

CS5800: Algorithms Spring 2018

Assignment 1

Out: 12 January 2018

Due: 18 January 2018, 8:59pm

Instructions:

- The assignment is due at the time and date specified. Late assignments will not be accepted.
- You must work on all the problems and write the solutions by yourself! Finding solutions to homework problems on the web, or by asking students in and outside the course is strictly prohibited. This would be defeat the purpose of learning by doing the assignment.
- You must submit typed solutions. You may use plain text or a word processor like Microsoft Word or LaTeX for your submissions. You may hand-sketch and scan any diagrams that you support your answer.
- If you are not comfortable with Word or Latex, please solve these problems by hand before devoting time to typing them. Do not waste precious time investigating typesetting up front!

1. (20 points) Ordering functions

Arrange the following functions in order from the slowest growing function to the fastest growing function. Briefly justify your answers.

$$\sqrt{n}, n\sqrt{\lg n}, 2^{\sqrt{\lg n}}, (\lg n)^2$$

2. (10 points) Properties of asymptotic notation

Let $f(n)$, $g(n)$, and $h(n)$ be asymptotically positive and monotonically increasing functions. For each of the following statements, decide whether you think it is true or false and give a proof or a counterexample.

- (a) If $f(n) = \Omega(h(n))$ and $g(n) = O(h(n))$, then $f(n) = \Omega(g(n))$.
- (c) If $f(n) = O(g(n))$, then $3^{f(n)}$ is $O(3^{g(n)})$.

3. (20 points)

The Body Mass Index (BMI) of a child at 5 years of age is a good indicator of future chances of childhood obesity. Arlington Pediatrics maintains a list of BMI for all their 5-year old patients for an entire year. Given a 5-year old's BMI *today*, it computes the child's BMI percentile as the ratio of the number of 5-year old patients *last year* whose BMI is lesser than it, and the total number of patients in last year's record. At the end of the year, it updates its BMI list to prepare for the next year.

Polly's intuition is that the binary search algorithm is a good inspiration for this problem.

- (a) Explain briefly in words how you will solve this problem.
- (b) Provide the pseudo-code for an algorithm to compute a child's percentile BMI. Be sure to state precise the inputs and outputs to the algorithm, and any assumptions about them.
- (c) Prove the correctness of your algorithm.

You will be graded on (a) the precision of how you have stated your algorithm (b) the validity and precision of your proof of correctness.