprocal vectors forms the basis of the curlometer technique: a four-point magnetic field measurement to evaluate the integral form of Ampere's law

$$k_4 = \frac{\mathbf{r}_{12} \times \mathbf{r}_{13}}{\mathbf{r}_{14} \cdot (\mathbf{r}_{12} \times \mathbf{r}_{13})} \quad \text{JUZ}$$

Chanteur et al., 1998

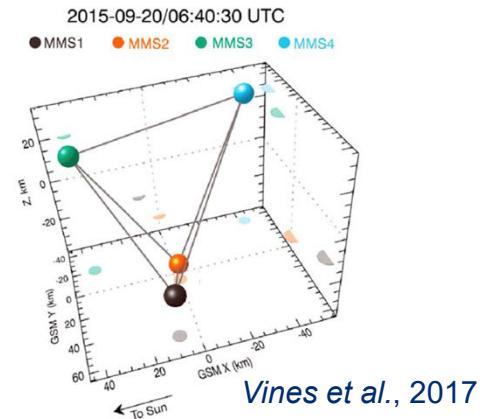
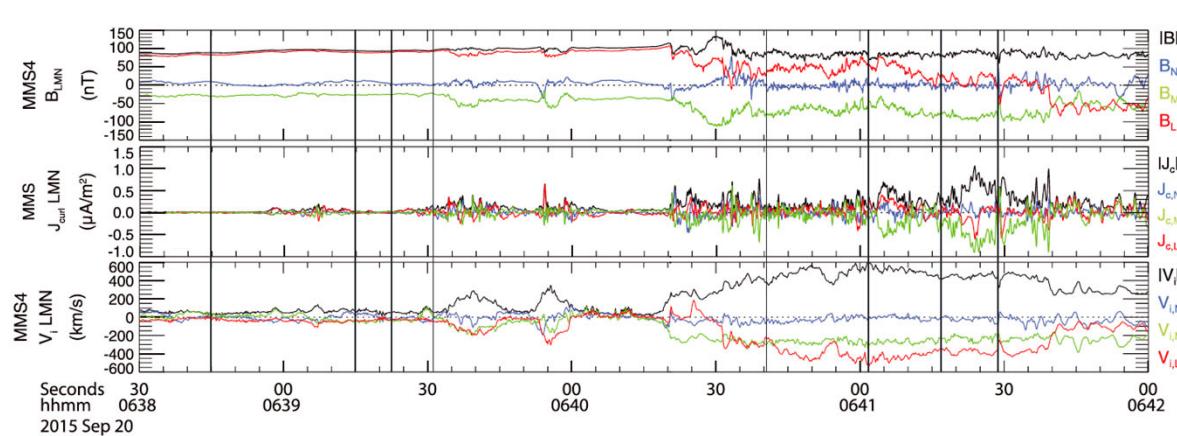
- This provides the average current density vector at the barycenter of the constellation



# Gradients in the Magnetic Field

## The Tetrahedron, Reciprocal Vectors, and the Curlometer

- The curlometer has been widely used to calculate the current density of various structures throughout the magnetosphere, in the magnetosheath, and in the solar wind using Cluster and MMS

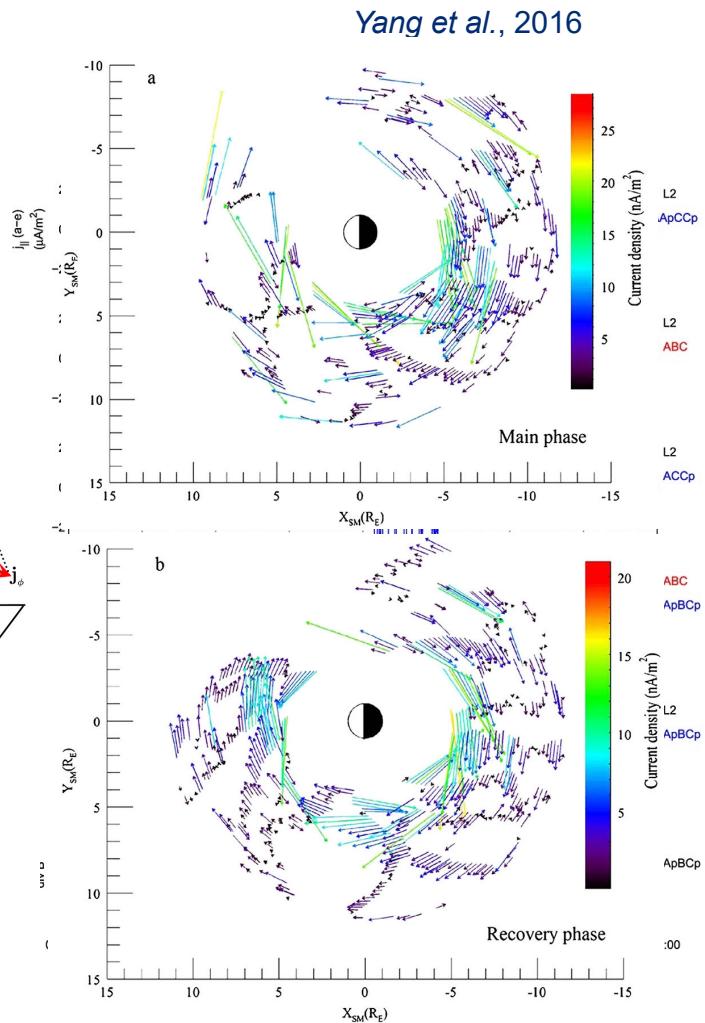
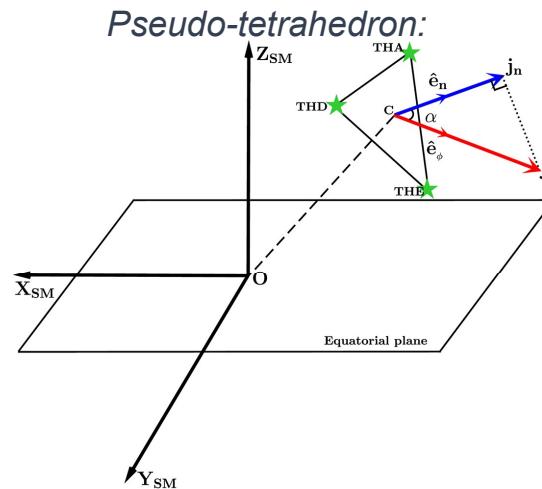


- The separations and configuration of the constellation limits the scale sizes and dimensionality of the current densities that can be obtained (i.e., where the linear gradient approximation is valid)
- A metric of “goodness” of curlometer results comes from  $|\nabla \cdot \mathbf{B}| / |\nabla \times \mathbf{B}|$ 
  - Since  $\nabla \cdot \mathbf{B}$  should ideally be 0, this ratio should be very small
- Another metric of goodness is the tetrahedral quality factor,  $Q$ , derived from the volume of the tetrahedron
  - For an ideal tetrahedron with MMS,  $Q = 1$  (i.e., the “ $Q_{RR}$  parameter” in Robert et al., 1998b)

# Gradients in the Magnetic Field

## Working with Other Configurations

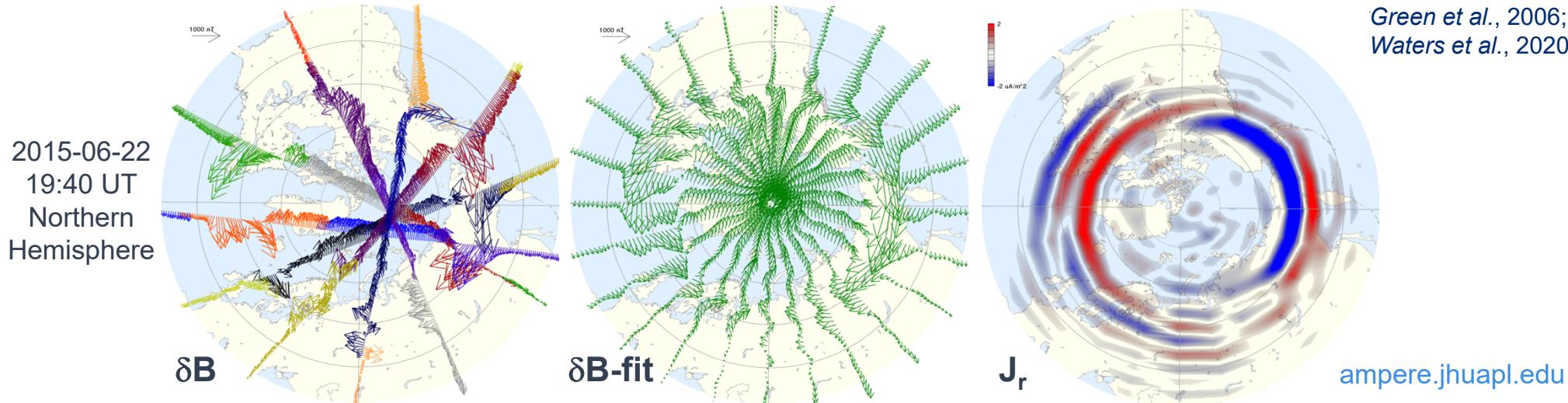
- Curlometer and gradient-based approaches have been used with other constellations having less than 4 satellites and/or in an irregular/non-tetrahedral configuration (e.g., *Dunlop et al.*, 2015; 2020; *Shen et al.*, 2012a,b; *Yang et al.*, 2016; *Artemyev et al.*, 2019)
- The less spacecraft, the more assumptions:
  - time stationarity of gradients or structure
  - direction and structure of the magnetic field and current density (i.e., force-free  $\mathbf{B}$  or magnetic field-aligned  $\mathbf{J}$ )
  - measure of only 1 or 2D current densities



# Gradients in the Magnetic Field

Making Use of Mega-constellations and Networks: The AMPERE dataset

- Collect all horizontal  $\delta\mathbf{B}$  observations in a 10-minute window from 66 Iridium satellites
  - Assumption of radial currents only, and so only a toroidal perturbation magnetic field
- Fit to a regular grid through spherical cap harmonic expansion
  - Linear interpolation to estimate the  $\delta\mathbf{B}$  values between tracks
- In solving the system to obtain FACs, the squared difference between the data and the estimated values from the SCHA basis functions are minimized using Singular Value Decomposition (SVD)



# Gradients in the Plasma

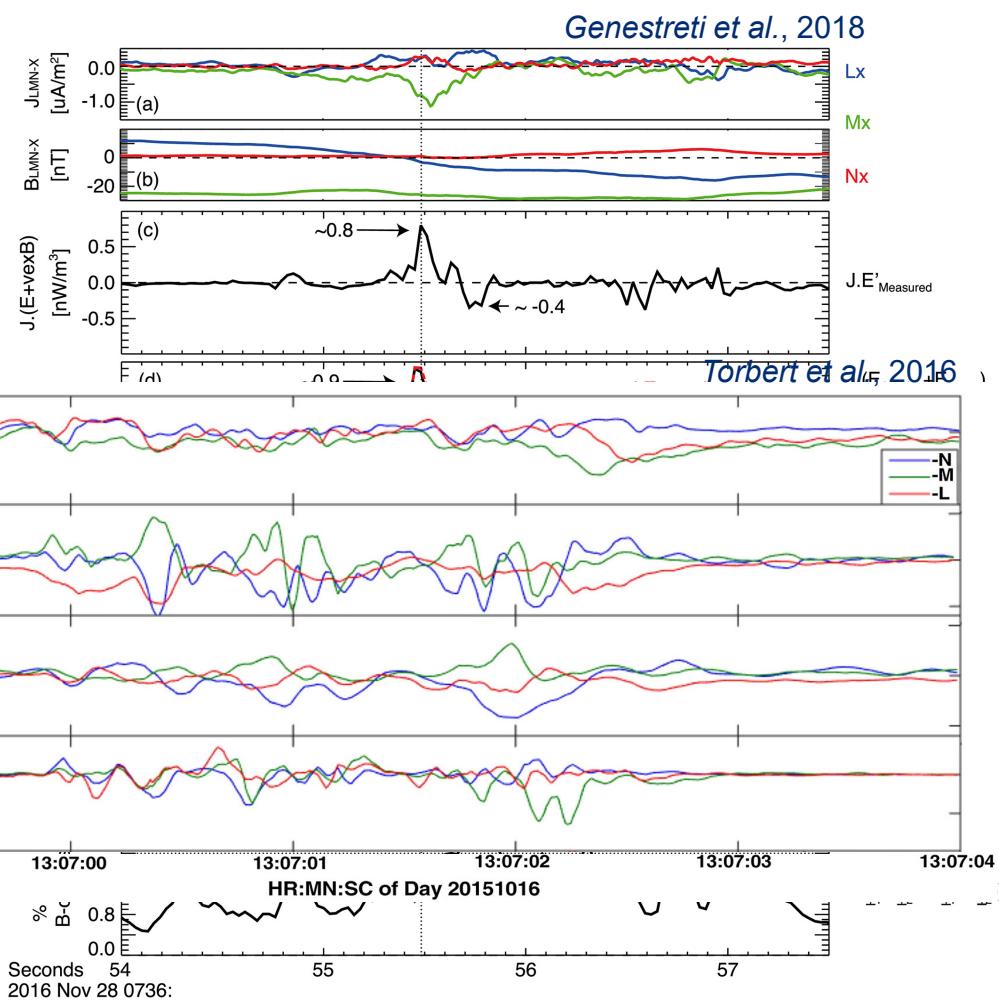
## Application to the Generalized Ohm's Law

- Extension of the classic Ohm's law relating current density, electric field, and conductivity:

$$\mathbf{E} + \mathbf{U}_e \times \mathbf{B} = \eta \mathbf{J} - \frac{1}{en} \nabla \cdot \mathbf{P}_e + \frac{m_e}{en} \left( \frac{\partial \mathbf{J}}{e\partial t} + \nabla \cdot n(\mathbf{U}_i \mathbf{U}_i - \mathbf{U}_e \mathbf{U}_e) \right)$$

Torbert et al., 2016; Hesse et al., 2014

- Using assumptions of linear gradients of the fields and plasma between the individual spacecraft, divergence of electron pressure and inertial terms are calculated
- For studies of magnetic reconnection, each term of the generalized form is evaluated to better understand what is contributing to energy conversion



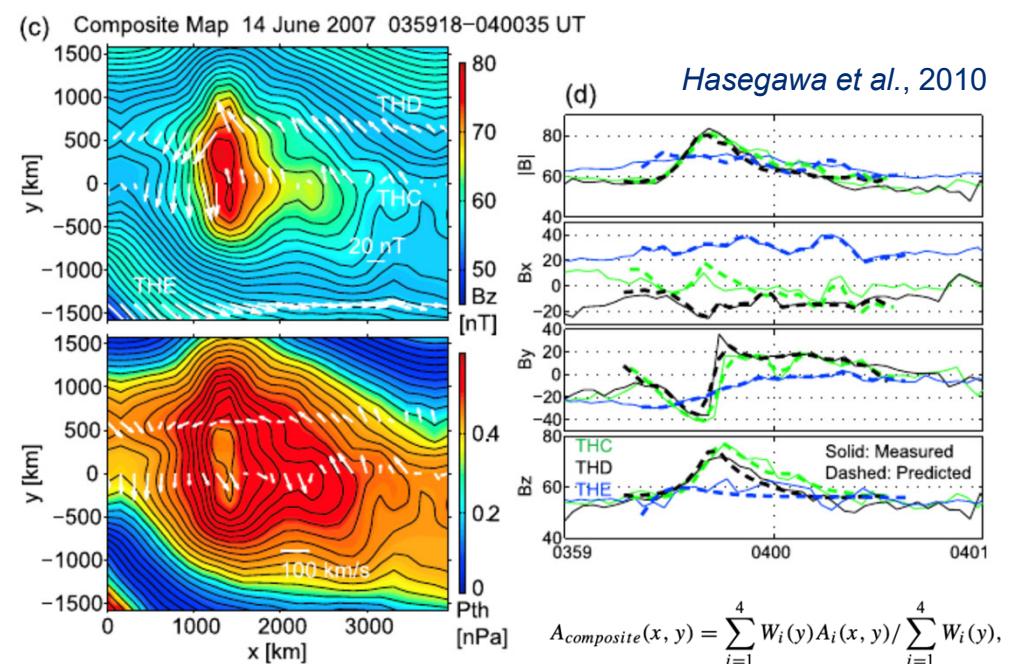
# Reconstructions from Constellation Data

## Grad-Shafranov Reconstruction

- Assuming structures are 2D and magnetohydrostatic (i.e., time independent and negligible inertial effects) in the frame of the moving structure (e.g., *Sonnerup et al.*, 2008)
- Solves the Grad-Shafranov equation describing force balance for this 2D structure:

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu_0 \frac{dP_t}{dA} = -\mu_0 j_z(A),$$

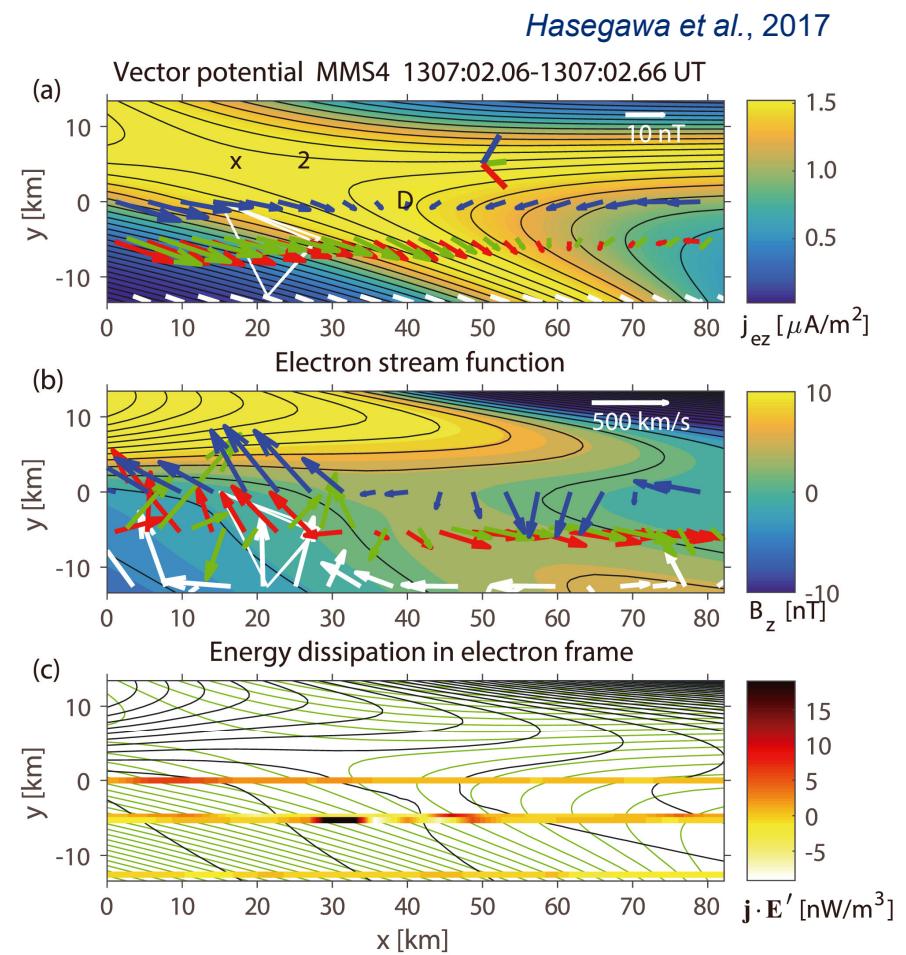
- Observations of the magnetic field and isotropic plasma pressure are used as initial values
- Initially developed for single-spacecraft measurements (e.g., *Sonnerup & Guo*, 1996; *Teh et al.*, 2011) and then extended to multi-point observations (e.g., *Hasegawa et al.*, 2005; 2007)
  - The multi-point method merges individual field and potential maps from each spacecraft
  - Individual maps used to constrain the invariant axis orientation



# Reconstructions from Constellation Data

## MHD-based Reconstruction

- Uses multi-point observations as initial inputs for MHD descriptions of steady, 2D magnetic and velocity fields
  - Solving the various forms of Ohm's law under different assumptions of plasma behavior (i.e., satisfying the frozen-in condition or allowing for different effects when that condition is violated)
- Depending on the structure or region of interest, and on the available observations, multiple forms of this type of reconstruction have been developed:
  - Ideal MHD (*Sonnerup et al.*, 2008; *Teh et al.*, 2009)
  - Hall and resistive MHD (*Teh et al.*, 2010; *Sonnerup & Teh*, 2009)
  - Electron MHD (EMHD) (*Sonnerup et al.*, 2016; *Hasegawa et al.*, 2017; 2018)



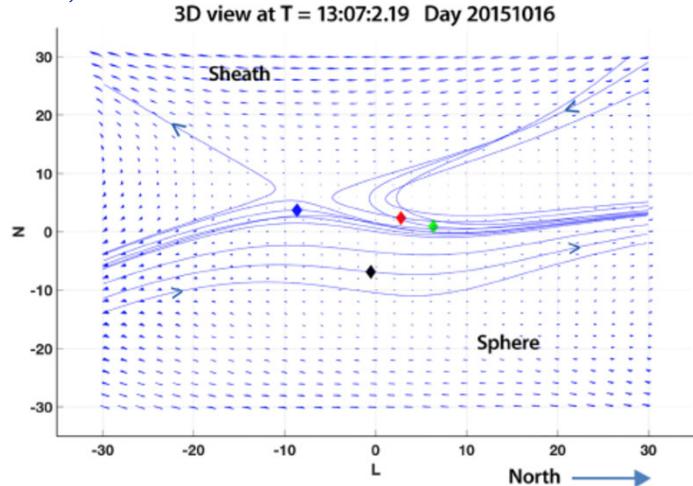
# Reconstructions from Constellation Data

## Polynomial Expansion Reconstruction

- Quadratic expansion of the magnetic field and current density measured at the 4 MMS spacecraft, with the constraint  $\nabla \cdot \mathbf{B} = 0$ 
  - Because of the ability to measure plasma moments to high fidelity,  $\mathbf{J}$  from each spacecraft can be used

$$\mathbf{B}_j = \mathbf{B}_{0j} + \sum_k x_k (\partial_k \mathbf{B}_j)_0 + \frac{1}{2} \sum_{k,l} x_k x_l (\partial_k \partial_l \mathbf{B}_j)_0$$

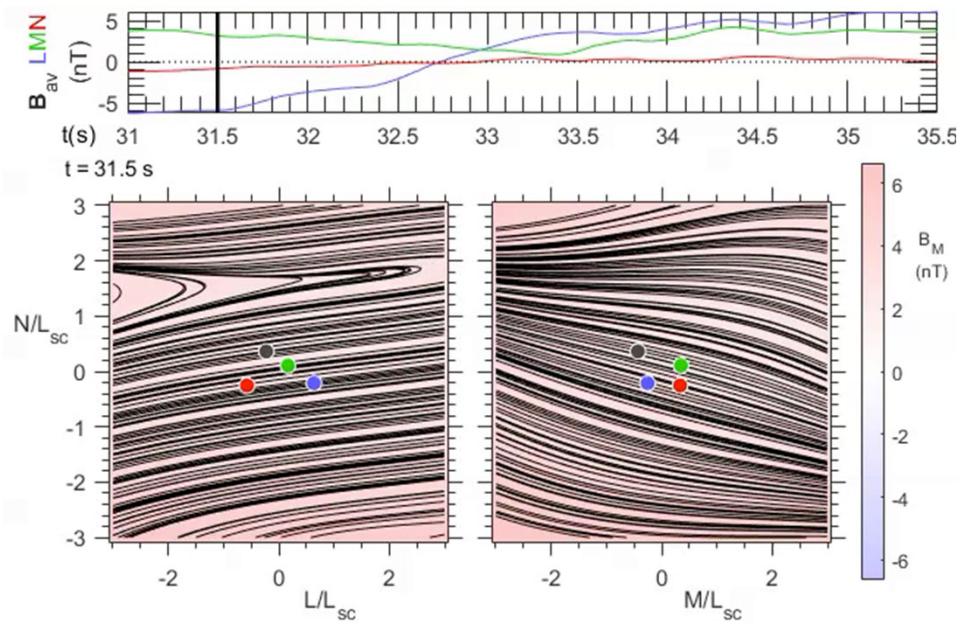
Torbert et al., 2020



- Expansions of varying complexity, from linear to fully 3D quadratic formulation with additions of cubic terms, and the use of least squares fits to the measurements are explored in Denton et al. 2020

## 3D Reduced Quadratic Model

Denton et al., 2020



# Multi-Point Observations for our Present and Future

## *Multi-mission/observatory considerations*

- Multi-mission and multi-observatory are incredibly useful for understanding cross-scale dynamics and coupling, and the more datasets we have, the more kinds of conjunctions we can consider in trying to understand larger-scale processes and responses of the magnetosphere
- As missions are extended and future missions are implemented, continue to increase collaboration between active missions and observatories, such as conducting campaign events and planning of opportune conjunctions
- Consider how to better incorporate remote sensing techniques in magnetospheric research, whether from satellites or ground-based observations, particularly for system-wide contextual observations

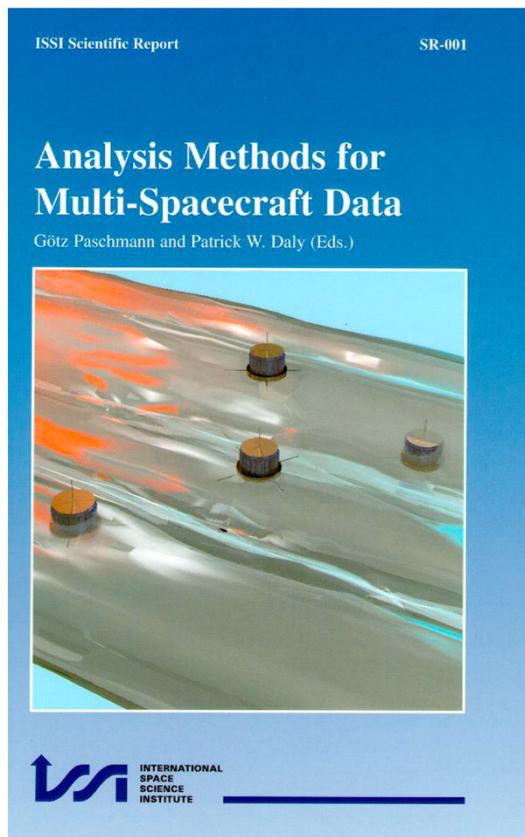
## *Satellite constellation considerations*

- Coupled with the increasing number of satellite missions and ground-based observatories, ever-larger constellations are on their way, especially with the advances in CubeSat/SmallSat capabilities and launch vehicle infrastructure
- As we think about constellations, recognize the value of copious good enough measurements versus just a few exquisite measurements
- Continue to push forward in how we process, analyze, and visualize all of the data streams from individual satellites/observatories, including reconstruction techniques and data mining and assimilation

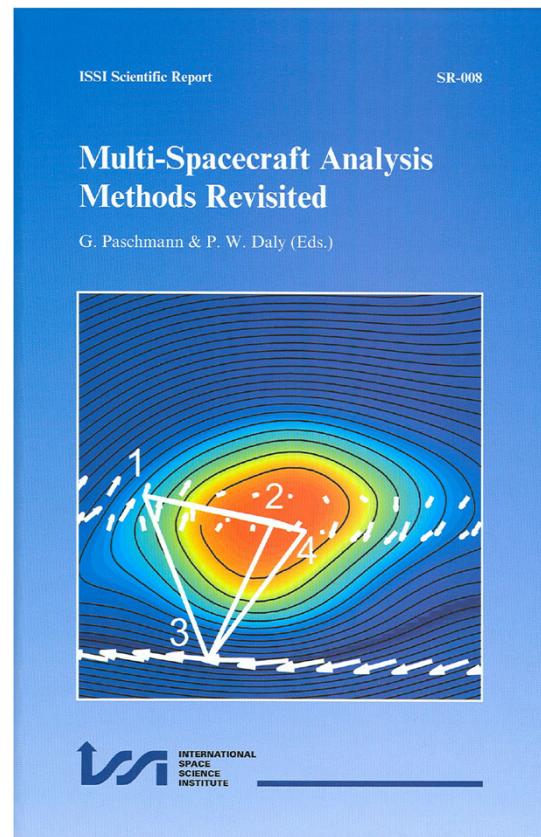
With multi-point measurements and techniques, we can explore system-wide dynamics and local variations beyond what can be done with just one observatory.



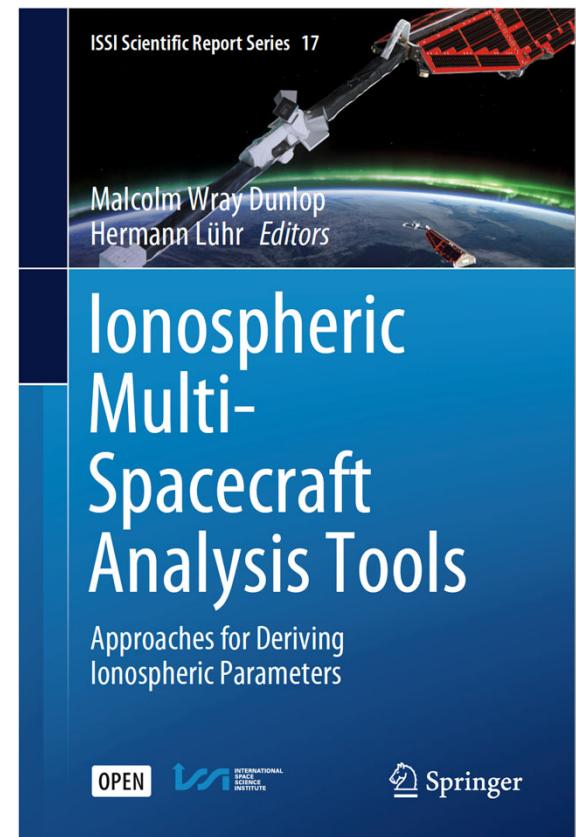
# Using multi-point techniques: Where to start?



1998, 2000



2008



2020

